Dynamic Adjustment to Trade Shocks

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Abstract

Global trade flows and supply chains adjust gradually. Empirical estimates of the trade elasticity for the short run are about half as large as those for the long run and suggest that trade is subject to substantive adjustment frictions. We develop a tractable framework that provides microfoundations for dynamic trade adjustment and rationalizes reduced-form estimation of a time-varying trade elasticity. The model features sticky sourcing, forward-looking firms, and nests the Eaton-Kortum model as the limiting long-run case. We calibratie the model to observed time-varying elasticities and quantify the welfare impacts of two events: the 2018 US-China trade war (an arguably unanticipated change) and the 2004 EU enlargement (an anticipated change). Our findings suggest that sourcing frictions and anticipation effects alter the time pattern of welfare changes, can result in short-term welfare losses but long-term gains, and can drive marked trade share adjustments before anticipated shocks occur.

Keywords: International trade; estimation of the elasticity of trade; dynamic trade adjustment; staggered sourcing decision; US-China trade war; 2004 EU Enlargement.

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1 Introduction

Innovations and disruptions to global supply chains lead to gradual adjustments in international trade flows. It has long been recognized that the trade elasticity, a key parameter that captures the substitution between imported goods from different countries in response to trade costs, varies by time horizon (e.g. Dekle, Eaton and Kortum 2008). Boehm, Levchenko and Pandalai-Nayar (2023, henceforth BLP) use plausibly exogenous tariff changes to measure the trade elasticity by time horizon and find that the short-run trade elasticity is about half the size of the long-run elasticity. This differential implies substantial frictions in trade adjustment that a static trade model cannot account for. A dynamic framework is needed to provide a rigorous and plausible quantification of the transitory and lasting impacts of shocks to global supply chains.

This paper proposes a dynamic general equilibrium model of trade with many countries and many industries, where staggered and forward-looking procurement decisions give rise to horizon-specific trade elasticities. Under the Ricardian trade tenet, products are sourced from the least expensive global supplier. However, the opportunity to switch to a new supplier arrives randomly following a Poisson process. As a consequence, only some buyers respond to a trade disruption by adjusting to optimal sourcing relations. Other buyers endure a suboptimal procurement choice until they can adjust. In this framework, disruptions put the world economy through a sustained period of adjustment.

The model preserves the analytical tractability of a class of quantitative Ricardian models based on Eaton and Kortum (2002, henceforth EK). We characterize impulse responses in the model using the dynamic hat algebra method. We establish closed-form expressions for the horizon-specific trade elasticity, showing that our model rationalizes empirical estimates of the trade elasticity at different time horizons as a convex combination of short- and long-run elasticity parameters, linked by transitory weights that shift at a constant rate of decay. Moreover, we derive a novel characterization of the horizon-specific gains from trade that sheds light on the importance of sourcing frictions. Our model shows how the original static welfare formula based on Arkolakis, Costinot and Rodríguez-Clare (2012) can be augmented to account for dynamic adjustment, so it delivers welfare predictions under time-varying and forward-looking trade elasticity.

Specifically, we assume that intermediate goods are produced using constant returns-to-scale technologies and producers differ by productivity drawn from a country-industry specific Fréchet distribution. Trade is subject to iceberg trade costs. To transact with producers, the local assembler of the final good of an industry at a destination d requires specialized local traders who have exclusive access to the competitive global Walrasian market for a particular intermediate good. Under profit maximization, each trader seeks to procure its good from the least expensive global supplier but may not be able to instantly switch from one supplier to another. This inability to switch suppliers, which renders the sourcing decisions of traders dynamic, is governed by a binary random process: an assembler is either in a position to choose the least expensive global supplier of an intermediate good from any source-industry, or the assembler has to continue purchasing from the same producer as in the preceding period. We can therefore characterize equilibrium as a set of measurable partitions of the space of intermediate goods for each supplier, and then derive the equilibrium distributions. An intermediate good's price at a moment in time equals the initial destination price adjusted for the cumulative changes in marginal costs since the supplier was last selected. A destination country's expenditure shares by source country across intermediate goods depend on trade costs in the past, present and future; characterized by tractable analytical expressions similar to EK and other Ricardian frameworks that are consistent with the gravity equation of trade.

For legacy varieties that are imported from the same supplier as in the preceding period, the expenditure shares in the augmented gravity equation encode the price that a buyer paid at the time of the last supplier change. Through this unmoved component, the short-term elasticity of trade governs the effects of substitution between prices on the intensive margin while buyer-supplier relationships last, similar to an Armington (1969) model. When supplier-buyer relationships are reset optimally, the gravity expression for trade flows incorporates two components. The first of these components captures familiar forces from the EK framework, namely the extensive margin adjustment of trade flows to contemporaneous global factor prices and trade cost. The second component incorporates how anticipations of future procurement costs impact optimal supplier choices in the present, leading traders to buy from contemporaneously more expensive suppliers.

With the equilibrium relationships at hand, we compute impulse responses recursively, and we analytically derive partial-equilibrium trade elasticities ε_i^h for varying time horizons h. For a shock now (at time $t = 0$), we compute the backward-looking elasticity of future trade responses to the present trade cost shock by future time horizon h

$$\varepsilon_i^h \equiv \frac{\partial \ln \lambda_{sdi,h}}{\partial \ln \tau_{sdi,0}} = -\theta_i \left[1 - (1 - \zeta_i)^{h+1}\right] - (\sigma_i - 1)(1 - \zeta_i)^{h+1},$$

where $\lambda_{sdi,t}$ is destination country d's expenditure share falling on intermediate goods from source country s in industry i at time t, $\tau_{sdi,t'}$ is the trade cost component that is shocked at time t', θ_i is the long-term trade elasticity as in EK, and $\sigma_i - 1$ is the short-term trade elasticity as in Armington. The frequency at which traders from industry i can switch suppliers is $\zeta_i \in (0, 1)$, which we call the *supplier adjustment probability*. The backward-looking trade elasticity ε_i^h increases over time in absolute value from the short-term level to the long-term level (for the common parametrization $\theta_i > \sigma_i - 1$).

Agents in our model have perfect foresight, so for a future shock (at some subsequent time horizon h), we can also derive the forward-looking elasticity of a trade response now (at time $t = 0$) to an anticipated future trade cost shock at h

$$\varepsilon_i^{-h} \equiv \frac{\partial \ln \lambda_{sdi,0}}{\partial \ln \tau_{sdi,h+1}/\tau_{sdi,h}} = -\left[\theta_i - (\sigma_i - 1)\right](1 - r - \zeta_i)\left(\frac{1 - \zeta_i}{1 + r}\right)^h \zeta_i,$$

were r is the interest rate. The forward-looking trade elasticity ε_i^{-h} reflects decreases in the horizon at which the shock is anticipated.

In the long run, the backward-looking trade elasticity converges to the familiar Fréchet parameter θ_i as in EK. The rate of convergence depends on the frequency with which buyers can establish a new sourcing relationship ζ_i. The key parameters of our model are therefore identifiable from reduced-form estimates of the trade elasticity at varying time horizons as in BLP. This characterization of the horizon-specific trade elasticity also implies a horizon-specific welfare formula, which we derive in closed form. The horizon-specific welfare formula features a dynamic adjustment component, which fades as the economy converges to steady state over time, and nests the well-known formula from Arkolakis, Costinot and Rodríguez-Clare (2012) as the limiting case in the long run.

We show how the above results can be used to derive a set of estimation equations for the relevant parameters governing short- and long-term trade elasticities, document how existing results from BLP can be employed, and quantify our trade model. With the tractability of our model and data on input-output relations, we consider a model world economy consisting of 32 industries across 77 regions. We apply the model to study the general equilibrium response of global trade and production to the US-China trade war started in 2018, an unanticipated trade shock; and to the announcement and implementation of the EU enlargement in 2004, which we treat as an anticipated trade shock. We show that rich industry-level dynamics can result, with consequential changes in welfare implications. First, despite the low trade elasticity in absolute value in the short-run, the United States may suffer smaller welfare losses from unanticipated trade disruptions over the short run relative to the long-run outcome when sourcing frictions are no longer relevant. China, on the other hand, may suffer a short-run welfare loss that exceeds the long-run loss. The effect of a low short-run trade elasticity for short-run welfare depends on the trade balance. Second, a direct application of the static welfare formula from Arkolakis, Costinot and Rodríguez-Clare (2012), using realized domestic trade shares, can result in qualitatively misleading predictions over finite time horizons. The reason is that sourcing frictions, and the resulting time-varying trade elasticities, can induce substantive and shifting deviations from the long-term welfare response. Third, gains from trade can differ between the short and the long run in both sign and magnitude. In the short-run, price disruptions caused by the US-China trade war propagate through the network of existing supply relationships, leading to a global reduction in economic welfare. Those short-run losses, in part, reflect the limited scope for third-party countries to gain from the trade dispute by forming new supply relationships with the United States or China immediately. Gains for third countries may materialize in the medium to long term, however. As a consequence, countries whose previous trade linkages leave them most exposed to the US-China trade war, such as Mexico and Vietnam, experience initial welfare losses in the short-run, but marked increases in welfare in the long-run.

The wide discrepancy between a low (short-run) trade elasticity in absolute value in international macroeconomics and a high (long-run) trade elasticity in absolute value in international trade has been documented in, for example, Ruhl (2008, who calls the discrepancy an "international elasticity puzzle") and Fontagné, Martin and Orefice (2018). Fontagné, Guimbard and Orefice (2022), BLP and Anderson and Yotov (2022) offer estimation procedures to separately identify short- and long-run trade elasticities. de Souza et al. (2024) obtain horizon-specific trade elasticity estimates in a difference-in-differences design for anti-dumping tariff changes. Anderson and Yotov (2022) rationalize their estimation procedure with firm heterogeneity in lag times from recognition to action in the spirit of Lucas and Prescott (1971). In an alternative approach from a macroeconomic perspective, Yilmazkuday (2019) proposes a framework with nested CES models and derives the trade elasticity as the weighted average of macro elasticities. Our general equilibrium model offers a rationalization for the existing estimation methods with a mixture of the Armington and EK elasticities. Beyond Ricardian trade, Boehm et al. (2024) show for a family of firm-level trade models that the short-run trade elasticity can co-determine the steady-state gains from trade even in the long term.

The importance of staggered contracts for trade and exchange rate dynamics has been recognized since at least Kollintzas and Zhou (1992) and shares features with staggered pricing (Calvo 1983). We generalize deterministic contract ages to supplier relationships that end stochastically. In a related approach, Arkolakis, Eaton and Kortum (2011) embeds a consumer without knowledge of the identity of the origin countries into an EK model. The consumer can switch to the lowest-cost supplier at random intervals, but cannot act strategically because the supplier is unknown. In comparison, we rationalize consumer behavior by assigning procurement to a special group of profit-seeking intermediaries, whom we call traders. Our model tractably accounts for the dynamic formation of supplier-buyer relations and the resulting equilibrium prices by contract age beyond a binary characterization in Arkolakis, Eaton and Kortum (2011).[1] Based on the tractable characterization of prices by contract age, we can fully characterize steady states and transition dynamics. As a result, we obtain the original EK model as the limit of the equilibria along the transition path. Our welfare formula therefore endogenously inherits the long-run elasticity as a special case when all supplier contracts are optimally set.

The remainder of the paper is organized as follows. We present the model in Section 2, with details on mathematical derivations relegated to the Appendix. In Section 3 we turn to the dynamic analysis of the model. Estimation of the key parameters follows in Section 4. To illuminate the novel dynamic features of the model for economic activity during the adjustment path and the welfare consequences, we present case studies of the US-China trade war in 2018 and the EU's Eastern enlargement in 2014 in Section 5. Section 6 concludes.

2 Model

2.1 Fundamentals

Consider a world economy with N destination countries $d \in \mathcal{N} := \{1, 2, \cdots, N\}$ of trade flows, $s \in \mathcal{N}$ source regions, and I industries $i, j \in \mathcal{I} := \{0, 1, 2, \cdots, I\}$. Time t is discrete and foresight is perfect. Subscripts sdi, t denote a trade flow from source region s to destination d in industry i at time t.

Households. In each period t, a mass of L_d infinitely-lived households in country d inelastically supplies one unit of the single production factor (labor) to domestic firms at a competitive wage $w_{d,t}$.[2] Household utility in country d at time t is given by $u(C_{d,t})$, where $C_{d,t}$ is the aggregate final good: a Cobb-Douglas aggregate over the composite goods $C_{di,t}$ from each industry i with

$$C_{d,t} = \prod_{i \in \mathcal{I}} (C_{di,t})^{\eta_{di}} . \tag{1}$$

The coefficient η_{di} is the consumption expenditure share of industry i's composite good, where $\sum_{i \in \mathcal{I}} \eta_{di} = 1$. We denote with $P_{di,t}$ the price index of industry i's composite good in d at time t. Country d's consumer price index is then given by $P_{d,t} = \prod_{i \in \mathcal{I}} (P_{di,t}/\eta_{d,i})^{\eta_{di}}$. We assume that households consume their income in every period and discount future utility flows at a rate $\beta_t \in (0, 1)$.

Intermediate goods. Every industry i consists of a continuum of potential producers of intermediate goods $\omega \in [0, 1]$. A producer of intermediate good ω in source country s has an individual productivity z and operates a constant-returns-to-scale technology to produce the good using domestic labor ℓ and composite goods M_{ji} from

[1] The underlying stochastic process shares features with the so-called Sisyphos Process (Montero and Villarroel 2016).

[2] Our model can accommodate varying labor supply. In Appendix B.2 we present a tractable extension that allows for a costly forward-looking labor allocation choice and a resulting upward-sloping labor supply curve by industry.

other industries with

$$y_i(\omega) = z\,(\ell)^{\alpha_{si}} \prod_{j \in \mathcal{I}} (M_{ji})^{\alpha_{sji}}, \tag{2}$$

where $y_i(\omega)$ is the output of intermediate good ω. The coefficient α_{si} is the value-added share of industry i in country s, and the coefficients $\alpha_{sij} \geq 0$ are such that $\alpha_{si} = 1 - \sum_{j \in \mathcal{J}} \alpha_{sji}$. Intermediate-goods producers are the only agents that employ labor.

Intermediate goods can be traded across countries subject to an iceberg transport cost, so that shipping one unit of a good in industry i from country s to country d at time t requires shipping out $u_{sdi,t} \geq 1$ units from s, where $u_{ddi,t} = 1$ for all d. In addition, goods imported by d from s at t may be subject to an *ad valorem* tariff $\bar{\tau}_{sdi,t} \geq 1$. We combine transport costs and tariffs into the single trade cost parameter $\tau_{sdi,t} \equiv u_{sdi,t}\bar{\tau}_{sdi,t}$. Only intermediate goods can be traded across country borders.

Given trade costs and technologies, there is a *common unit cost component* at destination d for all intermediate goods produced in country s, which we denote with

$$c_{sdi,t} \equiv \Theta_{si}\tau_{sdi,t}\,(w_{s,t})^{\alpha_{si}} \prod_{j \in \mathcal{J}} (P_{sj,t})^{\alpha_{sji}}, \tag{3}$$

where Θ_{si} is a collection of Cobb-Douglas coefficients. The resulting unit cost of good ω at destination d produced in country s with a productivity $z(\omega)$ is given by $c_{sdi,t}/z(\omega)$.

Production technologies for intermediate goods arrive stochastically and independently at a rate that varies by country and industry. Following EK, the number of potential producers for an intermediate good ω in country s's industry i with a productivity higher than z is distributed Poisson with mean $A_{si}z^{-\theta_i}$.

Assemblers of composite goods. In each industry i, assemblers bundle intermediate goods ω into a composite good for consumption or production. An assembler costlessly aggregates intermediates into $Y_{di,t}$ units of industry i's composite good using the technology

$$Y_{di,t} = \left(\int_{[0,1]} y_{di,t}(\omega)^{\frac{\sigma_i - 1}{\sigma_i}} \mathrm{d}\omega \right)^{\frac{\sigma_i}{\sigma_i - 1}}, \tag{4}$$

where $y_{di,t}(\omega)$ is the quantity purchased of an intermediate good ω by an assembler in country-industry di, and $(\sigma_i - 1)$ is the elasticity of substitution between intermediate goods in industry i. Assemblers take as given the price $p_{di,t}(\omega)$, at which an intermediate good ω can be purchased in destination d. We explain in detail below the exact price at which an intermediate good is available. Cost minimization given (4) yields the assembler's demand for an intermediate good ω:

$$y_{di,t}(\omega) = p_{di,t}(\omega)^{-\sigma_i} P_{di,t}^{\sigma_i} Y_{di,t}, \tag{5}$$

where

$$P_{di,t} = \left(\int_{[0,1]} p_{di,t}(d\omega)^{-(\sigma_i - 1)} \mathrm{d}\omega \right)^{-\frac{1}{\sigma_i - 1}} \tag{6}$$

is the price of industry i's composite good at destination d. Composite goods cannot be traded across country borders.

Traders of intermediate goods. An assembler of a composite good requires specialized local *traders* to transact with the domestic and foreign producers of intermediate goods. Concretely, access to each intermediate good variety ω is facilitated by a dedicated local trader at destination d who holds the exclusive license to procure ω from any of the potential global suppliers for assemblers at d. Under the exclusive license, a trader generates profits and therefore has the incentive to engage in forward-looking behavior.

The hallmark feature of Ricardian trade is arguably the single-sourcing property at the extensive margin: any destination d procures a given variety ω from a unique source country s.[3] We therefore assign procurement, the economic activity associated with this key Ricardian property, to a special group of economic agents who we call traders. We allow the procurement decision of these traders to be forward looking.

A trader is a firm that requires no labor but has exclusive access to the competitive global Walrasian market, where the intermediate good ω is sold by producers from around the world, effectively granting the trader local monopoly power at destination d over the intermediate good's procurement. Traders exploit this monopoly power to charge a price above marginal cost to assemblers and thereby turn a flow profit in every period, which is claimed by the local representative household at destination d. Traders discount future profits at a rate $1/(1 + r_t)$. We take the resulting market structure for traders to be monopolistic competition.

2.2 Sourcing friction

Under the Ricardian trade tenet, a trader seeks to procure an intermediate good from the least costly global supplier. However, a trader only has the opportunity to adjust its choice of supplier after a random interval of time under a sourcing friction, which we describe now.

For each intermediate good ω, a trader's choice of source country is governed by an i.i.d. random variable $a_{di,t}(\omega) \in \{0, 1\}$ which takes the value of 1 with probability $\Pr[a_{di,t}(\omega) = 1] = \zeta_i$, for all $\omega \in [0, 1], d \in \mathcal{N}, i \in \mathcal{I}$ and t. If $a_{di,t}(\omega) = 1$, then destination d's trader for an intermediate good ω in industry i has the green light to switch to its preferred source country. Else, if $a_{di,t}(\omega) = 0$, that is if the global draw for intermediate variety ω does not turn to green for the trader, then the trader must purchase variety ω from the same producer as in the preceding period $t - 1$. The parameter $\zeta_i \in (0, 1)$ is the industry-specific *supplier adjustment probability*, and

[3] In the original two-industry, two-country benchmark as well as in the two-country, many-industry Dornbusch-Fischer-Samuelson model (1977) any given good is made in one location only, resulting in "bang bang" solutions for countably many imports. In the many-country, many-industry EK framework with a continuum of imports, multiple countries may produce the same variety but a given country purchases each variety only from a unique source country. For example, while there can be automobile manufacturers in several source countries, say in South Korea and Germany, a given destination such as Australia will uniquely buy a specific type of car from one source country, say South Korea, while French consumers uniquely buy the same type of car from Germany.

it measures the frequency at which the sourcing friction is lifted.[4] While the identity of the source country does not change, the quantity procured and the price that the trader pays can differ from the preceding period if the factory gate price moves (because of changing factor costs) or the currently prevailing trade costs move (because of a policy or natural exogenous change).

This formulation of the sourcing friction captures search costs and other types of impediments that prevent the optimal rematch of supply relationships at a moment in time. The sourcing friction creates a lock-in effect that makes the optimal choice of supplier a dynamic and, in the presence of monopoly profits for traders, a forward-looking decision. Another implication of the sourcing friction is that price elasticities of demand will differ across intermediate goods according to when their suppliers were last chosen. Let $\Omega^k_{di,t}$ denote the set of industry i's intermediate goods whose supplier at time t was last chosen k periods ago:

$$\Omega^k_{di,t} = \left\{\omega : a_{di,t-k}(\omega) = 1,\ \prod_{\varsigma=t-k+1}^{t} a_{di,\varsigma}(\omega) = 0\right\}, \tag{7}$$

where $\cup_k \Omega^k_{di,t} = [0,1]$. We call the intermediate goods that were last sourced optimally $k > 0$ periods ago the *legacy varieties*. The sets $\Omega^k_{di,t}$ mutually exclusively and exhaustively partition the unit interval of intermediate goods for each industry i, where $k \geq 0$.

2.2.1 The trader's problem

We now characterize the optimal choice of supplier for intermediate goods $\omega \in \Omega^0_{di.t}$, beginning with a description of the problem of a trader.

Static problem. Consider a trader in industry i at destination d who at time t sources an intermediate good ω from a supplier in country s, and suppose the chosen supplier's realized productivity $z_{si}(\omega)$ is the highest among all potential producers of the good in country s. Under this sourcing strategy, the trader can procure the good at a constant marginal cost $c_{sdi,t}/z_{si}(\omega)$ that depends on equilibrium factor prices and parameters by the common unit cost component (3). Profit maximization subject to demand in (5) then yields the price $p_{sdi,t}(\omega)$ that an assembler in destination-industry di will pay for an intermediate good ω procured from source country s:

$$p_{sdi,t}(\omega) = \frac{\sigma_i}{\sigma_i - 1}\frac{c_{sdi,t}}{z_{si}(\omega)}.$$

In turn, the instantaneous profit that the trader earns from the sourcing strategy satisfies

$$\pi_{sdi,t}(\omega) = \frac{1}{\sigma_i}\left(\frac{\sigma_i}{\sigma_i - 1}\frac{c_{sdi,t}}{z_{si}(\omega)}\right)^{-(\sigma_i - 1)} P^{\sigma_i}_{di,t} Y_{di,t}. \tag{8}$$

Dynamic problem. Under the sourcing friction, the opportunity for supplier selection at the extensive margin does not follow a deterministic schedule but arrives randomly, with probability ζ_i for each trader in country-industry di. For an intermediate good $\omega \in \Omega^0_{di,t}$ that has the green light to be procured from a newly chosen

[4]The model can also accommodate country-industry-specific frequencies of supplier adjustment ζ_{di} but they would result in country-industry-specific trade elasticities, which are not commonly estimated.

supplier, a trader's optimal choice of supplier can therefore be characterized by the following Bellman equations:

$$V_{di,t}(\omega) = \max_{n\in\mathcal{N}} V_{di,t}(\omega, n), \tag{9}$$

where

$$V_{di,t}(\omega, n) = \pi_{ndi,t}(\omega) + \frac{\zeta_i}{1+r_t} V_{di,t+1}(\omega) + \frac{1-\zeta_i}{1+r_t} V_{di,t+1}(\omega, n), \text{ for } n \in \mathcal{N}, \tag{10}$$

is the net-present value of a trader in d that procures the intermediate good ω from country n at time t.

2.2.2 Legacy suppliers

Legacy intermediate goods $\omega \in \Omega^k_{di,t}$ with $k > 0$ receive no green light for optimal procurement at time t. To characterize supplier relations for these legacy intermediate goods, we denote changes over time for a variable x_t succinctly by $\hat{x}_t \equiv x_t/x_{t-1}$. Suppose a trader last optimally sourced intermediate good $\omega \in \Omega^k_{di,t}$ from s at time $t-k$ under the unit cost $c_{sdi,t-k}/z_{si}(\omega)$. If the intermediate good is still sourced from the same producer at time t, its price equals

$$p_{sdi,t}(\omega) = \frac{\sigma_i}{\sigma_i - 1}\frac{c_{sdi,t}}{z_{si}(\omega)} = p_{sdi,t-k}(\omega) \prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdi,\varsigma},$$

which is the initial destination price adjusted for the cumulative changes in trade costs and factor prices since $t-k$.

2.2.3 Optimal supplier choice

To characterize the optimal choice of supplier for an intermediate good $\omega \in \Omega^0_{di,t}$ with a green light for optimal procurement, we use the recursive nature of the Bellman equations (9)-(10). As we elaborate in Appendix A.2, forward iteration under a standard set of transversality conditions[5] yields the value of supplier choice for intermediate goods $\omega \in \Omega^0_{di,t}$:

$$V_{di,t}(\omega) = \max_{n\in\mathcal{N}} \Big\{ \pi_{ndi,t}(\omega) \left(1 + \Psi_{ndi,t}\right) \Big\} + \frac{\zeta_i}{1-\zeta_i} \sum_{u=1}^{\infty} \frac{(1-\zeta_i)^u}{\prod_{\varsigma=1}^{u}(1 + r_{t+\varsigma-1})} V_{di,t+u}(\omega), \tag{11}$$

where

$$\Psi_{ndi,t} = \sum_{u=1}^{\infty} (1-\zeta_i)^u \left[\prod_{\varsigma=1}^{u} \frac{1}{1+r_{t+\varsigma-1}} \left(\frac{\hat{c}_{ndi,t+\varsigma}}{\hat{P}_{di,t+\varsigma}} \right)^{-(\sigma_i - 1)} \hat{P}_{di,t+\varsigma} \hat{Y}_{di,t+\varsigma} \right]. \tag{12}$$

is a measure of the net rate of appreciation for the current value $\pi_{sdi,t}(\omega)$ of a supply relationship with a producer in country s given anticipated changes in trade costs and factor prices (through $\hat{c}_{nid,t+\varsigma}/\hat{P}_{di,t+\varsigma}$) and domestic demand (through $\hat{P}_{di,t+\varsigma}\hat{Y}_{di,t+\varsigma}$). We refer to $\Psi_{sdi,t}$ as the *option value of a procurement relation* for traders in destination d with a given source country s. The option value is specific to bilateral trade relationships between

[5] The transversality conditions ensure that the present-value of sourcing from a given country n vanishes in the distant future: $\lim_{T\to\infty} \left[\prod_{\varsigma=1}^{T}(1-\zeta_i)/(1+r_{t+\varsigma-1})\right] V_{di,T}(\omega, n) = 0$.

source and destination countries sd and common across intermediate goods producers and traders in an industry i at a moment in time t. This option value reflects the lock-in effect generated by the sourcing friction. In particular, if $\zeta_i \to 1$ so that there is no lock-in effect since supply relations fully reset in every period, then $\Psi_{sdi,t} \to 0$ and the procurement relation holds no option value for any pair of countries.

From equation (11) it follows that the optimal source country choice $s^*_{di,t}(\omega) = \arg\max_n V_{di,t}(\omega, n)$ for the supplier of an intermediate good $\omega \in \Omega^0_{di,t}$ solves

$$s^*_{di,t}(\omega) = \arg\min_{n\in\mathcal{N}} \frac{c_{ndi,t}}{z_{ni}(\omega)} \left(1 + \Psi_{ndi,t}\right)^{-\frac{1}{\sigma_i - 1}} .$$

In words, if the choice of supplier for intermediate good ω falls on a producer from country s, then the combination of the producer's productivity, the current unit cost in source country s, the current trade costs between s and d, and the anticipated future trajectories of these costs must make the supplier the most profitable at time t.

