Zombie Lending and Policy Traps*

Viral V. Acharya† Simone Lenzu‡ Olivier Wang§

September 2023

Abstract

We build a model with heterogeneous firms and banks to explain how accommodative policy can get trapped due to credit misallocation and its spillovers, as witnessed in Japan in the 1990s and in Europe in the 2010s. Conventional monetary policy is sufficient to achieve efficient production following small negative shocks, but large shocks necessitate unconventional policy such as regulatory forbearance towards banks. Excessive accommodation, however, induces "diabolical sorting", whereby low-capitalization banks lend to low-productivity "zombie" firms. Due to congestion externalities of zombie lending on healthier firms, policymakers avoiding short-term recessions can get trapped into protracted low rates, excessive forbearance, and permanent output losses.

Keywords: Credit misallocation, Evergreening, Forbearance, Bank capital, Conventional monetary policy, Unconventional monetary policy.
JEL: E44, E52, G21, G28.

---

*First draft: September 2021. Stefano Pastore provided excellent research assistance. We are grateful to two anonymous referees, as well as Juliane Begenau, Ricardo Caballero, Charles Calomiris, Tim Eisert, Kinda Hachem, Martina Jasova, Anil Kashyap, David Martinez-Miera, Simon Mayer, Cecilia Parlatore, Enrico Perotti, Silvio Petriconi, Giorgia Piacentino, Alexi Savov, Philipp Schnabl, Sascha Steffen, Johannes Stroebel, Javier Suarez, and seminar participants at Amsterdam Business School, NYU Stern, Norges Bank, the Virtual Finance Theory Seminar, the Fixed Income and Financial Institutions (FIFI) Conference at University of South Carolina, Richmond Fed, Bank of Italy, IMF, Northwestern Kellogg, Pompeu Fabra, CEMFI, FIRS, SED and EFA for comments. We are also highly indebted to Tim Eisert for providing us with data underlying the motivating charts for zombie lending, bank capitalization and aggregate productivity outcomes for Europe.

†NYU Stern, CEPR, ECGI and NBER. E-mail: vva1@stern.nyu.edu.
‡NYU Stern. E-mail: slenzu@stern.nyu.edu.
§NYU Stern. E-mail: owang@stern.nyu.edu.

1 Introduction

In this paper, we build a model with heterogeneous firms and banks to analyze how monetary and banking policies affect credit allocation and long-term economic outcomes. In particular, we explain why policy may get trapped into protracted low rates and excessive regulatory forbearance towards banks that are coincident with permanent output losses.

Since the housing and banking crisis in Japan in the early 1990s, regulatory forbearance towards banks, effectively allowing severely under-capitalized or insolvent banks to continue operating by backstopping bank creditors, has been increasingly used in conjunction with accommodative monetary policy in a bid to restore economic growth in the aftermath of aggregate shocks. This policy combination also found favor in the Eurozone following the global financial crisis of 2007–08 and especially after the European sovereign debt crisis in 2010–12.[1] In both cases, despite an operative period exceeding in length the initial intentions and expectations, this policy script's impact on economic growth remained relatively muted. Starting with Peek and Rosengren (2005) and Caballero, Hoshi and Kashyap (2008), the literature has attributed this ineffectiveness of employed policies (at least in part) to credit misallocation and, in particular, to the phenomenon of zombie lending: the provision of subsidized credit to poorly-performing firms by weakly-capitalized banks.[2]

Section 2 puts the experiences of Japan and Europe into perspective, presenting stylized facts that document the proliferation of zombie lending in the aftermath of the crisis outbreaks, the negative externalities that zombie lending generated, and its feedback loop with policy interventions. Importantly, the evidence suggests that adverse effects of credit misallocation due to the proliferation of zombie lending persist and even compound over time.

Motivated by these facts, we build a tractable model that provides a unified framework simultaneously explaining and conforming to all of them. We start with a static setting, before turning to the full dynamic model. The economy is populated by heterogeneous firms that differ in their productivity and risk. Firms' investments require credit, which is provided by banks that are themselves heterogeneous in their level of capitalization. Banks face a portfolio problem, whose solution depends on their capital: they decide whether to invest in safe assets (meant to capture a wide range of non-loan assets, such as central bank reserves, government bonds, or safe mortgage-backed securities) or lend to the productive sector, and if so, to which type of firms.

Policies play a crucial role in banks' incentives, and thereby the equilibrium allocation of

[1] In the U.S., several aspects of how the savings and loans institutions (or thrifts) crisis was resolved in the mid 1980's and how thrifts responded to it by gambling for resurrection also resemble these episodes.

[2] Poorly-capitalized banks supply credit to poorly-performing firms either because they are incentivized to engage in risk-shifting (gambling for resurrection) or because they attempt to avoid reporting losses on distressed positions, as documented in Giannetti and Simonov (2013), Acharya et al. 2019, Blattner et al. (2023), Gropp et al. 2020, Faria E Castro et al. 2021, Acharya et al. (2021a), and Schivardi et al. (2021), among others.

credit. We summarize all the components of policy that affect bank decisions into two simple instruments: the risk-free rate R^f set by conventional monetary policy, and an unconventional "forbearance policy" p that determines the level of government guarantees granted to banks that are willing to lend. Accommodative conventional monetary policy makes lending more attractive by lowering the return on safe assets R^f. This is a standard bank lending channel. Increasing forbearance also stimulates lending, by compressing the cost of funds associated with lending: a higher p lowers the cost of funds because a larger part of the bank loan risk is borne by public authorities (the government, the central bank, or the deposit insurance agency). However, excessive forbearance can shift banks' portfolios towards riskier loans to less productive firms, which we refer to as the "zombie lending channel".

The two-sided firm-bank heterogeneity opens the door to the "diabolical sorting" documented in the data: banks with low capital and high leverage end up lending to less productive firms, even though aggregate output would be raised by letting these firms exit and be replaced by more productive entrants. The reason is that the subsidy from forbearance increases with the interaction of banks' asset risk and leverage. This sorting between banks and firms leads to a delicate policy trade-off. While zombie lending and depressed creative destruction are the main perils on the side of poorly-capitalized banks, policymakers must also encourage well-capitalized banks to lend to the good firms. Well-capitalized banks are not tempted by zombie lending, but may simply invest in safe assets. The tension between inducing well-capitalized banks to lend and preventing poorly-capitalized banks from engaging in zombie lending is at the heart of our analysis of the optimal policy mix in response to exogenous shocks.

Importantly, firms' output depends on an aggregate productivity or demand shock.[3] If zombie loans and safe assets are always less productive than loans to good firms, output reaches its potential if and only if all banks lend and there is no zombie lending. As long as the risk-free rate is not constrained, conventional monetary policy alone without any forbearance can achieve this objective. Without forbearance, there is no zombie lending by weak banks, while a sufficiently low risk-free rate (i.e., the "natural interest rate" in our economy) encourages healthy banks to lend. However, larger negative shocks to fundamentals must be accommodated by increasingly lower interest rates. Hence, if the shocks are large enough, conventional monetary policy runs into an effective lower bound on interest rates (ELB), assumed exogenously in the model. This is where unconventional policy in the form of regulatory forbearance and its unintended consequences come into play.

We show that a small amount of forbearance is beneficial, as it can substitute for the constrained conventional monetary policy and help lower banks' funding costs, thereby stimulating

[3]Motivated by the empirical evidence (see Section 2), the shock can affect either the average productivity of firms or the cross-sectional dispersion between good (safer) and bad (riskier) firms.

lending and output. Pushing on the forbearance string, however, eventually spurs zombie lending by weak banks. Zombie lending reduces aggregate output and productivity because of the misallocation of credit and, as documented in the empirical literature, because it can generate congestion externalities in input and output markets that ultimately impair the productivity and growth prospects of healthy firms in the economy.[4]

If congestion externalities are large, we show that the optimal forbearance policy is non-monotonic in the size of the shocks: when shocks are moderate, forbearance should increase with the size of the shock as expected; but in the face of large shocks, policymakers should actually backtrack and *reduce* forbearance to avoid triggering zombie lending, even though this entails letting some banks invest in safe assets instead of lending. For large shocks, the loss from zombie lending can far exceed the opportunity cost of not lending to some healthy firms. Thus, there exists a "reversal" level of forbearance policy, a counterpart to the "reversal interest rate" below which conventional monetary policy turns contractionary (Abadi et al., 2023). However, if policymakers put a low welfare weight on congestion externalities (for instance, because they are politically aligned with current labor or lobbied by bank shareholders), then they will focus on maximizing output by maximizing bank lending, ignoring concerns about the composition of lending. This is achieved by responding to larger shocks with more forbearance which induces zombie lending.

To explore the intertemporal trade-offs posed by zombie lending, we expand on the static framework and model the dynamic interactions between policy choices, zombie lending, and aggregate outcomes. Since zombie lending also causes negative spillovers on the productivity of healthy firms in future periods via persistent congestion externalities, the policy trade-off is dynamic, between maximizing short-run output and hurting future productivity. We show that the crucial parameter (corresponding to the welfare weight on congestion externalities in the static model) is the *horizon of policymakers* and consider two polar cases: "patient" policymakers, seeking to avoid future output losses, and "myopic" policymakers willing to preserve incumbent firms at the expense of future productivity. Myopic policymakers are not naive and realize the effect of their policies on equilibrium outcomes, but are subject to capture, term limits, or reputational concerns that shorten their effective horizon of decision-making.[5]

In our main policy experiment, we consider a transitory exogenous shock to fundamentals, as in the static model. When policymakers are patient, the optimal response is exactly as in the

[4]There is now a large body of evidence showing that the broad adverse effects of zombie lending on healthier firms translate into lower economic outcomes such as depressed aggregate employment, investment, productivity, innovation, and output (see the survey of Acharya et al. (2021b) and references therein). The empirical estimates available to date suggest that not only can the spillover effects be substantial, but also that the output losses due to spillover effects might be persistent and even compound over time.

[5]See, for instance, Alesina and Tabellini (1990) and Boot and Thakor (1993).

static model: conventional monetary policy without forbearance achieves potential output for small shocks; some forbearance is optimal once the ELB binds when shocks remain moderate; and forbearance should decrease for large shocks to avoid any zombie lending and the associated congestion externalities. When policymakers are myopic, they implement the same joint policies for small and moderate shocks as patient policymakers, but respond very differently to large shocks. Since they are focused on maximizing short-term output, they accept zombie lending and respond to larger shocks with more forbearance.

The dynamic consequences of such myopic policy can be dire: we find that if spillovers from zombie lending to the productivity of healthy firms are strong enough, the optimal myopic policy response precipitates the economy into the following *policy trap*. Unlike in standard macroeconomic models, the natural rate becomes an endogenous variable. Although the exogenous shock is transitory, future policymakers face an endogenously low productivity due to the congestion externalities, and continue responding in the same accommodating way, keeping interest rates low and forbearance high. This keeps zombies alive, and productivity low, for at least another period. At the very least, this negative dynamic feedback generates endogenous persistence.

In the extreme, for large enough initial shocks, the pattern repeats itself until the economy converges to a *sclerosis* steady state, defined as featuring a permanent combination of interest rates stuck at the ELB, high forbearance, zombie lending, and low output. In our theory, forward-looking policymakers should accept a "V-shaped" (i.e., sharp but transitory) recession precisely when fundamental shocks are large, which is exactly the opposite of what myopic policymakers do and what is often argued in practice.[6] We discuss two ways to exit the sclerosis steady state: a bank recapitalization, which improves banks' incentives at some fiscal cost, and an improvement in productivity. Both need to be sufficiently strong, as timid interventions only have a transitory effect before the economy and the policy become trapped again.

