Inflation Indexation and Zero Lower Bound[*]

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Abstract

This study quantitatively assesses both the aggregate and disaggregate effects of inflation-indexed loan contracts using a heterogeneous agent New Keynesian (HANK) model with an occasionally binding zero lower bound (ZLB). Substituting real for nominal government bonds reduces the volatility of output and inflation and decreases the frequency of ZLB events. Real loans sever the link between real interest rates and inflation, preventing a rise in real interest rates at the ZLB. Accordingly, ZLB events become less costly, weakening precautionary savings against aggregate risk. This leads to higher average nominal rates and a reduced frequency of ZLB occurrences, further reducing aggregate volatility. Although inflation indexation improves aggregate welfare, at the disaggregate level, the wealthy lose while the poor gain. Inflation indexation outperforms suggested policies aimed at providing more room for monetary policy, such as increasing the inflation target and implementing an asymmetric Taylor rule.

Key Words: Zero lower bound, HANK model, Inflation indexation, Welfare
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1 Introduction

Most loan agreements specify a nominal interest rate. For example, in the U.S., more than 90% of new loans to households specify fixed nominal interest rates (Badarinza, Campbell and Ramadorai, 2018). If nominal rates respond sluggishly, unexpected changes in inflation cause a fluctuation in real interest rates, thus destabilizing the economy through intertemporal substitution or income effects. This destabilization can be amplified by the presence of nonlinear features such as the zero lower bound (ZLB) on interest rates and household inequality. When the ZLB binds, deflationary pressure increases real rates more than when it does not bind, thereby increasing macroeconomic volatility. Income and wealth inequality increase the household demand for precautionary savings, which lowers the real interest rate. Consequently, the average nominal rate—the sum of the real interest rate and inflation—decreases, causing the ZLB to bind more frequently and thus further destabilizing the economy. Significantly high macroeconomic volatility caused by nominal loans in the presence of the ZLB and inequality poses a serious challenge for central banks, which have limited room to stabilize the economy when policy rates are near the ZLB.

In this paper, we show that inflation-indexed loan contracts can reduce output and inflation volatility arising from the interplay between household inequality and the ZLB, thus easing the challenges central banks face in normal times. We build on the model of Fernández-Villaverde et al. (2023), who construct and solve a heterogeneous agent New Keynesian (HANK) model with an occasionally binding ZLB constraint, by incorporating inflation-indexed government bonds. We calibrate the model to match the realistic income and wealth distribution and the historical frequency of hitting the ZLB in the U.S. and solve the model using the Krusell and Smith (1998) method.

To quantify the stabilization effect of inflation indexation, we compare the variance of output and inflation, as well as the frequency of hitting the ZLB, computed from the ZLB-HANK model with nominal contracts to those implied by the ZLB-HANK model with real contracts. The latter model assumes that government bonds are fully indexed to inflation, implying that real interest rates are unaffected by inflation variability. If nominal contracts are replaced with their real counterparts, the volatility of output and inflation falls by 33% and 38%, respectively. Additionally, the frequency of the ZLB decreases from 10.5% to 2.8%, indicating a substantial stabilization effect of inflation indexation.

The mechanism through which real contracts stabilize aggregate fluctuations works as follows. In a world with nominal contracts, contractionary demand shocks induce excess savings that cannot be cleared by a fall in real interest rates due to the ZLB. The only way to clear the excess supply of savings is through a significant contraction in output, which leads to a fall in expected inflation as the decline in goods prices is expected to persist due to sticky prices. The resulting rise in ex-ante real interest rates further aggravates the contraction. However, when government bonds are indexed to inflation, a fall in expected inflation does not affect ex-ante real rates. As households no longer face an increase in ex-ante real rates, ZLB episodes become much less costly. Accordingly, the demand for precautionary savings against aggregate risk weakens substantially, leading to higher average nominal rates than under nominal contracts, thus contributing to the reduced ZLB frequency.

At the disaggregate level, real contracts benefit the wealth-poor but hurt the wealth-rich during recessions. Consider a ZLB episode. When bonds are inflation-indexed, the wealth-rich can no longer benefit from the unexpectedly high ex-post real interest payments that would have arisen under nominal contracts. However, such losses in benefits apply less to wealth-poor households whose income is mostly from labor. Instead, because of a less contracted recession resulting from the prevalence of real contracts, their consumption is higher than it would be under nominal contracts.

Our main result, which indicates that real contracts reduce the ZLB frequency by raising the average nominal interest rates, suggests that issuing inflation-linked bonds can be one way to create more leeway for central banks to adjust nominal interest rates. Alternative policies to expand the scope of conventional monetary policy include increasing the inflation target (Blanchard, Dell'Ariccia and Mauro, 2010) or following the asymmetric Taylor rule. The former increases the average nominal rate by raising the steady-state inflation rate. The latter keeps the steady-state inflation rate unchanged but corrects the deflationary bias—the distance between the mean inflation and the inflation target—by calling for a more aggressive policy rate response when inflation is below the target than when it is above the target. This asymmetric strategy increases the average inflation and nominal rates. We show that, under the same ZLB frequency or the same deflationary bias, real contracts result in smaller macroeconomic volatility and higher welfare compared to the high inflation target policy and the asymmetric monetary policy. This is because, during ZLB episodes, the high inflation target policy and the asymmetric rule fail to sever the link between expected inflation and real interest rates, whereas real contracts do.

The present paper is connected to the literature that studies the propagation of aggregate shocks or policy evaluation in HANK models with the ZLB. McKay and Reis (2016) study the role of automatic stabilizers at the ZLB using a perfect foresight solution method. However, in contrast to the present paper, their model does not include households' expectations regarding the risk of hitting the ZLB. Studies that do incorporate the risk of hitting the ZLB include Schaab (2020) and Fernández-Villaverde et al. (2023). Schaab (2020) examines the propagation of macro uncertainty near the ZLB through the interaction with countercyclical unemployment risk, whereas our model does not include unemployment risk. Fernández-Villaverde et al. (2023) investigate how inflation targets influence households' precautionary savings by altering the frequency of ZLB events. In contrast, our focus is on the extent to which indexing loans to inflation matters for the ZLB frequency.

The current work also aligns with the literature that evaluates the redistributive or aggregate effects under nominal contracts. Doepke and Schneider (2006*b*) and Adam and Zhu (2016) study the redistributive effects of unanticipated inflation shocks, while Doepke and Schneider (2006*a*) and Meh, Ríos-Rull and Terajima (2010) focus on the aggregate effect in a partial equilibrium model with heterogeneity in household net nominal positions. Iacoviello (2005) and Kuncl and Ueberfeldt (2023) study the aggregate effects of monetary policy shocks in a general equilibrium New Keynesian model that operates through the debt deflation channel. Carrillo and Poilly (2013) analyze the fiscal multiplier under nominal contracts when monetary policy is constrained at the ZLB, using a perfect foresight solution method. However, none of these papers investigates how nominal contracts affect household precautionary savings and the ZLB frequency. Moreover, all of these papers assume loan contracts between households or between households and firms, whereas we assume contracts between households and the government.

Our work contributes to the literature that discusses ways to remove the deflationary bias. Gust, López-Salido and Meyer (2017) show that the deflationary bias can be eliminated if policymakers view output losses as asymmetric, placing more weight on the negative output gap than the positive output gap. Nakata and Schmidt (2019) show that the deflationary bias can be mitigated by appointing a conservative central banker.

The rest of the paper is structured as follows. Section 2 presents the ZLB-HANK model, in which the degree of inflation indexation can be parameterized. In Section 3, we calibrate the model and describe our solution method and its accuracy. Section 4 compares business cycle moments and welfare under nominal and real contracts to quantify the stabilization effects of the latter. Section 5 compares real contracts with a high inflation target policy and an asymmetric monetary policy rule in terms of their stabilizing effect and welfare. Section 6 concludes.

2 Model

In this section, we present the ZLB-HANK model, which incorporates both nominal and real contracts. It comprises four main components: a continuum (measure one) of households with identical preferences but different productivity levels, firms, a central bank, and a government. In this model economy, individual households are subject to uninsurable labor productivity risk due to the incompleteness of asset markets, as in Huggett (1993) and Aiyagari (1994). Asset market incompleteness, combined with borrowing constraints, results in substantial heterogeneity in households' asset holdings, income, and consumption. The government supplies public bonds and collects taxes from households to finance interest payments on these bonds. Notably, government bonds can be indexed to inflation or not. These two extreme forms of indexation enable us to examine the extent to which inflation indexation stabilizes aggregate fluctuations. The remaining model elements are standard in the New Keynesian literature: sticky prices, monopolistically competitive goods markets, and a conventional Taylor rule that is bounded by zero nominal interest rates.

2.1 Households

Households maximize expected lifetime utility by selecting a sequence of consumption, c_t, labor supply, h_t and real bonds, b_{t+1}:

$$\max_{\{c_t,h_t,b_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \zeta_t \left(\frac{c_t^{1-\sigma}-1}{1-\sigma} - \Xi \frac{h_t^{1+1/\gamma}}{1+1/\gamma} \right) \right]$$

subject to the sequence of budget constraints,

$$c_t + b_{t+1} = (1+r_t)b_t + w_t z_t h_t - T_t + d_t, \tag{1}$$

and the borrowing constraint,

$$b_{t+1} \geq \underline{b},$$

where $0 < \beta < 1$ is a discount factor, σ is the degree of relative risk aversion, $\Xi > 0$ is a parameter for disutility from labor, and γ is the Frisch elasticity of labor supply. Each household is endowed with one unit of time in each period, which can be allocated between work and leisure. Additionally, households trade one-period non-contingent bonds, denoted as b, which offer a real rate of return, r_t. The bond position of households is subject to an exogenous limit, denoted as $\underline{b}$. When a household contributes h_t units of labor, it receives $w_t z_t h_t$ as income for its labor. Here, w_t represents the wage rate per unit of effective labor, and z_t is the household's labor productivity. In addition to labor income, households receive profit income, d_t, from monopolistic firms, and they are also obligated to pay taxes, T_t, to the government.