Similar to EK, our distributional assumptions imply that the productivity $z_{si}(\omega)$ of a potential supplier in country s has a country-industry specific Fréchet distribution given by[6]

$$\Pr\left[z_{si}(\omega) \leq z \middle| A_{si}, \theta_i \right] = \exp\left\{ -A_{si} z^{-\theta_i} \right\}.$$

As a consequence, the fraction of intermediate goods $\omega \in \Omega^0_{di,t}$, for which a trader in destination d optimally chooses a supplier from source country s, satisfies

$$\upsilon^0_{sdi,t} \equiv \Pr\left[\omega \in \Omega^0_{i,t} : s^*_{di,t}(\omega) = s \right] = \frac{A_{si} c^{-\theta_i}_{sdi,t} \left(1 + \Psi_{sdi,t}\right)^{\frac{\theta_i}{\sigma_i - 1}}}{\Upsilon_{di,t}}, \tag{13}$$

where

$$\Upsilon_{di,t} = \sum_n A_{ni} c^{-\theta_i}_{ndi,t} \left(1 + \Psi_{ndi,t}\right)^{\frac{\theta_i}{\sigma_i - 1}} \tag{14}$$

is a measure of destination d's market access for intermediate goods $\omega \in \Omega^0_{di,t}$, given current and future trade cost along with factor prices behind the common unit cost component $c_{sdi,t}$ and the option value $\Psi_{sdi,t}$. We report the derivation of these results in Appendix A.2.

Equation (13) states the extensive margin demand of destination d for potential sources of intermediate goods $\omega \in \Omega^0_{di,t}$ in explicit form. The familiar EK characterization of the optimal supplier choice follows as a limiting case when $(1 - \zeta_i)/(1 + r_{t+\varsigma-1}) \to 0$ so that $\Psi_{sdi,t} \to 0$ for all s. Except for this limiting case, optimal supplier choices also incorporate future changes in procurement costs to account for the lock-in effect generated by the sourcing friction.

[6]The model could also accommodate productivity change over time with a country-industry-time specific Fréchet distribution and resulting $z_{si,t}(\omega)$ realizations that vary over time and across the sets $\Omega^k_{i,t}$. We develop this extension formally in Appendix B.1. To focus on adjustment to policy-related or natural trade cost shocks, we do not specify time-varying productivity shocks in the main text.

2.3 Trade flows

We now describe the global demand for intermediate goods in each set $\Omega^k_{di,t}$, including legacy varieties ($k > 0$) but beginning with those whose supply relationships are formed concurrently ($k = 0$). For well defined relationships, we assume that the shape parameter of the productivity distribution exceeds the elasticity of substitution: $\theta_i > \sigma_i - 1$ in all industries.

2.3.1 Demand for intermediate goods with newly formed supply relationships

Under optimal forward-looking procurement, the distribution of prices paid by a local assembler for goods $\omega \in \Omega^0_{di,t}$ sourced from a country s at time t satisfies

$$G^0_{sdi,t}(p) = \Pr\left(p_{d,t}(\omega) \le p \,\middle|\, s^*_{di,t}(\omega) = s\right) = 1 - \exp\left\{-\left(\frac{\sigma_i}{\sigma_i - 1}\right)^{-\theta_i} (1 + \Psi_{sdi,t})^{-\frac{\theta_i}{\sigma_i - 1}} \Upsilon_{di,t} p^{\theta_i}\right\}, \quad (15)$$

as we show in Appendix A.3. Hence, if option values $\Psi_{sdi,t}$ vary between potential source countries s, then so will the distributions of prices paid by assemblers and consumers, depending on whether a country's suppliers are chosen based on current or future cost. If the option value for a country s is high, then traders are more likely to source goods from suppliers that are not currently the least expensive, so the current average price for goods procured from country s by forward-looking traders will be higher than the choice of a myopic agent would be. Formally, if $\Psi_{sdi,t} > \Psi_{s'di,t}$, then $G_{sdi,t} \succ_1 G_{s'di,t}$, so the distribution of prices paid across goods sourced from s first-order stochastically dominates that of goods procured from s'. If $\Psi_{sdi,t} = \Psi_{s'di,t}$, then the distribution of the prices paid will be the same for countries s and s', as is the case in EK.

From the distributions of prices paid in (15), we can readily derive a destination d's expenditure share for each potential source country s across intermediate goods $\omega \in \Omega^0_{di,t}$:

$$\lambda^0_{sdi,t} = \frac{A_{si} c_{sdi,t}^{-\theta_i} (1 + \Psi_{sdi,t})^{\frac{\theta_i}{\sigma_i - 1} - 1}}{\Phi^0_{di,t}}, \quad (16)$$

where

$$\Phi^0_{di,t} = \sum_{n \in \mathcal{N}} A_{ni} c_{nid,t}^{-\theta_i} (1 + \Psi_{ndi,t})^{\frac{\theta_i}{\sigma_i - 1} - 1} \quad (17)$$

is the measure of country d's market access for intermediate goods $\omega \in \Omega^0_{di,t}$ based on current trade costs and factor prices. See Appendix A.3.

Equation (16) clarifies how current and future costs impact trade flows differently, depending on the underlying margins of adjustment. Note that the option value $\Psi_{sdi,t}$ is exclusively based on anticipated future unit cost changes by (12), relative to the preceding unit cost, while the current level of the unit cost component $c_{sdi,t}$ only matters at the present time t. At the extensive margin, trade flows therefore respond to current cost, as in EK, with an elasticity of $-\theta_i$. For intermediate goods in the set $\Omega^0_{sdi,t}$, with a green light for optimal procurement, the partial-equilibrium elasticity of trade flows with respect to the current trade cost is governed by the familiar

Fréchet shape parameter:

$$\left.\frac{\partial \ln \lambda^0_{sdi,t}}{\partial \ln \tau_{sdi,t}}\right|_{\Phi^0_{di,t},\Psi sdi,t} = -\theta_i.$$

In contrast, anticipated procurement choices in response to changes in future unit costs can result in uneven shifts in prices across source countries and can thereby trigger additional anticipated adjustments along the intensive margin. Suppose trade costs increase permanently after a future horizon h (for all $h' > h$), and that the interest rate is constant ($r_t = r$). For intermediate goods in the set $\Omega^0_{sdi,t}$, the partial-equilibrium response of trade flows to future trade costs is governed by the elasticity of substitution $(\sigma_i - 1)$ at the intensive margin, in addition to the Fréchet shape parameter at the extensive margin, and by a horizon-h specific discount factor:

$$\left.\frac{\partial \ln \lambda^0_{sdi,t}}{\partial \ln \{\tau_{sd,t+h'}\}_{h'>h}}\right|_{\Phi^0_{di,t},c_{sdi,t}} = -\left[\theta_i - (\sigma_i - 1)\right]\left(\frac{1-\zeta_i}{1+r}\right)^h \frac{\Psi_{sdi,t+h}}{1+\Psi_{sdi,t}}.$$

The future option value $\Psi_{sdi,t+h}$ incorporates the permanent change in trade costs after h (for all $h' \geq h$).

Importantly, the response of trade flows to anticipated future trade costs reflects not just the extensive-margin adjustment abut also adjustment of demand at the intensive margin. For $\theta_i > \sigma_i - 1$, the elasticity of substitution $(\sigma_i - 1)$ reduces the prevailing elasticity θ_i at the extensive margin and dampens the impact of future trade cost on current trade flows. The role of the option value for a trade-flow response mirrors the effect of fixed operation costs in heterogeneous-firm models with monopolistic competition when there is no firm entry. In fact, equation (16) coincides with the gravity equation in a Melitz (2003)-Chaney (2008) model with no entry and Pareto-distributed firm-level productivity, where the endogenous gross option value $(1 + \Psi_{sdi,t})$ replaces the bilateral fixed export costs from Chaney (2008).

2.3.2 Demand for intermediate goods with legacy supply relationships

Legacy intermediate goods $\omega \in \Omega^k_{di,t}$ with $k > 0$ are purchased from a supplier that was last optimally chosen at time $t - k$. We show in Appendix A.4 that country d's expenditure share by source country across intermediate goods $\omega \in \Omega^0_{di,t}$ equals

$$\lambda^k_{sdi,t} = \frac{\lambda^0_{sdi,t-k}\left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdi,\varsigma}\right)^{-(\sigma_i-1)}}{\Phi^k_{di,t}}, \tag{18}$$

where

$$\Phi^k_{di,t} = \sum_{n\in\mathcal{N}} \lambda^0_{ndi,t-k}\left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndi,\varsigma}\right)^{-(\sigma_i-1)} \tag{19}$$

reflects the mean price paid for the set of intermediate goods $\Omega^0_{di,t-k}$ at time $t-k$ given trade shares $\{\lambda^0_{ndi,t-k}\}_{n\in\mathcal{N}}$.

A comparison of equations (16) and (18) shows how the effect of unit costs on trade flows differs by the timing of the procurement relationship. If a trader can optimally source an intermediate good at time t, the effect of the cost change on trade flows is governed by the Fréchet shape parameter θ_i in (16), and trade flows reflect the effects of current and future changes to comparative advantage patterns. Conversely, if a trader is unable to switch suppliers, then the extensive margin of adjustment is inoperative. The only remaining margin of adjustment is the intensive one, which is governed by the terms that collect the cumulative changes in past unit costs under the elasticity of substitution $(\sigma_i - 1)$ in (18). In effect, trade flows follow a pattern as if the varieties were differentiated by country of origin and time of last procurement choice, where the measure of varieties from each origin and procurement time is defined by legacy partition $\Omega^k_{di,t}$. For each legacy partition $\Omega^k_{i,t}$ with $k > 0$, Armington (1969) forces determine trade flows. The partial-equilibrium response of trade flows with respect to a concurrent trade cost change is governed by the elasticity of substitution $\sigma_i - 1$:

$$\left.\frac{\partial \ln \lambda^k_{sdi,t}}{\partial \ln \hat{\tau}_{sdi,t}}\right|_{\Phi^k_{di,t},\lambda^0_{sdi,t-k}} = -(\sigma_i - 1).$$

To close the model, we now show how aggregate demand for the composite good of industry i follows from the aggregation of trade shares in (16) and (18).

2.4 Aggregation

To derive aggregate demand, we rely on the homotheticity of assembly. The partial price index for the composite of legacy intermediate goods purchased at time t from suppliers chosen $t-k$ periods ago satisfies $(P^k_{di,t})^{-(\sigma_i-1)} = \int_{\omega\in\Omega^0_{i,t}} p_{di,t}(\omega)^{-(\sigma_i-1)} \mathrm{d}\omega$. The sets $\Omega^k_{di,t}$ partition industry i's product space, so we can obtain country d's price index for industry i goods at time t by aggregating these partial price indices across all partitions and find $P^{-(\sigma_i-1)}_{di,t} = \sum_k (P^k_{di,t})^{-(\sigma_i-1)}$.

We establish in Appendix A.3 that the partial price index for the set of intermediate goods whose suppliers are chosen optimally at time t takes the form

$$P^0_{di,t} = \gamma_i \left(\mu_{di,t}(0)\frac{\Phi^0_{di,t}}{\Upsilon_{di,t}}\right)^{-\frac{1}{\sigma_i-1}} (\Upsilon_{di,t})^{-\frac{1}{\theta_i}}\ , \tag{20}$$

where $\gamma_i \equiv [\sigma_i/(\sigma_i-1)]\,\Gamma([\theta_i-(\sigma_i-1)]/\theta_i)^{-1/(\sigma_i-1)}$, $\Upsilon_{di,t}$ is given by (14), $\Phi^0_{di,t}$ is given by (17), and $\mu_{di,t}(0)$ denotes the measure of the set $\Omega^0_{di,t}$. Following our previous discussion, the endogenous market access term $\Upsilon_{di,t}$ represents the mean present value of the newly formed supply relationships to traders in d at time t, and $\Phi^0_{di,t}$ is the value these relationships provide to assemblers and consumers. The proportion of $\Phi^0_{di,t}$ to $\Upsilon_{di,t}$ therefore informs the transitory price effects of the sourcing friction between goods in the basket $\Omega^0_{di,t}$.

The measure $\mu_{di,t}(0)$ accounts for gains from variety. This measure recursively evolves over time according to a stochastic process that governs the traders' optimal procurement decisions, similar to the so-called Sisyphos Process (Montero and Villarroel 2016):

$$\mu_{di,t}(k) = \begin{cases} \zeta_i & \text{for } k = 0, \\ (1-\zeta_i)\mu_{di,t-1}(k-1) & \text{for } k > 0. \end{cases} \tag{21}$$

The parameter $\zeta_i \in (0,1)$ is the supplier adjustment probability, and it measures the frequency at which traders from industry i can switch suppliers.

We establish in Appendix A.4 that the partial price index across legacy intermediate goods, whose suppliers were chosen at time $t-k$, satisfies:

$$P_{di,t}^{k} = P_{di,t-k}^{0} \left(\frac{\mu_{di,t}(k)}{\mu_{di,t-k}(0)} \Phi_{di,t}^{k} \right)^{-\frac{1}{\sigma_i - 1}}, \tag{22}$$

which is the period $t-k$ price index of the basket of intermediate goods $\Omega_{i,t-k}^{0}$ adjusted for subsequent changes in variety composition, captured by $\mu_{di,t}(k)$, and prices, captured by $\Phi_{di,t}^{k}$.

Given equations (20) and (22), we can solve for the composite price index of industry i goods in country d at time t

$$P_{di,t} = P_{di,t}^{0} \left[1 + \sum_{k=0}^{\infty} \frac{\mu_{di,t}(k)}{\mu_{di,t}(0)} \left(\frac{P_{di,t-k}^{0}}{P_{di,t}^{0}} \right)^{-(\sigma_i - 1)} \Phi_{di,t}^{k} \right]^{-\frac{1}{\sigma_i - 1}} \tag{23}$$

as well as for country d's expenditure share on industry i goods sourced from country s

$$\lambda_{sdi,t} = \sum_{k=0}^{\infty} \left(\frac{P_{di,t}^{k}}{P_{di,t}} \right)^{-(\sigma_i - 1)} \lambda_{sdi,t}^{k}, \tag{24}$$

where $\lambda_{sdi,t}^{k}$ is given by (16) if $k = 0$ and by (18) if $k > 0$. Appendix A.5 provides details.

The set of trade shares $\{\lambda_{sdi,t}\}_{s,d \in \mathcal{N}, i \in \mathcal{I}}$ fully characterizes the demand in the world economy at time t. To close the model, we now describe the conditions for market clearing and define the general equilibrium.

2.5 Equilibrium

We denote total revenues of industry i from a source country s at time t with $X_{si,t}$. To define equilibrium, we express each industry's total revenues in terms of trade shares by (24), total expenditures on consumption $E_{d,t}$, and sales $X_{dj,t}$ anywhere in the world

$$X_{si,t} = \sum_{d \in \mathcal{N}} \lambda_{sdi,t} \left[\eta_{di} E_{d,t} + \sum_{j \in \mathcal{I}} \alpha_{dij} X_{dj,t} \right]. \tag{25}$$

Under constant markup pricing, a fraction $1/\sigma_i$ of the sales of a local assembler is earned as a profit by local traders in industry i. A destination d's total profit income $\Pi_{d,t}$ can therefore be expressed as a function of its local expenditures on composite goods:

$$\Pi_{d,t} = \sum_{i \in \mathcal{I}} \frac{1}{\sigma_i} \left[\eta_{di} E_{d,t} + \sum_{j \in \mathcal{I}} \alpha_{dij} X_{dj,t} \right]. \tag{26}$$

A country's national expenditure equals the sum of its factor income, profit income and trade deficit so that $E_{d,t} = w_{d,t} L_{d,t} + \Pi_{d,t} + D_{d,t}$, under the adding-up constraint $\sum_{d \in \mathcal{N}} D_{d,t} = 0$. We follow the conventional approach in the international trade literature and treat aggregate trade deficits as exogenous. To clear the factor market, wages adjust to ensure that total factor income in each country equals the factor expenditures of all producers:

$$w_{d,t}L_{d,t} = \sum_{i\in\mathcal{I}}(1-\alpha_{di})\frac{\sigma_i - 1}{\sigma_i}X_{di,t}, \tag{27}$$

and goods market clearing is guaranteed by Walras' law.

We are now ready to define a dynamic general equilibrium and a steady state.

Definition 1. *An economy is a set of time-invariant parameters for technologies, preferences and factor endowments* $\boldsymbol{\Theta} = \{\theta_i, \sigma_i, \{\alpha_{dji}\}_{j\in\mathcal{I}}, \varphi_{di}, A_{di}, \eta_{di}, L_d\}_{d\in\mathcal{N}}\}_{i\in\mathcal{I}}$*, sourcing frictions* $\boldsymbol{\zeta} = \{\zeta_i\}_{i\in\mathcal{I}}$*, as well as measures* $\boldsymbol{\mu}_0 = \{\mu_{di,t_0}(k)\}_{d\in\mathcal{N},i\in\mathcal{I},k\in\{0,1,\cdots\}}$ *for some* t_0*. Given a path of trade costs* $\boldsymbol{\tau} \equiv \{\boldsymbol{\tau}_t\}_{t\in\mathbb{N}} = \{\tau_{sdi,t}\}_{s,d\in\mathcal{N},i\in\mathcal{I},t\in\mathbb{N}}$ *and interest rates* $\boldsymbol{r} \equiv \{r_t\}_{t\in\mathbb{N}}$*:*

1. *A static equilibrium at time* t *is a vector of wages* $\boldsymbol{w}_t = w(\tau_t, \hat{\boldsymbol{w}}_{-t}, \hat{\boldsymbol{\tau}}_{-t}, \boldsymbol{\Theta}, \boldsymbol{\zeta}, \boldsymbol{\mu}_0, \boldsymbol{r})$ *that jointly solves equations (24), (25), (26) and (27) given* $s, d \in \mathcal{N}$*,* $i \in \mathcal{I}$*,* $\hat{\boldsymbol{w}}_{-t} = \{\hat{w}_{d,\varsigma}\}_{d\in\mathcal{N},\varsigma\in\mathbb{N}\backslash\{t\}}$ *and* $\hat{\boldsymbol{\tau}}_{-t} = \{\hat{\tau}_{sid,\varsigma}\}_{s,d\in\mathcal{N},i\in\mathcal{I},\varsigma\in\mathbb{N}\backslash\{t\}}$*.*
2. *A dynamic equilibrium is a path of wages* $\{\boldsymbol{w}_t\}_{t\in\mathbb{N}}$ *so that, for all* t*,* $\boldsymbol{w}_t = w(\tau_t, \hat{\boldsymbol{w}}_{-t}, \hat{\boldsymbol{\tau}}_{-t}, \boldsymbol{\Theta}, \boldsymbol{\zeta}, \boldsymbol{\mu}_0, \boldsymbol{r})$*.*
3. *A dynamic equilibrium is a steady state if* $\boldsymbol{w}_{t'} = \boldsymbol{w}_t$ *for all* $t' > t$*.*

2.6 Dynamic hat algebra

The forward-looking and gradual formation of supply relationships in our model generates transitional dynamics with substantive cross-sectional heterogeneity in procurement strategies and prices between goods in each country-industry cell. Nevertheless, we can efficiently solve for the economy's dynamic response to an anticipated sequence of changes in fundamentals using dynamic exact-hat algebra techniques.

Suppose that we observe a dynamic equilibrium allocation somewhere along the transition path towards a steady state equilibrium. As we verify in Appendix A.6, we can then solve for the transition path of endogenous variables in terms of time changes $\hat{x}_t$ for any anticipated convergent sequence of future changes in trade costs, without having to solve for the economy's structural fundamentals (productivity parameters and trade costs). Consequently, we can apply the so-called "hat algebra" of Dekle, Eaton and Kortum (2007) to efficiently compute perfect foresight counterfactuals in our model.

3 Dynamic Adjustment to Trade Shocks

We now use the model to study the economy's dynamic response to present and anticipated future changes in trade costs. We derive new structural estimating equations for the trade elasticity at different time horizons as well as a new formula for the dynamic welfare gains from trade. To anchor the dynamics, we first turn to steady state. For the steady state to be well defined, we consider a time invariant interest rate r from now on.

3.1 Steady-state properties

Our model preserves the properties of quantitative trade models based on EK in the limit when the economy is in steady state, regardless of the magnitude of the frictions underlying imperfect supplier adjustment. Intuitively, the supplier adjustment probability $\zeta_i \in (0,1)$ regulates the speed of adjustment to the long-term limit but does not affect the limit itself. The transitory effects of trade disruptions that arise in our model reflect how opportunities for finding new suppliers are limited in the short run. Assemblers get to adjust all supply relationships in the long run, so we obtain a version of the EK model as the limit of the equilibria along the transition path.

Formally, let $w^{EK}\left(\tau, \mathbf{1}, \mathbf{1}, \mathbf{\Theta}, \mathbf{1}, \boldsymbol{\mu}^{EK}\right)$ represent the equilibrium allocation in an economy in which suppliers can be flexibly selected for all intermediate goods, so $\zeta_i = 1$ for all industries i and $\boldsymbol{\mu}^{EK} = \{1, 0, 0, ...\}$. We can then establish

Proposition 1. *If w^* is a steady state equilibrium, then*

1. *For any $\boldsymbol{\zeta}$, $w^* = w\left(\tau, \mathbf{1}, \mathbf{1}, \mathbf{\Theta}, \boldsymbol{\zeta}, \boldsymbol{\mu}_0\right) = w^{EK}\left(\tau, \mathbf{1}, \mathbf{1}, \mathbf{\Theta}, \mathbf{1}, \boldsymbol{\mu}^{EK}\right)$.*
2. *For all $k \in \{0, 1, ...\}$, the measure of goods $\omega \in \Omega^k_{di,t}$ equals $\mu^*_i(k) = \zeta_i(1-\zeta_i)^k$ for all $d \in \mathcal{N}$, and trade flows are given by $\lambda^k_{sdi,*} = \lambda^{EK}_{sid}$ where λ^{EK}_{sid} denotes the trade shares in the frictionless economy.*

Proof. See Appendix A.7. □

Proposition 1 establishes the relationship between our model with sourcing frictions and existing models, and characterizes the age distribution of legacy variables in steady state. The first statement is a reminder that the tools developed in the literature to study the equilibrium properties of static quantitative trade models can be used to establish the existence and uniqueness of steady state in our model. The second part of Proposition 1 highlights properties of the steady state that we can later revisit to quantify our model. In particular, the second statement shows that the process governing the evolution of the age distribution of supply relationships over time has a simple geometric stationary distribution. Moreover, the second part shows that steady state expenditure allocations equalize across intermediate goods within an industry in steady state, irrespective of when s supplier was last chosen optimally.

3.2 Trade elasticities by time horizon

We begin by showing how the trade elasticity (the elasticity of trade flows with respect to current transport cost) varies over time in our model. The trade elasticity $\varepsilon^h_{sdi,t}$ at time horizon h is defined as

$$\varepsilon^h_{sdi,t} \equiv \left.\frac{\partial \ln X_{sdi,t+h}}{\partial \ln \tau_{sdi,t}}\right|_{\{\Phi^k_{di,t+\varsigma}, \Psi_{sdi,t+\varsigma}\}_{t \le \varsigma \le h, k}}, \text{ for } \quad h \ge 0, \tag{28}$$

for trade flows in industry i from country s to d at time $t+h$ after a permanent trade cost change at time t. The elasticity measures the proportional change in trade flows $X_{sdi,t+h}/X_{sdi,t-1}$, compared to the pre-shock period, with respect to the change in trade costs at t: $\mathrm{d} \ln \tau_{sdi,t} = \ln \hat{\tau}_{sdi,t}$. The elasticity definition holds fixed the general equilibrium terms that summarize changes in current market access and future procurement cost for industry i goods at destination d. Proposition 2 provides an analytical characterization of this elasticity.

Proposition 2. *Suppose that the economy is in steady state at $t = -1$. Then, up to first order, the elasticity of the horizon-h trade flows with respect to a shock to trade cost at time $t = 0$ is*

$$\varepsilon_i^h = -\theta_i \left[1 - (1 - \zeta_i)^{h+1}\right] - (\sigma_i - 1)(1 - \zeta_i)^{h+1}. \tag{29}$$

If $\zeta_i \in (0, 1)$, then $\lim_{h\to\infty} \varepsilon_i^h = -\theta_i$, and the rate of convergence equals

$$\lim_{h\to\infty} \ln \frac{\varepsilon_i^{h+1} + \theta_i}{\varepsilon_i^h + \theta_i} = \ln (1 - \zeta_i).$$

Proof. See Appendix A.8. □

As $h \to 0$, the trade elasticity $\varepsilon_i^h > -\theta_i$ strictly decreases toward the long-run limit of $-\theta_i$ (strictly increases over time in absolute value) if $\theta_i > \sigma_i - 1$. In the long-run, the trade elasticity equals the Fréchet parameter θ_i in absolute value. The rate of convergence to the industry's long-run elasticity depends on the industry-specific supplier adjustment probability ζ_i, the frequency at which assemblers can establish a new sourcing relationship.

The definition of the horizon-specific trade elasticity in (28) is consistent with reduced-form estimates at varying time horizons as in BLP. For estimation, we will leverage this equivalence to identify the structural parameters governing the trade elasticity.

3.3 Anticipatory trade adjustment

We now describe how trade flows respond to anticipated changes in future trade cost. The anticipatory trade elasticity is defined as

$$\varepsilon_{sdi,t}^{-h} = \left.\frac{\partial \ln X_{sdi,t}}{\partial \ln \hat{\tau}_{sdi,t+h}}\right|_{\{\Phi^0_{sdi,t+\varsigma}\}_{\varsigma\geq t}} \quad \text{for } h > 0 \tag{30}$$

for source-industry-destination sdi trade flows at time t and an anticipated change in trade cost at time $t + h$. This elasticity captures the partial equilibrium response of trade flows in the present to an anticipated permanent shift in trade cost h periods in the future. The following proposition provides an analytical characterization of this partial equilibrium elasticity.

Proposition 3. *Suppose that the economy is in steady state at $t = -1$. Then, up to first order, the elasticity of trade flows at $t = 0$ with respect to an anticipated change in future trade cost at time $h > 0$ is*

$$\varepsilon_i^{-h} = -\left[\theta_i - (\sigma_i - 1)\right](1 - r - \zeta_i)\left(\frac{1 - \zeta_i}{1 + r}\right)^h \zeta_i. \tag{31}$$

Proof. See Appendix A.9. □

Following our earlier discussion, the forward-looking trade elasticity $\epsilon_i^h > -\theta_i$ decreases (in absolute value) in the elasticity of substitution, since anticipation implies buying from contemporaneously more expensive suppliers and giving up part of current sales of a given variety ω due to demand substitution; and in the interest rate r and the anticipation horizon h, since both decrease the present discounted value of traders.

Equation (31) also clarifies that ε_i^{-h} is non-monotonic in the supplier adjustment probability ζ_i, which affects the anticipatory trade elasticity through two opposing forces. If $\zeta_i = 0$, there is no anticipation because no trader can ever adjust, the mass of varieties from each country is fixed, and all changes in demand occur along the intensive margin. If $\zeta_i = 1$, there is no anticipation because traders can readjust in every period with probability one, and therefore it is always optimal to readjust when the shock hits. The action lies between. As ζ_i increases, the measure of the set $\Omega^0_{di,t}$, which represents the mass of adjusting firms, increases, favoring anticipation; but also the probability of readjustment prior to shock arrival increases and therefore the value of current adjustment decreases, favoring no anticipation. The second channel is stronger for shocks farther away in the future, as seen by the discount factor $\left[(1-\zeta_i)/(1+r)\right]^h$.