Finally, the central role of bank capital in our analysis raises important questions: Do under-capitalized banks have incentives to issue more equity? And if not, can regulators eliminate the zombie lending problem by simply increasing capital requirements? We address these questions in an extension of our model, allowing for costly equity issuance, legacy lending, and capital requirements. We find that the same risk-shifting incentives affecting bank lending decisions also apply to capital structure decisions, preventing under-capitalized banks from raising enough capital to avoid zombie lending. Imposing high capital requirements can then deter zombie lending if the costs of breaking the relationship with a legacy zombie borrower (e.g., recognizing losses) are low enough. However, if these costs are high, we show that zombie lending becomes inevitable,

[6]While this argument resembles some of the classic "liquidationist" views of Hayek and Schumpeter, in our framework this conclusion is contingent on many factors such as the size of the shock, the policy space available to address the shock with conventional tools, and the state of capitalization of the banking sector when the shock hits.

in the sense that some banks will evergreen (lend to legacy zombie firms) for *any* level of capital requirement. Furthermore, there is a zombie-minimizing level of capital requirement and going beyond this level leads to more zombie lending.

Overall, our model consistently explains a set of empirical facts connecting bank capitalization, credit misallocation, policy choices, and aggregate growth and productivity, following adverse economic shocks. In particular, it makes three important contributions.

First, it helps understand why in the face of large shocks, the policy response to restore economic growth may feature a combination of conventional policy in the form of monetary accommodation and unconventional policy in the form of regulatory forbearance towards banks. Forbearance becomes useful in our model only when the conventional policy hits an effective or zero lower bound. This is a distinctive feature of our model relative to the banking literature that models regulatory forbearance as arising from a time-inconsistency problem of regulation (Mailath and Mester, 1994).

Secondly, our model derives the empirically documented phenomenon that regulatory forbearance leads to zombie lending and a diabolical sorting, whereby low-capitalization banks extend new credit or evergreen existing loans to low-productivity firms. It is this positive implication of the model that then allows for a meaningful normative analysis of the policies affecting bank incentives to engage in such lending.

Thirdly, by examining a dynamic setting in which zombie lending imposes congestion externalities in the form of adverse productivity spillovers on healthier firms, the model explains why economies facing large, but only transitory, shocks may jointly feature thereafter (i) a phase of delayed recovery and potentially permanent output losses, which we call economic sclerosis; and, (ii) a policy trap whereby monetary accommodation and regulatory forbearance aimed at avoiding short-term recessions become entrenched even as they persistently fail to restore long-term economic health. The possibility that a transitory shock turns into permanent stagnation when policymakers favor avoiding recessions in the short run is the most salient feature of our analysis. A key policy implication is that to avoid zombie lending and associated economic sclerosis, it is important to maintain a well-capitalized banking system but also that bank capital requirements need to be raised upfront rather than upon realization of economic shocks.

Related Literature

Our model builds on the seminal contribution of Caballero et al. (2008) (henceforth CHK) and extends it in two key dimensions. First, CHK features negative spillovers generated by zombie firms due to congestion in input and output markets, but it does not explicitly model the role of credit markets and financial intermediaries, and their incentives to extend credit to low productivity firms. By contrast, the credit market, banks and their capital structures are front and center in our framework. Second, our model stresses the nexus between policy interventions, credit allocation, and aggregate outcomes. To the best of our knowledge, ours is the first theoretical treatment emphasizing the zombie lending-policy feedback loop, where macro-financial policies dynamically affect and are affected by banks' credit supply choices. In this respect, our results on economic sclerosis and policy traps also speak to the stagnation traps analyzed by Benigno and Fornaro (2018), who highlight the ineffectiveness of conventional monetary policy alone in stimulating the economy.

Also related to our model are the theoretical contributions of Bruche and Llobet (2013), Hu and Varas (2021), and Begenau et al. (2021) investigating the role of bank incentives as driver of zombie lending, with a particular emphasis on the role of hidden losses and asymmetric information. We share with these papers the emphasis on developing a micro-founded model of bank lending, although we do not directly model asymmetric information frictions. Rather, we focus on developing a tractable general equilibrium framework to study how credit allocation and policy actions shape aggregate outcomes. In this regard, Tracey (2021) also shows that excessive forbearance may ultimately play a significant role in explaining the slow economic growth observed in the aftermath of aggregate shocks.

More broadly, our paper is related to the macroeconomic literature on financial frictions and misallocation. Gopinath et al. (2017), Banerjee and Hofmann (2018), Asriyan et al. (2024) and Jafarov and Minnella (2023) stress how a low interest rate environment and financial frictions can induce capital misallocation and aggregate losses. Our focus is on the central role played by financial intermediaries and how their actions depend on and influence macro policies. Buera et al. (2013) study how policies aimed at stimulating output can lead to long-run productivity losses. Their focus is on targeted industrial policies such as credit subsidies directly aimed at firms. In contrast, we are interested in studying stabilization policies and bank lending incentives in response to macroeconomic shocks.

Finally, motivated by the policies adopted during of the COVID 19 pandemic, Crouzet and Tourre (2021) and Li and Li (2021) also highlight how government interventions can have negative long-run effects, by exacerbating debt overhang or worsening the quality distribution of firms, respectively. Li and Li (2021) shows how public liquidity support might be able to preserve a country's production capacity in the short run but also dampen creative destruction in a self-perpetuating way. Complementary to these papers, our model focuses more on the zombie lending channel operating via weakly-capitalized banks.

The remainder of the paper is organized as follows. We first present stylized empirical facts about zombie lending in Section 2 and then develop our baseline model in Section 3. In Section 4 we analyze optimal policy and turn to the dynamic model in Section 5. Section 6 presents extensions around the role of bank capital and capital requirements. Section 7 concludes with some directions for future research.

2 Zombie Lending: Incidence, Consequences, and Policy Responses

We present a set of key stylized facts about the incidence of zombie lending, its consequences for the real economy, and the feedback loops with policy responses that motivate and guide our theoretical framework. We draw on the historical experiences of Japan in the early 1990s and peripheral European countries (Italy, Spain, and Portugal) after 2010, following significant aggregate shocks. For Japan, the aggregate shock was the burst of a real estate crisis; for the peripheral European countries the outbreak of a sovereign debt crisis. In all countries, the burst of the crisis was followed by a substantial and protracted slowdown in economic activity, accompanied by the proliferation of zombie lending. Table 1 and Figure 1 show salient aggregate statistics for both episodes.

Table 1: Zombie lending, aggregate outcomes, and bank capital in Japan and European periphery.

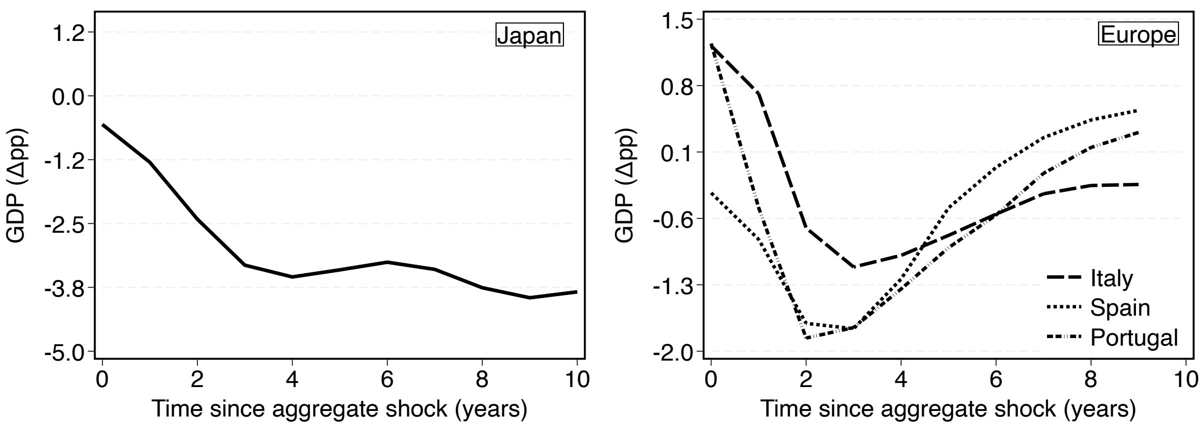
	pp. change between 1986–1989 and 1990–2001	pp. change relative to Germany during 2010–2019		
	Japan	*Italy*	*Spain*	*Portugal*
Share of zombie firms	6.1	7.7	2.4	3.8
Annualized GDP growth	-3.1	-18.1	-11.6	-14.4
Annualized aggregate TFP growth	-6.3	-10.7	-7.7	-7.1
Banking system capitalization	-3.3	-3.5	-3.1	-3.6

Note: The table reports the following statistics: the change (in percentage points) of the average share of zombie firms by assets after the aggregate shock relative to the average share of zombie firms in the benchmark (defined below), where the definition of zombie firm follows the one in Caballero et al. (2008); the change (in percentage points) in the annualized growth rate of real GDP and aggregate total factor productivity (TFP) relative to the annualized growth rates of real GDP and aggregate TFP in the benchmark; and the change (in percentage points) in the average capital ratio of the Japanese banking system relative to the average capital ratio in the benchmark. For Japan, the aggregate shock is the burst of the real estate crisis in 1989–1990, the post-shock period is 1990–2001, and the benchmark is Japan itself in the pre-shock period (1986–1989). For Italy, Portugal, and Spain, the aggregate shock is the burst of the European sovereign debt crisis in 2010, the post-shock period is 2010–2019, and the benchmark is Germany in 2010–2019. See Appendix B for additional information on data sources and variables' definition.

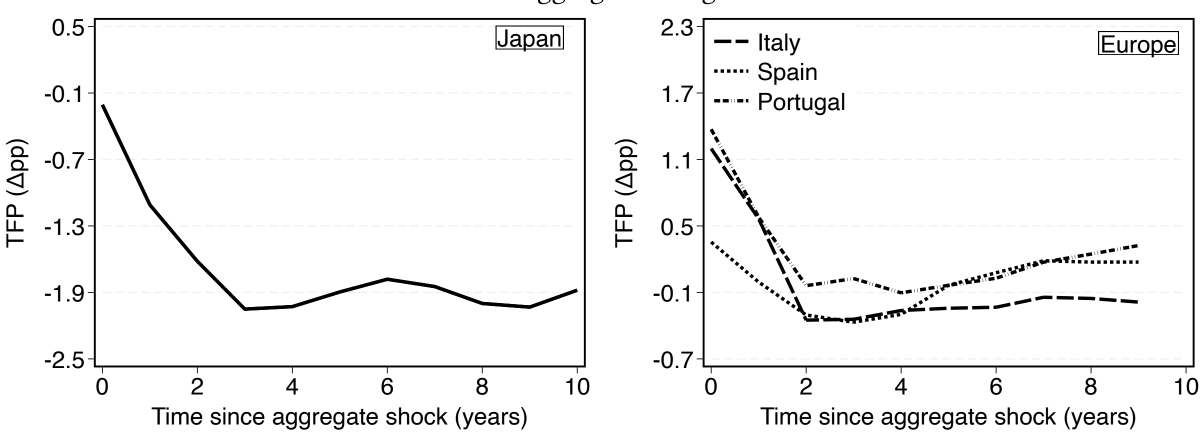
The Japanese stagnation is a textbook case study of the phenomenon of zombie lending and its implications. Figure 1A (left-side plots) shows that the growth rate of GDP and aggregate total factor productivity (TFP) slowed down dramatically relative to the period that preceded the real estate crisis (a reduction of 3.1 and 6.3 percentage points, respectively, as shown in Table 1). A similar pattern emerges when looking at the GDP and productivity dynamics of peripheral European countries in the aftermath of the sovereign debt crisis (Figure 1B, right-side plots). Italy, Spain, and Portugal all experienced sharp GDP and TFP slowdowns in the post-sovereign crisis period when compared to Germany (the reference country) over the same period. Both in Japan and the European periphery, the burst of the crisis marked a steady increase in the number of poorly-performing firms receiving subsidized credit.

