A Theory of International Official Lending*

Qing Liu †, Zanhui Liu ‡and Vivian Zhanwei Yue §

December 2024

Abstract

This paper develops a theoretical framework to explain the role of international official lending. We study a repeated game model of sovereign debt in the presence of default risk and moral hazard. First, the sovereign country can renege on the debt. Second, lenders cannot perfectly observe whether the sovereign uses the borrowed funds for consumption or export production. The constrained optimal allocation (COA) prescribes imperfect insurance and provides dynamic incentives. The COA is decentralized by a competitive equilibrium with official lending and private lending. Numerical exercises demonstrate that official debt is countercyclical, while market debt is procyclical. This study offers insights into the interaction between different types of creditors and the dynamics of sovereign debt.

Keywords: International official lending; sovereign debt; default risk; moral hazard

*We would like to thank Marina Azzimonti, Yan Bai, Carlos Esquivel, Xiang Fang, Yi Huang, Stelios Fourakis, Jinill Kim, Yan Liu, Veronica Rappoport, Helene Rey, Zheng Song, Kjetil Storesletten, Xuan Tam, Fabrizio Zilibotti, and conference participants at the 8th International Macroeconomics and Finance Conference at HKUST, 2024 City University of Hong Kong Conference on International Economics, first annual Center for Advancing Women in Economics Conference at Richmond Fed, Insightful Minds in International Macro Online Seminar, and VIII Santiago Macro Workshop.

†School of Economics and Management, Tsinghua University.Email: liuqing@sem.tsinghua.edu.cn.

‡School of Economics and Management, Tsinghua University. Email: liuzanhu22@mails.tsinghua.edu.cn.

§Department of Economics, Emory University, NBER, and CEPR. Email: vivianyue1@gmail.com.

1 Introduction

International official lending, provided by multinational institutions such as the International Monetary Fund (IMF) and the World Bank, as well as through intergovernmental agreements, serves as a crucial source of external financing. Notably, international official lending tends to surge during periods of economic distress, natural disasters, or wars. Furthermore, sovereign defaults on market debt often lead to a marked increase in official lending.[1]

This paper examines international official lending in the presence of limited enforcement and informational frictions. Our focus is motivated by the increasing influence of official lending. Official lending has resurged post global financial crisis and the COVID-19 pandemic. Furthermore, China has played an important role in the provision of official loans to developing countries in recent years.[2] Like multinational institutions, China typically designs lending contracts to mitigate moral hazard. Our theory rationalizes these practices and studies the implications of official lending for risk sharing and market debt dynamics.

Using a repeated game model of international lending with production, we develop a theoretical framework to explain the role of official lending. The model incorporates two primary frictions: default risk and moral hazard in international lending. First, sovereign debt is subject to limited enforcement as countries can renege on their obligation as in Eaton and Gersovitz (1981),Arellano (2008), and Aguiar and Gopinath (2006). Second, international lending is subject to moral hazard and information frictions as highlighted in Atkeson (1991). Sovereign borrowers can engage in unobservable and undesirable activities which create a moral hazard problem.

In particular, in our model, sovereign borrowers produce domestic consumption goods and

[1] Horn et al. (2020) document the patterns and determinants of official international lending from 1790 to 2015. It finds that official lenders often step in when private flows retreat, providing a stabilizing force during crises. Arellano and Barreto (2024) finds that the official debt tends to increase during default episodes.

[2] China had become the world's largest official creditor, surpassing the World Bank and the IMF as of 2017. China's share of low-income country debt has increased from 18% in 2010 to 49% in 2021, Horn et al. (2021) study the size, terms, and destination of Chinese official international lending at the loan level.

tradable export goods using labor and intermediate goods. The economy is subject to relative productivity shocks for the export sector, modeled as a two-state Markov process. The sovereign borrows intermediate inputs for production, but the productivity and allocation of resources between domestic consumption and export production are private information. Lenders receive noisy signals about the unobservable states.

We begin by solving two planning problems: the first-best allocation and the constrained optimal allocation (COA). In the first-best scenario, the planner provides resources to achieve efficiency. Risk-neutral lenders completely insure the risk-averse sovereign borrower against productivity shocks. The realization of a productivity shock has no effect on the continuation of the allocation. However, such risk sharing is unfeasible due to the limited enforcement and moral hazard. Due to limited enforcement, the sovereign can deviate from the planner's allocations by increasing current consumption and going to autarky if the continuation value is low. Furthermore, due to moral hazard, the sovereign has an incentive to allocate more resources towards domestic consumption. A constrained planning problem takes these frictions into account. The optimal contract is subject to incentive compatibility and participation constraints. Lack of commitment interacts with the incentive problem.

We characterize the constrained optimal allocation using a promise utility approach as in Kocherlakota (1996). The planner provides dynamic incentives for the sovereign. Due to the participation constraint, the lending and intermediate goods the sovereign received is increasing in the sovereign's continuation value. When the continuation value is low, the sovereign is tempted to deviate from the efficient allocation by increasing current consumption and living in autarky thereafter. To prevent such an outcome, the planner must provide a sufficiently low amount of intermediate goods so that such deviation is unprofitable. Hence, the planner imposes a cost associated with a low repayment of export goods by reducing the continuation value of the sovereign borrower as the borrower's future consumption levels are lower. Furthermore, when both the state and the noisy signal indicate poor performance, the sovereign's continuation value is higher compared to when the signal is positive. Our

model, therefore, accommodates excusable defaults and imposes harsher penalties for repudiation occurring in the presence of good signals.[3] The solution to the constrained planner's problem is history dependent, with continuation value, consumption, and lending increasing with promised utility. Unlike Dovis (2019), the stationary distribution of the model does not support a continuation value for the sovereign close to efficiency because of the interaction between the participation constraint and the incentive compatibility constraint. The level of intermediate goods under COA is always below the level in the first-best case.

We then study the decentralization of the constrained optimal allocations. We show that the COA can be implemented as an equilibrium outcome of a model with sovereign debt. The sovereign borrower issues long-term noncontingent debt and borrows from three types of lenders with different information and seniority. First, two types of official lender can lend to the sovereign and receive noisy signals about the underlying state. Multilateral official lending is non-defaultable, which captures the seniority of such official lending. Bilateral official lending is defaultable and concessional with an exogenous recovery rate. The third type of creditors are private lenders. The sovereign can borrow from private creditors through market debt, which is defaultable and less concessional than bilateral official debt.[4] With such a seniority protocol, the three types of debt are sufficient to decentralize the COA.[5]

In a numerical exercise, we show that consumption and GDP fall when the bad shock hits. The model features an endogenous default cost due to the reduction in imported intermediate goods as in Mendoza and Yue (2012).[6] Official multilateral debt increases during defaults, while market debt decreases, consistent with empirical findings on the countercyclicality of official debt and the procyclicality of market debt. The reason is that multilateral official debt has a low interest rate and thus can provide insurance in a bad state. However, the

[3] Grossman and Huyck (1988) studies a theoretical model in which lenders differentiate excusable default from unjustifiable repudiation.

[4] Schlegl et al. (2019) find that the IMF and other multilateral creditors are the most senior. Official bilateral debt is more junior to private sovereign debt and loses more in default.

[5] The assumption about the recovery rates of the three types of debt can be microfounded using a bargaining model where the multilateral official lender has a very high bargaining power, followed by the private creditors and then the bilateral official lender. The appendix provides details of the bargaining model.

[6] The drop in trade during default is in line with the data as in Rose (2005)

market debt and bilateral official debt are defaultable and thus provide a contingency that the sovereign values more in good state. Overall, the total debt burden increases and the portfolio shifts towards the official debt in default. In the model, the default decision on the market debt is endogenous. The sovereign defaults when the sustainability constraint binds. Consistent with the data, the model can replicate the dynamics of sovereign spreads on market debt.

We also consider the economy with only one or two types of creditors. If there is no official lending or no multilateral official lending, the COA cannot be implemented. The sovereign attains less consumption smoothing and the allocation is less efficient. A special case is when no creditors receive the noisy signals about the underlying state and the sovereign's action. The welfare frontier is within the one for the baseline model. The associated COA can be implemented with two types of debt, but is less efficient. Our baseline model with the noisy signal captures a key feature of international official lending. The official creditor can better monitor the use of funds, and thus provide better insurance for the sovereign borrower. Our study therefore can explain the common practice of attaching a conditionality to official lending, as well as the close monitoring of fund usage by official lenders.[7]

Literature

Our study contributes to a substantial body of literature on sovereign debt, including works by Eaton and Gersovitz (1981), Arellano (2008), Aguiar and Gopinath (2006), and Yue (2010), which explore incomplete markets and sovereign debt dynamics. Following this line of research, several papers incorporate official lending in the quantitative analysis of sovereign debt, such as Boz (2011), Fink and Scholl (2016), Dellas and Niepelt (2016), and Kirsch and Rühmkorf (2017). These papers study the role of non-defaulting multinational

[7]Multilateral official lenders such as IMF often require borrowers to provide periodic financial and operational reports and may conduct visits to assess the progress of funded projects or programs. Some bilateral official lenders such as China also adopt a similar approach and may even impose requirements on the borrower to set up accounts under the lender's surveillance.

loans. Recently, Arellano and Barreto (2024) examines the role of long-term concessional official debt. Our work is different from all these papers, which assume that markets are incomplete and the set of assets is exogenously given. We study the best equilibrium outcome of a sovereign debt game and shed light on the role of official lending in the presence of moral hazard. In addition, we do not need to impose any exogenous default cost as in these papers. Our model is a production economy and generates endogenous default cost which is in line with Mendoza and Yue (2012).