3.4 Welfare gains from trade by time horizon

Having drawn out the forces that govern the transitory dynamics of trade flows in partial equilibrium, we now turn to describing the general equilibrium welfare ramifications of these trade dynamics. In the proposition below, we show that our model yields a dynamic welfare formula that generalizes existing sufficient statistic results for the welfare gains from trade in static models with CES demand from Arkolakis, Costinot and Rodríguez-Clare (2012) to incorporate a forward-looking and gradual formation of supply relationships.

Proposition 4. *Suppose that the economy is in steady state at $t = -1$. Then, the change of real wages $\hat{W}_d^h = C_{d,h}/C_{d,-1}$ in country d at time horizons $h = \{0, 1, ...\}$ in response to an arbitrary convergent sequence of trade shocks is given by*

$$\hat{W}_d^h = \prod_{j\in\mathcal{I}} \left[\left(\frac{\lambda_{ddj,h}}{\lambda_{ddj,-1}}\right)^{-\frac{1}{\theta_j}} \left(\Xi_{dj,h}\right)^{\frac{1}{\sigma_j - 1}} \right]^{\sum_{i\in\mathcal{I}} \bar{a}_{dji}\eta_i}, \tag{32}$$

where

$$\Xi_{dj,h} \equiv (1-\zeta_j)^{h+1} \left(\frac{\lambda_{ddi,-1}}{\lambda_{ddj,h}}\right)^{\frac{\theta_j - \sigma_j + 1}{\theta_j}} + \sum_{\varsigma=0}^{h} \zeta_j (1-\zeta_j)^{\varsigma} \left(\frac{\nu_{ddj,h-\varsigma}}{\lambda_{ddj,h}}\right)^{\frac{\theta_j - \sigma_j+1}{\theta_j}} \tag{33}$$

and $\bar{a}_{dji}$ is the (j,i)-th element of the Leontief inverse $(\mathbf{I} - \boldsymbol{A}_d)^{-1}$, with the elements of $\boldsymbol{A}_d$ given by α_{dji}.

If $\zeta_i \in (0,1)$, then $\lim_{h\to\infty} \hat{W}_d^h = \prod_{j\in\mathcal{I}} (\lambda_{ddj,t+h}/\lambda_{ddj,-1})^{-\sum_{i\in\mathcal{I}} \bar{a}_{dji}\eta_i/\theta_j}$.

Proof. See Appendix A.10. □

Following (32), welfare analysis can be conducted using ex-post sufficient statistics. These statistics inform how the different margins of demand adjustment contribute to the overall change in real wages between period $t = -1$ and period $t = h$.

The aggregate change in a country's real wage incorporates shifts in its realized comparative advantage via the terms $(\lambda_{ddj,h}/\lambda_{ddj,-1})^{-1/\theta_j}$ on the right-hand side of (32). Since the Fréchet parameter θ_j is the price elasticity of demand for newly sourced goods, these terms measure the change in an industry's consumer price index as if all varieties were being sourced optimally. For finite horizons $h < \infty$, the home expenditure shares $\lambda_{ddj,h}$ are distorted under the sourcing frictions. All goods are optimally sourced when $h \to \infty$, only then do the home expenditure shares capture the entire real wage response as in EK.

The exact real wage distortion at any finite horizon $h < \infty$ depends on the terms $\Xi_{dj,h}$ in (33). These factors $(\Xi_{dj,h})^{1/(\sigma_j - 1)}$ incorporate distortions in a country's terms of trade brought about by the sourcing friction. There are two aspects to the distortions. First, for legacy varieties that have not yet been optimally sourced between $t = 0$ and $t = h$, the current share of home expenditures $\lambda_{ddj,h}$ deviate from the initial steady state at $t = -1$. Second, assemblers' current home expenditure shares for legacy varieties that were at least once optimally sourced between $t = 0$ and the current horizon $t = h$ deviate from the extensive margin demand for domestic producers of traders behind $\nu_{ddj,h-\varsigma}$. Rearrangement of equations (13) and (16) shows how the importance of these two effects varies across legacy varieties depending on when their current supplier was chosen between $t = 0$ and $t = h$:

$$\frac{\nu_{ddi,h-\varsigma}}{\lambda_{ddi,h}} = \frac{\Phi^0_{di,h-\varsigma}}{\Upsilon_{di,h-\zeta}}(1 + \Psi_{ddi,h-\varsigma})\frac{\lambda^0_{ddi,h-\varsigma}}{\lambda_{ddi,h}}. \text{ for } 0 \leq \varsigma \leq h$$

The first two terms on the right incorporate how traders' choices to procure goods based on option values rather than from the least expensive global supplier at time $t = h - \zeta$ continue to distort prices paid for goods $\omega \in \Omega^{h-\zeta}_{di,h}$ at time $t = h$. In turn, the ratio $\lambda^0_{ddi,h-\zeta}/\lambda_{ddi,h}$ traces how the impact of these initial extensive margin distortions on a country's terms-of-trade evolves over time as traders cannot adjust their initial supplier choice to changes in global factor prices and trade costs that occur between $t = h - \zeta$ and $t = h$.

From the above discussion, it follows intuitively that the relevant trade elasticity for welfare analysis differs from the structural elasticity in (29). Consequently, the welfare effects of trade shocks can vary both quantitatively and qualitatively over time, even conditional on the structural parameters that govern the time variation in the trade elasticity. Proposition 4 allows us to summarize these dynamic effects in terms of a small set of key statistics.

4 Estimation

We turn to the quantitative implications of our theory for the responses of production and welfare to trade shocks. In preparation for that, we outline and implement our approach to estimating the structural parameters that govern the horizon-specific trade elasticity in this section. In the next section, we will use these estimates for a quantitative evaluation of how the 2018 US-China trade war and the 2004 EU enlargement have impacted trade, production and welfare through the lens of our model.

4.1 Methodology

To obtain the structural parameters governing the horizon-specific trade elasticity, we leverage the explicit analytical characterization from Proposition 2. Intuitively, the parameter σ_i governs the empirical behavior of the trade elasticity in the short-run; while θ_i pins down its long-run level. The rate at which the trade elasticity changes from the short-run to the long-run level informs the structural parameter ζ_i, governing the stickiness of supplier relationship in our model. Below, we lay out the empirical methodology for obtaining estimates of structural parameters $\Theta_i \equiv \{\theta_i, \sigma_i, \zeta_i\}$.

Using Proposition 2, we can state the following structural equation for the h-horizon elasticity of tariff-exclusive bilateral exports $X_{sdi,t+h}$ to an unanticipated change in tariffs $\ln \bar{\tau}_{sdi,t}/\bar{\tau}_{sdi,t-1}$ at year t

$$\ln\left(\frac{X_{sdi,t+h}}{X_{sdi,t-1}}\right) = \left(\epsilon_i^h\left(\Theta_i\right) - 1\right)\ln\left(\frac{\bar{\tau}_{sdi,t}}{\bar{\tau}_{sdi,t-1}}\right) + \gamma_i^h \Gamma_{sdi,t} + \delta_{si,t+h} + \delta_{di,t+h} + u_{sdi,t+h} \tag{34}$$

where $\epsilon_i^h(\Theta_i)$ is the model-implied horizon-specific trade elasticity, written as a function of the vector of structural parameters Θ_i using equation (29); $\Gamma_{sdi,t}$ denotes a vector of lagged controls for trade volumes and tariff changes

$$\Gamma_{sdi,t} \equiv \left[\ln\left(\frac{X_{sdi,t-1}}{X_{sdi,t-2}}\right),\ \ln\left(\frac{\bar{\tau}_{sdi,t-1}}{\bar{\tau}_{sdi,t-2}}\right)\right] \tag{35}$$

following BLP; δ denotes fixed effects; and $u_{sdi,t+h}$ is a residual.[7]

Identification of the structural parameters involves the same endogeneity challenges discussed in BLP. For this reason, we adopt the instruments from BLP and consider the following form of moment conditions under a standard framework of generalized method of moments (GMM) for a given horizon h

$$m_{sdi,t}^h(\Theta_i) \equiv \mathrm{E}[u_{sdi,t+h} Z_{sdi,t}] = 0 \tag{36}$$

where $Z_{sdi,t}$ denotes the instrumental variable constructed by BLP. The pursuit of exogenous tariff movements by BLP is appealing to us as the derivation of Proposition 2 requires that the shocks arrive as surprise. Specifically, they use tariff changes resulting from binding changes in the most favored nation (MFN) tariffs for non-major trade partners to obtain plausibly exogenous variation in trade policy. Such tariff changes can arguably be viewed as unanticipated permanent shocks, and thus proxy for structural shocks in our model environment.

For simplicity, we assume that $\Theta_i = \Theta$ for all i and therefore that there is no heterogeneity of the trade elasticity by industry. This restriction can be relaxed by repeating the estimation with industry-specific estimates for each industry i. Estimation of the vector of structural parameters Θ_i is therefore boiled down to solving a GMM problem with the moment conditions $\{m_{sdi,t}^h\}_{h\in\mathcal{H}}$ for some set $\mathcal{H}$ of horizons with cardinality of at least three.[8]

4.2 Implementation and results

In principle, the set of horizons involved in estimation can be as large as what the data allow. Yet, for the baseline estimation, we restrict our attention to just-identified GMM and pick three horizons from the range between 1 and

[7] The adjustment of coefficient by 1 in $\epsilon_i^h\left(\Theta_i\right) - 1$ arises because the trade data employed are tariff exclusive while the model-consistent definition for trade elasticity requires including the changes in tariff payments in trade volumes.

[8] Here, we capitalize on the fact that the estimating equation is linear in independent variables. Consequently, the Frisch-Waugh logic continues to apply and we can residualize the model before running standard GMM.

Table 1: Parameter Estimates for Trade Elasticity

Parameter	Targeted Moments		
	$h = 1, 5, 10$ (Baseline)	$h = 1, 4, 10$	$h = 1, 4, 5, 10$
Long-run Trade Elasticity (θ)	3.24	2.67	2.90
	(2.31)	(1.00)	(1.44)
Short-run Trade Elasticity (σ)	1.35	1.27	1.32
	(0.41)	(0.44)	(0.42)
Supplier adjustment rate per annum (ζ)	0.10	0.15	0.12
	(0.12)	(0.13)	(0.11)

Notes: Parameter estimates for θ, σ and ζ are based on GMM. Standard errors in parenthesis are cluster-robust at the level of importer-exporter-HS4 triplets.

10 to identify our structural parameters, namely $h = 1$, 5, and 10. This choice of horizons is a natural baseline, given that the parameter σ largely governs the empirical behavior of the trade elasticity in the short-run ($h = 1$), while θ_i pins down its long-run level ($h = 10$). The transition from the short-run to the long-run level informs ζ_i.

Table 1 presents our estimates of the structural parameters governing the trade elasticity. Estimates in the first column are those to be used for the quantification exercise in Section 5.2. For the other columns, we alter the set of horizons covered by the moment conditions for the GMM estimator. Our estimate for the Fréchet parameter implies that the long-run tariff-inclusive elasticity of trade flows to exogenous tariff changes is between -3.24 and -2.7. The estimates for σ imply that existing intermediate-good varieties are net substitutes from the perspective of assemblers, but suggest limited scope for expenditure switching when adjustment happens at the intensive margin. Finally, we find that the adjustment rate for supplier relationships ζ ranges between about 10% and 15% per annum, indicating substantial stickiness in supplier relationship.

Given estimates for Θ, it is straightforward to translate them into estimates for model-implied horizon-specific elasticity using the expression from Proposition 2 and the delta method. Figure 1 plots 10 of these horizon-specific trade elasticity estimates based on the baseline parameter estimates in the first column of Table 1. Notice that the three structural parameters are governing all 10 horizon-specific elasticity estimates. To get a sense on how well such model-implied elasticity estimates fit the data, Figure 1 additionally includes 10 elasticity estimates that are obtained without imposing the structural restrictions based on Proposition 2. That is, a separate parameter ϵ^h is estimated for each individual horizon h under a linear GMM framework based on the same sample and IV. The results from this unrestricted version look similar to our model-based approach that only involves three parameters, indicating that our structural estimates fit the data reasonably well.

5 Quantitative Applications

We illustrate the features of the model in two quantitative applications: the 2018 US-China trade war and the 2004 Enlargement of the European Union (EU). We treat the former as an example of an unanticipated trade shock that highlights how our model diverges from the canonical EK benchmark by explicitly incorporating transitional dynamics. The latter was a pre-announced accession process for which both new member states (NMS) and EU

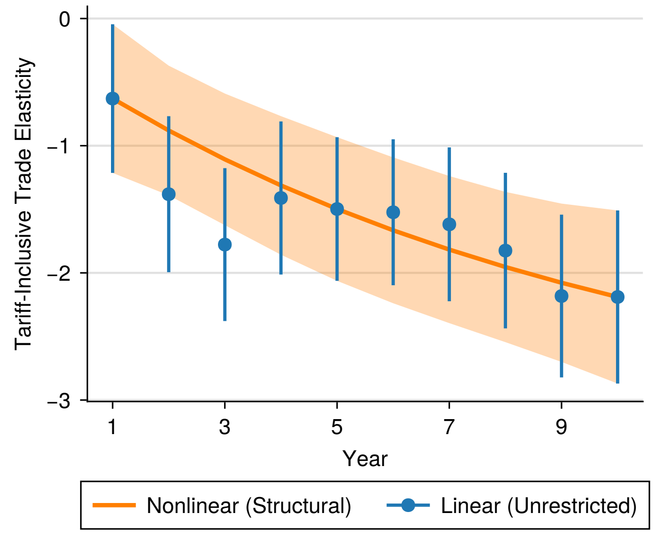

Figure 1: Horizon-Specific Trade Elasticity

Notes: The nonlinear estimates are translated from the three structural parameter estimates from the first column in Table 1 using Proposition 2 and the delta method. The linear estimates are obtained by allowing a separate parameter for each horizon without imposing any structural restrictions among them.

countries knew many years before the actual accession date what the liberalization schedule looked like, giving rise to anticipatory behavior and allowing us to compare model responses with and without anticipation.

We recognize that in episodes of global trade integration or disintegration other dynamics may come into play, including corporate reallocations and multinational production shifts (Ding et al. 2022, Arkolakis, Eckert and Shi 2023, Freund et al. 2024), migration (Caliendo et al. 2021), and innovation (Góes and Bekkers 2022). We hold those aspects constant and quantify welfare changes induced by tariff changes in the presence of anticipation and sourcing frictions.

5.1 Calibration of the initial steady state

To calibrate the model, we assume that the world economy is in a steady state prior to the announcement of a shock. As mentioned, we use the 2023 edition of the OECD Inter-Country Input-Output (ICIO) tables (OECD 2023). The ICIO database covers 45 industries in 76 economies along with a constructed rest of the world (ROW). In the model, we allow 77 economies corresponding to those in the data.[9] From the 45 ICIO industries,

[9]For China (mainland) and Mexico, the data separately record the input-output relations for a subset of manufacturing activities only intended for export. To take advantage of this additional detail for China and Mexico in the model, we implement these two economies as consisting of two types of producers for each industry. Concretely, for each industry-specific good in these two economies, there is a set of regular producers delivering output for both domestic and foreign use; an additional set of producers produce special varieties that are only delivered abroad. The technological parameters including those governing trade shares are allowed to be different across these two types of producers. However, the value added generated from all these producers is pooled for computation of aggregate income. Labor inputs are assumed to be perfectly mobile across the two types of producers. The producers therefore face possibly different prices for intermediate inputs but identical wages.

Table 2: Model Parameters and Variable Levels for Initial Steady State

Parameters or Initial Levels	Notation	Level of Variation
Matching Input-Output Data Exactly		
Producer expenditure shares across inputs	$\alpha_{sij}, \alpha_{sj}$	Producer region-industry
Initial import shares by source region	λ_{sdi}	User region
Initial level of bilateral trade flows	X_{sdi}	Industry-specific bilateral pair
Household expenditure shares across industries	η_{dj}	User region
Derived from Model Equilibrium		
Initial aggregate labor income and profits	$w_s L_s, \Pi_{si}$	Producer region
Deficit (difference between expenditure and income)	D_{di}	User region-industry

we exclude three that are primarily for public expenditure or services that are hard to classify. We then aggregate the remaining 42 industries into 32 industries by combining non-manufacturing industries. Table 2 summarizes the parameters and initial levels for our calibration.

We set the technological parameters $\{\{\alpha_{sij}\}_i, \alpha_{sj}\}_{sj}$ so that the expenditure shares across production inputs match those in the data exactly.[10] We match the initial import shares $\{\lambda_{sdi}\}_{sdi}$ and intermediate expenditure $\{X_{sdi}\}_{sdi}$ exactly. We ensure that the aggregate expenditure (total trade flows) for a given region-industry pair $\{X_{si}\}_{si}$ are as in the data before imposing the markup charged by the traders in model.

To derive final goods consumption while maintaining the accounting identities, we proceed as follows. We first compute the expenditure shares across the final use of goods from each industry among the 32 aggregated industries in order to obtain the household expenditure shares η_{si}. We then compute the aggregate wage and profit income received by households which can be derived from the aggregate expenditures of each country-industry pair and technological parameters.[11] Lastly, we back out hypothetical trade deficits that ensure all equilibrium relations to hold. These trade deficits enter into the resource constraints as discrepancies between the level of household consumption and goods arrived in the destination country for each industry good so that the η_{si} from data can be respected while maintaining the model structure.

5.2 The 2018 US-China Trade War

Measuring the shock. On 1 March 2018, the Trump administration announced global tariffs on steel (25%) and aluminum (10%), citing national security concerns – these would go into effect after temporarily exempting some allied countries. In April 2018, China would retaliate imposing tariffs on agricultural products and aluminum waste[12]. By September 2018, average applied tariffs had increased from 3.8 percent to 12.0 percent, covering 56% of total Chinese imports from the US; and from 7.2 percent to 18.3 percent, covering 47% of US imports

[10] The ICIO tables account for taxes and subsidies. We treat taxes and subsidies as special expenditures that are not contributing to any part of disposable income. For this reason, the sum of expenditure shares across inputs is smaller than one.

[11] With the region-industry specific value added shares at hand, we compute the initial levels of aggregate labor income as $w_d L_d = \sum_{i \in \mathcal{I}} (1 - \alpha_{di}) \cdot (\sigma_i - 1)/\sigma_i \cdot X_{di}$, which is our model's labor market clearing condition. We compute trader profits based on the markup and total value of industry goods arriving in each destination.

[12] See blog post by Zhiyao (Lucy) Lu and Jeffrey J. Schott on the Peterson Institute for International Economics website: *How Is China Retaliating for US National Security Tariffs on Steel and Aluminum?* link.

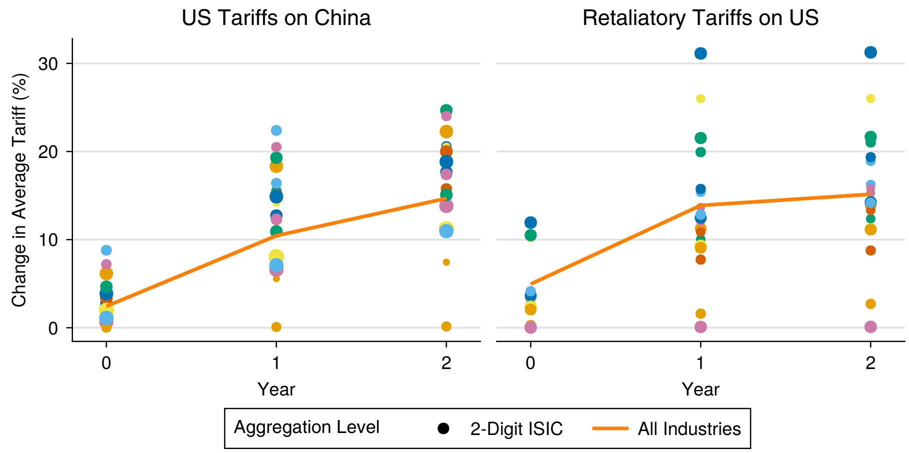

Figure 2: Tariff Changes for Trade War Counterfactuals

from the China, respectively (Bown 2021). By late 2019, average applied tariffs would have stabilized at around 20%, covering between 58-66% of bilateral trade flows.

The time span between the announcement of tariff increases and its implementation was short, often just a few months. Additionally, with a tariff distribution across sectors that ranges from 0-30%, depending on the sector and initial tariff, we take this episode as a largely unanticipated trade shock.

The timing of our anticipated shock experiment works as follows: prior to the announcement, the system is at its steady state, calibrated according to what was described in Section 5.1; on 2018, the US and China reveal the tariff rates they will impose against each other, with increases in 2018 through 2020. We then trace out the transition dynamics that happen after the tariff shock until convergence to a new steady state.

Tariff changes during the US-China trade war are from Fajgelbaum et al. (2020). Tariffs are reported at an eight-digit Harmonized System (HS) code level. We compute weighted averages of these tariff changes within each of the model industries over time. Tariff changes are aggregated across different HS codes and over the months when they take effect. For aggregation across product categories, we determine the most relevant model industry based on the associated industry classification and use the annual bilateral trade volume of each product in 2017 as weight. Fajgelbaum et al. (2020) offer aggregates over months, using the shares of months within a year for which the tariff changes are in effect as weights. Figure 2 shows the evolution of tariffs imposed by either party at a 2-digit ISIC level. Tables C.1 and C.2 in Appendix C.3.1 present the underlying data.[13]

Effects on trade flows. We begin with a description of how the trade war impacted trade flows between the United States and China. Figure 3 depicts the counterfactual response of imports, in tariff-inclusive terms and aggregated across industries. Tariff-inclusive import values drop slightly in the first periods, reflecting the low degree of substitutability among suppliers implied by the short-run structural trade elasticity. However, as the

[13]For model calibration, we rely on the OECD ICIO data, which reconcile trade data with national accounts To aggregate tariff changes, we use the 8-digit HS-code level trade data from the US Census as weights. Given discrepancies in industry classifications between OECD ICIO and the US Census, there are some discrepancies in import and export volumes. Many products were targeted by trade-war tariffs only during the second half of 2018, so the aggregate changes at the annual level in 2018 are smaller than those in 2019. By the end of 2020, all tariff changes associated with the trade war were in place.

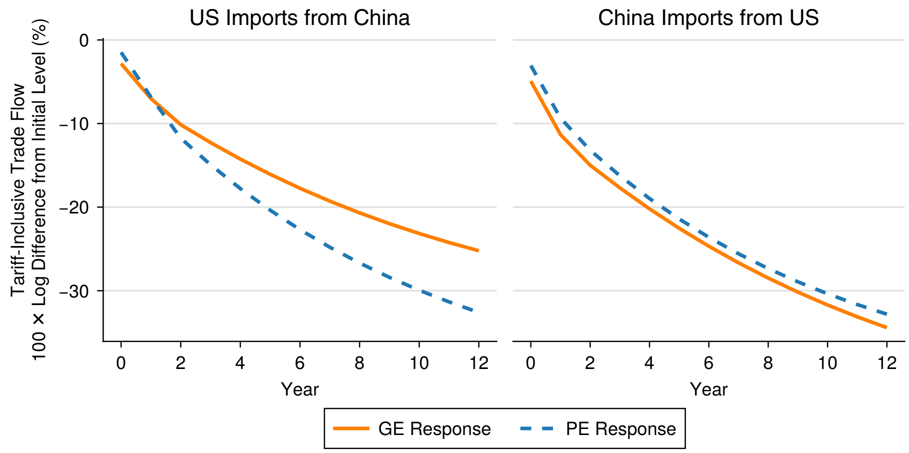

Figure 3: Changes in Tariff-Inclusive Trade Flows

Notes: Tariff changes implemented gradually over the first two years. The model determines changes in trade shares at the industry level. The country-level outcomes are based on aggregate trade flows summed across industries. "GE Response" and "PE Response" refer to results generated from the full model involving factor price changes and results only based on the PE trade elasticity respectively. "Regular Sectors" and "Export-Only Sectors" are only relevant to China and Mexico for according ICIO tables, as explained in Section 5.1.

trade elasticity increases over time, trade flows start to decline sharply. In the long-run, US imports from China settle at a level that is 25 percent lower than in 2017, while Chinese imports from the US decrease by over 30 percent. As we will describe in further detail below, this discrepancy in the magnitude of trade adjustment in part reflects differences in how shifts in world factor prices brought about by the trade war affect production and trade.

Effects on prices. Figure C.1 in Appendix C.3.1 depicts the industry-level price effects of the trade war in China and the United States. In the US, the increase in trade costs leads to an increase in prices across all industries and time horizons. This pattern is qualitatively consistent with empirical studies that found that products targeted by retaliatory tariffs from China experienced a high pass-through to prices in the United States [14]. Due to sluggish adjustment of demand, these price increases are particularly pronounced in the short-run, when traders could not yet adjust the extensive margin. Some industries—notably textiles, basic metals, and electrical equipment—see prices rise by over 4 percent once all retaliatory tariffs are in place. As sourcing decisions gradually adjust to the initial rise in trade cost, prices partially decline but remain high.

In contrast to the substantial and uneven price hikes in the United States, domestic prices in China decline across all industries. Price changes are smaller in absolute terms in the short-run, reflecting the limited scope for demand reallocation in the short-run. There are many potential explanations for these divergences. Importantly, we simulate both US imposed tariffs and Chinese retaliatory tariffs simultaneously – and tariffs imposed by the

[14]Fajgelbaum and Khandelwal (2022) is a comprehensive review of the economic effects of the US-China trade war. The bulk of the evidence shows that the pass-through of retaliatory tariffs to import prices was complete in the Trade War episode. Fajgelbaum et al. (2020), Amiti, Redding and Weinstein (2019), Amiti, Redding and Weinstein (2020), and Cavallo et al. (2021) all estimate a complete pass through.

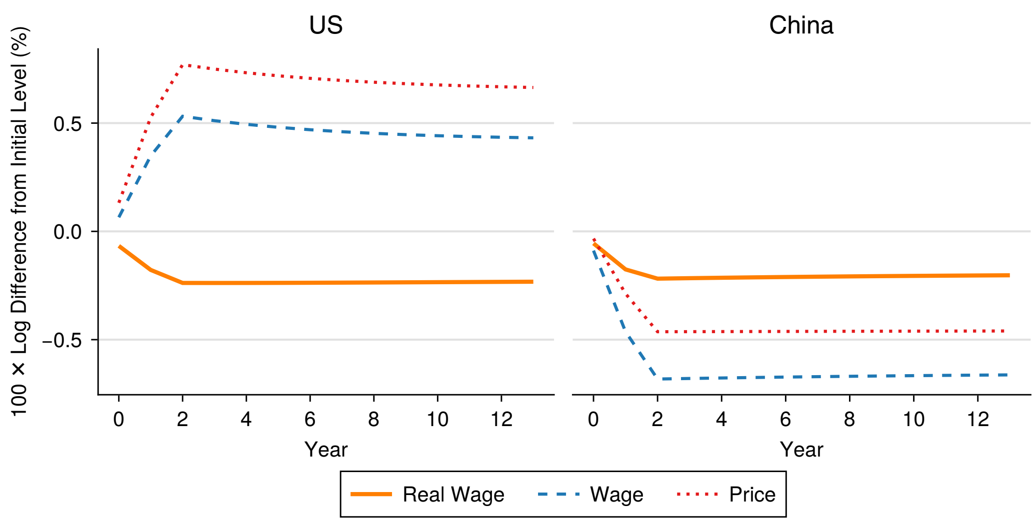

Figure 4: Changes in Real Wages, Wages and Consumer Prices

Notes: Tariff changes implemented gradually over the first two years. "Real Wage" in year t refers to real wage changes between t and the initial steady state generated by the full model. "Wage" refers to the corresponding change in the nominal wage. "Price" is the change in the aggregate consumer price index.

