Belief Updates and Asset Mispricing

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Abstract

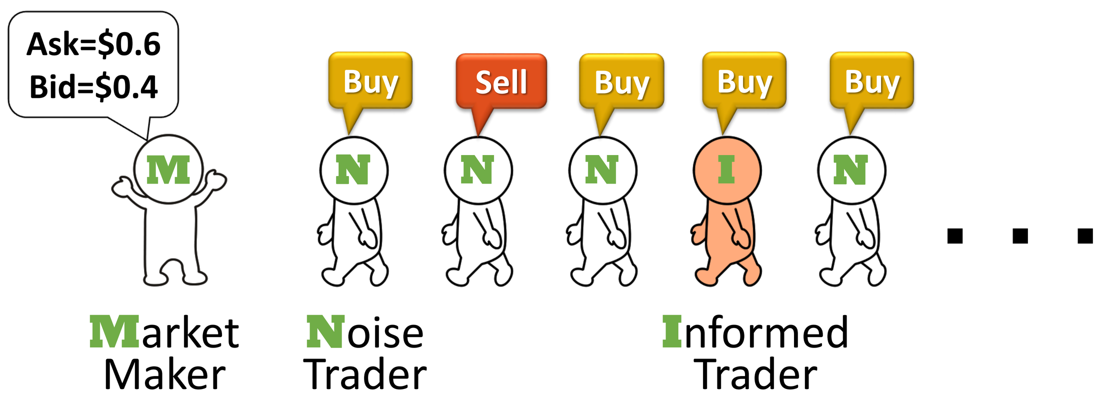
- We assume an **asymmetric information** environment.
- There are two uncertainties: **uncertainty of the asset value** and **uncertainty of the existence of informed traders**.
- When a market maker does not know whether informed traders exist or not and informed traders actually do not exist, the asset price systematically deviates from the fair value, causing **asset mispricing**.
- This situation is close to the **information mirage** that Camerer and Weigelt (1991) found in their asset market experiments.
- After the market maker sufficiently updates his belief, he adequately finds the non-existence of informed traders. Then asset mispricing shrinks, and the market becomes **efficient**.

Market

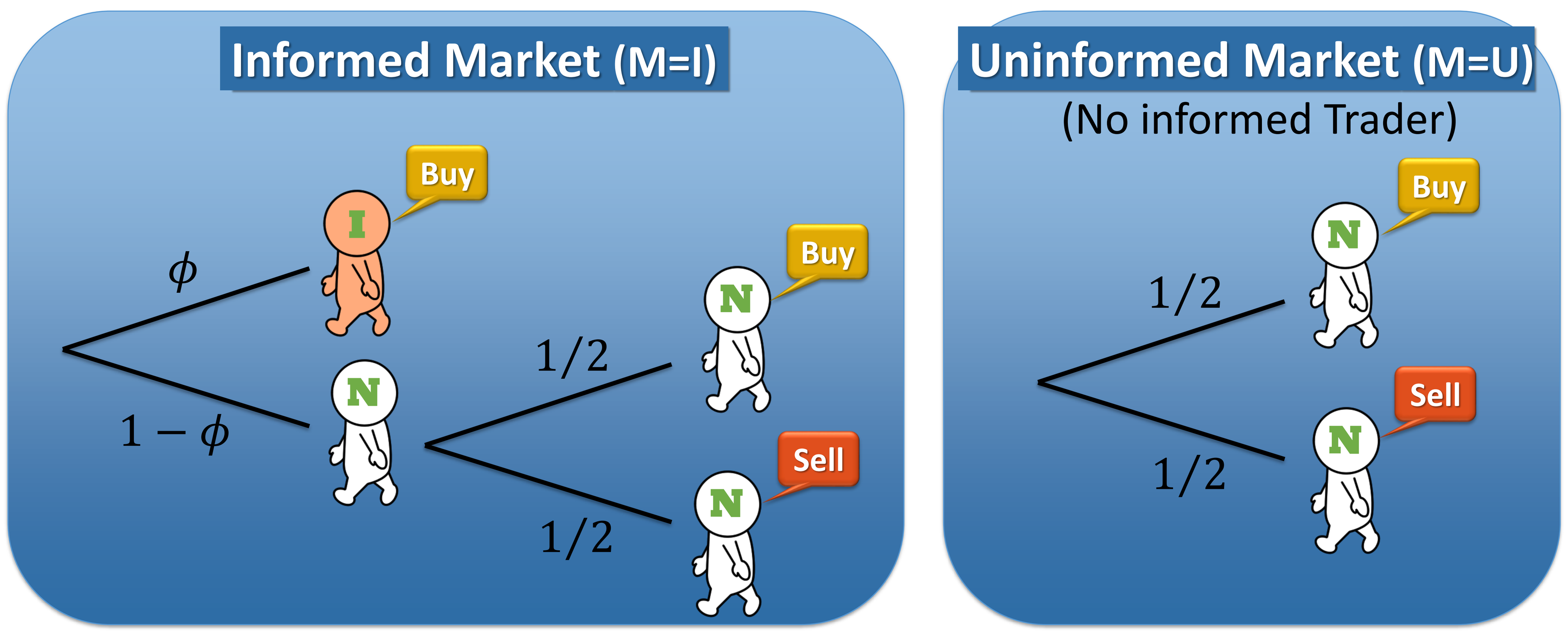
1) Risky Asset

Asset value : $\theta = \begin{cases} 1 & (prob. = 1/2) \\ 0 & (prob. = 1/2) \end{cases}$

