# EMPIRICAL TESTS OF A SEQUENTIAL DECISION PROBLEM APPROACH TO MARKETABILITY DISCOUNTS

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**Abstract**

Many equity investments in businesses have restrictions on resale. Both economics and law have long recognized that such restrictions cause a "discount for lack of marketability" on the value of these investments. However, extant estimating methods have been roundly criticized by US courts and produce greatly divergent estimates for the same businesses.

We outline a new approach makes use of a theory of business as a sequential decision problem. This method avoids some of the structural problems of traditional menu-choice and financial-option models, although is computationally demanding. We model representative businesses facing varying economic conditions and scales of operations using this approach, comparing the estimated values with, and without, the restrictions on resale.

We provide a set of tables of marketability discounts for these representative firms, which demonstrate discounts that range from close to zero to over 60 percent. These confirm the preliminary results from an earlier use of this method, which is that commonly-used marketability discounts of 20% to 30% are too high for many well-traded operating companies, and too low for many entrepreneurial, franchisee, and early-stage companies.

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# I. INTRODUCTION

## The discount for lack of marketability

Many equity investments have embedded restrictions on resale. This includes "restricted stock," franchisee businesses with franchisor approval requirements; partner approval requirements for closely held professional businesses and limited liability companies; real estate interests with restrictive covenants; and numerous other situations.

In theory, an asset that cannot be sold must have lower value than an identical asset that could be sold, because the owner lacks important rights. Consistent with this theory, the existence of a "discount for lack of marketability" has long been recognized in both practical work and in United States tax law. As IRS (2009) put it:

> Given two identical business interests, a higher price will be paid by investors in the market for the business interest that can be converted to cash most rapidly, without risk of loss in value. An example is publicly-traded stock on the New York Stock Exchange, where the owner can order the sale and the proceeds are deposited in a bank account in three days.
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> In the alternative, a lesser price is expected for the business interest that cannot be quickly sold and converted to cash. A primary concern driving this price reduction is that, over the uncertain time frame required to complete the sale, the final sale price becomes less certain and with it a decline in value is quite possible. Accordingly, a prudent buyer would want a discount for acquiring such an interest to protect against value loss in a future sale scenario.

## Huge variation in estimates; complaints from the courts

The concept of a marketability discount is well accepted. However, the magnitude of such a discount has been the subject of recurring criticism, and the extant models have been the topic of recurring debate.

In the prior version of this paper, we summarized the following criticisms and found them amply justified:[1]

1. There is a huge variation of estimates produced by practitioners.
   Using the traditional techniques and citing publications and court decisions, it is possible to estimate discounts ranging as far as 5% to 40% an equity interest in the same company.

2. US courts complain about unjustified discounts.
   US courts have complained that the range of estimates is unacceptably large, and repeatedly admonished those estimating marketability discounts to provide a better justification for the magnitude of the estimate.[2]

1. See Anderson (2024) in "References" on page 19.

 

3. Two broad approaches, and no agreement on their use.

Professional and academic literature describes at least two broad approaches, and numerous techniques within them. There is no agreement, or even a tentative consensus, on the proper method to use. As a result, analysts are given the opportunity to "mix and match" and take advantage of choices of convenience.

4. Menu-choice approaches allows nearly untrammeled subjective choice.

The first approach relies on the analyst to choose from a menu of different potential discounts. This approach includes the use of "benchmark" discounts from prior court cases; studies of "restricted stock" done decades ago;[3] "Mandlebaum" factors from a Court decision;[4] and factors from the professional literature.[5] The obvious problem with this approach is that the analyst's choice, not the underlying company or the current economic conditions facing it, is the dominant force in determining the discount.

5. Financial option approach requires heroic assumptions.

A second broad approach is the use of financial option models, including numerous variations of plain vanilla put options, lookback options Asian options, and other variations. This approach, pioneered by Chafee (1993) and Longstaff (1995), relies upon the value of a "put" option priced at the time of the restriction.[6] Financial option pricing models commonly presume a economy in which equity investments are widely traded, riskless borrowing and lending are available, and counter-party risk can be ignored. In the option pricing approach to estimating marketability risks, these assumptions are heroically applied to equity investments on which no trading is allowed, no "volatility" can be observed, and where borrowing is risky.

6. Economic and company factors routinely downplayed or ignored.

In practice, "benchmarks" and other menu-choice models, as well as financial option models,

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2. Key among these decisions are *Mandlebaum* (1995), *Litman* (2007), and *Murphy* (2009). Complete citations are in "References" on page 19. For extensive discussion of these criticisms and citations to court decisions, see Pratt, 6th edition (2022), and IRS (2009).
3. There are at least a dozen such studies, including SEC (1971); Silber (1991); and Trout (1997, 2003). Hertzel & Smith (1993) similarly looked at private placements.
4. These factors emerged from an opinion by Judge David Laro (now deceased) in *Bernard Mandelbaum v. Comm*., TC Memo 1995-255 (1995). Judge Laro was one of the leading jurists on valuation topics, and co-authored a book with Shannon Pratt on valuation and taxes

   Here, the judge reviews evidence presented by experts that include restricted stock studies, studies of IPOs, and interviews with bankers and investors. The judge repeatedly declares he is "unpersuaded," and "troubled" with the evidence presented. Evidencing considerable frustration, the judge then outlines what are now known as Mandlebaum factors.

   See "Innovations Driving a New Approach" on page 8.
5. Other menu-choice methods include the "Quantitative Marketability Discount Model" outlined by Mercer (1997), and a CAPM extension due to Tabak (2002).
6. A "call" option is the right (but not the obligation) to buy a specific asset at a specific price. A "put" option is the right (but not obligation) to sell an asset at a specific price. See "Notes on the Literature" on page 12.

   Option pricing methods applied to marketability discounts have been proposed by Finnerty (2002), Ghaidorov (2009), Comment (2012), and Elmore (2017).

 

can also produce a wide variation in estimated discounts for the same company, without substantial justification involving the underlying company and economic conditions. Here, we repeat the criticism of the tax court in *Mandlebaum* (1995):

> Ascertaining the appropriate discount for limited marketability is a factual determination. Critical to this determination is an appreciation of the fundamental elements of value that are used by an investor in making his or her investment decision.

Making use of these "fundamental elements" is a key element of the new approach.

## High stakes should motivate improved methods.

The stakes involved for taxpayers, business owners, and even practitioners are serious. Given this, leaders in the professions involved with estimating business value should pay more attention to this issue. This includes those tasked with estimating commercial damages, preparing business value estimates for estate taxation, and those involved with buying and selling investments in companies.

In addition, a benefit of better understanding of the causes of large discounts could be the improvement in regulations and contracts that cause such discounts.

 

# II. NEW APPROACH

## Two Arguments for a New Approach

We motivate the new approach with the following arguments.

*1. Investors in closely-held businesses are active, not passive, investors.*

Investors in closely-held businesses are not properly modeled as portfolio investors.[7] These entrepreneurs make business decisions under conditions of uncertainty; commonly operate under financial and operational constraints.[8] Furthermore, such companies themselves are typically organized as limited liability companies, partnerships, or other business forms. In general, such business forms do not allow for traded shares in the manner of the traditional "C" corporation.[9]

Typically, a restriction on sale arises with personal involvement and investment in a company. As a rule, the existence of a restriction on sale eliminates the pretense of a passive investor managing a portfolio of marketable securities.[10] Consistent with both the prevalence of closely-held firms in the economy, we do not adopt the perfect-market presumptions employed in the common financial option pricing models. These assumption, including zero transaction costs, continuous trading, and the availability of risk-free borrowing, are pillars of modern Finance.[11] However, they are all inconsistent with the reality of equity in closely-held firms, costly borrowing, liquidity constraints, and limitations on resale of equity.

*2. Economic conditions and scale affect the value of the enterprise, and therefore should affect discounts on that valuation.*

It is obvious that economic conditions and the scale of the enterprise affect its ability to acquire resources, hire workers, attract financing capital, invest in R&D, and take advantage

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7. As noted in Anderson (2012), the assumption that businesses are managed by entrepreneurs sharply contrasts with the passive-investor model that underlies much of modern finance. He notes that Irving Fisher's study of interest rates in the 1930s already recognized the reliance (and, in Fisher's words, deference) of the entrepreneur to the needs of the family for income and assets. See "Notes on the Literature" on page 12.
8. Dixit & Pindyck (1994) describe the failure of the neoclassical investment rule when uncertainty exists, and the role of management flexibility. Anderson (2012) amplifies this for businesses. See "Notes on the Literature" on page 12.
9. This is for both practical and legal reasons. First, in practice such forms are designed for a business managed and owned by a handful of private individuals. Second, laws in countries like the United States establish a corporate form of organization allowing for investor disclosure and other elements of publicly-traded securities, and these laws and their compliance burdens establish a barrier to most small companies from entering.
10. While we do not explore this question here, it is worth observing that a restriction on sale could have significant effects on the finances of an un-diversified investor for whom the investment is a major part of their net worth and is related to their regular income. Many entrepreneurs fall within this definition.
11. See "Notes on the Literature" on page 12.

 

of opportunities that arise periodically. If this is the case, then such conditions should be integral to any value and discount on that value due to marketability.

It is remarkable that this same insight was asserted in the seminal *Mandlebaum* tax court decision nearly 30 years ago, which stated the "fundamental elements of value that are used by an investor in making his or her investment decision" were "critical to this determination" of a marketability discount.[12]

## Innovations Driving a New Approach

We propose a new approach that makes use of two innovations, which directly address the motivating arguments. They establish a structure that is different from the passive investor model that underlies much of modern finance, and which natively considers economic conditions and the scale of the enterprise:

1. *We model business decisions as the entrepreneur's sequential decision problem.*

In a sequential decision problem, a person if faced with a set of conditions that represent the state of the world. The person then evaluates what decision options they have, noting that some are more costly than others, and that the consequences of today's decision will affect the state of the world tomorrow. The person makes that decision with the intention of maximizing value, where value is part of the solution to a functional equation taking into account current and future benefits.

The concept of a sequential decision model is much closer to the actual decisions facing the manager and investors in a closely-held company than the neoclassical notion of a profit-maximizing firm. It is also much closer than the concept of a dispassionate investor maximizing the value of a portfolio of securities. Thus, it is sharply different from financial option models based on the notion of constantly-traded securities, and also different from idealized notions of profit-maximizing firms.

2. *We explicitly include business and industry-specific factors in the valuation model.*

In particular, we consider the investor-manager as operating in a market with at least three state dimensions: the underlying economy that determines aggregate demand for a company's products; the scale of the business itself, which determines its ability to produce and earn; and an idiosyncratic factor that is the restriction on sale. This makes broad use of one of the attractive features of the menu-choice approach, which is the recognition of different factors related to specific businesses.