Figure 1: GDP and aggregate TFP growth of Japan and Europe after the aggregate shock.

Panel A: GDP growth



Panel B: Aggregate TFP growth



Note: On the y-axis, the figures reports the difference (in percentage points) between the annualized growth rate of GDP (panel A) and aggregate TFP (panel B) in each country between $t = 0$ an t and the benchmark annualized GDP and TFP growth rate for that country. The x-axis reports time (in years) since the aggregate shock. For Japan, the aggregate shock is the burst of the real estate crisis (time $t = 0$ is 1990) and the benchmark is the annualized GDP (TFP) growth rate of Japan itself in the pre-shock period (1986–1989). For Italy, Portugal, and Spain, the aggregate shock is the burst of the European sovereign debt crisis (time $t = 0$ is 2010) and the benchmark is the annualized GDP (TFP) growth rate of Germany between t and $t = 0$. See Appendix B for additional information on data sources and variables' definition.

For instance, Table 1 shows that in Japan and Italy, the proportion of zombie firms by assets (relative to the total active firms) increased by 6.1 and 7.7 percentage points, respectively, representing a doubling of their pre-crisis levels.

Table 1 also documents that the (officially recorded) capitalization of the banking system was over 3 percent lower during the crisis compared to the pre-crisis period, for Japan, and 3 percent lower during the crisis relative to Germany over the same period, for Italy, Spain, and Portugal.

Initial shocks and subsequent zombie lending—Data suggests that the proliferation of zombie lending is related to both the size of the initial economic shock and the cross-sectional incidence of the shock (whether it reduced or widened the profitability gap between good and poorly-performing firms). Both of these relationships will be featured in our theoretical framework.

First, we document the role of large negative shocks in igniting zombie lending. The binned scatter plot in Figure 2A (left) relates the *average* impact of the crisis on firms for a given country-industry pair in Europe (measured by the change in average industry productivity before and after the burst of the crisis) to the growth rate of zombie lending in that country-industry pair in the aftermath of the sovereign debt crisis. The negative correlation between the two variables indicates that the industries more affected by an initial negative shock are those that display a greater incidence of zombie lending.

A second empirical regularity is the connection between zombie lending and the heterogeneous impact of the crisis on different firms *within* an industry-country pair. Figure 2A (right) shows that the growth rate of zombie lending is particularly pronounced in industry-country pairs in which the initial negative shock reduced the productivity of better-performing firms *relative* to the productivity of poorly-performing ones.[7]

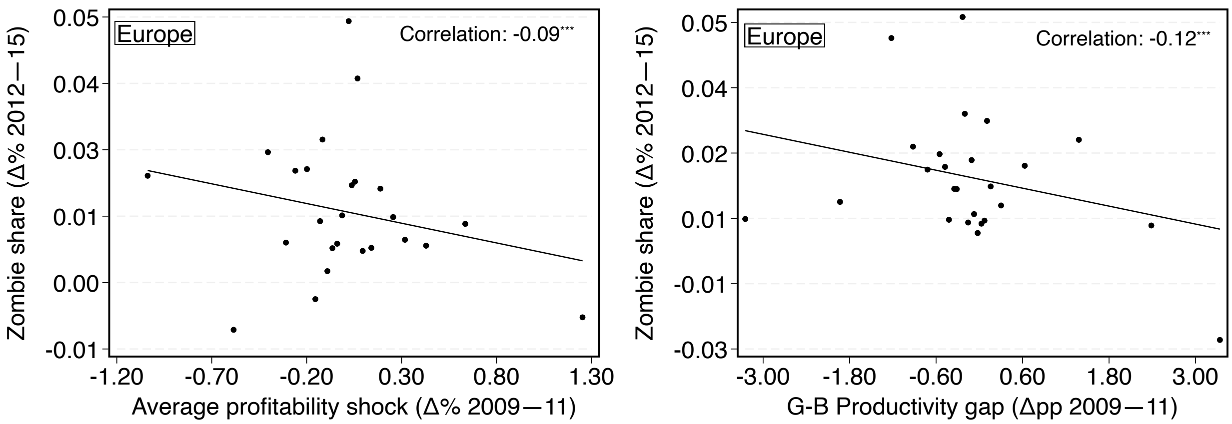
Negative spillovers—Previous studies have highlighted various channels through which the provision of subsidized credit to poorly-performing firms impedes the economy's ability to grow and thrive.[8] For one, zombie lending tames creative destruction, by distorting both entry and exit margins. Favorable lending terms keep unproductive firms afloat, trapping bank capital that could otherwise be allocated toward more productive newcomers. For another, the presence of zombie firms can impact the performance of healthier firms that compete with them, due to congestion in labor and other input markets (Caballero et al., 2008), congestion in output markets due to price competition (Acharya et al., 2024), or reduced innovation incentives (Schmidt et al., 2023). In line with these findings, Figure 2B (left) shows, for peripheral European countries, a strong negative correlation between the change in the zombie share of an industry and its *post-crisis* long-run TFP growth. We observe a similar pattern in Japan in the aftermath of the real

[7] The correlations shown in Figure 1A are robust to the inclusion of industry fixed effects and country fixed effects.

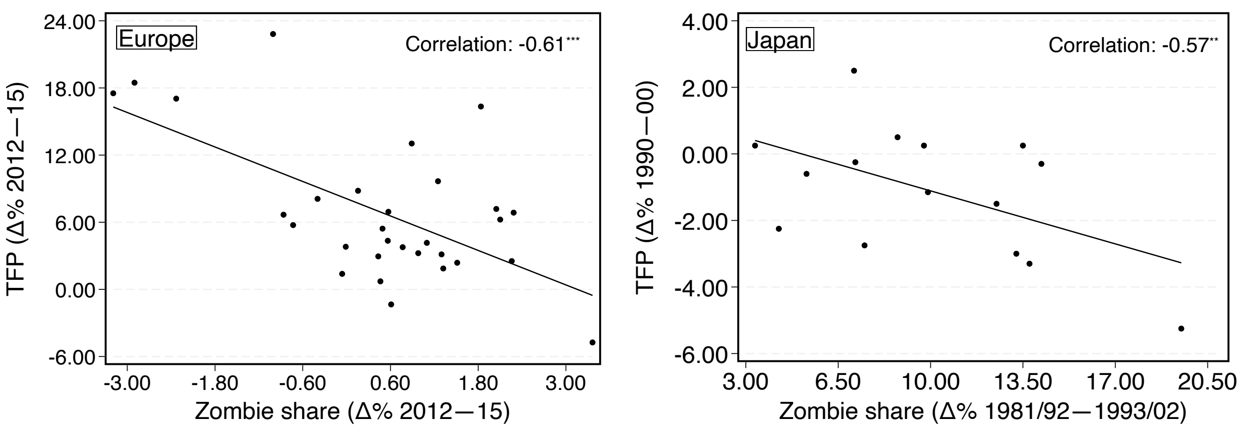
[8] See, e.g., the discussion in Hoshi and Kashyap (2015) and the review in Acharya et al. (2021b).

Figure 2: Economic shocks, zombie lending, and productivity growth: Cross-sectional evidence.

Panel A: Initial shocks and subsequent zombie lending (Europe).



Panel B: Negative spillovers of zombie lending (Europe and Japan).

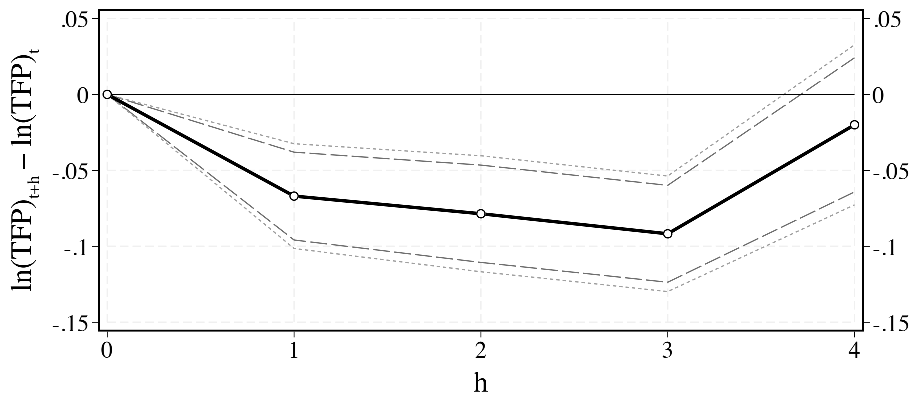


Notes: Panel A studies the role of productivity shocks in igniting the proliferation of zombie lending in Europe. The graph on the left is a binned scatter plot that displays on the x-axis the percentage change in the asset-weighted average productivity (percentage change in EBITDA/Assets) of a country-industry pair between 2009 and 2011, and the percentage change in the (asset-weighted) share of zombie firms of the country-industry pair between 2012 and 2015 on the y-axis. The graph on the right displays on the x-axis the change in productivity of good firms relative to poorly-performing firms in a country-industry pair, and the percentage change in the share of zombie firms of the country-industry pair on the y-axis. The sample includes Italy, Spain, Portugal—whose industries had been strongly hit by the burst of the sovereign debt crisis—and also France, and Germany—whose industries had been less affected by the shock and are therefore used as benchmark.

Panel B studies the implication of the presence of zombie firms for an industry productivity growth. The binned scatter plot displays the growth rate (in percent) of TFP of a given country×industry (asset-weighted EBITDA/Assets), on the y-axis, and the percentage change in the (asset-weighted) industry share of zombie firms on the x-axis. A dot represent either an macro-industry in Japan (right panel) or a group of country×4-digit industry- pairs in Europe displaying a similar change in the zombie share (left panel). For Italy, Spain, and Portugal, the change in industry share of zombie firms is the difference between 2012 and 2015; the percentage change in industry productivity of European industries is measured between 2012 and 2015; The TFP growth rate is measured between 2012 and 2015. For Japan, the graph replicates Figure 6 in Caballero et al. (2008). The change in the (asset-weighted) industry share of zombie firms is the difference between average share of zombie firms in the 1981–1992 period and the average share of zombie firms in the 1993–2002 period; the TFP growth rate is the average annual growth rate between 1990 and 2000. See Appendix B for additional information on data sources and variables' definition.

estate crisis (Figure 2B, right).[9]

Figure 3: Dynamic spillovers of zombie lending (Europe)



Notes: This figure reports the estimates of the coefficients β_h from the local linear projection $\ln(\text{TFP})_{ic,t+h} - \ln(\text{TFP})_{ic,t} = \alpha_i + \alpha_c + \beta_h \,\text{Zombie Share}_{ic,t} + \gamma_h \boldsymbol{X}_{ic,h} + \epsilon_{ic,t}$. The sample includes country × 4-digit industry pairs in southern Europe (Italy, Spain, and Portugal) between 2009 and 2015. TFP is the asset-weighted average EBITDA/Assets of non-zombies in a given country × 4-digit industry pair. The variable Zombie Share is the (asset-weighted) share of zombie in the country-industry pair. We standardize the zombie share to be mean zero and standard deviation one. The dotted and dashed lines represent the 90 and 95 percent confidence intervals of the estimates, respectively. Standard errors are clustered at the country-industry level. See Appendix B for additional information on data sources and variables' definition.

