Housing and the Welfare Cost of Inflation *

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Abstract

We show that standard nominal fixed amortization mortgages and borrowing constraints lead to the level of anticipated and stable (trend) inflation having real effects on life cycle consumption profiles. To quantify these welfare costs of trend inflation, we develop an incomplete market life-cycle model of housing tenure choice. In the model, nominal interest rates and nominal wage growth vary one-for-one with trend inflation. With standard mortgage contracts, higher rates of inflation tilt the profile of real mortgage payments so that they are higher (lower) earlier (later) in the amortization period, which tightens debt payments to income constraints that are more binding for younger homeowners. We calibrate the model to 2019 US homeownership rates, loan-to-value ratios, and debt-to-income ratios over the life-cycle. In our counterfactual experiments, we vary steady state inflation between 0 and 10 percent and find that inflation has significant welfare costs as it distorts life cycle consumption and housing. These welfare costs are nearly four times larger for upper middle income than lower income households and can have different impacts across cities.

Keywords: Mortgage Debt, Debt-to-income, Inflation, Homeownership rate, Life-cycle

JEL: E21, E60, R21

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1 Introduction

The rapid rise of inflation since 2021 which followed a period of below target inflation has rekindled debate over the appropriate target level of inflation (e.g., see Blanchard, Dell'Ariccia, and Mauro (2010), Gagnon and Collins (2019)). Although this debate acknowledges practical challenges such as the impact on credibility from changing an inflation target, proponents of increasing inflation targets often argue that the welfare cost of sustained but moderate inflation is small. This view typically draws on estimates of the welfare costs of inflation resulting from the costs of holding (non-interest) bearing money or distortions in relative prices.[1]

In this paper, we show that standard nominal fixed amortization mortgage contracts provide a mechanism via which anticipated and stable (trend) inflation can impact consumer's choice of non-housing and housing consumption over the life-cycle. Central to our analysis is the observation that increases in steady state inflation tilts up the growth rate of nominal income while increasing the nominal interest rate. For a fixed real interest rate, higher rates of inflation map directly into higher nominal interest rates. With a standard constant nominal payment mortgage schedule, this implies that higher rates of inflation translates into higher (nominal) monthly mortgage payments. Correspondingly, the real monthly mortgage payments – the nominal mortgage payment adjusted by inflation – decline faster over time, which implies that higher rates of inflation tilt the profile of real mortgage payments to earlier in the amortization period. Following the literature (see e.g. Lessard and Modigliani (1975)), we refer to this as front-loaded real mortgage payments. With standard mortgage contracts and borrowing constraints, higher inflation tightens credit constraints for prime-age buyers, and thus distorts life-cycle consumption decisions.

To evaluate the quantitative welfare cost of sustained (steady state) inflation that result from standard fixed amortization mortgage contracts and borrowing constraints, we develop an incomplete market life cycle tenure choice model.[2]We show, for reasonable parameter

[1]Following the seminal framework proposed Bailey (1956) and Friedman (1969) where higher rates of inflation increase the cost of holding real monetary balances, welfare loss are measured under money demand curve (see e.g. Lucas (2000)). The second channel focuses on how the inflation interacts with nominal rigidities that slow the adjustment of relative prices to result in shifts in relative prices that distort output (e.g., Burstein and Hellwig (2008), Blanco (2021)).

[2]Despite the extensive discussions about inflation-indexed mortgage contracts in the literature (e.g., see Campbell and Cocco (2003)), fixed payment mortgage contracts (including fixed-rate mortgage contracts

values, that these distortions can result in welfare losses for even modest levels of inflation.

Our focus on the interplay between inflation, borrowing constraints and mortgage contracts reflects the fact that housing is both the most important asset in many households' portfolio and accounts for a large share of consumption. In addition, almost all first-time buyers choose to finance their home purchase with mortgage loans. The most popular mortgage contracts generally specify a fixed amortization period (typically 30 years in the U.S. and 25-30 years in Canada and UK) at a fixed interest rate (albeit with an option to refinance) with **constant nominal payments** (typically monthly). As most buyers are in their 30s and 40s, they face an earning profile that features real income growth. Hence, in the presence of limited ability to borrow from their future income, front-loaded real mortgage payments due to higher inflation can significantly impact households non-housing and housing consumption over the life-cycle.

Using aggregate U.S. data, in Section 2 we document suggestive empirical evidence supporting the key channels we embed in our model. First, we document that nominal income grows faster (conditional on age) when inflation is higher by comparing the earning profile of two cohorts, the cohort born in 1956 who were exposed to high inflation in their 20s and the cohort born in 1986 who experienced stable, low inflation in their 20s. We also document that average inflation is passed through into nominal mortgage rates. These empirical patterns combined with fixed amortization mortgages implies that higher inflation results in mortgage payments as a share of income starting at a higher level and declining faster over time. In section 2.3 we use data from the Census and American Community Survey to show that the ratio of mortgage payments to income declined faster with a borrower's age in 1980 when inflation was high than in 2000 or 2016 when inflation was lower. We use data from the Consumer Expenditure Survey to compare cross-sectional consumption growth by age in 1982 and 2013. Consistent with our mechanism via which front loaded housing costs tighten the credit constraints, we show that non-durable consumption grows faster with age in 1982 when inflation is high than in 2013.

To illustrate the key mechanism via which inflation distorts both intra-temporal and inter-temporal allocation of consumption and housing services, we develop a two-period model where households consume a non-storable good and housing services. Households finance their housing purchase with fixed amortization and constant nominal payments mortgage contracts. We show that without borrowing constraints, inflation has no impact on

and adjustable-rate mortgage contracts) are the most common contracts in the US.

consumption, housing demand or welfare. We also show that in the presence of credit constraints that the welfare cost of inflation depends on the growth rate of real income and the housing supply elasticity. When households see a larger increase in their real income as they grow older, higher inflation creates a larger gap in consumption and housing services between young and old and therefore results in a higher welfare loss. We also find that the housing price adjustment has important implications on the welfare cost of inflation. Higher inflation increases the cost of housing and therefore reduces the aggregate housing demand. As a result, equilibrium house prices fall which partially alleviate the welfare loss of households due to inflation.

To quantitatively evaluate the welfare cost of inflation, we develop a general equilibrium life-cycle housing tenure choice model where households choose whether to rent or purchase a home and the amount of mortgage debt. Households face a life-cycle earnings profile and stochastic shocks to income, and have preferences over a non-storable consumption good as well as housing services. Households choose between owning and renting different sized houses based on their age, income, and wealth.

The only credit contract in the model is a standard thirty-year amortization mortgage with fixed nominal payments. The nominal mortgage interest rate varies proportionately with the rate of inflation. Borrowers face both a maximum loan-to-value constraint as well as a cap on the payment-to-income ratio when a mortgage is issued. The buying and selling of houses incur a cost proportional to the value of the house. In addition, there is a fixed costs to obtain a mortgage. To avoid tracking the change of prices and interest rates due to inflation, we map the nominal budget constraint of a household into real terms. As a result, the real mortgage payment depends on inflation and the mortgage age.

We calibrate the baseline model to match homeownership rates, loan-to-value ratios, and debt-to-income ratios over the life-cycle in 2019. In our baseline calibration, we assume the economy is in steady state with inflation stable at 2%. We use the calibrated model to evaluate the welfare cost of alternative steady state inflation rates. In our experiments, we consider both a partial equilibrium setting where house prices do not adjust and a general equilibrium setting where house prices responds to demand changes.

We find that when house prices are fixed, a 1 percentage point increase in inflation (i.e, from 2 % to 3 %) lowers welfare (in consumption equivalent variation) by roughly 0.05%. When we endogenize the relative price of housing, we find that inflation reduces aggregate housing demand and thus (real) house prices. This partially mitigates the tighter borrowing constraints of younger homebuyers resulting from inflation, which reduces the welfare cost of inflation.

Since welfare primarily distorts housing choices of homeowners, one would expect that the welfare costs are larger for higher income households who are generally homeowners than for low income households who typically rent. To evaluate this conjecture, we compute the welfare costs for households based on their initial persistent income draw. We find that the welfare costs of inflation are much higher for the upper middle income than lower income earners. In our baseline, the welfare costs of inflation for a household who starts life in the second highest earnings septile are roughly roughly times larger than those of a household born into the bottom septile.

To assess the sensitivity of these welfare losses to financial innovations that relax how higher inflation front-loads the path of real payments, we replicate our baseline experiments with a maximum amortization of 36 years.[3] Since households in the model can prepay their mortgage without penalty, a longer amortization relaxes the borrowing constraint of younger households and allows them to make larger payments later in life. We find that lengthening mortgage amortization only leads to a small decline in the welfare costs of inflation.

This paper contributes to a large literature on the welfare cost of inflation. Our steady state analysis features perfectly anticipated inflation. The extant analysis of perfectly anticipated inflation has largely focused on costs arising from inflation acting as a tax on money balances or price stickiness (e.g., Driffill, Mizon, and Ulph (1990), Schmitt-Grohe and Uribe (2011)). Lucas (2000) uses U.S. data from 1900-94 to estimate a money demand equation, and finds that reducing inflation from 10 to 0 percent would see a gain in income of roughly 1 percent. While our results are similar in magnitude, they result from a different channel. Since our mechanism is independent from money demand, our results suggest that the welfare cost of inflation could be more than twice as large as the commonly cited estimates based on money demand.[4]

A related literature examines the implications of nominal rigidities for the welfare cost of inflation. Ascari, Phaneuf, and Sims (2018) examine a New Keynesian model with multiple nominal rigidities, and report that a two percentage point rise in trend inflation (from 2

[3]Since each model period corresponds to two years this involves adding three model periods to the maximum amortization period.

[4]We share with Cao et al. (2021) the view that life-cycle motives are important for assessing the welfare costs of inflation.

to 4 percent) implies a nearly 4 percent consumption-equivalent welfare loss. Nakamura et al. (2018) however argue that relative price dispersion during the U.S. great inflation was modest.

The discussion of the welfare costs of trend inflation is related to a broader debate around whether money is neutral or superneutral. Afrouzi et al. (2024) note that in the standard New Keynesian model money (trend inflation) is not superneutral as changes in steady state inflation affects real variables. The mechanism in our paper points to another mechanism via which money is not supernetutral, as in our model steady state inflation changes the mix of housing and non-housing consumption over the life-cycle and household's debt to income ratio.

Despite the long debate over the consequences of high inflation on mortgage borrowing (e.g., see Lessard and Modigliani (1975) and Garriga, Kydland, and Šustek (2017)), to the best of our knowledge, there is no quantitative evaluation on the welfare cost of inflation arising from housing finance. This is surprising as the implications of inflation for mortgages and housing choice were illustrated by Kearl (2009). In this paper, we highlight that in the presence of long-term fixed-nominal-payment debt, inflation affects household credit constraint and leads to a significant welfare loss.[5] Consistent with our mechanism, Backman, Moran, and van santen (2024) find that a Swedish regulatory reform that eliminated interest only mortgages for mortgages with LTVs above 50% resulted in bunching at the LTV threshold and lower mortgage borrowing. They point to the effect of higher nominal mortgage payments for newly originated mortgages above the LTV threshold as the key force behind the change in borrower behaviour. This is consistent with the mechanism via which inflation impacts households borrowing in our model.