US affected a larger share of imports from China than the retaliatory tariffs affected US exports to China[15]. Trade diversion to the domestic market is likely to decrease domestic prices.

Figure C.2 shows that the price responses of industries not directly exposed to tariff changes follow the same qualitative patterns as those described above. This outcome reflects two main channels: the impact of the trade war on both intermediate goods prices, since our model includes an input-output structure, and domestic factor prices. We further elaborate on the latter below.

Effects on real wages and welfare. Figure 4 traces the counterfactual response of real wages, as well as nominal wages and consumer prices in the United States and China. In the long run, the trade war reduces the real wage in both countries—albeit for different reasons.

In the United States, the trade war increases prices and nominal wages. However, price levels increase more than nominal wages, decreasing real wages. Conversely, in China, both nominal wages and prices decrease. The counterfactual decrease in nominal wages is stronger than that of prices, leading real wages to decrease. In both countries, the real wage responds gradually, achieving about a -0.24 percent decrease for the United States and -0.22 for China over the medium term.

In Figure 4, we compare the transitory dynamics of real wages in our baseline economy to the equilibrium response of real wages in a variant of the model with equivalent fundamentals but instantaneous trade adjustment ($\zeta = 1$).[16] In both countries, the presence of sticky sourcing and sluggish adjustment exacerbate real wage losses

[15]The U.S. "raised tariffs on import transactions corresponding to about 2.6% of GDP [...] trade partners imposed retaliations on exports corresponding to about 1% of US GDP" Fajgelbaum and Khandelwal (2022)

[16]From Proposition 1 we know that shocks have identical long-run effects in both economies but cause transitory dynamics in the

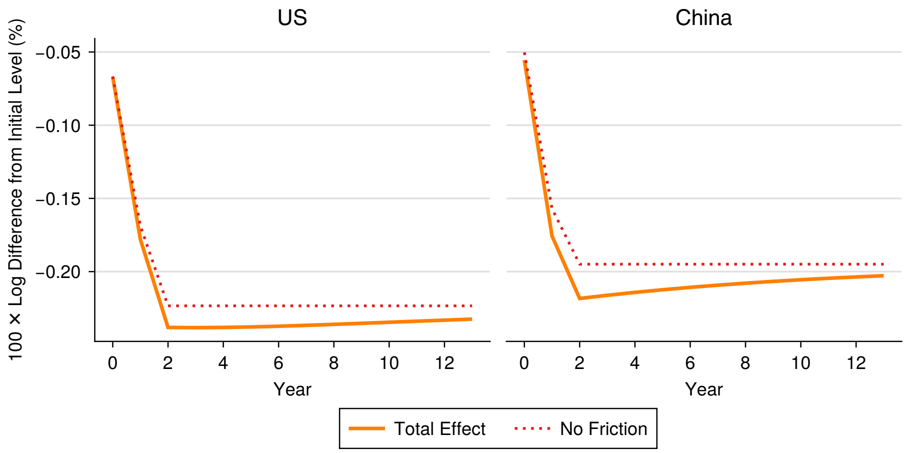

Figure 5: Horizon-Specific Impact on Real Wages

Notes: Tariff changes implemented gradually over the first two years. "Total Effect" in year t refers to the real wage change between t and the initial steady state generated by the full model. "No Friction" refers to the change in real wages under the assumption that the economy reaches the long-run outcomes instantly ($\zeta = 1$).

over the transition path.

In Figure 5 we use Proposition 4 to further elucidate how adjustment frictions contribute to the transitory dynamics of real wages. We present the changes in welfare inferred from trade shares in our augmented ACR formula. We plot in the dotted line the changes in welfare in the usual ACR formula, setting $\Xi = 1$ and contrast it to the changes in welfare in the presence of frictions. Intuitively, most of the deviations of welfare from its long-run equilibrium are explained by frictions. Additionally, these deviations are larger over the short- and medium-term. In the long run, as trade flows gradually realign and start to be driven increasingly by comparative advantage, price distortions ultimately fully reflect the welfare effects of the trade war for US households, while ameliorating the shot-term excess impact on real wages in China.

Effects on third-party countries. We conclude the discussion of the simulations by highlighting the importance of short-run adjustment frictions for assessing the welfare effects of the US-China trade war in third-party countries. In Figure 6, we display the counterfactual real income responses in Mexico and Vietnam. Figure C.3 in the Appendix depicts the underlying aggregate price and wage responses.

Due to trade diversion, both countries ultimately stand to benefit from the US-China trade war. However, traders are unable to freely adjust their extensive margins. Therefore, they have to adjust down their intensive margins over the short-run and these countries face short-run losses despite long-run gains.

These initial income losses are substantial and, in the case of Mexico, the largest among all third-party countries. Over time Vietnam's real wage increases by more than 0.15 percent between steady states—a magnitude that is comparable in absolute terms to the corresponding losses borne by households in China and the United

baseline economy. The fact that the impact of the trade war on the friction-less economy is smaller during the first two years reflects the fact that the tariff changes are not fully implemented until the end of 2019.

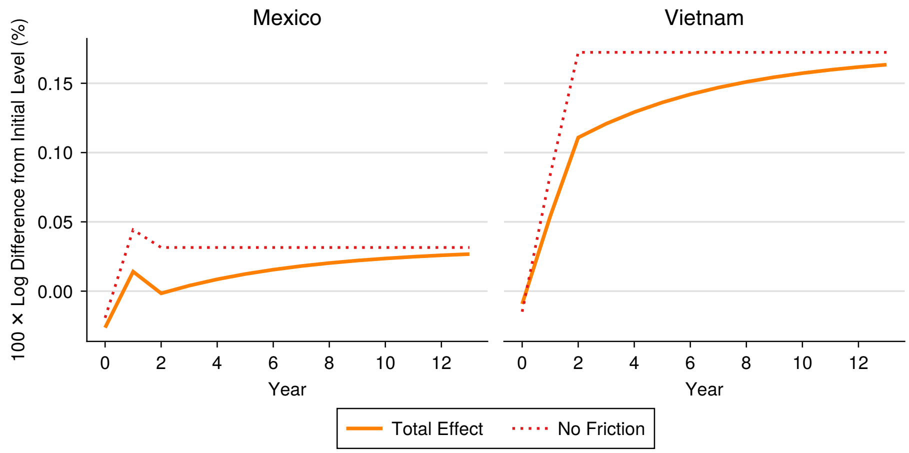

Figure 6: Welfare Impact in Mexico and Vietnam

Notes: Tariff changes implemented gradually over the first two years. "Total Effect" in year t refers to the real wage change between t and the initial steady state generated by the full model. "No Friction" refers to the change in real wages under the assumption that the economy reaches the long-run outcomes instantly ($\zeta = 1$).

States.

The cases of Mexico and Vietnam also illustrate a broader point: the welfare effects of trade disruptions can *change sign over time* when adjustment is subject to frictions. Adjustment frictions imply that third-party countries stand to gain little from bilateral trade disruptions in the short-run when supply relationships are sticky and respond little to shocks. In the long-run, however, some countries stand to benefit from the realignment of supply relationships. In the case of Mexico and Vietnam, this realignment leads to sustained increases in domestic factor prices, reflecting both the reallocation of US and Chinese demand as well as their favorable positions in the international production network.

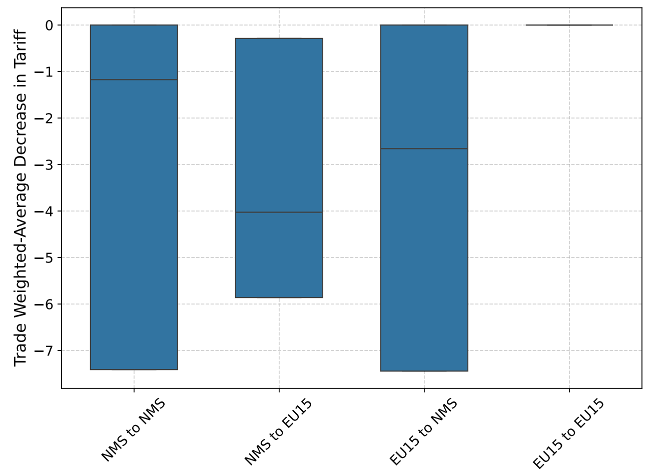

Figure 7: Distribution of Tariff Shocks Across Source-Destination-Industry Tuples

Notes: Interquartile range of tariff shock source-destination-industry tuples. Data are from WITS with MFN and Preferential Tariff filled according to the procedure described in the Appendix. Whenever data for 1995 were missing, we used the closest year to 1995 available in the sample. More detailed summary statistics can be found in Table C.3 in the Appendix.

5.3 The 2004 Enlargement of the European Union

Measuring the shock. On May 1st 2004, ten countries joined the European Union (EU): Czechia, Estonia, Cyprus, Latvia, Lithuania, Hungary, Malta, Poland, Slovenia, and Slovakia (the New Member States or "NMS" for short). In order to become a member state, a country has to adhere to the corpus of the legislation of the EU, called the *acquis communautaire*. After initial approval[17], the candidate country and the EU start negotiations over a schedule to adhere to European legislation. As part of this process, the NMS have signed association agreements with the EU that established a tariff liberalization schedule about a decade before their accession in 2004[18]. Therefore, the EU Enlargement is a long, preannounced process. This is a useful scenario to illustrate the forward looking aspects of our dynamic trade model.

The timing of our anticipated shock experiment works as follows: prior to the announcement, the system is at its steady state, calibrated according to what was described in Section 5.1; on 1995, the EU and NMS governments announce that on 2004, bilateral tariff rates across all members of the will go to zero, remaining unchanged between 1995 and 2004[19]. We then trace out the purely anticipatory responses – induced entirely by the announcement of the tariff changes and realized before the tariff actually changes; and the transition dynamics

[17]"A country that wishes to join the EU addresses its application to the Council, which asks the Commission to submit an opinion. Parliament is notified of this application. If the Commission's opinion is favourable, the European Council may decide – by unanimity – to grant the country candidate status. Following a recommendation by the Commission, the Council decides – again by unanimity – whether negotiations should be opened." Source: `https://www.europarl.europa.eu/factsheets/en/sheet/167/the-enlargement-of-the-union`.

[18]Each NMS signed an association "Europe Agreement" between 1991 and 1996. For instance, Czechia and Estonia signed their Europe Agreements on 4 October 1993 and 12 June 1995, respectively. The full list of Agreements are available on EUR-lex.

[19]We chose to keep tariff rates constant between 1995 and 2004 rather than feed in the whole liberalization schedule between those countries in order to be able to disentangle the anticipatory responses from those induced by actual tariff changes.

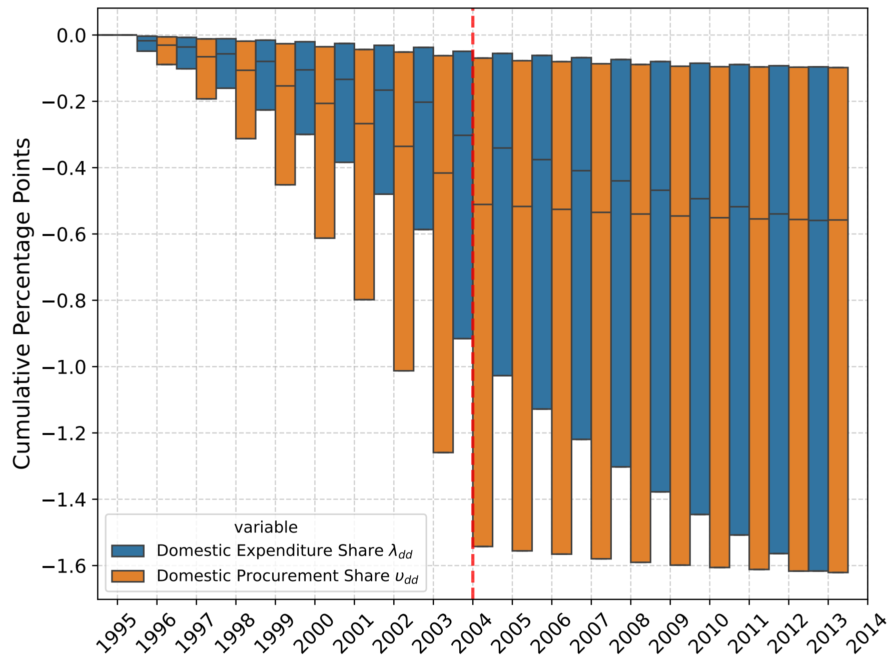

Figure 8: New Member States: Cumulative Changes in Domestic Expenditure Shares and Domestic Procurement Shares, in percentage points

Notes: The Figure shows the interquartile range of changes in domestic expenditure shares $\{\lambda_{ddi,h}-\lambda_{ddi,1995}\}$ and domestic procurement shares $\{\upsilon_{ddi,h}-\upsilon_{ddi,1995}\}$ across industries i and destinations d, for each horizon $h \in \{1995, \cdot, 2014\}$.

that happen after the tariff shock arrives until convergence to a new steady state.

Figure 7 plots the distribution of tariff shocks fed into to the model. These consist of a reduction of tariffs from the baseline year (1995) to zero. NMS experience tariff drops with respect to the EU but also among themselves. Further summary statistics can be found on Table C.3 in Appendix C.2. In the remainder of this section, we explore how these shocks pre-announced shocks induce anticipatory adjustments.

Effects on trade flows. We first explore how the anticipated shock affects sourcing of varieties and import shares before and after the tariff changes actually happens. Figure 8 shows how the share of varieties and import shares evolve after the shock is announced. Note that there are relevant anticipatory dynamics, as the share of varieties and import shares move much before the shock is actually realized.

Outside of the steady state, the share of varieties imported from a given source-industry pair $\upsilon_{sdi,t}$ will in general differ from the expenditure shares on imported varieties source-industry pair $\lambda_{sdi,t}$. They will only be equal if the option values are the same across all sources. The intuition is that whenever the option value is sufficiently high, traders will be willing to choose a source country s even if varieties from that source are not the cheapest and hurt their short-term flow profits. Therefore, in anticipated shocks, changes in the share of varieties $\upsilon_{sdi,t}$ will be weakly larger than changes in expenditure shares $\lambda_{sdi,t}$ before the shock arrives.

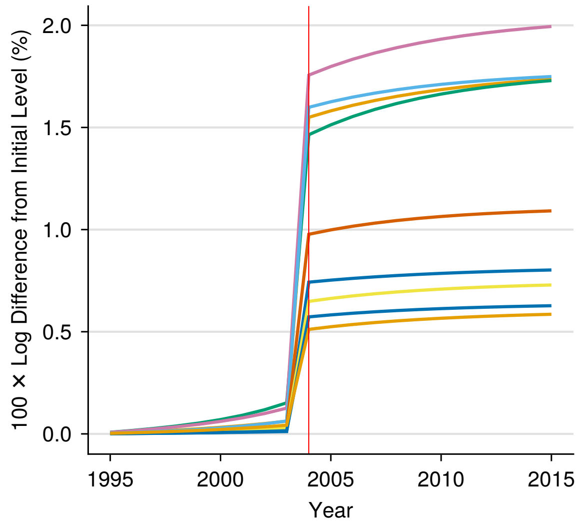

Figure 9: Changes in Real Wages Among NMS

In Figure 8, we plot the cumulative changes in domestic expenditure shares $\{\lambda_{ddi,h} - \lambda_{ddi,1995}\}$ and domestic procurement shares $\{\upsilon_{ddi,h} - \upsilon_{ddi,1995}\}$ across industries i and destinations d, for each horizon $h \in \{1995, \cdots, 2014\}$. Two facts stand out. First, there is non-negligible movement before the tariff changes, with a median cumulative drop of expenditure and procurement shares of -0.13 and -0.25 percentage points, respectively, by 2003. Second, procurement shares move before expenditure shares. Intuitively, assemblers solve a static problem and shift demand from varieties with higher relative prices while traders might find it optimal to forgo current period profits for higher anticipated future profits. To see that, note that $\frac{\upsilon_{sdi,t}/\lambda^0_{sdi,t}}{\upsilon_{kdi,t}/\lambda^0_{kdi,t}} = \frac{1+\Psi_{sdi,t}}{1+\Psi_{kdi,t}}$, such that relative deviations between the share of varieties and expenditure shares currently being resourced depend only on the ratio of option values. As the system moves towards the new steady state, the two converge to the same level.

Effects on real Wages and welfare. A perhaps surprising outcome of our experiment is that, despite much movement in trade flows, real wages among NMS do not change substantially before the trade flows arrive. As seen in Figure 5.3, real wages increase slightly prior to the tariff change and increase sharply when the measure is actually implemented.

These results underline that observed changes in trade flows are insufficient to infer the welfare gains from trade in our model. Proposition 4 shows how the ACR decomposition needs to be adjusted by the terms $(\Xi_{dj,h})^{1/(\sigma_j-1)}$, which accounts for the deviations between observed expenditure shares and their optimal levels if they were to be freely procured from the cheapest source.

Our model hence implies that if observed domestic trade shares were decrease but welfare does not move, then it must be the case that the terms $(\Xi_{dj,h})^{1/(\sigma_j-1)}$ are moving in the opposite direction. To illustrate that fact, we plot in Figure 5.3 a decomposition of this sort.

In blue, we show what one would estimate the changes in welfare to be if we evaluated the observed changes in domestic expenditure shares with the traditional ACR formula. The dotted orange shows welfare adjustment

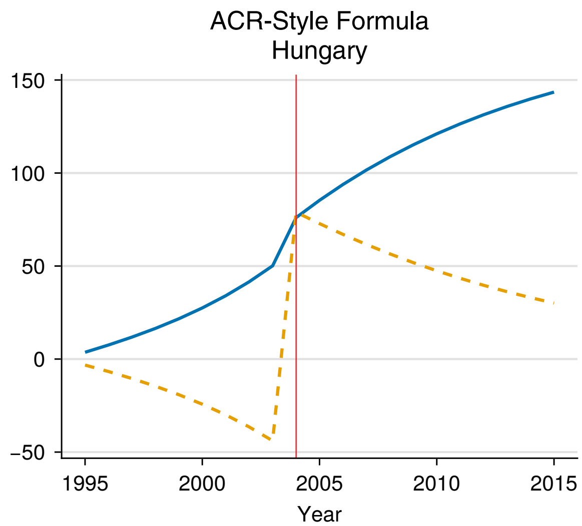

Figure 10: Decomposition of Real Wage Changes for Hungary

necessary due to frictions to map observe domestic expenditure shares to welfare. As observed, the adjustment term is negative ($\Xi_{dj,h} < 1$ for some j) before the tariff changes happen and turns positive thereafter.

The intuition is the following: before the the tariff decreases, the traders in the destination country are procuring from countries that do not necessarily have the cheapest variety. Therefore, while it looks as if welfare is improving, due to decreases in domestic expenditure shares, that is not the case.

6 Concluding Remarks

To account for imperfect adjustment to global supply chain shocks, we develop a Ricardian trade framework with frictions that result from staggered decisions of producers to change global suppliers. We obtain extended formulas for welfare changes to trade openness and trade shocks over time, derive novel estimation equations for a horizon-specific trade elasticities, and quantify the model. Counterfactual experiments of the US-China trade in 2018-19 war and the Eastern enlargement of the EU in 2004 suggest that rich sectoral dynamics ensue, resulting in considerable short-term reallocations and substantive welfare fluctuations at variance with long-term welfare predictions for economies with no sourcing frictions.

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Appendix A Model Details and Proofs

A.1 Ideal Price Indexes and Generic Trade Shares

The composite good in industry j is

$$Y_{dj,t} \equiv \left(\int_{[0,1]} y_{dj,t}(\omega)^{\frac{\sigma_j - 1}{\sigma_j}} \mathrm{d}\omega \right)^{\frac{\sigma_j}{\sigma_j - 1}}.$$

Product space $\Omega_j = [0,1]$ can be exhaustively partitioned into disjoint sets with $\Omega_j = \bigcup_{k=0}^{\infty} \Omega_{dj,t}^k$, so we can rewrite the composite good as

$$Y_{dj,t} \equiv \left(\sum_{k=0}^{\infty} \int_{\Omega_{dj,t}^k} y_{dj,t}(\omega)^{\frac{\sigma_j - 1}{\sigma_j}} \mathrm{d}\omega \right)^{\frac{\sigma_j}{\sigma_j - 1}}. \tag{A.1}$$

The assembler's associated cost minimization problem is

$$\begin{aligned} \min_{\{y_{dj,t}(\omega)\}_{\omega \in \Omega_{j,t}}, \{Y_{dj,t}^k\}} P_{dj,t} Y_{dj,t} \quad &= \quad \sum_{k=0}^{\infty} P_{dj,t}^k Y_{dj,t}^k \\ s.t. \qquad & Y_{dj,t} = \left[\sum_{k=0}^{\infty} \left(Y_{dj,t}^k \right)^{\frac{\sigma_j - 1}{\sigma_j}} \right]^{\frac{\sigma_j}{\sigma_j - 1}}, Y_{dj,t}^k \equiv \left(\int_{\Omega_{dj,t}^k} y_{dj,t}(\omega)^{\frac{\sigma_j - 1}{\sigma_j}} \mathrm{d}\omega \right)^{\frac{\sigma_j}{\sigma_j - 1}}, \\ & P_{dj,t}^k Y_{dj,t}^k = \int_{\Omega_{dj,t}^k} p_{dj,t}(\omega) y_{dj,t}(\omega) \, \mathrm{d}\omega, \end{aligned}$$

where we define the partial composite good $Y_{dj,t}^k \equiv (\int_{\Omega_{dj,t}^k} y_{dj,t}(\omega)^{(\sigma_j - 1)/\sigma_j} \mathrm{d}\omega)^{\sigma_j/(\sigma_j - 1)}$ for each partition k as a helpful construct for derivations and implicity define the associated partial ideal price index $P_{dj,t}^k$ that satisfies $P_{dj,t}^k Y_{dj,t}^k = \int_{\Omega_{dj,t}^k} p_{dj,t}(\omega) y_{dj,t}(\omega) \mathrm{d}\omega$.

Under homotheticity of the assembler's production function, this problem can be solved in two steps. First, the assembler decides which share of cost it allocates to each partial composite good $Y_{dj,t}^k$. Given those choices, the assembler then decides the optimal cost for each intermediate good $y_{dj,t}(\omega)$. Optimal demand satisfies

$$Y_{dj,t}^k = \left(\frac{P_{dj,t}^k}{P_{dj,t}} \right)^{-\sigma_j} Y_{dj,t} \quad \text{and} \tag{A.2}$$

$$y_{dj,t}^k(\omega) = \left(\frac{p_{dj,t}(\omega)}{P_{dj,t}^k} \right)^{-\sigma_j} Y_{dj,t}^k = \left(\frac{p_{dj,t}(\omega)}{P_{dj,t}} \right)^{-\sigma_j} Y_{dj,t} \quad \text{for each } \omega \in \Omega_{dj,t}^k, \tag{A.3}$$

where the last equality also shows that the partitioned solution equals the standard solution under a constant elasticity of substitution. Replacing the demand functions above in the definition of the budget constraint delivers the ideal price indices:

$$P_{dj,t} = \left(\int_{[0,1]} p_{dj,t}(\omega)^{1-\sigma_j} \mathrm{d}\omega \right)^{\frac{1}{1-\sigma_j}}, \qquad P^k_{dj,t} = \left(\int_{\Omega^k_{dj,t}} p_{dj,t}(\omega)^{1-\sigma_j} \mathrm{d}\omega \right)^{\frac{1}{1-\sigma_j}}. \tag{A.4}$$

We have now established that partitioning the product space into disjoint sets results in well-behaved demand functions such that, given optimal choices within each set, we can analyze demand for each intermediate good independently and then aggregate. In subsequent derivations, expenditure shares within each partition k will play a crucial role, so we state a general definition here:

$$\lambda^k_{sdj,t} \equiv \frac{X^k_{sdj,t}}{X^k_{dj,t}} = \frac{\int_{\Omega^k_{dj,t}} \mathbf{1}_d \left\{ s \text{ is } \omega\text{'s source country} \right\} p_{dj,t}(\omega) y_{dj,t}(\omega)\, \mathrm{d}\omega}{\sum_n \int_{\Omega^k_{dj,t}} \mathbf{1}_d \left\{ n \text{ is } \omega\text{'s source country} \right\} p_{dj,t}(\omega) y_{dj,t}(\omega)\, \mathrm{d}\omega}. \tag{A.5}$$

A.2 Optimal Sourcing

A.2.1 Value functions

For $k = 0$, traders have the option to readjust their extensive margin decisions. Recall from Section 2.2.1 that a trader's value function is

$$V_{di,t}(\omega) = \max_{n \in \mathcal{N}} V_{di,t}(\omega, n), \tag{A.6}$$

where

$$V_{di,t}(\omega, n) = \pi_{ndi,t}(\omega) + \frac{\zeta_i}{1+r_t} V_{di,t+1}(\omega) + \frac{1-\zeta_i}{1+r_t} V_{di,t+1}(\omega, n), \text{ for } n \in \mathcal{N}, \tag{A.7}$$

is the net-present value of a trader in d that procures an intermediate good ω from a country n at time t. Equations (A.6) and (A.7) restate the value functions (9) and (10) from the main text. Substituting $V_{di,t+1}(\omega, n)$ into $V_{di,t}(\omega)$, we obtain

$$V_{di,t}(\omega) = \max_{n \in \mathcal{N}} \left\{ \pi_{ndi,t}(\omega) + \frac{\zeta_i}{1+r_t} V_{di,t+1}(\omega) + \frac{1-\zeta_i}{1+r_t} V_{di,t+1}(\omega, n) \right\}.$$

Iterate forward by one period

$$\begin{aligned} V_{di,t}(\omega) \quad = \quad & \max_{n \in \mathcal{N}} \left\{ \pi_{ndi,t}(\omega) + \frac{\zeta_i}{1+r_t} V_{di,t+1}(\omega) \right. \\ & \left. + \frac{1-\zeta_i}{1+r_t} \left(\pi_{ndi,t+1}(\omega) + \frac{\zeta_i}{1+r_{t+1}} V_{di,t+2}(\omega) + \frac{1-\zeta_i}{1+r_{t+1}} V_{di,t+2}(\omega, n) \right) \right\} \end{aligned}$$

and rearrange to obtain

$$\begin{aligned}V_{di,t}(\omega) &= \max_{n\in\mathcal{N}}\Bigg\{\pi_{ndi,t}(\omega)+\frac{1-\zeta_i}{1+r_t}\pi_{ndi,t+1}(\omega)\\ &\quad+\left(\frac{\zeta_i}{1+r_t}\right)V_{di,t+1}(\omega)+\left(\frac{\zeta_i}{1+r_{t+1}}\right)\left(\frac{1-\zeta_i}{1+r_t}\right)V_{di,t+2}(\omega)\\ &\quad+\left(\frac{1-\zeta_i}{1+r_t}\right)\left(\frac{1-\zeta_i}{1+r_{t+1}}\right)V_{di,t+2}(\omega,n)\Bigg\}.\end{aligned}$$