Our model is also related to the literature using the optimal contracting approach to study sovereign debt, such as Atkeson (1991), Dovis (2019), Jie and Wang (2018), Aguiar et al. (2009).[8] In particular, this paper is related to the literature on dynamic contracting with limited commitment and informational frictions. Dovis (2019) studies the dynamic theory of sovereign debt with a lack of commitment and private information. He proposes an implementation of the constraint optimal allocation using sovereign debt of different maturities. Muller et al. (2019) study sovereign debt and structural reforms with limited enforcement and moral hazard. The COA in their model is decentralized using renegotiable GDP-linked one-period debt. Compared to these papers, we develop a dynamic theory of sovereign debt in the presence of limited commitment and moral hazard. We show that a portfolio of multilateral official lending, bilateral official lending, and private debt can implement the COA. Lastly, Abraham et al. (2024) shows the Financial Stability Fund for a union of sovereign countries is long-term contracts that are optimal in the presence of limited commitment and moral hazard problem. Our model share similar frictions, yet have different focuses.

Furthermore, our research is related to studies on official debt and China's lending. There has been an extensive discussion regarding the role of IMF. Jeanne et al. (2008) provides a detailed account of the history of IMF lending and presents a theoretical model to examine the role of the IMF, the rationale for conditionality, and the conditions under which IMF-

[8]In addition, several papers in the literature builds theoretical models of sovereign debt with asymmetric information. See Sandleris (2008).

induced moral hazard can arise. Our paper shares the view of the IMF as "delegated monitor" as in Tirole (2002) and treats the official debt as part of the risk-sharing arrangement as in Chami et al. (2004). In our model, official lenders can observe a noisy signal about the sovereign borrower's state. More recent empirical studies provide a comprehensive analysis of multilateral and bilateral official debt. Schlegl et al. (2019) examines the different seniority of official debt. Horn et al. (2020) find that the official lending is countercyclical. Lastly, Horn et al. (2021) provides a detailed account of the features of the loan from China's government. Kondo et al. (2022) particularly focus on the impact of Chinese loans on private market debt.

Lastly, our paper draws on the literature on endogenous incomplete markets due to limited enforcement, such as Alverez and Jermann (2000), Kehoe and Perri (2004), Albuquerque and Hopenhayn (2004). Our analysis shows that the interaction of limited enforcement and moral hazard determines the dynamic incentive provision. The COA can be decentralized using official lending. We also conduct welfare analysis to provide policy insights related to international official lending.

The remainder of the paper is organized as follows. Section 2 describes the model environment. Section 3 characterizes the first-best and constrained optimal allocation. Section 4 provides the main result of decentralization. Section 5 presents the implementation of the COA with official lending. Section 6 contains a quantiative analysis of the model economy. Section 7 concludes.

2 Model Environment

In this section, we present the model environment of a small open production economy.

Time is discrete and indexed by $t = 0, 1, ...$ There are three types of agents in the economy: a representative risk-averse domestic household, a benevolent sovereign government, and a large number of risk-neutral foreign lenders. The benevolent sovereign government makes decisions on behalf of the domestic representative household, and we will call domes-

tic agents "the borrower" when it causes no confusion. In addition, there are three types of goods, as in Dovis (2019): a domestic consumption good (non-tradable), an export good, and an intermediate good. The borrower uses intermediate goods as input for the production of consumption and export goods. However, the borrower can privately allocate imported intermediate inputs and labor input to the non-tradable sector, which creates a moral hazard problem as in Atkeson (1991) and Muller et al. (2019). To monitor the use of intermediate goods, foreign lenders can receive a noisy signal about the borrower's allocation decisions.

2.1 Preferences

All agents are infinitely-lived, with the same discount factor, $\beta < 1$. The domestic household values a stochastic sequence of consumption of the domestic good, $\{c_t\}_{t=0}^{\infty}$, according to

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t\right),$$

where the period utility function $U(c)$ has constant relative risk aversion, $U(c) = \dfrac{c^{1-\gamma}}{1-\gamma}$, with $\gamma > 0$.

Risk-neutral foreign lenders value their consumption of the export good, $\{x_t\}_{t=0}^{\infty}$, according to

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t x_t.$$

2.2 Endowments and technology

Foreign lenders have a large endowment of export good, which can be transformed into intermediate good in a one-to-one ratio[9]. In period t, the supply of intermediate goods to domestic production is m_t, and the representative household is endowed with one unit of labor. A domestic production technology transforms the intermediate good and labor into

[9]This is to ensure that the relative price between the export and the intermediate good is fixed at one in the international market.

the domestic consumption good, c_t, and export good, x_t, as follows:

$$c_t \leq F\left(m_{c,t}, l_{c,t}\right), \quad \text{and} \quad x_t \leq A_t F\left(m_{x,t}, l_{x,t}\right), \tag{2.1}$$

$$m_{c,t} + m_{x,t} \leq m_t, \quad l_{c,t} + l_{x,t} \leq 1, \tag{2.2}$$

where $m_{c,t}$ and $m_{x,t}$ are the units of the intermediate good allocated to the production of the consumption and export good, and $l_{c,t}$ and $l_{x,t}$ are the units of domestic labor allocated to consumption and export production, respectively.

The production function F has constant return to scale; it is increasing and twice continuously differentiable, with $F_m(m,l) > 0$ and $F_{mm}(m,l) < 0$; and it satisfies the Inada condition $\lim_{m\to 0} F_m(m,l) = +\infty, \forall l > 0$. Finally, $F(0,1) > 0$, so strictly positive output can be produced in autarky (without imported intermediate goods). For notational simplicity, we denote $f(m) \equiv F(m,1)$.

Due to the properties of the production function, (2.1)-(2.2) can be summarized by the following aggregate resource constraint

$$x_t = A_t F\left(m_{x,t}, l_{x,t}\right) = A_t l_{x,t} f\left(m_t\right) = A_t\left(f\left(m_t\right) - l_{c,t} f\left(m_t\right)\right) = A_t\left(f\left(m_t\right) - c_t\right), \tag{RC}$$

where c_t and x_t are nonnegative. This is because the optimal allocation of the intermediate good and labor requires that $m_{c,t}/l_{c,t} = m_{x,t}/l_{x,t} = m_t$. Hence, we can assume that the borrower directly chooses domestic consumption c_t, and x_t is produced according to (RC). The value of c_t fully characterizes the borrower's allocation decision.

The relative productivity of the export sector, A_t, is independently and identically distributed over time. As in Dovis (2019), we let A_t take only two values, $A_t \in \{A_L, A_H\}$ with $0 < A_L < A_H$. Denote $p = Pr\left(A_t = A_H\right)$. We normalize the expected relative productivity $\mathbf{E}(A_t)$ to one.

2.3 Information and the noisy signal

The supply of m_t and the export x_t are public information. However, the productivity shock A_t is unobservable for lenders. This indicates that lenders cannot infer the value of c_t from export x_t. The borrower privately decides the allocation of inputs between the consumption and export sectors before the realization of A_t. That is, c_t is privately decided by the borrower and cannot be conditional on A_t. This creates a moral hazard problem: The borrower is always willing to serve domestic needs (c_t) at the expense of the export sector (x_t).

To mitigate the moral hazard problem, there exists a public noisy signal $s_t \in \{L, H\}$ in each period after the realization of A_t. This signal is informative about the productivity shock A_t, with $Pr(s_t = L|A_t = A_L) = Pr(s_t = H|A_t = A_H) = a \in (0.5, 1)$, where a refers to the accuracy of the signal. Hence, according to (RC), foreign lenders can form a posterior belief about c_t using their information about m_t, x_t and s_t. The introduction of a noisy signal to provide additional information is standard in the literature, as in Holmström (1979). We apply this setting to study the monitoring role of official lenders, which will be made clear in Section 5.

It is worth mentioning two extreme cases. The first case is the uninformative case where $a = 0.5$. In this circumstance, the noisy signal does not reveal additional information on the realization of A_t, and is therefore irrelevant. The second case is the complete information case, where $a = 1$. In this circumstance, foreign lenders know A_t for sure and can therefore infer the value of c_t correctly. Hence, there will be no moral hazard problem.

2.4 Timing

The timing of events within period t is as follows:

1. Foreign lenders supply intermediate goods $m_t \geq 0$.
2. The borrower chooses c_t.
3. The productivity shock A_t is realized (and x_t is realized as $A_t(f(m_t) - c_t)$).

4. The noisy signal s_t is realized.

Define $h^{-1} = \emptyset$ and $h^t = h^{t-1} \cup \{A_t, s_t\}$ for $t \geq 0$. An allocation is a stochastic process

$$\sigma \equiv \left\{ m\left(h^{t-1}\right), c\left(h^{t-1}\right) \right\}_{t=0}^{+\infty},$$

which describes the dynamics of imports and domestic consumption. According to (RC), exports under the allocation are given by

$$x\left(A_t, h^{t-1}\right) = A_t \left[f\left(m\left(h^{t-1}\right)\right) - c\left(h^{t-1}\right) \right].$$

3 Planning Allocation

In this section, we characterize the constrained optimal allocation as the solution to the social planner's problem, which maximizes the lenders' expected profits subject to incentive constraint and limited enforcement constraints. First, define the continuation utility for the borrower and lenders associated with an allocation σ after h^t as

$$v\left(\sigma, h^t\right) = \sum_{j=1}^{\infty} \sum_{h^t \subseteq h^{t+j}} \beta^{j-1} Pr\left(h^{t+j} | h^t\right) U\left(c\left(h^{t+j-1}\right)\right)$$

and

$$L\left(\sigma, h^t\right) = \sum_{j=1}^{\infty} \sum_{h^t \subseteq h^{t+j}} \beta^{j-1} Pr\left(h^{t+j} | h^t\right) \left[x\left(A_{t+j}, h^{t+j-1}\right) - m\left(h^{t+j-1}\right) \right],$$

respectively. σ will be omitted whenever it causes no confusion. The planning problem has a recursive formulation, where the borrower's continuation utility is the state variable, as in Abreu et al. (1990). We then show how the moral-hazard problem creates inefficiency in the sovereign debt market and how additional information can be used to mitigate the problem.