This paper is also related to the growing literature that uses quantitative models to study the housing market. Boar, Gorea, and Midrigan (2022) argues that the liquidity constraints are severe in the housing market. Their analysis are conducted in the low inflation environment and our paper complements their discussion by showing that high inflation may further tighten the constrains and results in a significant welfare loss. Our modeling approach shares a number of features with other recent papers that study the housing market including: Yang (2009), Ma and Zubairy (2021) Chambers, Garriga, and Schlagenhauf (2009), Sommer, Sullivan, and Verbrugge (2013). In contrast to these papers, which typically assume constant

[5] In related work, **?** examine the implications of changes in inflation over time for the ratio of mortgage debt to income.

low inflation or no inflation, we discuss the implication of constant high inflation on the housing market and estimate the welfare cost.

The rest of this paper is organized as follows. Section 2 presents the empirical evidence that supports the relationship between inflation and household mortgage debt. Section 3 presents the two-period model which illustrate the key mechanism. Section 4 outlines the quantitative model. Section 5 discusses the calibration strategy and presents the calibration results. Section 6 discusses the welfare implications of varying steady state inflation. Section 7 concludes.

2 Key Empirical Regularities

In this section, we document that sustained shifts in the level of inflation translate into roughly proportional shifts in nominal borrowing rates and nominal wage growth. We also provide a simple numerical example of how an increase the level of inflation can tilt the real payments profile of standard mortgage contracts and the consumption profile over life time.

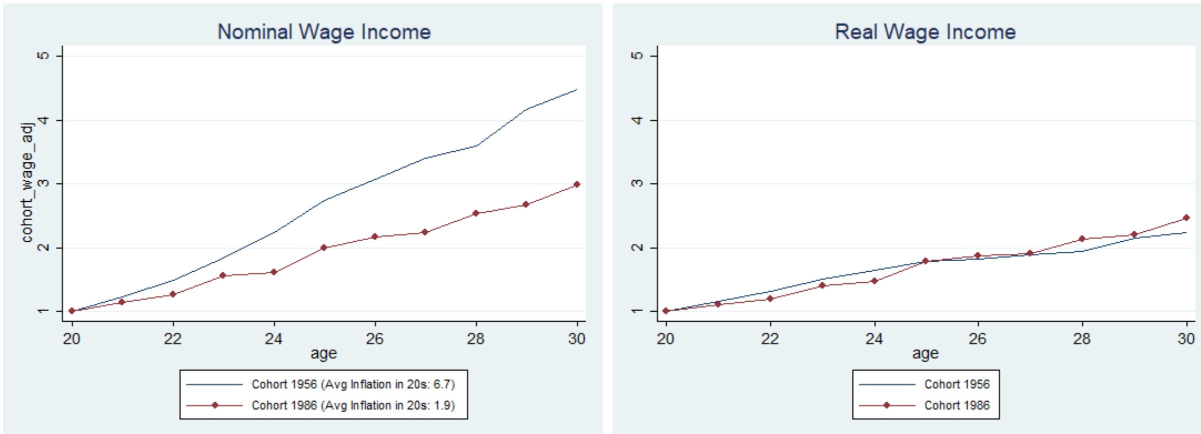
2.1 Inflation, Nominal Wage Growth and Nominal Borrowing Rates

Standard economic intuition suggests that average nominal wage growth and nominal interest rates should move with average inflation rates.[6] This relationship plays an important role in our model since both nominal mortgage payments and nominal wages (growth) are important components in determining eligible loans and households' borrowing decisions.

To illustrate the implication of inflation on wage growth, we compare the wage growth for two cohorts, one born in 1956 and one born in 1986. Those born in 1956 experienced the high inflation during the 1970s and 1980s early in their work careers, with an average inflation rate of 6.7% in their 20s. The cohort 1986, in contrast, entered the labour market when inflation was low during the 2000s, and experienced an average inflation rate of 1.9% in their 20s. Figure 1 compares the nominal wage growth and real wage growth for these two cohorts, constructed using the Current Population Survey (CPS). Despite very similar real wage growth for both cohorts, the nominal wage growth for the 1956 cohort was 50% higher than the 1986 cohort. In other words, on average nominal wage tracks inflation.

[6] A positive correlation between U.S. inflation and nominal wage growth has been documented by Sanchez (2015).

Figure 1: Nominal Wage Growth and Real Wage Growth



Note: Constructed using Current Population Survey

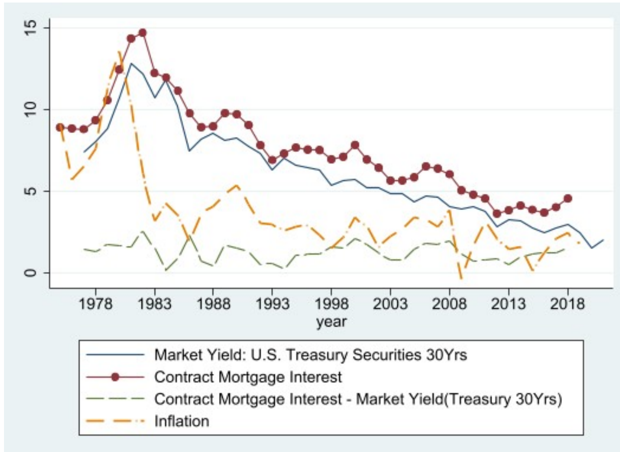
In addition to nominal wage growth rate, nominal mortgage interest rates and nominal interest rates on savings also vary with inflation. Figure 2 presents the time trend for inflation, contract mortgage interest and the market yield of U.S. 30 years treasury securities, which we use as a proxy for the safe return. Both the mortgage rate and bond yields move closely with inflation, which leaves the mortgage spread roughly stable.

2.2 Inflation and Mortgage Payments

To provide some intuition on how inflation impacts housing, we use a simple numerical example to illustrate how inflation twists the path of mortgage payments. In this example, we make use of the empirical (and theoretical) observation that anticipated and stable inflation shifts nominal wages and interest rates proportionately.

The example compares the path of real mortgage payments on a $ 300,000 30-year amortization mortgage for annual inflation rates between two and fifteen percent (see Figure 3). Real mortgage payments are the nominal mortgage payment adjusted by inflation since the date of the mortgage origination. Since the level if inflation shifts nominal rates, the nominal mortgage payment rises with inflation. Thus, the nominal annual payment mortgage rises from $ 17,433 when inflation is 2%, to nearly double at $ 32,249 when inflation is 8%. However, the effect of inflation means that real payments (adjusted for cumulative inflation since origination) are higher in the initial years but lower in later years of the mortgage term for higher inflation. In this example, after 12 years real payments with 8% inflation fall below

Figure 2: Mortgage Interest, Nominal Interest and Inflation

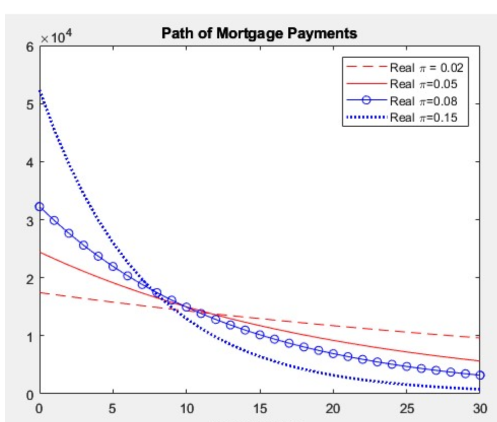


Notes: Contract mortgage interest rate is the contract interest rate for conventional single-family mortgages extracted from Federal Housing Finance Agency (FHFA); Nominal interest rate is measured by the market yield on U.S. Treasury Securities at 30-Year Constant Maturity provided by Federal Reserve Economic Data. Inflation is the consumer prices for the U.S., provided by Federal Reserve Economic Data.

those of 2% inflation.

An alternative way to assess the implication of inflation for the profile of mortgage payments is to examine the mortgage payment to income ratio. Consistent with the empirical evidence discussed in Section 2.1 (e.g., see Figure 1), assume that inflation does not affect real income growth but proportionately shifts nominal income growth. For the same nominal income at origination, the mortgage-to-income ratio with 8% inflation is almost twice as much as the mortgage-to-income ratio under 2% inflation rate. However, after 12 years, the mortgage to income ratio with 8% inflation rate is lower than with 2% inflation.

Figure 3: Inflation and Real Mortgage Payments

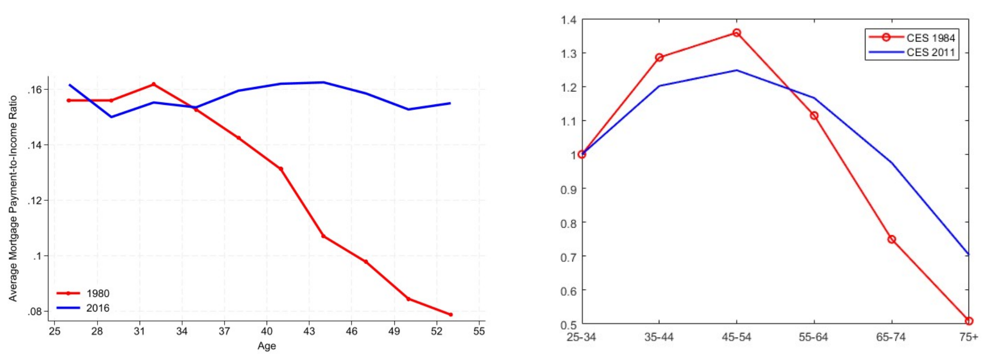


2.3 Inflation, Payment-to-Income and Nonhousing Consumption

The simple example above illustrated that constant nominal mortgage payments lead to higher inflation resulting in more front-loaded real mortgage payments. In addition, higher inflation steepens the nominal life cycle income profile of households. These forces imply that the mortgage payment to income ratio should fall faster over the lifecycle when inflation is higher than when inflation is lower. The tighter borrowing constraints and higher PTI for younger homeowners resulting from higher inflation should also lead to a steeper profile for non-housing consumption expenditures.

To assess these implications of inflation on life-cycle PTI, we compare the age profile of PTI in 1980 and 2016 among all owners. We construct PTI using data on mortgage payments and household income from Census (1980 and 2000) and American Community Survey (2016) (see Figure 4). PTI declines faster with age in 1980 when inflation rate was high compared to 2000 and 2016 when inflation was stabled at a lower level.

Figure 4: Payment to Income and Nonhousing Consumption



(a) Payment to Income (b) Lifecycle Consumption

Mortgage payment to income ratios among owners: computed using data from Census (wave 1980) and American Community Survey (2016). Life-cycle non-housing expenditure in 1984 and 2011. Data comes from consumer expenditure survey. Household expenditure for the youngest group (25-34) is normalized to 1.

In our model, higher inflation rates results in tighter borrowing constraints for younger households which impacts the timing of non-housing consumption over the life-cycle. As an illustrative empirical example, we compare consumer expenditure on non-durables by age in 1982 when inflation was 8% and in 2013 when inflation was roughly 2% using data from the Consumer Expenditure Survey. To isolate the impact of inflation from economic growth, we normalize the non-housing expenditure of different age groups to that of the youngest group in our data which is under 25 years old. As we can see from Figure 4, non-durable consumption grows faster with age in 1982 when inflation was high than in 2013. This is consistent with the narrative that inflation can twist the life-cycle consumption profile.