Households face both individual and aggregate risks. Firstly, they are subject to an uninsurable idiosyncratic productivity shock. Labor productivity, z, follows a log-AR(1) process: $\log z_{t+1} = \rho_z \log z_t + \varepsilon_{z,t+1}, \quad \varepsilon_{z,t+1} \sim N(0, {\sigma_z}^2)$. Secondly, concerning aggregate risk, households are affected by a common exogenous preference shock, ζ_t. As discussed by Christiano, Eichenbaum and Rebelo (2011) and others, shifts in households' preferences have a significant impact on aggregate demand, potentially leading to the ZLB if the shock is sizable. ζ_t is assumed to follow an AR(1) process in logs: $\log \zeta_{t+1} = \rho_\zeta \log \zeta_t + \varepsilon_{\zeta,t+1}, \quad \varepsilon_{\zeta,t+1} \sim N(0, {\sigma_\zeta}^2)$.

2.2 Final Good Firm

A competitive firm combines a continuum of intermediate inputs, $y_t(j)$, indexed by $j \in [0, 1]$ to produce a homogeneous final good, Y_t, according to a CES production function:

$$Y_t = \left(\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{2}$$

where $\epsilon > 1$ is the input demand elasticity. The profit maximization problem of the final good firm implies the demand for intermediate good j:

$$y_t(j) = \left(\frac{p_t(j)}{P_t}\right)^{-\epsilon} Y_t, \tag{3}$$

where $P_t = \left(\int_0^1 p_t(j)^{1-\epsilon}\, dj\right)^{\frac{1}{1-\epsilon}}$.

Each intermediate good firm produces a different type of intermediate good $y_t(j)$ using $n_t(j)$ units of effective labor, by means of a production function:

$$y_t(j) = n_t(j) - f,$$

where $f \geq 0$ is the fixed cost of production. Each intermediate goods firm j pays quadratic nominal price adjustment costs à la Rotemberg (1982). The problem for intermediate goods firms is to choose a sequence of prices that maximizes their expected discounted profits net of pricing costs:

$$\max_{p_{t+s}(j)} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \left(\prod_{i=0}^{s} \frac{1}{1+r_{t+i}} \right) \left\{ \left(\frac{p_{t+s}(j)}{P_{t+s}} - mc_{t+s} \right) y_{t+s}(j) - \frac{\theta}{2} \left(\frac{p_{t+s}(j)}{p_{t+s-1}(j)} - \overline{\Pi} \right)^2 Y_{t+s} \right\} \right], \tag{4}$$

subject to (3), where θ measures the degree of price stickiness, mc_{t+s} is the firm's real marginal cost, and $\overline{\Pi}$ is the steady-state gross inflation. The first-order condition associated with the optimal price gives rise to a New Keynesian Phillips curve:

$$\theta \left(\Pi_t - \overline{\Pi}\right) \Pi_t + \epsilon \left(\frac{\epsilon - 1}{\epsilon} - mc_t\right) = \theta \mathbb{E}_t \left[\frac{1}{1+r_t} \left\{\Pi_{t+1} - \overline{\Pi}\right\} \Pi_{t+1} \frac{Y_{t+1}}{Y_t} \right], \tag{5}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$.

2.3 Central Bank and Government

The central bank operates under a ZLB constraint. It determines the policy rate based on a Taylor rule when the shadow rates are greater than zero, but sets the policy rate at zero if the shadow rates are 0 or below. Specifically, the gross nominal interest rate, R_t, is set according to:

$$R_t = \max\left\{1, \widetilde{R_t}\right\}. \tag{6}$$

$\widetilde{R_t}$ is the desired (or shadow) interest rate, which is set according to the Taylor rule:

$$\log \widetilde{R_t} = \log \overline{R} + \phi_\pi \left(\log \Pi_t - \log \overline{\Pi}\right) + \phi_y \left(\log Y_t - \log \overline{Y}\right), \tag{7}$$

where $\overline{X}$ is the deterministic steady-state value of variable X_t, and ϕ_π and ϕ_y are coefficients on inflation and the output gap, respectively.

The government plays three roles in the economy: i) collecting taxes from households, ii) issuing public bonds, and iii) redistributing profits from intermediate good firms to households. Following McKay, Nakamura and Steinsson (2016), we assume that taxes increase with households' labor productivity, z_t:

$$T(z_t) = \tau_t z_t, \tag{8}$$

where τ_t is the tax rate.[1] The government supplies bonds with a real face value of B_t, and adjusts taxes to cover interest payments on public debt. Specifically, in line with McKay, Nakamura and Steinsson (2016), we assume a constant level of public debt, i.e., $B_t = \overline{B}$, and assume that the government maintains a balanced budget in each period:

$$r_t \overline{B} = \int T_t(z_t) d\mu_t. \tag{9}$$

The government also has the responsibility of distributing monopoly profits to households. We assume that aggregate profits, D_t, are proportionally distributed according to produc-

[1] While McKay, Nakamura and Steinsson (2016) assume a non-linear tax system with a positive tax rate applicable only to the highest productivity levels, we employ a linear tax system in which the tax rate is proportional to individual productivity. Given that individual labor productivity follows an exogenous process, this assumption does not influence or distort households' optimal choices.

tivity:[2]

$$d_t(z_t) = \frac{z_t}{\int z_t d\mu_t} D_t. \tag{10}$$

Importantly, government bonds can be indexed to inflation. Following Carrillo and Poilly (2013), we assume that nominal public debt is indexed to inflation at a rate of $\chi \in [0, 1]$. Formally, the real interest rate that households face follows a simple indexation rule:

$$r_t = \log\left(\frac{R_{t-1}}{\Pi_t}\left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\chi}\right), \tag{11}$$

where $\left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\chi}$ is a term that determines the adjustment of real rates to inflation. When $\chi = 0$, it implies that government bonds are purely denominated in nominal terms. Consequently, the real return on the bond becomes lower (greater) than expected in the presence of unexpectedly high (low) inflation. On the other hand, with $\chi = 1$, government debt is fully indexed to inflation, and the real return paid to households is unaffected by surprise inflation.[3]

3 Calibration and Numerical Solution

3.1 Calibration

We calibrate the parameters of the model following the existing literature. Table 1 provides a summary of the parameter values used in the model.

The time discount factor, β, is chosen to yield a steady-state real interest rate of 1 percent per year. The risk aversion parameter, σ, is assigned a value of one. The Frisch elasticity of labor supply, γ, is set to one. We determine the disutility parameter of working, Ξ, to match the steady-state hours, which are equal to 0.233.[4]

Regarding individual labor productivity shocks, we adopt the values used in Debortoli and Gali (2018), with ρ_z set to 0.978 and σ_z set to 0.193. These parameter values imply that individual wages exhibit an autoregressive coefficient of 0.914 and an innovation standard

[2]It should also be noted that $\int z_t d\mu_t = Z_t = \overline{Z}$ by construction.

[3]When χ is 1, Walras' law does not hold, so we use a value very close to 1 for the real contract case.

[4]This value is obtained by multiplying the average hours conditional on working (1/3) by the long-run employment rate (70 percent).

Table 1. Model Parameters

Parameter	Value	Description	Source/Target Moments
		Households	
β	0.974477	Time discount factor	1% real return to bond
σ	1.0	Risk-aversion	Standard
Ξ	15.2	Disutility parameter	See text
γ	1.0	Labor supply elasticity	Standard
ρ_z	0.978	Persistence of z shocks	Debortoli and Gali (2018)
σ_z	0.193	Standard deviation of z shocks	Debortoli and Gali (2018)
$\underline{b}$	0	Borrowing limit	McKay, Nakamura and Steinsson (2016)
ρ_ζ	0.6	Persistence of ζ shocks	Fernández-Villaverde et al. (2023)
σ_ζ	0.0048	Standard deviation of ζ shocks	11% ZLB frequency
		Firms	
f	0.0412	Production fixed cost	Zero profits
ϵ	10	Elasticity of substitution	Standard
θ	100	Price adjustment cost	See text
		Government and Monetary Authority	
ϕ_π	2.0	Coefficient on inflation dev.	Standard
ϕ_y	0.1	Coefficient on output dev.	Standard
$\overline{B}/\overline{Y}$	1.4	Public debt to annual GDP	McKay, Nakamura and Steinsson (2016)
$\overline{\Pi}$	1.0025	Steady state gross inflation	2% inflation target

deviation of 0.258 at an annual frequency, similar to the estimates provided by Floden and Linde (2001). The AR(1) process of individual labor productivity is converted into a 17-state Markov chain using Tauchen (1986)'s method. The borrowing limit, $\underline{b}$, is set to zero, following McKay, Nakamura and Steinsson (2016) and Hagedorn et al. (2019). Regarding the demand shock process, we adopt $\rho_\zeta = 0.6$ as suggested by Fernández-Villaverde et al. (2023) and set $\sigma_\zeta = 0.0048$, leading to an 11% frequency of hitting the ZLB.[5] We approximate the demand shock process using a discrete Markov chain, according to the Rouwenhorst (1995) method.

We choose the fixed cost of production, f, to ensure zero profits for intermediate goods firms in the steady state. The elasticity of substitution across intermediate goods, ϵ, is set to 10. The Rotemberg adjustment cost parameter, θ, is set to be consistent with the Calvo stickiness parameter of 0.75.[6]

[5]This frequency is consistent with US interest rates at the ZLB during 2009Q1-2015Q4 and 2020Q2-2020Q4, where the total sample period is from 1952Q1 until late 2020Q4.

[6]Given a Calvo parameter ϱ, the Rotemberg adjustment cost parameter can be recovered from $\theta =$

Table 2. Accuracy of Forecasting Rule for Inflation

	R^2	Den Haan Error (pp)	
		Mean	Max.
Non-ZLB Sample	0.9995	0.0455	0.4157
ZLB Sample	0.9993	0.0723	0.4479

Note: The accuracy of forecasting rules is evaluated based on the statistics proposed by Den Haan (2010). The unit is expressed in percentage points on an annualized basis. 'Non-ZLB Sample' refers to simulated periods excluding the ZLB, while 'ZLB Sample' denotes periods when the ZLB binds. The errors are computed using the parameters estimated from all simulated periods.

In accordance with McKay, Nakamura and Steinsson (2016), we set the debt-to-annual GDP ratio at 1.4, while the tax rate, τ_t, is chosen to ensure a balanced budget for the government each period. The Taylor rule coefficients for inflation and output, ϕ_π and ϕ_y, are set to 2.0 and 0.1, respectively. These choices are conventional values in the New Keynesian DSGE literature and are consistent with estimates from the empirical literature. The steady-state gross inflation rate, $\overline{\Pi}$, is set so that the annual inflation rate is 2% in the steady state, in line with the target inflation rate set by the Fed. In the baseline model, we assume that public bonds are purely denominated in nominal terms, i.e., $\chi = 0$.