Importantly, further analysis of the European data underscores the intertemporal nature of these congestion externalities. We estimate local linear projections that capture the post-crisis, i.e., after the burst of the sovereign crisis ($t = 2011$), effect of the zombie share in a narrowly defined country × 4-digit industry pair on the cumulative percentage change in TFP of non-zombie firms in that country-industry pair in the subsequent years:

$$\ln(\text{TFP})_{ic,t+h} - \ln(\text{TFP})_{ic,t} = \alpha_i + \alpha_c + \beta_h \,\text{Zombie Share}_{ic,t} + \gamma_h \boldsymbol{X}_{ic,h} + \epsilon_{ic,t},$$

where α_i and α_c represent industry and country fixed effects, respectively. To isolate the effect of the lagged zombie share, the vector $\boldsymbol{X}_{ic,h}$ includes industry-country controls, which comprise two lagged values of the dependent variable ($\ln(\text{TFP})_{ic,t-1}$, $\ln(\text{TFP})_{ic,t-2}$) to control for predetermined heterogeneity in productivity as well as several leads of the independent variable ($\text{Zombie Share}_{ic,t+\tau}$, with $\tau = 1, \ldots, h$).

Figure 3 reports the estimates of the coefficients of interest, β_h for $h = 1, 2, 3, 4$, measuring the semi-elasticity of non-zombie firms' productivity to the penetration of zombies, as measured by

[9] Quantitatively, in Japan, CHK finds that, depending on the industry, the presence of zombies reduced other firms' cumulative investment and employment by 14 to 50 pp. and 5 to 19 pp., respectively. In the aftermath of the European sovereign crisis, Acharya et al. (2019) and Blattner et al. (2023) estimate that the reallocation of credit toward zombies can explain a 3 to 11 pp. employment loss of non-zombie firms experienced and a substantial portion of the observed decline of aggregate TFP.

the zombie share. We standardize the variable Zombie $\text{Share}_{ic,t}$ so that the coefficient captures the effect of a one–standard deviation increase in zombie's penetration in a country-industry pair. The data indicates that congestion externalities are persistent and unfold dynamically, gradually compounding over time. A one–standard deviation increase in the industry's zombie share leads to a cumulative 8 percent decrease in the productivity of non-zombies three years later.

The role of policy interventions—Finally, these Japanese and European episodes also offer insights into the role played by policy interventions. In both countries, policymakers implemented a series of unprecedented macro-financial measures in an effort to shield the real economy from the adverse effects of the real estate and sovereign debt crisis. These interventions featured capital injections into their national banking systems as well as incisive packages of forbearance measures in the form of implicit or explicit government guarantees, central bank liquidity support facilities, and delayed loss-recognition schemes. As shown in Table 1, equity injections were not able to adequately recapitalize the banking sector. The forbearance policies did help lower banks' cost of capital but also allowed weakly-capitalized banks to extend new credit or evergreen existing loans to borrowers who should have otherwise been deemed insolvent (Peek and Rosengren 2005; Giannetti and Simonov 2013; Acharya et al. 2019), generating negative spillovers.

3 A Model of Zombie Lending

We begin with a static model building on the empirical evidence presented in Section 2. This static model can be viewed as one period of the dynamic model presented in Section 5.

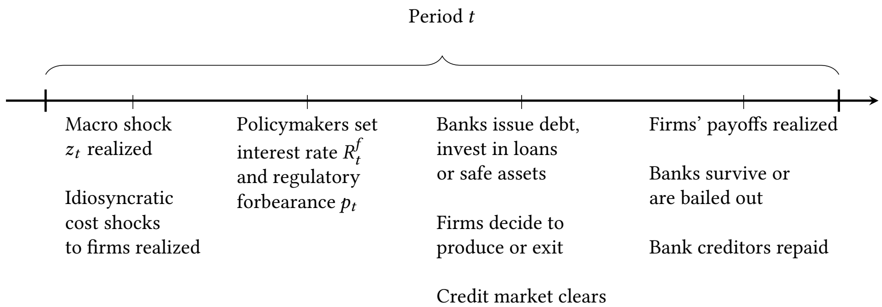
3.1 Environment: Firms, Banks, and Policies

The economy is populated by heterogeneous firms that differ in their productivity and risk. These firms' investments require credit, which is provided by heterogeneous banks that differ in their level of capitalization. Figure 4 shows a timeline of the events in a period.

3.1.1 Firms

There are two types of firms, G or B. Initially, the economy is populated by a unit mass of incumbent firms. A mass $(1 - \lambda)$ of incumbents are endowed with an indivisible project of type G, that yields revenues y^g in case of success and zero otherwise. A mass λ of incumbents are endowed with type B projects, yielding revenues y^b in case of success and zero otherwise. The success probability is θ^g for type G projects and θ^b for type B projects. Success is independent across firms, but the payoffs y^g and y^b are exposed to an aggregate shock z; we omit the dependence in z until Section 4. There are also potential entrants, each endowed with a type G project.

Figure 4: Timeline of events within a period.



Without loss of generality, we assume the mass of potential entrants to be equal to λ to simplify expressions.

Both types of projects require \$1 in capital to be implemented. Firms have no wealth, and need to finance their project entirely via bank debt. Firm types are observable to banks. Therefore, the debt contracts feature type-specific interest rates: G firms borrow at a rate R^g and B firms borrow at a rate R^b.[10]

In addition, all firms incur a production cost $\epsilon \in [0, \bar{\epsilon}]$ distributed according to the same c.d.f. H for both types of firms. The realization ϵ_i is known to the firm (but not to the bank) before production and financing decisions are made. Before their entry decision, potential entrants also observe their idiosyncratic cost ϵ, drawn from the same distribution H as incumbents.

Given the binary payoff structure, the project and the loan share the same risk: firms repay their loan entirely if their project succeeds, and default on the full loan if their project fails. We make the following assumption on payoffs:

Assumption 1. $\Delta\theta = (\theta^g - \theta^b) > 0$ *and* $0 \leq \theta^b y^b < \theta^g y^g - \bar{\epsilon}$.

In words, type B projects are riskier, which captures the fact that B firms have more outstanding debt and are thus more likely to default on their new loans. Moreover, even the least productive type G firms with a draw $\epsilon = \bar{\epsilon}$ are better than the most productive type B firms with a draw $\epsilon = 0$. The greater risk and lower profitability of type B projects mirror the characteristics of "zombie firms."[11]

[10] We call these "loans" even though given the binary project payoffs there is no difference between equity and debt contracts.

[11] Empirical studies document that zombie firms are riskier borrowers, as they tend to have higher leverage, lower net worth, higher interest rate coverage ratio, and lower profitability ratios than healthy firms (Hoshi 2006; Acharya et al. 2019).

3.1.2 Banks

There is a unit mass of heterogenous financial intermediaries (hereafter, banks) with a balance sheet scale of \$1. Banks are indexed by their exogenous equity e, distributed in the interval $[e_{\min}, e_{\max}]$ according to the c.d.f. F, with $0 \leq e_{\min} \leq e_{\max} < 1$. In Section 6 we allow e to be chosen endogenously subject to equity issuance frictions and capital requirements.

Each bank can invest its entire \$1 in a single asset, which can be either a risky corporate loan or a safe asset. Banks can lend to a type $j \in \{b, g\}$ firm at rate R^j, earning an expected return equal to $\theta^j R^j$. Credit markets are competitive: loan rates R^j are taken as given by both firms and banks, and determined in general equilibrium. Alternatively, banks can invest in "safe assets". We interpret safe assets as a broad class of assets held in banks' portfolios that are generally safer than corporate loans, such as mortgages, reserves, Treasuries, or asset-backed securities. Safe assets are supplied elastically and pay a risk-free return R^f set by monetary policy. Investment in safe assets does not produce output; results are unchanged if investments in safe assets yields a positive output lower than what is produced by lending to firms.[12]

On the liability side, a bank with capital e needs to raise $(1 - e)$ of debt in order to invest. In equilibrium, debt holders require an expected return equal to R^f. The actual contractual rate paid to debt holders by each bank, $\tilde{R}^j$, depends on the riskiness of banks' asset choice j and on the degree of regulatory forbearance indexed by a parameter p set by policy, as we describe next. Specifically, we assume that, under the forbearance policy, debt holders are able to recover their principal with probability $p \in [0, 1]$ if the bank defaults.[13] Thus the contractual rate $\tilde{R}^j$ on the debt of a bank that invests in asset $j \in \{g, b, f\}$ needs to satisfy

$$R^f = \theta^j \tilde{R}^j + \left(1 - \theta^j\right) p. \tag{1}$$

A key observation is that a positive p makes riskier investments more attractive. The expected payoff from choosing investment of type i bank with capital e is

$$\theta^j [R^j - \tilde{R}^j (1 - e)] = \theta^j R^j - R^f (1 - e) + \underbrace{p(1 - \theta^j)(1 - e)}_{\text{subsidy}}, \tag{2}$$

[12]The assumption that banks invest in a single asset captures more broadly bank specialization, which is common in the data, see e.g. Berger et al. (2017), Paravisini et al. (2020), and Blickle et al. (2023). Loans can be reinterpreted as being portfolios of loans to the sector in which individual banks have acquired information and competences. What matters is that banks with different leverage end up with different portfolios, instead of all investing in the same diversified loan portfolio. The assumption of full specialization could be relaxed by allowing banks to hold a portfolio of projects with correlated risks a la Vasicek (1977) without affecting the key message of the model.

[13]An alternative formulation would assume that the net interest $(\tilde{R}^j - 1)$ is also guaranteed with probability p. Our formulation yields simpler expressions throughout and is consistent with the typical insurance scheme offered to depositors (e.g., by the FDIC in the U.S).

where the last term is the policy-induced subsidy to type-j investments. Note that the subsidy is positive only if banks have some leverage *and* if they take positive risk; thus there is no subsidy for safe investments ($\theta^j = 1$) or for fully equity-funded banks ($e = 1$). Outside these extreme cases, the subsidy is increasing in p, in leverage $(1 - e)$ and in risk $(1 - \theta^j)$.

Relationship Lending and Evergreening. In the baseline model we focus on risk-shifting as a driver of zombie lending, and abstract from the role of relationship lending, and in particular from weak banks willing to "extend and pretend" by rolling over loans at subsidized rates to *legacy* borrowers that should be declared non-performing. We incorporate this important "evergreening" channel in Section 6.