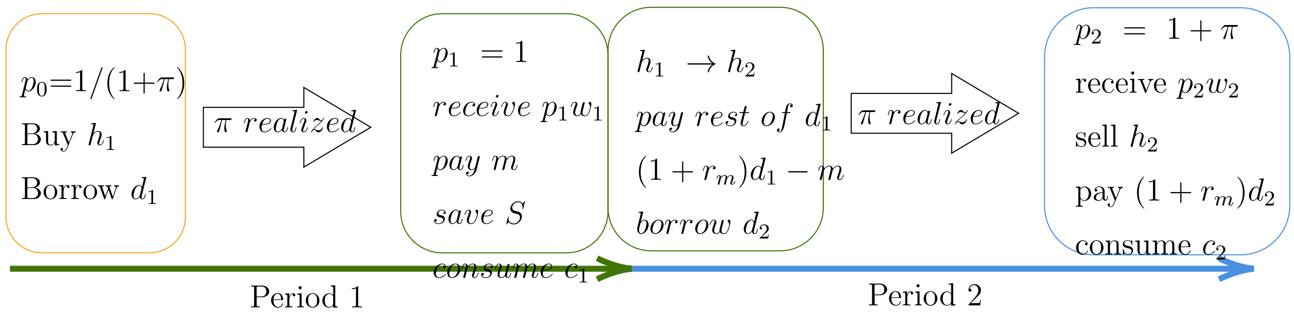
3 Two Period Example

The welfare costs of trend inflation that we examine arise from the interaction between nominal fixed amortization contracts and borrowing constraint-housing. Since this mechanism that links trend inflation to household decisions on housing and non-housing consumption is relatively novel, in this section we provide intuition by developing a 2-period small open economy with a standard mortgage contract. We show analytically that anticipated inflation can tighten borrowing constraints and distort intra-temporal and intertemporal consumption decisions.

The economy is populated by a two-period lived household with preferences defined over a non-durable consumption good and housing services. The flow of housing services is proportional to the housing stock. The only borrowing instrument is a standard fixed amortization mortgage. The real interest rate is fixed exogenously, and inflation (which we vary exogenously) moves nominal wages, prices and interest rates by factor π.

To show how borrowing restrictions linked to (collateralized) mortgage debt can lead to trend inflation having real effects, we solve the model for two household borrowing constraints. In the first, the household can borrow up to their intertemporal budget constraint. In the second, we impose that households can only borrow using debt collateralized by the value of their house. We show that inflation has no impact on household's decisions (or welfare) when households can borrow at the risk-free nominal rate against their entire future income. However, in the presence of borrowing constraints inflation can distort household's consumption decisions, with higher rates of inflation having a larger negative impact on welfare.

Figure 5: Time Line



3.1 Households

A mass 1 of two period lived households are born young. They receive an endowment of 1 when young and $g > 1$ when old units of the consumption good. Household preferences are represented by a separable utility function defined on consumption and housing services. The price P denotes the price of a unit of housing in units of the non-durable good which young households are endowed with.

$$\ln(c_1) + \theta \ln(h_1) + \beta \ln(c_2) + \beta\theta \ln(h_2) \tag{1}$$

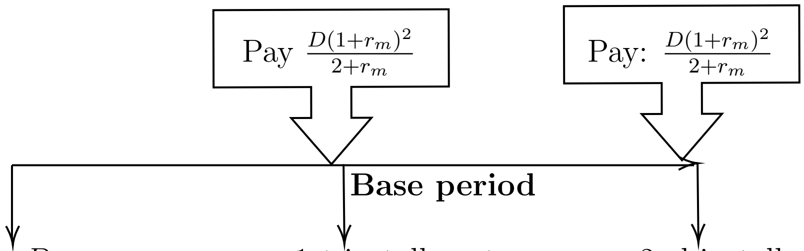
where c_1 and h_1 are the consumption and housing consumption of young households and c_2 and h_2 are the consumption and housing consumption of old households. The nominal price of housing and wage income between periods move proportionately with inflation π.

At the beginning of the first period, the young choose the size of house h_1 to buy at price $\frac{P_1}{1+\pi}$ which they finance with a mortgage d_1 which amortizes over two periods with **fixed-nominal** mortgage payments. At the end of the first period, households receive their wage income 1, pay the mortgage interest and the first of two principal payments. We use the end of the first period (the beginning of the second period) as the reference period for all real values.

Old households enter the second period with housing h_1 and their remaining debt. They choose a new housing level h_2 and the associated level of borrowing. At the end of the second period, households make mortgage interest and principal payments. Finally, they sell the house (after consuming the flow of services from owning the house during the period) and consume their wealth.

The timing of transactions is summarized in Figure 5.

Figure 6: Debt Evolution: Nominal and Real



Debt taken D	1st installment	2nd installment
Nominal Debt: D	$\frac{D(1+r_m)}{2+r_m}$	0
Real Debt: $D(1+\pi)$	$\frac{1+r_m}{2+r_m}D$	0
Nominal Equity: $\frac{P}{1+\pi}h_1 - D$	$Ph_1 - \frac{D(1+r_m)}{2+r_m}$	$P(1+\pi)h_1$
Real Equity: $Ph_1 - D(1+\pi)$	$Ph_1 - \frac{D(1+r_m)}{2+r_m}$	Ph_1

3.2 Financial Markets

We examine a small open economy with exogenous incomplete markets. Mortgages are secured against the housing good, and are fixed amortization at a fixed interest rate. The nominal interest rate r_m on a mortgage is: $(1+r_m) = \underbrace{(1+\bar{r})}_{\text{real rate}}\underbrace{(1+\pi)}_{\text{inflation}}$

This can be interpreted as a small open economy where the real rate is exogenous and inflation is determined by monetary policy.

To make it easier to see how shifts in the nominal interest rate (which varies with inflation) impacts mortgage payments we incorporate some simplifying algebra in how we introduce mortgages in the household budget constraint. Specifically, for a loan amount of D, the nominal mortgage payment per period paid in two future consecutive periods is $\frac{D(1+r_m)^2}{2+r_m}$.

The implications for the evolution of real and nominal debt and housing equity across periods are summarized in Figure 6.

3.3 Households' problem

The households' problem can be characterized by:

$$\max_{c_1,h_1,c_2,h_2,S_1,d_2} \ln(c_1) + \theta\ln(h_1) + \beta\ln(c_2) + \beta\theta\ln(h_2) \quad (2)$$

subject to

1. Budget constraint for the young:

$$c_1 + \delta p h_1 + \frac{(1+r_m)^2}{2+r_m}\frac{p}{1+\pi}h_1 + S = 1$$
$$S > 0 \tag{3}$$

2. At the beginning of the second period, households can adjust their house size with a new mortgage contract:

$$d_2 = Ph_2 - S - (Ph_1 - \frac{1+r_m}{2+r_m}\frac{Ph_1}{1+\pi}) \tag{4}$$

3. Budget constraint for old households:

$$(1+\pi)c_2 + \frac{(1+r_m)^2}{2+r_m}d_2 + (1+\pi)\delta Ph_2 = (1+\pi)Ph_2 - \frac{1+r_m}{2+r_m}d_2 + (1+\pi)g \tag{5}$$

where c_1 is non-durable consumption, δph_1 is depreciation, $\frac{(1+r_m)^2}{2+r_m}\frac{p}{1+\pi}h_1$ is the scheduled mortgage payment, S is saving, Ph_2 is the cost of purchasing a new house of size h_2 and $(Ph_1 - \frac{1+r_m}{2+r_m}\frac{Ph_1}{1+\pi})$ is the home equity after the first mortgage payment for the house purchased in period 1. After the realization of inflation in the second period, house prices and income are both scaled up by $(1+\pi)$.

3.4 With Fixed Nominal Payments Mortgages Anticipated Inflation Matters for Consumption

In our two-period economy, both the nominal interest rate and the relative prices of goods across periods are fully indexed to inflation. Thus, any real effects of inflation households' optimal choices arise via how inflation impacts the household's budget constraints. In this section, we show that higher rates of inflation can tighten the borrowing constraint on young households with fixed nominal payment mortgages.

It is straightforward to show that inflation has no direct impact on the budget constraint for old households. Mapping the nominal budget constraint of an old household (equation 5) into real values using the end of the first period (beginning of the second period) as the reference period:

$$c_2 + (1+r)d_2 + \delta Ph_2 = Ph_2 + g \tag{6}$$

The level of inflation, however, does affect the first period budget constraint (Equation 3), and the second period indirectly through its impact on the household's housing choice

h_2 and debt d_2 at the beginning of the second period (see Equation 4). For any given housing choice h_1, a higher inflation rate increases the period 1 nominal mortgage payment $\frac{D(1+r_m)^2}{2+r_m}$ since r_m increases with inflation. This also implies that the real balance after the first period payment decreases in inflation, which results in higher real home equity at the beginning of the second period. In the presence of borrowing constraints — which result from all borrowing taking place via mortgage with a maximum LTV constraint — inflation can impact the timing and mix of household consumption. This is formalized in Lemma 3.1 and Proposition 3.2.

Lemma 3.1 *Households with wage growth $g \geq r(1+\beta)$ are credit constrained when $S_1 = 0$.*

Proof. The budget constraint in period 1 can be rewritten as

$$\begin{aligned} & c_1 + (\delta + \frac{(1+r_m)(1+r)}{2+r_m})ph_1 + S = 1 \\ & S \geq 0 \end{aligned} \tag{7}$$

Using the consumption and housing choices from Equation 9, we obtain:

$$\frac{1+\frac{g}{1+r}}{(1+\beta)(1+\theta)} + \frac{(1+\frac{g}{1+r})(\delta+r+1-\frac{1+r}{2+r_m})}{(1+\beta)(1+\theta)(\delta+r)} > \frac{1+\frac{g}{1+r}}{(1+\beta)(1+\theta)} + \frac{(1+\frac{g}{1+r})(\delta+r)}{(1+\beta)(1+\theta)(\delta+r)} > 1$$

so $S = 0$ and consumers are credit constrained. Note that the housing cost in the first period $(\delta + \frac{(1+r_m)(1+r)}{2+r_m})ph$ increases in π as r_m increases in π. So an increase in inflation further tightens the constraint in period 1.

Theorem 3.2 *Assume wage growth $g \geq r(1+\beta)$ such that households are credit constrained without a positive inflation. An increase in inflation tightens the constraint in period 1 and leads to*

1. *a decrease in housing consumption in the first period and an increase in the housing consumption in the second period: $\frac{\partial h_1}{\partial \pi} < 0; \frac{\partial h_2}{\partial \pi} > 0$.*
2. *an increase in consumption for both periods $\frac{\partial c_1}{\partial \pi} \leq 0; \frac{\partial c_2}{\partial \pi} > 0$*
3. *a decrease in welfare*

Proof. See Appendix

Intuitively, when the credit constraint binds in the first period, an increase in inflation acts to shift more of the cost of housing to the first period. This further tightens the credit constraint and results in a decrease in h_1. This tightening of the borrowing distorts both the intra and inter temporal consumption allocations, and thus lowers welfare.