We explicitly include fundamental business indicators for economic reasons. These economic reasons match the Court's opinion in *Mandlebaum* which asserted such factors must be considered.[13]

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12. *Mandlebaum* (1995, "Opinion", 1A).

### Benefits of the New Approach

This approach has a significant benefits compared to traditional methods:

1. First, it produces numerical estimates that are different for firms facing different economic conditions, and for those with different scale of operations. This matches our intuition that economic and scale factors affect business enterprise value and therefore discounts to that value. It also addresses the criticism of the *Mandlebaum* court that prevalent methods ignored these differences.

2. Second, it produces discounts that decline in magnitude as the restriction period moves toward expiration. This also matches our intuition that an expiring restriction should have little effect on the value.

3. Third, it produces discounts that are higher for firms where holding the equity for the restriction periods means giving up significant opportunities or accepting unusual risks. This again matches intuition.

## Implementing the New Approach

We implement this new approach as follows:

1. Create a set of representative businesses for which profits that vary with economic conditions, scale of the operation, and random events. Include within these firms an idiosyncratic factor which is a restriction on sale that applies to some equity interests. See "Representative Companies" on page 10.

2. Create an economy that incorporates variation in economic demand, and allows business managers to invest to increase the scale of their operation, while subjecting them to business risk related to unknown events. See "Creating an Economy" on page 10.

3. Using the sequential decision problem formulation of business value outlined in Anderson (2012), calculate the equity value of the businesses for multiple combinations of economic and industry conditions, and across different remaining durations of restriction on sale. See "Composing and Solving the Sequential Decision Model" on page 11.

4. Compare these values for the same businesses and same economic conditions across different remaining restrictions on sale of the equity.

The differences in value, for the same business with the same scale of operation and under the same economic conditions, represent a discount for lack of marketability ("DLOM"). See

13. Mandlebaum (1995, "Opinion", 1A) lists the following "non exclusive" factors: value of privately-traded vs. publicly traded stock, if both exist; analysis of the company's financial statements; the company's dividend-paying capacity, history of paying dividends; nature of the company, its history, its position in the industry, and its economic outlook; company management; degree of control in the transferred stock; the restriction on the transferability of the stock; the period of time for which an investor must hold the subject stock to realize a sufficient profit; the company's redemption policy; the cost of effectuating a public offering, including legal and accounting fees.

 

"Extracting the Discount for Lack of Marketability" on page 11.

## Representative Companies

We generate a set of representative companies, each with different underlying cost structure, trend growth rate, cost of investment, likelihood of that investment producing an increase in scale. For each company, we also assume a different trend growth rate and investor discount rate.

Some companies are in stable industries and economies; some companies are entrepreneurial and have substantial risks of both upside and downside, with growth and discount rates that match that presumption. In addition, all companies are presumed to generate operating profits in a baseline economy and scale, when not making major investments.

### *Parameters Varying Across Companies*

The important parameters that vary among companies include:

- growth and discount rates.
- resale price of company equity, as multiple of earnings
- power of investment, in terms of likelihood of investment producing a growth in scale
- income statement ratios, and therefore baseline profitability in normal economic times

We label these as "CompanyA", "CompanyB," etc.

## Creating an Economy

We create an economy with three state variables. This implies the "dimension" of the state space is three. The three state dimensions are:

- The underlying economic demand, which we designate as "Y." This variable can be seen as a macroeconomic indicator, but for the purposes of estimating the value of any specific company should be an economic indicator that is relevant for that industry.
- The scale of the business, which we designate as "M." Here the scale can indicate the number of employees, size of operating revenue, number of stores or factories, or another indicator that is relevant for operating companies in a particular industry.
- The duration of any restriction on sale of equity, which is the idiosyncratic factor and is designated as "I."

We do not presume that the economy and scale variables affect the value of the company in a linear, log-linear, or other restrictive fashion. In the implementation used for these representative companies, a better economy and larger scale improve the value of the company, but with lesser effect as the variable increases.

The use of three state variables produces a very rich fabric of potential situations in which a company owner-manager could find themselves. The number of possible states represents all combinations of each state variable. In this model, we allow 4 levels of the economy (Y) state variable;

 

5 levels of the scale (M) variable; and 5 levels of the idiosyncratic restriction variable (I). The size of the state space is therefore 4x5x5=100.

### *Comparing sequence of multi-dimensional states with DCF scenarios*

It is useful to compare this with the standard discounted cash flow model (income approach) in valuation. It is common for such models to presume a static set of income statement ratios and continued operation in the same industry and in the same manner. Arithmetically, such models routinely distill to a base year level of earnings and an assumption about the discount rate and earnings growth rate.[14]

One can easily produce a number of scenarios using this method, where each scenario involves a different growth and discount rate. However, all these trajectories carry with them the same underlying—and extremely restrictive—assumption that the company continues to operate in identical manner proportionate to revenue. The sequential decision model approach used here, with a dimension of the state for the scale of the business, breaks out of this restrictive assumption in a way that allows for a much better representation of closely-held businesses.

## Composing and Solving the Sequential Decision Model

We compose the investor's sequential decision model and solve it using the value functional approach to business valuation outlined in Anderson (2012) and Anderson (2019). This value represents the benefits (earnings from holding the equity or selling it), discounted across time and taking into account risk and incomplete information.

The value is estimated for each state. In this implementation of a three-dimensional state space, each state has a different combination of the Y, M, and I state variables. The steps in composing and solving the decision problem are described further in "Appendix A: Value of Representative Companies" on page 1 of the Appendix.

## Extracting the Discount for Lack of Marketability

Once the value of the representative companies have been calculated for each state, we can extract the discounts for lack of marketability that are implied. We extract these by comparing values of the same business in the same industry, facing the same economic conditions, and with the same scale, but where one equity investment has a restriction on sale and the other does not.

We calculate this as a share of the no-restriction value.

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14. This is most apparent when a "terminal value" is calculated using a perceptual capitalization of current earnings, such as using the Gordon Growth formula *pi/(d-g)*. The same result can be shown to occur in a discounted cash flow schedule where the expense and dividend ratios remain the same and revenue grows at a constant rate.

    Of course, analysts can and do adjust for known or anticipated changes, such as assuming "two stage growth" or other discrete changes in underlying revenue growth or expenses. They can also add adjustments (such a working capital allowance and new or retired financing) that make the resulting discounted earnings stream differ from a standard capitalization of income formula.

## Notes on the Literature

### *Sequential decision problems*

The mathematics underlying the sequential decision problem approach to economic decisions were outlined by Stokey & Lucas (1989), making use of innovations from Blackwell (1965) and antecedents in control theory, including the seminal work of Richard Bellman (1957). Dixit & Pindyck (1994) explored sequential decision models as a method to evaluate investment decisions that had elements of real options, discussed further below. Ljungvist & Sargent (2000, with later editions) presented extensive uses of sequential decision models in macroeconomics.

The application of this approach to businesses is more recent. Anderson (2012) outlines specific restrictions (including the human transversality conditions) applicable to privately-held businesses. These provide the mathematical bases for composing the sequential decision problem and solving the related functional equation to represent the value of a business in different economic conditions. We follow the general method outlined in Anderson (2012) and also demonstrated in Anderson (2019) for businesses in different locations.

### *Functional equations; value functional method*

In the sequential decision model of a business, the value of the business is calculated using a *value functional* equation. A *functional* equation is a function with other functions as arguments. This particular functional is sometimes called a "Bellman equation" after Richard Bellman. It is a *recursive* equation in that the value of the business can be represented as a recursion across future time periods, with the value at one time period involving the value in future time periods.

The solution of this value functional equation is a mapping from states to decisions. These decisions represent the value-maximizing option for the subject of the decision problem, given the current state. The mathematics behind this are stated in Stokey & Lucas (1989) and Ljungvist & Sargent (2000).

As outlined by Anderson (2012), the value-maximizing solution for a business will often not be the same as the "profit maximizing" decision. The empirical results shown here demonstrate that the value-maximizing decision is quite often not the same as the profit-maximizing decision.[15]

### *Financial and real options*

Seminal articles in this now-vast literature of financial options include Black & Scholes (1973), Merton (1973) and Cox, Ross & Rubenstein (1979). Hull (2005) is a standard reference to the many varieties of options now used. In general, financial option models (including the Black-Scholes-Merton model and its many extensions) presume perfect markets, including the availabil-

15. This result is demonstrated with this method whenever the optimal management decision is to invest, or sell.
We do not present the results of comparable financial option models in this article. However, extended results obtained by the author using these methods amply demonstrate that option methods indicate rational equity investors in businesses should, in some cases, sell equity in companies that are making operating profits.

ity of riskless borrowing and lending, zero transaction costs, and the non-existence of counter-party risk. These and other strong assumptions allow for the principle of "no arbitrage." Such a principle cannot be asserted for equity in privately-held companies that is not widely traded, and for which transaction costs are often considerable.

Some option texts recognize "real options" as a related concept, where the underlying "security" is something real, and not a publicly-traded security. Dixit & Pindyck (1994) is the seminal reference to understanding how the valuation of real options requires different methods than financial options traded in near-perfect markets. Schwartz & Trigeorgis (2001) collect important articles in this field, including the article by Stewart Myers (1977) which coined the term "real options."

# III. EMPIRICAL RESULTS

We compose the sequential decision model for a set of representative companies as described above in "Representative Companies" on page 10 and "Creating an Economy" on page 10. We then solve for the value of each company in all states, as outlined in "Composing and Solving the Sequential Decision Model" on page 11.

We provide more information on this methodology, including a table presenting the value for all representative companies, in a selection of the states, in "Appendix A: Value of Representative Companies" on page 1 of the Appendix.

## Consolidated Table of Discounts, All Companies for Selected Y-M Combinations

To illustrate the range of discounts calculated among these six representative companies, each with different operating and investment parameters, and 100 different states for each company, we provide the table below. This table lists, for the selected company-state combinations, the best policy in each state (which could indicate operating, investing, or selling); the value in that state (given the restriction on equity resale, if still in place), and the marketability discount.

- For all states where the restriction has expired, the marketability discount is zero.
- For some companies (notably Company A), the marketability discount is zero or close to it. As noted in "Company A" on page 1 in Appendix B, this firm is quite profitable and holding the equity investment involves very little risk.

  Such firms should, our intuition suggests, have low marketability discounts as holders of the equity are likely to be rewarded with consistent earnings, and potential buyers would presumably be ready to purchase these on the basis of recurring earnings.
- For some companies (notably Company C and Company D), there are significant discounts for lack of marketability. As noted in the detailed results for these companies shown in "Company C" on page 4 and "Company D" on page 6 of Appendix B, these firms have more volatility, higher risk, and a good chance that equity holders that are prevented from selling will miss opportunities.
- As motivated by the arguments in "New Approach" on page 7, the discounts vary considerably. Furthermore, they vary in the manner we would expect from economic intuition. See "Advice Based on these Results" on page 16.