3.1.3 Policy Instruments: R^f and p

Policymakers affect banks decisions through the choice of the two variables R^f and p. They directly control the level of the risk-free rate R^f through conventional monetary policy, but are subject to an "effective lower bound" (ELB):

Assumption 2. *There is an effective lower bound $R^f_{\min} > 0$ on the risk-free rate:*

$$R^f \geq R^f_{\min}. \tag{3}$$

The ELB can be interpreted as a standard zero lower bound due to the presence of cash.[14] It can arise more broadly as a constraint on monetary policy due to conflicting objectives. For instance, the central bank may be unable to further lower R^f to stabilize banks when it is already busy fighting inflation. Here we focus on a "bank lending channel" of monetary policy working through portfolio rebalancing and abstract from other channels, such as the aggregate demand channel emphasized in New Keynesian models. We will show that the interesting regime for zombie lending is when shocks z are large enough to make the ELB bind, hence other channels of monetary policy also become inoperative.

Policymakers also set the parameter p, which influences banks' cost of capital through the debt pricing equation (1): a higher degree of insurance p encourages risky lending by decreasing the associated cost of funds. We focus on the case of a guarantee p that applies independently of banks' choice of investment, and discuss alternative policies below.

Assumption 3. *p is independent of banks' portfolio choices.*

The level of p captures the extent of explicit and implicit government guarantees to banks, which are often reinforced when bad macroeconomic or financial shocks hit. It can also be viewed

[14] The lower bound on nominal rates translates into a lower bound on real rates if inflation expectations are sticky.

as indexing the leniency of bank closure policy: higher p means more regulatory forbearance. Closure can be avoided by rescuing a distressed bank, which in turns reduces the ex-ante risk-sensitivity of the bank's creditors. The relevant common thread of these unconventional policies is that a higher p undermines the market discipline imposed by bank creditors by decoupling the cost of funds from asset risk, which can become a valuable tool to stimulate lending once the constraint (3) starts binding.[15]

We use state-contingent guarantees as a convenient modeling device for a range of interventions meant to support lending by absorbing risk away from the private sector, which we broadly refer to p as "regulatory forbearance". One standard example is the state-contingent enforcement of capital requirements. Suppose that capital requirements are binding and bank equity is given by $e = \hat{e}$, where $\hat{e}$ is set by regulators (as we will model in Section 6). Holding the capital requirement $\hat{e}$ fixed, increasing p in bad times allows policymakers to stimulate risky corporate lending relative to investments in safe assets by increasing the subsidy $p(1 - \hat{e})(1 - \theta^j)$ defined in (2). Alternatively, suppose that there is a fixed level of guarantees (i.e., not state-contingent), but policymakers lower the capital requirement $\hat{e}$ in bad times. This is an equivalent way to increase the effective subsidy earned by banks, as a higher leverage $(1 - \hat{e})$ makes risky assets more attractive and thus stimulates lending just like an increase in p. Since the *product* $p \times (1 - \hat{e})$ is what matters for the subsidy and hence bank incentives, framing the model in terms of state-contingent guarantees allows us to illustrate the main mechanism without having to model banks' equity issuance decisions, which we leave for Section 6.[16]

Both forms of forbearance are widely used in practice. Governments and regulators indeed expand the coverage and depth of guarantees in bad times, without conditioning on the asset risk of individual banks. In the U.S., in the run-up to the Savings and Loans (S&L) crisis in the 1980s, the Depository Institutions Deregulation and Monetary Control Act increased deposit insurance from \$40,000 to \$100,000 per account with the purpose of curbing deposit outflows. The cap increased to \$250,000 in 2008 and the FDIC introduced the Transaction Account Guarantee Program guaranteeing corporate checking accounts without limit, and the Debt Guarantee Program providing debt guarantees to banks, until 2012. In the recent 2023 banking crisis, the FDIC effectively guaranteed all uninsured deposits of distressed banks and the Federal Reserve introduced the Bank Term Funding Program allowing banks to borrow against eligible securities at par.

[15] Acharya et al. (2019) provides evidence showing how central bank policies in Europe helped lowering banks funding costs at the cost of reducing market discipline, with substantial effects on banks' asset composition.

[16] There we also show how the simple equivalence between guarantees and regulatory forbearance is altered when banks face large costs of switching from old zombie borrowers to new healthy borrowers, including the costs of loss recognition.

3.2 Bank Portfolio Choice

A bank with equity e chooses among the three investment options (safe assets, lending to type G firms, lending to type B firm) to maximize expected profits. Taking as given p, the loan rates R^g, R^b and the risk-free rate R^f, the bank solves

$$\max_{i\in\{g,b,f\}} \theta^i \left[R^i - \tilde{R}^i (1-e)\right] \quad \text{s.t. } \tilde{R}^i = \frac{R^f - \left(1-\theta^i\right) p}{\theta^i}. \tag{4}$$

The subsidy defined in (2) is not only increasing in risk and leverage but also *supermodular* in these two variables, meaning that risk-shifting incentives induced by regulatory forbearance increase with bank leverage. When banks are heterogeneous in leverage, this complementarity between leverage and risk implies a natural "diabolical" sorting between poorly capitalized banks and riskier firms, as captured by the following lemma (proved in Appendix D) characterizing the solution of banks' problem as a function of their level of capitalization:

Lemma 1. *Define the following equity levels:*

$$e^* = 1 - \frac{(\theta^g R^g - \theta^b R^b)}{p\Delta\theta} \quad \textit{and} \quad e^{**} = 1 - \frac{(R^f - \theta^g R^g)}{p\,(1-\theta^g)}.$$

With parameters such that $e^* < e^{**}$, banks invest as follows:[17]

(i) Banks with equity $e < e^*$ lend to a type B borrower at rate R^b.

(ii) Banks with equity $e \in (e^*, e^{**})$ lend to a type G borrower at rate R^g.

(iii) Banks with equity $e > e^{**}$ do not lend and invest in safe assets at rate R^f.

Lemma 1 shows that the solution of banks' problem features a *diabolical sorting* of poorly-capitalized banks with low productivity firms. The lemma takes loan rates as given, but will continue to hold once rates are determined in general equilibrium. Figure A.1 offers a graphical intuition for the result, showing the expected profits from the three available investments as a function of bank capital e.

Heterogeneity in bank capital is not essential for our main dynamic result on policy traps in Section 5, and Lemma 1 also applies in the special case of homogeneous banks. Allowing for heterogeneous banks allows us to connect to the evidence cited in the introduction that poorly-capitalized banks are more likely to extend credit to risky and unproductive borrowers. In that sense, Lemma 1 provides indirect support for the kind of guarantees we assume. In this regard,

[17] Condition (A.3) in the Appendix ensures $e^* < e^{**}$. The only difference if $e^* \geq e^{**}$ is that region (ii) does not exist, i.e., no bank lends to a G firm. This leads to an even more extreme diabolical sorting: low equity banks lend to type B borrowers, and high equity banks invest in safe assets.

it is useful to contrast the effects of a uniform guarantee p with alternative policies, such as (i) subsidies that do not reward risk-taking, and (ii) more targeted subsidies that seek to stimulate lending while avoiding zombie lending.

Risk-Insensitive Subsidies. Equation (2) shows that the uniform guarantee p implies a "risk-based" subsidy, in the sense that the subsidy increases with the risk of bank assets. Consider instead a policy that subsidizes banks at a rate s that does *not* depend on risk, so that an asset of type i yields an expected payoff

$$\Pi^i(s) = \theta^i R^i - R^f(1-e) + s \tag{5}$$

where the subsidy s is the same across assets i. Such a policy would not induce risk-shifting, because banks' portfolio choice would be the same as without subsidy:

$$\Pi^i(s) \geq \Pi^j(s) \Leftrightarrow \Pi^i(0) \geq \Pi^j(0) \text{ for two assets } i \neq j.$$

The problem, however, is that a blanket subsidy s would also fail to encourage lending.[18] Thus *some* level of risk-dependence is required in order to stimulate good lending.

This point is consistent with, e.g., the evolution of the ECB's long-term refinancing operations (LTRO) policies since the Great Financial Crisis. The early design in 2008-2009 simply allowed banks to borrow at favorable terms against eligible collateral such as government bonds. Since the loan terms were not directly tied to corporate loans and those loans were not eligible as collateral, the policy was closer to the broad-based subsidy in (5) from the perspective of business lending. The risk-shifting was concentrated in government bonds instead, as treating a wide range of sovereign bonds as equivalent collateral made the riskier ones particularly attractive, just like the uniform p in our model (Drechsler et al., 2016; Acharya et al., 2021a). Muted lending effects led to a shift towards more risk-based subsidies, closer in spirit to our model's (2). In 2012, the ECB introduced the "Additional Credit Claims" framework allowing banks to pledge risky corporate loans as collateral. ECB-eligibility makes such loans more attractive, and the implicit subsidy is larger for riskier and more illiquid loans.[19] Individual countries also had ways to replicate this outcome on their own through "Government Guaranteed Bank Bonds" (Carpinelli

[18] Over time, however, the direct transfers s could potentially increase equity and decrease risk-shifting, but only if banks are forced to retain those earnings instead of paying them out as dividends. If the goal is to recapitalize banks, however, we show in Section 5.4 that a rapid and sufficiently large direct intervention would be more effective.

[19] See https://www.ecb.europa.eu/ecb-and-you/explainers/tell-me-more/html/acc_frameworks.en.html for a description of the ACC framework. The stated effect is to further stimulating lending: "*ACC frameworks have incentivised the acceptance of loans to smaller businesses and self-employed and private individuals as Eurosystem collateral for years. The temporary extension of ACC frameworks now allows the further easing of certain requirements for the acceptance of such loans. This can help banks to provide loans to the real economy.*"

and Crosignani, 2021).[20] In 2014, the ECB introduced *targeted* longer-term refinancing operations (TLTRO), which make the subsidized rate at which each bank can borrow a decreasing function of how much the bank lends to firms and households, without penalizing riskier lending, but excluding mortgage lending which is considered safer and less productive.[21]

Risk-Sensitive Guarantees and Capital Requirements. Risk-based regulation is a natural solution to the risk-shifting induced by government guarantees. Making p or capital requirements $\hat{e}$ a function of θ could allow policymakers to target more precisely their interventions. For instance, setting

$$p(\theta^g) > 0, \quad p(\theta^b) = 0$$

would allow policymakers to stimulate lending to G firms without subsidizing loans to B firms. An equivalent implementation would be to set a uniform guarantee p, but require banks to pay fairly priced deposit insurance premiums equal to the subsidy $p(1-\theta^b)(1-e)$ for loans to B firms, while waiving the premium for loans to G firms. Alternatively, regulators could impose capital requirements $\hat{e}(\theta)$ (i.e., requiring banks to raise sufficient equity to meet the constraint $e \geq \hat{e}$) such that the resulting subsidy $p(1-\hat{e}(\theta))(1-\theta)$ does not incentivize zombie lending at the expense of healthy lending. As stated in our Assumption 3, our results rely on policymakers being unable to design fully risk-adjusted policy instruments. Put differently, the forces in our paper reinforce the case for risk-sensitive regulation.