3.2 Numerical Solution

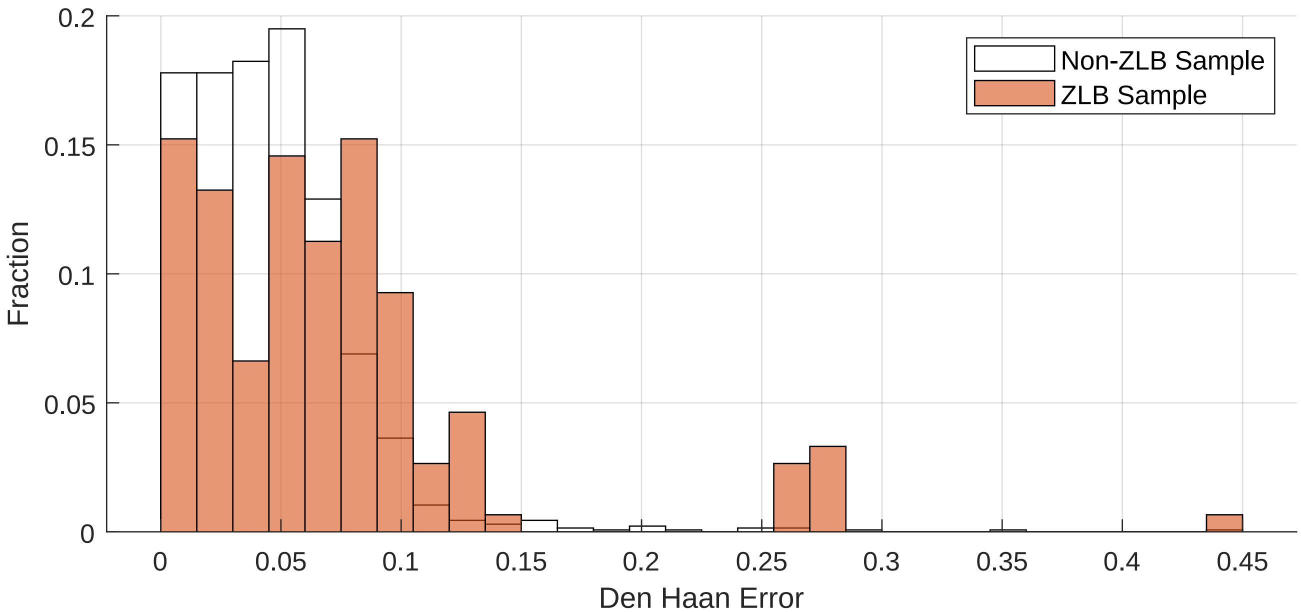
To solve the model with wealth distribution, we adopt the widely recognized approach proposed by Krusell and Smith (1998). Given the inherent nonlinearity of our model due to the presence of the ZLB, we modify the Krusell and Smith approach as follows. We distribute grid points for $\widetilde{R}_t$ in a non-uniform manner, allocating more points near the zero shadow rates. Furthermore, for a more accurate fit, we extend the forecasting functions by incorporating interaction and quadratic terms. Remarkably, our modified technique yields a highly accurate fit, even when the ZLB constraint is active.[7]

Table 2 presents a summary of the goodness of fit and precision of the inflation forecasting rule. The accuracy of the forecasting rule is assessed in two scenarios: i) simulated periods that exclude ZLB episodes and ii) periods when the ZLB binds. Evidently, the R^2 values for the forecasting function are notably high in both sets of time periods. When evaluating the accuracy of the forecasting rule, we utilize the metrics introduced by Den Haan (2010). In the non-ZLB sample, the mean errors are sufficiently small, staying be-

$\frac{\varrho(\epsilon-1)}{(1-\varrho)(1-\beta\varrho)}$.

[7] Further details on the computational procedures can be found in Appendix B.

Figure 1. Distribution of Den Haan Errors



Note: On the x-axis, errors are expressed in percentage points on an annualized basis, while the y-axis represents the fraction of errors (normalized to sum to one). 'Non-ZLB Sample' refers to simulated periods excluding the ZLB period, while 'ZLB Sample' denotes periods when the ZLB is binding. The errors are computed using the parameters estimated from all simulated periods.

low 0.05 percentage points on an annualized basis. Furthermore, the maximum errors remain reasonably modest, at around 0.42 percentage points. Regarding the periods when the ZLB constraint binds, our methodology effectively captures inflation dynamics. In this case, the mean and maximum errors are around 0.07 and 0.45 percentage points, respectively—slightly larger than those in the non-ZLB sample but still showcasing reasonable accuracy.[8] Figure 1 depicts the distribution of the Den Haan errors for the inflation forecasting rule in both ZLB and non-ZLB periods. The x-axis represents errors in percentage points on an annualized basis, while the y-axis illustrates the frequencies of errors, normalized to sum to one. Remarkably, the non-ZLB samples exhibit a relatively high number of zero or very small errors. However, the forecasting rule maintains its accuracy in the ZLB sample. Although there are some relatively large outliers in ZLB cases, they do not significantly undermine the overall accuracy of the forecasting rule, as these outliers are rare. This observation underscores the robustness of our methodology, showcasing its ability to sustain accurate predictions even in ZLB scenarios. We note that this level of performance is comparable to that achieved by neural networks in the work of Fernández-Villaverde et al. (2023).

[8]The Den Haan errors in both non-ZLB and ZLB periods are computed using the parameters estimated from all simulated periods.

Table 3. Income and Wealth Distributions

	Quintile					Gini
	1st	2nd	3rd	4th	5th	
U.S. Data						
Share of Income	2.8	6.7	11.3	18.3	60.9	0.58
Share of Wealth	-0.2	1.1	4.5	11.2	83.4	0.82
Model Economy						
Share of Income	3.1	8.0	8.9	19.7	60.4	0.56
Share of Wealth	0.0	0.1	1.7	11.6	86.6	0.82

Note: Income and wealth data are from the Survey of Consumer Finances (SCF) 2007 (source: Diaz-Gimenez, Glover and Rios-Rull (2011)).

4 Results

4.1 Cross-sectional Distributions and MPC

We examine whether the model economy effectively reproduces income and wealth distribution among households observed in the data. Table 3 reveals the comparison between the income and net asset holdings in the model and their data counterparts in the U.S.[9] The model economy demonstrates a reasonable reproduction of the income distribution found in the data, resulting in an income Gini coefficient of 0.56, which closely aligns with its empirical counterpart (0.58). Similarly, wealth inequality, characterized by a Gini coefficient of 0.82, is accurately replicated by the model economy. Overall, the model economy successfully achieves a realistic representation of heterogeneity across households.

We calculate the marginal propensity to consume (MPC) in our model by measuring the change in average consumption on date 0 in response to a transfer on date 0. The MPC from giving transfers of $400, $800, $1,200, and $1,600 are 0.13, 0.11, 0.10, and 0.09, respectively. These values closely align with the quarterly MPC in the HANK model of Hagedorn, Manovskii and Mitman (2019).

[9]Information for income and wealth in the data is from the Survey of Consumer Finances (SCF) 2007 in Diaz-Gimenez, Glover and Rios-Rull (2011).

Table 4. Business Cycle Statistics

	Std(Y)	Std(Π)	Pr($\tilde{R} < 1$)	Mean($\tilde{R}$)	Mean(r)	Mean(Π)
Steady State	-	-	-	3.00	1.00	2.00
ZLB-HANK	0.61	0.24	10.55%	2.58	0.79	1.79
HANK w/o ZLB	0.53	0.20	8.30%	2.90	0.95	1.95
ZLB-HANK+Index	0.41	0.15	2.80%	2.98	0.98	2.00

Note: Y, Π, $\tilde{R}$, and r denote output, inflation, the shadow nominal interest rate, and the real interest rate, respectively. When computing the standard deviations, all variables are logged and then detrended using the HP filter.

4.2 Aggregate Effects of Inflation Indexation

In this subsection, we discuss the extent to which inflation-indexed loan contracts stabilize business cycle fluctuations. We assess the stabilization by examining how much output and inflation volatilities are reduced when nominal contracts are replaced with contracts indexed to inflation. We demonstrate that the stabilizing effect of inflation indexation is particularly powerful in the presence of the ZLB. To do so, we compare the volatility of output, inflation, and the frequency of shadow nominal interest rates being less than zero, computed from the time series of HANK models with and without indexation when the ZLB is present.

Rows 2 to 4 of Table 4 illustrate the standard deviations and the mean of variables of interest across different model specifications. Row 1 reports nominal interest rate, real interest rate, and inflation in the deterministic steady state.[10] Compare these values with those shown in Row 3, which represents the HANK model without the ZLB constraint. Unlike linearly solved heterogeneous agent models such as those in McKay and Reis (2016) and Auclert, Rognlie and Straub (2020), the ergodic mean implied by the model is not equal to the deterministic steady state. The precautionary savings motive in response to aggregate uncertainty drives the average nominal and real interest rates below their steady-state counterparts. With sticky prices, the contraction in aggregate demand associated with precautionary motives results in mean output and inflation being lower than their steady-state levels. This nonlinear effect of aggregate uncertainty is also present in nonlinear representative agent models, but it is stronger in HANK models where aggregate income volatility creates larger consumption volatility due to the well-known interaction between MPCs and nominal rigidities.