See Table 1, "Value and Discounts as Restriction Expires, Selected Companies," on page 15.

**Table 1: Value and Discounts as Restriction Expires, Selected Companies**

| Company, State | *State number* | Best Policy in State | Value in State | Discount Share |
|---|---|---|---|---|
| A_Y100_Scale1_Restrict4 | *26* | 2 | 5710 | 0 |
| A_Y100_Scale1_Restrict3 | *27* | 2 | 5710 | 0 |
| A_Y100_Scale1_Restrict2 | *28* | 2 | 5710 | 0 |
| A_Y100_Scale1_Restrict1 | *29* | 2 | 5710 | 0 |
| A_Y100_Scale1_Restrict0 | *30* | 2 | 5710 | 0 |
| B_Y150_Scale1_Restrict4 | *51* | 1 | 3133 | 0.004 |
| B_Y150_Scale1_Restrict3 | *52* | 1 | 3140 | 0.002 |
| B_Y150_Scale1_Restrict2 | *53* | 1 | 3145 | 0 |
| B_Y150_Scale1_Restrict1 | *54* | 1 | 3146 | 0 |
| B_Y150_Scale1_Restrict0 | *55* | 1 | 3146 | 0 |
| C_Y150_Scale1.5_Restrict4 | *61* | 1 | 2096 | 0.343 |
| C_Y150_Scale1.5_Restrict3 | *62* | 1 | 2294 | 0.281 |
| C_Y150_Scale1.5_Restrict2 | *63* | 1 | 2529 | 0.207 |
| C_Y150_Scale1.5_Restrict1 | *64* | 1 | 2807 | 0.12 |
| C_Y150_Scale1.5_Restrict0 | *65* | 3 | 3189 | 0 |
| D_Y75_Scale2.25_Restrict4 | *16* | 1 | 1943 | 0.393 |
| D_Y75_Scale2.25_Restrict3 | *17* | 1 | 2284 | 0.287 |
| D_Y75_Scale2.25_Restrict2 | *18* | 1 | 2698 | 0.157 |
| D_Y75_Scale2.25_Restrict1 | *19* | 1 | 3202 | 0 |
| D_Y75_Scale2.25_Restrict0 | *20* | 1 | 3202 | 0 |

*Value expressed in $thousands, for companies A, B, C, D. See Appendix C for extended results.*
*Table shows companies in states with same Y and M, with declining restrictions.*
*Policies are: 1=operate; 2=invest; 3=sell equity.*

## Advice Based on these Results

The detailed results in "Discounts for Representative Companies" on page 1 of Appendix B, combined with the additional information on the characteristics of the underlying firms also included in the appendix, provide strong indicators of who marketability discounts vary among firms. In particular:

- Higher investor discount rates; lower trend growth rates; a bigger spread between growth and discount rates; and the potential to lose resale opportunities all contribute to higher marketability discounts.
- Stable growth prospects and consistent earnings suggest lower marketability discounts, which in some cases are close to zero.
- The need to rapidly secure specific rights are elements common to franchisee and technology firms. When these conditions are present in a company and industry, it suggests a higher marketability discount.

With this information and the extended results summarized in the appendices, we summarize the implied guidance in Table 2, "Preliminary Advice on Marketability Discounts, from Sequential Decision Model Approach," on page 16.

**Table 2: Preliminary Advice on Marketability Discounts, from Sequential Decision Model Approach**

| Company and Industry | Stable Economies and Scale | Volatile Economies and Scale |
|---|---|---|
| Well capitalized, income-earning companies with solid industry position | 0-10% | 5-20% |
| Companies with typical risk profiles, including financing, market position and technology | 5-25% | 10-50% |
| Entrepreneurial, technology, and companies with significant franchise or IP risks | 10-40% | 25-75% |
| | | |

*Source: Author's advice from preliminary results from limited set of representative firms, intended as comparison with menu choice (including "benchmark") and option methods for estimating marketability discounts of one to two years in duration.*

# IV. CONCLUSIONS

We conclude the following:

*1. Courts are correct: current literature creates license*

We find substantial basis for the criticism levied by U.S courts: literature and common practice allow for an unacceptably large variation in the estimation of marketability discounts. Furthermore, we observe that the same literature and common practice allow for methods that are untethered from rationales originating in the subject business, industries, or potential buyers.

Using the extant approaches, practitioners can estimate a marketability discount of up to 40% on nearly any company, and assert a basis for doing so within the professional literature and available case law.

*2. Loss of value to the underlying business investment is the logical basis for marketability discounts.*

Logic dictates that a change in the value of a business will affect the value of an equity investment in that business. Thus, a proper method to estimate the loss in value due to a marketability restriction should take into account the fundamental factors that drive business value. This same principle was asserted in a seminal tax court decision nearly 30 years ago.

Unfortunately, much of the literature supporting traditional approaches focuses on financial market factors such as "volatility" that are not primary determinants of the value of a business, and menu-choice methods that can ignore the underlying economic conditions, management, distress and opportunities facing a business.

*3. A Sequential Decision Model approach overcomes serious deficiencies in extant approaches*

The approach we recommend here overcomes a number of serious deficiencies in the extant approaches, including:

- Business and industry factors are explicitly included in the model, and substantially determine the results. As a result, different companies in different industries, and companies in the same industry with different risks and investment opportunities, will have different estimated discounts for lack of marketability.
- Abstract financial market factors; interpretations of past court rulings; and mechanical use of "factors" do not determine the discount.
- While professional judgment is involved in creating the model and its parameters, there is much less subjective judgment that is allowed within the current tableau of menu-choice and financial option models.
- An estimated discount cannot exceed 100%, nor be negative, as can occur in some option

models.

- Discounts can be close to zero for equity in companies that steadily produce income and face low risk. They can be high for companies that face significant risks and opportunities, including entrepreneurial, technology, and franchisee companies.
- Matching intuition, the level of discount drops as the restriction period closes.

4. The SDP approach has its difficulties. These include:

- Novelty, which is probably the biggest barrier.
- Difficulty in applying, which is significant but not insurmountable.
- Sensitivity to input parameters related to potential investments, discounts, and growth rates, and possible sale prices. This is both a difficulty, and an opportunity, because these factors should affect the value of the company, and the reduction in value for an equity holder that is prevented from selling.

5. *Value loss can be small or large.*

There is ample theoretical basis to presume a significant loss in value when the owner of an asset gives up important rights, such as the right to sell. There is also theoretical basis to presume only a tiny fraction of value is lost when a marketability restriction is about to expire or does not prevent an investor from earning a regular cash return. This method, as demonstrated in Table 2, "Preliminary Advice on Marketability Discounts, from Sequential Decision Model Approach," on page 16, provides advice consistent with these strong and logical theoretical conclusions.

# V. REFERENCES

## References: Scholarly Works

Anderson, Patrick, 2024. "Loss in Business Value Due to Restrictions on Sale: A Necessary Re-Visitation," working paper presented at NAFE International Conference, June 1, 2024, Reykjavik, Iceland.

Anderson, Patrick, 2012. *Economics of Business Valuation*, Stanford University Press.

Antoniades, Harris, and Ionannis Micholpoulous, 2017. "DLOMs: Common Valuation Approaches to the Illiquidity Discount," blog post, retrieved from Stout website at https://www.stout.com/en/insights, February 2024. Link in February 2024: https://www.stout.com/en/insights/article/sj17-dloms-common-valuation-approaches-to-the-illiquidity-discount.

Bajaj, M., D.J. Denis, S.P. Ferris, and A. Sarin. 2001. "Firm Value and Marketability Discounts", *Journal of Corporation Law*, vol. 27, no. 1 (Fall):89-115.

Barenbaum, Lester; Schubert, Walter; and Garcia, Kyle, 2015. "Determining Lack of Marketability Discounts: Employing an Equity Collar," *The Journal of Entrepreneurial Finance,* vol. 17, no. 1, pp. 65-81. DOI: https://doi.org/10.57229/2373-1761.1254. Retrieved from: https://digitalcommons.pepperdine.edu/jef/vol17/iss1/3.

Black, F. and M. Scholes, 1973. "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* 81, No. 3 (May-June 1973), pp. 637-654.

Comment, Robert, 2012. "Revisiting the Illiquidity Discount for Private Companies: A New (and Skeptical) Restricted Stock Study," *Journal of Applied Corporate Finance,* vol. 24 no. 1, (Winter): 80-92.

Cox, J., S. Ross, and M. Rubenstein, 1979. "Option Pricing: A Simplified Approach." *Journal of Financial Economics* vol. 7, Sept. 1979, pp. 229–263. ISSN 0304-405X, https://doi.org/10.1016/0304-405X(79)90015-1. Available from: https://www.sciencedirect.com/science/article/pii/0304405X79900151.

David Chaffe, 1993. "Option Pricing as a Proxy for Discount for Lack of Marketability in Private Company Valuations." *Business Valuation Review* (December 1993), 12, 4:182-188.

Dixit & Pindyck, 1994. *Investment Under Uncertainty*. Princeton University Press.

Elmore, John, 2017. "Determining the Discount for Lack of Marketability with Put Option Pricing Models in View of the Section 2704 Proposed Regulations." *Valuation Practices and Procedures Insights*, (Winter 2017). Retrieved from: willamette.com.

Finnerty, John D., 2002. "An Average-Strike Put Option Model of Marketability Discount." *The Journal of Derivatives* (Summer 2002), 19, 4:53-69.

Ghaidarov, Stillian, 2009. "Analysis and Critique of the Average Strike Put Option Marketability Discount Model" (September 24, 2009). Available at SSRN: https://ssrn.com/

abstract=1478266 or http://dx.doi.org/10.2139/ssrn.1478266

Hertzel, M., and R.L. Smith. 1993. "Market Discounts and Shareholder Gains from Placing Equity Privately." *Journal of Finance*, vol.48, no.2 (June): 459-485.

Hitchner, James R, (editor), 2003. *Financial Valuation: Application and Models*. Wiley.

Hull, John C., 2003. *Options, Futures, and Other Derivatives. 5th edition*, Prentice Hall.

IRS, 2009. "Discount for Lack of Marketability," Job Aid for IRS Valuation Professionals, September 25, 2009. No author indicated. Retrieved in November 2023 from: https://www.irs.gov/businesses/valuation-of-assets,

Longstaff, Francis A., 1995. "How Much Can Marketability Affect Security Values?" *The Journal of Finance*, vol. L, No. 5 (1995), pp.1767-1774.

Ljungvist, Lars, & Thomas Sargent, 2000, 2004, 2012, 2018. *Recursive Macroeconomic Theory.* MIT Press.

Mercer, Z. Christopher, 1997. *Quantifying Marketability Discounts*, Peabody Publishing.

Merton, R.C., 1973. "The Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science* 4, No. 1 (Spring 1973), pp. 141-183.

Myers, Stewart C., 1977. "Determinants of corporate borrowing". *Journal of Financial Economics,* 5: 147–175.