From a theoretical perspective, Chan, Greenbaum and Thakor (1992) show that risk-sensitive guarantees may not be incentive-compatible in a general environment with private information about assets and/or moral hazard in monitoring. In fact, our model highlights an additional challenge relative to the standard setting in which guarantees lead to over-investment in a single risky asset and regulators "only" need to find a way to tax unobserved risk. Our setting features two closely related risky assets, with under-investment in one (loans to G firms) and over-investment in the other (loans to B firms). Inducing risk-taking in G loans is the intended effect of the guarantees, but the key tension is that the government's objective is *non-monotonic* in terms of risk: the goal is to subsidize an intermediate level of risk-taking (lending to good firms), while avoiding activities that are "too safe" (safe assets) or "too risky" (zombie lending). Thus simply taxing risk

[20] Carpinelli and Crosignani (2021) study how in 2011, "*right after the LTRO announcement, the Italian government offered banks a guarantee on securities otherwise ineligible at the ECB by paying a fee. As the ECB accepts all government-guaranteed assets as collateral, the program effectively gave banks a technology to "manufacture" ECB-eligible collateral.*" Italian banks used this guarantee to essentially pledge their entire illiquid assets "*by issuing and retaining unsecured bank bonds [...] banks could then obtain a government guarantee on these newly created bonds (called Government Guaranteed Bank Bonds) so that they became eligible to be pledged at the LTRO.*"

[21] See https://www.ecb.europa.eu/mopo/implement/omo/tltro/html/index.en.html for details on the TLTRO programs.

is too blunt, and subsidizing good loans creates strong incentives for banks to make zombie loans appear performing.

In our view, the empirically salient assumption is that informational or institutional frictions prevent regulators from tailoring policies to bank assets. The information available to regulators to monitor a bank's risk-taking is mostly backward-looking, and the risk metrics available are often coarse due to banks' opacity and assets complexity. Moreover, information is often not available in a timely manner, or is too costly to collect at a high frequency. Stress tests are only conducted on a semi-annual basis and significant implementation costs restrict the number of scenarios considered (Parlatore and Philippon, 2024), whereas on-site inspections are typically randomized to economize supervisory resources (Passalacqua et al., 2020). Institutional frictions also play a role: p could be constrained to be the same across U.S. states or European countries in spite of large observable differences in the quality of local banks' balance sheets, as discussed previously in the case of the ECB's policies, and studied in the case of U.S. conforming mortgages by Hurst et al. (2016). While a large number of countries have established deposit insurance funds, banks frequently don't pay a deposit insurance premium (see Saunders et al. 2021 for a detailed discussion and international comparison). In the U.S., banks do pay a premium, but they stop paying after the Deposit Insurance Fund reaches a limit (decided by Congress and seldomly revised). Even when banks do pay a premium, the payment is only marginally connected to the risk of the bank's assets or liabilities. Under the Dodd-Frank Act, the FDIC forbearance toward failing systemically important institutions may even be charged to the healthier surviving institutions.

Remark 1 (Pricing of risk by bank debt holders.). Equation (1) implies that banks' funding costs increase with risk, albeit less so in the presence of government guarantees. Risk-sensitive pricing relies on debt holders being able to observe banks' asset risk. One explanation is that the market for uninsured deposits and subordinated debt can incorporate information unavailable to regulators in real time (Flannery 1998, Berger, Davies and Flannery 2000). In any case, the assumption that bank debt holders can discriminate between risky and safer banks is not crucial, as removing risk-sensitive pricing of bank debt would in fact strengthen banks' risk-shifting motive. To see this, suppose that banks' funding costs become completely insulated of their asset risk, that is $\tilde{R}^g = \tilde{R}^b = \tilde{R}$ with $R^f \leq \tilde{R} \leq R^f/\theta^g$.[22] Comparing to expression (2) shows that this is equivalent to an implicit guarantee $p(\theta^i) = R^f - \frac{\theta^i}{1-\theta^i}(\tilde{R} - R^f)$ which is risk-sensitive, but *in the wrong direction*, as $p(\theta^i)$ decreases with θ^i. Thus funding costs that are insensitive to risk imply an even stronger subsidy towards riskier assets. This configuration, which is empirically plausible, would reinforce risk-shifting towards type-B loans relative to our benchmark with a common p.

[22]The inequality $\tilde{R} \leq R^f/\theta^g$ means that the funding cost for a bank lending to G firms is weakly lower than without government guarantees.

3.3 Equilibrium, Congestion Externalities, and Policy Objective

We need to determine both the equilibrium allocation of bank capital (aggregate lending and aggregate investment in safe assets) and the composition of lending (good versus bad types of firms). As we explain below, the highest level of aggregate output is achieved when there is maximal *creative destruction*. That is, all the type B incumbent firms exit, and are replaced by more productive type G entrants. We model the entry and exit process building on CHK, with the additional layer of banks' portfolio choices. Equilibrium loan interest rates are the variables that adjust to bring about, or hinder, creative destruction.

Firms' Entry and Exit Decisions. Given the realization of production costs ϵ and the borrowing rates offered by banks, incumbent firms decide whether to produce or exit, and potential entrants decide whether to enter or not. Incumbents remain in business and undertake their project if and only if they expect positive profits, which happens if and only if the idiosyncratic cost realization ϵ is lower than a type-specific threshold $\tilde{\epsilon}^i$, $i = g, b$. A type i incumbent drawing ϵ produces if

$$\epsilon \leq \tilde{\epsilon}^g = \theta^i \left(y^i - R^i\right) \tag{6}$$

and exits otherwise. The masses of active firms of type G and B are respectively

$$\begin{aligned} m^g &= \underbrace{(1-\lambda)\, H\left(\tilde{\epsilon}^g\right)}_{\text{incumbents}} + \underbrace{\lambda H\left(\tilde{\epsilon}^g\right)}_{\text{entrants}} = H\left(\theta^g\left(y^g - R^g\right)\right), \\ m^b &= \lambda H\left(\theta^b\left(y^b - R^b\right)\right). \end{aligned} \tag{7}$$

m^g and m^b are the aggregate loan demands from each type of firm. There is no intensive margin adjustment as projects are all of unit size, but higher loan rates decrease aggregate loan demand at the extensive margin.

Equilibrium. Given policies $\left(R^f, p\right)$, the static general equilibrium of the model is characterized by loan rates $\left(R^g, R^b\right)$ such that agents optimize and the two market clearing conditions hold:

$$F\left(e^*\right) = m^b = \lambda H\left(\theta^b\left(y^b - R^b\right)\right), \tag{8}$$

$$F\left(e^{**}\right) - F\left(e^*\right) = m^g = H\left(\theta^g\left(y^g - R^g\right)\right), \tag{9}$$

where the thresholds e^* and e^{**} are defined in Lemma 1.

Equation (8) equalizes the supply of zombie loans, by banks with equity $e < e^*$, to the demand from type B firms with idiosyncratic shocks $\epsilon \leq \theta^b\left(y^b - R^b\right)$. Similarly, equation (9) describes market-clearing for good loans.

Aggregate Output. Given equilibrium loan rates, aggregate output can be written as

$$Y = \int_0^{\theta^g(y^g - R^g)} [\theta^g y^g - \epsilon] dH(\epsilon) + \lambda \int_0^{\theta^b(y^b - R^b)} [\theta^b y^b - \epsilon] dH(\epsilon) \,. \tag{10}$$

The first term in (10) captures the net contribution of type G firms (both incumbents and entrants). The second term is the net contribution of type B firms. By Assumption 1 both terms are positive, thus lower lending rates R^g and R^b increase aggregate output by stimulating the entry and continuation of productive firms. Note that type B firms are not negative-NPV from a partial equilibrium perspective. However, they are relatively worse firms that prevent scarce resources (such as bank loans but also other inputs, as we discuss next) from going to more efficient firms, with a negative impact in general equilibrium.

We define *potential output* Y^* as the highest possible aggregate output the economy can achieve given its fundamentals, given by $Y^* = \theta^g y^g - \mathbf{E}[\epsilon]$. According to equation (10), the economy attains Y^* when all bank capital is used to finance the productive sector (i.e., there is no investment in bonds) and, within the productive sector, the most productive firms (i.e., there is no zombie lending).

Congestion Externalities and Policy Objective. When analyzing optimal policy, we assume policymakers set policies (p, R^f) to maximize output net of *congestion externalities* resulting from zombie lending. A substantial body of empirical evidence, discussed in Section 2, highlights that zombie firms can impact the performance of healthier firms in the economy through various channels, such as congestion in labor and input markets, congestion in output markets due to price competition, or reduced innovation incentives.

We start by introducing these various congestion externalities through a reduced-form deadweight loss in the policy objective; in Section 5 we revisit the role of externalities and provide a dynamic specification that more closely matches the empirical evidence. The policy objective is

$$Y - \beta\Gamma(m^b) - \zeta p \int_{e_{\min}}^{e_{\max}} (1 - \theta^{j(e,R^f,p)})(1-e) dF(e), \tag{11}$$

where Γ is an increasing and convex function of the extent of zombie lending m^b, and there is no externality if $m^b = 0$, that is, $\Gamma(0) = 0$. The parameter $\beta \geq 0$ denotes the policy weight on congestion externalities (which we will map to policymakers' discount factor in the dynamic model).

The third term in (11) denotes the fiscal costs of insuring banks at the expense of taxpayers, where $j(e, R^f, p) \in \{g, b, f\}$ is the optimal portfolio choice for a bank with capital e given policy (R^f, p) and $\zeta \geq 0$ is the shadow cost of public funds. We assume that ζ is infinitesimal, hence fiscal costs are irrelevant except to break ties: if different combinations of p and R^f can achieve the same level of output net of congestion externalities (11), policymakers strictly prefer policies that minimize p. A non-negligible fiscal cost ζ would lead to a lower optimal p and thus be similar to increasing β. Therefore we define the *optimal* policy as follows:

Definition 1. The optimal policy is the combination $\left(p, R^f\right)$ that minimizes p among the set of policies that maximize $Y - \beta\Gamma(m^b)$.

4 Optimal Policy Response to Aggregate Shocks

Having set up the model and the policy objective, we now analyze how policymakers can optimally combine their instruments R^f and p to maximize their objective (11), and how the optimal policy mix should respond to shocks to fundamentals.

The two variables R^f and p impact banks' decisions—and therefore credit allocation—through two different channels. The first channel is a standard *bank lending channel*, that is, the choice between investing in safe assets versus lending to the productive sector. A lower R^f stimulates lending to both types of firms by decreasing the return of investing in safe assets relative to loans. Government guarantees subsidize riskier investments, thus a higher p also stimulates lending to both types of firms, by lowering the cost of funds.

The second channel is the *zombie lending channel*, operating through the choice between lending to different types of borrowers. A higher p not only makes lending in general more appealing, but it also increases the profits from loans to B firms relatively more. These loans are riskier, thus a given subsidy p lowers the cost of funds by more when lending to B firms, through the term $\left(1-\theta^b\right)p$ in (1). As we will show, the incentives to lend to one type of firm or the other are bank-specific, as they depend on bank capitalization.

4.1 Shocks

We assume that firm outputs $y^g(z)$ and $y^b(z)$ are *decreasing* functions of an aggregate productivity or demand shock z

$$y^i(z) = \bar{y}^i(1-z) \quad \text{for } i = g, b, \tag{12}$$

where z lies between 0 and $z_{\max}$ such that Assumption 1 holds even for $z = z_{\max}$. Therefore potential output Y^* also decreases with z. This shock to aggregate productivity corresponds to the empirical evidence in Figure 2 (left plot) in Section 2.[23]

[23]In Appendix C.1 we generalize the shock structure to allow for asymmetric shocks, and show that shocks that reduce the profitability gap between good and bad firms lead to even more zombie lending, consistent with Figure 2 (right plot) in Section 2.