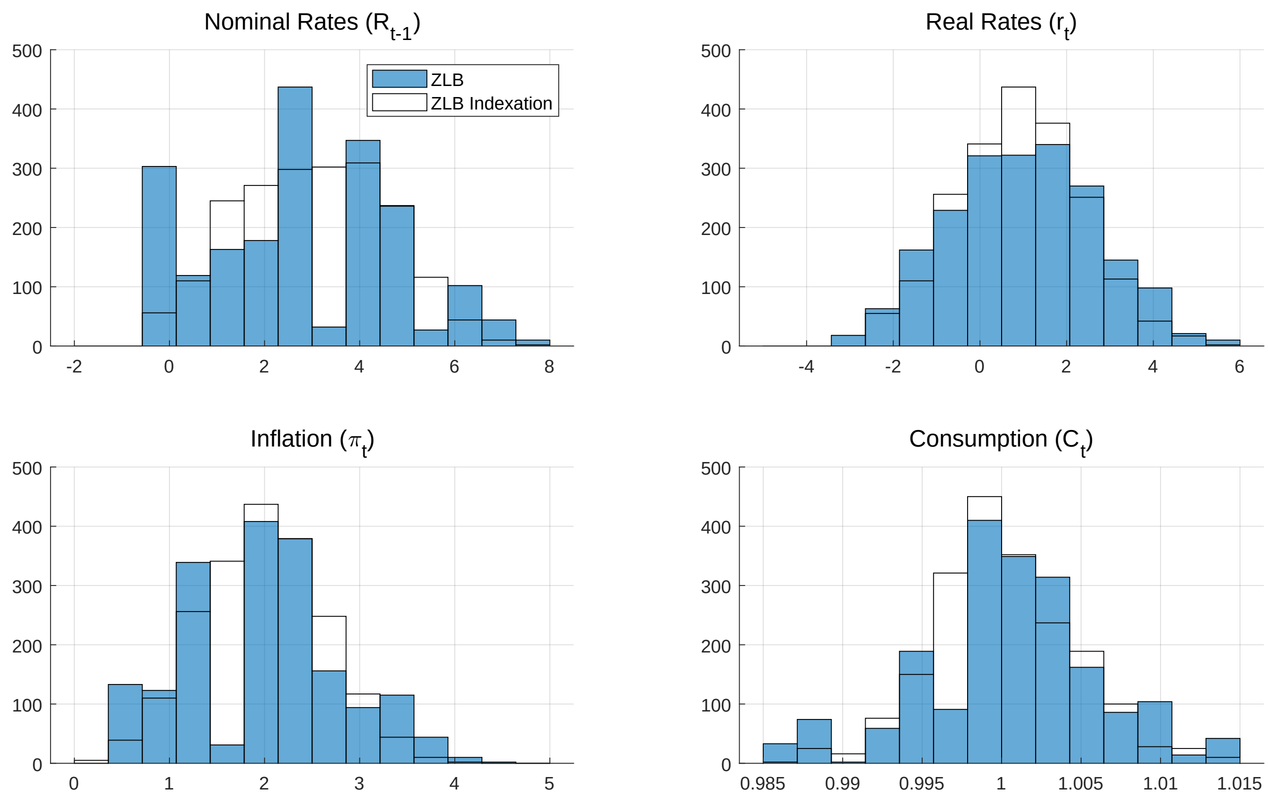
[10]The real interest rate reported in the table is computed as $\log\left(\frac{\widetilde{R}_{t-1}}{\Pi_t}\right)$.

Compare the moments presented in row 2, which correspond to the ZLB-HANK economy with nominal contracts, with those in row 3. Row 3 shows that the volatility of inflation and output is larger when the HANK economy faces occasionally binding ZLB constraints. Additionally, the presence of the ZLB further lowers the long-run mean of nominal interest rates, real interest rates, and inflation. This outcome confirms the findings of Fernández-Villaverde et al. (2023). Since income is expected to drop more in a recession when monetary policy is constrained than when it is unconstrained, wealth-poor households attempt to accumulate a larger buffer stock of savings to hedge against their consumption drop during ZLB episodes. This stronger precautionary motive, caused by the risk of hitting the ZLB, puts further downward pressure on mean output, inflation, and interest rates.

To assess the extent to which full inflation indexing stabilizes macroeconomic volatility when the ZLB is present, compare rows 2 and 4. Expressing the stabilization effect in percentage terms, the volatility of output and inflation is reduced by 33% $(= (0.61 - 0.41)/0.61 \times 100)$ and 38% $(= (0.24 - 0.15)/0.24 \times 100)$, respectively. A substantial decrease in macroeconomic volatility owing to inflation indexation weakens the precautionary savings motives against aggregate risk, and thus the average real and nominal interest rates are higher than in the world without the indexation, as observed in columns 5 and 6. A higher mean nominal interest rate makes the future occurrence of ZLB events less likely. As observed in the table, the frequency of hitting the ZLB changes from 10.6% to 2.8%, indicating that indexing loan contracts to inflation greatly expands the room for maneuvering policy rates by central banks.

The decreased variations in output and inflation under inflation indexation are not merely a result of the less asymmetric responses of nominal rates due to the decreased probability of the ZLB per se. That is, output and inflation in the presence of indexation not only decline less but also increase less. One way to illustrate this point is by comparing the ergodic distribution of the aggregate variables in ZLB-HANK models with and without inflation indexation. Figure 2 shows that inflation indexation reduces the frequency of the ZLB binding. This decreased frequency is associated with less frequent drops in inflation and consumption, as evident from the thinner left tail of the distribution of these variables, and less frequent rises in real interest rates, as seen in the thinner right tail of the distribution of real rates. Additionally, the figure illustrates that increases in inflation and consumption occur less often with indexation. The thinner left and right tails of the distribution for consumption and inflation indicate that inflation indexation not only limits

Figure 2. Ergodic Distributions: Indexation vs. No-Indexation



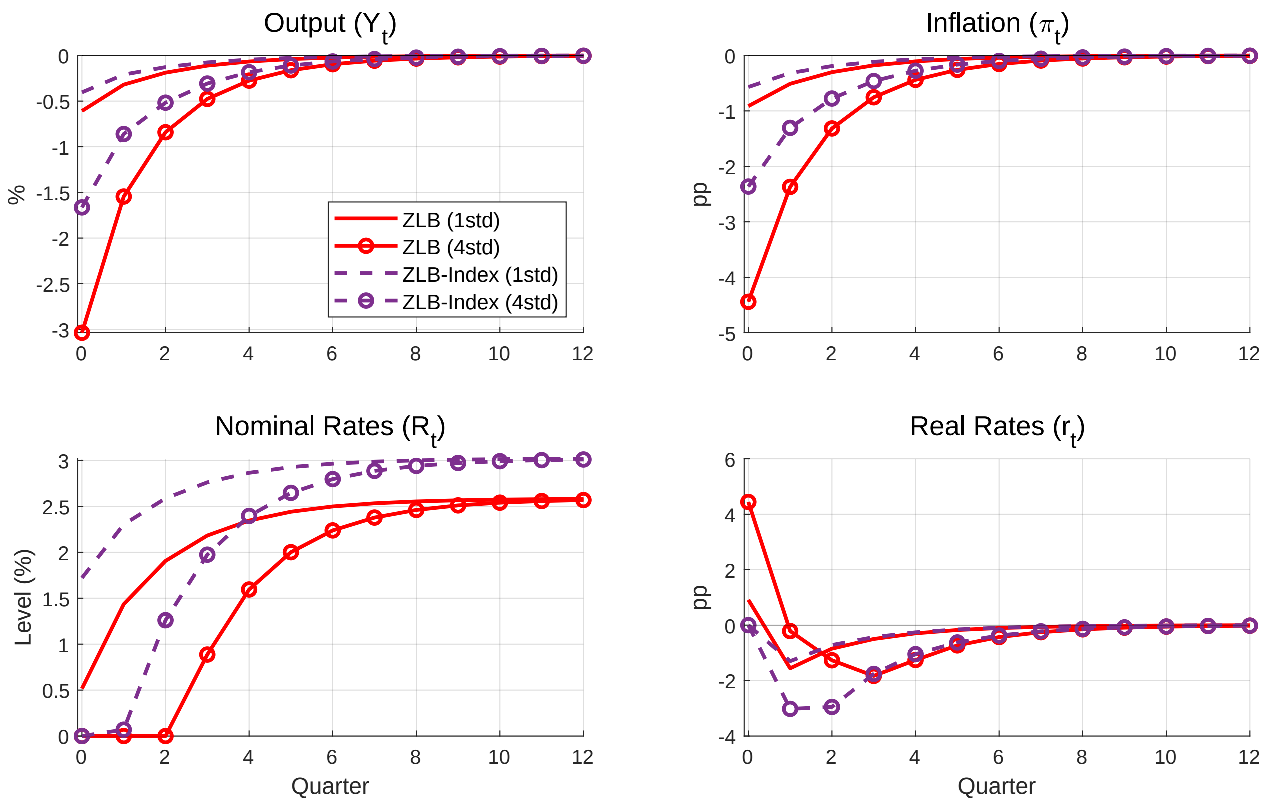
Note: The ergodic distributions of aggregate variables in the ZLB-HANK models with and without inflation indexation.

contractions but also expansions.

What is the mechanism that explains the stabilizing effect of inflation indexation? Consider a world without the ZLB first. A fall in inflation during contractions leads to a decline in nominal interest rates via a Taylor rule. When loans are fully indexed to inflation, as seen in equation (11), a decrease in nominal rates is entirely translated into a decrease in ex-ante real rates, boosting aggregate demand through the Euler equation. The decrease in ex-ante real rates mitigates the drop in inflation and output during contractions. However, in a world without inflation indexation, ex-ante real rates are affected not only by nominal rates but also by inflation expectations, which decrease during contractions. A fall in expected inflation weakens the drop in ex-ante real rates, contracting aggregate demand more than in a world with inflation indexation. The disconnection between real rates and inflation expectations is a key mechanism in the stabilizing effect of inflation indexation.

In a world with the ZLB, inflation indexation is even more stabilizing because it ameliorates the feedback loop between aggregate demand and deflation, a popular transmission mechanism of demand shocks at the ZLB documented in the New Keynesian literature

Figure 3. IRFs: Indexation vs. No-Indexation



Note: The impulse responses to demand shocks in the ZLB-HANK models with and without inflation indexation. The responses represent the deviations from the long-run mean, which differs across model economies.

(Christiano, Eichenbaum and Rebelo, 2011). Excess savings induced by adverse demand shocks cannot be cleared by a fall in real interest rates due to the constraint on nominal interest rates. In this setting, output must decline significantly to eliminate the excess supply of savings. In the absence of inflation indexation, the fall in output leads to expected deflation, which, in conjunction with zero nominal rates, drives up ex-ante real interest rates. The increase in real rates depresses aggregate demand further. However, when loans are indexed to inflation, expected deflation does not influence real rates. Thus, households no longer face the increase in ex-ante real rates, making ZLB episodes less contractionary. Accordingly, the precautionary savings motive against the risk of hitting the ZLB diminishes substantially, exerting upward pressure on nominal rates. The resulting decrease in the frequency of hitting the ZLB further reduces macroeconomic volatility.