OTC, 2023. Descriptions of "Pink" markets, and "15c2-11 Resource Center."

Pratt, Shannon and ASA Foundation, 2022. *Valuing a Business, 6th edition,* McGraw-Hill.

Roll, Richard, 1977. "A Critique of the Asset Pricing Theory's Tests Part I: On Past and Potential Tstability of the Theory." *Journal of Financial Economics*, vol. 4, no. 2, March 1977, pp. 129-176.

Silber, William L., 1991, "Discounts on restricted stock: The impact of illiquidity on stock prices," *Financial Analysts Journal* 47, 60 - 64.

Securities and Exchange Commission, 1971. "Institutional investor study report of the Securities and Exchange Commission," U.S. Government Printing Office, Washington, D.C.

Stokey, Nancy L., & Robert E. Lucas, 1989. *Recursive Methods in Economic Dynamics*, Harvard University Press, 1989.

Schwartz, Eduardo & Lenos Trigeorgis (editors), 2004. *Real Options and Investment under Uncertainty*. MIT Press.

Stout, 2023. "Stout Restricted Stock Study Companion Guide," BVresources. (no author indicated). Retrieved from https://www.bvresources.com/, April 2024.

Tabak, David, 2002. "A CAPM-Based Approach to Calculating Illiquidity Discounts," mono-

graph, NERA. Nov. 11, 2002.

Trout, Robert R., 2003. "Minimum Marketability Discounts," *Business Valuation Review*, September 2003 pp. 124-126.

## References: Court Decisions

*Litman v. United States*, 78 Fed, Cl. 90, (2007). An errata was issued by the same court on a later date that does not affect the criticisms of methods included in the original decision.

*Mandelbaum v. Commissioner*, 1995. T.C. Memo. 1995-255, aff'd. 91 F.3d 124 (3rd Circuit, 1996).

*Murphy v. United States*, 94923, 2009-2

# A. APPENDIX A: VALUE OF REPRESENTATIVE COMPANIES

## Model of Economy and Business

### *Periodicity*

This model is quarterly. Discount, growth, and likelihood of events are all calculated on a per-time-period basis.

### *State Dimensions*

Three state dimensions: Y (economic demand), M (scale of company), and I (idiosyncratic dimension for the company). Y is expressed as an index, with 100 as the base; M is expressed as a ratio to 1, with 1 as the base; and I is a time counter that mechanically declines over the duration of the prohibited transfer period, reaching zero when the restriction expires.

### *Actions*

Actions available to the investor-manager: operate in the normal manner; make discretionary investments in the hopes of expanding the scale of the enterprise; or sell equity in the company.

### *Rewards*

Rewards when operating or investing are calculated using the cost ratios for each company, and if required the additional cost of discretionary investment. Revenue for the company is a function of the underlying economy and the scale of the company. Selling the equity produces a large one-time reward, and afterwards zero rewards.

### *Transitions*

Investing costs money at the time of the investment, but potentially increases the scale of the company and therefore the likely future operating earnings and the likely future resale price for equity in the company. Selling equity in the company leads to no future rewards.

### *Composing and Solving the Model*

We compose the sequential decision model and solve it using the value functional approach outlined in Anderson (2012), as also applied in Anderson (2019). This method includes composing the problem so that the business fulfills certain restrictions, outlined in Anderson (2012), that guarantee that the decision problem can be solved. These restrictions include growth rates and discount rates, bounded earnings and losses, and limited liability for the investor.

After composing the decision problem, we check it to ensure that it meets the requirements for a solution. We then attempt to solve it. We use Matlab® software, which allows vector mathematics, and the Rapid Recursive Toolbox®, which is designed for sequential decision problems, for this purpose.

## Values for different companies in zero-restriction states

Table 1, "Value of Companies, Zero-Restriction States," on page 3 below records the calculated value for each of the representative companies, in selected states representing different economic conditions (variable Y) and scale (variable M). These are calculated for the companies in zero-restriction states, where the equity owners have no restriction on selling.

As one indication of the validity of the method when applied without regard to discounts for lack of marketability, we can see the Company A is more valuable than the other firms in all combinations of economic conditions and scale. Consistent with economic intuition, Company A has a higher operating profit margin than the other firms (as shown in Table 2, "Parameters for Representative Companies," on page 4 of this Appendix).

Similarly, companies E and F have very similar profit margins and growth rates. However, the discount rate is higher for company F, due to higher volatility or other risks. Consistent with intuition, Company F is worth less than Company E in every state shown.

**Figure 1. Value of Companies in Zero-Restriction States**

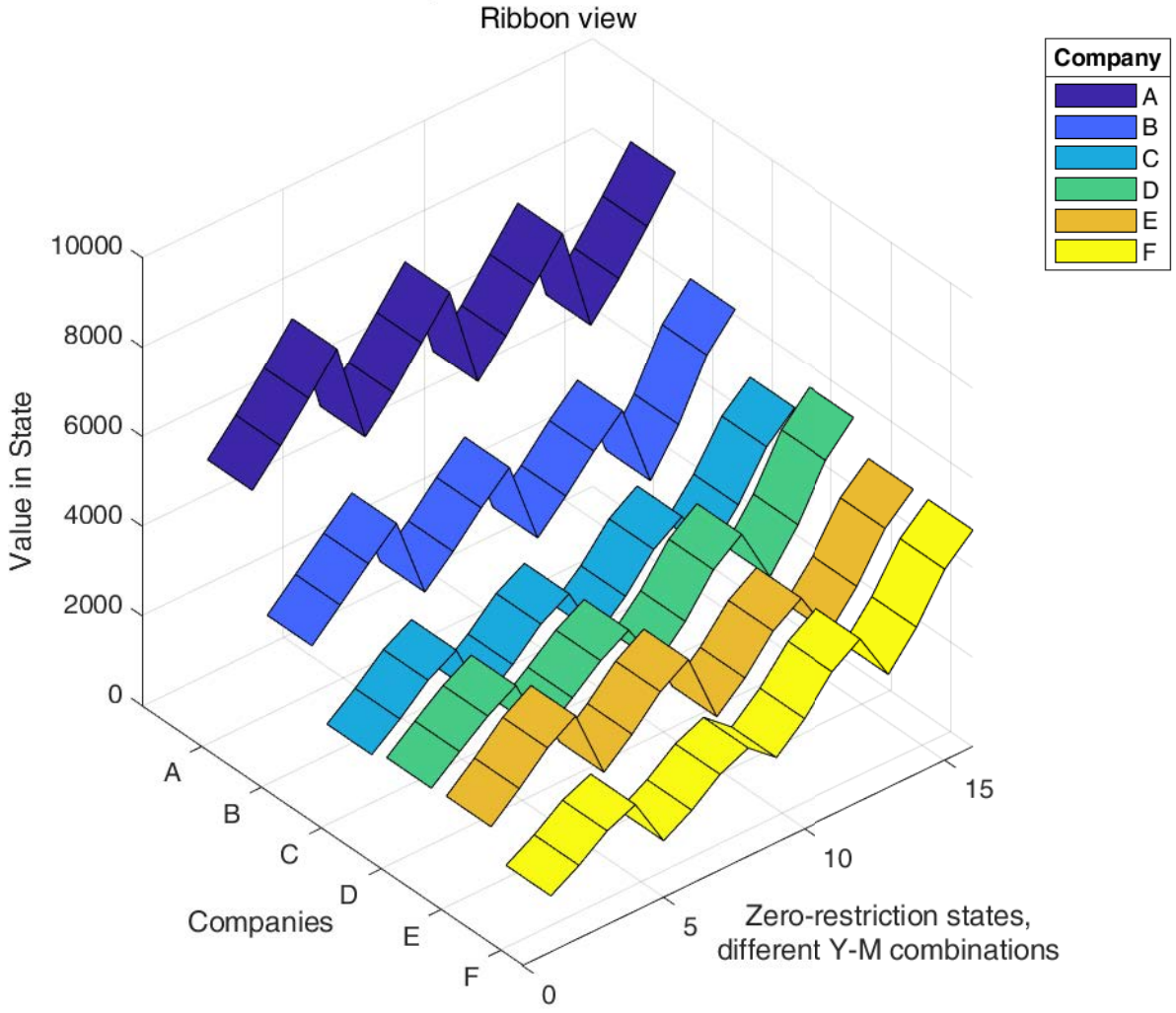


**Table 1: Value of Companies, Zero-Restriction States**

| State | CoA | CoB | CoC | CoD | CoE | CoF |
|---|---|---|---|---|---|---|
| **Y75_Scale1** | 5740 | 3185 | 1671 | 1833 | 1883 | 1252 |
| **Y75_Scale1.2** | 6430 | 3790 | 2160 | 2342 | 2428 | 1626 |
| **Y75_Scale1.5** | 7185 | 4404 | 2719 | 2829 | 3029 | 2090 |
| **Y75_Scale2.25** | 7988 | 4995 | 3081 | 3202 | 3433 | 2368 |
| **Y100_Scale1** | 5710 | 3163 | 1664 | 1816 | 1876 | 1260 |
| **Y100_Scale1.2** | 6391 | 3760 | 2150 | 2319 | 2418 | 1633 |
| **Y100_Scale1.5** | 7208 | 4425 | 2748 | 2840 | 3058 | 2126 |
| **Y100_Scale2.25** | 8031 | 5019 | 3115 | 3216 | 3467 | 2410 |
| **Y150_Scale1** | 5727 | 3146 | 1890 | 2160 | 1890 | 1890 |
| **Y150_Scale1.2** | 6448 | 3774 | 2449 | 2799 | 2449 | 2449 |
| **Y150_Scale1.5** | 7284 | 4468 | 3189 | 3645 | 3189 | 3189 |
| **Y150_Scale2.25** | 8117 | 5066 | 3615 | 4131 | 3615 | 3615 |
| **Y200_Scale1** | 5772 | 3200 | 2520 | 2880 | 2520 | 2520 |
| **Y200_Scale1.2** | 6506 | 4147 | 3266 | 3732 | 3266 | 3266 |
| **Y200_Scale1.5** | 7360 | 5400 | 4253 | 4860 | 4253 | 4253 |
| **Y200_Scale2.25** | 8262 | 6120 | 4820 | 5508 | 4820 | 4820 |

Value in $thousands. “Y” indicates economy; “M” indicates scale.

## Parameters for Representative Companies

Table 2, “Parameters for Representative Companies,” on page 4 records the key parameters for these companies. They vary in purposeful ways, to allow us to observe the effects of factors we know affect value, such as the underlying discount rate investors apply to a firm of this type, and the trend growth rate and profitability of the company.

Parameters vary for the representative companies, and are summarized for each company after the table for that company. *Rbar* is restriction period; *lambda* is chance good economy; *gamma* is recession chance; *phi* is likelihood investment subsequently increases scale; *BS* is blue sky multiple on sale.