3.1 Incentive constraints

Consider the restriction imposed by the fact that c_t is observed privately by the borrower. This generates the problem of moral hazard. If the borrower chooses to consume $c_t' = (1 - A_L/A_H) f(m_t) + A_L/A_H c_t \geq c_t$, then $x_t = A_L(f(m_t) - c_t)$ when $A_t = A_H$. Hence, when $x_t = A_L(f(m_t) - c_t)$, foreign lenders cannot tell if there is a low productivity shock or if the borrower deviates and chooses to consume more. Therefore, to incentivize the borrower to produce export goods, the promised continuation utility to the borrower should be lower if x_t is low. This serves as a punishment to the borrower. At the same time, foreign lenders can rely on the noisy signal s_t to monitor the behaviors of the borrower: if $s_t = L$, it is more likely that there is a bad shock; if $s_t = H$, it is more likely that the borrower deviates. Hence, the promised continuation utility to the borrower should differ if the noisy signal differs. The borrower is not expected to be punished harshly if low exports are "excusable": that is, x_t is low, but $s_t = L$. The associated incentive compatibility constraint for all t and h^t is:

$$\begin{aligned} & U\left(c\left(h^{t-1}\right)\right) + \beta p\left[av\left(h^{t-1}, A_H, H\right) + (1-a)v\left(h^{t-1}, A_H, L\right)\right] \\ & + \beta(1-p)\left[(1-a)v\left(h^{t-1}, A_L, H\right) + av\left(h^{t-1}, A_L, L\right)\right] \\ & \geq U\left(\left(1 - \frac{A_L}{A_H}\right) f\left(m\left(h^{t-1}\right)\right) + \frac{A_L}{A_H} c\left(h^{t-1}\right)\right) \\ & + \beta p\left[av\left(h^{t-1}, A_L, H\right) + (1-a)v\left(h^{t-1}, A_L, L\right)\right] + \beta(1-p)v_a, \end{aligned} \tag{IC}$$

where v_a is the value of permanent autarky given by

$$v_a = \frac{U(f(0))}{1-\beta}.$$

According to the RHS of (IC), if the borrower deviates and chooses to consume c_t', then this deviation is undetected if $A_t = A_H$ (with probability p). The promised utility will be the same as if $A_t = A_L$ and the borrower does not deviate (that is, $v(h^{t-1}, A_L, H)$ and $v(h^{t-1}, A_L, L)$) and only depends on the realization of the signal s_t.

However, if the borrower chooses to consume c_t' and it turns out that $A_t = A_L$, then the

value of x_t will reveal that the borrower deviates. It is without loss of generality to assume that detectable deviations are punished by the worst equilibrium payoff v_a regardless the realization of the noisy signal. The incentive compatibility (IC) requires that the utility under deviation is not larger than the utility without deviation.

Second, the restriction imposed by the lack of commitment of the borrower is captured by the sustainability constraint. The borrower can renege on the relationship, choose not to produce export goods and stay in autarky starting from the next period. The associated sustainability constraint for all t and h^t is:

$$\begin{aligned} & U\left(c\left(h^{t-1}\right)\right)+\beta p\left[av\left(h^{t-1},A_H,H\right)+(1-a)v\left(h^{t-1},A_H,L\right)\right] \\ & +\beta(1-p)\left[(1-a)v\left(h^{t-1},A_L,H\right)+av\left(h^{t-1},A_L,L\right)\right] \\ & \geq U\left(f\left(m\left(h^{t-1}\right)\right)\right)+\beta v_a. \end{aligned} \tag{SUST}$$

Lastly, lenders can always choose not to lend. Hence, the continuation utility for lenders is no smaller than 0 since lenders can choose to leave the international financial market at any time. The associated participation constraint for all t and h^t is:

$$L\left(h^t\right)\geq 0. \tag{PC}$$

Definition 1 An allocation σ is implementable if and only if it satisfies (IC), (SUST), and (PC).

3.2 Recursive formulation

Define $\Sigma=\{\sigma|\sigma$ as an implementable allocation$\}$.[10] Abusing notation, we define $L(\sigma)$ and $v(\sigma)$ the continuation utility of lenders and the borrower evaluated at h^{-1}, respectively. Define $\overline{v}=sup\{v(\sigma)|\sigma\in\Sigma\}$. We assume that $\overline{v}>v_a$. As in Muller et al. (2019) and Dovis

[10] Σ is nonempty as the autarkic allocation with $m\left(h^{t-1}\right)=0$, $c\left(h^{t-1}\right)=f(0)$ and $x\left(A_t,h^{t-1}\right)=0$ is always implementable.

(2019), we focus on the constrained optimal allocation (COA) which is implementable and maximizes the continuation utility of lenders given the continuation utility of the borrower.[11] A COA is therefore a solution to the following program, where $v \in [v_a, \overline{v}]$ is the period 0 utility promised to the borrower:

$$L(v) = \max_{\sigma \in \Sigma} L(\sigma), \quad s.t. \quad v(\sigma) = v. \tag{L}$$

Following Abreu et al. (1990), the problem in (L) admits a recursive formulation using the borrower's promised utility, v, as a state variable. Let $v^{i,j}$ $(i, j \in \{L, H\})$ represent the promised continuation utility to the borrower, where i refers to the realization of the productivity shock A_t, and j refers to the realization of the noisy signal s_t. A COA solves the following recursive problem for $v \in [v_a, \overline{v}]$:

$$\begin{aligned} L(v) = \max_{m, c, (v^{i,j})_{i,j=L,H}} & f(m) - c - m + \beta p \left[aL\left(v^{H,H}\right) + (1-a)L\left(v^{H,L}\right) \right] \\ & + \beta(1-p)\left[(1-a)L\left(v^{L,H}\right) + aL\left(v^{L,L}\right) \right], \end{aligned} \tag{3.1}$$

subject to

$$\begin{aligned} v = U(c) + \beta p \left[a v^{H,H} + (1-a) v^{H,L} \right] \\ + \beta(1-p)\left[(1-a) v^{L,H} + a v^{L,L} \right], \end{aligned} \tag{3.2}$$

$$v \geq U\left(\left(1 - \frac{A_L}{A_H}\right) f(m) + \frac{A_L}{A_H} c \right) + \beta p \left[a v^{L,H} + (1-a) v^{L,L} \right] + \beta(1-p) v_a, \tag{3.3}$$

$$v \geq U(f(m)) + \beta v_a, \tag{3.4}$$

$$L(v) \geq 0 \quad \forall v \in [v_a, \overline{v}], \tag{3.5}$$

$$v^{i,j} \in [v_a, \overline{v}]. \tag{3.6}$$

Equation (3.2) is the constraint that the recursive allocation delivers a value of v to the borrower (the promise-keeping constraint). Inequalities (3.3), (3.4) and (3.5) are recursive

[11] There exist multiple implementable allocations.

versions of the incentive compatibility constraint (IC), the sustainability constraint (SUST) and the participation constraint (PC), respectively. The promised continuation utility must be generated by an implementable allocation, which gives (3.6). The function $L(v)$ is the largest continuation utility that lenders can achieve given that the continuation utility of the borrower is v. Hence, $L(v)$ trace the frontier of utility possibilities.

The solution to the above recursive problem is $\left\{m(v), c(v), \left(v^{i,j}(v)\right)_{i,j=L,H}\right\}$.

3.3 First-best allocation

We start by characterizing the first-best allocation given by the program (3.1)-(3.2). The result is summarized in the following proposition. The proof (and all the following proofs) can be found in the online Appendix.

Proposition 1 (First-Best Allocation) Under (3.2), the solution to (3.1) satisfies:

(i) $m_{FB}(v) = m^*$, where m^* is given by $f'(m^*) = 1$.

(ii) $c_{FB}(v)$ satisfies $U(c(v)) = (1-\beta)v$.

(iii) $v_{FB}^{i,j}(v) = v$ $(i, j = L, H)$.

The first-best allocation yields production efficiency and full insurance: the supply of intermediate goods m reaches the efficient level m^*, and the borrower enjoys constant consumption irrespective of the productivity shock.

Introducing additional constraints, the sustainability constraint (3.4) and the participation constraint (3.5), will not change the main implications of Proposition 1. In sum, limited enforcement alone does not necessarily give rise to undesirable results. This highlights the importance of moral hazard and the incentive compatibility constraint (3.3) in generating distortions in allocations.

3.4 Constrained optimal allocation (COA)

Next, we characterize the COA. The planning problem in (3.1)-(3.2) is subject to the IC (3.3), the SUST (3.4), the PC (3.5) and the feasibility constraint (3.6). To establish our results, we first make the following assumptions:

Assumption 1 $\dfrac{f'(m)-1}{U'(f(m))f'(m)}$ decreases in m.

Assumption 2 p is sufficiently large.

Assumption 1 is to ensure that the problem is convex so that randomization is not optimal[12]. Assumption 2 is to ensure that the incentive compatibility constraint is always binding, so that our results can differ from the first-best allocation. When p is large, it is less likely that a deviation is detected, and the borrower is more willing to deviate. The detailed threshold for p in Assumption 2 is given in the online appendix. In Section 5 we will show that Assumption 2 is satisfied for realistic parameters.

The COA satisfies the following properties:

Proposition 2 (Constrained Optimal Allocation) If $a \in (0.5, 1)$, under (3.2)-(3.6), the solution to (3.1) satisfies:

(i) $m_{COA}(v) < m^* \quad \forall v \in [v_a, \overline{v}]$.