To show that this feature arises from the interaction between the borrowing constraint and the impact of inflation on nominal mortgage payments, we remove these features by allowing households to borrow subject to a standard intertemporal budget constraint. In this case, we obtain the standard result that inflation has no impact on a households choices with nominal indexation of interest rates and intetemporal goods prices. We formalize this result in the following Lemma 3.3:

Lemma 3.3 *If a household can borrow up to their intertemporal budget constraint, inflation has no impact on their choice of non-durable consumption and housing consumption.*

Proof. If we remove the the positive saving constraint in Equation 3, such that households can borrow against their future income, and combine equation 3, 4 and 6, we obtain the life-time budget constraint:

$$c_1 + \frac{c_2}{1+r} + \delta P h_1 + rPh_1 + \frac{\delta P h_2}{1+r} + \frac{rPh_2}{1+r} = 1 + \frac{g}{1+r} \tag{8}$$

where $\delta Ph_1 + rPh_1$ and $\frac{\delta Ph_2}{1+r} + \frac{rPh_2}{1+r}$ are the user cost of h_1 and h_2, respectively which consist of the depreciation cost and the forgone interest on the house. Noticeably, the life-time budget constraint is independent of inflation. As a result, the consumer's optimal consumption and housing consumption are independent of inflation. This can be seen in the solution to the households' optimization problem:

$$\begin{aligned} c_1^* &= \frac{1 + \frac{g}{1+r}}{1+\beta+\theta+\beta\theta} \\ c_2^* &= \frac{\beta(1+r)(1+\frac{g}{1+r})}{1+\beta+\theta+\beta\theta} \\ h_1^* &= \frac{\theta(1+\frac{g}{1+r})}{(1+\beta+\theta+\beta\theta)(\delta+r)} \\ h_2^* &= \frac{\beta(1+r)\theta(1+\frac{g}{1+r})}{(1+\beta+\theta+\beta\theta)(\delta+r)} \end{aligned} \tag{9}$$

■

Although the consumption allocation is independent of inflation, the level of nominal savings, S, does vary with inflation. When the scheduled mortgage payment, $\frac{(1+r_m)^2}{2+r_m}\frac{p}{1+\pi}$ increases with inflation, S rises to cover the higher (nominal) mortgage payment.

3.4.1 An Illustrative Numerical Example

To further illustrate the intuition behind how fixed nominal payment mortgages can give rise to real effects of inflation, we use the two period model to numerically show how the welfare cost of inflation varies with real wage growth and inflation. Intuitively, higher rates of real wage growth should amplify the distortions associated with higher levels of inflation via a tighter borrowing constraint.

We set each period to correspond to 15 years and $\theta = 0.5$, $\delta = 0.15$ so the annual depreciation cost is 0.01, $r = 0.345$ so the annual risk free return is 0.02, and $\beta = 0.74$ such that the annual discount rate is 0.98. We vary the (real) wage growth rate g from 1 to 2 and annual inflation from 0 to 25 percent.

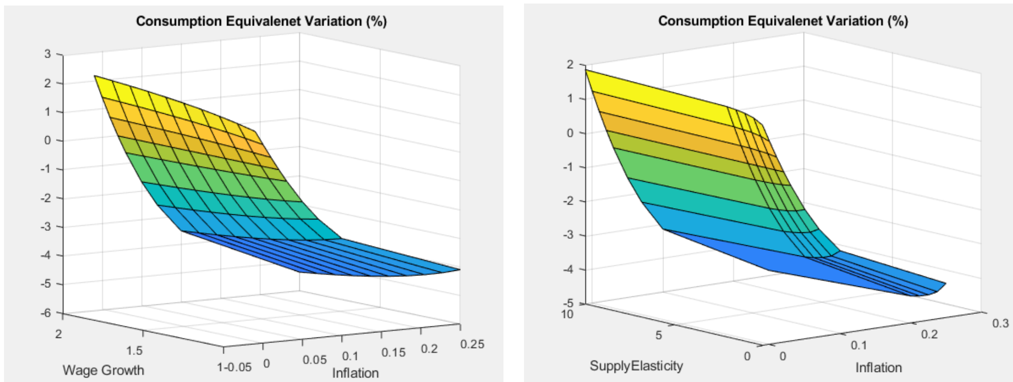
As we can see from Figure 7, for a given real wage growth rate, welfare declines with inflation. Figure 7 also shows that the welfare costs of inflation are increasing in the rate of real income growth. This implies that the welfare cost of inflation market may vary with the steepness of life-cycle income growth.

General equilibrium effects via the price of housing could ameliorate the partial equilibrium estimates presented in Figure 7 (where we hold fixed the relative price of housing). To evaluate the welfare cost of inflation in general equilibrium, allow the housing price to respond to inflation via an exogenous supply elasticity (see Figure 7). Intuitively, higher inflation reduces the demand for housing which lowers the equilibrium price of housing relative to the non-durable consumption good. A decline in house prices relaxes the borrowing constraint, which should ameliorate the welfare cost. As we can see from Figure 7, with elastic housing supply (elasticity =10), an increase in inflation from 2 to 5% leads to a 1.49% welfare loss. With inelastic housing supply, the price decline is larger which reduces the welfare loss reduces to 1.2% for an increase in inflation from 2 to 5%.

4 Quantitative Model

To quantitatively evaluate the welfare cost of trend inflation arising from standard mortgage contracts, we develop a life-cycle housing tenure choice model with standard 30 year amortization fixed nominal payment mortgage contract. In the steady state comparisons in Section 6 where inflation is fully anticipated, what matters is the amortization period since fixed rate mortgage contracts are effectively the same as the adjustable rate mortgage contracts

Figure 7: Inflation and Welfare in Two Period Example



(a) Partial Equilibrium (b) General Equilibrium

Note: One period corresponds to 15 years. Preference towards housing $\theta = 0.5$, annual depreciation cost $\delta = 0.15$, annual interest rate $r = 0.34$, wage growth $g = 1.5$ and annual discount rate is $\beta = 0.78$.

when there is no change in inflation rates or mortgage rates over a borrower's life. However, in our experiments where we examine the implications of transitions between steady states we explore the implications of fixed versus variable rate mortgages.

The economy is populated by finitely lived households who are subject to stochastic earnings shocks.Households' preferences are defined over a perishable consumption good and housing services. Households can obtain housing services by owning or renting. We capture the segregation between owner-occupied markets and rental market with a ownership premium, which is calibrated to the homeownership rates. [7]

Homeowners can borrow via 30-year fixed-amortization mortgage contracts secured against their house. The 30-year fixed amortization (and fixed rate) is the most common mortgage contract in the US. Buyers and sellers of houses incur transaction costs which makes adjusting housing costly for owners. When a new mortgage is issued, borrowers incur a mortgage closing cost. On the supply side, there is a competitive construction sector that combines final goods and land together to produce houses. The land supply is exogenously determined by a government.

To simplify our quantitative analysis, we embed the impact of inflation in the budget

[7] The ownership premium captures two channels via which owning is attractive relative to renting. First, owners can customize their homes according to their own taste. Second, owner-occupied units tend to be in better condition compared to a standard rental unit. According to Yao (2019), 93% of owners live in detached home while 63% of renters live in apartments.

constraint via how it affects the path of real mortgage payments. Specifically, the real mortgage payment depends on inflation as well as the age of the mortgage. This is equivalent to expressing all payments in nominal terms when the price of the numeraire good, house prices, rents, and interest rates grow at the same rate as inflation while nominal mortgage payments do not change over time.

4.1 Households

The economy is populated by J period lived households who work from period 1 to Jr and then enter retirement for the remainder of their life. Working-age households face both stochastic mortality risks and labour income uncertainty, while retired households only face mortality risks. The initial wealth of a household at birth is drawn from a distribution Γ.

Each household has preferences defined over a non-durable good and housing service represented by $u = \frac{((1-\eta)C^{1-\xi}+\eta d^{1-\xi})^{\frac{1-\sigma}{1-\xi}}}{1-\sigma}$ each period. The value function of a household of age j is:

$$V_j(\cdot) = \frac{((1-\eta)C^{1-\xi}+\eta d^{1-\xi})^{\frac{1-\sigma}{1-\xi}}}{1-\sigma} + \beta e_j s_j V_{j+1}(\cdot) + \beta e_j (1-s_j) B(W_{j+1}). \qquad (10)$$

where β is the discount factor and e_j is the consumption expenditure equivalence scale which captures the change in household size over the life cycle (Muellbauer, 1979), s_j is the probability a household survives from age j to age $j+1$, c_j is age j consumption, and h_j is age j housing services. The ownership premium, θ, captures the value to a household of the quality difference between a standard owner-occupied house and a standard rental apartment. The elasticity of substitution between non-durable consumption and housing is determined by ξ, and σ is the elasticity of intertemporal substitution. $B(W)$ captures the bequest motive, and W_{j+1} is the wealth bequested by a household who dies at age $j+1$, which is the sum of savings a and the home equity (net of any outstanding mortgage debt) of owners. Following the literature (see e.g. (Guren et al., 2021)), the functional form for the bequest motive is:

$$B(W) = \frac{B_0(B_1+W)^{1-\sigma}}{1-\sigma} \qquad (11)$$

4.1.1 Household Income

At the beginning of each period, working-age households receive exogenous real wage income that depends on their age $\bar{w}_j$, and a persistent idiosyncratic shock ϵ_{ij}:

$$w_j(\epsilon) = \bar{w}_j \epsilon_{ij} \text{ if } j \leq Jr \tag{12}$$

The individual-level idiosyncratic shock follows an AR1 process.

$$ln(\epsilon_{ij}) = \rho ln(\epsilon_{ij-1}) + \xi_i \tag{13}$$

where ξ_i is drawn from a normal distribution with mean 0 and standard deviation σ_ϵ.

Retired households earnings depend on their age and their income before retirement.

$$w_j(\epsilon) = \bar{w}_j + rpc\bar{w_{Jr}}\epsilon_{iJr} \text{ if } j > Jr \tag{14}$$

where rpc is the replacement ratio.

Households are subject to a progressive income tax schedule which is represented by a function $\tau(\bar{w}_j\epsilon_i)$. Following Kaplan, Mitman, and Violante (2020), we assume

$$\tau(\bar{w}_j\epsilon_i) = \tau_0(\bar{w}_j\epsilon_i)^{1-\tau_1} \tag{15}$$

where τ_0 captures the average level of tax rate and τ_1 represents how steep tax rate increases with income.

4.2 Assets

There are three assets: housing, risk-free bonds that pay a constant real interest rate, and mortgages. Households can only borrow via mortgages using the housing asset as collateral.

The housing asset provides housing consumption and serves as an investment. Owning a house is costly since housing depreciates at rate δ and owners have to pay a proportional property tax τ_h on the value of their house Ph. Buying and selling housing is costly. A buyer incurs a transaction cost of k_bPh and a seller a total transaction cost of k_sPh. There is a minimum size for an owner occupied house $\underline{h}$, so owners can only buy a house larger than $\underline{h}$.

Households can save through a risk-free bond, which pays a real interest r. The nominal return $(1+r)(1+\pi)$ varies with the inflation rate π.