See Table 2, “Parameters for Representative Companies,” on page 4 in the Appendix.

**Table 2: Parameters for Representative Companies**

| Company | Operating parameters | Restriction |
|---|---|---|
| Company A | Baseline Operating margin 0.09, lambda 0.6, phi 0.35, BS 24, d 0.05, g 0.025 | 8 periods |
| Company B | Baseline Operating margin 0.05, lambda 0.6, phi 0.3, BS 32, d 0.0625, g 0.03 | 8 periods |
| Company C | Baseline Operating margin 0.07, lambda 0.7, phi 0.25, BS 18, d 0.3, g 0.1 | 8 periods |
| Company D | Baseline Operating margin 0.03, lambda 0.7, phi 0.35, BS 48, d 0.4, g 0.15 | 8 periods |
| Company E | Baseline Operating margin 0.07, lambda 0.7, phi 0.25, BS 18, d 0.225, g 0.1 | 8 periods |
| Company F | Baseline Operating margin 0.07, lambda 0.7, phi 0.25, BS 18, d 0.525, g 0.1 | 8 periods |

# B. APPENDIX B: DISCOUNTS FOR REPRESENTATIVE COMPANIES

## Discounts for Representative Companies

We present below discounts for marketability in two forms:

1. Holding the economic conditions and scale constant, discounts as the duration of the restriction on sale goes toward zero.

2. Holding the duration of the restriction constant, discounts for combinations of economic conditions (Y) and scale (M).

We also provide a comment on each, indicating how our economic intuition compares with the calculated result.

### Company A

Company A is a profitable company (with a significant operating margin), and a good chance that investment will result in a larger scale (phi). In addition, it has a healthy resale market for its equity (BS ratio). The discount rate is fairly low given the low risk, and the growth rate is steady.

Our economic intuition is that the penalty for holding on to this equity for a one or two year restriction is very little, as the company is likely to do well and have substantial resale value. Holding this stock is a solid investment.

We also graph the value of the restricted equity over time as the restriction lapses, and the related discount. See “Company A, discount as restriction lapses” on page 2. Both are flat as there is close to zero discount.

**Figure 2. Company A, discount as restriction lapses**

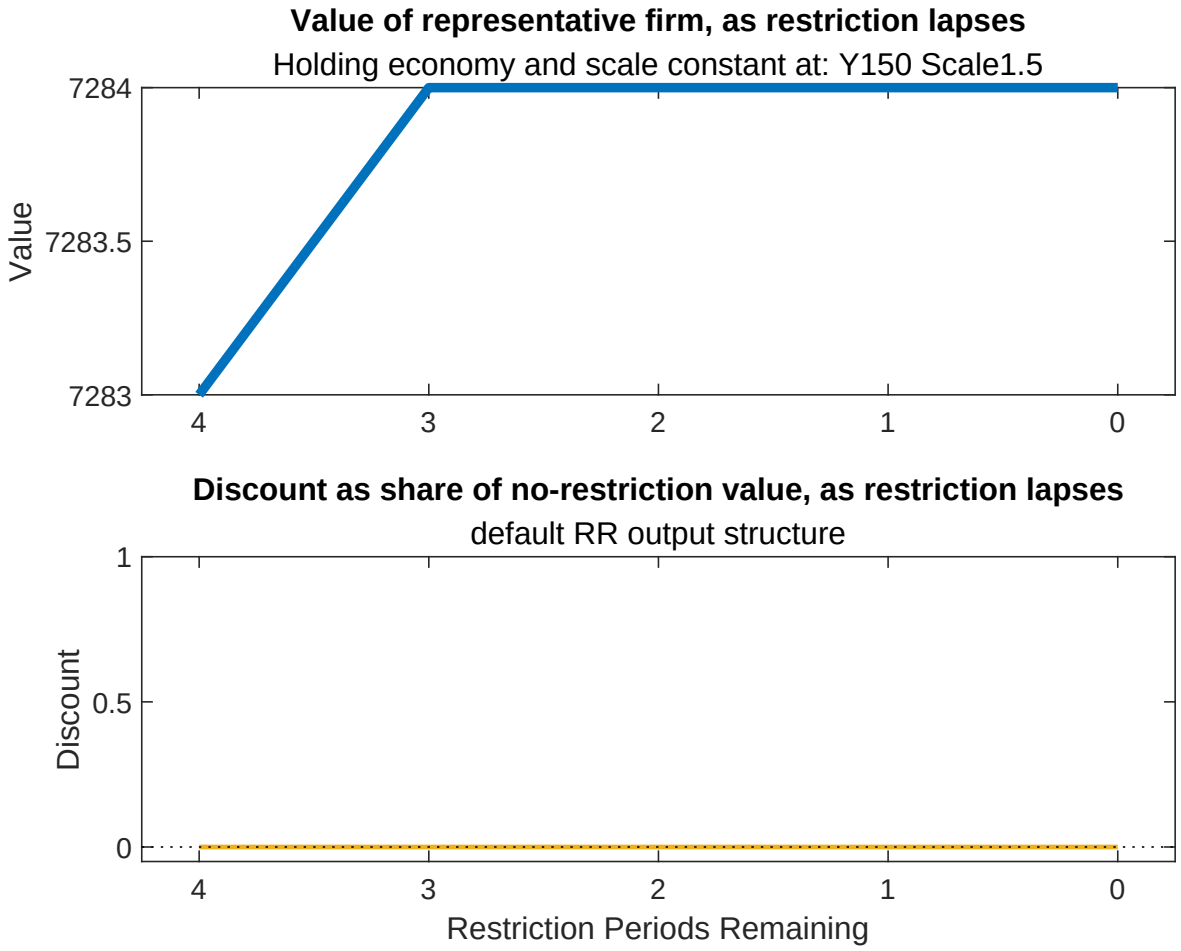



**Table 3: Company A Discounts**

| | Scale1 | Scale1.2 | Scale1.5 | Scale2.25 |
|---|---|---|---|---|
| **Y75** | 0 | 0 | 0 | -0.001 |
| **Y100** | 0 | 0 | 0 | -0.001 |
| **Y150** | 0 | 0 | 0 | -0.001 |
| **Y200** | 0 | 0 | 0 | -0.008 |

"Company A: Baseline Operating margin 0.09, lambda 0.6, phi 0.35, BS 24, d 0.05, g 0.025"

"Restriction: 8 periods"

"Difference calculated over 4 periods, and expressed as share of zero-restriction value."

## Company B

Company B is less profitable (operating margin is smaller than Company A), and the discount rate and growth rate are both somewhat higher than with Company A, indicating higher risk. The resale value of the equity is good, but not as good as Company A.

Our economic intuition is that there is small discount, unless the company faces some obstacle, such as reaching the upper end of the scale it can achieve with regular investment. In this case, there is a significant discount as the equity holder cannot sell at a good price and reinvest in another company with more growth opportunities.

We also graph the value of the restricted equity over time as the restriction lapses, and the related discount. See "Company B, discount as restriction lapses" on page 3. Our intuition suggests the value of the restricted stock will become higher as the restriction moves toward zero duration. This does occur, but since the discount is very small the improvement is also very small.

**Figure 3. Company B, discount as restriction lapses**

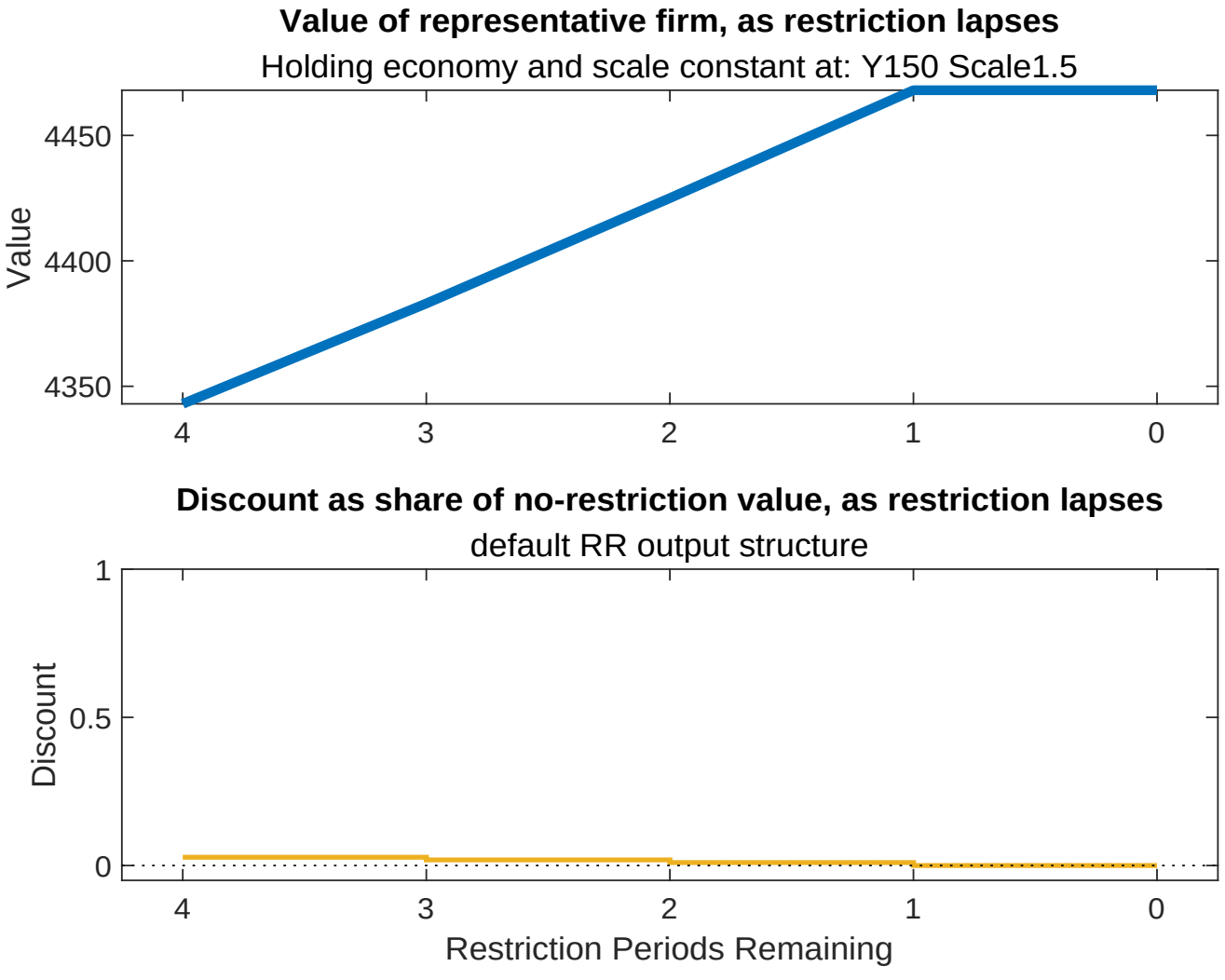


**Table 4: Company B Discounts**

| | Scale1 | Scale1.2 | Scale1.5 | Scale2.25 |
|---|---|---|---|---|
| **Y75** | -0.009 | -0.022 | -0.028 | -0.028 |
| **Y100** | -0.009 | -0.021 | -0.028 | -0.028 |
| **Y150** | -0.004 | -0.016 | -0.028 | -0.028 |
| **Y200** | -0.013 | -0.097 | -0.188 | -0.188 |

"Company B: Baseline Operating margin 0.05, lambda 0.6, phi 0.3, BS 32, d 0.0625, g 0.03"

"Restriction: 8 periods"

"Difference calculated over 4 periods, and expressed as share of zero-restriction value."