(ii) $m_{COA}(v)$ is strictly increasing in v with $m_{COA}(v_a) = 0$.

(iii) $c_{COA}(v)$ is strictly increasing in v with $0 < c_{COA}(v) < f\left(m_{COA}(v)\right)$.

(iv) $v^{H,H}_{COA}(v) = v^{H,L}_{COA}(v) > v^{L,L}_{COA}(v) > v^{L,H}_{COA}(v) > v_a, \quad \forall v \in [v_a, \overline{v}]$.

Part (i) explains that the supply of intermediate goods is inevitable inefficient due to the

[12] Dovis (2019) uses a similar assumption.

moral hazard problem. To prevent borrowers from misusing resources, imports are limited because the temptation to consume more increases with more intermediate goods (the gain in consumption from deviation is proportional to $f(m) - c$, which increases in m). Parts (ii) and (iii) show that imports and domestic consumption increase as the borrower's promised future utility improves. Part (iv) highlights the dynamic nature of the constrained optimal allocation (COA): high exports lead to higher continuation utility, encouraging more imports and consumption, while low exports reduce future utility and lead to harsher consequences. If low exports are "excusable" likely due to external shocks (indicated by a signal $s_t = L$), lenders are more lenient, allowing better risk-sharing. However, if the signal $s_t = H$ suggests borrower misuse, stricter penalties are imposed to deter bad behavior. Notably, $v^{i,j}_{COA}(v) > v_a$ regardless of v. COA avoids complete autarky, even temporarily, justifying "rescue lending" during crises when borrowers face exclusion from markets. This feature of the COA is a key reason why the implementation involves the multilateral official lending as shown in the next section.

To clarify the role of the noisy signal, two extreme scenarios are considered. With complete information ($a = 1$), all deviations are detected and penalized, eliminating the moral hazard problem and enabling efficient outcomes. The first-best allocation is achieved. On the other hand, with uninformative signals ($a = 0.5$), decisions cannot rely on the signal, $v^{L,H} = v^{L,L}$. The uninformative case is Pareto-inferior to the COA. This aligns with the informativeness principle, which states that incorporating informative signals into the optimal contract improves welfare. (Holmström, 1979).

The above results are summarized in the following figures (Figure 3.1). The figure in the upper left trace the frontier of utility possibilities $L(v)$. The figure in the upper right shows the policy function for intermediate imports, $m_{COA}(v)$. The figure in the bottom left shows the policy function for domestic consumption, $c_{COA}(v)$. The figure in the bottom right depicts the law of motion for the borrower's value, $v^{i,j}_{COA}(v)$.

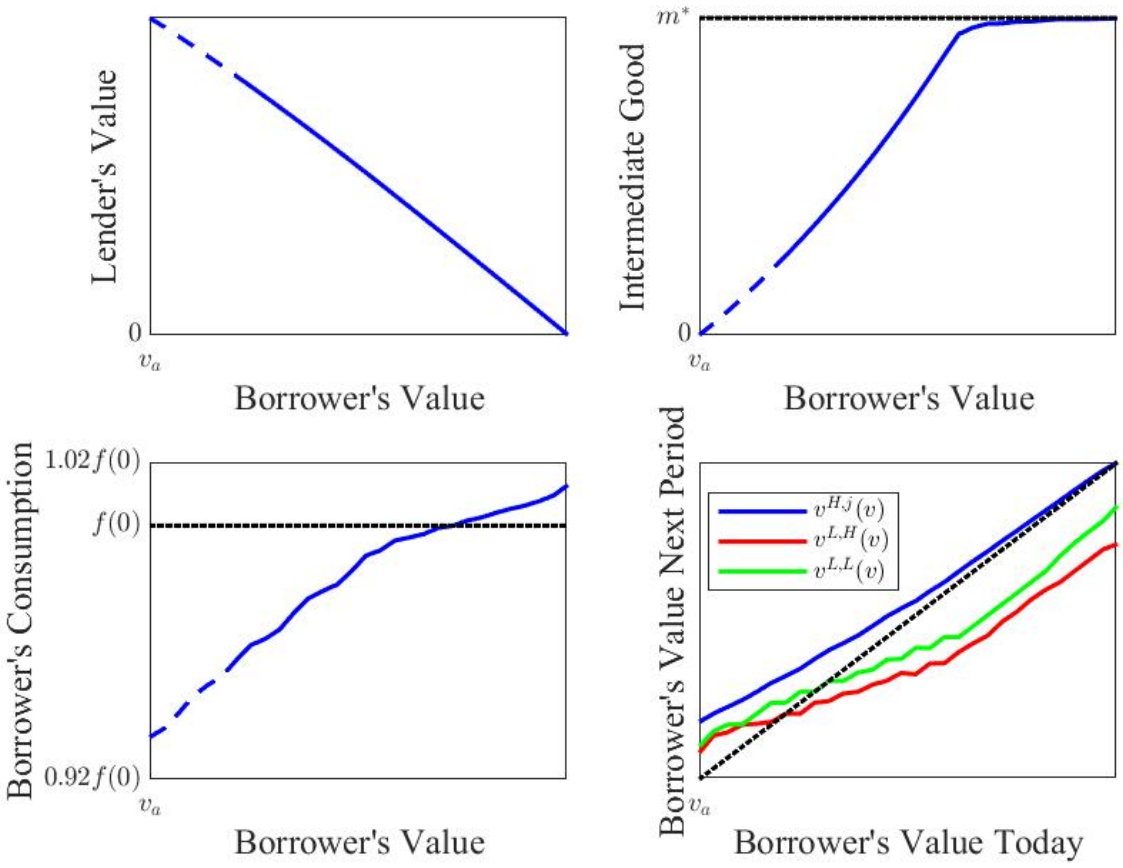

Figure 3.1: The dynamics of the COA.

4 Decentralization

In this section, we describe how the COA can be decentralized in a sustainable market equilibrium. In the market equilibrium, the sovereign government borrows internationally to import intermediate goods and buys exports goods to service debt repayment. In the domestic market, the government collects lump sum tax to finance its budget and to direct the allocation of inputs (intermediate goods and labor) between consumption and export sectors. To implement the COA, the government borrows from two types of official lenders and market lenders. The government actively manages its debt portfolio to maximize domestic welfare, taking the price of debt as given. The government can also default on its market debt obligations, and the interest spread reflects the probability of default. The key distinction between official debt and market debt is that market debt is subject to the government's default risk, whereas official debt is non-defaultable (though official lenders can choose to provide debt reductions). We will illustrate how the combination of different types of lenders helps implement the COA.

4.1 The domestic market

In the domestic market, competitive domestic firms purchase intermediate inputs from the government (which imports intermediate goods from the international market) and hire labor to produce consumption and export goods.[13]

The profit-maximizing problems for firms in the consumption and export sectors are

$$\max_{m_{c,t},l_{c,t}} p_{c,t}F\left(m_{c,t},l_{c,t}\right)-p_{i,t}m_{c,t}-w_t l_{c,t}, \tag{4.1}$$

and

$$\max_{m_{x,t},l_{x,t}} A_t F\left(m_{x,t},l_{x,t}\right)-p_{i,t}m_{x,t}-w_t l_{x,t}, \tag{4.2}$$

[13]We implicitly assume a trade barrier between the domestic and the international market. The government is the only domestic agent which can access both markets.

where $p_{c,t}$, $p_{i,t}$, and w_t are the relative price of consumption good, the relative price of intermediate good, and the competitive wage for labor, respectively. Export good is assumed to be the numeraire.

The government collects lump-sum tax on labor to achieve the desired allocation of inputs (intermediate goods and labor) between consumption and export sectors.[14] As there is no saving decision in our setting and the benevolent government is the only buyer of export good in the domestic market, the market clear condition for consumption and export goods are

$$w_t - p_{c,t}\tau_t = p_{c,t}F\left(m_{c,t}, l_{c,t}\right), \tag{4.3}$$

and

$$p_{i,t}m_t + p_{c,t}\tau_t = A_t F\left(m_{x,t}, l_{x,t}\right), \tag{4.4}$$

where τ_t is the lump-sum tax on labor. Given the tax, the consumption of the representative household is $c_t = \left(w_t - p_{c,t}\tau_t\right)/p_{c,t}$, and the export is $x_t = A_t F\left(m_{2,t}, l_{2,t}\right)$. Finally, markets for intermediate inputs and labor clear.

$$m_{c,t} + m_{x,t} = m_t, \quad l_{c,t} + l_{x,t} = 1. \tag{4.5}$$

Based on (4.1)-(4.5), we can solve for the relative prices and the output of each sector. The results are summarized in the following lemma.

Lemma 1 (Domestic Market)

(i) Prices: $p_{c,t} = A_t$, $p_{i,t} = A_t f'\left(m_t\right)$.

(ii) Domestic consumption: $c_t = f\left(m_t\right) - m_t f'\left(m_t\right) - \tau_t$.

(iii) Exports: $x_t = A_t\left(f\left(m_t\right) - c_t\right)$.

[14] We assume that the tax cannot be observed by foreign lenders. Thus there still exists an information asymmetry between lenders and the borrower.

Part (i) states the relative prices of the consumption good and the intermediate good in the domestic market. The relative price of the consumption good represents the real exchange rate. As $p_{c,t} = A_t$, real exchange rate depreciates after a negative productivity shock. Parts (ii) and (iii) derive domestic consumption c_t and export x_t. Given m_t, c_t is uniquely determined by the lump sum tax τ_t, and x_t is determined by (RC). This suggests that the government's tax (or subsidy) on domestic consumption affects the allocation of resources between the consumption and export sectors.

In sum, given the government's tax policy and the imports of the economy, the allocation of intermediate goods and labor between the consumption and export sectors is determined by the competitive market.