Households can (only) borrow via mortgages collateralized by housing.[8] Mortgages take the form of a 30-year amortization fixed-interestrate mortgage, which is the most common mortgage contract in the U.S. The nominal borrowing interest rate on a mortgage, r_m, equals the real interest rate r plus a spread ζ_m, adjusted by the inflation rate π.

$$r_m = (1 + r + \zeta_m)(1 + \pi). \tag{16}$$

The nominal mortgage annual payment schedule m of a mortgage of value L is given by:

$$m = \frac{r_m L}{1 - (1 + r_m)^{-30}}. \tag{17}$$

Consequently, the real mortgage payment for a loan L issued $n \in \{1, 2, ..., 30\}$ years previously is $\frac{r_m L}{(1-(1+r_m)^{-30})(1+r)^n}$. The evolution of the nominal balance of a mortgage contract issued n years ago, which specifies a nominal payment m, is described by:

$$\begin{aligned} D(m, 0) &= \frac{m(1 - (1 + r_m)^{-30})}{r_m} \\ D(m, n) &= (1 + r_m)D(m, n - 1) - m, \quad n \in \{1, 2, ..., 30\} \end{aligned} \tag{18}$$

A closing cost k_m, proportional to the face value of the mortgage L, is incurred every time a (new) mortgage contract is signed. In addition, households are subject to a payment-to-income requirement when applying for a new mortgage.

Households can prepay their mortgage without penalty, but it is costly to refinance. When households refinance, they are subject to the mortgage closing cost and the payment-to-income requirement.

4.3 Household's Recursive Problem

The state variables (Λ) of a household are their age j, housing asset h, saving a, mortgage contract with a mortgage payment m that was issued n years previously, and preference towards owning θ ($\Lambda = (a, h, m, n, \epsilon, j, \theta)$). At the beginning of each period j, a household receives wage income $y(\epsilon, j)$, makes decisions about housing services to obtain either though renting or purchasing, mortgage, saving, and consumption of non-durable good. We group households into three cases with respect to housing investment and mortgage status.

[8]This is a common assumption in the housing literature, see e.g., Yang (2009) and Sommer, Sullivan, and Verbrugge (2013).

1. Households who choose to rent: A household who was a renter in the previous period can costly adjust their rental unit if they continue as a renter. If the household owned a house in the previous period ($h > 0$), choosing to rent means selling the house. In this case, he/she chooses the size of rental unit d, consumption c, and saving a' to maximize:

$$V^1(a,h,m,n,\epsilon,j,\theta) = \max_{c,d,a'} \frac{((1-\eta)C^{1-\xi}+\eta d^{1-\xi})^{\frac{1-\sigma}{1-\xi}}}{1-\sigma} + \beta e_j s_j E_{\epsilon'|\epsilon,\theta'}(V(a',0,0,0,\epsilon',j+1,\theta'))$$
$$+ \beta e_j(1-s_j)B(a')$$
$$s.t. \;\; c + a' + Rd = (1-\tau(w_j(\epsilon))w_j(\epsilon)$$
$$+ (1+r)a - (\delta+\tau_h)Ph - \frac{m}{(1+\pi)^n} + (1-k_s)Ph - \frac{D(m,n)}{(1+\pi)^n} \tag{19}$$

where R is the rental rate and $(1-\tau(w_j(\epsilon))w_j(\epsilon)$ is the real after-tax household wage income. $(\delta+\tau_h)Ph$ is the depreciation costs and property tax if the household carries the dwelling h and $\frac{m}{(1+\pi)^n}$ is the real mortgage payment. When an owner chooses to be a renter, they must sell their house. In this case, $(1-k_s)Ph - \frac{D(m,n)}{(1+\pi)^n}$ represents the net profit from selling the house, net of the selling cost,$(1-k_s)Ph$ and the real value of the outstanding debt $\frac{D(m,n)}{(1+\pi)^n}$.[9] $E(.)$ represents the expected value given current housing and mortgage decisions. $\beta e_j(1-s_j)B(a')$ captures the value of leaving a bequest of value a' as savings are the only asset available to renters.

2. Owners choose to continue with the current mortgage contract: An owner with house $h > 0$ with a nominal mortgage payment m of age n who decides to keep their house and mortgage chooses consumption c and saving a' to maximize:

$$V^2(a,h,m,n,\epsilon,j,\theta) = \max_{c,a'} \frac{((1-\eta)C^{1-\xi}+\eta d^{1-\xi})^{\frac{1-\sigma}{1-\xi}}}{1-\sigma} + \beta e_j s_j E_{\epsilon'|\epsilon,\theta'}(V(a',h,m,n+1,\epsilon',j+1)\mathbb{1}_{n<30}$$
$$+ V(a',h,0,0,\epsilon',j+1)\mathbb{1}_{n=30}) + \beta e_j(1-s_j)B(W_T)$$
$$s.t. \;\; c + a' = (1-\tau(\bar{w}_j\epsilon_i))\bar{w}_j + (1+r)a - (\delta+\tau_h)Ph - \frac{m}{(1+\pi)^n}. \tag{20}$$

where $(\delta+\tau_h)Ph$ is property tax and the cost to fix depreciation of the house and $\frac{m}{(1+\pi)^n}$ is the real mortgage payment. Note that $n = 30$ means that this is the last

[9] The budget constrains are specified in real terms. Therefore, we convert the nominal mortgage payments m and nominal debt outstanding D to real values by the real interest rate r.

period of the mortgage contract. In other words, the mortgage is cleared within the current period and therefore the household does not have any mortgage debt in the following period if he/she decides not to get a new loan, i.e. $m' = 0$ and $n' = 0$. W_T is the total wealth left by the households which is sum of saving and home equity.

3. Households choose to sign a new mortgage contract: The final case is where a household decides to enter a new mortgage contract. This includes the case where a renter becomes an owner, an owner decides to change their house size, or an owner decides to refinance. In this case, a household choose housing asset h', new mortgage contract m', saving a', and consumption c to maximize the current utility and expected future values.

$$V^3(a,h,m,n,\epsilon,j,\theta) = \max_{c,h',m',a'} \frac{((1-\eta)C^{1-\xi} + \eta(\theta h')^{1-\xi})^{\frac{1-\sigma}{1-\xi}}}{1-\sigma} + \beta e_j s_j E_{\epsilon'|\epsilon}(V(a',h',m',1,\epsilon,j+1))$$
$$+ \beta e_j (1-s_j) B(W_T)$$
$$s.t.\ \ c + a' + Ph' = (1-\tau(\bar{w_j}\epsilon_i))\bar{w_j} + (1+r)a - (\delta+\tau_h)Ph - \frac{m}{(1+\pi)^n} - (k_s Ph + k_b Ph')\mathbb{1}_{h\neq h'}$$
$$+ Ph - \frac{D(m,n)}{(1+\pi)^n} + (1-\tau_m)\frac{m(1-(1+r_m)^{-30})}{r_m} \tag{21}$$

$$\frac{m'(1-(1+r_m)^{-30})}{r_m} \leq (1-\chi)Ph'$$
$$\frac{m'}{w_j\epsilon_i} \leq \varphi. \tag{22}$$

Equation 21 specifies the budget constraint. Specifically, $(k_s Ph + k_b Ph')\mathbb{1}_{h\neq h'}$ represents the transaction costs for buying and selling a house. $\mathbb{1}_{h\neq h'}$ is an indicator that takes value 1 only if the household adjusts house size, i.e. $h \neq h'$. In other words, for a household doing refinancing without changing houses size, this term will be 0 as $h = h'$. $Ph - \frac{D(m,n)}{(1+\pi)^n}$ is the real value of home equity which is the real value of the house net of the real value of the mortgage debt. Recall $D(m,n)$ is the nominal value of the a mortgage debt with a schedule mortgage payment m after n periods.

According to Equation 18, $\frac{m'(1-(1+r_m)^{-30})}{r_m}$ is the real total amount debt of a standard 30-year fixed-rate mortgage contract with a mortgage payment m' per year. τ_m is the refinancing cost or closing cost of a mortgage contract.

Similar to Greenwald (2018), we have two constraints: one on Loan-to-Value ratio (LTV) and the other one is on Payment-to-Income ratios (PTI) when a new mortgage contract is issued. Specifically, the first line in Equation 22 detailed the LTV constraint, which implies that the a χ percent downpayment is required. The second line in Equation 22 shows that mortgage payment has to be lower than φ of the household income.

Finally, at the beginning of each period, households choose among the three options, base on the state variable $\Lambda = (a, h, m, n, \epsilon, j, \theta)$. That is

$$V(a, h, m, n, \epsilon, j, \theta) = max\{V^1(a, h, m, n, \epsilon, j, \theta), V^2(a, h, m, n, \epsilon, j, \theta), V^3(a, h, m, n, \epsilon, j, \theta)\} \tag{23}$$

4.4 Housing Supply

Following the recent literature (see e.g. Kaplan, Mitman, and Violante (2020) and Favilukis, Ludvigson, and Van Nieuwerburgh (2017)), we assume that a competitive construction sector combines final goods K and land to produce houses. A government determines the number of new permits which affects the amount of land available for new construction, $\bar{L}$. Land is owned by an absentee landlord who lives outside of the economy and consumes the profit of selling the land. The construction sector earns zero profits in equilibrium

$$H = K^\alpha L^{1-\alpha} \tag{24}$$

The developers solve the following problem.

$$\max_{K,L} PK^\alpha L^{1-\alpha} - K - qL \tag{25}$$

where q is the equilibrium price of land. The developers' problem and market clearing condition for land yield the housing supply function:

$$H = (\alpha P)^{\frac{\alpha}{1-\alpha}} \bar{L} \tag{26}$$

where the supply elasticity of housing is $\frac{\alpha}{1-\alpha}$.

5 Parameterization

We calibrate the model to match key moments of the U.S. housing market in 2019, including the age profile of homeownership rates, loan-to-value ratios, debt-to-income ratios and payment-to-income ratios.The calibration is conducted in two stages. In the first stage, we determine the parameters which are directly estimated using the data or taken from the literature (see Table 1). In the second stage, key parameters are estimated using the generalized method of moments.

5.1 Parameters Determined Outside of the Model

Each period in our model corresponds to two years in the data.

Demographics: Households are born at age 23 (model period 1) and live up to age 80 (model period 31). Households retire at age 64 (model period 23). The survival probabilities are taken from the National Center for Health Statistics, United States Life Tables, 2016.

Preference: Intertemporal elasticity of substitution σ is set to 2. The elasticity of substitution between housing and non-housing $\frac{1}{\xi}$ is set to be 1.25, following Guren et al. (2021) and Kaplan, Mitman, and Violante (2020). The age-specific discount factor is the product of the time discount factor and an adjustment for the evolution of household size.[10] The time discount factor β, housing share in the utility η, and the high ownership premium θ are to be estimated.

Asset: The risk-free interest r is set to 2% per year. The average annual inflation in the benchmark is set to 2%, similar to its value between the late 1990s and 2019. This implies an annual nominal return on the risk free asset of roughly 4.04%. The nominal mortgage rate is the risk free rate plus the mortgage spread $\zeta_m = 0.015\%$, which is the average difference between the contract mortgage interest rate reported by the Federal Housing Finance Agency (FHFA) and the 30-year treasury market yield.

Housing The annual property tax and depreciation cost are set to 1% and 2%, respectively based on the average property taxes and owner costs reported in the American Community Survey (2016). Transaction costs for buyers and sellers are set to $k_b = 6\%$ and $k_s = 2\%$ (see e.g. Sommer, Sullivan, and Verbrugge (2013)). The mortgage financing cost is set to $\tau_m = 2\%$, the average initial fees between 2010 and 2019 reported by the FHFA.