2) Sequential Trading



3) Market Type (Given $\theta = 1$)



Example

1) θ **=0.5 3Markets**

$M = U$	$M = I, \theta = 1$	$M = I, \theta = 0$
Prob(ω_k= B)=0.5 Prob(ω_k= B)=0.5	Prob(ω_k= B)=0.75 Prob(ω_k= B)=0.25	Prob(ω_k= B)=0.75 Prob(ω_k= B)=0.25

2) History $\mathcal{H}_n = BSBBB$

3) Problem $\text{Prob}(M = I \mid \mathcal{H}_n)$?

4) Thinking without strict calculation

Two step estimation : The first step $\text{Prob}(\theta = 1 \mid M = I, \mathcal{H}_n)$?

Prob(ω_k= B)=0.5 Prob(ω_k= B)=0.5	Prob(ω_k= B)=0.75 Prob(ω_k= B)=0.25 ?	Prob(ω_k= B)=0.75 Prob(ω_k= B)=0.25

In particular, thinking $\text{Prob}(\theta = 1 \mid M = I, \mathcal{H}_n)$=1, this is probability pruning.

The second step $\text{Prob}(M = I \mid \mathcal{H}_n)$?

Prob(ω_k= B)=0.5 Prob(ω_k= B)=0.5 ?	Prob(ω_k= B)=0.75 Prob(ω_k= B)=0.25	~~Prob(ω_k= B)=0.75 Prob(ω_k= B)=0.25~~

Belief Update

1) Market Maker's belief

$$\mu_n = \text{Prob}(\theta = 1 \mid M = I, \mathcal{H}_n) \qquad \mathcal{H}_n = \omega_1\omega_2\cdots\omega_n \ : \ \textit{Buy/Sell history}$$

$$\xi_n = \text{Prob}(M = I \mid \omega_n, \ \mu_n) \qquad \textit{Suspicion of informed traders}$$

2) Quasi-Bayesian Update

$$\xi_{n+1} = \begin{cases} \dfrac{0.5 + \phi(\mu_{n+1} - 0.5)}{0.5 + \phi(\mu_{n+1} - 0.5)\xi_n} \cdot \xi_n & (\omega_{n+1} = B) \\[2ex] \dfrac{0.5 - \phi(\mu_{n+1} - 0.5)}{0.5 - \phi(\mu_{n+1} - 0.5)\xi_n} \cdot \xi_n & (\omega_{n+1} = S) \end{cases}$$

Theorem

1) $\xi_n \xrightarrow{a.s.} \textit{True Value}$ ***Quasi-Bayesian markets become efficient.***