4.2 The international market

In the international market, there are two types of official lenders (official bilateral lenders and official multilateral lenders) and market lenders. We denote different types of debt as d_t^i, their prices as q_t^i, and the repayment decisions as X_t^i ($i \in \{M, OB, OM\}$).

4.2.1 Market lenders

The government can issue long-term market debt in period t that promises an infinite and decaying stream of coupons at a constant rate $\delta^M \in [0, 1]$. In particular, the coupon payment on the market debt issued at t is $\left(1 - \delta^M\right)^{k-1}$ units of the export good in period $t + k$, with $k \geq 1$, following Hatchondo and Martinez (2009). δ^M is the parameter that determines the duration of the long-term market debt. Market debt cannot be used for savings, so $d_t^M \geq 0$ in any period t. The borrower can default on the market debt. By defaulting, the government does not pay the coupon due in that period.[15] The price of the market debt price reflects the probability of future default. Private lenders of market debt are assumed to be competitive

[15]This assumption is without loss of generality. The present value of the market debt is lower in default which implies a debt reduction.

and risk-neutral. Therefore the price of the market debt, q_t^M, is given by

$$q_t^M = \beta \mathbf{E}_t \left(X_{t+1}^M + \left(1 - \delta^M\right) q_{t+1}^M \right), \tag{PR-M}$$

where $X_{t+1}^M = 0$ if the borrower defaults at period $t+1$, and $X_{t+1}^M = 1$ if the borrower does not default. It important to note that it is the borrower who makes the default decision X_t^M, consistent with the "willingness to pay" problem in the study of sovereign debt on market debt.

4.2.2 Official lenders

There are two types of official lenders: official multilateral lenders and official bilateral lenders. Similarly to market debt, official debts are long-term debts with duration $\delta^{OM} \in [0,1]$ for official multilateral debt and duration $\delta^{OB} \in [0,1]$ for official bilateral debt. The liquidity provided by official multilateral lenders is risk-free: the borrower simply cannot default on official multilateral debt. Hence, $X_t^{OM} = 1$, and the pricing rule of the official multilateral debt is given by

$$q_t^{OM} = \beta \mathbf{E}_t \left(X_{t+1}^{OM} + \left(1 - \delta^{OM}\right) q_{t+1}^{OM} \right) = \frac{\beta}{1 - (1 - \delta^{OM})\,\beta}. \tag{PR-OM}$$

This price is constant and hence denoted as q^{OM}. On the other hand, official bilateral lenders offer a debt reduction of $1 - r_{OB}$ ($r_{OB} \in [0,1)$) to the borrower if the low export is "excusable". That is, a debt reduction is granted if the noisy signal $s_t = L$ and the export x_t are lower than expected. Hence, $X_t^{OB} = r_{OB}$ if low export is "excusable", and $X_t^{OB} = 1$ otherwise. The pricing rule of the official bilateral debt is given by

$$q_t^{OB} = \beta \mathbf{E}_t \left(X_{t+1}^{OB} \left[1 + \left(1 - \delta^{OB}\right) q_{t+1}^{OB} \right] \right) = \frac{\beta \left[1 - (1-p) a (1 - r_{OB}) \right]}{1 - (1 - \delta^{OB})\,\beta \left[1 - (1-p) a (1 - r_{OB}) \right]}. \tag{PR-OB}$$

This price is also time invariant. We denote it by q^{OB} whenever it causes no confusion. Similarly to market debt, official bilateral lenders do not borrow from the sovereign government, $d_t^{OB} \geq 0$. However, the official multilateral debt can be negative, which is the case if the sovereign government accumulates foreign reserves, so $d_t^{OM} \in \mathbf{R}$.

4.3 The government

The borrowing government is benevolent and maximizes the utility of the representative household. We assume that the imports of intermediate goods are subject to the working capital constraint. To import m_{t+1} in period $t+1$, the government is required to make the payment βm_{t+1} at period t.[16] According to Lemma 1, the government's revenue equals $A_t\left(f\left(m_t\right)-c_t\right)$, and the budget constraint of the government is given by

$$\begin{aligned} & A_t\left(f\left(m_t\right)-c_t\right)+q_t^M\left[d_{t+1}^M-\left(1-\delta^M\right)d_t^M\right] \\ & +q_t^{OB}\left[d_{t+1}^{OB}-\left(1-\delta^{OB}\right)X_t^{OB}d_t^{OB}\right]+q_t^M\left[d_{t+1}^{OM}-\left(1-\delta^{OM}\right)d_t^{OM}\right] \\ & =\sum_{i\in\{M,OB,OM\}} X_t^i d_t^i+\beta m_{t+1}. \end{aligned} \tag{BC}$$

4.4 Timing

The sequence of events within each period t is as follows:

1. The government imports intermediate goods, m_t, and sells them to domestic firms at the price $p_{i,t}$.
2. The government sets the lump-sum tax rate τ_t.
3. The productivity shock A_t is realized and the production of c_t and x_t occurs.
4. The noisy signal s_t is realized.

[16] Consistent with Mendoza and Yue (2012), the interest rate on the working capital credit is risk free rate.

5. The government chooses the default decision X_t^M, borrowing decisions d_{t+1}^i, and next period's imports m_{t+1}.

4.5 The sustainable equilibrium

The government's decision on default, borrowing and import decision is summarized as the policy $\pi_t \equiv \left(X_t^M, \left(d_{t+1}^i\right)_{i\in\{M,OB,OM\}}, m_{t+1}\right)$. The government also makes the tax decision, τ_t.

Let $h_p^{-1} = \left(\left(d_0^i\right)_{i\in\{M,OB,OM\}}, m_0\right)$ be the initial outstanding debt and imports. The public history up to period $t \geq 0$ is $h_p^t = \left(h_p^{t-1}, x_t, s_t, \pi_t\right)$. We further define the history when the government decides its policy π_t as $h_s^t = \left(h_p^{t-1}, x_t, s_t\right)$. The strategy for the government and the debt prices are all functions of the relevant histories. Specifically, the tax decision, τ_t, is a function of public history h_p^{t-1}; the default, borrowing and import decision, π_t, is a function of the history h_s^t, because π_t is made after the realization of production x_t and the noisy signal s_t.

The problem for the government is to maximize the consumption of the representative household, taken as given the price schedule for debts, $(q_t^i)_{i\in\{M,OB,OM\}}$, after any history h_p^t. The strategy for the government solves the government's problem

$$W\left(h_p^{t-1}\right) = \max_{\tau_t, \pi_t(x_t,s_t)} U\left(c_t\right) + \beta \mathbf{E}\left[W\left(h_p^t\right) | h_p^{t-1}, \pi_t\left(x_t, s_t\right)\right] \tag{G}$$

subject to $d_{t+1}^i < \overline{D}$ and the budget constraint for the government (BC). Borrowings are bounded by a large positive constant $\overline{D}$ to rule out a Ponzi scheme. Domestic consumption c_t is a function of m_t and τ_t based on Lemma 1.

Now we can define the sustainable equilibrium. Given initial outstanding debt and imports, a sustainable equilibrium is a strategy for the government, $\left(\tau_t\left(h_p^{t-1}\right), \pi_t\left(h_s^t\right)\right)_{t=0}^{\infty}$ and prices for the government debt, $(q_t^i)_{i\in\{M,OB,OM\}}$, such that (i) given debt prices $(q_t^i)_{i\in\{M,OB,OM\}}$, the government's strategy is the optimal policy associated with (G) for all histories h_p^{t-1} and

h_s^t; (ii) given the government's strategy, the debt prices $(q_t^i)_{i\in\{M,OB,OM\}}$ satisfy (PR-M), (PR-OB), and (PR-OM).

The associated equilibrium result is denoted by $\mathbf{y} = (\sigma, \mathbf{g}, \mathbf{q})$, where σ is the equilibrium allocation of m, c, and x, $\mathbf{g}$ is the government policy τ and π, and $\mathbf{q}$ is the bond prices. Given the government policy $\mathbf{g}$, the equilibrium allocation σ is a stochastic process in terms of h^t, as defined in Section 2.4.

We characterize the sustainable equilibrium outcome $\mathbf{y}$ in the following lemma.

Lemma 2 (Sustainable Equilibrium) Given the initial history h_p^{-1}, $\mathbf{y}$ is a sustainable equilibrium outcome if and only if:

(i) The associated equilibrium allocation σ is an implementable allocation.

(ii) The associated government policy g satisfies the budget constraint (BC).

(iii) The debt prices q satisfy (PR-M), (PR-OB), and (PR-OM).

There may exist multiple equilibrium. We focus only on the efficient equilibrium. Lemma 2 allows for the characterization of the efficient equilibrium based on the results in Section 3: Lemma 2 shows that the associated allocation of a sustainable equilibrium is an implementable allocation with the budget constraint of the government. If the budget constraint does not impose an additional constraint on the problem, the COA can be achieved under efficient sustainable equilibrium. We will show next that the structure of the international financial market and the set of debt instruments available is crucial to achieve the constrained efficiency because the government's budget relies heavily on external financing opportunities.

5 Implementation with Official Lenders

In this section, we will illustrate how the combination of market lenders and official lenders helps to implement the COA under a sustainable equilibrium. We construct a sustainable

equilibrium that replicates the COA, using the continuation utility of the government, v, as the state variable. As the COA is implementable, Lemma 2 suggests that we only need to verify whether or not the government budget constraint is satisfied.