## Company C

Company C is solidly profitable (operating margin is similar to Company A), but the discount rate is much higher than the growth rate, indicating higher risk. The resale value of the equity is good, but not as good as Company A.

Our economic intuition is that there is a significant discount for being forced to hold this equity, as there are the equity holder cannot sell at a good price and reinvest in another company with more growth opportunities.

We graph the value of the restricted equity over time as the restriction lapses, and the related discount. See "Company C, discount as restriction lapses" on page 5. Our intuition suggests the value of the restricted stock will become higher as the restriction moves toward zero duration, and the discount will steadily fall. This occurs as our intuition suggests.

**Figure 4. Company C, discount as restriction lapses**

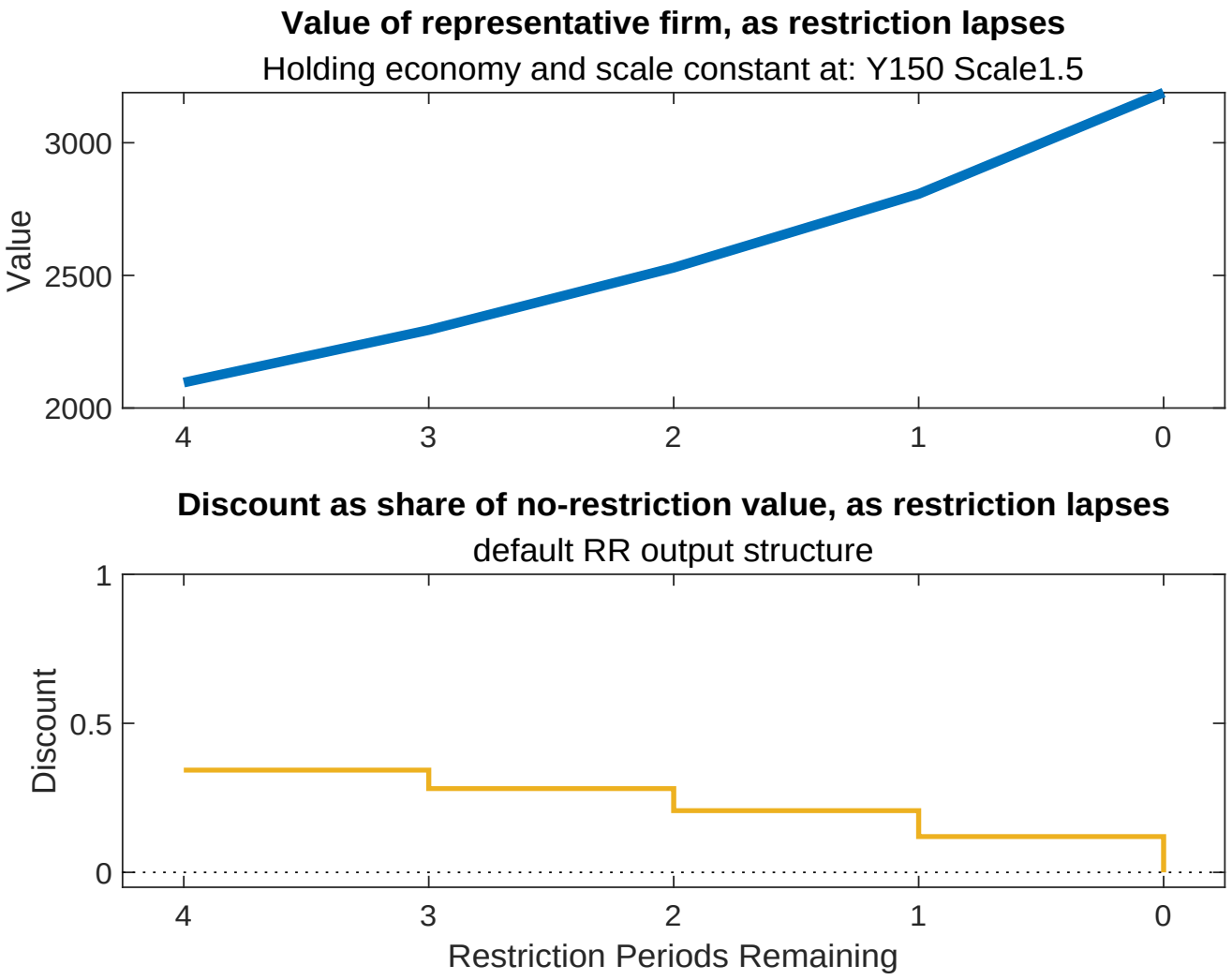



**Table 5: Company C Discounts**

| | Scale1 | Scale1.2 | Scale1.5 | Scale2.25 |
|---|---|---|---|---|
| **Y75** | -0.268 | -0.27 | -0.261 | -0.262 |
| **Y100** | -0.26 | -0.26 | -0.259 | -0.259 |
| **Y150** | -0.33 | -0.332 | -0.343 | -0.343 |
| **Y200** | -0.483 | -0.485 | -0.493 | -0.493 |

Company C: Baseline Operating margin 0.07, lambda 0.7, phi 0.25, BS 18, d 0.3, g 0.1
Restriction: 8 periods. Difference calculated over 4 periods, and expressed as share of zero-restriction value.

## Company D

Company D has a thinner operating margin, a slow growth rate, and a discount rate considerably higher than its growth rate. The company may be able to increase its scale, but is subject to significant risk.

Our economic intuition suggests preventing resale of equity in this company would foreclose opportunities to take advantage of opportunities to sell, thus indicating a relatively high discount for lack of marketability.

We graph the effect of that restriction as it lapses in "Company D, discount as restriction lapses" on page 6. As our intuition suggests, the restricted equity is worth more as the restriction lapses, and the discount for lack of marketability moves toward zero as the restriction expires.

**Figure 5. Company D, discount as restriction lapses**

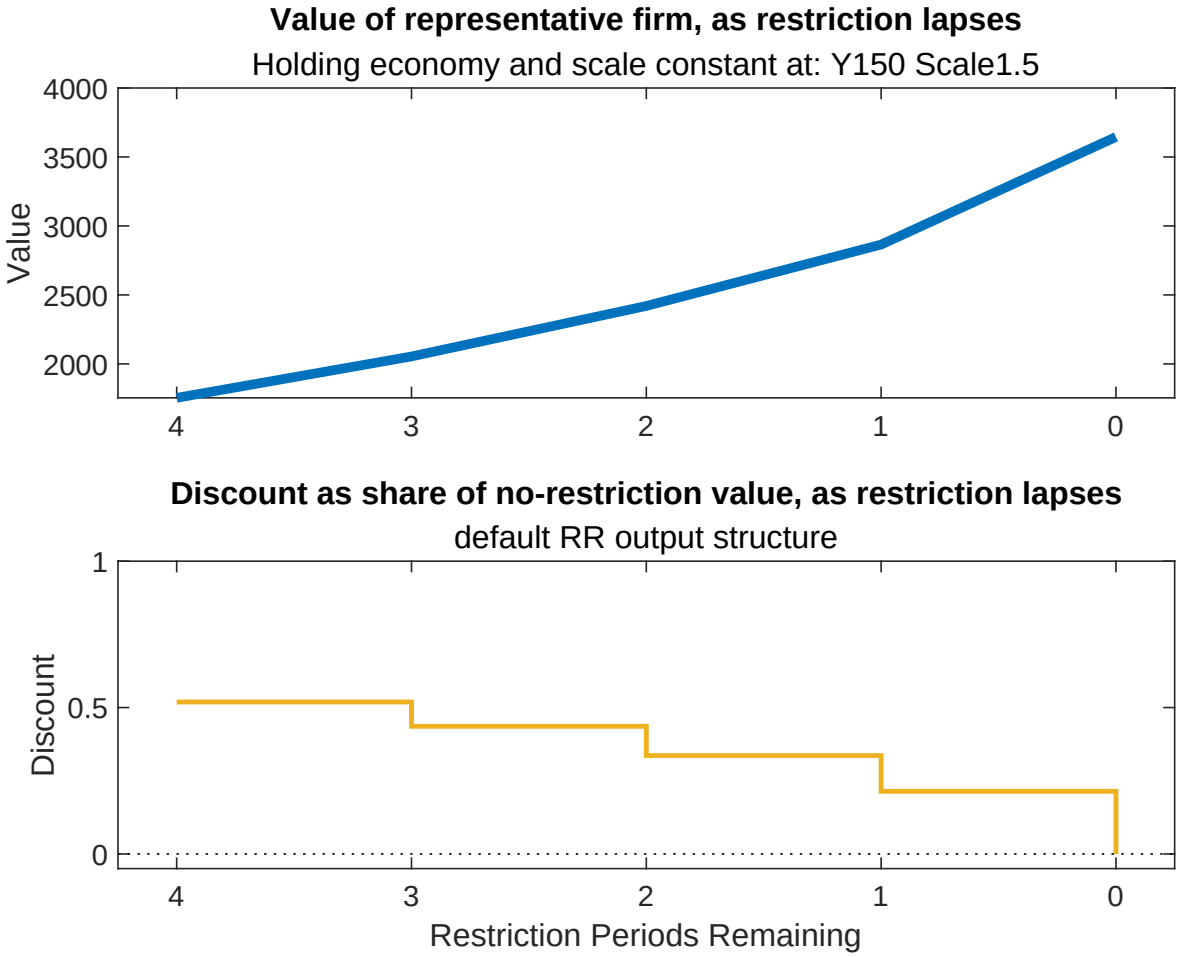


**Table 6: Company D Discounts**

| | Scale1 | Scale1.2 | Scale1.5 | Scale2.25 |
|---|---|---|---|---|
| **Y75** | -0.383 | -0.399 | -0.393 | -0.393 |
| **Y100** | -0.385 | -0.393 | -0.391 | -0.391 |
| **Y150** | -0.476 | -0.491 | -0.519 | -0.519 |
| **Y200** | -0.602 | -0.613 | -0.634 | -0.634 |

Company D: Baseline Operating margin 0.03, lambda 0.7, phi 0.35, BS 48, d 0.4, g 0.15
Restriction: 8 periods. Difference calculated over 4 periods, and expressed as share of zero-restriction value.