Similarly, iterating the value function forward up to period T results in

$$\begin{aligned}V_{di,t}(\omega) &= \max_{n\in\mathcal{N}}\Bigg\{\pi_{ndi,t}(\omega)+\sum_{\varsigma=1}^{T}\left(\prod_{\varsigma'=1}^{\varsigma}\frac{1-\zeta_i}{1+r_{t+\varsigma'-1}}\right)\pi_{ndi,t+\varsigma}(\omega)\\ &\quad+\sum_{\varsigma=1}^{T}\left(\frac{\zeta_i}{1+r_{t+\varsigma-1}}\right)\left(\prod_{\varsigma'=1}^{\varsigma-1}\frac{1-\zeta_i}{1+r_{t+\varsigma'-1}}\right)V_{di,t+\varsigma}(\omega)\\ &\quad+\left(\prod_{\varsigma=1}^{T}\frac{1-\zeta_i}{1+r_{t+\varsigma-1}}\right)V_{di,T}(\omega,n)\Bigg\}.\end{aligned}$$

Recall that we can express a trader's anticipated profits for procuring intermediate good ω from source country n as

$$\begin{aligned}\pi_{ndi,t+\varsigma}(\omega) &= \frac{1}{\sigma_i}\left(\frac{p_{ndi,t+\varsigma}(\omega)}{P_{di,t+\varsigma}}\right)^{-(\sigma_i-1)}P_{di,t+\varsigma}Y_{di,t+\varsigma}\\ &= \frac{1}{\sigma_i}\left(\frac{p_{ndi,t}(\omega)}{P_{di,t}}\right)^{-(\sigma_i-1)}P_{di,t}Y_{di,t}(\omega)\cdot\prod_{\varsigma'=1}^{\varsigma}\left(\frac{\hat{p}_{ndi,t+\varsigma'}(\omega)}{\hat{P}_{di,t+\varsigma'}}\right)^{-(\sigma_i-1)}\hat{P}_{di,t+\varsigma'}\hat{Y}_{di,t+\varsigma'}\\ &= \pi_{ndi,t}(\omega)\cdot\prod_{\varsigma'=1}^{\varsigma}\left(\frac{\hat{p}_{ndi,t+\varsigma'}(\omega)}{\hat{P}_{di,t+\varsigma'}}\right)^{-(\sigma_i-1)}\hat{P}_{di,t+\varsigma'}\hat{Y}_{di,t+\varsigma'}\\ &= \pi_{ndi,t}(\omega)\cdot\prod_{\varsigma'=1}^{\varsigma}\left(\frac{\hat{c}_{ndi,t+\varsigma'}}{\hat{P}_{di,t+\varsigma'}}\right)^{-(\sigma_i-1)}\hat{P}_{di,t+\varsigma'}\hat{Y}_{di,t+\varsigma'},\end{aligned}$$

where the last equality follows from the fact that

$$\hat{p}_{ndi,t+\varsigma}(\omega)\equiv\frac{p_{ndi,t+\varsigma}(\omega)}{p_{ndi,t+\varsigma-1}(\omega)}=\frac{\sigma_i}{\sigma_i-1}\frac{c_{ndi,t+\varsigma}}{z_{ndi}(\omega)}\left(\frac{\sigma_i}{\sigma_i-1}\frac{c_{ndi,t+\varsigma-1}}{z_{ndi}(\omega)}\right)^{-1}=\frac{c_{ndi,t+\varsigma}}{c_{ndi,t+\varsigma-1}}=\hat{c}_{ndi,t+\varsigma}.$$

We can therefore write the limit $T \to \infty$ of the value function as

$$\begin{aligned} V_{di,t}(\omega) \quad = \quad \max_{n \in \mathcal{N}} \Bigg\{ \pi_{ndi,t}(\omega) \left[1 + \sum_{\varsigma=1}^{\infty} \left(\prod_{\varsigma'=1}^{\varsigma} \frac{1-\zeta_i}{1+r_{t+\varsigma'-1}} \left(\frac{\hat{c}_{ndi,t+\varsigma'}}{\hat{P}_{di,t+\varsigma'}} \right)^{-(\sigma_i-1)} \hat{P}_{di,t+\varsigma'} \hat{Y}_{di,t+\varsigma'} \right) \right] \\ + \sum_{\varsigma=1}^{\infty} \left(\frac{\zeta_i}{1+r_{t+\varsigma-1}} \right) \left(\prod_{\varsigma'=1}^{\varsigma-1} \frac{1-\zeta_i}{1+r_{t+\varsigma'-1}} \right) V_{di,t+\varsigma}(\omega) \\ + \lim_{T \to \infty} \left(\prod_{\varsigma=1}^{T} \frac{1-\zeta_i}{1+r_{t+\varsigma-1}} \right) V_{di,T}(\omega, n) \Bigg\}. \end{aligned} \quad \text{(A.8)}$$

Finally, we invoke the transversality condition

$$\lim_{T \to \infty} \left(\prod_{\varsigma=1}^{T} \frac{1-\zeta_i}{1+r_{t+\varsigma-1}} \right) V_{di,T}(\omega, n) = 0$$

to ensure that the present anticipated continuation value of having source country n as the origin of variety ω vanishes infinitely many periods into the future. As stated in equation (12), we define the option value of a procurement relation between a trader in destination d and the variety producer in source country n with

$$\Psi_{ndi,t} \equiv \sum_{\varsigma=1}^{\infty} (1-\zeta_i)^{\varsigma} \left[\prod_{\varsigma'=1}^{\varsigma} \frac{1}{1+r_{t+\varsigma'-1}} \left(\frac{\hat{c}_{ndi,t+\varsigma'}}{\hat{P}_{di,t+\varsigma'}} \right)^{-(\sigma_i-1)} \hat{P}_{di,t+\varsigma'} \hat{Y}_{di,t+\varsigma'} \right].$$

The option value represents a trader's discounted anticipated profits from being locked-in with a supplier from n under the anticipated equilibrium path of prices. Using the transversality conditions and the option values for countries n in the value function above, we arrive at equation (11) in the text

$$V_{di,t}(\omega) \quad = \quad \max_{n \in \mathcal{N}} \left\{ \pi_{ndi,t}(\omega) \left(1 + \Psi_{ndi,t}\right) \right\} + \sum_{\varsigma=1}^{\infty} \left(\frac{\zeta_i}{1+r_{t+\varsigma-1}} \right) \left(\prod_{\varsigma'=1}^{\varsigma-1} \frac{1-\zeta_i}{1+r_{t+\varsigma'-1}} \right) V_{di,t+\varsigma}(\omega),$$

which only depends on n through flow profits $\pi_{ndi,t}(\omega)$ and the option value $\Psi_{ndi,t}$.

If we impose a time-invariant interest rate $r_t = r$ for all t, then the value function further simplifies to

$$V_{di,t}(\omega) \quad = \quad \max_{n \in \mathcal{N}} \left\{ \pi_{ndi,t}(\omega) \left(1 + \bar{\Psi}_{ndi,t}\right) \right\} + \frac{\zeta_i}{1-\zeta_i} \sum_{\varsigma=1}^{\infty} \left(\frac{1-\zeta_i}{1+r} \right)^{\varsigma} V_{di,t+\varsigma}(\omega)$$

with

$$\bar{\Psi}_{ndi,t} \equiv \sum_{\varsigma=1}^{\infty} \left(\frac{1-\zeta_i}{1+r} \right)^{\varsigma} \left[\prod_{\varsigma'=1}^{\varsigma} \left(\frac{\hat{c}_{ndi,t+\varsigma'}}{\hat{P}_{di,t+\varsigma'}} \right)^{-(\sigma_i-1)} \hat{P}_{di,t+\varsigma'} \hat{Y}_{di,t+\varsigma'} \right].$$

For $k > 0$, define $V^k_{di,t}(\omega, n)$ as the value function of a trader in destination d who sources intermediate good $\omega \in \Omega^k_{di,t}$ from the same supplier in source country n since time period $t-k$. $V^k_{di,t}(\omega, n)$ can be expressed in terms of past changes in trade cost, factor prices and demand as

$$V_{di,t}^{k}(\omega,n)=\pi_{ndi,t-k}(\omega)\left[\prod_{\varsigma=t-k+1}^{t}\left(\frac{\hat{c}_{ndi,\varsigma}}{\hat{P}_{di,\varsigma}}\right)^{-(\sigma_i-1)}\hat{P}_{di,\varsigma}\hat{Y}_{di,\varsigma}\right]+\frac{\zeta_i}{1+r_t}V_{di,t+1}(\omega)+\frac{1-\zeta_i}{1+r_t}V_{di,t+1}^{k+1}(\omega,n).$$

A.2.2 Sourcing probabillity

The second term in the value function (A.8) is independent of the trader's choice of source country at time t, and the third term vanishes by the transversality conditions. The trader's optimal choice of supplier $s_{di,t}^{*}(\omega)$ therefore solves

$$s_{di,t}^{*}(\omega)=\arg\max_{n\in\mathcal{N}}\left\{\pi_{ndi,t}(\omega)\left(1+\Psi_{ndi,t}\right)\right\}=\arg\min_{n\in\mathcal{N}}\left\{\frac{c_{ndi,t}}{z_{ni}(\omega)}\left(1+\Psi_{ndi,t}\right)^{-1/(\sigma_i-1)}\right\},$$

where flow profits are given by (8). The probability that a trader in country d sources an intermediate good $\omega\in\Omega_{di,t}^{0}$ from source country s hence equals

$$\begin{aligned}\upsilon_{sdi,t}^{0}&=\Pr\left[\arg\min_{n\in\mathcal{N}}\left\{\frac{c_{ndi,t}}{z_{ni}(\omega)}\left(1+\Psi_{ndi,t}\right)^{-\frac{1}{\sigma_i-1}}\right\}=s\right]\\&=\Pr\left[\frac{c_{sdi,t}\left(1+\Psi_{sdi,t}\right)^{-\frac{1}{\sigma_i-1}}}{z_{si}(\omega)}\leq\frac{c_{ndi,t}\left(1+\Psi_{ndi,t}\right)^{-\frac{1}{\sigma_i-1}}}{z_{ni}(\omega)}\ \forall n\neq s\right]\\&=\Pr\left[\frac{\nu_{sdi,t}}{z_{si}(\omega)}\leq\frac{\nu_{ndi,t}}{z_{ni}(\omega)}\ \forall n\neq s\right],\end{aligned}$$

using the shorthand

$$\nu_{sdi,t}\equiv c_{sdi,t}(1+\Psi_{sdi,t})^{-1/(\sigma_i-1)}.$$

As in EK, the Poisson arrival rate of Pareto distributed productivity implies that the highest realized productivity $z_{si}(\omega)$ for an intermediate good ω in a country-industry si is an i.i.d. Fréchet distributed random variable with the distribution function

$$\Pr\left[z_{si}(\omega)\leq z\right]=\exp\left\{-A_{si}z^{-\theta_i}\right\}.$$

Maximum productivity $z_{si}(\omega)$ is distributed Fréchet, so the random variable $c_{sdi,t}\left(1+\Psi_{sdi,t}\right)^{-1/(\sigma_i-1)}/z_{si}(\omega)=\nu_{sdi,t}/z_{si}(\omega)$ is Weibull distributed with the distribution function

$$\Pr\left[\frac{c_{sdi,t}\left(1+\Psi_{sdi,t}\right)^{-\frac{1}{\sigma_i-1}}}{z_{si}(\omega)}\leq v\right]=\Pr\left[\nu_{sdi,t}/v\leq z_{si}(\omega)\right]=1-\exp\left\{-A_{si}(\nu_{sdi,t})^{-\theta_i}v^{\theta_i}\right\}.$$

The minimum from a draw of N i.i.d. Weibull distributed random variables with common shape parameter θ_i is also Weibull distributed with the same shape parameter and

$$F_{di,t}(v)=\Pr\left[\min_{n\in\mathcal{N}}\frac{\nu_{ndi,t}}{z_{ni}(\omega)}\leq v\right]=1-\exp\left\{-v^{\theta_i}\Upsilon_{di,t}\right\},$$

where

$$\Upsilon_{di,t} = \sum_{n \in \mathcal{N}} A_{ni}(\nu_{ndi,t})^{-\theta_i}.$$

The properties of the Weibull distribution then yield the probability that an intermediate good $\omega \in \Omega^0_{di,t}$ in destination d is optimally sourced from a supplier in country s

$$\upsilon^0_{sdi,t} = \Pr\left[\frac{\nu_{sdi,t}}{z_{si}(\omega)} \leq \frac{\nu_{ndi,t}}{z_{ni}(\omega)} \ \forall n \neq s\right] = \frac{A_{si}(\nu_{sdi,t})^{-\theta_i}}{\sum_{n \in \mathcal{N}} A_{ni}(\nu_{ndi,t})^{-\theta_i}} = \frac{A_{si} c_{sdi,t}^{-\theta_i}(1+\Psi_{sdi,t})^{\frac{\theta_i}{\sigma_i - 1}}}{\sum_{n \in \mathcal{N}} A_{ni} c_{ndi,t}^{-\theta_i}(1+\Psi_{ndi,t})^{\frac{\theta_i}{\sigma_i - 1}}}$$
$$= \frac{A_{si} c_{sdi,t}^{-\theta_i}(1+\Psi_{sdi,t})^{\frac{\theta_i}{\sigma_i - 1}}}{\Upsilon_{di,t}}.$$

By the law of large numbers, the probability $\upsilon^0_{sdi,t}$ coincides with the share of intermediate goods $\omega \in \Omega^0_{di,t}$ that a destination d optimally procures from a country s at time t.

As another implication of the Weibull distribution, the term $\Upsilon_{di,t}$ encodes the average net return per unit sold across intermediate goods $\omega \in \Omega^0_{di,t}$:

$$\upsilon^0_{sdi,t} = \int_{\Omega^0_{di,t}} \arg\max_n \frac{z_{ni}(\omega)}{\nu_{ndi,t}} \,\mathrm{d}\omega = \int_0^\infty v \,\mathrm{d}(1 - F_{di,t}(v)) = \Gamma\left(\frac{\theta_i - (\sigma_i - 1)}{\theta_i}\right) \Upsilon_{di,t}.$$

A.3 Demand for Optimally Sourced Intermediate Goods

Consider the partition of intermediate-goods space $k = 0$, where traders can adjust their sourcing choice at the extensive margin. To characterize destination country d's expenditure allocation across source countries for goods $\omega \in \Omega^0_{di,t}$, we start with the following auxiliary lemma.

Lemma 1. *Let $\{a_n\}_{n=1}^N > \mathbf{0}$ be a vector of non-negative constants and $\{X_n\}_{n=1}^N$ a vector of i.i.d. Weibull-distributed random variables, each with cumulative distribution function $\Pr[X_n \leq x] = 1 - \exp\{-T_n x^\theta\}$. Then the distribution of X_s conditional on s being the minimum realization among a set of draws from $\{a_n X_n\}_{n=1}^N$ is given by*

$$\Pr\Big[X_s \leq x \big| \arg\min_n\{a_n X_n\} = s\Big] = 1 - \exp\left\{-x^\theta a_s^\theta \sum_n T_n a_n^{-\theta}\right\}.$$

Proof. For any value of $\overline{x}$, $\Pr\left[X_s \geq \overline{x}\frac{a_s}{a_n}\right] = \exp\left\{-T_n\left(\overline{x}\frac{a_s}{a_n}\right)^\theta\right\}$, so

$$\Pr\left[X_n \geq \overline{x}\frac{a_s}{a_n} \ \forall n \neq s\right] = \prod_{n \neq s} \Pr\left[X_n \geq \overline{x}\frac{a_s}{a_n}\right] = \exp\left\{-\overline{x}^\theta \sum_{n \neq s} T_n \left(\frac{a_n}{a_s}\right)^{-\theta}\right\}.$$

We can therefore state the following joint probability

$$\begin{aligned}
\Pr\left[X_s \le \overline{x}, s = \arg\min_n\{a_n X_n\}\right] &= \int_0^{\overline{x}} \Pr\left[X_s \le t\right] \cdot \Pr\left[s = \arg\min_n\{a_n X_n\} \mid X_s = t\right] \mathrm{d}t \\
&= \int_0^{\overline{x}} T_s \theta t^{\theta-1} \exp\left\{-t^\theta T_s\right\} \exp\left\{-t^\theta \sum_{n\neq s} T_n \left(\frac{a_n}{a_s}\right)^{-\theta}\right\} \mathrm{d}t \\
&= \int_0^{\overline{x}} T_s \theta t^{\theta-1} \exp\left\{-t^\theta a_s^\theta \sum_n T_n a_n^{-\theta}\right\} \mathrm{d}t.
\end{aligned}$$

From Bayes' rule and the fact that $\Pr\left[s = \arg\min_n\{a_n X_n\}\right] = (T_s a_s^{-\theta})/(\sum_n T_n a_n^{-\theta})$, it follows that the conditional distribution $\Pr\left[X_s \le \overline{x} | \arg\min_n\{a_n X_n\} = s\right]$ satisfies

$$\begin{aligned}
\Pr\left[X_s \le \overline{x} \mid \arg\min_n\{a_n X_n\} = s\right] &= \frac{\Pr\left[\arg\min_n\{a_n X_n\} = s \mid X_s \le \overline{x}\right] \cdot \Pr\left[X_s \le \overline{x}\right]}{\Pr\left[\arg\min_n\{a_n X_n\} = s\right]} \\
&= \frac{\Pr\left[X_s \le \overline{x}, s = \arg\min_n\{a_n X_n\}\right]}{\Pr\left[\arg\min_n\{a_n X_n\} = s\right]} \\
&= \frac{\sum_n T_n a_n^{-\theta}}{T_s a_s^{-\theta}} \int_0^{\overline{x}} T_s \theta t^{\theta-1} \exp\left\{-t^\theta a_s^\theta \sum_n T_n a_n^{-\theta}\right\} \mathrm{d}t \\
&= \int_0^{\overline{x}} \theta t^{\theta-1} \left[a_s^\theta \sum_n T_n a_n^{-\theta}\right] \exp\left\{-t^\theta a_s^\theta \sum_n T_n a_n^{-\theta}\right\} \mathrm{d}t \\
&= 1 - \exp\left\{-\overline{x}^\theta a_n^\theta \sum_s T_s a_s^{-\theta}\right\}.
\end{aligned}$$

□

A trader in d faces the following distribution of prices for intermediate goods $\omega \in \Omega^0_{di,t}$ sourced countries s:

$$\begin{aligned}
G^0_{sdi,t}(p) &= \Pr\left(p_{di,t}(\omega) \le p \,\middle|\, s^*_{di,t}(\omega) = s\right) \\
&= \Pr\left(\frac{\sigma_i}{\sigma_i - 1} \frac{c_{sdi,t}}{z_{ni,t}(\omega)} \le p \,\middle|\, \arg\min_n \left\{\frac{c_{ndi,t}}{z_{ni,t}(\omega)} \left(1 + \Psi_{ndi,t}\right)^{-\frac{1}{\sigma_i - 1}}\right\} = s\right).
\end{aligned}$$

Using Lemma 1 and substituting $\bar{x} = \frac{\sigma_i - 1}{\sigma_i} p$, $T_n = A_{ni}$, $a_n = \left(1 + \Psi_{ndi,t}\right)^{-\frac{1}{\sigma_i - 1}} c_{ndi,t}$ the distribution of intermediate-goods prices becomes

$$\begin{aligned}
G^0_{sdi,t}(p) &= 1 - \exp\left\{-p^{\theta_i} \left(\frac{\sigma_i - 1}{\sigma_i}\right)^{\theta_i} \left(1 + \Psi_{sdi,t}\right)^{-\frac{\theta_i}{\sigma - 1}} \sum_n A_{ni} c_{sdi,t}^{-\theta_i} \left(1 + \Psi_{sdi,t}\right)^{\frac{\theta_i}{\sigma_i - 1}}\right\} \\
&= 1 - \exp\left\{-p^{\theta_i} \left(\frac{\sigma_i - 1}{\sigma_i}\right)^{\theta_i} \left(1 + \Psi_{sdi,t}\right)^{-\frac{\theta_i}{\sigma_i - 1}} \Upsilon_{di,t}\right\}.
\end{aligned}$$

To derive country $d's$ expenditure share for every potential source country of goods $\omega \in \Omega^0_{it}$, we can invoke the law of large numbers and show

$$\lambda^0_{sdi,t} = \frac{\int_{\omega\in\Omega^0_{di,t}} \mathbf{1}_d\left\{s^*_{di,t}(\omega)=s\right\} p(\omega)^{-(\sigma_i-1)}\,\mathrm{d}\omega}{\sum_n \int_{\omega\in\Omega^0_{di,t}} \mathbf{1}_d\left\{s^*_{di,t}(\omega)=n\right\} p(\omega)^{-(\sigma_i-1)}\,\mathrm{d}\omega} = \frac{A_{si}\nu_{sdi,t}^{-\theta_i}\int_0^\infty p^{-(\sigma_i-1)}\,\mathrm{d}G_{sdi,t}(p)}{\sum_n A_{ni}\nu_{ndi,t}^{-\theta_i}\int_0^\infty p^{-(\sigma_i-1)}\,\mathrm{d}G_{ndi,t}(p)}, \tag{A.9}$$

where

$$\nu_{sdi,t} \equiv c_{sdi,t}(1+\Psi_{sdi,t})^{-1/(\sigma_i-1)}.$$

Use a change in variables

$$x = p^{\theta_i}\left(\tfrac{\sigma_i-1}{\sigma_i}\right)^{\theta_i}(1+\Psi_{ndi,t})^{-\frac{\theta_i}{\sigma_i-1}}\Upsilon_{di,t}, \quad \text{so } \mathrm{d}x = \theta_i p^{\theta_i-1}\left(\tfrac{\sigma_i-1}{\sigma_i}\right)^{\theta_i}(1+\Psi_{ndi,t})^{-\frac{\theta_i}{\sigma_i-1}}\Upsilon_{di,t}\,\mathrm{d}p.$$

Plug these results into the conditional distribution $G^0_{sdi,t}(p)$ above to find

$$\begin{aligned}
\int_0^\infty p^{-(\sigma_i-1)}\,\mathrm{d}G^0_{sdi,t}(p) &= \int_0^\infty p^{-(\sigma_i-1)}\,\theta_i p^{\theta_i-1}\left(\frac{\sigma_i-1}{\sigma_i}\right)^{\theta_i}(1+\Psi_{sdi,t})^{-\frac{\theta_i}{\sigma_i-1}}\Upsilon_{d,t} \\
&\qquad \cdot\exp\left\{-p^{\theta_i}\left(\frac{\sigma_i-1}{\sigma_i}\right)^{\theta_i}(1+\Psi_{sdi,t})^{-\frac{\theta_i}{\sigma_i-1}}\Upsilon_{di,t}\right\}\mathrm{d}p \\
&= \int_0^\infty \left(\frac{x}{\left(\frac{\sigma_i-1}{\sigma_i}\right)^{\theta_i}(1+\Psi_{sdi,t})^{-\frac{\theta_i}{\sigma_i-1}}\Upsilon_{di,t}}\right)^{-\frac{\sigma_i-1}{\theta_i}}\mathrm{d}x \\
&= \left(\frac{\sigma_i-1}{\sigma_i}\right)^{\sigma_i-1}(1+\Psi_{sdi,t})^{-1}\left(\Upsilon_{di,t}\right)^{\frac{\sigma_i-1}{\theta_i}}\int_0^\infty x^{-\frac{\sigma_i-1}{\theta_i}}e^{-x}\,\mathrm{d}x \\
&= \left(\frac{\sigma_i-1}{\sigma_i}\right)^{\sigma_i-1}(1+\Psi_{sdi,t})^{-1}\left(\Upsilon_{di,t}\right)^{\frac{\sigma_i-1}{\theta_i}}\Gamma\left(\frac{\theta_i-(\sigma_i-1)}{\theta_i}\right).
\end{aligned}$$

Using this solution in $\lambda^0_{sdi,t}$ above establishes equation (16) in the text.

From equation (16) we can compute the partial elasticity of trade flows for goods $\omega\in\Omega^0_{di,t}$ with respect to a permanent change in trade cost $\tau_{sdi,t}$ from time t on. We ignore the general-equilibrium response of factor prices and isolate trade cost changes from unit cost $c_{si,t}$, we condition on market access $\Phi^0_{di,t}$, and we hold the future path of factor costs and prices behind the option value $\Psi_{sdi,t}$ constant, so

$$\left.\frac{\partial \ln \lambda^0_{sdi,t}}{\partial \ln \tau_{sdi,t}}\right|_{\Phi^0_{di,t},\Psi sdi,t} = -\theta_i.$$

Agents have perfect foresight. Beyond the concurrent effect of a change in trade costs on trade flows, we can therefore also derive the effect of a future trade cost change by Δ percent on trade flows today. Suppose that, starting at horizon $t+h+1$, trade costs permanently increase. To compute the partial-equilibrium response of trade flows at time t to such a future change from equation (16), we condition on current market aggregates

$\{\Phi^0_{di,t}, c_{sdi,t}\}$ and on changes in trade and unit cost at horizons $0 < h' < h+1$:

$$\left.\frac{\partial \ln \lambda^0_{sdi,t}}{\partial \ln\{\tau_{sd,t+h'}\}_{h'>h}}\right|_{\Phi^0_{di,t}, c_{sdi,t}} = \frac{\theta_i - (\sigma_i - 1)}{\sigma_i - 1} \left.\frac{\partial \ln (1+\Psi_{sd,t})}{\partial \ln\{\tau_{sd,t+h'}\}_{h'>h}}\right|_{\Phi^0_{di,t}},$$

where

$$\begin{aligned}
\left.\frac{\partial \ln (1+\Psi_{sd,t})}{\partial \ln\{\tau_{sd,t+h'}\}_{h'>h}}\right|_{\Phi^0_{di,t}} &= \lim_{\Delta\to 0} \frac{\partial \ln \left[1 + \sum_{\varsigma=1}^{\infty} \left(\frac{1-\zeta_i}{1+r}\right)^{\varsigma} \left(X_\varsigma \prod_{\varsigma'=1}^{\varsigma} \left(\frac{\hat{c}_{ndi,t+\varsigma'}}{\hat{P}_{di,t+\varsigma'}}\right)^{-(\sigma_i-1)} \hat{P}_{di,t+\varsigma'} \hat{Y}_{di,t+\varsigma'}\right)\right]}{\partial \ln(1+\Delta)} \\
&= -\frac{\sum_{\varsigma=h+1}^{\infty} \left(\frac{1-\zeta_i}{1+r}\right)^{\varsigma} \left(\prod_{\varsigma'=1}^{\varsigma} \left(\frac{\hat{c}_{ndi,t+\varsigma'}}{\hat{P}_{di,t+\varsigma'}}\right)^{-(\sigma_i-1)} \hat{P}_{di,t+\varsigma'} \hat{Y}_{di,t+\varsigma'}\right)}{1+\Psi_{sd,t}} (\sigma_i - 1) \\
&= -\left(\frac{1-\zeta_i}{1+r}\right)^{h} \frac{\Psi_{sd,t+h}}{1+\Psi_{sd,t}} (\sigma_i - 1)
\end{aligned}$$

for $X_\varsigma \equiv \left((1+\Delta)^{\mathbf{1}\{\varsigma \geq h+1\}}\right)^{-(\sigma_i-1)}$.