5.1 Dynamics of domestic economy

We first construct the allocations that determine the dynamics of domestic consumption and the allocation of imported intermediate goods. First, we need to determine the continuation utility. Given the continuation utility today, v, the continuation utility of the next period, v', is given by

$$v' = \begin{cases} v_{COA}^{H,H}(v), & A = A_H, s = H; \\ v_{COA}^{H,L}(v), & A = A_H, s = L; \\ v_{COA}^{L,H}(v), & A = A_L, s = H; \\ v_{COA}^{L,L}(v), & A = A_L, s = L. \end{cases} \tag{5.1}$$

Now let the imports of the government be given by

$$m(v) = m_{COA}(v). \tag{5.2}$$

Set the tax of the government as

$$\tau(v) = f\left(m_{COA}(v)\right) - m_{COA}(v) f'\left(m_{COA}(v)\right) - c_{COA}(v). \tag{5.3}$$

Under this import and tax policy, it is easy to verify that today's domestic consumption $c(v)$ is given by

$$c(v) = c_{COA}(v). \tag{5.4}$$

Hence, the dynamics of consumption and allocations replicate the COA.

5.2 Dynamics of repayment and debts

We now characterize the dynamics of debt and default. We have shown in Section 4.4 that the government's default decision $X^M(v, A, s)$ is chosen after the shock A and the signal s are realized. The government chooses to default, $X^M(v, A, s) = 0$, if and only if the (SUST) constraint is binding in the next period. That is,

$$v' = U(f(m(v'))) + \beta v_a,$$

. This default setting can be rationalized by the following protocol: if the government defaults today, there is a positive probability that the government is excluded from the international financial market tomorrow. Hence, the government will default only when the (SUST) is binding. As (SUST) is more likely to be binding when v' is small, default is more likely to occur after the realization of a negative productivity shock since $v' = v_{COA}^{L,j}(v)$ is lower.

Given the default decision of the government, the price for market debt is derived from (PR-M):

$$q^M(v) = \beta \mathbf{E}\left(X^M(v, A, s) + \left(1 - \delta^M\right) q^M\left(v'\right)\right). \tag{5.5}$$

Hence, the price of the debt reflects the probability of default.

Lemma 3 (Debt Price) For any continuation utility $v \in [v_a, \overline{v}]$, price of market debt $q^M(v) < \beta / \left[1 - \left(1 - \delta^M\right)\beta\right]$ and is increasing in v.

Lemma 3 implies that the interest rate of market debt is strictly higher than the risk-free rate. Moreover, the spread of market debt increases after a negative shock due to the higher probabilities of future default.

Next, the following lemma about the budget constraint of the government is key to characterize the debt portfolio that can implement COA.

Lemma 4 (Budget Constraint) The budget constraint of the government (BC) is satisfied if and only if the government's debt holding $d^i(v)$ ($i \in \{M, OB, OM\}$) is such that:

$$\begin{aligned}
&\left[X_t^M + \left(1-\delta^M\right) q_t^M\right] d_t^M + X_t^{OB}\left[1 + \left(1-\delta^{OB}\right) q^{OB}\right] d_t^{OB} + \left[X_t^{OM} + \left(1-\delta^{OM}\right) q^{OM}\right] d_t^{OM} \\
=&\mathbf{E}_t\left[\sum_{j=0} \beta^j \left(A_{t+j}\left(f(m_{t+j}) - c_{t+j}\right) - m_{t+j}\right)\right] + m_t, \quad \forall t.
\end{aligned}$$

The RHS of the above equation is the total expected profit of lenders plus m_t. From Section 3, we know that the total expected profit can be written as $L(v)$, which is a function of the promised utility of the borrower.

Applying Lemma 4, we can determine the debt holdings, $d^i(v)$ ($i \in \{M, OB, OM\}$), of the government.

$$\begin{aligned}
&\left[X^M(v, A_H, s) + \left(1-\delta^M\right) q^M\left(v_{COA}^{H,H}(v)\right)\right] d^M(v) + D^{OB}(v) + D^{OM}(v) \\
=&A_H\left[f\left(m(v)\right) - c(v)\right] + \beta L\left(v_{COA}^{H,H}(v)\right); \\
&\left[X^M(v, A_L, H) + \left(1-\delta^M\right) q^M\left(v_{COA}^{L,H}(v)\right)\right] d^M(v) + D^{OB}(v) + D^{OM}(v) \\
=&A_L\left[f\left(m(v)\right) - c(v)\right] + \beta L\left(v_{COA}^{L,H}(v)\right); \\
&\left[X^M(v, A_L, L) + \left(1-\delta^M\right) q^M\left(v_{COA}^{L,L}(v)\right)\right] d^M(v) + r_{OB} D^{OB}(v) + D^{OM}(v) \\
=&A_L\left[f\left(m(v)\right) - c(v)\right] + \beta L\left(v_{COA}^{L,L}(v)\right),
\end{aligned} \tag{5.6}$$

where $D^{OB}(v) = \left[1 + \left(1-\delta^{OB}\right) q^{OB}\right] d^{OB}(v)$ and $D^{OM}(v) = \left[1 + \left(1-\delta^{OM}\right) q^{OM}\right] d^{OM}(v)$.

Specifically, if $A = A_H$ or $s = H$, the official lenders do not offer a debt reduction. The government responds to the shock through its default decision, and the price of the debt may move. Hence, with the first and second equations, we can solve for $d^M(v)$. As long as the signal is noisy (that is, the precision a is not large enough), lenders will share some risk of low productivity even if the signal suggests that the borrower might deviate ($s = H$). This implies that the RHS of the first equation is greater than that of the second equation and

the corresponding $d^M(v) > 0$. Moreover, official bilateral lenders will offer a reduction in debt to the borrower when the low export is "excusable", and lenders will assume a greater loss under this circumstance. This solves $d^{OB}(v) > 0$. In sum, we have the following lemma regarding debt holdings of the government.

Lemma 5 (Debt Holdings) There exists a debt portfolio $d^i(v)$ $(i \in \{M, OB, OM\})$ that satisfies (5.6), with $d^M(v) > 0$ and $d^{OB}(v) > 0$ when a is not too high.

We have now constructed a sustainable equilibrium, summarized in Proposition 3.

Proposition 3 The strategy constructed by (5.1)-(5.6) constitutes a sustainable equilibrium. The associated equilibrium allocation is the COA.

Proposition 3 shows that the combination of market lenders and official lenders in the international financial market helps implement the COA. We now discuss in detail how the implementation works.

Under a negative productivity shock, lenders should bear some losses of the borrower as lenders are risk neutral. The decline in the price of the market debt after the negative shock imposes a capital loss on the lenders and a capital gain on the borrower. The borrower's gain is based on the fact that the borrower is more likely to default after a negative productivity shock. Hence, the existence of defaultable market debt provides some state contingencey on the realization of productivity shock and helps achieve some risk-sharing between players. Second, because of the moral hazard problem, it is necessary to impose restrictions on the borrower's consumption. Borrowing from official multilateral lenders is a credible way to constrain consumption since the borrower cannot default on official multilateral debts and the borrower's budget becomes more restrictive. Hence, the role of nondefaultable official multilateral debt is to provide an instrument to the borrower to improve its commitment.

Lastly, the repayment of official bilateral debt is conditional on the signal, serving the role of monitoring the borrower. In sum, all types of debt are necessary for the implementation of the COA. Without market debt, risk-sharing is hard to achieve; without official multilateral debts, the borrower cannot commit to follow the equilibrium path; without official bilateral debt, the signal plays no role, and there is little room for monitoring. These three types of debt help the borrower achieve flexibility in managing its budget.

6 Numerical Exercise

We next take the model to the data to show its empirical relevance. Table 6.1 shows the calibration of parameters. We assume the risk-aversion coefficient $\gamma = 2$ and the discount factor $\beta = 0.98$ as in the standard RBC literature. The production function $F(m, l)$ take the standard CES form $F(m, l) = \left[\alpha m^{\rho} + (1-\alpha) l^{\rho}\right]/\alpha$. Hence, the statically efficient level of intermediate goods $m^* = 1$. As in Mendoza and Yue (2012), during default, intermediate good imports drop by 20% and GDP falls about 5% below trend. ρ is set to 0.974 to match the contraction of imports and α is set to 0.26 to match the contraction of domestic production. p is set to 0.945 to match the default probability of market debt at 3%, and A_L is set to 0.7 to match the volatility of real exchange rate (Lemma 1 shows that A_t reflects the real exchange rate). The recovery rate for bilateral official lending, r_{OB}, is set to 0.4 to match the empirical literature on official lending (Schlegl et al., 2019). Schlegl et al. (2019) documents that the share of official bilateral debt is around 17% on average. We set the precision of the noisy signal, a, to 0.55 to match the share of official bilateral lending. Finally, the duration of market debt, δ^M, is set to 0.045 as in Hatchondo and Martinez (2009). The duration of official debts can take any values without affecting any results in our model.

We compute the equilibrium and examine the impulse responses of imports, domestic output, domestic consumption, debt prices, and debt holdings around default. The results are summarized in Figure 6.2 and Figure 6.3. The figure shows the average of simulated

Calibrated parameters	Value	Source
Risk aversion γ	2	
Discount factor β	0.98	
Production function para ρ	0.974	Drop of intermediate inputs
Production function para α	0.26	Output drop
Productivity p	0.945	Default probability
Productivity A_L	0.7	Real depreciation rate
Recovery rate r_{OB}	0.4	Schlegl et al. (2019)
Signal precision a	0.55	Share of official bilateral debt
Duration δ^M	0.045	Hatchondo and Martinez (2009)

Table 6.1: Calibrated parameters.

dynamics around defaults on market debt. The country typically defaults after a series of negative productivity shocks, and imports, output, and consumption all decline during default. In our model, the duration of a default period is 3.8 years on average. Debt price also declines during default due to higher future default probability. As market financing becomes increasingly costly to the borrower, it is more appealing to borrow from official multilateral lenders that offer cheap financing. This switch to official debt is consistent with the finding in the literature (Boz, 2011; Kirsch and Rühmkorf, 2017; Cordella and Powell, 2021). Hence, our model replicates the stylized facts that market debt is procyclical, whereas official debt is countercyclical, as shown in Figure 6.3. This is in line with the empirical study on official lending (Horn et al., 2020; Arellano and Barreto, 2024), which empirically discovers that one of the main features of official lending is its countercyclicality. Because more resources are required to repay the non-defaultable official debt, the imports of intermediate goods are constrained, serving as an endogenous fiscal rule. At the same time, bilateral official lending is essentially acyclical in the model.[17] Finally, the total debt-to-GDP ratio also increases during default.