## Company E

Company E has a bigger operating margin than company D, but similar slow growth rate and discount rate considerably higher than its growth rate. The company may be able to increase its scale, and its risk is lower because it is more profitable.

Our economic intuition suggests preventing resale of equity in this company would foreclose opportunities to take advantage of opportunities to sell. However, the shareholder is likely to enjoy consistent earnings during the restriction period. Thus, intuition suggests a modest discount for lack of marketability.

**Table 7: Company E Discounts**

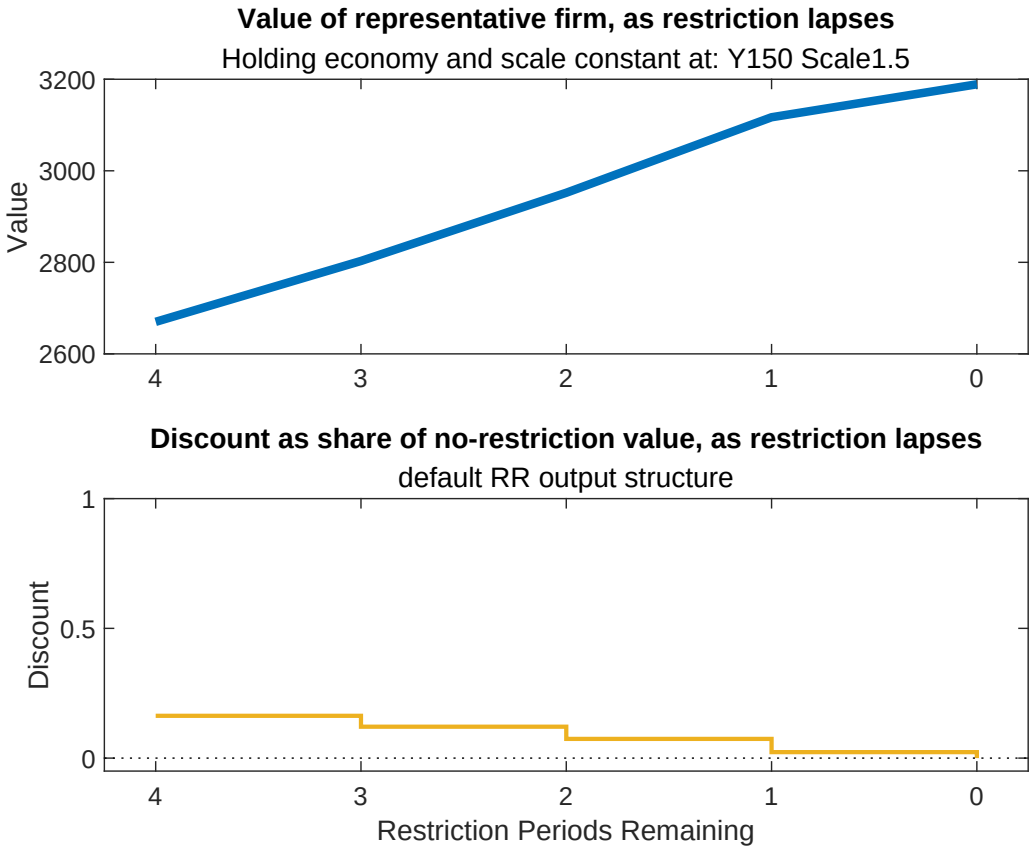
| | Scale1 | Scale1.2 | Scale1.5 | Scale2.25 |
|---|---|---|---|---|
| **Y75** | -0.136 | -0.145 | -0.148 | -0.148 |
| **Y100** | -0.137 | -0.145 | -0.146 | -0.146 |
| **Y150** | -0.13 | -0.137 | -0.163 | -0.163 |
| **Y200** | -0.333 | -0.339 | -0.358 | -0.358 |

"Company E: Baseline Operating margin 0.07, lambda 0.7, phi 0.25, BS 18, d 0.225, g 0.1"

"Restriction: 8 periods"

"Difference calculated over 4 periods, and expressed as share of zero-restriction value."

**Figure 6. Company E: discount as restriction lapses**



## Company F

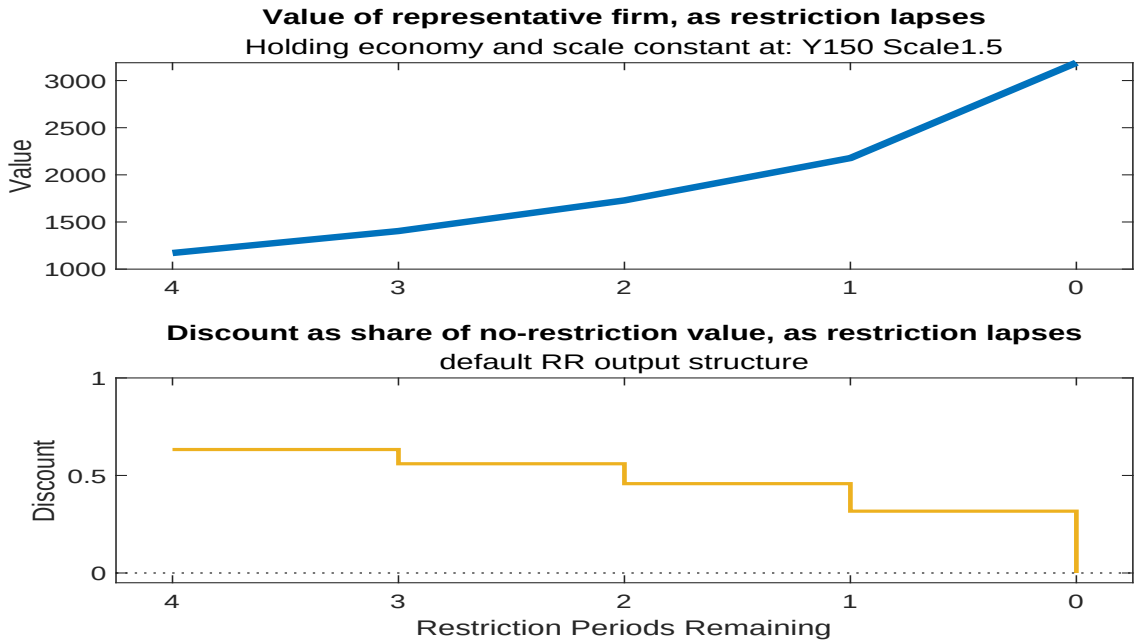
Company F is similar to company E, but has a higher discount rate given its more volatile industry (or other cause). Our economic intuition suggests preventing resale of equity in this company would foreclose opportunities to sell, and possibly to avoid downturns. Our intuition suggests a higher discount for lack of marketability than Company E.

**Table 8: Company F Discounts**

| | Scale1 | Scale1.2 | Scale1.5 | Scale2.25 |
|---|---|---|---|---|
| **Y75** | -0.486 | -0.488 | -0.482 | -0.482 |
| **Y100** | -0.476 | -0.476 | -0.477 | -0.477 |
| **Y150** | -0.632 | -0.632 | -0.633 | -0.633 |
| **Y200** | -0.71 | -0.71 | -0.711 | -0.711 |

Company F: Baseline Operating margin 0.07, lambda 0.7, phi 0.25, BS 18, d 0.525, g 0.1
Restriction: 8 periods. Difference calculated over 4 periods, and expressed as share of zero-restriction value.

**Figure 7. Company F: discount as restriction lapses**




### *Reminder of company parameters*

Parameters vary for the representative companies, and are summarized for each company after the table for that company. *Rbar* is restriction period; *lambda* is chance good economy; *gamma* is recession chance; *phi* is likelihood investment subsequently increases scale; *BS* is blue sky multiple on sale.

# C. APPENDIX C.

## Exhibit C-1. Comparative Income Statements, Representative Companies

*At baseline economy and scale. Amounts in thousands.*

| | CoA | CoB | CoC | CoD | CoE | CoF |
|---|---|---|---|---|---|---|
| **Rev** | $ 1,000.0 | $ 1,000.0 | $ 1,000.0 | $ 1,000.0 | $ 1,000.0 | $ 1,000.0 |
| **COGS** | $ 350.0 | $ 350.0 | $ 350.0 | $ 350.0 | $ 350.0 | $ 350.0 |
| **GrossProfit** | $ 650.0 | $ 650.0 | $ 650.0 | $ 650.0 | $ 650.0 | $ 650.0 |
| **OpExp** | $ 400.0 | $ 400.0 | $ 400.0 | $ 400.0 | $ 400.0 | $ 400.0 |
| **ActionCost** | $ - | $ - | $ - | $ - | $ - | $ - |
| **OpIncome** | $ 250.0 | $ 250.0 | $ 250.0 | $ 250.0 | $ 250.0 | $ 250.0 |
| **OtherExp** | $ 160.0 | $ 200.0 | $ 180.0 | $ 220.0 | $ 180.0 | $ 180.0 |
| **Pretax** | $ 90.0 | $ 50.0 | $ 70.0 | $ 30.0 | $ 70.0 | $ 70.0 |
| **Div** | $ 90.0 | $ 50.0 | $ 70.0 | $ 30.0 | $ 70.0 | $ 70.0 |
| ***OperatingMargin*** | *9.0%* | *5.0%* | *7.0%* | *3.0%* | *7.0%* | *7.0%* |

*Baseline income statements for representative companies, with parameters:*

Company A: Baseline OpMargin 0.09, lambda 0.6, phi 0.35, BS 24, d 0.05, g 0.025
Restriction: 8 periods
Company B: Baseline OpMargin 0.05, lambda 0.6, phi 0.3, BS 32, d 0.0625, g 0.03
Restriction: 8 periods
Company C: Baseline OpMargin 0.07, lambda 0.7, phi 0.25, BS 18, d 0.3, g 0.1
Restriction: 8 periods
Company D: Baseline OpMargin 0.03, lambda 0.7, phi 0.35, BS 48, d 0.4, g 0.15
Restriction: 8 periods
Company E: Baseline OpMargin 0.07, lambda 0.7, phi 0.25, BS 18, d 0.225, g 0.1
Restriction: 8 periods
Company F: Baseline OpMargin 0.07, lambda 0.7, phi 0.25, BS 18, d 0.525, g 0.1
Restriction: 8 periods

*Explanation: Rbar is restriction period; lambda is chance good economy; gamma is chance of recession; phi is likelihood investment subsequently increases scale; BS is blue sky multiple on sale.*