Figure 3 illustrates the stabilizing power of inflation indexation via impulse responses to demand shocks of different magnitudes. Except for the nominal rates, which are plotted in levels, the responses of variables represent deviations from their long-run means, which differ across model specifications. In our ZLB-HANK models, with and without indexa-

Table 5. Welfare Gains of Inflation Indexation

Wealth Percentile				Total
1-40	40- 80	80-99	99-100 (Top 1%)	
0.4460	0.2675	-0.3127	-0.9028	0.2189

Note: The welfare difference between the ZLB-HANK with and without indexation, expressed as a fraction of steady-state consumption in the ZLB-HANK without indexation. The welfare in each economy is computed as the welfare conditional on the highest preference level and the lowest shadow policy rate. The positive number indicates that the ZLB-HANK with indexation is more desirable.

tion, the ZLB does not bind in response to a 1-standard deviation shock, as the shock does not lead to a sufficiently large reduction in nominal interest rates. In this unbinding ZLB case, the inflation indexation helps to alleviate the contraction to some extent. In response to a 4-standard deviation shock, the ZLB binds for three periods when loans are denominated in nominal terms and binds for one period when they are denominated in real terms. This is because, without inflation indexation, the long-run mean of nominal interest rates is lower, as shown in Table 4. The shorter periods of ZLB under inflation indexation are associated with a substantial stabilization of inflation and output. Overall, the figure illustrates that irrespective of the magnitude of the demand shock, both output and inflation exhibit less contraction when loans are indexed to inflation, reducing the likelihood of policy rates being constrained.

4.3 Disaggregate Effects of Inflation Indexation

Having demonstrated that the contribution of inflation indexation to stabilizing aggregate fluctuations is significant, we now study the distributional consequences of inflation indexation. Specifically, we investigate whether inflation indexation is beneficial for all households. To answer this question, we first compute the welfare of business cycles in the ZLB model without indexation for each wealth level group, and then compare it with the welfare in the model with indexation. The welfare metric we use is conditional welfare, which is conditioned on the highest preference level and the lowest shadow policy rate. We then use impulse responses for each group to explain the intuition behind the welfare gains or losses associated with inflation indexation.

Table 5 describes the welfare difference between the economy with and without indexation, where a positive number indicates that the latter is more welfare-enhancing. Clearly, in terms of aggregate welfare, the economy with indexation is better off than the one without indexation. This is consistent with the lower macro-volatility in the former, as shown in

Table 4. However, the table illustrates that not all households benefit from inflation indexation. While the welfare gain from indexation is greater for households with lesser amounts of wealth, those in the 80th percentile of the wealth distribution and above are worse off due to indexation.

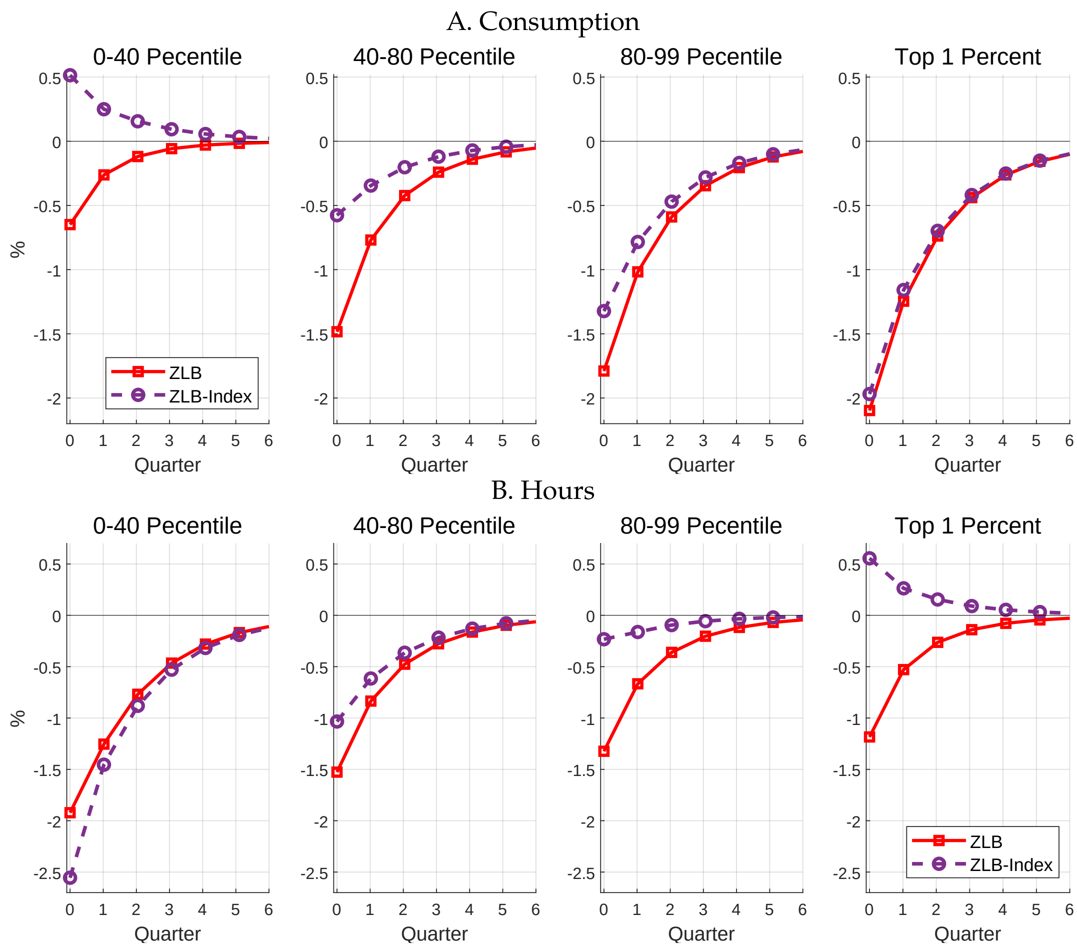
To interpret these different welfare outcomes across households, it is useful to refer to Figure 4, which displays the impulse responses of consumption and hours to negative demand shocks for each wealth group. In response to a 4-standard-deviation demand shock, households in the bottom 80th percentile of the distribution experience a larger stabilization in consumption compared to those in the top 20%, explaining the substantial welfare gain of indexation for households in the bottom 80%. Because the bottom 80% are relatively poorly insured against aggregate output fluctuations, their consumption is more sensitive to changes in aggregate conditions. Accordingly, indexing loan contracts to inflation, which mitigates the drop in aggregate income, stabilizes the consumption of wealth-poor households more than that of wealth-rich households.

Notice that the consumption of households in the 0-40th percentile increases when indexation is in place due to two factors. First, a large fraction of these households is borrowing-constrained and thus exhibits high MPCs. Second, in sticky price models, profits are countercyclical, so households receive positive profit income during recessions.[11] The increased profit income and the mitigated drop in labor income under inflation indexation boost the consumption of high MPC households.

Observe from Table 5 that the top 20% of the wealth distribution are worse off when nominal contracts are replaced with real contracts. The adverse effect of real contracts can be explained by the forgone interest income. Under nominal contracts, realized real interest rates that households face increase during recessions as nominal rates are predetermined and inflation falls. The increased realized real interest rate in response to adverse demand shocks, as shown in Figure 3, benefits wealthy households, for whom a significant portion of income is derived from interest income. However, under inflation indexation, such a beneficial effect disappears. To compensate for the loss of real interest income, wealthy households increase their hours more than they would under nominal contracts, which

[11] One can modify the distribution of countercyclical profits to rectify the positive consumption response among individuals with lower wealth, as outlined in Section C of the Appendix. These modifications result in individuals within the 0-40th percentile range showing a negative consumption response under inflation indexation. Even with the modified profit distribution, households at the bottom of the wealth distribution experience greater stabilization in consumption than their wealthier counterparts when inflation indexation is in place.

Figure 4. IRFs: Indexation vs. No-Indexation



Note: The impulse responses to demand shocks in the ZLB-HANK models with and without inflation indexation. The responses represent the deviations from the long-run mean, which differs across model economies.

reduces their welfare. Figure 4 shows that households in the 80th percentile of the wealth distribution and above increase hours more when nominal loan contracts are replaced with real contracts for a given size of demand shock. Observe that the increase in hours worked is particularly pronounced for the top 1% of households, who are hurt the most by the loss of real interest income. In contrast, households in the 0-40th percentile of the distribution do not increase hours worked when nominal contracts are replaced with real contracts, as interest income is not a large component of their total income.

4.4 The Role of Inequality

In this subsection, we discuss how household heterogeneity matters when assessing the stabilizing effect of inflation indexation. We show that the stabilization effect of indexation

Table 6. Inequality and Business Cycle Statistics

	Std(Y)	Std(Π)	Pr($\tilde{R} < 1$)
ZLB-HANK (high inequality)	0.81	0.32	30.00%
ZLB-HANK	0.61	0.24	10.55%
ZLB-HANK+Index (high inequality)	0.41	0.15	3.70%
ZLB-HANK+Index	0.41	0.15	2.80%

Note: Y, Π, and $\tilde{R}$ denote output, inflation, and the shadow nominal interest rate, respectively. When computing the standard deviations, all variables are logged and then detrended using the HP filter.

increases with the degree of labor income risk. Table 6 reports the standard deviations of output, inflation, and the ZLB frequency in ZLB-HANK models and those in ZLB-HANK models with inflation indexation across different degrees of income risk. To construct the highly unequal economy, we increase the standard deviation of the idiosyncratic productivity shock from 0.193 to 0.198, implying a 2% increase in the income Gini coefficient.

As found in Fernández-Villaverde et al. (2023), aggregate volatility depends substantially on income inequality. Even a small increase in idiosyncratic income risk results in increased aggregate volatility, with output and inflation rising by 33% ($= (0.81 - 0.61)/0.61 \times 100$) and 33% ($= (0.32 - 0.24)/0.24 \times 100$), respectively. The reason for the increased volatility is as follows. Higher idiosyncratic risk leads to more household precautionary savings in a steady state, implying a lower steady-state nominal interest rate. The lower steady-state nominal rates make the economy more prone to encountering the ZLB when aggregate demand shocks occur. As households anticipate more frequent deep recessions associated with the ZLB, the precautionary savings motive against aggregate uncertainty becomes more potent. Consequently, the average nominal interest rates decrease further, leading to an increase in both the frequency of encountering the ZLB and aggregate volatility. The ZLB hitting frequency is 30% in our high-inequality benchmark, whereas it is 10.5% under the baseline calibration.