We also examine the impacts of the noisy signal on the real variables. Under a negative

[17](Horn et al., 2020) provides some evidence to show that bilateral official lending increases during economic crisis and natural disasters. Using exculsively the bilateral loan data(Avellán and Lotti, 2024) find that the bilateral official loans are acyclical with respect to the business cycle of the borrower.

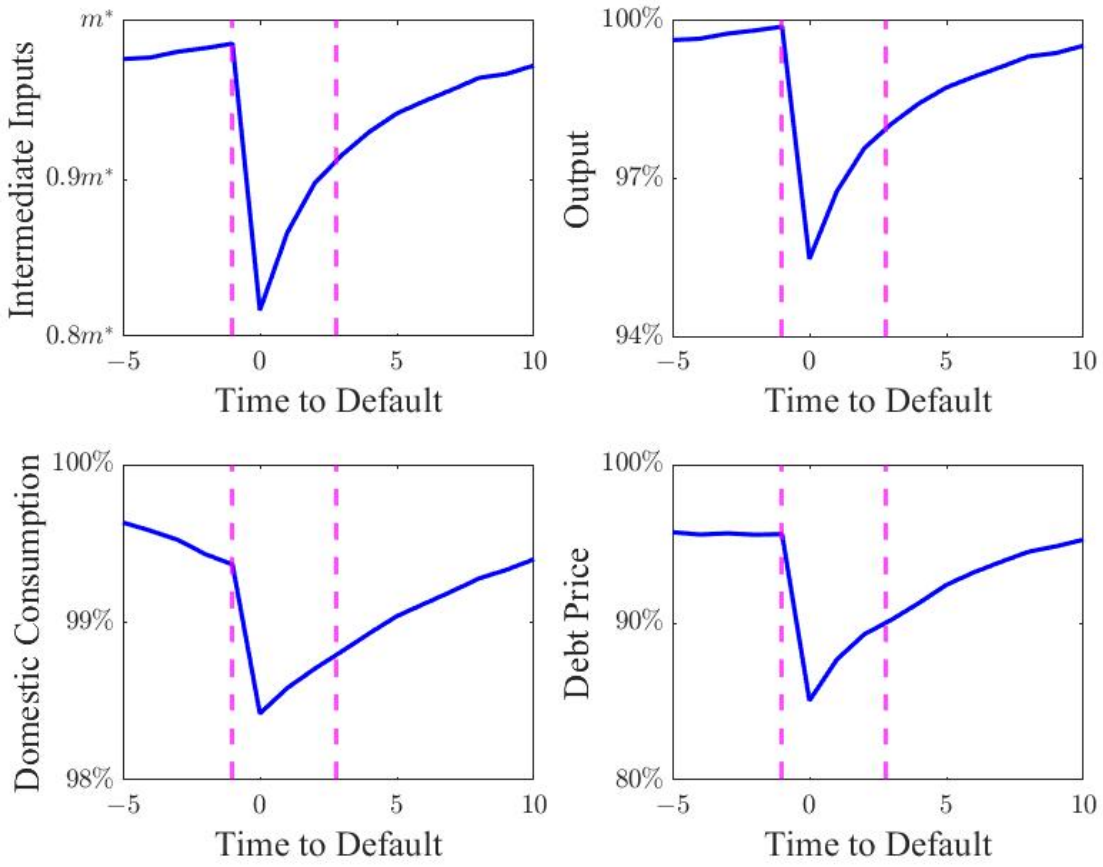

Figure 6.2: Impulse responses of imports, domestic output $f(m)$, domestic consumption, and the debt price around default. The pink lines represent the start and end of the default.

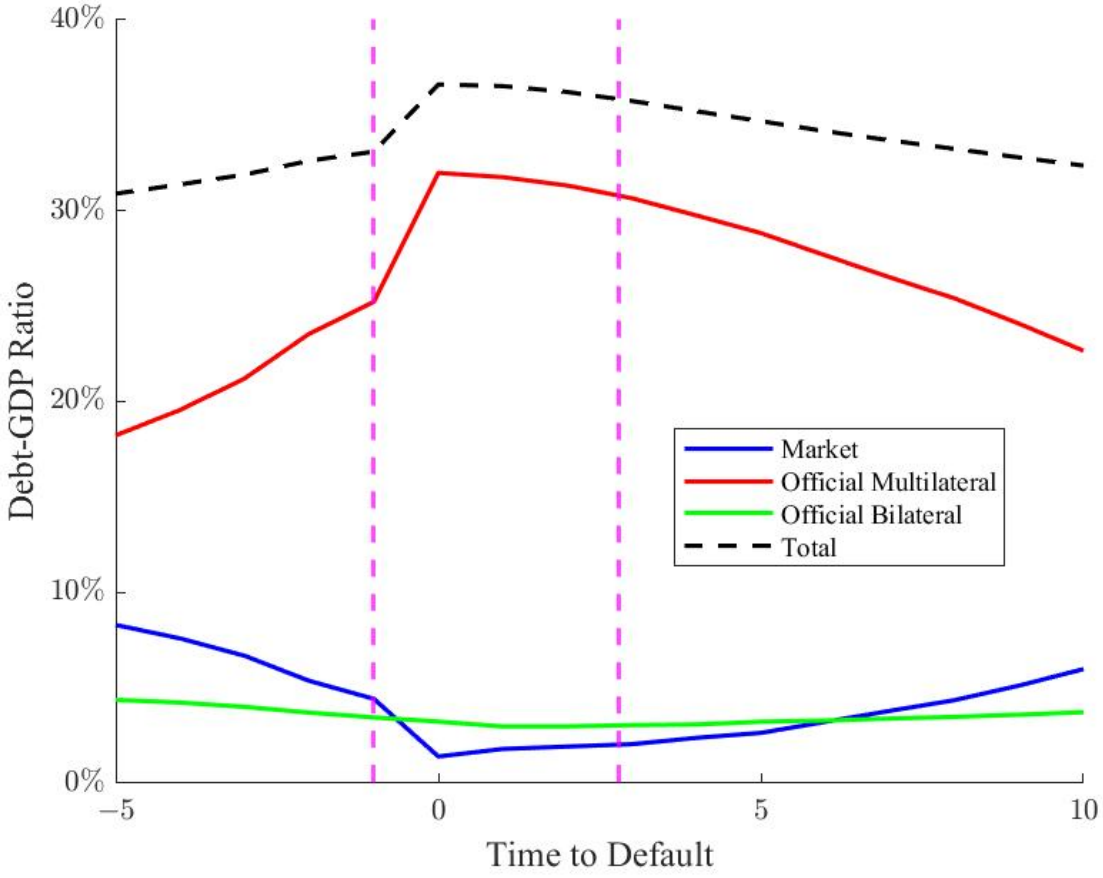

Figure 6.3: Impulse response of debt holdings around default. The pink lines represent the start and end of the default.

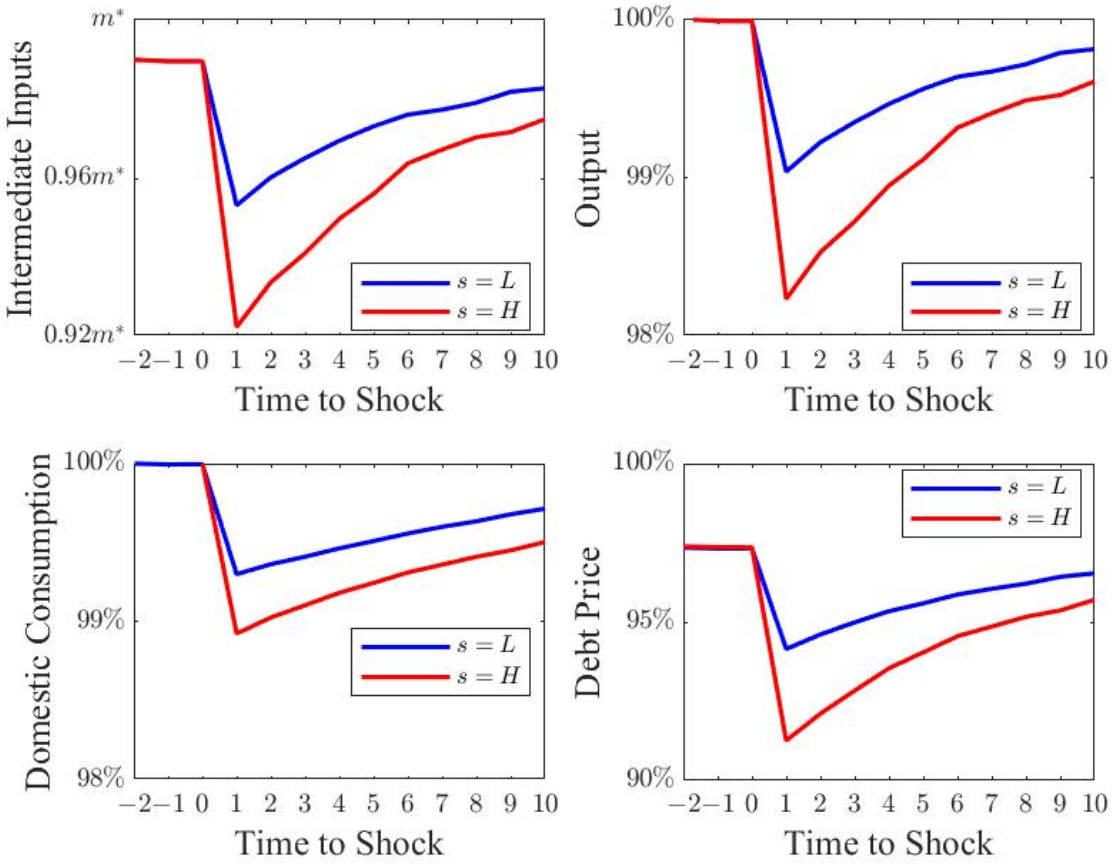

Figure 6.4: Impulse responses of imports, domestic output $f(m)$, domestic consumption, and the debt price around a negative productivity shock.

productivity shock, the responses of the economy depend on the signals, as shown in Figure 6.4 and Figure 6.5. The realization of an unfavorable signal ($s = H$) suggests that the borrower could deviate and negatively affect the economy more. The increase in the debt-to-GDP ratio and the share of official multilateral lending is more prominent under $s = H$.