4.2 Optimal Policy

We first characterize when potential output can be attained, as any policy that yields $Y = Y^*(z)$ must be optimal according to criterion (11). When there is no binding ELB (3), this can be achieved through a sufficiently low risk-free rate R^f that discourages substitution towards safe assets, together with a low subsidy p that curbs the risk-shifting incentives of poorly-capitalized banks:

Proposition 1. *Given z, there exist a threshold $\bar{p}(z) > 0$ and a function $\bar{R}^f(p; z)$ increasing in p and decreasing in z, such that output reaches its potential, $Y = Y^*(z)$, if and only if the following two conditions hold:*

$$\begin{cases} R^f \leq \bar{R}^f(p; z) & \textit{(full lending)} \\ p \leq \bar{p}(z) & \textit{(no zombie lending)} \end{cases}$$

For any $p > \bar{p}(z)$, zombie lending necessarily emerges in equilibrium.

The "full lending" condition $R^f \leq \bar{R}^f$ ensures that the return on safe assets is sufficiently low to make lending attractive to the least leveraged banks ($e = e_{\max}$), who benefit the least from government guarantees. The "no zombie lending" condition $p \leq \bar{p}$ ensures that the most leveraged banks ($e = e_{\min}$), who benefit the most from guarantees, still prefer to lend to type G firms.

If $\bar{R}^f(0; z)$ is above the ELB $R^f_{\min}$, then the optimal policy is to set $R^f(z) = \bar{R}^f(0; z)$ together with $p(z) = 0$. This achieves potential output without any congestion externality, since the mass of zombie firms m^b remains at zero. Thus $\bar{R}^f(0; z)$ provides a notion of the "natural interest rate", that is the interest rate required to achieve Y^* without subsidy. The natural rate fluctuates with fundamentals: larger z shocks must be accommodated by a lower risk-free rate, exactly as in standard macroeconomic models.

The most interesting case is when R^f cannot be made arbitrary low. We focus on two polar cases that are sufficient to illustrate the economic mechanisms: (i) when the weight β on congestion externalities is low, in which case the optimal policy is just to maximize bank lending; and, (ii) when β is high, in which case the optimal policy features "No Zombie lending". Between these extremes, the optimal policy may be characterized by an interior solution that trades off the marginal congestion externality $\beta\Gamma'(m^b)$ with the output loss from reducing bank lending, and thus requires an intermediate level of accommodation.

Low congestion β: Output-maximizing policy. If the welfare weight on congestion externalities is low enough relative to the output gains from zombie lending

$$\beta\Gamma'(1) < \theta^b \bar{y}^b (1 - z_{\max}) - \bar{\epsilon}, \tag{13}$$

then the optimal policy simply maximizes bank lending and output, by setting p at a high enough level to ensure that no bank invests in safe assets.

There are two thresholds $\underline{z}$ and $\bar{z}$. In line with Proposition 1, following small shocks $z \leq \underline{z}$, an accommodative conventional monetary policy can achieve Y^* at no costs ($p = 0$). Moderate shocks $z \in \left[\underline{z}, \bar{z}\right]$, however, require combining conventional monetary policy and forbearance policy in order to keep the economy at its full capacity. Specifically, a positive p helps stabilize output once conventional monetary policy is constrained by the lower bound ($R^f = R^f_{\min}$). In this region, the optimal forbearance increases in response to more severe shocks. The increase in p subsidizes bank lending as much as possible, but all the lending is to type G firms. Thus if shocks are moderate, some forbearance $p > 0$ is sufficient to attain Y^*.

Once the shock is severe enough, $z > \bar{z}$, stimulating aggregate lending necessarily triggers some zombie lending by banks at the bottom of the equity distribution.

High congestion β: No-Zombie Lending policy. If congestion externalities are costly enough:

$$\beta\Gamma'(0) > \theta^b \bar{y}^b, \tag{14}$$

then the marginal output gain from zombie lending is not worth bearing congestion externalities. The optimal policy is then to maximize output while preventing zombie lending, i.e., keeping $m^b = 0$. Our main result in this case is that the optimal forbearance policy $p(z)$ is non-monotonic in the size of the shock.

As in the previous case, for shocks $z < \bar{z}$, the economy can achieve its potential $Y = Y^*(z)$. If the economy is hit by severe aggregate shocks $z > \bar{z}$, conventional monetary policy is still constrained by the effective lower bound, but now the optimal forbearance needs to balance two opposite forces. On the one hand, an increase in regulatory forbearance (higher p) spurs lending at the expense of investment in safe assets. On the other hand, if forbearance p is too high, poorly-capitalized banks engage in zombie lending, which creates congestion externalities.

As a result, for large enough shocks, policymakers must optimally *reduce* the degree of regulatory forbearance p as shock size z increases, and allow some banks to retrench from lending and invest in safe assets instead. Aggregate output Y necessarily falls short of its potential $Y^*(z)$. Put differently, when severe aggregate shocks hit the economy, policy should allow healthy banks to start hoarding safe assets, rather than "pushing on a string": more accommodation would only trigger more zombie lending by the poorly-capitalized banks. The key point is not that policy should not increase $p(z)$ given a high weight β on congestion externalities, but that even holding p fixed would trigger more zombie lending, hence p should actually be reduced when z is larger. Our result thus shows that there exists a "reversal" level of forbearance p above which further accommodation becomes harmful, a counterpart to the "reversal interest rate" for conventional monetary policy (Abadi et al., 2023).

Proposition 2 formalizes these results. The proof, including the definitions of the thresholds $\underline{z}$ and $\bar{z}$, is in Appendix D.

Proposition 2 (Optimal policy). *There exist thresholds $\underline{z} > 0$ and $\bar{z} > \underline{z}$ such that the optimal policy response to an aggregate shock z is the following:*

(i) *For small shocks $z \leq \underline{z}$, conventional monetary policy alone achieves Y^*. The optimal policy features $p = 0$ and an interest rate $R^f(z)$ that decreases with the size of the shock.*

(ii) *For moderate shocks $z \in (\underline{z}, \bar{z}]$, forbearance policy p can achieve Y^*. The ELB binds, $R^f = R^f_{\min}$, and the optimal $p(z)$ increases with the size of the shock.*

(iii) *For larger shocks $z > \bar{z}$, Y^* is not attainable. The ELB binds and the optimal $p(z)$*

(a) increases with the size of the shock in the case of a low β (13);

(b) decreases with the size of the shock in the case of a high β (14).

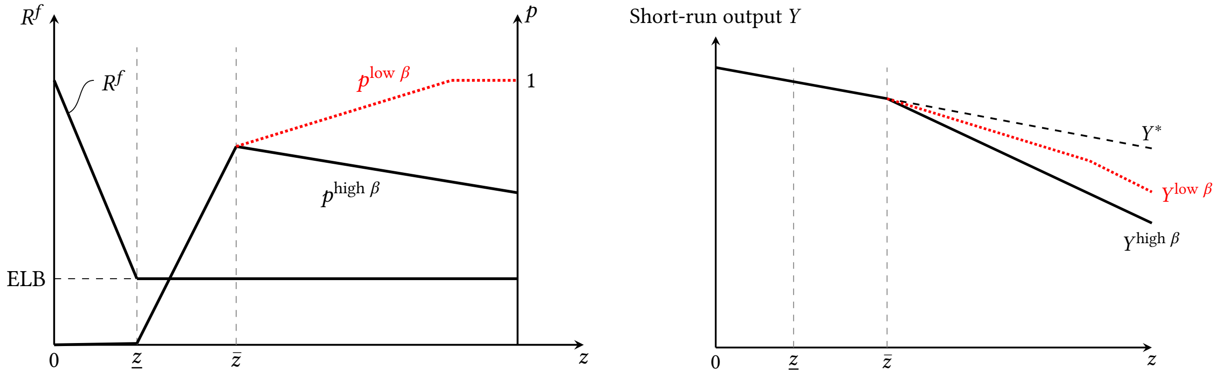
Figure 5 illustrates the result by contrasting the two policy regimes and the level of output achieved by the two policies in the short run. The only difference arises for p in the case of large shocks $z > \bar{z}$. The high-β policy backtracks and reduces forbearance p as shocks grow larger, whereas the low-β policy keeps accommodating more and more until it hits the upper bound $p = 1$.

Proposition 2 especially highlights the role of *large shocks*. A lack of profitable investment opportunities for good firms is not only detrimental per se, but it also makes zombie lending more attractive to banks. Thus zombie lending tends to emerge after large shocks that hit economies with a weak banking sector. While this is optimal in the case of a low β, it must be prevented in the case of a high β, by tightening policy in spite of large shocks z.[24]

Complementarity between bank capital and stabilization policy. Our model also highlights that the capitalization of the banking system not only plays a crucial role in determining the allocation of credit—as illustrated in Proposition 1—but also mediates the effectiveness of policy interventions following real economic shocks. The threshold shock size $\bar{z}$ in Proposition 2 depends on the minimal level of equity $e_{\min}$:

[24] Note that our results abstract from the frictions in the bankruptcy system that may also follow such large shocks. A massive wave of bankruptcies may lead to court congestion, fire sales, and widespread financial stress, calling for a richer set of policies than those we consider, such as those analyzed in Gourinchas et al. (2020) and Greenwood, Iverson and Thesmar (2020).

Figure 5: Optimal policy as a function of shock z under high-β and low-β policy regimes.



Note: The left panel illustrates the optimal joint policy response (R^f and p) when β is high (solid black line) and when β is low (red dotted line) as a function of the size of the shock z. The right panel illustrates the aggregate output Y under the policy regimes and potential output Y^* (dashed line).

Corollary 1. *An improvement in the health of weak banks (higher $e_{\min}$) leads to a more resilient economy, in the sense that policy can achieve Y^* in response to a larger range of shocks $z \in [0, \bar{z}]$.*

This result links the effectiveness of accommodative policy to the level of capitalization of the banking system, consistent with the evidence in Acharya et al. (2020). Our emphasis on widespread zombie lending as a central constraint on policy complements other contexts where weak bank balance sheets undermine policy effectiveness (e.g., Bernanke and Gertler 1995, Kashyap and Stein 2000, Van den Heuvel 2002, Bolton and Freixas 2006, Gambacorta and Shin 2018).

To summarize, the single-period theoretical framework reproduces some key empirical findings relating the allocative efficiency of credit markets, optimal policy actions, and the capitalization of the banking system, recognizing that zombie lending has real spillover effects in the form of negative externalities imposed by unproductive firms on the other firms in the economy. Next, we study the dynamic implications of zombie lending in the presence of these externalities.

5 Dynamic Model: Policy Traps and Sclerosis

Zombie lending is far from being a temporary problem. As discussed in Section 2, a substantial body of empirical evidence from the historical experiences of Japan and Southern European countries suggests that the adverse effects of credit misallocation on healthy firms due to the proliferation of zombie lending practices might be persistent and even compound over time.

To incorporate these features we turn to a dynamic version of our model that emphasizes how the interplay of accommodative policies and zombie lending can lead to persistent output losses and policy traps. Our main result shows that in response to transitory shocks and policies seeking to avoid a recession in the short run, the economy can get stuck in a state of permanent low productivity and output (which we call "sclerosis") with policymakers forced to implement a combination of low interest rates and high forbearance (which we call a "policy trap"). We then discuss how the economy can exit such a trap through a large bank recapitalization or an improvement in the productivity of good firms.