## Exhibit C-2. Value of Companies in Selected Zero-Restriction States

| Economy and Scale | CoA | CoB | CoC | CoD | CoE | CoF |
|---|---|---|---|---|---|---|
| Y75_Scale1 | $ 5,740.0 | $ 3,185.0 | $ 1,671.0 | $ 1,833.0 | $ 1,883.0 | $ 1,252.0 |
| Y75_Scale1.2 | $ 6,430.0 | $ 3,790.0 | $ 2,160.0 | $ 2,342.0 | $ 2,428.0 | $ 1,626.0 |
| Y75_Scale1.5 | $ 7,185.0 | $ 4,404.0 | $ 2,719.0 | $ 2,829.0 | $ 3,029.0 | $ 2,090.0 |
| Y75_Scale2.25 | $ 7,988.0 | $ 4,995.0 | $ 3,081.0 | $ 3,202.0 | $ 3,433.0 | $ 2,368.0 |
| Y100_Scale1 | $ 5,710.0 | $ 3,163.0 | $ 1,664.0 | $ 1,816.0 | $ 1,876.0 | $ 1,260.0 |
| Y100_Scale1.2 | $ 6,391.0 | $ 3,760.0 | $ 2,150.0 | $ 2,319.0 | $ 2,418.0 | $ 1,633.0 |
| Y100_Scale1.5 | $ 7,208.0 | $ 4,425.0 | $ 2,748.0 | $ 2,840.0 | $ 3,058.0 | $ 2,126.0 |
| Y100_Scale2.25 | $ 8,031.0 | $ 5,019.0 | $ 3,115.0 | $ 3,216.0 | $ 3,467.0 | $ 2,410.0 |
| Y150_Scale1 | $ 5,727.0 | $ 3,146.0 | $ 1,890.0 | $ 2,160.0 | $ 1,890.0 | $ 1,890.0 |
| Y150_Scale1.2 | $ 6,448.0 | $ 3,774.0 | $ 2,449.0 | $ 2,799.0 | $ 2,449.0 | $ 2,449.0 |
| Y150_Scale1.5 | $ 7,284.0 | $ 4,468.0 | $ 3,189.0 | $ 3,645.0 | $ 3,189.0 | $ 3,189.0 |
| Y150_Scale2.25 | $ 8,117.0 | $ 5,066.0 | $ 3,615.0 | $ 4,131.0 | $ 3,615.0 | $ 3,615.0 |
| Y200_Scale1 | $ 5,772.0 | $ 3,200.0 | $ 2,520.0 | $ 2,880.0 | $ 2,520.0 | $ 2,520.0 |
| Y200_Scale1.2 | $ 6,506.0 | $ 4,147.0 | $ 3,266.0 | $ 3,732.0 | $ 3,266.0 | $ 3,266.0 |
| Y200_Scale1.5 | $ 7,360.0 | $ 5,400.0 | $ 4,253.0 | $ 4,860.0 | $ 4,253.0 | $ 4,253.0 |
| Y200_Scale2.25 | $ 8,262.0 | $ 6,120.0 | $ 4,820.0 | $ 5,508.0 | $ 4,820.0 | $ 4,820.0 |

*Values for Y-M combinations, in selected zero-restriction states.*
*Values in $thousands*

## Exhibit C-3. Discounts for Each Company, for Different Y and M States

*Showing discount with 4 quarters of restriction on sale, for different combinations of economy and scale.*

| Company A, Economy | Scale1 | Scale1.2 | Scale1.5 | Scale2.25 |
|---|---|---|---|---|
| Y75 | 0 | 0 | 0 | -0.001 |
| Y100 | 0 | 0 | 0 | -0.001 |
| Y150 | 0 | 0 | 0 | -0.001 |
| Y200 | 0 | 0 | 0 | -0.008 |

**Company A: Baseline OpMargin 0.09, lambda 0.6, phi 0.35, BS 24, d 0.05, g 0.025**
Restriction: 8 periods
Difference calculated over 4 periods,
and expressed as share of zero-restriction value.

| Company B, Economy | Scale1 | Scale1.2 | Scale1.5 | Scale2.25 |
|---|---|---|---|---|
| Y75 | -0.009 | -0.022 | -0.028 | -0.028 |
| Y100 | -0.009 | -0.021 | -0.028 | -0.028 |
| Y150 | -0.004 | -0.016 | -0.028 | -0.028 |
| Y200 | -0.013 | -0.097 | -0.188 | -0.188 |

**Company B: Baseline OpMargin 0.05, lambda 0.6, phi 0.3, BS 32, d 0.0625, g 0.03**
Restriction: 8 periods
Difference calculated over 4 periods,
and expressed as share of zero-restriction value.

| Company C, Economy | Scale1 | Scale1.2 | Scale1.5 | Scale2.25 |
|---|---|---|---|---|
| Y75 | -0.268 | -0.27 | -0.261 | -0.262 |
| Y100 | -0.26 | -0.26 | -0.259 | -0.259 |
| Y150 | -0.33 | -0.332 | -0.343 | -0.343 |
| Y200 | -0.483 | -0.485 | -0.493 | -0.493 |

**Company C: Baseline OpMargin 0.07, lambda 0.7, phi 0.25, BS 18, d 0.3, g 0.1**
Restriction: 8 periods
Difference calculated over 4 periods,
and expressed as share of zero-restriction value.

**Appendix C**

| Company D, Economy | Scale1 | Scale1.2 | Scale1.5 | Scale2.25 |
|---|---|---|---|---|
| Y75 | -0.383 | -0.399 | -0.393 | -0.393 |
| Y100 | -0.385 | -0.393 | -0.391 | -0.391 |
| Y150 | -0.476 | -0.491 | -0.519 | -0.519 |
| Y200 | -0.602 | -0.613 | -0.634 | -0.634 |

**Company D: Baseline OpMargin 0.03, lambda 0.7, phi 0.35, BS 48, d 0.4, g 0.15**
Restriction: 8 periods
Difference calculated over 4 periods,
and expressed as share of zero-restriction value.

| Company E, Economy | Scale1 | Scale1.2 | Scale1.5 | Scale2.25 |
|---|---|---|---|---|
| Y75 | -0.136 | -0.145 | -0.148 | -0.148 |
| Y100 | -0.137 | -0.145 | -0.146 | -0.146 |
| Y150 | -0.13 | -0.137 | -0.163 | -0.163 |
| Y200 | -0.333 | -0.339 | -0.358 | -0.358 |

**Company E: Baseline OpMargin 0.07, lambda 0.7, phi 0.25, BS 18, d 0.225, g 0.1**
Restriction: 8 periods
Difference calculated over 4 periods,
and expressed as share of zero-restriction value.

| Company F, Economy | Scale1 | Scale1.2 | Scale1.5 | Scale2.25 |
|---|---|---|---|---|
| Y75 | -0.486 | -0.488 | -0.482 | -0.482 |
| Y100 | -0.476 | -0.476 | -0.477 | -0.477 |
| Y150 | -0.632 | -0.632 | -0.633 | -0.633 |
| Y200 | -0.71 | -0.71 | -0.711 | -0.711 |

**Company F: Baseline OpMargin 0.07, lambda 0.7, phi 0.25, BS 18, d 0.525, g 0.1**
Restriction: 8 periods
Difference calculated over 4 periods,
and expressed as share of zero-restriction value.

## Exhibit C-4. Discounts as Restriction Expires, Selected Y-M States

*For each company and one combination of Y (economic conditions) and M (scale). showing marketabilty discount as restriction expires.*

| Row | State | Policy | Value | DiscountShare |
|---|---|---|---|---|
| A_Y100_Scale1_Restrict4 | 26 | 2 | $ 5,710.0 | 0.0% |
| A_Y100_Scale1_Restrict3 | 27 | 2 | $ 5,710.0 | 0.0% |
| A_Y100_Scale1_Restrict2 | 28 | 2 | $ 5,710.0 | 0.0% |
| A_Y100_Scale1_Restrict1 | 29 | 2 | $ 5,710.0 | 0.0% |
| A_Y100_Scale1_Restrict0 | 30 | 2 | $ 5,710.0 | 0.0% |
| B_Y150_Scale1_Restrict4 | 51 | 1 | $ 3,133.0 | 0.4% |
| B_Y150_Scale1_Restrict3 | 52 | 1 | $ 3,140.0 | 0.2% |
| B_Y150_Scale1_Restrict2 | 53 | 1 | $ 3,145.0 | 0.0% |
| B_Y150_Scale1_Restrict1 | 54 | 1 | $ 3,146.0 | 0.0% |
| B_Y150_Scale1_Restrict0 | 55 | 1 | $ 3,146.0 | 0.0% |
| C_Y150_Scale1.5_Restrict4 | 61 | 1 | $ 2,096.0 | 34.3% |
| C_Y150_Scale1.5_Restrict3 | 62 | 1 | $ 2,294.0 | 28.1% |
| C_Y150_Scale1.5_Restrict2 | 63 | 1 | $ 2,529.0 | 20.7% |
| C_Y150_Scale1.5_Restrict1 | 64 | 1 | $ 2,807.0 | 12.0% |
| C_Y150_Scale1.5_Restrict0 | 65 | 3 | $ 3,189.0 | 0.0% |
| D_Y75_Scale2.25_Restrict4 | 16 | 1 | $ 1,943.0 | 39.3% |
| D_Y75_Scale2.25_Restrict3 | 17 | 1 | $ 2,284.0 | 28.7% |
| D_Y75_Scale2.25_Restrict2 | 18 | 1 | $ 2,698.0 | 15.7% |
| D_Y75_Scale2.25_Restrict1 | 19 | 1 | $ 3,202.0 | 0.0% |
| D_Y75_Scale2.25_Restrict0 | 20 | 1 | $ 3,202.0 | 0.0% |
| E_Y200_Scale2.25_Restrict4 | 91 | 1 | $ 3,094.0 | 35.8% |
| E_Y200_Scale2.25_Restrict3 | 92 | 1 | $ 3,245.0 | 32.7% |
| E_Y200_Scale2.25_Restrict2 | 93 | 1 | $ 3,413.0 | 29.2% |
| E_Y200_Scale2.25_Restrict1 | 94 | 1 | $ 3,601.0 | 25.3% |
| E_Y200_Scale2.25_Restrict0 | 95 | 3 | $ 4,820.0 | 0.0% |
| F_Y150_Scale1.5_Restrict4 | 61 | 1 | $ 1,171.0 | 63.3% |
| F_Y150_Scale1.5_Restrict3 | 62 | 1 | $ 1,404.0 | 56.0% |
| F_Y150_Scale1.5_Restrict2 | 63 | 1 | $ 1,729.0 | 45.8% |
| F_Y150_Scale1.5_Restrict1 | 64 | 1 | $ 2,178.0 | 31.7% |
| F_Y150_Scale1.5_Restrict0 | 65 | 3 | $ 3,189.0 | 0.0% |

Prefixes "A", "B",…denote companies; "Y" is economy, "M" is scale."
Restrictions are expressed on calendar quarters; Policies represent discretionary business decisions, with "1" meaning operate; "2" invest; "3" sell equity.
*Values in $thousands. Discount shown as share of value in zero-restriction state.*