Overall, our model generates an endogenous default cost of 5% of outputs and accounts for the cyclicality of market lending and official lending, which is in line with the empirical literature. We argue that the haircut of official bilateral lending depends on the realization of the noisy signal, which serves the role of monitoring the sovereign borrower. On the other hand, non-defaultable official multilateral lending imposes fiscal rule on the sovereign

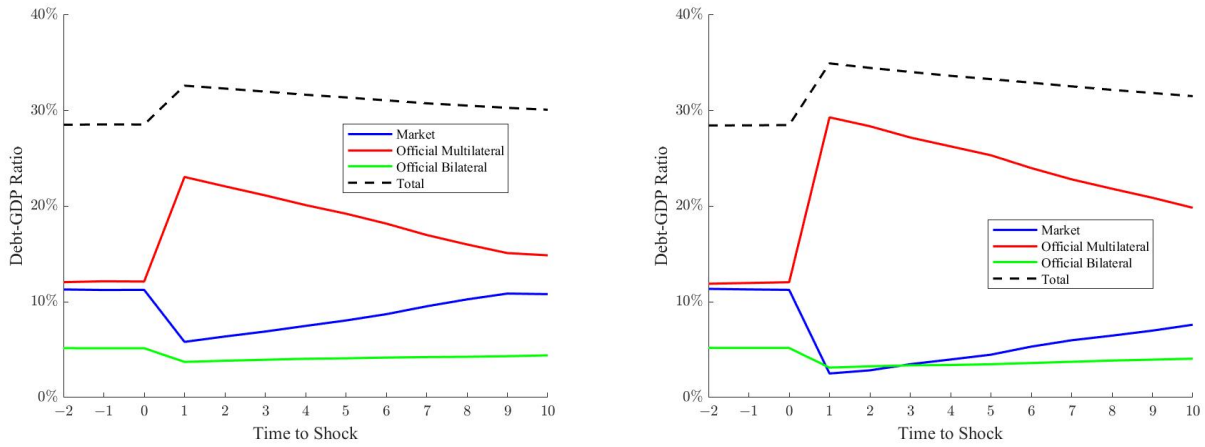

Figure 6.5: Impulse responses of debt to GDP ratio under different realizations of the noisy signal around a negative productivity shock. Panel (a) is the case where $s = L$, whereas Panel (b) is the case where $s = H$.

borrower to mitigate the moral hazard problem. The reason why the borrower switches to official lending is that market lending becomes more costly during default. The combination of market lending and official lending helps implement the COA.

7 Conclusion

The importance of official lending for developing and emerging economies, along with China's growing role, highlights the need to understand the dynamics of international official lending. This paper studies a repeated game of international lending under moral hazard. It illustrates the role of official lenders as liquidity providers and as monitors. We show that the existence of a moral hazard problem on the part of the sovereign borrower constrains international lending, preventing it from achieving efficiency. The constrained optimal allocation can be implemented through a combination of private lenders, official bilateral lenders, and official multilateral lenders. Both types of official lenders act as monitors in the presence of informational friction. Official bilateral lenders mitigate the moral hazard problem by tailoring punishment based on the signal received and accept a haircut only when the default

is excusable. Official multilateral lenders provide cheap credit and serve as liquidity providers for the sovereign borrower. Moreover, since official multilateral lending is non-defaultable, borrowing from official multilateral lenders restricts the borrower's future consumption, thus serving as an endogenous fiscal rule during bad times. The model accounts for countercyclical lending from the official multilateral creditors and replicates the aggregate dynamics around default including endogenous reduction in output. This theory of international official lending offers new insight into the role of official lenders and the dynamics of sovereign debt.

References

Abraham, A., Carceles-Poveda, E., , Liu, Y., and Marimon, R. (2024). On the optimal design of a financial stability fund. *Review of Economic Studies*.

Abreu, D., Pearce, D., and Stacchetti, E. (1990). Towards a theory of discounted repeated games with imperfect monitoring. *Econometrica*, 58(5):1041–1063.

Aguiar, M., Amador, M., and Gopinath, G. (2009). Investment cycles and sovereign debt overhang. *The Review of economic studies*, 76(1):1–31.

Aguiar, M. and Gopinath, G. (2006). Defaultable debt, interest rates and the current account. *Journal of International Economics*, 69(1):64–83.

Albuquerque, R. and Hopenhayn, H. (2004). Optimal lending contracts and firm dynamics. *The Review of Economic Studies*, 71:285–315.

Alverez, F. and Jermann, U. (2000). Efficiency, equilibrium, and asset pricing with risk of default. *Econometrica*, 68(4):775–797.

Arellano, C. (2008). Default risk and income fluctuations in emerging economies. *American Economic Review*, 98(3):690–712.

Arellano, C. and Barreto, L. (2024). Official sovereign debt. *Working paper*.

Atkeson, A. (1991). International lending with moral hazard and risk of repudiation. *Econometrica*, 59(4):1069–1089.

Avellán, Leopoldo, A. J. G. T. G. and Lotti, G. (2024). The cyclicality of official bilateral lending: Which cycle do flows follow? *Emerging Markets Review*, 59:101120.

Boz, E. (2011). Sovereign default, private sector creditors, and the ifis. *Journal of International Economics*, 83(1):70–82.

Chami, R., Sharma, S., and Shim, I. (2004). A model of the imf as a coinsurance arrangement. *IMF Working Paper*, 04/219.

Cordella, T. and Powell, A. (2021). Preferred and non-preferred creditors. *Journal of International Economics*, 132:103491.

Dellas, H. and Niepelt, D. (2016). Sovereign debt with heterogeneous creditors. *Journal of International Economics*, 99:S16–S26.

Dovis, A. (2019). Efficient sovereign default. *Review of Economic Studies*, 86:282–312.

Eaton, J. and Gersovitz, M. (1981). Debt with potential repudiation: Theoretical and empirical analysis. *The Review of Economic Studies*, 48(2):289–309.

Fink, F. and Scholl, A. (2016). A quantitative model of sovereign debt, bailouts and conditionality. *Journal of International Economics*, 98:176–190.

Grossman, H. I. and Huyck, J. B. V. (1988). Sovereign debt as a contingent claim: Excusable default, repudiation, and reputation. *The American Economic Review*, 78(5):1088–1097.

Hatchondo, J. C. and Martinez, L. (2009). Long-duration bonds and sovereign defaults. *Journal of International Economics*, 79(1):117–125.

Holmström, B. (1979). Moral hazard and observability. *The Bell Journal of Economics*, pages 74–91.

Horn, S., Reinhart, C. M., and Trebesch, C. (2020). Coping with disasters: two centuries of international official lending. *National Bureau of Economic Research*.

Horn, S., Reinhart, C. M., and Trebesch, C. (2021). China's overseas lending. *Journal of International Economics*, 133:103539.

Jeanne, O., Ostry, J. D., and Zettelmeyer, J. (2008). A theory of international crisis lending and imf conditionality. *IMF Working Paper*, 08/236.

Jie, L. and Wang, C. (2018). Optimal sovereign lending and default. *Journal of International Economics*, 111:190–213.

Kehoe, P. and Perri, F. (2004). Competitive equilbria with limited enforcement. *Journal of economic theory*, 119:184–206.

Kirsch, F. and Rühmkorf, R. (2017). Sovereign borrowing, financial assistance, and debt repudiation. *Economic Theory*, 64(4):777–804.

Kocherlakota, N. (1996). Implications of efficient risk sharing without commitment. *Review of Economic Studies*, 63(4):595–609.

Kondo, I., Mkhitaryan, A., Sosa-Padilla, C., and Swaziek, Z. (2022). Borrowing in the shadow of china. *Working paper*.

Mendoza, E. G. and Yue, V. Z. (2012). A general equilibrium model of sovereign default and business cycles. *The Quarterly Journal of Economics*, 127(2):889–946.

Muller, A., Storesletten, K., and Zilibotti, F. (2019). Sovereign debt and structural reforms. *American Economic Review*, 109(12):4220–4259.

Rose, A. K. (2005). One reasons countries pay their debts: renegotiation and international trade. *Journal of International Economics*, 77:189–206.

Sandleris, G. (2008). Sovereign defaults: Information, investment and credit. *Journal of International Economics*, 76(2):267–275.

Schlegl, M., Trebesch, C., and Wright, M. L. (2019). The seniority structure of sovereign debt. *National Bureau of Economic Research*.

Tirole, J. (2002). Financial crises, liquidity, and the international monetary system. *Princeton: Princeton University Press*.

Yue, V. Z. (2010). Sovereign default and debt renegotiation. *Journal of international Economics*, 80(2):176–187.