2) $E[\xi_n]$ *has Local Maximum* ***Temporal Miss price (Bubble) occurs.***

Simulation

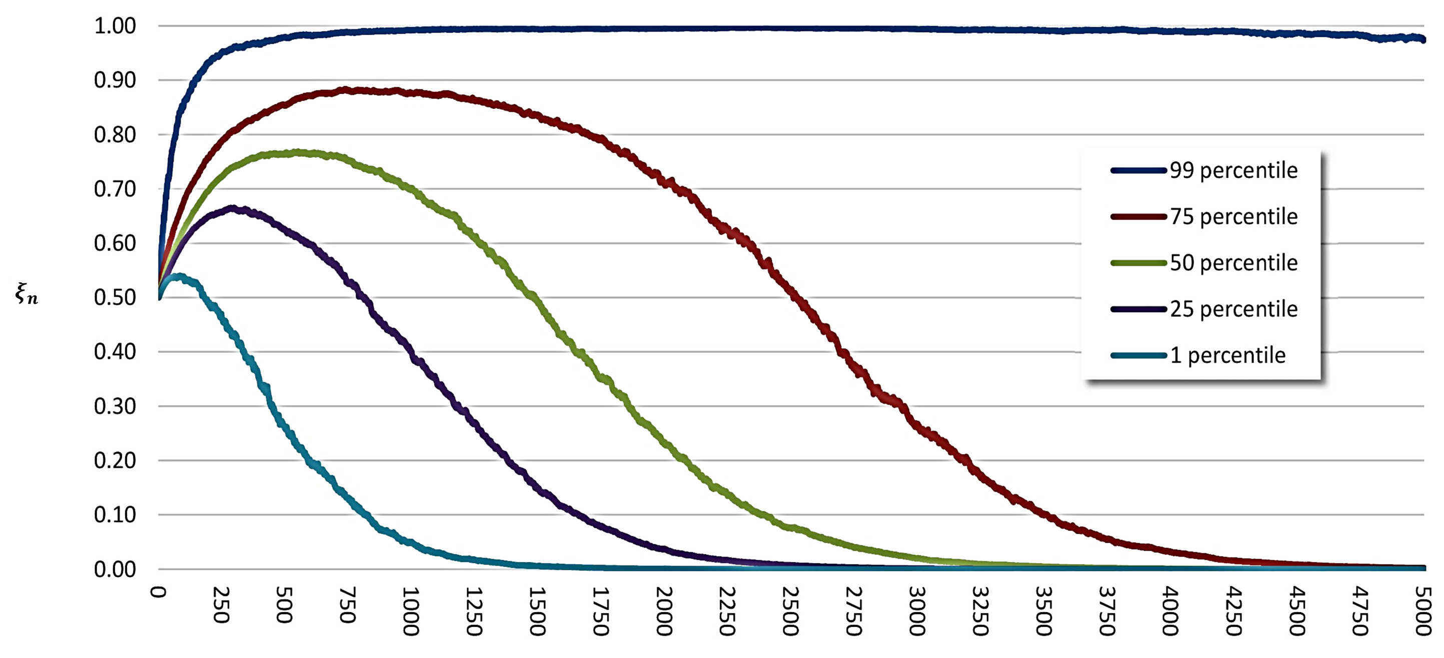


Figure 1. Transition of ξ_n.

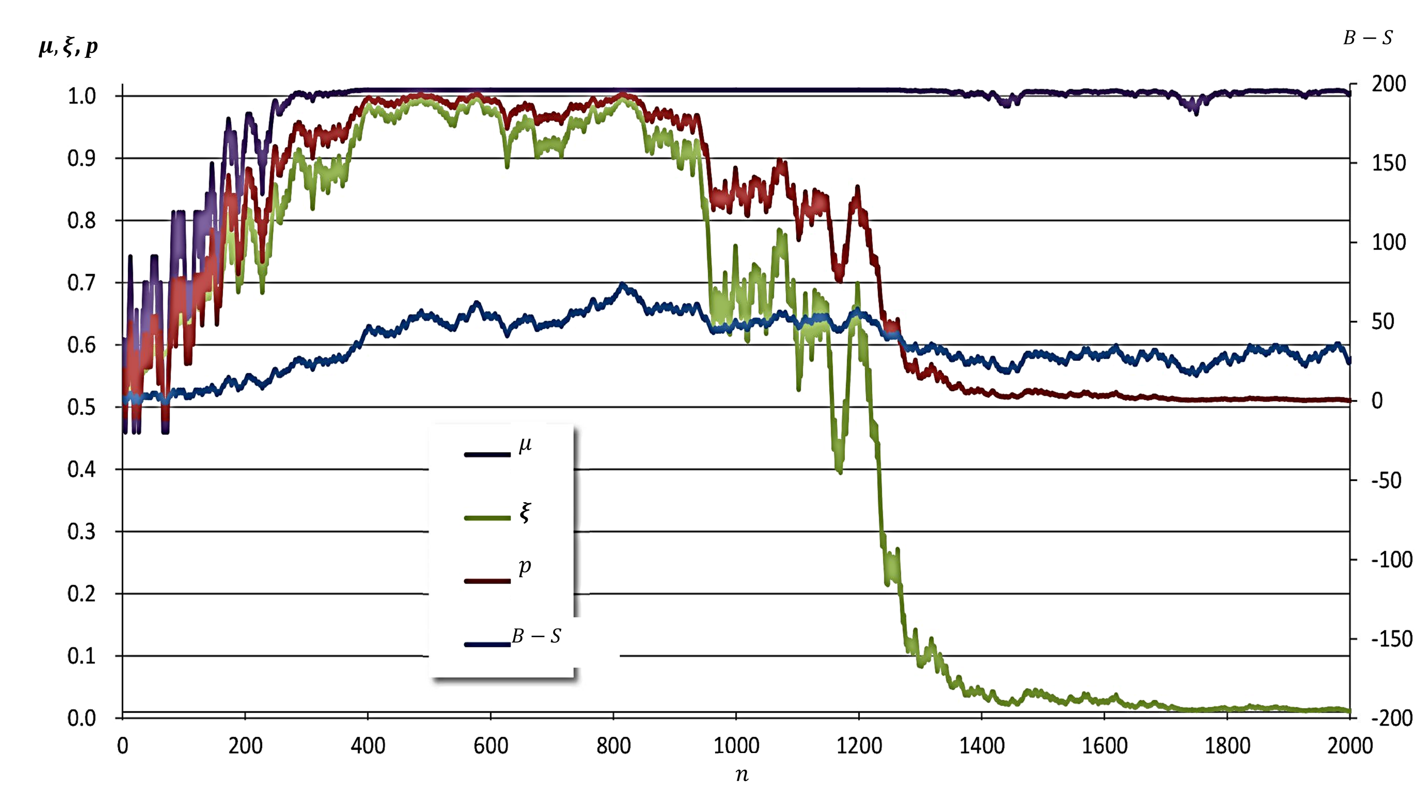


Figure 2. A typical pass of miss pricing.

Conclusion

Our results suggest that mispricing may occur when investors believe that private information exists in stock markets and trade on the basis of their own belief even if private information does not exist. For stock markets to be efficient, controlling the information flow is important for policy makers and regulators.