Notice that the effect of inflation indexation in reducing aggregate volatility increases with the level of inequality. When loans are indexed to inflation, the volatilities of output and inflation are reduced by 49% ($= (0.81 - 0.41)/0.81 \times 100$) and 53% ($= (0.32 - 0.15)/0.32 \times 100$), respectively, under high productivity risk, compared to 33% ($= (0.61 - 0.41)/0.61 \times 100$) and 38% ($= (0.24 - 0.15)/0.24 \times 100$) under baseline productivity risk. This striking difference implies that the stabilizing effect of inflation indexation becomes more pronounced in a more unequal economy, which has a much higher probability of encountering the ZLB.

One interesting observation is that, despite the higher frequency of hitting the ZLB in an economy with higher idiosyncratic risk, the volatilities of output and inflation are almost identical across models with different degrees of idiosyncratic risk when loans are indexed to inflation, as observed in rows 3 and 4. This indicates that in a setting where deflationary expectations do not affect ex-ante real interest rates, the risk of hitting the ZLB is no longer a concern for policymakers.

5 Horse Race

In the previous section, we demonstrated that the stabilizing effect of inflation indexation is powerful. Indexing loan contracts to inflation decreases macroeconomic volatility by severing the link between real interest rates and inflation, thus reducing households' demand for precautionary savings. Consequently, it raises the average level of real and nominal interest rates, decreasing the likelihood of encountering the ZLB. In this regard, loan contracts indexed to inflation can be an effective way to provide more room for monetary policy to ease during a crisis. In this section, we compare inflation indexation with two other policies proposed to enlarge the space for conventional monetary policy: i) increasing the inflation target and ii) implementing an asymmetric Taylor rule. Unlike the so-called makeup strategies (e.g., price-level targeting and average inflation targeting), these two policies and inflation indexation do not require the central bank to commit to overheating the economy after a ZLB episode.

5.1 High Inflation Target vs. Inflation Indexation

One proposed policy in academic and policy circles to reduce the risk of hitting the ZLB is to raise the inflation target (Blanchard, Dell'Ariccia and Mauro, 2010). Coibion, Gorodnichenko and Wieland (2012) show that raising the inflation target in a representative agent model shifts up the nominal interest rates in the deterministic steady state, reducing the frequency of hitting the ZLB. Fernández-Villaverde et al. (2023) show that raising the inflation target in a heterogeneous agent model reduces the ZLB frequency to a greater extent than in a representative agent model because of a larger reduction in households' precautionary savings demand. A natural question to ask is: between raising the inflation

Table 7. Business Cycle Statistics: High Inflation Target vs. Inflation Indexation

	Std(Y)	Std(Π)	Pr$(\tilde{R} < 1)$
ZLB-HANK	0.61	0.24	10.55%
ZLB-HANK+Infl. target	0.52	0.20	2.80%
ZLB-HANK+Index	0.41	0.15	2.80%

Note: Y, Π, and $\tilde{R}$ denote output, inflation, and the shadow nominal interest rate, respectively. When computing the standard deviations, all variables are logged and then detrended using the HP filter.

target and indexing loans to inflation, which policy is more desirable? The goal of this subsection is to compare the stabilization effect and welfare of these two policies at both the aggregate and disaggregate levels.

For a fair comparison, we raise the inflation target in the ZLB-HANK model without indexation so that the implied frequency of hitting the ZLB is equal to that in the ZLB-HANK model with indexation. When raising the inflation target, we also adjust the steady-state inflation that appears in the price adjustment cost term of equation (4) to ensure that the welfare between the two policies is not affected by differences in steady-state price adjustment costs. The two model economies we compare exhibit the same quantities and cross-sectional distribution in a deterministic steady state. The inflation target in the model where the target is increased is 2.8%, whereas the model with inflation indexation has an inflation target of 2%. This difference implies that the steady-state nominal rate is 3.8% in the former, while it is 3% in the latter.

Rows 1 and 2 in Table 7 illustrate that raising the inflation target from 2% to 2.8% reduces the volatility of output and inflation, consistent with the findings of Coibion, Gorodnichenko and Wieland (2012) and Fernández-Villaverde et al. (2023). Rows 2 and 3 show that the model with a high inflation target is less stabilizing than the model with indexation, indicating that raising the inflation target is less effective in reducing volatility than inflation indexing, despite the equal frequency of the ZLB. Why does the model with a higher inflation target exhibit greater aggregate volatility than the model with indexation? The reason is that while raising the inflation target increases the average nominal rates, it does not eliminate the link between expected inflation and real interest rates. Unlike the model with inflation indexation, households in the high inflation target model still face a rise in the ex-ante real rate caused by expected deflation when the economy is at the ZLB, making the high inflation target economy more contractionary. Moreover, during expansions, the rise in expected inflation mitigates the increase in real rates faced by households,

Table 8. Welfare Gains of High Inflation Target and Inflation Indexation

	Wealth Percentile				Total
	1-40	40- 80	80-99	99-100 (Top 1%)	
ZLB-HANK+Infl. target	0.2687	0.1600	-0.1885	-0.5402	0.1314
ZLB-HANK+Index	0.4460	0.2675	-0.3127	-0.9028	0.2189

Note: The first row shows the welfare difference between the ZLB-HANK with a 2.8% inflation target and the ZLB-HANK, while the second row shows the welfare difference between the ZLB-HANK with indexation and the ZLB-HANK, expressed as a fraction of steady-state consumption in the ZLB-HANK. The welfare in each economy is computed as the welfare conditional on the highest preference level and the lowest shadow policy rate. The positive number indicates that the ZLB-HANK with indexation or ZLB-HANK with 2.8% inflation target is more desirable.

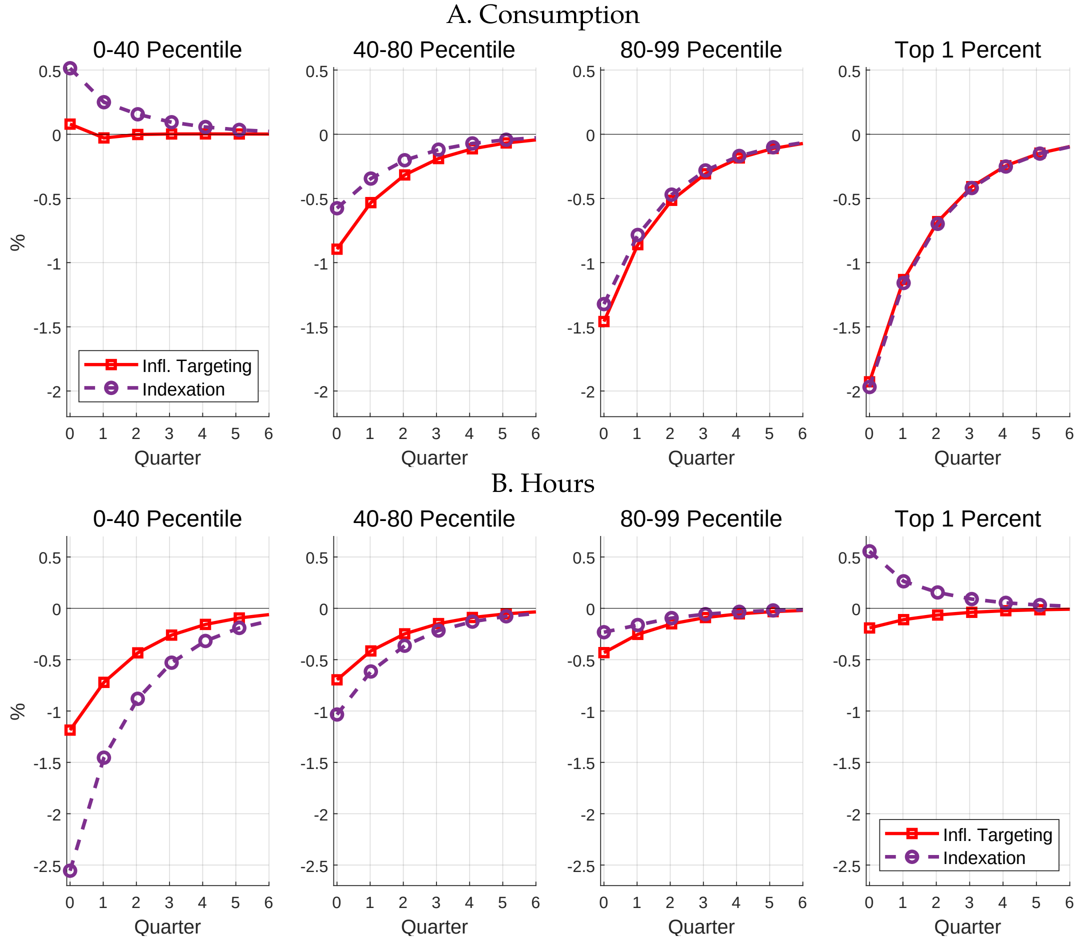
leading to larger increases in output and inflation compared to the model with indexation.

We compare the welfare effect of business cycles in the model with a high inflation target to that in the model with inflation indexation. The first row in Table 8 reports the welfare change when the target inflation rate is raised from 2.0% to 2.8% in the ZLB-HANK model across different wealth levels, where positive numbers indicate that, all else being equal, households prefer a higher inflation target. As indicated by the total welfare, a positive trend in inflation enhances overall welfare thanks to the reduction in aggregate volatility resulting from the decreased ZLB frequency. The second row in the table reproduces the result reported in Table 5, showing the welfare change when nominal contracts are replaced with real contracts, holding the inflation target at 2%. The comparison between rows 1 and 2 indicates that inflation indexation is more welfare-enhancing than raising the inflation target in the aggregate, reflecting the lower aggregate volatility in the former.

However, the welfare gain of inflation indexation over increasing the inflation target is not uniform across households. While households below the 80th percentile of the wealth distribution prefer indexation over a high inflation target, the top 20% are worse off. The reason for this disproportionate gain can be understood from Figure 5, which compares the impulse responses under indexation and those under a 2.8% inflation target to a 4-standard deviation shock. As noted in the figure, the bottom 80% of households enjoy a higher level of consumption under indexation than under a high inflation target for a given magnitude of demand shock. Since the consumption of these households is closely tied to individual labor and dividend income, which moves proportionately with aggregate income, a less severe recession under indexation allows these households to consume more than under a high inflation target.