For the baseline calibration in 2019, we set the downpayent requirement χ to be 10%, which is standard in the literature and consistent with the distribution of loan-to-value ratio

[10] See appendix for more details. This approach is quantitatively similar to assuming the discount factor does not vary with age, but instead, the ownership premium is age-specific, which is an alternative way to capture the impact of changes in household size over the life cycle on housing consumption.

for mortgages issued in 2019. Meanwhile, the constraint on payment-to-income ratio (PTI) φ is set to be 0.42, following the 2010 Dodd-Frank legislation.

The initial wealth distribution is constructed using data from Survey of Consumer Survey (2019) for households aged from 20 to 22. We take the value of all their assets to construct the initial wealth distribution.

The income process is approximated by a seven-state Markov chain using the Tauchen (1986) methodology. The annual autocorrelation of earnings is set to 0.97 and the standard deviation of earning shocks is 0.20, following Kaplan, Mitman, and Violante (2020). Parameters in the tax schedule $\tau_0 = 4.787$ and $\tau_1 = 0.151$ come from Kaplan, Mitman, and Violante (2020). The replacement rate after retirement is calibrated to match the average income of retired households. The deterministic age income profile $\bar{w}_j$ is calibrated to match the average household labor income for different age groups using data from Survey of Consumer Finances (2019).

The house price is the average house value reported by owners in the 2019 Survey of Consumer Finance and the rent is the median gross rent from the Census.

5.2 Calibration Strategy

We calibrate the ownership premium θ, discount factor β, minimum house size $\underline{h}$, housing share η, and the two parameters that shape the bequest motive, B_0 and B_1, to the age profile (for 31 age groups) of the homeownership rate, the Loan-to-Value ratios and mortgage, Debt-to-Income ratios and Payment-to-income ratios among owners. Thus, we have 124 moments in total. We estimate these parameters using the Simulated Method of Moments.

Although we jointly calibrate six parameters, each parameter is most closely related to one or two moments. The ownership premium θ is mainly targeted at the level of homeownership rates. Intuitively, an increase in θ makes owning more attractive to renting which pushes up the ownership rate. The minimum house size, $\underline{h}$, is most sensitive to the homeownership rates of young and old households with relatively low wealth as these households are more likely to be constrained by the minimum house size. The housing share in the utility functionη, is most closely related to the payment to income ratio, which is mainly determined by the mortgage-to-income ratio of owners. The discount factor, β, influences the the debt-to-income ratio and the loan-to-value ratio. The parameters of the bequest motive, B_0 and B_1, are most impactful on the ownership rates and mortgage debts of older households. A

Table 1: Directly Calibrated Parameters

Parameter	Value	Source
Maximum of life length J	31	
Working life Jr	23	
Intertemporal elasticity of substitution σ	2	Standard in the literature
Elasticity of substitution ξ	0.8	stnadard in the literature
Equivalence scale e_j		Square root of family size
Survival Probabilities s_j		National Center for Health Statistic
Annual risk free interest rate	2%	Standard in the literature
Annual inflation rate π	2%	Inflation target and realized inflation rates (2010-201
Mortgage spread ζ_m	1.5%	Federal Housing Finance Agency & Federal Reserve I
Annual property tax	1%	American Community Survey
Annual depreciation cost	2%	American Community Survey
Transaction cost for buyers	6%	Sommer, Sullivan, and Verbrugge (2013)
Mortgage closing cost	1%	Federal Housing Fiance Agency
Annual auto correlation of earnings ρ	0.97	Kaplan, Mitman, and Violante (2020)
Standard deviation of earning σ_ϵ	0.2	Kaplan, Mitman, and Violante (2020)
Downpayment requirement χ	10%	LTV distribution
Cap on PTI φ	0.42	Dodd-Frank legislation
Tax schedule τ_0	4.787	Kaplan, Mitman, and Violante (2020)
Tax schedule τ_1	0.151	Kaplan, Mitman, and Violante (2020)
Income Profile $\bar{w}_j$		Survey of Consumer Finances

stronger bequest motive leads to older households being more likely to continue owning and less likely to use mortgage refinancing to extract equity. In particular, $B1$ mainly captures the extent to which a bequest is a luxury good.

5.3 Calibration Results

The calibrated parameters are reported in Table 2. As expected, the ownership premium is greater than 1. This is consistent with the market segregation between owner occupied and rental units. As Yao (2019) points out, most owners live in detached homes while most renters live in apartments.[11] As the price and rent we observe in the data reflect the difference in owned and rented units, the estimated ownership premium, 1.8, captures the difference in the utility generated by owner-occupied and rental units.

The calibrated annual discount factor is 0.9 . The housing share in the utility is 0.27, similar to other estimates in the literature (see e.g. Davis and Ortalo-Magné (2011)).

Table 2: Calibration Results

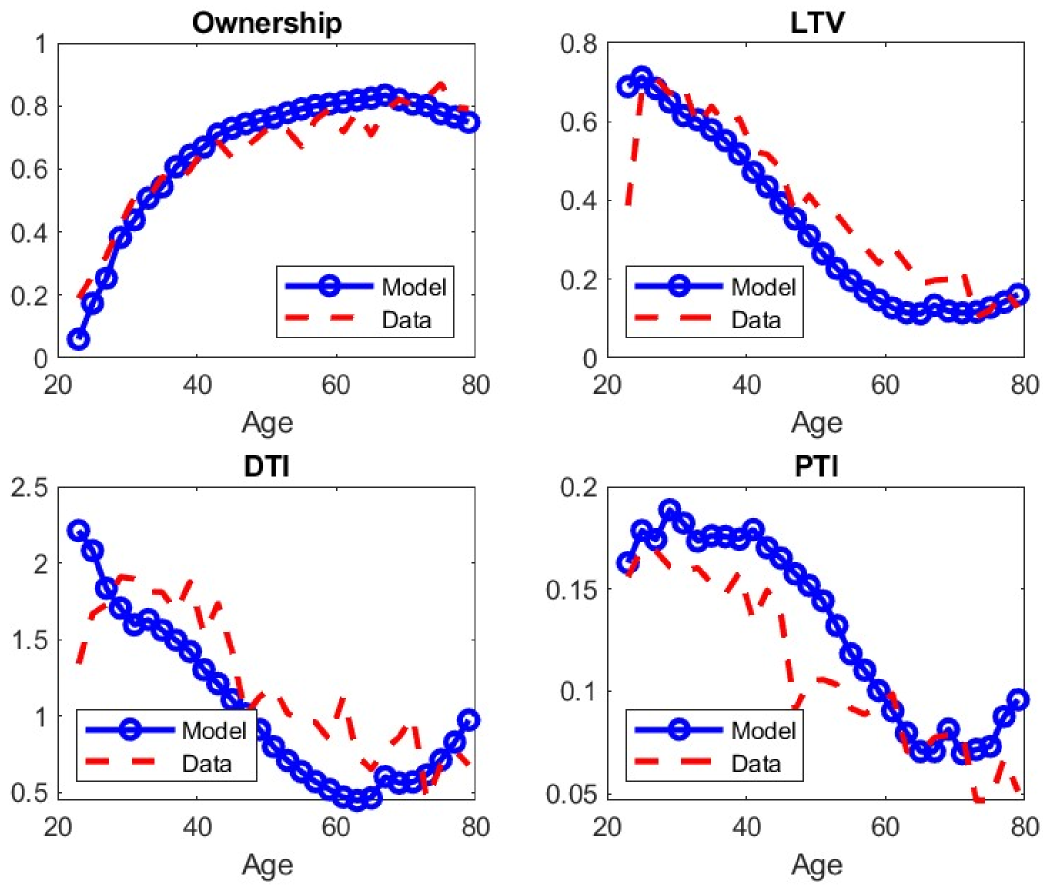
Parameter		Value	Target
η	Housing Share	0.27	DTI
θ	Ownership Premium	1.8	Homeownership Rate
$\underline{h}$	Minimum House Size	1.1	Homeownership Rate
B_0	Bequest Motive	13	Homeownership Rate & Mortgage Loan
B_1	Bequest Motive	0.1	of Senior Households

Figure 8 plots the model generated moments and the data. Our model matches the homeownership rates, average Loan-to-Value ratios and average Debt-to-Income ratios by age reasonably well. It captures the increase in homeownership rate and decreasing mortgage balances by age. This pattern plays an important role in our welfare analysis as we focus on how inflation distorts life-cycle housing and non-housing consumption through the front-loading of real mortgage payments.

We use moments that are not targeted in the calibration to further assess the performance of our baseline. The firstpanel figure in 9 shows that our model tracks the average rent-to-income ratio of renters over the life-cycle, which validates our calibrated housing expenditure

[11] According to the American Housing Survey (2007), the average unit size for owner-occupied units is around 2262 square feet while the average unit size for rental units is 1209 square feet.

Figure 8: Calibration Results



Note: One model period is equivalent to 2 years. Loan to value ratio is ratio of total outstanding mortgage debt to the total housing value of all owners in the economy. Debt to Income ratio is the ratio of total outstanding mortgage debt to the total household income of owners in the economy. Payment to Income ratio is the ratio of total mortgage payments made by owners to the total household income of owners in the economy. Moments are constructed using data from SCF (2019).

share η. In addition to matching the ratio of mortgage loan to house value, our model captures the level of average house value and average outstanding mortgage debt among the owners.

Figure 9: Non-targeted Moments

Note: One model period is equivalent to 2 years. Rent to income ratio is ratio of total rental payment to the total household labor income of all renters in the economy. Average house value is the average value of the primary residence among the owners. Average mortgage debt is the average value of the mortgage loans secured by the primary residence among the owners. Moments are constructed using data from SCF (2019).

6 Welfare Cost of Inflation

We use the calibrated model to quantitatively evaluate the steady-state welfare cost of anticipated and stable inflation. In our counterfactual steady-state exercises, we compare the expected utility of a household born in an economy with different levels of inflation. Although we hold all parameters (except for inflation) fixed in our experiments, we compare the welfare costs both in partial equilibrium where house prices and rents are held fixed across steady states and in general equilibrium where house prices may adjust when housing demand varies with inflation. The general equilibrium analysis highlights two competing effects of inflation on households. First, higher steady state inflation tightens the credit constraint of prime age buyers and reduces their welfare. Second, as the total housing demand goes down due to higher inflation, house prices go down which could be welfare improving from the households' perspective.

In addition to the steady state comparisons, we discuss the welfare implications on different cohorts for an economy that transits between steady states in the partial equilibrium setting. Specifically, we focus on the transition from a 8% inflation steady state to a 2% inflation steady state completed in 18 years, which resembles inflation fall from early 1980s to 2000s. In our robustness analysis, we examine two potential mortgage finance innovations – extending amortization and removing payment-to-income ratio cap – which could relax borrowing constraints on younger households and thus ameliorate the welfare loss due to higher inflation.

We find that anticipated and stable inflation has significant welfare costs. On average, a one percentage point increase in inflation lowers welfare by 0.053% of lifetime consumption in the partial equilibrium setting. The welfare cost of inflation in the general equilibrium analysis depends on the housing supply elasticity. In economies with low housing supply elasticity where house price respond more to demand changes, mild inflation can be welfare enhancing. This implies that the welfare cost of inflation can vary significantly across cities.