Turning to the price index for goods $\omega \in \Phi^0_{di,t}$, we use our previous results to show that

$$\begin{aligned}
\left(P^0_{di,t}\right)^{-(\sigma_i-1)} &= \int_{\Omega^0_{di,t}} \sum_n \mathbf{1}_d \left\{n = s^*_{d,t}(\omega)\right\} p_{d,t}(\omega)^{-(\sigma_i-1)} \, \mathrm{d}\omega \\
&= \mu_{di,t}(0) \sum_n \frac{A_{ni} \nu_{ndi,t}^{-\theta_i}}{\Upsilon_{di,t}} \int_0^\infty p^{-(\sigma_i-1)} \, \mathrm{d}G^0_{ndi,t}(p) \\
&= \mu_{di,t}(0) \left(\frac{\sigma_i - 1}{\sigma_i}\right)^{\sigma_i - 1} \Gamma\left(\frac{\theta_i - (\sigma_i-1)}{\theta_i}\right) \sum_n \frac{A_{ni}\nu_{ndi,t}^{-\theta_i}}{\Upsilon_{di,t}} \left(1+\Psi_{nd,t}\right)^{-1} \left(\Upsilon_{di,t}\right)^{\frac{\sigma_i-1}{\theta_i}} \\
&= \mu_{di,t}(0) \left(\frac{\sigma_i - 1}{\sigma_i}\right)^{\sigma_i - 1} \Gamma\left(\frac{\theta_i - (\sigma_i-1)}{\theta_i}\right) \left(\sum_n A_{ni} c_{ndi,t}^{-\theta} \left(1+\Psi_{ndi,t}\right)^{\frac{\theta_i}{\sigma_i-1}-1}\right) \left(\Upsilon_{di,t}\right)^{\frac{\sigma_i-1}{\theta_i}-1}. \\
&= \mu_{di,t}(0)\, \gamma_i^{-(\sigma_i-1)} \frac{\Phi^0_{di,t}}{\Upsilon_{di,t}} \left(\Upsilon_{di,t}\right)^{\frac{\sigma_i-1}{\theta_i}},
\end{aligned}$$

where $\nu_{sdi,t} \equiv c_{sdi,t}(1+\Psi_{sdi,t})^{-1/(\sigma_i-1)}$, $\gamma_i \equiv \frac{\sigma_i}{\sigma_i-1}\Gamma\left(\frac{\theta_i-(\sigma_i-1)}{\theta_i}\right)^{-1/(\sigma_i-1)}$. Alternatively, we can use the fact that $\lambda^0_{ddi,t} = \left[A_{di} c_{ddi,t}^{-\theta} \left(1+\Psi_{sdi,t}\right)^{\frac{\theta_i}{\sigma_i-1}-1}\right] / \left(P^0_{di,t}\right)^{-(\sigma_i-1)}$ to obtain:

$$\left(P^0_{di,t}\right)^{-(\sigma_i-1)} = c_{ddi,t}^{-\theta} \left(1+\Psi_{sdi,t}\right)^{\frac{\theta_i}{\sigma_i-1}-1} \frac{A_{di}}{\lambda^0_{ddi,t}}.$$

A.4 Demand for Legacy Intermediate Goods

We turn to the partitions of intermediate-goods space $k > 0$, where traders cannot adjust their sourcing choice at the extensive margin. For intermediate goods $\omega \in \Omega^k_{di,t}$ with $k > 0$, traders last chose the optimal supplier $t-k$ periods ago. In order to account for changes in trade shares and price levels, we need to recall optimal sourcing choices at period $t-k$ and trace changes in parameters and prices from period $t-k$ onwards.

Suppose that in period $t-k$ intermediate good ω was optimally sourced from country s to country d in industry i. Then the destination price in period t for this intermediate good is

$$p_{sdi,t}(\omega) = \frac{\sigma_i}{\sigma_i - 1} \frac{c_{sdi,t}}{z_{si}(\omega)} = \frac{\sigma_i}{\sigma_i - 1} \frac{\prod_{\varsigma=t-k+1}^{t} c_{sdi,t-k}\, \hat{c}_{sdi,\varsigma}}{z_{si}(\omega)} = p_{sdi,t-k}(\omega) \prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdi,\varsigma}, \tag{A.10}$$

which is the initial destination price adjusted for the cumulative changes in trade costs and factor costs. Using this result, we can derive country d's expenditure share by source country across intermediate goods $\omega \in \Omega^k_{di,t}$

$$\begin{aligned}
\lambda^k_{sdi,t} &= \frac{\int_{\Omega^k_{di,t}} \mathbf{1}_d \left\{ s = s^*_{di,t-k}(\omega) \right\} p_{sdi,t-k}(\omega)^{-(\sigma_i-1)} \prod_{\varsigma=t-k+1}^{t} \left(\hat{c}_{sdi,\varsigma}\right)^{-(\sigma_i-1)} \,\mathrm{d}\omega}{\sum_n \int_{\Omega^k_{di,t}} \mathbf{1}_d \left\{ n = s^*_{di,t-k}(\omega) \right\} p_{ndi,t-k}(\omega)^{-(\sigma_i-1)} \prod_{\varsigma=t-k+1}^{t} \left(\hat{c}_{ndi,\varsigma}\right)^{-(\sigma_i-1)} \,\mathrm{d}\omega} \\
&= \frac{\left[A_{si} \nu_{sdi,t-k}^{-\theta_i} \int_0^\infty p^{-(\sigma_i-1)} \,\mathrm{d}G^0_{sdi,t-k}(p) \right] \prod_{\varsigma=t-k+1}^{t} \left(\hat{c}_{sdi,\varsigma}\right)^{-(\sigma_i-1)}}{\sum_n \left[A_{ni} \nu_{ndi,t-k}^{-\theta_i} \int_0^\infty p^{-(\sigma_i-1)} \,\mathrm{d}G^0_{ndi,t-k}(p) \right] \prod_{\varsigma=t-k+1}^{t} \left(\hat{c}_{ndi,\varsigma}\right)^{-(\sigma_i-1)}} \\
&= \frac{\lambda^0_{sdi,t-k} \prod_{\varsigma=t-k+1}^{t} \left(\hat{c}_{sdi,\varsigma}\right)^{-(\sigma_i-1)}}{\sum_n \lambda^0_{ndi,t-k} \prod_{\varsigma=t-k+1}^{t} \left(\hat{c}_{sdi,\varsigma}\right)^{-(\sigma_i-1)}},
\end{aligned}$$

where $\mu_{di,t}(k)$ is the measure of the set $\Omega^k_{di,t}$ and $\nu_{sdi,t} \equiv c_{sdi,t}(1 + \Psi_{sdi,t})^{-1/(\sigma_i-1)}$. The final equality follows after dividing both numerator and denominator by $\sum_m A_{mi} \nu_{mdi,t}^{-\theta_i} \int_0^\infty p^{-(\sigma_i-1)} \,\mathrm{d}G_{mdi,t}(p)$ and using (A.9) for $\lambda^0_{sdi,t-k}$.

By the properties of CES demand, the ideal price index for goods in the partition $\Omega^k_{di,t}$ follows from the initial price $P^0_{di,t-k}$ at time $t-k$ and subsequent changes in prices and varieties:

$$\begin{aligned}
\left(P^k_{di,t}\right)^{-(\sigma_i-1)} &= \int_{\Omega^k_{di,t}} \sum_n \mathbf{1}_d \left\{ n = s^*_{d,t-k}(\omega) \right\} p_{ndi,t}(\omega)^{-(\sigma_i-1)} \,\mathrm{d}\omega \\
&= \mu_{di,t}(k) \left(\frac{\sigma_i - 1}{\sigma_i} \right)^{\sigma_i - 1} \sum_n \left[\frac{A_{ni} \nu_{ndi,t}^{-\theta_i}}{\Upsilon_{di,t}} \int_0^\infty p^{-(\sigma_i-1)} \,\mathrm{d}G^0_{ndi,t-k}(p) \right] \prod_{\varsigma=t-k+1}^{t} \left(\hat{c}_{ndi,\varsigma}\right)^{-(\sigma_i-1)} \\
&= \frac{\mu_{di,t}(k)}{\mu_{di,t-k}(0)} \left(P^0_{di,t-k}\right)^{-(\sigma_i-1)} \sum_n \lambda^0_{ndi,t-k} \left(\prod_{\varsigma=t-k+1}^{t} \left(\hat{c}_{ndi,\varsigma}\right)^{-(\sigma_i-1)} \right) \\
&= \frac{\mu_{di,t}(k)}{\mu_{di,t-k}(0)} \left(P^0_{di,t-k}\right)^{-(\sigma_i-1)} \Phi^k_{di,t}.
\end{aligned}$$

A.5 Aggregation

The aggregate ideal price level of the final good i can be rewritten as a combination of the price levels of the partial price indices for the composites of intermediate goods purchased at time t from suppliers chosen $t-k$ periods ago:

$$
\begin{aligned}
P_{di,t} &= \left[\int_{[0,1]} p_{d,t}(\omega)^{-(\sigma_i-1)}\,\mathrm{d}\omega\right]^{-\frac{1}{\sigma_i-1}} \\
&= \left(\sum_{k=0}^{\infty}\left(P_{di,t}^{k}\right)^{-(\sigma_i-1)}\right)^{-\frac{1}{\sigma_i-1}} \\
&= \left(P_{di,t}^{0}\right)\left[1+\sum_{k=1}^{\infty}\frac{\mu_{di,t}(k)}{\mu_{di,t}(0)}\left(\frac{P_{di,t-k}^{0}}{P_{di,t}^{0}}\right)^{-(\sigma_i-1)}\Phi_{di,t}^{k}\right]^{-\frac{1}{\sigma_i-1}}.
\end{aligned}
$$

Under CES demand, total expenditure shares are simply the weighted average of trade shares across partitions:

$$
\lambda_{sdi,t} = \sum_{k=0}^{\infty}\left(\frac{P_{di,t}^{k}}{P_{di,t}}\right)^{-(\sigma_i-1)}\lambda_{sdi,t}^{k}.
$$

A.6 Dynamic Hat Algebra

We now restate the conditions for equilibrium at time t in terms of past sourcing probabilities, trade shares, expenditures as well as exogenous changes in trade costs and endogenous changes in factor prices.

Bilateral trade shifters.

$$
\hat{c}_{sdi,t} = \hat{\tau}_{sdi,t}\left(\hat{w}_{si,t}\right)^{\alpha_{si}}\prod_{j\in\mathcal{I}}\left(\hat{P}_{sj,t}\right)^{\eta_{sji}}, \tag{A.11}
$$

$$
\widehat{1+\Psi_{sdi,t}} = \frac{1+\Psi_{sdi,t}}{1+\Psi_{sdi,t-1}} = \frac{1+\sum_{u=1}^{\infty}\left(\frac{1-\zeta_i}{1+r}\right)^{u}\prod_{u'=1}^{u}\left[\hat{c}_{ndi,t+u'}^{-(\sigma_i-1)}\hat{P}_{di,t+u'}^{\sigma_i}\hat{Y}_{di,t+u'}\right]}{1+\sum_{u=1}^{\infty}\left(\frac{1-\zeta_i}{1+r}\right)^{u}\prod_{u'=1}^{u}\left[\hat{c}_{ndi,t+u'-1}^{-(\sigma_i-1)}\hat{P}_{di,t+u'-1}^{\sigma_i}\hat{Y}_{di,t+u'-1}\right]}. \tag{A.12}
$$

Supplier choice, trade flows and prices for optimally sourced goods.

$$
\hat{v}_{sdi,t} = \frac{\hat{c}_{sdi,t}^{-\theta_i}\left(\widehat{1+\Psi_{sdi,t}}\right)^{\frac{\theta_i}{\sigma_i-1}}}{\hat{\Upsilon}_{di,t}}, \tag{A.13}
$$

$$
\hat{\Upsilon}_{di,t} = \sum_{n}\nu_{ndi,t-1}\hat{c}_{ndi,t}^{-\theta_i}\left(\widehat{1+\Psi_{ndi,t}}\right)^{\frac{\theta_i}{\sigma_i-1}}, \tag{A.14}
$$

$$
\hat{\lambda}_{sdi,t}^{0} = \frac{\hat{c}_{sdi,t}^{-\theta_i}\left(\widehat{1+\Psi_{sdi,t}}\right)^{\frac{\theta_i}{\sigma_i-1}-1}}{\hat{\Phi}_{di,t}^{0}}, \tag{A.15}
$$

$$\hat{\Phi}^0_{di,t} = \sum_n \lambda^0_{ndi,t-1}\hat{c}^{-\theta_i}_{ndi,t}(\widehat{1+\Psi_{ndi,t}})^{\frac{\theta_i}{\sigma_i-1}-1}, \tag{A.16}$$

$$\hat{P}^0_{di,t} = \left(\hat{\Upsilon}_{di,t}\right)^{-1/\theta_i}\left(\frac{\hat{\Phi}^0_{di,t}}{\hat{\Upsilon}_{di,t}}\right)^{-1/(\sigma_i-1)}. \tag{A.17}$$

Trade flows for legacy goods with $k > 0$.

$$\lambda^k_{sdi,t} = \frac{\lambda^0_{sdi,t-k}\prod_{\varsigma=t-k+1}^{t}\hat{c}^{-(\sigma_i-1)}_{sdi,\varsigma}}{\Phi^k_{di,t}}, \tag{A.18}$$

$$\Phi^k_{di,t} = \sum_n \lambda^0_{ndi,t-k}\prod_{\varsigma=t-k+1}^{t}\hat{c}^{-(\sigma_i-1)}_{ndi,\varsigma}, \tag{A.19}$$

$$\left(P^k_{di,t}\right)^{-(\sigma_i-1)} = \left(P^0_{di,t-k}\right)^{-(\sigma_i-1)}\frac{\mu_{i,t}(k)}{\mu_{i,t-k}(0)}\Phi^k_{di,t}. \tag{A.20}$$

Industry price index and trade flows.

$$\hat{P}_{di,t} = \hat{P}^0_{di,t}\frac{\left[1+\sum_{k=1}^{\infty}\frac{\mu_{i,t}(k)}{\mu_{di,t}(0)}\left(\frac{P^0_{di,t-k}}{P^0_{di,t}}\right)^{-(\sigma_i-1)}\Phi^k_{di,t}\right]^{-1/(\sigma_i-1)}}{\left[1+\sum_{k=1}^{\infty}\frac{\mu_{i,t-1}(k)}{\mu_{i,t-1}(0)}\left(\frac{P^0_{di,t-k-1}}{P^0_{di,t-1}}\right)^{-(\sigma_i-1)}\Phi^k_{di,t}\right]^{-1/(\sigma_i-1)}}. \tag{A.21}$$

Market clearing conditions.

$$X_{si,t} = \sum_{d\in\mathcal{N}}\lambda_{sdi,t-1}\hat{\lambda}_{sdi,t-1}\left[\eta_{di}E_{d,t-1}\hat{E}_{d,t} + \sum_{j\in\mathcal{I}}\alpha_{dij}X_{di,t}\right], \tag{A.22}$$

$$\Pi_{d,t} = \sum_i \frac{1}{\sigma_i}\left[\eta_{di}E_{d,t-1}\hat{E}_{d,t} + \sum_j \alpha_{dij}X_{di,t}\right], \tag{A.23}$$

$$\hat{E}_{d,t} = \frac{w_{d,t-1}\hat{w}_{d,t-1}L_d + \Pi_{d,t} + D_{d,t}}{E_{d,t-1}}, \tag{A.24}$$

$$w_{d,t-1}\hat{w}_{d,t-1}L_d = \sum_{i\in\mathcal{I}}\frac{\sigma_i}{\sigma_i-1}(1-\alpha_{di})X_{si,t}. \tag{A.25}$$

A.7 Proof of Proposition 1

In an economy that is in steady state factor prices and trade costs are constant so $\hat{c}_{sdi,t} = 1$ and $1+\Psi_{sdi,t} = (1+r)/(r+\zeta_i)$. Then, a destination d's sourcing probability for different source countries s across intermediate goods $\omega \in \Omega^0_{i,t}$ equals its expenditure shares, $\upsilon^0_{sdi,t} = \lambda^0_{sdi,t} = \lambda^0_{sdi,t-1} = \cdots = \lambda^0_{sdj,0}$. For traders who cannot adjust their suppliers in steady state ($k > 0$), we can evaluate (24) using the same logic as above: $\lambda^k_{sdi,t} = \lambda^0_{sdi,t-k} = \lambda^0_{sdi,0}$ for all k. From (24), it is straightforward to recognize that $\nu_{sdi,t} = \lambda_{sdj,t} = \lambda^0_{sdj,t}$, which coincides with the static equilibrium of an EK economy with the same fundamentals $\boldsymbol{\Theta}$, the same trade costs $\boldsymbol{\tau}$, and no sourcing frictions ($\boldsymbol{\zeta} = \mathbf{1}$).

To derive the stationary distribution of partition measures, start with the case $k = 0$. Note that $\mu^*_i(0) = \Pr\left[\omega \in \Omega^0_{di,t}\right] = \zeta_i$ does not vary across destinations or over time. Now consider cases $k > 0$. Note that

$$\begin{aligned}
\mu^*_i(k) = \Pr\left[\omega \in \Omega^k_{di,t}, k > 0\right] &= \sum_{\ell=0}^{\infty} \Pr\left[\omega \in \Omega^k_{di,t}, k > 0 \middle| \omega \in \Omega^\ell_{di,t-1}\right] \Pr\left[\omega \in \Omega^\ell_{di,t-1}\right] \\
&= (1-\zeta_i) \Pr\left[\omega \in \Omega^{k-1}_{di,t-1}\right].
\end{aligned}$$

The remaining proof for $k > 0$ then follows by induction. For $k = 1$, $\Pr\left[\omega \in \Omega^1_{di,t}\right] = (1-\zeta_i)\zeta_i$, and for $k = 2$, $\Pr\left[\omega \in \Omega^2_{di,t}\right] = (1-\zeta_i)\Pr\left[\omega \in \Omega^1_{di,t}\right] = (1-\zeta_i)^2\zeta_i$, and so forth recursively. For an arbitrary $k > 0$ we must therefore have $\Pr\left[\omega \in \Omega^k_{di,t}\right] = (1-\zeta_i)^k\zeta_i$. This is the probability density function of a geometric distribution with mean $(1-\zeta_i)/\zeta_i$ and standard deviation $\sqrt{1-\zeta_i}/\zeta_i$. By the Markov property, the distribution of partition measures is stationary for all $k \in \mathbb{N}_0$ with

$$\mu^*_i(k) = \zeta_i \left(1-\zeta_i\right)^k.$$

A.8 Proof of Proposition 2.

For ease of notation, we suppress industry subscripts throughout the derivation. Let $\left\{\lambda^k_{sd,t-1}\right\}_{k=0}^{\infty}$ denote the vector of destination $d's$ expenditure shares for a source country s across goods in each of the partitions $\left\{\Omega^k_{d,t}\right\}_{k=0}^{\infty}$ at time t. Now, consider a one-time permanent change in trade costs such that $\hat{\tau}_{sd,t} \neq 1$ and $\hat{\tau}_{sd,t+h} = 1\ \forall h > 0$. To characterize the partial trade elasticity at horizon h, we first characterize the elasticity for trade shares of each partition, then aggregate them up using the consumption shares derived from the CES preferences over partitions. The change in expenditure shares on intermediate goods in the k-th partition in period $t+h$, relative to period t is given by

$$\ln \frac{\lambda^k_{sd,t+h}}{\lambda^k_{sd,t-1}} = \begin{cases} -(\sigma-1)\ln\hat{\tau}_{sd,t} + \ln \frac{\lambda^0_{sd,t+h-k}}{\lambda^k_{sd,t-1}} \left(\frac{(c_{s,t+h}/P^k_{d,t+h})}{(c_{s,t+h-k}/P^k_{d,t+h-k})}\right)^{-(\sigma_i-1)} & , k > h \\ \ln \frac{\lambda^0_{sd,t+h-k}}{\lambda^k_{sd,t-1}} \left(\frac{c_{s,t+h}/P^k_{d,t+h}}{c_{s,t+h-k}/P^k_{d,t+h-k}}\right)^{-(\sigma_i-1)} & , 1 \leq k < h \\ \ln \frac{\lambda^0_{sd,t+h-1}}{\lambda^k_{sd,t-1}} \left(\frac{(c_{s,t+h}/\Phi^0_{d,t+h})}{(c_{s,t-1}/\Phi^0_{d,t-1})}\right)^{-\theta} \left(\frac{1+\Psi_{sd,t+h}}{1+\Psi_{sd,t-1}}\right)^{\theta/(\sigma-1)} & , k = 0 \end{cases}$$

The first line denotes intermediate goods that have not updated suppliers since the shock arrived. For such intermediate goods, changes in expenditure shares still explicitly depend on the shock to trade costs. The remaining intermediate goods have updated at least once, and a "new" optimal sourcing share $\lambda^0_{sd,t+h-k}$ from a time period between t and $t+h$ encodes the "initial price index" relative to which changes in expenditure shares are updated as well as the effect of the shock in trade costs. Unit costs are the relevant GE variables.

To capture general equilibrium effects, we introduce the variables

$$\Delta \boldsymbol{G}^{EK}_{sd,t,t+h} = -\theta \ln \prod_{\varsigma=0}^{h} \frac{\hat{c}_{sd,t+\varsigma}}{\hat{P}^{0}_{sd,t+\varsigma}},$$

and

$$\Delta \boldsymbol{G}^{\Psi}_{sd,t-1,t+h} = \frac{\theta}{\sigma-1} \ln \prod_{\varsigma=0}^{h} \left(\widehat{1+\Psi_{sd,t+\varsigma}}\right) \left(\approx \frac{\theta}{\sigma-1}\left(\Psi_{sd,t+h} - \Psi_{sd,t-1}\right)\right)$$

to summarize how current and future changes in unit cost affect trade flows for optimally sourced goods, respectively; and

$$\Delta \boldsymbol{G}^{k}_{sd,\varsigma,t+h} = (1-\sigma) \ln \prod_{\varsigma'=\varsigma+1}^{t+h} \frac{\hat{c}_{sd,\varsigma'}}{P^{k}_{sd,\varsigma'}},$$

to summarize how past changes in unit cost continue to affect trade flows for goods sourced from suppliers chosen at $t-k$. Then we can solve backwards to express all changes in trade shares above in terms of $\lambda^{0}_{sd,t-1}$, if possible:

$$\ln \frac{\lambda^{k}_{sd,t+h}}{\lambda^{k}_{sd,t-1}} = \begin{cases} -(\sigma-1)\ln \hat{\tau}_{sd,t} + \ln \frac{\lambda^{0}_{sd,t+h-k}}{\lambda^{k}_{sd,t-1}} + \Delta \boldsymbol{G}^{k}_{sd,t,t+h} & ,k > h \geq 0 \\ -\theta \ln \hat{\tau}_{sd,t} + \ln \frac{\lambda^{0}_{sd,t-1}}{\lambda^{k}_{sd,t-1}} + \Delta \boldsymbol{G}^{EK}_{sd,t,t+h-k} + \Delta \boldsymbol{G}^{\Psi}_{sd,t,t+h-k} + \Delta \boldsymbol{G}^{k}_{sd,t+h-k+1,t+h} & ,1 \leq k < h \\ -\theta \ln \hat{\tau}_{sd,t} + \Delta \boldsymbol{G}^{EK}_{sd,t,t+h} + \Delta \boldsymbol{G}^{\Psi}_{sd,t,t+h} & ,k = 0 \end{cases}$$

Use the fact that outcomes determined at $t-1$ and earlier do not respond to the change in trade costs. Hence, the elasticity of $\lambda^{k}_{sd,t+h}$ with respect to a change in trade costs at t is hence given by,

$$\frac{\mathrm{d}\ln(\lambda^{k}_{sd,t+h}/\lambda^{k}_{sd,t-1})}{\mathrm{d}\ln \tau_{sd,t}} = \begin{cases} -(\sigma-1) + \frac{\mathrm{d}\Delta \boldsymbol{G}^{k}_{sd,t-1,t+h}}{\mathrm{d}\ln \tau_{sd,t}} & ,k > h \geq 0 \\ -\theta + \frac{\Delta \boldsymbol{G}^{EK}_{sd,t-1,t+h-k}}{\mathrm{d}\ln \tau_{sd,t}} + \frac{\mathrm{d}\Delta \boldsymbol{G}^{\Psi}_{sd,t-1,t+h-k}}{\mathrm{d}\ln \tau_{sd,t}} + \frac{\mathrm{d}\Delta \boldsymbol{G}^{k}_{sd,t+h-k+1,t+h}}{\mathrm{d}\ln \tau_{sd,t}} + & ,1 \leq k < h \\ -\theta + \frac{\Delta \boldsymbol{G}^{EK}_{sd,t-1,t+h}}{\mathrm{d}\ln \tau_{sd,t}} + \frac{\mathrm{d}\Delta \boldsymbol{G}^{\Psi}_{sd,t-1,t+h}}{\mathrm{d}\ln \tau_{sd,t}} & ,k = 0 \end{cases}$$

To derive the change in trade flows across all goods $\omega \in [0,1]$, denote

$$\Delta \boldsymbol{G}^{P^k}_{d,t-1,t+h} \equiv \ln \frac{P^{k}_{d,t+h}/P_{d,t-1}}{P^{k}_{d,t-1}/P_{d,t+h}}$$

To a first order, the change in overall expenditures at time $t+h$ caused by a one-time permanent shock to trade costs at t $d\ln\tau_{sd,t}=\ln\hat{\tau}_{sd,t}$ is given by

$$
\begin{aligned}
\frac{\mathrm{d}\ln\frac{\lambda_{sd,t+h}}{\lambda_{sd,t-1}}}{\mathrm{d}\ln\tau_{sd,t}} =&\sum_{k=0}^{\infty}\omega_k\left\{\frac{\mathrm{d}\ln\lambda^k_{sd,t+h}/\lambda^k_{sd,t-1}}{\mathrm{d}\ln\tau_{sd,t}}+(1-\sigma)\frac{\mathrm{d}\Delta\boldsymbol{G}^{P^k}_{d,t-1,t+h}}{\mathrm{d}\ln\tau_{sd,t}}\right\}\\
=&\sum_{k=0}^{h}\omega_k\left\{-\theta+\frac{\Delta\boldsymbol{G}^{EK}_{sd,t-1,t+h}}{\mathrm{d}\ln\tau_{sd,t}}+\frac{\Delta\boldsymbol{G}^{\Psi}_{sd,t-1,t+h-k}}{\mathrm{d}\ln\tau_{sd,t}}+\frac{\mathrm{d}\Delta\boldsymbol{G}^{k}_{sd,t+h-k+1,t+h}}{\mathrm{d}\ln\tau_{sd,t}}+(1-\sigma)\frac{\mathrm{d}\Delta\boldsymbol{G}^{P^k}_{d,t-1,t+h}}{\mathrm{d}\ln\tau_{sd,t}}\right\}\\
&+\sum_{k=h+1}^{\infty}\omega_k\left\{(1-\sigma)+\frac{\Delta\boldsymbol{G}^{k}_{sd,t-1,t+h}}{\mathrm{d}\ln\tau_{sd,t}}+(1-\sigma)\frac{\ln\Delta\boldsymbol{G}^{P}_{d,t-1,t+h,k}}{\mathrm{d}\ln\tau_{sd,t}}\right\}\\
=&-\theta\sum_{k=0}^{h}\omega_k+(1-\sigma)\sum_{k=h+1}^{\infty}\omega_k\\
&+\sum_{k=0}^{h}\omega_k\left\{\frac{\mathrm{d}\Delta\boldsymbol{G}^{EK}_{sd,t-1,t+h}}{\mathrm{d}\ln\tau_{sd,t}}+\frac{\mathrm{d}\Delta\boldsymbol{G}^{\Psi}_{sd,t-1,t+h}}{\mathrm{d}\ln\tau_{sd,t}}\right\}\\
&+\sum_{k=0}^{h}\omega_k(1-\sigma)\left\{\frac{\sum_{\varsigma=t+h-k+1}^{t+h}\mathrm{d}\ln c_{sd,\varsigma}}{\mathrm{d}\ln\tau_{sd,t}}+\frac{\sum_{\varsigma=t}^{t+h-k}\mathrm{d}\ln P^k_{d,\varsigma}}{\mathrm{d}\ln\tau_{sd,t}}\right\}\\
&+\sum_{k=h+1}^{\infty}\omega_k(1-\sigma)\left\{\frac{\sum_{i=0}^{t+h}\mathrm{d}\ln c_{sd,t+i}}{\mathrm{d}\ln\tau_{sd,t}}\right\}\\
&-(1-\sigma)\frac{\sum_{i=0}^{h}\mathrm{d}\ln P_{d,t+i}}{\mathrm{d}\ln\tau_{sd,t}},
\end{aligned}
$$

where $\omega_k\equiv\frac{\left(\frac{P^k_{dj,t}}{P_{dj,t}}\right)^{-(\sigma_i-1)}\lambda^k_{sdj,t}}{\sum_k\left(\frac{P^k_{dj,t}}{P_{dj,t}}\right)^{-(\sigma_i-1)}\lambda^k_{sdj,t}}=\frac{\mu_t(k)\lambda^k_{sdj,t}}{\sum_k\mu_t(k)\lambda^k_{sdj,t}}$.