Notice that the consumption stabilization effect of inflation indexation relative to the high inflation target is smaller for households in the 80-99th percentile of the wealth distri-

Figure 5. IRFs: Indexation vs. High Inflation Target



Note: The impulse responses to demand shocks in the ZLB-HANK with inflation indexation and the ZLB-HANK with a 2.8% inflation target. The responses represent the deviations from the long-run mean, which differs across model economies.

bution compared to those below the 80th percentile. Again, by severing the link between realized real interest rates and inflation, indexation makes the real interest income of wealth-rich households smaller than under a high inflation target during deflationary episodes. This smaller real interest income under indexation explains the reduced consumption stabilization for wealth-rich households. To compensate for the lower interest income under indexation, wealthy households work more than they would under a 2.8% inflation target, explaining why the top 20% are worse off under indexation than under a high inflation target.

The desirability of inflation indexation relative to increasing the inflation target to create more room for conventional monetary policy may increase if the two are compared in models where positive steady-state inflation is costly. As discussed in Coibion, Gorodnichenko and Wieland (2012), the most prominent welfare cost associated with higher trend inflation under staggered price setting is greater price dispersion in a steady state, which leads to an inefficient allocation of factor inputs and lowers aggregate output. In this context, inflation indexation may be even more beneficial, as it does not involve the steady-state cost of price dispersion.

5.2 Asymmetric Taylor Rule vs. Inflation Indexation

Another suggested monetary policy to reduce the probability of encountering the ZLB is to implement an asymmetric Taylor rule, where the central bank responds less aggressively to above-target inflation and more aggressively to below-target inflation (Bianchi, Melosi and Rottner, 2021). This rule corrects the deflationary bias, reduces the risk of hitting the ZLB, and lowers the welfare cost of business cycles. The deflationary bias refers to a situation in which average inflation is below the inflation target, a phenomenon observed between 2001 and 2020. In New Keynesian models with symmetric Taylor rules subject to the ZLB, such a phenomenon is an artifact of nonlinearities. Since the ZLB makes the likelihood of a decrease in inflation caused by a large negative shock greater than the increase caused by a positive shock of the same magnitude, the expected inflation is lower than the inflation target. The lower expected inflation induces current inflation to fall below the target even when away from the ZLB, thus increasing the frequency of encountering the ZLB. Given that inflation indexation limits the downside risk of inflation by mitigating the increase in real rates faced by households, it can be an alternative strategy to correct the deflation bias.

In this subsection, we compare an asymmetric Taylor rule and inflation indexation with respect to macroeconomic stabilization and welfare. Since the model with indexation already generates no deflationary bias, we calibrate the degree of asymmetry in the Taylor rule to match zero deflationary bias. We define the deflationary bias as the difference between the model's unconditional mean of inflation and the central bank's inflation target.[12] We introduce an asymmetric rule adopted by Bianchi, Melosi and Rottner (2021):

$$\log \widetilde{R_t} = \begin{cases} \log \overline{R} + \phi_\pi^N \left(\log \Pi_t - \log \overline{\Pi}\right) + \phi_y \left(\log Y_t - \log \overline{Y}\right) & \text{if } \Pi_t < \overline{\Pi} \\ \log \overline{R} + \phi_\pi^P \left(\log \Pi_t - \log \overline{\Pi}\right) + \phi_y \left(\log Y_t - \log \overline{Y}\right) & \text{if } \Pi_t \geq \overline{\Pi}, \end{cases} \tag{12}$$

[12] Alternatively, one can define the deflationary bias as the difference between the rate of inflation at the stochastic steady-state equilibrium and the central bank's inflation target. Since mean inflation can be observed more directly in the data than inflation at the stochastic steady-state, we define the deflationary bias as the difference between the mean of inflation and the central bank's inflation target.

Table 9. Business Cycle Statistics: Asymmetric Taylor Rule vs. Inflation Indexation

	Std(Y)	Std(Π)	Pr($\tilde{R} < 1$)	Mean(Π)	Total Welfare
ZLB-HANK	0.61	0.24	10.55%	1.87	0
ZLB-HANK+Asymm.	0.62	0.24	8.30%	2.00	0.0274
ZLB-HANK+Index	0.41	0.15	2.80%	2.00	0.2189

Note: Y, Π, and $\tilde{R}$ denote output, inflation, and the shadow nominal interest rate, respectively. When computing the standard deviations, all variables are logged and then detrended using the HP filter. Total welfare denotes the difference in aggregate welfare between the economy of interest and ZLB-HANK, expressed as a fraction of steady-state consumption in the ZLB-HANK.

where $\phi_\pi^P < \phi_\pi^N$. We set $\phi_\pi^N = 2$ and search for ϕ_π^P that delivers zero deflationary bias. It turns out that $\phi_\pi^P = 0.7$, which implies a smaller response of the policy rate to inflation when inflation is above target.

Row 1 of Table 9 shows that mean inflation in ZLB-HANK is lower than the inflation target of 2%, indicating the presence of a deflationary bias. The table also illustrates that both the asymmetric rule (row 2) and inflation indexation (row 3) correct the deflationary bias, thereby reducing the ZLB frequency more than in a model without these policies. However, despite the absence of deflationary bias, these two policies result in very different macroeconomic volatility and welfare. Inflation indexation exhibits lower variations in output and inflation and higher welfare. The reason inflation indexation is more effective in stabilizing macroeconomic conditions than the asymmetric rule is due to the lower probability of hitting the ZLB and the more responsive policy rates when inflation increases. Again, at the ZLB, households under inflation indexation face lower ex-ante real rates than those under the asymmetric rule, as ex-ante real rates are unaffected by deflation expectations under the former but not under the latter. This property makes the ZLB less contractionary in an economy with indexation. Consequently, the duration of a ZLB spell under inflation indexation is shorter than under the asymmetric rule in response to a given negative demand shock, explaining the lower ZLB-hitting probability in the former. Moreover, by construction, the asymmetric rule predicts a less responsive monetary policy than inflation indexation when inflation is above target, implying a greater upside swing in inflation and output at the cost of correcting the deflationary bias. However, such a trade-off between correcting the deflationary bias and mitigating the rise in inflation and output is absent under inflation indexation.

6 Conclusion

In this paper, we employed a ZLB-HANK model to assess the extent to which inflation-indexed loan contracts can reduce macroeconomic volatility. We found that if loans were fully indexed to inflation, output and inflation would stabilize significantly, with a notable reduction in ZLB frequency. When government bonds held by households are indexed to inflation, a fall in expected inflation does not affect ex-ante real rates. This indicates that households do not experience an increase in ex-ante real rates at the ZLB, which would have occurred under nominal bonds. Consequently, the reduced downside aggregate risk under real contracts significantly weakens the demand for precautionary savings, leading to higher average nominal rates and a reduction in ZLB frequency.

The stabilization effect of real contracts increases when idiosyncratic income risk is high. With higher income risk, steady-state nominal interest rates fall, raising the likelihood of hitting the ZLB. In this case, the benefits of reducing downside risk by replacing nominal contracts with real contracts increase substantially.

We demonstrated that inflation indexation is more effective in stabilizing aggregate volatility than increasing the inflation target or adopting an asymmetric Taylor rule. This result suggests that issuing inflation-linked bonds, or at least combining these bonds with alternative strategies, might be a more desirable approach than changing monetary policy strategies alone in response to the declining natural rate of interest observed in many advanced countries.

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Appendix

A Definition of Recursive Competitive Equilibrium

A.1 The Recursive Problem of the Household

The household's problem can be recursively written as follows. Define x and X as the vectors of individual and aggregate state variables, respectively: $x \equiv (b, z)$ and $X \equiv (\mu, \zeta, \widetilde{R}_{-1})$, where $\mu(x)$ is the type distribution of households, and $\widetilde{R}_{-1}$ is the shadow interest rate set in the previous period.[13] The value function for an individual household, denoted by $V(x, X)$, is defined as:

$$V(x, X) = \max_{c,b',h} \left\{ \frac{c^{1-\sigma} - 1}{1 - \sigma} - \Xi \frac{h^{1+1/\gamma}}{1 + 1/\gamma} + \zeta\beta\mathbb{E}\left[V(x', X')|z, \zeta\right] \right\} \tag{A.1}$$

subject to

$$c + b' = w(X)zh + (1 + r(X))b - T + d, \; b' \geq \underline{b},$$

and

$$\mu' = \mathbb{T}(X),$$

where $\mathbb{T}$ denotes the law of motion for μ. Time subindices are suppressed to simplify notation, and primes denote variables in the next period.

A.2 Definition of Equilibrium

A recursive competitive equilibrium is a value function $V(x, X)$, a transition operator $\mathbb{T}(X)$, a set of policy functions $\{c(x, X), b'(x, X), h(x, X), n_j(X), p_j(X), y_j(X)\}$, and a set of prices $\{w(X), r(X), R(X), P(X)\}$ such that:

1. Individual households' optimization: given $w(X)$ and $r(X)$, optimal decision rules $c(x, X)$, $b'(x, X)$, and $h(x, X)$ solve the Bellman equation, $V(x, X)$.

[13] Denote $\mathcal{B}$ and $\mathcal{Z}$ as the sets of all possible realizations of b and z, respectively. Then, the measure $\mu(b, z)$ is defined over a σ-algebra of $\mathcal{B} \times \mathcal{Z}$.

2. Intermediate goods firms' optimization: given $w(X)$, $r(X)$, and $P(X)$, the associated optimal decision rules are $n_j(X)$ and $p_j(X)$.

3. Final good firm's optimization: given a set of prices $P(X)$ and $p_j(X)$, the associated optimal decision rules are $y_j(X)$ and $Y(X)$.

4. The gross nominal interest rate, $R(X)$, satisfies the interest rate rule (6).

5. Balanced budget of the government: $r(X)\overline{B} = \int T(x, X)d\mu$.

6. For all Ω,

   - (Labor market) $N(X) = \int zh(x, X)d\mu = \int n_j(X)dj$
   - (Bond market) $\overline{B} = \int b'(x, X)d\mu$
   - (Goods market) $Y(X) = C(X)$ where $Y(X) = N(X) - f$ and $C(X) = \int c(x, X)d\mu$.

7. Consistency of individual and aggregate behaviors: for all $A^0 \subset \mathcal{A}$ and $Z^0 \subset \mathcal{Z}$,

$$\mu'(A^0, Z^0) = \int_{A^0, Z^0} \left\{ \int_{\mathcal{A},\mathcal{Z}} \mathbf{1}_{b'=b'(x,X)} d\mathit{\Gamma}_z(z'|z)d\mu \right\} db'dz',$$

   where $\mathit{\Gamma}_z(z'|z)$ is a transition probability distribution function for z.

B Computational Procedures

B.1 Steady-State Equilibrium

We summarize the steps required to solve for the stationary measure, $\overline{\mu}$.