We also show that the welfare losses/gain vary across different income groups. We find that when inflation is welfare reducing, the upper-middle class households tend to suffer more from higher inflation than the highest income group and the relatively low income groups.

Along the transition from high to low inflation targets, we find that all the existing cohorts before the transition starts suffer. The cohort with highest amount of average mortgage loans suffer the most. This is consistent with Doepke and Schneider (2006) that unexpected reduction in inflation increases the real debt balance. Cohorts born after the transition starts all enjoy a welfare gain.

6.1 Welfare Cost of Inflation with Fixed House Prices

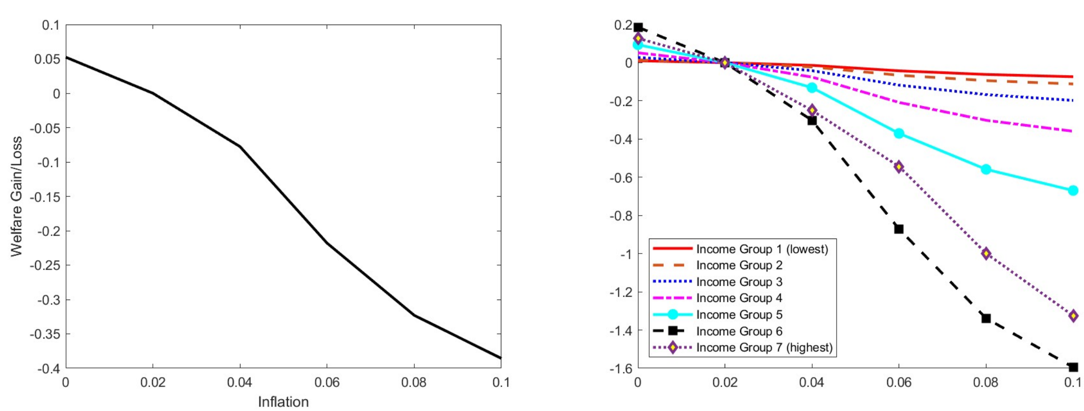
We begin by examining the welfare cost of inflation when we hold fixed the relative price of housing and non-housing goods. This exercise allows us to isolate the effect of inflation through the credit tightening channel. In our counterfactuals we vary annual inflation from 0 to 10 percentage points. Then we move to the transition analysis between 8% annual inflation steady state to 2% annual inflation steady state.

6.1.1 Steady State Comparison

In each experiment we compute the value of a new born which we compare to our benchmark economy with annual inflation of 2%. We calculate the consumption equivalent variation, which is the fraction of consumption and housing consumption required to leave the new born indifferent between the benchmark and the counterfactual economy.

The results are plotted in the left panel of Figure 10. On average, a 1 percentage point increase in inflation leads to welfare cost comparable to 0.05% drop in life-time consumption. This is about half of estimated welfare cost based on money demand models. For instance, according to Lucas (2000), a 1 percentage point increase in inflation leads to a welfare loss slightly smaller than 0.1% consumption loss. This implies that the welfare cost of inflation can be at least 50% higher adding in friction in the housing market due to the typical mortgage design.

Figure 10: Welfare Cost of Inflation: Fixed Real House Prices



(a) Welfare Cost of Inflation (b) Welfare Cost of Inflation by Income

We perform welfare analysis for different income groups. In the quantitative analysis, we discretize the persistent income process using a seven state Markov process. As a result, within each age, we have seven income groups. We calculate and compare the value of a new born with each of these seven income realizations (see the right panel in Figure 10). Interestingly, the upper-middle households see the largest expected lifetime costs from higher inflation (group 5 and 6), while households with the lowest and highest income realizations see the lowest lifetime costs. This difference is due to the differential impact of tighter borrowing constraints across income. Since income is persistent, people who start with the low income realizations are likely to remain as renters throughout their life. As a result, they are less impacted by tighter borrowing constraints resulting from higher inflation. On the other hand, people who start with the highest income realization are less likely to be credit constrained when buying a home due to their high income. Upper-middle income households, in contrast, have both high homeownership rates and are likely to be credit constrained when first buying a house. As a result, their welfare losses from higher inflation are roughly 3 to 4 times larger than the economy-wide average.

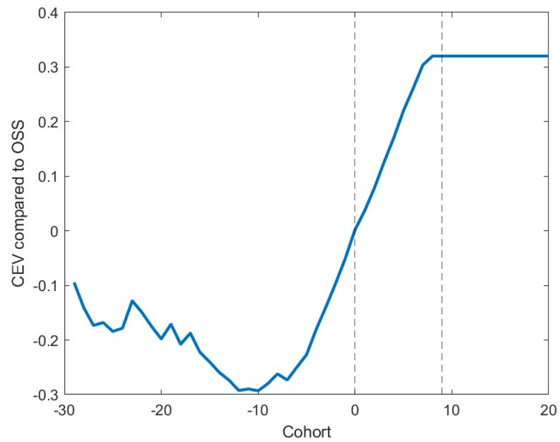
6.1.2 Transitional Cost to Lower Inflation

The previous analysis shows that steady state welfare is higher with 8% than 2 % inflation. This is a change roughly comparable to the fall in the level of inflation in the late 1970s to the late 1990s in the US examined in MacGee and Yao (2024). However, the transition between inflation steady states may result in welfare losses for some age cohorts as the fall in inflation results in higher real mortgage rates on outstanding mortgages (see e.g. Doepke and Schneider (2006)).

We examine the welfare implications for a transition between steady states. The experiment assumes that the economy is originally at the 8% inflation steady state and there is an unexpected change in the steady state level of inflation to 2%. The change will be completed in 8 period (16 years). Once this change is announced, households in the economy know the whole path of future inflation. Along the transition in inflation the mortgage interest rate offered each period incorporates inflation in the period when the mortgage contract was signed. We assume that households can refinance their mortgage along the transition after paying the mortgage interest and with a refinancing cost of 2%.

We report the change in expected welfare for each age cohort in Figure 11. The cohort born at the date the transition is announced is labeled cohort 0, and cohort 9 is the first generation born after the transition in inflation to 2 % is complete. Once the transition in

Figure 11: Welfare by Cohort along Transition



inflation complete, nominal mortgage rates are at the steady state level tied to 2% inflation. As a result, the welfare of cohorts born following the transition are at the 2% inflation steady state, above the pre-tansition steady state welfare. Similarly, cohorts born during the transition between steady states gain. However, the magnitude of the gains for these cohorts are smaller. This is due to the assumed path of nominal mortgage rates which track the path of inflation along the transition.

The cohorts born before the transition see a loss in welfare. There are two points worth noting. First, the loss in welfare for the most impacted cohorts are less than the gains received by cohorts born following the transition. Second, the largest losses are for cohorts in their forties when the transition is announced. This reflects the fact that this age group both has high ownership rates and also high debt levels. As a result, the rise in real rates is most costly for this age group.

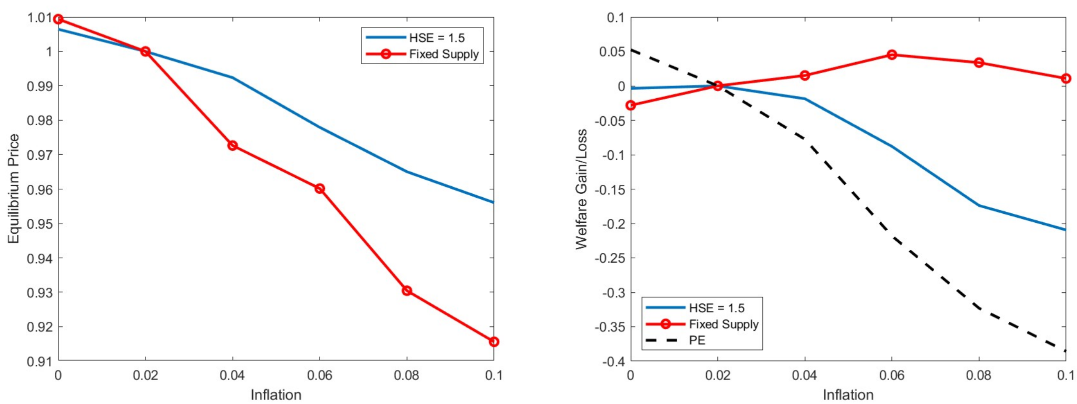
6.2 Welfare Costs of Inflation and Endogenous House Prices

Since the level of inflation influences housing demand, if housing supply is not perfectly elastic, it is also likely to shift the (real) price of housing relative to non-housing. Intuitively, since the direct effect of higher inflation is to tighten borrowing constraints and lower housing demand, it should result in lower real house prices. This shift in relative prices (which we refer to as the indirect effect of inflation) should dampen the welfare effects of higher inflation since lower real house prices relaxes the borrowing constraint of first time home buyers.

To quantify the general equilibrium effect of inflation, we repeat our counterfactuals where we endogenize the relative price of housing. In our baseline we assume a housing supply elasticity of 1.5, following Kaplan, Mitman, and Violante (2020), although we also report the results for when the supply of housing is fixed (i.e., completely inelastic supply).

The indirect effects of inflation via house prices are significant. In the baseline with a supply elasticity of 1.5, a one percentage increase in inflation lowers equilibrium house prices (relative to the non-durable good) by roughly 0.5% (see the left panel in Figure 12). The effect of inflation on house prices with a fixed supply of housing is roughly twice as large.

Figure 12: Welfare Cost/Gain of Inflation: Endogenous Real House Prices



(a) Equilibrium House Prices

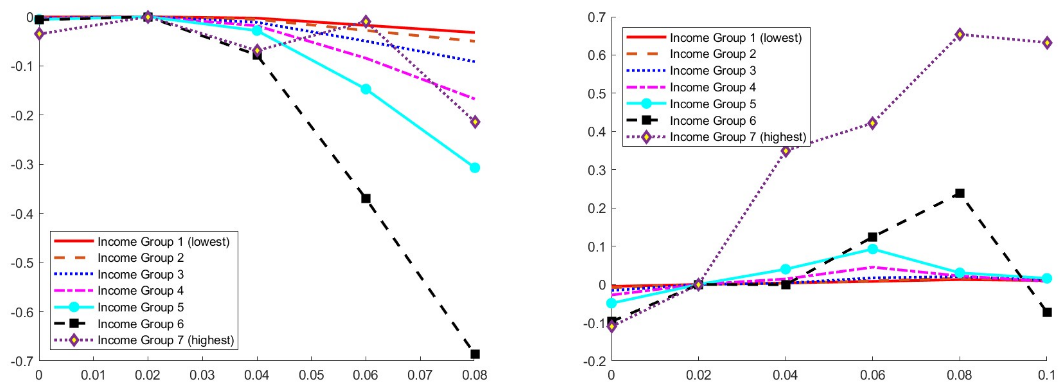
(b) Welfare Cost/Gain of Inflatioin

The indirect effect of inflation on house prices ameliorates the welfare costs of inflation. With a housing supply elasticity of 1.5, the welfare cost of moderate levels of inflation is reduced by almost half compared to when the relative price of housing is fixed. Interestingly, there is little impact of inflation below 4 percent on welfare. An increase in annual inflation from 2 to 4 percentage point leads to a 1% decline in house prices and a welfare loss comparable to 0.02% of lifetime consumption. However, the impact of inflation rises as one moves above 4 percent. An increase in annual inflation from 4 to 6 percentage points leads to a 0.08% welfare loss which is 4 time higher. With a housing supply elasticity of 1.5, welfare peaks at 2% annual inflation.