If $t-1$ was a steady state, then $\omega_k=\mu^*(k)=\zeta\left(1-\zeta\right)^k$ (following Proposition 1). Hence, the partial horizon-h trade elasticity equals:

$$
\begin{aligned}
\varepsilon^{t+h}_{sd}\equiv\frac{\partial\ln\lambda_{sdj,t+h}}{\partial\ln\tau_{sd,t}}=&\left.\frac{\mathrm{d}\ln(\lambda_{sd,t+h}/\lambda_{sd,t})}{\mathrm{d}\ln\tau_{sd,t}}\right|_{\Delta\boldsymbol{G}^{EK},\Delta\boldsymbol{G}^{\Psi},\{\Delta\boldsymbol{G}^k\}_{k=1}^{\infty}=0}\\
=&-\theta\sum_{k=0}^{h}\mu(k)+(1-\sigma)\sum_{k=h+1}^{\infty}\mu^*(k)\\
=&-\theta\left[1-(1-\zeta)^{h+1}\right]+(1-\sigma)\left(1-\zeta\right)^{h+1}.
\end{aligned}
$$

When $\zeta=1$, so that supply relations reset instantaneously, in every period, then $\varepsilon^{t+h}_{sd}=-\theta$, so trade is governed by Ricardian forces alone. If $\zeta=0$, so that the extensive margin of supplier adjustment at all times, then $\varepsilon^{t+h}_{sd}=1-\sigma$, so trade is governed by Armington forces. In the intermediate case where $\zeta\in(0,1)$, then trade flows are influenced by both forces $\sigma-1<\left|\varepsilon^{t+h}_{sd}\right|<\theta$. As more supply relations get to reset, Armington forces become weaker over time and, in the long-run limit, trade is governed by Ricardian forces alone, $\left|\varepsilon^{t+h}_{sd}\right|<\left|\varepsilon^{t+h+1}_{sd}\right|$ and

$\lim_{h\to\infty}\left|\varepsilon_{sd}^{t+h}\right| = \theta$, with rate of convergence governed by the rate of supplier adjustment:

$$\lim_{h\to\infty}\left|\frac{-\theta\left[1-(1-\zeta)^{h+1}\right]+(1-\sigma)(1-\zeta)^{h+1}+\theta}{-\theta\left[1-(1-\zeta)^{h}\right]+(1-\sigma)(1-\zeta)^{h}+\theta}\right| = 1-\zeta.$$

A.9 Proof of Proposition 3

We again suppress industry subscripts throughout the derivations. Let $\{\lambda_{sd,t-1}\}_{k=0}^{\infty}$ denote the equilibrium vector of destination $d's$ expenditure shares for a source country s across goods in each of the partitions $\left\{\Omega_{d,t-1}^{k}\right\}_{k=0}^{\infty}$ at time $t-1$, given a path $\boldsymbol{\tau} = \boldsymbol{\tau}_{t,-}\cup\tau_t\cup\boldsymbol{\tau}_{t,+}$ for trade cost. Now, at time t, suppose that there is a change in the path of trade cost to $\boldsymbol{\tau}'_f$ so that $f=\{t:\tau_t\neq\tau'_t\}$. To characterize the anticipatory trade elasticity at horizons $t+f-a$ for $0\le a<f$, we follow steps similar to those we took to establish Proposition A.8.

For $0<a<f$, we have that

$$\ln\frac{\lambda_{sd,t+f-a}^{k}}{\lambda_{sd,t-1}^{k}} = \begin{cases}\ln\frac{\lambda_{sd,t-k}^{0}}{\lambda_{sd,t-1}^{k}}+\Delta\boldsymbol{G}_{sd,t,t+h}^{k} & k>f-a\ge 0\\ \frac{\theta+1-\sigma}{\sigma-1}\ln\frac{1+\Psi_{sd,t+f-a-k}}{1+\Psi_{sd,t-1}}+\ln\frac{\lambda_{sd,t-1}^{0}}{\lambda_{sd,t-1}^{k}}+\Delta\boldsymbol{G}_{sd,t-1,t+f-a-k}^{EK}+\Delta\boldsymbol{G}_{sd,t+f-a-k+1,t+f-h}^{k} & ,1\le k\le f-a\\ \frac{\theta+1-\sigma}{\sigma-1}\ln\frac{1+\Psi_{sd,t+f-a}}{1+\Psi_{sd,t-1}}+\Delta\boldsymbol{G}_{sd,t,t+f-a}^{EK} & ,k=0\end{cases}$$

where $\Delta\boldsymbol{G}_{sd,t,t+h}^{EK}\equiv-\theta\ln\prod_{\varsigma=0}^{h}\frac{\hat{c}_{sd,t+\varsigma}}{\hat{P}_{sd,t+\varsigma}^{0}}$ and $\Delta\boldsymbol{G}_{sd,t,t+h}^{k}=(1-\sigma)\ln\prod_{\varsigma'=t+1}^{t+h}\frac{\hat{c}_{sd,\varsigma'}}{P_{sd,\varsigma'}^{k}}$. Using the expression for $\partial\ln(1+\Psi_{sd,t})/\partial\ln\{\tau_{sd,t+h'}\}_{h'\ge h}$ derived earlier, we obtain

$$\frac{\partial\ln\left(\lambda_{sd,t+f-a}^{0}/\lambda_{sd,t-1}^{0}\right)}{\partial\ln\{\tau_{sd,t+f}\}_{h\ge f}} = (-\theta+\sigma-1)\frac{\Psi_{sd,t+f}}{1+\Psi_{sd,t+f-a}}\left(\frac{1-\zeta}{1+r}\right)^{a},\quad \text{for } 1<a\le f.$$

For $k>0$, approximating up to a first order around an initial steady state yields:

$$\begin{aligned}\frac{\partial\ln\lambda_{sd,t+f-a}^{k}/\lambda_{sdi,t-1}^{k}}{\partial\ln\{\tau_{sd,t+h}\}_{h\ge f}} &\approx \begin{cases}0, & \text{if } k>f-a\\ \frac{\partial\ln\left(\lambda_{sd,t+f-a-k}^{0}/\lambda_{sd,t-1}^{0}\right)}{\partial\ln\{\tau_{sd,t+h'}\}_{h'\ge f}}+\frac{\partial\ln\frac{\lambda_{sd,t-1}^{0}}{\lambda_{sd,t-1}^{k}}\prod_{\varsigma=t+f-a-k+1}^{t+f-a}\hat{c}_{sd,\varsigma}^{-(\sigma_i-1)}/\Phi_{d,\varsigma}^{0}}{\partial\ln\{\tau_{sd,t+h}\}_{h\ge f}}, & \text{if } 0<k\le f-a\end{cases}\\ &= \begin{cases}0, & \text{if } k>f-a\\ -(\theta+1-\sigma)\frac{\Psi_{sd,t+f}}{1+\Psi_{sd,t+f-a-k}}\left(\frac{1-\zeta}{1+r}\right)^{a+k}, & \text{if } 0<k\le f-a\end{cases}\end{aligned}$$

Letting $h=f-a$, the partial equilibrium response of bilateral expenditures at time $t+h$ to a one-time permanent shock to anticipated trade cost at time $t+f$ can then be written

$$\frac{\partial\ln(\lambda_{sd,t+h}/\lambda_{sd,t-1})}{\partial\ln\{\tau_{sd,t+h'}\}_{h'\ge f}} = -(\theta+1-\sigma)\frac{\Psi_{sd,t+f}}{1+\Psi_{sd,t-1}}\left(\frac{1-\zeta}{1+r}\right)^{f-h}\sum_{k=0}^{h}\omega_k\left(\frac{1-\zeta}{1+r}\right)^{k}\frac{\Psi_{sd,t-1}}{\Psi_{sd,t+h-k}}$$

where $\omega_k \equiv \frac{\left(\frac{P^k_{dj,t}}{P_{dj,t}}\right)^{-(\sigma_i-1)}\lambda^k_{sdj,t}}{\sum_k \left(\frac{P^k_{dj,t}}{P_{dj,t}}\right)^{-(\sigma_i-1)}\lambda^k_{sdj,t}} = \frac{\mu_t(k)\lambda^k_{sdj,t}}{\sum_k \mu_t(k)\lambda^k_{sdj,t}}$.

If $h = 0$, this expression simplifies to

$$\varepsilon^0_{sdi,t-1,t+f} = -\left(\theta + 1 - \sigma\right) \frac{\Psi_{sd,t+f}}{1 + \Psi_{sd,t}} \left(\frac{1-\zeta}{1+r}\right)^f \zeta$$

When $t-1$ is a steady state, we can use Proposition A.8 to further simplify the terms $\omega_k = \mu^*(k) = \zeta\left(1-\zeta\right)^k$, and $\frac{\Psi_{sd,t+f}}{1+\Psi_{sd,t-1}} = 1 - r - \zeta$. Hence:

$$\varepsilon^0_{i,f} = -\left(\theta + 1 - \sigma\right)\left(1 - r - \zeta\right) \left(\frac{1-\zeta}{1+r}\right)^f \zeta$$

For the general case with $=> 0,$:

$$\begin{aligned}
\varepsilon^h_{sdi,} &\equiv \sum_{\varsigma=f}^{\infty} \frac{\partial \ln X_{sdi,t}}{\partial \ln \tau_{sd,t+\zeta}}\Bigg| \\
&= -\left(\theta + 1 - \sigma\right)\zeta \left(\frac{1-\zeta}{1+r}\right)^{f-h} \sum_{k=0}^{h} \left(\frac{(1-\zeta)^2}{1+r}\right)^k \\
&= -\left(\theta + 1 - \sigma\right)\left(1 - r - \zeta\right)\zeta \left(\frac{1-\zeta}{1+r}\right)^{f-h} \frac{1 - \left(\frac{(1-\zeta)^2}{1+r}\right)^{h+1}}{1 - \frac{(1-\zeta)^2}{1+r}}.
\end{aligned}$$

and when $h = f - 1$, then

$$\lim_{f\to\infty} \varepsilon^{f-1,f}_i = -\lim_{f\to\infty} \left(\theta + 1 - \sigma\right)\left(1 - r - \zeta\right)\zeta \frac{(1-\zeta)}{1 + r - (1-\zeta)^2}$$

A.10 Proof of Proposition 4

We begin by rearranging equation (24) to express the prices of composite goods in terms of home expenditure shares

$$\begin{aligned}
\lambda_{ddi,t} P^{-(\sigma_i-1)}_{di,t} &= \gamma_i \mu_i(0) \frac{\Phi^0_{di,t}}{\Upsilon_{di,t}} \left(\Upsilon^0_{di,t}\right)^{-\frac{1-\sigma_i}{\theta_i}} \lambda^0_{ddi,t} + \sum_{k\geq 1} \gamma_i \mu_i(k) \frac{\Phi^0_{di,t-k}}{\Upsilon_{di,t-k}} \left(\Upsilon^0_{di,t-k}\right)^{-\frac{1-\sigma_i}{\theta_i}} \Phi^k_{di,t} \lambda^k_{ddi,t} \\
&= \gamma_i \mu_i(0) \frac{\Phi^0_{di,t}}{\Upsilon_{di,t}} \left(\Upsilon^0_{di,t}\right)^{-\frac{1-\sigma_i}{\theta_i}} \lambda^0_{ddi,t} + \sum_{k\geq 1} \gamma_i \mu_i(k) \frac{\Phi^0_{di,t-k}}{\Upsilon_{di,t-k}} \left(\Upsilon^0_{di,t-k}\right)^{-\frac{1-\sigma_i}{\theta_i}} \lambda^0_{ddi,t-k} \left(\frac{c_{ddi,t}}{c_{ddi,t-k}}\right)^{-(\sigma_i-1)} \\
&= \gamma_i \mu_i(0) \left(\frac{c^{-\theta_i}_{dd,t}}{\nu_{ddi,t}}\right)^{-\frac{1-\sigma_i}{\theta_i}} \nu_{ddi,t} + \sum_{k\geq 1} \gamma_i \mu_i(k) \left(\frac{c^{-\theta_i}_{dd,t-k}}{\nu_{ddi,t-k}}\right)^{-\frac{1-\sigma_i}{\theta_i}} \nu_{ddi,t-k} \left(\frac{c_{dd,t}}{c_{dd,t-k}}\right)^{-(\sigma_i-1)},
\end{aligned}$$

where the third equality obtains from substituting $\frac{\Phi^0_{di,t}}{\Upsilon_{di,t}} = \frac{\nu_{ddi,t}}{\lambda^0_{ddi,t}}(1+\Psi_{ddi,t})$ and $\Upsilon^0_{di,t} = \frac{c_{dd,t}^{-\theta_i}\left(1+\Psi_{ddi,t}\right)^{\theta_i/(\sigma_i-1)}}{\nu_{ddi,t}}$. It follows that

$$P_{di,t}^{(1-\sigma_i)} = c_{dd,t}^{(1-\sigma_i)} \left(\nu_{ddi,t}\right)^{\frac{1-\sigma_i}{\theta_i}} \frac{1}{\lambda_{ddi,t}} \gamma_i \left[\mu_i(0)\nu_{ddi,t} + \sum_{k\geq 1} \mu_i(k) \left(\frac{\nu_{ddi,t}}{\nu_{ddi,t-k}}\right)^{\frac{\sigma_i-1}{\theta_i}} \nu_{ddi,t-k}\right]$$

where the price index is expressed in terms of unit cost, the share of domestic expenditures on domestically produced goods and the allocation of extensive margin demand across source countries in each of the goods baskets $\{\Omega^k_{i,t}\}_{k=0}^{\infty}$. With the unit cost under Cobb-Douglas technology, the above equation can be rewritten as

$$\frac{P_{di,t}}{w_{d,t}} = \left(\nu_{ddi,t}\right)^{\frac{1}{\theta_i}} \left(\lambda_{ddi,t}\right)^{1/(\sigma_i-1)} \left(\gamma_i \xi_{di,t}\right)^{-1/(\sigma_i-1)} \alpha_{di}^{-\alpha_{di}} \prod_j \left(\frac{P_{dj,t}}{\alpha_{dji} w_{d,t}}\right)^{\alpha_{dji}}$$

where

$$\xi_{di,t} \equiv \mu_i(0)\nu_{ddi,t} + \sum_{k\geq 1} \mu_i(k) \left(\frac{\nu_{ddi,t}}{\nu_{ddi,t-k}}\right)^{\frac{\sigma_i-1}{\theta_i}} \nu_{ddi,t-k}.$$

Taking logs yields

$$\ln \frac{P_{di,t}}{w_{d,t}} = \ln B_{si,t} + \sum_j \alpha_{sji} \ln \frac{P_{sj,t}}{w_{s,t}},$$

where $B_{di,t} \equiv \alpha_{di}^{-\alpha_{di}} \left(\prod_j \alpha_{dji}^{-\alpha_{dji}}\right) \left(\nu_{ddi,t}\right)^{\frac{1}{\theta_i}} \left(\lambda_{ddi,t}\right)^{1/(\sigma_i-1)} \left(\gamma_i \xi_{di,t}\right)^{-1/(\sigma_i-1)}$. In matrix notation, this leads to

$$(\mathbf{I} - A_d) \ln \hat{\boldsymbol{P}}_{d,t} = \ln \boldsymbol{B}_{d,t},$$

where $A_d = \{\alpha_{dji}\}$ and $\ln \hat{\boldsymbol{P}}_{d,t}$ and $\ln \boldsymbol{B}_{d,t}$ are $I \times 1$ vectors. Inverting this system of equations, we obtain

$$\frac{P_{di,t}}{w_{d,t}} = \prod_j B_{dj,t}^{\bar{a}_{dji}},$$

where $\bar{a}_{dji}$ is the (j,i) entry of the Leontief matrix $(\mathbf{I} - A_d)^{-1}$. The consumer price index in country d can be written as

$$P_{d,t} = \prod_i \left(P_{di,t}\right)^{\eta_i} = w_{d,t} \prod_{i,j} B_{dj,t}^{\bar{a}_{dji}\eta_i} = w_{d,t} \prod_j B_{dj,t}^{\sum_i \bar{a}_{dji}\eta_i}$$

It follows that the real wage is

$$W_{d,t} \equiv \frac{w_{d,t}}{P_{d,t}} = \prod_j B_{dj,t}^{-\sum_i \bar{a}_{dji}\eta_i}.$$

Taking the ratio between real wages in $t-1$ and $t+h$ yields

$$\frac{W_{d,t+h}}{W_{d,t-1}} = \prod_j \left[\left(\frac{\nu_{ddj,t+h}}{\nu_{ddj,t-1}}\right)^{-\frac{1}{\theta_j}} \left(\frac{\lambda_{ddj,t+h}}{\lambda_{ddj,t-1}}\right)^{-\frac{1}{\sigma_j-1}} \left(\frac{\xi_{dj,t+h}}{\xi_{dj,t-1}}\right)^{\frac{1}{\sigma_j-1}}\right]^{\sum_i \bar{a}_{dji}\eta_i}.$$

If $t-1$ is a steady state, then $\nu_{ddj,t-1} = \lambda^k_{ddj,t-1} = \lambda_{ddj,t-1}$ for all $k \in \{0,1,2,...\}$ and the above expression simplifies to

$$\begin{aligned}
\frac{W_{d,t+h}}{W_{d,t-1}} &= \prod_j \left[\left(\frac{\nu_{ddj,t+h}}{\lambda_{ddj,t-1}} \right)^{-\frac{1}{\theta_j}} \left(\frac{\lambda_{ddj,t+h}}{\xi_{dj,t+h}} \right)^{-\frac{1}{\sigma_j - 1}} \right]^{\sum_i \bar{a}_{dji}\eta_i} \\
&= \prod_j \left[\left(\frac{\lambda_{ddj,t+h}}{\lambda_{ddj,t-1}} \right)^{-\frac{1}{\theta_j}} \left(\frac{\nu_{ddj,t+h}}{\lambda_{ddj,t+h}} \right)^{-\frac{1}{\theta_j}} \left(\frac{\lambda_{ddj,t+h}}{\xi_{dj,t+h}} \right)^{-\frac{1}{\sigma_j - 1}} \right]^{\sum_i \bar{a}_{dji}\eta_i} \\
&= \prod_j \left[\left(\frac{\lambda_{ddj,t+h}}{\lambda_{ddj,t-1}} \right)^{-\frac{1}{\theta_j}} \left(\Xi_{dj,h} \right)^{\frac{1}{\sigma_j - 1}} \right]^{\sum_i \bar{a}_{dji}\eta_i}, \qquad \text{(A.26)}
\end{aligned}$$

where

$$\begin{aligned}
\Xi_{dj,h} &\equiv \frac{\xi_{dj,t+h}}{\lambda_{ddj,t+h}} \left(\frac{\nu_{ddj,t+h}}{\lambda_{ddj,t+h}} \right)^{\frac{1-\sigma_j}{\theta_j}} \\
&= \left(\frac{\nu_{ddj,t+h}}{\lambda_{ddj,t+h}} \right)^{\frac{1-\sigma_j}{\theta_j}} \left\{ \zeta_j \frac{\nu_{ddj,t+h}}{\lambda_{ddj,t+h}} + \sum_{k \geq 1} \zeta_j (1-\zeta_j)^k \left(\frac{\nu_{ddj,t+h}}{\nu_{ddj,t+h-k}} \right)^{\frac{\sigma_i - 1}{\theta_i} - 1} \frac{\nu_{ddj,t+h}}{\lambda_{ddj,t+h}} \right\} \\
&= \sum_{k=0}^{h} \zeta_j (1-\zeta_j)^k \left(\frac{\nu_{ddj,t+h-k}}{\lambda_{ddj,t+h}} \right)^{\frac{\theta_j - \sigma_j + 1}{\theta_j}} + (1-\zeta_j)^h \left(\frac{\lambda_{ddj,t-1}}{\lambda_{ddj,t+h}} \right)^{\frac{\theta_j - \sigma_j + 1}{\theta_j}}
\end{aligned}$$

follows from combining the last two factors in the second line of (A.26). This completes the proof.

Appendix B Theoretical Extensions

B.1 Technology Change

In this section, we consider an extension that allows for productivity change. We provide explicit microfoundations for time-varying Fréchet scale parameters and the resulting $z_{si,t}(\omega)$ that vary over time. Under this microfoundation, the highest-realized productivity $z_{si,t}(\omega)$ among producers for a good may evolve for two reasons, either because of innovation within existing producers; or, due to producer entry. As will become clear once we integrate this theory into our model, this distinction between within- and across-producer technology change is important when supplier adjustment happens gradually over time.

B.1.1 Innovation and Diffusion

Suppose that at time $t = 0$, the highest-realized productivity $z_{si,0}(\omega)$ among all potential producers for a good ω in country-industry si follows a Fréchet distribution given by

$$F_{si,0}(z) = \Pr[z_{si,0}(\omega) < z] = e^{A_{si,0} z^{-\theta_i}}.$$

The distribution $F_{si,0}(z)$ evolves over time, governed by two processes; one that determines the productivity of new potential producers and another governing the productivity of the incumbent producer behind the realization $z_{si,t}(\omega)$. We now describe each of these processes in turn, starting with the arrival process for new potential producers.

Produtivity of new potential producers. Following Kortum (1997) and Buera and Oberfield (2020), new potential producers arrive stochastically and exogenously. The productivity z of a new potential producer for a good ω in industry i arriving in location s at time t is $z = q\,(z')^{\beta_i^E}$, which has two random components. There is an insight derived from an existing technique, z', which is drawn from the distribution $F_{sdi,t-1}$, and an original component q, which is drawn from an exogenous distribution. We assume that the arrival rate of new techniques with original component greater than q' is distributed Poisson with mean $\alpha_{si,t}^E q'^{-\theta_i}$.

The diffusion parameter $\beta_i \in [0, 1)$ governs the importance of existing knowledge for the productivity of entrants. The parameter $\alpha_{si,t}^E$ controls the contribution of entrants to overall technological progress.

Productivity shocks to incumbent producers. We extend Buera and Oberfield (2020) to allow for innovation by incumbent producers. In contrast to entrants, incumbent producers receive multiple potential ideas for how to improve their productivity.

When a new idea arrives to an incumbent at time t, its productivity is z is $q\,[z']^{\beta_i^I}$, with the existing component $z' \sim F_{si,t-1}$, $\beta_i^I \in [0, 1)$ and the original component q drawn from an exogenous distribution. For incumbents, the arrival rate of new ideas with original component greater than q' is distributed Poisson with mean $\alpha_{si,t}^I q'^{-\theta_i}$, with $\alpha_{si,t}^I$ governing the contribution of incumbents to overall technological progress.

Evolution of the frontier of knowledge. We characterize the evolution of supplier productivity the intermediate goods in each of the partitions $\{\Omega_{i,t}\}_{k=0}^{\infty}$. Let $F_{si,t+\Delta}^k(z)$ denote the probability that an incumbent in period $t-k$ will have a productivity less than z at time $t+\Delta$, which will coincide with source country s's productivity distribution for potential suppliers across goods $\omega \in \Omega_{i,t+\Delta}^k$.

Given the frontier distribution at time $F_{si,t}^0$, the exogenous arrival rate of new producers $\alpha_{si,t}^E z^{-\theta_i}$ and the exogenous arrival rate of ideas to incumbents $\alpha_{si,t}^I z^{-\theta_i}$, the frontier distribution at time $t+\Delta$ satisfies

$$1 - F_{si,t+\Delta}^0(z) = 1 - F_{si,t}^0(z) + \Delta F_{si,t}^0(z) \int_0^{\infty} \left[\alpha_{si,t}^E \left(z / z'^{\beta_i^E} \right)^{-\theta} + \alpha_{si,t}^I \left(z / z'^{\beta_i^E} \right)^{-\theta} \right] dz'$$

In words, the fraction of goods for which the frontier productivity exceeds z at $t+1$ is given by those for which the frontier exceeds z at t, $1 - F_{si,t}^0(z)$, and, among the remainder, those for which a new idea with productivity exceeding z is received by either an entrant or an incumbent between t and $t+1$. To find the arrival rate of these ideas, note that given an insight z', the arrival rate of ideas that, in combination with that insight, would produce a productivity greater than z is $\alpha^E_{si,t}\left(z/z'^{\beta^E_i}\right)^{-\theta_i}$, for entrants; and $\alpha^I_{si,t}\left(z/z'^{\beta^E_i}\right)^{-\theta_i}$, for incumbents. Integrating over possible insights gives the arrival rate of ideas with productivity greater than q.

From the same logic, it follows that the productivity distribution among country s's potential producers for a good $\omega \in \Omega^k_{i,t+\Delta}$ with $k>0$ will satisfy

$$1-F^k_{si,t+1}(z) = 1-F^{k-1}_{si,t}(z) + \Delta F^{k-1}_{si,t}(z)\int_0^\infty \left[\alpha^I_{si,t}\left(z/z'^{\beta^E_i}\right)^{-\theta}\right] dz', \text{ for } k>0.$$

The following proposition shows that $F^k_{si,t}$ follows a Fréchet distribution for all k, with $k-$specific location parameter depending on entry.