Step 1. Have guesses for endogenous parameters, β, Ξ, τ, and w.

Step 2. Construct grids for individual-state variables–bond holdings, b, and logged individual labor productivity, $\hat{z} = \ln z$. Letting N_b and N_z, denote the number of grids for b and z, respectively, we choose $N_b = 101$ and $N_z = 17$. The range of b is $[0, 40]$. More bond grid points are assigned on the lower range, while $\widehat{z}$ is equally spaced in the range of $[-3\sigma_{\hat{z}}, 3\sigma_{\hat{z}}]$, where $\sigma_{\hat{z}} = \sigma_z/\sqrt{1-\rho_z^2}$.

Step 3. Approximate the transition probability matrices for individual labor productivity, Γ_z, following Tauchen (1986).

Step 4. Solve for the individual value function, $V(b, z)$. In this step, we obtain the optimal decision rules for saving, $b'(b, z)$, and hours worked, $h(b, z)$.

(a) Make an initial guess for the value function, $V_0(b, z)$, for every grid point.

(b) Solve the individual household's problem and obtain $V_1(b, z)$:

$$V_1(b, z) = \max_{\{b',h\}}$$
$$\left\{\log\left(wzh + (1 + r)b - T + d - b'\right) - \Xi \frac{h^{1+1/\nu}}{1+1/\nu} + \beta \sum_{z'} \Gamma_z(z'|z)V_0(b', z')\right\}$$

(c) If V_0 and V_1 are close enough for each grid point, go to the next step. Otherwise, update the value functions ($V_0 = V_1$) and go back to (b).

Step 5. Obtain the time-invariant measure, $\overline{\mu}$, using the optimal decision rules for bond holdings and transition probabilities for z, Γ_z.

Step 6. Compute aggregate variables and moments using $\overline{\mu}$. If the moments become sufficiently close to the targeted ones, then the steady-state equilibrium of the economy is found. Otherwise, update the endogenous parameters, and go back to Step 4.

B.2 Equilibrium with Aggregate Fluctuations

We summarize the computational algorithm for solving the model with aggregate preference shocks. To address the intricacies of our model, we employ the well-established methodology introduced by Krusell and Smith (1998).

Step 1. Construct grids for aggregate and individual state variables. For preference shocks, ζ, construct 15 grid points in the range of $[-3\widetilde{\sigma}_\zeta, 3\widetilde{\sigma}_\zeta]$, where $\widetilde{\sigma}_\zeta = \sigma_\zeta/\sqrt{1-\rho_\zeta^2}$. Distribute grid points for $\widetilde{R}_{-1}$ in a non-uniform manner, allocating more points around the zero shadow rates. The grids for individual state variables are the same as those in the steady-state economy.

Step 2. Parameterize the forecasting functions for Y, Π, w, $\widetilde{R}$, and d. The forecasting model for the variable of interest, X, is parameterized as follows:

$$\log X = b_0 + b_1 \log \widetilde{R}_{-1} + b_2 \log \zeta + b_3 \log \zeta * \log \widetilde{R}_{-1} + b_4 (\log \zeta)^2 + b_5 \left(\log \widetilde{R}_{-1}\right)^2.$$

Step 3. Given the forecasting functions, solve the optimization problems for individual households. Obtain the policy functions for asset holdings, $b'(b, z, R_{-1}, \zeta)$, and hours, $h(b, z, R_{-1}, \zeta)$.

Table A.1. Estimates and Accuracy of Forecasting Rules

Dependent Variable	Coefficient b_0	b_1	b_2	b_3	b_4	b_5	R^2	Den Haan Error (%)
$\log \widetilde{R}$	0.008	-0.026	-0.988	0.098	-15.892	-2.523	0.9971	0.0965
$\log Y$	-0.990	-0.062	-1.153	-0.673	-21.459	-5.799	0.9994	0.0545
$\log \Pi$	0.005	-0.010	-0.436	0.080	-6.868	-0.972	0.9963	0.0482
$\log w$	-0.102	-0.120	-2.189	-0.937	-42.016	-11.863	0.9838	0.5036
$\log(1+d)$	-0.001	0.026	0.461	0.109	7.645	2.478	0.9837	0.1073

Step 4. Generate simulated data for 2,500 periods using the policy and value functions obtained in Step 4. The details are as follows.

(a) Set the initial conditions for $\widetilde{R}_{-1}, \zeta$, and $\mu(a, z)$.

(b) Given the forecasting functions, the evaluated value function obtained in Step 3, and the obtained new prices, solve the optimization problems for individual households to get the policy functions for asset holdings, $b'(b, z)$, and the hours decision rule, $h(b, z)$.

(c) Using μ and the forecasting functions, compute aggregate variables: $C = \int c(b, z) d\mu$, $N = \int z h(b, z) d\mu$, $H = \int h(b, z) d\mu$, $Y = N - f$, Π, w, $\widetilde{R}$, and d.

(d) Save the time series of Y, Π, w, $\widetilde{R}$, and d.

(e) Obtain the next period measure $\mu'(b, z)$ using $b'(b, z)$ and transition probabilities for z.

Step 5. Obtain the new coefficients of the forecasting functions by the OLS estimation using the simulated time series.[14] If the new coefficients are close enough to the previous ones, we obtain the equilibrium forecasting functions. Otherwise, update the coefficients and go to Step 3.

Table A.1 presents a summary of the estimated coefficients, goodness-of-fit metrics, and the accuracy of the forecasting rules in the ZLB-HANK model. The results indicate that the R^2 values for all forecasting functions are consistently high, demonstrating strong model performance. Additionally, the accuracy of the forecasting rules has been evaluated

[14]We drop the first 500 periods to eliminate the impact of the arbitrary choice of initial aggregate state variables.

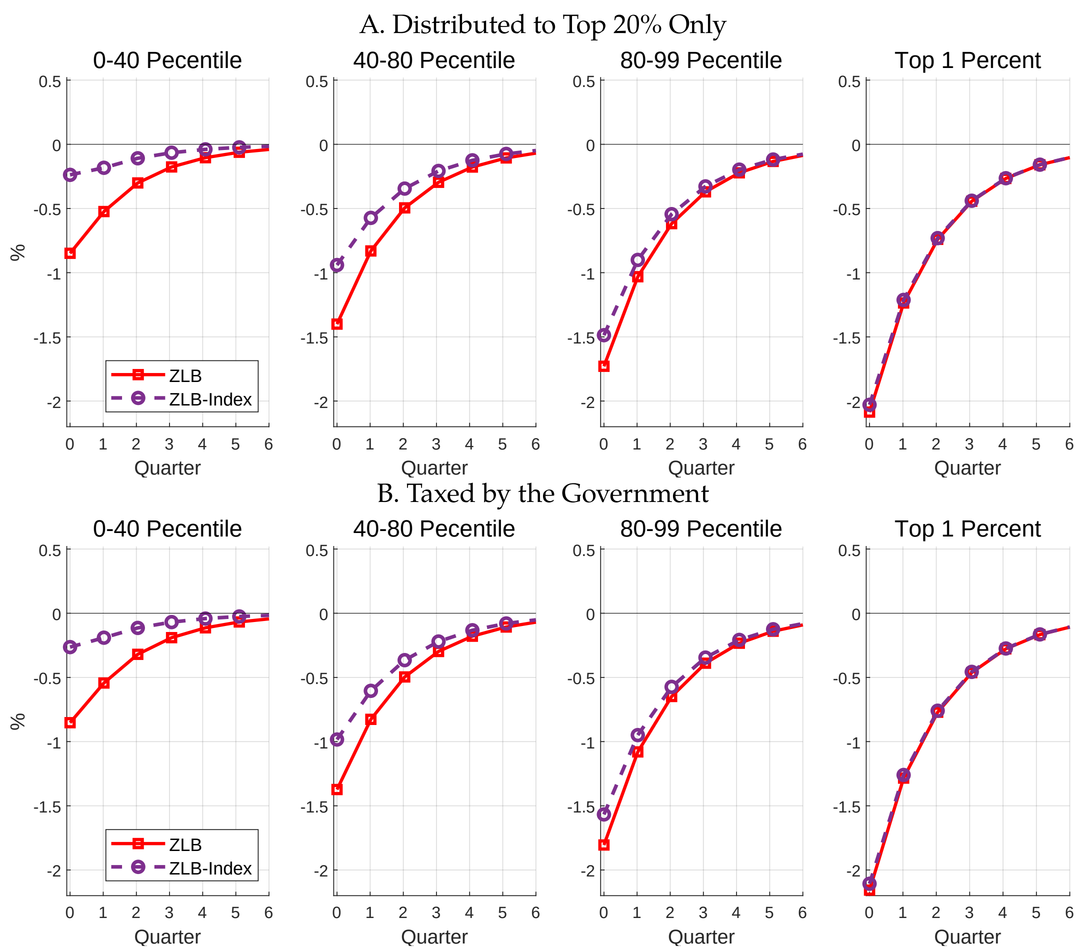
using the statistical measures proposed by Den Haan (2010). The findings show that the mean Den Haan errors are sufficiently small across all forecasting functions, further confirming the reliability of the model's predictive capabilities.

C Distribution of Countercyclical Profits

In models with monopolistic competition and nominal price rigidities, markups exhibit countercyclical behavior, leading to positive profits in response to negative demand shocks. This phenomenon significantly influences consumption dynamics, especially for households with low amounts of wealth. In this section, we conduct a comparative analysis of two distinct economic scenarios: (i) an economy wherein profits are exclusively distributed to the top 20% of households in the productivity distribution, and (ii) an economy where profit incomes are subjected to a 100 percent tax rate, with the government utilizing the proceeds for wasteful government consumption.

Figure A.1 illustrates the consumption responses across the wealth distribution in these two economies. Two noteworthy findings emerge. Firstly, the distribution of countercyclical profits has a profound impact on consumption dynamics, particularly for households with lower wealth. Specifically, irrespective of inflation indexation, households in the 0-40th percentile exhibit a decline in consumption, which was not observed in a model where profits are distributed to households proportionally to productivity. Secondly, the heterogeneous consumption stabilization due to inflation indexation persists in both cases, with households at the bottom of the wealth distribution exhibiting greater stabilization in consumption compared to their counterparts at the top.

Figure A.1. Consumption IRFs across Wealth Distribution



Note: The impulse responses to demand shocks in the ZLB-HANK models with and without inflation indexation. The responses represent the deviations from the long-run mean, which differs across model economies.