The economy with fixed housing supply features a larger impact of inflation on house prices and this a smaller welfare cost of inflation. Indeed, for moderate levels of inflation the price decline is large enough to more than offset the impact of higher nominal interest rates on payment to income ratios of younger homebuyers. As a result, welfare peaks at an annual inflation of 6% (see the red connected line in the right panel of figure 12).

The mitigating effect of the change in house prices is broad based across households born with different income realizations. However, households with the highest income realization benefit relatively more from the price decline due to high inflation than other income groups. This reflects the fact that these households are less likely to be borrowing constrained at moderate inflation levels. As a result, they mainly benefit from the lower price of housing that results from higher inflation.

Figure 13: Welfare Cost/Gain of Inflation of Income Groups: Endogenous Real House Prices



(a) Housing Supply Elasticity =1.5

(b) Fixed Housing Supply

6.3 Changing Mortgage Finance to Lower the Welfare Costs of Inflation

In this section, we examine two innovations in mortgage finance that could potentially ameliorate the welfare cost of inflation by relaxing borrowing constraints. First, we consider extending mortgage amortization from 30 years to 36 years. A longer amortization spreads mortgage payments over a longer period which helps to reduce the annual payments. This decline in the annual payment can relax the credit constraint for younger buyers. Second, we relax the cap on the initial payment-to-income ratio. This is more likely to be binding when inflation is high so increasing the cap for higher rates of inflation should reduce the impact on household mortgage (**?**).

In our counterfactual experiments we compare our baseline with 2% inflation to 8% inflation for three supply elasticities: $1.5, 0.6$ and fixed prices (i.e. infinite housing supply elasticity). In these experiments, we compare extending the amortization from 30 to 36

Table 3: Welfare Cost of a Rise in Inflation from 2% to 8%: Two Policies

Housing Supply Elasticity	Baseline	Amortization Extension	Remove cap on PTI
0.6	-0.77%	-0.77%	0.27%
1.5	-0.94%	-0.92%	0.29%
Infinity (Price fixed)	-1.22%	-1.19%	0.29%

years as well as removing the 42% cap on payment-to-income ratio.

Table 3 summarizes the implications for the welfare costs of inflation. In the baseline analysis, a rise in inflation from 2% to 8% lowers welfare by 0.94% of income (for a supply elasticity of 1.5). If we extend the amortization from 30 years to 36 years at 8% inflation, the welfare loss is only slightly smaller at 0.92% of income, which a reduction of only 2.1% smaller ($\frac{-0.92+0.94}{-0.92}$). The gains from extending amortization is even smaller (nearly zero) for a low supply elasticity of 0.6 (see the first row in Table 3), although the impact is slightly larger when house prices are held fixed. This finding is intuitive, as a relaxation in the amortization sees an increase in demand for housing that reduces the decline on house prices at higher levels of inflation.

Removing the cap on payment-to-income ratio successfully overcomes the welfare cost of inflation. Note that when price is fixed, removing the cap in payment-to-income ratio under 2% inflation generates a welfare gain equivalent to 0.73% increase in consumption. If we take the base model of which inflation is set to 2% with the 42% cap on payment-to-income ratio, raise the inflation to 8% while simultaneously remove the 42% payment-to-income ratio, we will see a slight welfare gain comparable to 0.27-0.29% consumption increase.

7 Conclusion

We examine a novel housing market finance channel via which anticipated inflation can impact households. Our main mechanism takes as given the standard mortgage contract which specifies a constant nominal mortgage payment. This implies that variations in inflation tilt the profile of real mortgage payments over the term of a mortgage. Higher levels of anticipated inflation result in more front-loaded real mortgage payments, which tightens the credit constraints for young households.

Our quantitative analysis uses a standard housing tenure choice model to evaluate the welfare cost of inflation. We find that the cost is at least as high as estimates based on money demand approaches. This suggests that the total welfare cost of inflation is likely larger than the commonly cited estimates. Our analysis also suggests that the welfare costs of inflation may vary across cities, with cities with a low supply elasticity likely to see smaller welfare costs from higher inflation that cities with more elastic housing supply due to differences in how housing prices respond to inflation.

Our analysis also suggests that further work on the role of mortgage finance innovations in mitigating the costs of high inflation could be useful. While we find that extending the amortization on mortgages has little impact, relaxing the cap on the payment-to-income ratio has a larger impact.

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Appendix

.1 Proof of Theorem 3.2

Lemma .1 *When $g > \beta(1+r)$, the consumption of housing in the first period h_1 decreases in inflation.*

When the budget constraint binds in the first period, the household's problem becomes

$$\begin{aligned} &\max_{c_1,h_1,c_2,h_2} \ln(c_1) + \theta\ln(h_1) + \beta\ln(c_2) + \beta\theta\ln(h_2) \\ s.t.\ \ & c_1 + (\delta + \frac{(1+r_m)(1+r)}{2+r_m})ph_1 = 1 \\ & c_2 + (\delta + r)ph_2 = g + [(1+r) - \frac{(1+r)^2}{2+r_m}]ph_1 \end{aligned} \tag{27}$$

Denote $x = \delta + \frac{(1+r_m)(1+r)}{2+r_m}$ as the effective housing cost and $y = (1+r) - \frac{(1+r)^2}{2+r_m}$ as the return to housing asset in the first period. Both x and y are increasing in inflation π. Specifically

$$\begin{aligned} \frac{\partial x}{\partial \pi} &= \frac{(1+r)^2}{(2+r_m)^2} \\ \frac{\partial y}{\partial \pi} &= \frac{(1+r)^3}{(2+r_m)^2} \end{aligned} \tag{28}$$

The Household's optimal choice is characterized by:

$$\begin{aligned} c_1 &= 1 - xph_1 \\ h_2 &= \frac{\theta(g + yph_1)}{(1+\theta)(\delta+r)p} \\ c_2 &= \frac{(g+yph_1)}{(1+\theta)} \end{aligned} \tag{29}$$

where

$$\frac{\theta}{h_1} + \frac{\beta(1+\theta)yp}{g+yph_1} = \frac{xp}{1-xph_1} \tag{30}$$

When the budget constraint binds in the first period, the optimal share of housing consumption is larger than $\frac{\theta}{1+\theta}$.

$$\begin{aligned} \frac{xph_1}{1-xph_1} &= \theta + \frac{\beta(1+\theta)yph_1}{g+yph_1} \geq \theta \\ xph_1 &\geq \frac{\theta}{1+\theta} \end{aligned} \tag{31}$$

Rewriting equation 30

$$(1+\beta)(1+\theta)xyp^2h^2 + [(1+\theta)xg - (\theta + \beta(1+\theta))y]ph - \theta g = 0 \tag{32}$$

and taking the total differential of:

$$\frac{\partial h}{\partial \pi} = -\frac{(1+\beta)(1+\theta)(p^2h^2)[x\frac{\partial y}{\partial \pi} + y\frac{\partial x}{\partial \pi}] + (1+\theta)phg\frac{\partial x}{\partial \pi} - [\theta + \beta(1+\theta)]ph\frac{\partial y}{\partial \pi}}{2(1+\beta)(1+\theta)xyp^2h + (1+\theta)xgp - [\theta+\beta(1+\theta)]py} \tag{33}$$

The denominator

$$\begin{aligned}
&2(1+\beta)(1+\theta)xyp^2h_1 + (1+\theta)xgp - [\theta+\beta(1+\theta)]py \\
&= 2(1+\beta)(1+\theta)xyp^2h_1 + (1+\theta)xgp - [(1+\beta)(1+\theta)xyp^2h_1 + (1+\theta)xgp - \frac{\theta g}{h_1}] \\
&= (1+\beta)(1+\theta)xyp^2h_1 + \frac{\theta g}{h_1} > 0
\end{aligned}$$

The numerator

$$\begin{aligned}
&(1+\beta)(1+\theta)(p^2h_1^2)[x\frac{\partial y}{\partial \pi} + y\frac{\partial x}{\partial \pi}] + (1+\theta)ph_1g\frac{\partial x}{\partial \pi} - [\theta+\beta(1+\theta)]ph_1\frac{\partial y}{\partial \pi} \\
&\geq (1+\beta)\theta ph_1\frac{\partial y}{\partial \pi} + (1+\beta)(1+\theta)(p^2h_1^2)y\frac{\partial x}{\partial \pi} + (1+\theta)ph_1g\frac{\partial x}{\partial \pi} - [\theta+\beta(1+\theta)]ph_1\frac{\partial y}{\partial \pi}[\text{ as } xph_1 \geq \frac{\theta}{1+\theta}] \\
&\geq (1+\beta)(1+\theta)(p^2h_1^2)y\frac{\partial x}{\partial \pi} + (1+\theta)ph_1g\frac{\partial x}{\partial \pi} - \beta ph_1\frac{\partial y}{\partial \pi} \\
&\geq (1+\beta)(1+\theta)(p^2h_1^2)y\frac{\partial x}{\partial \pi} + (1+\theta)ph_1\frac{g}{1+r}\frac{\partial y}{\partial \pi} - \beta ph_1\frac{\partial y}{\partial \pi} \\
&\geq (1+\beta)(1+\theta)(p^2h_1^2)y\frac{\partial x}{\partial \pi}\theta ph_1\frac{g}{1+r} + \frac{\partial y}{\partial \pi}(\frac{g}{1+r} - \beta)ph_1\frac{\partial y}{\partial \pi} \\
&\geq 0
\end{aligned}$$

Thus $\frac{\partial h}{\partial \pi} < 0$: Housing consumption in the first period decreases in inflation.

Lemma .2 *When $g > \beta(1+r)$, household's welfare decreases in inflation.*

Combining Equation 27 and Equation 29, we can write household's welfare

$$V(\pi) = ln(1-xph_1) + \theta ln(h_1) + \beta(1+\theta)ln(y+gph) - \beta(1+\theta)ln(1+\theta) + \beta\theta ln(\theta) - \beta\theta ln((\delta+r)p) \tag{34}$$

Applying the envelope theorem,

$$\begin{aligned}
\frac{\partial V}{\partial \pi} &= -\frac{ph_1}{1-xph_1}\frac{\partial x}{\partial \pi} + \frac{ph_1\beta(1+\theta)}{g+yph_1}\frac{\partial y}{\partial \pi} \\
&= [\frac{\beta(1+r)(1+\theta)}{(g+yph_1)} - \frac{1}{1-xph_1}]ph_1\frac{\partial x}{\partial \pi} \\
&= [\frac{\beta(1+r)(1+\theta)(1-xph_1) - (g+yph_1)}{(g+yph_1)(1-xph_1)}]ph_1\frac{\partial x}{\partial \pi} \\
&\leq [\frac{g(1+\theta)(1-xph_1) - (g+yph_1)}{(g+yph_1)(1-xph_1)}]ph_1\frac{\partial x}{\partial \pi} \\
&\leq [\frac{g - (g+yph_1)}{(g+yph_1)(1-xph_1)}]ph_1\frac{\partial x}{\partial \pi} < 0
\end{aligned} \tag{35}$$