Proposition 5. *The frontier distribution at time t satisfies*

$$F^0_{si,t}(z) = e^{-z^{-\theta_i}} A_{si,t-1}\hat{A}^0_{si,t},$$

where

$$\hat{A}^0_{si,t} = 1 + \frac{1}{A^0_{si,t-1}}\int_0^1 \left[\alpha^E_{si,t}\Gamma(1-\beta^E_i)\left(A^0_{si,t-\Delta}\right)^{\beta^I_i} + \alpha^E_{si,t}\Gamma(1-\beta^I_i)\left(A^0_{si,t-\Delta}\right)^{\beta^E_i}\right] d\Delta.$$

The productivity distribution among potential producers for each good $\omega\in\Omega^k_{i,t}$ with $k>0$ satisfies

$$F^0_{si,t}(z) = e^{-z^{-\theta_i} A^0_{si,t-k}\prod_{\varsigma=t-k+1}^{t}\hat{A}^I_{si,\varsigma}},$$

where

$$\hat{A}^I_{si,t} = 1 + \Gamma(1-\beta^I_i)\frac{\alpha^I_{si,t}}{A^0_{t-1}}\int_0^1 \left(A^0_{si,t-\Delta}\right)^{\beta^I_i} d\Delta$$

Proof. The result follows from using Proposition 1 in Buera and Oberfield (2020) to evaluate the functional difference equation for $F^k_{si,t-1}$ at the limit $\Delta\to 0$, under the imposition that $\alpha_{si,t-\Delta}=\alpha_{si,t}$ to evaluate $\int_0^1 \frac{d\ln F_{si,t-\Delta}}{d\ln\Delta} d\Delta$.

□

Hence, we can flexibly accommodate different models technological change in the model by allowing for country-industry-time-specific Fréchet productivity distributions across partitions of good $\{F^k_{si,t}\}^\infty_{k=0}$. A model where technological change is driven by incumbents only corresponds to the special case special case where $F^k_{si,t}=F_{si,t}$ for all k. A model where all progress is due to entry corresponds to the special case where $F^k_{si,t}=F^0_{si,t-k}$ for all $k>0$.

B.1.2 Equilibrium characterization

Under Proposition B.1.1, we can specify the distribution of producer-level productivity across intermediate goods $\omega \in \Omega^k_{si,t}$ as follows:

$$F^k_{si,t} = \Pr\left[\omega \in \Omega^k_{i,t} : z_{si,t}(\omega) < z\right] = \begin{cases} e^{-A^0_{si,t} z^{-\theta_i}}, & k = 0 \\ e^{-A^0_{si,t-k} \prod_{\varsigma=t-k+1}^{t} \hat{A}_{si,t} z^{-\theta_i}}, & k > 0 \end{cases},$$

Under this specification, the prices $p_{di,t}$ paid in destination d for goods $\omega \in \Omega^k_{i,t}$ sourced from country s now incorporate changes in incumbent producer productivity between t and $t-k$:

$$p_{di,t}(\omega) = p_{di,t-k}(\omega) \prod_{\varsigma=t-k+1}^{t} \left(\hat{A}^I_{si,\varsigma}\right)^{\theta_i} \left(\hat{c}_{sdi,\varsigma}\right)^{1-\sigma_i}, k > 0$$

Moreover, the productivity frontier among the potential producers for each good now evolves over time.

Demand for optimally sourced goods. Incorporating changes in incumbent's productivity, the option value of supply relations to country s for industry-i traders in d at time t now accounts for future productivity changes:

$$\Psi_{sdi,t} = \sum_{u=1}^{\infty} \left(\frac{1-\zeta_i}{1+r}\right)^u \left\{ \prod_{\varsigma=t-k+1}^{t} \hat{A}^I_{si,\varsigma} \left(\frac{\hat{c}_{sdi,\varsigma}}{\hat{P}_{di,\varsigma}}\right)^{1-\sigma_i} P_{di,\zeta} \hat{}\hat{}_{di,\zeta} Y_{di,\varsigma} \right\}.$$

From the above expression for option values, we obtain the same characterization of each country d's extensive margin demands, $\{\nu^0_{sdi,t}\}_{s\in\mathcal{N},i\in\mathcal{N}}$, and expenditure allocations, $\{\lambda^0_{sdi,t}\}_{s\in\mathcal{N},i\in\mathcal{N}}$, as in the main text. Setting $A_{si} = A^0_{si,t}$, the same is true for the expressions for the price indices $\{P^0_{di,t}\}_{d\in\mathcal{N},i\in\mathcal{N}}$ of each of the goods baskets $\Omega^0_{i,t}$. .

Demand for legacy goods. A destination d's trade shares and ideal price index for industry-i legacy varieties with $k > 0$ obtain from adjusting the trade shares $\{\lambda^0_{sdi,t-k}\}_{s\in\mathcal{S}}$ and price index $P^0_{di,t-k}$ at time $t-k$ for subsequent changes in supplier productivities, trade cost and factory gate pricese:

$$\lambda^k_{sdi,t} = \frac{\lambda^0_{sdi,t-k} \prod_{\varsigma=t-k+1}^{t} \hat{A}^I_{si,\varsigma} \left(\hat{c}_{sdi,\varsigma}\right)^{1-\sigma_i}}{\Phi^k_{di,t}}, k > 0$$

and

$$P^k_{di,t} = P^0_{di,t-k} \left(\frac{\mu_{i,t}(k)}{\mu_{i,t-k}(0)} \Phi^k_{di,t}\right)^{1/(1-\sigma_i)}, k > 0$$

where

$$\Phi^k_{di,t} = \sum_n \lambda^0_{ndi,t-k} \left[\prod_{\varsigma=t-k+1}^{t} \hat{A}^I_{ni,\varsigma} \left(\hat{c}_{ndi,\varsigma}\right)^{1-\sigma_i}\right], k > 0.$$

Aggregation and equilibrium. Given the trade shares and price index for the goods in each of the sets $\{\Omega_{i,t}^{k}\}_{k=0}^{\infty}$ described above, the vector of trade shares $\{\lambda_{sdi,t}\}_{s\in\mathcal{I}}$ in (24) and the price index $P_{di,t}$ in (23) of the main text to characterize each destination d's total demand for industry-i goods. Therefore, the conditions for goods and factor market clearing within each period t also remain unchanged, given by equations (25)-(27).

B.2 Imperfect factor mobility across industries

In this section, we consider an extension that allows for imperfect factor mobility across industries and resulting transitional dynamics in labor supply as another source of gradual trade adjustment. Incorporating the dynamic formulation of industry-specific labor supply curves by Caliendo, Dvorkin and Parro (2019), we show that our model can accommodate transitional equilibrium dynamics in factor supply at no loss in tractability.

B.2.1 Households

We follow Caliendo, Dvorkin and Parro (2019) in assuming that households must choose their industry of employment one period in advance, under full information about the economic conditions in all industries and subject to time-varying mobility costs. That is, a household in country s now starts each period t working in a sector s that was chosen during the previous period $t-1$. In period t, this household supplies its unit of labor at competitive sector-wide wage $w_{di,t}$, consumes the local final good and, then, chooses which industry to work for in the subsequent period $t+1$. The value of a d resident employed in sector i at time t is given by:

$$v_{di,t} = \ln\left(\frac{w_{di,t}}{P_{d,t}}\right) + \max_{j\in\mathcal{I}}\left\{\beta\mathbb{E}\left[v_{di,t+1}\right] - m_{dij,t} + \chi\epsilon_{dj,t}\right\},$$

where $m_{dij,t}$ is the utility cost of moving from sector i to j facing each household in country d at time t, and $\epsilon_{dj,t}$ are stochastic i.i.d. preference shocks drawn from a Gumbel distribution with zero mean and dispersion parameter equal to 1. In writing this value function, we implicitly assumed that workers hold log preferences over consumption of the aggregate final good whose local price is denoted by $P_{d,t}$, as in the main text.

The well-known characterization of the optimal discrete choice under Gumbel distributed shocks yields the expected lifetime utility of a worker employed in country d's industry i at time t:

$$\overline{v}_{di,t} \equiv \mathbb{E}\left[v_{di,t}\right] = \ln\left(\frac{w_{di,t}}{P_{s,t}}\right) + \chi\ln\left(\sum_{j\in\mathcal{N}}\exp\left(\beta\overline{v}_{dj,t+1} - m_{dij,t}\right)^{1/\chi}\right). \tag{B.27}$$

Moreover, the share of households in d transitioning from working for a producer in industry i to working for a producer in industry j is given by

$$\varphi_{dij,t} = \frac{\exp\left(\beta\overline{v}_{dj,t+1} - m_{dij,t}\right)^{1/\chi}}{\sum_{k\in\mathcal{I}}\exp\left(\beta\overline{v}_{dk,t+1} - m_{dik,t}\right)^{1/\chi}}. \tag{B.28}$$

From the initial distribution of labor across industries and the labor flows at time t, we obtain the labor supply curve at $t+1$:

$$L_{di,t+1} = \sum_{j\in\mathcal{I}} \varphi_{dji,t} L_{dj,t}.$$

B.2.2 Industry-level trade flows and prices

Under the same formulation of technologies and sourcing frictions as in the main text, the vector of trade shares $\{\lambda_{sdi,t}\}_{s\in\mathcal{I}}$ in (24) and the price index $P_{di,t}$ in (23) continue to characterize each destination d's optimal within-period demand for industry-i goods. The unit cost components behind these demand components now incorporate industry-specific wage rates and can be written as:

$$c_{sdi,t} = \Theta_{si}\tau_{sdi,t}\left(w_{si,t}\right)^{\alpha_{si}} \prod_{j\in\mathcal{J}} \left(P_{sj,t}\right)^{\alpha_{sji}},$$

for all $s,d\in\mathcal{S}$, $i\in\mathcal{I}$ and t.

B.2.3 Within-period market clearing

At time t, the vector of labor supply $\{L_{di,t}\}_{i\in\mathcal{I}}$ is predetermined by the optimal labor allocation decisions of households in the prior period, described by (B.2.1). The vector of wages $\{w_{di,t}\}_{d\in\mathcal{N},i\in\mathcal{I}}$ is concurrently determined with the vector of trade shares $\{\lambda_{sdi,t}\}_{s,d\in\mathcal{N},i\in\mathcal{I}}$. To define the conditions for within-period factor and goods market clearing, we can follow the exposition in the main text and express the total value of each industry's sales $X_{si,t}$ as

$$X_{si,t} = \sum_{d\in\mathcal{N}} \lambda_{sid,t}\left[\eta_{di}E_{d,t} + \sum_{j\in\mathcal{I}} \alpha_{dij}X_{dj,t}\right], \tag{B.29}$$

A destination d's total consumption expenditures now incorporate industry-specific wages and labor supplies:

$$E_{d,t} = \sum_{i}\left\{w_{di,t}L_{di,t} + \Pi_{di,t}\right\} + D_{d,t}. \tag{B.30}$$

As before, each destination's profit income derives from the sales of local assemblers and can be written:

$$\Pi_{d,t} = \sum_{i\in\mathcal{I}} \frac{1}{\sigma_i}\left[\eta_{di}E_{d,t} + \sum_{j\in\mathcal{I}} \alpha_{dij}X_{dj,t}\right]. \tag{B.31}$$

Wages then solve the following set of factor market clearing conditions:

$$w_{di,t}L_{di,t} = (1-\alpha_{di})\frac{\sigma_i - 1}{\sigma_i}X_{di,t}, \text{ for } i\in\mathcal{I} \tag{B.32}$$

B.2.4 Equilibrium

Definition 2. *An economy is described by a set of time-invariant parameters summarizing technologies, preferences,* $\boldsymbol{\Theta} = \{\theta_i, \sigma_i, \{\alpha_{dji}\}_{j\in\mathcal{I}}, \varphi_{di}, A_{di}, \eta_{di},\}_{i\in\mathcal{I}}$*, sourcing frictions* $\boldsymbol{\zeta} = \{\zeta_i\}_{i\in\mathcal{I}}$ *as well as measures* $\boldsymbol{\mu}_0 =$

$\{\mu_{di,t_0}(k)\}_{i\in\mathcal{N},d\in\mathcal{S},k\mathbb{N}_0}$ *and* $\boldsymbol{L}_0 = \{L_{di,t_0}\}_{i\in\mathcal{N},d\in\mathcal{S}}$ *for some* t_0. *Given a path for trade costs* $\boldsymbol{\tau} \equiv \{\tau_t\}_{t\in\mathbb{N}} = \{\tau_{sid,t}\}_{s,d\in\mathcal{N},i\in\mathcal{I},t\in\mathbb{N}}$:

1. *A static equilibrium at time* t *is a vector of wages* $\boldsymbol{w}_t = w\left(\tau_t, \hat{\boldsymbol{w}}_{-t}, \hat{\boldsymbol{\tau}}_{-t}, \boldsymbol{\Theta}, \boldsymbol{\zeta}, \boldsymbol{\mu}_0, \boldsymbol{L}_t\right)$ *that solves equations* (24), (25), (26) *and* (B.32) *for all* $s, d \in \mathcal{N}$, $i \in \mathcal{I}$, *given* $\boldsymbol{L}_t$, $\hat{\boldsymbol{w}}_{-t} = \{\hat{w}_{di,\varsigma}\}_{d\in\mathcal{N},i\in\mathcal{I}\varsigma\in\mathbb{N}\setminus\{t\}}$ *and* $\hat{\boldsymbol{\tau}}_{-t} = \{\hat{\tau}_{sid,\varsigma}\}_{s,d\in\mathcal{N},i\in\mathcal{I},\varsigma\in\mathbb{N}\setminus\{t\}}$.
2. *A dynamic equilibrium at time* t *is a path of wages* $\{\boldsymbol{w}_t\}_{t\in\mathbb{N}}$ *and labor allocations* $\{\boldsymbol{L}_t\}_{t\in\mathbb{N}}$ *satisfying* $\boldsymbol{w}_t = w\left(\tau_t, \hat{\boldsymbol{w}}_{-t}, \hat{\boldsymbol{\tau}}_{-t}, \boldsymbol{\Theta}, \boldsymbol{\zeta}, \boldsymbol{L}_t, \boldsymbol{\mu}_0\right)$ *and* (B.2.1) *for for all* t.
3. *A dynamic equilibrium at time* t *is a steady state if* $\boldsymbol{w}_t = \boldsymbol{w}_{t'}$ *and* $\boldsymbol{L}_t = \boldsymbol{L}_{t'}$ *for all* $t > t$.

Appendix C Detail on Quantitative Applications

The quantitative analysis proceeds in two steps. We first calibrate the model economy to an initial steady state. We then feed in a path of shocks into the model and compute the path of counterfactual outcomes.

C.1 2018 US China Trade War Tariffs

Table C.1: US Tariff Increases on Imports from China

	2017 Imports in Total (%)		Cumulative Increases in Tariffs (%)		
Affected Sector in Model	OECD ICIO	US Census	2018	2019	2020–
Agriculture, forestry and fishing	0.5	0.6	2.5	14.7	20.6
Mining and quarrying	0.0	0.1	1.0	5.6	7.4
Food products, beverages and tobacco	1.9	0.8	2.6	15.5	22.3
Textiles, textile products, leather and footwear	17.7	12.8	0.6	6.6	13.8
Wood and products of wood and cork	1.3	0.8	2.9	16.4	22.1
Paper products and printing	1.2	1.3	2.1	11.0	15.8
Coke and refined petroleum products	0.2	0.1	2.4	14.2	20.5
Chemical and chemical products	3.2	3.1	2.7	12.7	17.7
Pharmaceuticals, medicinal and botanical products	1.3	0.5	0.0	0.1	0.1
Rubber and plastics products	3.7	3.6	2.2	10.9	15.1
Other non-metallic mineral products	2.4	1.7	2.1	12.3	17.4
Basic metals	1.0	0.9	8.8	22.4	24.5
Fabricated metal products	3.8	4.1	3.4	15.0	20.0
Computer, electronic and optical equipment	29.0	36.3	2.0	8.1	11.2
Electrical equipment	9.1	8.9	3.9	14.9	18.8
Machinery and equipment, nec	6.9	7.3	6.1	18.3	22.3
Motor vehicles, trailers and semi-trailers	4.2	3.2	4.7	19.3	24.7
Other transport equipment	0.8	0.7	7.2	20.5	24.0
Furniture and other manufacturing	11.7	13.3	1.1	7.1	11.0

Notes: "Imports in Total" are the shares of industry-specific US imports in total imports from China. "OECD ICIO" refers to the input-output table used for calibrating the model. "US Census" refers to the HS-level bilateral trade data accessed via USA Trade Online. The tariff changes are aggregated based on weights derived from the US Census data. Tariff changes from Fajgelbaum et al. (2020).

Table C.2: Retaliatory Tariff Increases on US Exports to China

	2017 Exports in Total (%)		Cumulative Increases in Tariffs (%)		
Affected Sector in Model	OECD ICIO	US Census	2018	2019	2020–
Agriculture, forestry and fishing	14.7	15.1	11.9	31.1	31.3
Mining and quarrying	7.7	7.0	3.5	11.2	14.0
Food products, beverages and tobacco	4.2	2.8	10.4	19.9	21.0
Textiles, textile products, leather and footwear	0.4	0.9	2.5	12.1	15.3
Wood and products of wood and cork	1.5	1.5	2.7	12.9	16.3
Paper products and printing	2.0	2.5	2.1	7.7	8.8
Coke and refined petroleum products	2.6	1.0	10.2	26.0	26.0
Chemical and chemical products	10.6	9.6	3.7	12.5	14.3
Pharmaceuticals, medicinal and botanical products	2.5	2.8	0.3	1.6	2.7
Rubber and plastics products	1.4	1.3	2.3	10.0	12.4
Other non-metallic mineral products	0.6	0.8	4.0	13.7	15.8
Basic metals	10.7	1.9	4.1	15.4	18.9
Fabricated metal products	1.2	1.4	2.5	10.9	13.3
Computer, electronic and optical equipment	10.4	13.8	2.4	9.3	11.2
Electrical equipment	1.4	2.5	3.8	15.8	19.4
Machinery and equipment, nec	6.0	8.0	2.1	9.1	11.1
Motor vehicles, trailers and semi-trailers	8.6	11.0	10.5	21.5	21.7
Other transport equipment	12.4	13.3	0.0	0.1	0.1
Furniture and other manufacturing	1.1	2.8	4.1	12.8	14.2

Notes: "Imports in Total" are the shares of industry-specific US imports in total imports from China. "OECD ICIO" refers to the input-output table used for calibrating the model. "US Census" refers to the HS-level bilateral trade data accessed via USA Trade Online. The tariff changes are aggregated based on weights derived from the US Census data. Tariff changes from Fajgelbaum et al. (2020).

C.2 2004 Enlargement of the EU Tariffs

Table C.3: Distribution of Tariff Shocks Across Source-Destination-Industry Tuples

source $\rightarrow$ destination	Mean	Std. Dev.	min	Q1	Q2	Q3	max
NMS $\rightarrow$ NMS	4.2	6.0	0.0	0.0	1.2	7.4	60.0
NMS $\rightarrow$ EU15	4.0	3.2	0.0	0.3	4.0	5.9	13.4
EU15 $\rightarrow$ NMS	4.3	5.2	0.0	0.0	2.7	7.4	84.2
EU15 $\rightarrow$ EU15	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Notes: Mean, standard deviation, min, max, and top boundaries for the first, second, and third quartiles of the tariff shock source-destination-industry tuples. Data are from WITS with MFN and Preferential Tariff filled according to the procedure described in the Appendix. Whenever data for 1995 were missing, we used the closest year to 1995 available in the sample.

C.3 Additional Figures

C.3.1 2018 US-China Trade War

Figure C.1: Price changes in industries directly impacted by the trade war

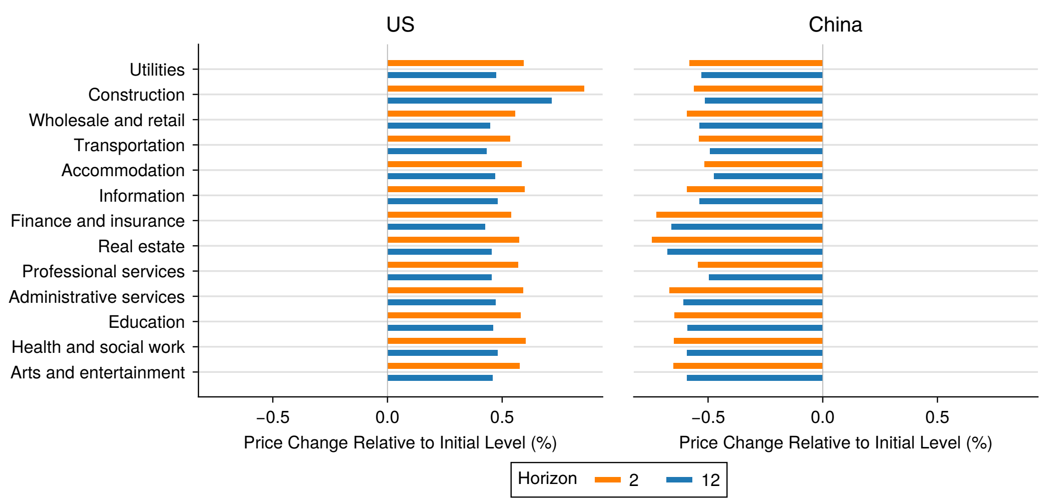

Figure C.2: Price changes in industries not directly impacted by the trade war

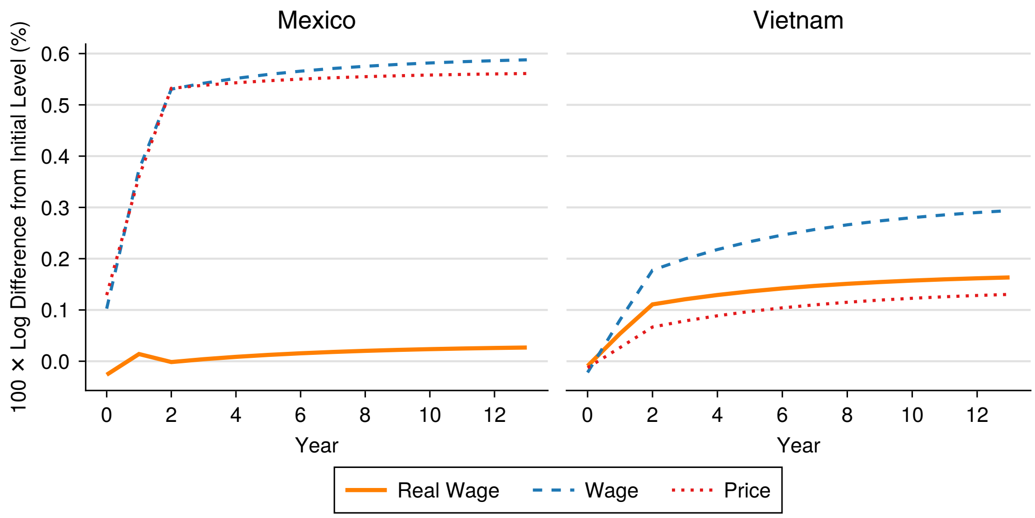

Figure C.3: Changes in Real Wages, Wages and Consumer Prices

Notes: Tariff changes implemented gradually over the first two years. "Real Wage" in year t refers to real wage changes between t and the initial steady state generated by the full model. "Wage" refers to the corresponding change in the nominal wage. "Price" is the change in the aggregate consumer price index.

C.4 Computational Algorithm

Given a path of changes in tariffs $\{\hat{\tau}_{sdi,t}\}_{t=0}^{H}$ from time 0 to H, the counterfactual outcomes are obtained by solving a nested system of nonlinear equations that determine four sets of equilibrium paths: (1) the country-industry-level total expenditure $\{X_{dj,t}\}_{t=0}^{H}$; (2) the changes in country-industry-level price indices faced by producers $\{\hat{P}_{dj,t}\}_{t=0}^{H}$; (3) the changes in country-level wages $\{\hat{w}_{d,t}\}_{t=0}^{H}$; and (4) the changes in country-industry-level option value of supplier relations $\{\widehat{1+\Psi_{sdi,t}}\}_{t=0}^{H}$. The computational algorithm is outlined below.

Step 0 Start with an initial guess of $\{X_{dj,t}\}_{t=0}^{H}$, $\{\hat{w}_{d,t}\}_{t=0}^{H}$, and $\{\widehat{1+\Psi_{sdi,t}}\}_{t=0}^{H}$. We use the values attained in the initial steady state as a staring point.

Step 1 For each period t from 0 to H, solve $X_{dj,t}$, $\hat{P}_{dj,t}$, and $\hat{w}_{d,t}$ sequentially given $\{\widehat{1+\Psi_{sdi,t}}\}_{t=0}^{H}$ following the steps below.

Step PW0 Given the shocks $\{\hat{\tau}_{sdi,t}\}_{t=0}^{H}$, $\{\hat{P}_{dj,t}\}_{t=0}^{H}$, and $\{\hat{w}_{d,\varsigma}\}_{\varsigma=0}^{t}$ obtained for any earlier period, compute the cumulative changes in unit cost for each variety with legacy $k > 0$, $\{\prod_{\varsigma=t-k}^{t} \hat{c}_{sdi,\varsigma}\}_{t=0}^{H}$.

Step PW1 Update the changes in price indices $\hat{P}_{dj,t}$ and wages $\hat{w}_{d,t}$ for the current period using a fixed-point solver.

Step PW2 Compute the changes in unit cost $\hat{c}_{sdi,t}$ for the current period using Equation (A.11).

Step PW3 Compute the legacy-specific trade shares $\lambda_{sdi,t}^{k}$ for varieties with legacy $k \geq 0$, using Equations (A.15), (A.16), (A.18) and (A.19).

Step PW4 Compute the shares of varieties from country s chosen by traders in each destination country d, $\upsilon_{sdi,t}$, using Equations (A.13) and (A.14).

Step PW5 Compute the implied legacy-specific price indices $P^k_{di,t}$ for each k and country-industry-level price indices $\hat{P}_{dj,t}$ using Equations (A.17), (A.20) and (A.21).

Step PW6 Compute the country-industry-level trade shares $\lambda_{sdi,t}$ based on $\lambda^k_{sdi,t}$, $P^k_{di,t}$, and $P_{dj,t}$.

Step PW7 Compute the country-industry-level expenditures on goods for final use $E_{d,t} - \sum_j \Pi_{dj,t}$ based on the income computed from the guess on wage changes, trade deficits, before counting profits from traders using Equation (B.30).

Step PW8 Solve $X_{dj,t}$ with the variables computed so far using Equation (B.29) and a fixed-point solver.

Step PW9 Compute the country-industry-level output $Y_{si,t}$ based on trade shares, expenditures, tariffs and markup.

Step PW10 Compute the profits earned by traders, wage income earned by workers and the implied wage changes $\hat{w}_{d,t}$ using Equation (A.25).

Step PW11 Compare the price indices $\hat{P}_{dj,t}$ from **Step PW5** and wages $\hat{w}_{d,t}$ from **Step PW10** with the previous updates from **Step PW1**. If they are not sufficiently close, go back to **Step PW1** and repeat.

Step 2 Update aggregate price indices faced by households based on preferences.

Step 3 Compute the changes in country-industry-level option value of supplier relations $\{\widehat{1 + \Psi_{sdi,t}}\}_{t=0}^{H}$ using variables computed so far and Equation (A.12).

Step 4 Compare the $\{\widehat{1 + \Psi_{sdi,t}}\}_{t=0}^{H}$ obtained from **Step 3** with those from **Step 1**. If they are not sufficiently close, update the guess for $\{\widehat{1 + \Psi_{sdi,t}}\}_{t=0}^{H}$ using a fixed-point solver and go back to **Step 1**.