5.1 Dynamic Environment

To analyze the short-run and long-run implications of zombie lending, we provide a dynamic foundation for the congestion externalities $\Gamma(m^b)$ imposed by the presence of zombie firms. As in Section 4.1, we assume the economy is hit by an adverse aggregate shock z_0 at time $t = 0$, which affects the productivity of all firms as in (12): $y_0^i = \bar{y}^i(1 - z_0)$ for $i = g, b$. To capture that the full cost of keeping zombie firms alive materializes over time, we assume the presence of type B firms hurts the productivity of healthy firms in the next period:

Assumption 4. *For $t \geq 0$, type-G firms' productivity follows $y_{t+1}^g = \bar{y}^g(1 - z_{t+1})$, where the endogenous output loss z_{t+1} increases with the extent of zombie lending in the previous period*

$$z_{t+1} = \alpha m_t^b \quad \textit{for } t \geq 0. \tag{15}$$

The parameter $\alpha \geq 0$ is the counterpart of Γ' in the static model, capturing the strength of the congestion externalities. The key point is that zombie lending has a *persistent* effect on healthy firms.[25] Several complementary mechanisms could generate persistence, through different notions of "capital", including customer bases, labor forces, and intellectual capital. Zombie lending also allows type-B firms to make investments whose impact on other firms only materializes at $t + 1$, as in time-to-build models (Kydland and Prescott, 1982), or persists due to other forms of slow equilibrium adjustment. For example, Asriyan et al. (2024) shows that relaxing unproductive firms' financial constraints allows them to bid up the price of capital, which can ultimately crowd out investment by more productive firms. If the supply of capital responds slowly to the higher price (due to, e.g., standard adjustment costs), then the misallocation of scarce capital propagates the initial misallocation of scarce bank lending over time. Secondly, as in the literature on customer markets (Phelps and Winter 1970, Ravn, Schmitt-Grohé and Uribe 2006, Gourio and Rudanko 2014) or, relatedly, customer switching costs (Klemperer, 1987), if customers become "attached" to type-B firms at date t, it becomes more difficult for type-G firms to compete in future periods. Recognizing the persistence of customer bases, type-B firms could even take advantage of zombie loans to charge especially low prices to attract more customers at t, consistent with the evidence in Acharya et al. (2024) on the disinflationary effects of zombie lending on product

[25] Remark 2, following our results, explains how Assumption 4 could be relaxed.

prices. Similarly, zombie lending could facilitate labor hoarding or low-quality matches between firms and workers, with lasting negative productivity effects in the presence of labor market frictions (Bertola and Caballero, 1994; Barlevy, 2002). Lastly, Schmidt et al. (2023) find that zombie lending reduces patent applications and R&D spending, particularly in R&D-intensive and highly competitive sectors.

Empirically, Caballero et al. (2008) and Acharya et al. (2019) find evidence of spillover effects imputable to the presence of zombie firms. The motivating evidence we present in Section 2 underscores the intertemporal nature of these effects, which appear to be persistent and gradually compound over time, as shown in Figure 3.

Bank and firm dynamics. To close the model, we need to specify the dynamics of the banking sector. Bank returns are stochastic, with some banks failing and others making large profits. In general, accounting for bank entry and exit and tracking the evolution of the full distribution of bank equity presents significant technical challenges, similar to the ones encountered in macroeconomic models with heterogeneous households and incomplete markets. We thus make the following assumptions to make the dynamic model tractable:

Assumption 5 (Bank dynamics)**.** *There are overlapping generations of bankers: bank managers at t are replaced after one period and earn a fraction ρ of the income accruing at $t+1$. The manager of a bank with date-t equity e_t chooses project $i \in \{b, g, f\}$ to maximize*

$$\rho\theta^i \left[R_t^i - \tilde{R}_t^i (1 - e_t)\right].$$

At the beginning of each period $t+1$, after date-t bank managers have been paid and replaced, failing banks are replaced by new banks and the profits of all surviving banks are pooled together and redistributed to all banks equally and banks raise equity $\iota > 0$.

This simplification allows us to keep track of the evolution of the aggregate capitalization of the banking system, rather than the entire distribution of bank equity. Lemma 1 continues to apply as we collapse the distribution of banks: even though the portfolio of individual banks is indeterminate in the limit of homogeneous banks, the aggregate portfolio of the banking system is well-defined, which is all we need to study the output effects of zombie lending.

The short-term nature of bank managers' contracts implies that banks' franchise value does not enter the bank investment problem, therefore banks' portfolio choice is the same as static problem of Section 3. In particular, given date-t equilibrium rates, the optimal portfolio choice is characterized by the same thresholds e_t^* and e_t^{**} stated in Proposition 1. In a more general setting, banks would have to consider their franchise value when choosing their portfolios, which would then feed back into the equilibrium thresholds e_t^* and e_t^{**}. Accounting for the effect of the franchise value on bank's portfolio choices is an interesting extension that we leave for future research.[26]

Like before, in each period, a fraction λ of incumbent G firms turn permanently into B firms, and there is a mass λ of type-G potential entrants.

Equilibrium. Given a path of policies $\left\{R_t^f, p_t\right\}$ and fundamentals $\{y_t^g, y_t^b\}$, a dynamic equilibrium is a sequence of masses $\left\{m_t^b, m_t^g, m_t^f\right\}$, equity e_t, and loan rates $\{R_t^g, R_t^b\}$ such that for all t, banks sort optimally, bank equity e_t follows Assumption 5, markets clear, and productivity follows (15).[27]

Next, we describe how policies are determined depending on policymakers' objectives, and characterize the resulting equilibria.

5.2 Policymakers' Objective and Policy Rules

The dynamic equilibrium depends on the path of policies $\left\{p_t, R_t^f\right\}$, which in turn are set by policymakers depending on their objective function. We assume that the policy objective is to maximize the present discounted value of aggregate output:

$$\max_{\left\{p_t, R_t^f\right\}} \sum_t \beta^t Y_t.$$

Unlike in Section 3, policymakers only care about output. The reduced-form externalities $\Gamma(m_t^b)$ in the static model can be interpreted as the present value of future productivity losses for good firms due to current zombie lending. Lending to type B firms has short-term benefits but possible long-term costs, and the optimal policy depends on how much weight policymakers put on current lending relative to future productivity, as captured by β. Therefore the welfare weight β put on congestion externalities in the static model can be micro-founded as the discount factor of policymakers.

As in the static model, we focus on two polar cases: a "No Zombie lending" policy under high β, chosen by patient policymakers concerned about long-run productivity, and a short-termist or "myopic" policy under low β, chosen by policymakers who are effectively more impatient. We interpret a low policy horizon as arising from term limits, regulatory capture by incumbents, or reputational concerns that create a wedge between the public and regulatory objectives, as an-

[26] We also assume that firms are focused on short-term profits, hence their entry and exit decisions are the same as in the static model; unlike the assumption on the bank side which simplifies the dynamic model considerably, the assumption on firms is mostly for exposition and can be relaxed to allow for forward-looking firms, see Appendix C.4.

[27] The full expressions defining optimal sorting, bank dynamics, and market clearing are in Appendix C.3.

alyzed, for example, by Boot and Thakor (1993). Another interpretation is to think of the short policy horizon as a reflection of policymakers' inability to implement policies that have immediate fiscal costs. Fiscally constrained governments tend to help financial institutions in distress by deploying guarantees and/or engaging in some form of forbearance, rather than promptly intervening with capital injections or restructuring and resolution measures (Acharya et al., 2021a).

Long horizon: No Zombie lending policy. The *No Zombie lending (NZ) policy*

$$p^{NZ}(z_t, e_t)$$

is the high-β optimal policy in the static model described in Proposition 2, in the special case of a degenerate equity distribution with $e_{\min} = e_{\max} = e_t$. In particular, p^{NZ} is non-monotonic in z_t. For moderate shocks (as long as Y^* can be reached), regulatory forbearance p increases with the shock z_t; for large shocks, the optimal p decreases with z_t (see Figure 5). Preventing zombie lending has a cost: it leads to a lower short-run output Y_t than under the policy that maximizes short-run output (described next), as some healthy banks end up investing in safe assets instead of lending. A policymaker with a high enough discount factor β is willing to bear this cost to maintain future productivity.

Short horizon: Myopic policy. Conversely, a policymaker with a sufficiently low discount factor β chooses to minimize the short-term costs of the shock z. This might require allowing zombie lending in equilibrium, even if doing so jeopardizes future productivity and output. Specifically, the optimal *myopic policy*

$$p^m(z_t, e_t)$$

is the low-β optimal policy in Proposition 2. It maximizes short-run output at each point in time by ensuring that all banks lend ($m_t^g + m_t^b = 1$) but ignoring congestion externalities and future productivity losses. As a result, the optimal myopic p is increasing in the size of the shock z: larger shocks are accommodated with a higher p, until p reaches its upper bound of 1.

5.3 Persistence of Output Losses under Different Policy Regimes

We now turn to our main dynamic experiment and result: transitory shocks can generate permanent output losses and policy traps due to the dynamic externalities imposed by zombie lending. Suppose the economy starts in a "good" steady state in which the zero lower bound is not binding: $R^f = (\theta^g \bar{y}^g - \bar{\epsilon}) > R^f_{\min}$. Thus no forbearance is needed ($p = 0$), there is no zombie lending, aggregate output is $Y = Y^*$, and equity is $e_0 = \frac{\iota}{1-(1-\rho)R^f}$.

At date-0 a transitory shock $z_0 > 0$ hits, so that $y_0^g = \bar{y}^g (1 - z_0)$. We contrast the paths of the economy under the No Zombie lending and myopic policy rules. Recall from Proposition 2 that there exists a threshold $\bar{z}$ such that for shocks $z_0 \leq \bar{z}$, optimal policy can still attain the potential output Y^* without triggering any zombie lending. Therefore the NZ and myopic policies only differ for large shocks $z_0 > \bar{z}$. Let us then restrict attention to large enough shocks $z_0 > \bar{z}$. Under both policy stances, the optimal conventional policy implies setting the minimal risk-free rate $R_t^f = R_{\min}^f$ as long as as $z_t > \bar{z}$.[28] However, the paths of p_t will differ across policy stances markedly. In fact, we show that seemingly small within-period differences between the NZ and myopic policies can lead to completely different long-run outcomes.

No Zombie Lending Policy: Transitory Recession and Full Recovery. Under the NZ policy (high β), congestion externalities never materialize since there is no zombie lending in any period in equilibrium. The endogenous component of productivity losses is always zero, and since there are no further exogenous shocks, z reverts immediately to zero starting from date-1 ($z_t = 0 \quad \forall t \geq 1$). The date-0 recession is "V-shaped": it can be quite deep, but remains short-lived. Output recovers immediately from the transitory aggregate shock. The following proposition formally describes the full equilibrium path:

Proposition 3. *Under the No Zombie lending policy, the transitional dynamics for policies and aggregate output following the shock z_0 are given by*

$$\begin{array}{ll}
\underline{t = 0} & \underline{\textit{for all } t \geq 1} \\
R_0^f = R_{\min}^f & R_t^f = \theta^g \bar{y}^g - \bar{\epsilon} \\
p_0 = \dfrac{R_{\min}^f - \theta^b \bar{y}^b (1 - z_0)}{(1 - e_0)\left(1 - \theta^b\right)} & p_t = 0 \\
Y_0 = Y_0^{NZ} < Y^*(z_0) & Y_t = Y^*(0)
\end{array}$$

Myopic Policy: Policy Trap and Sclerosis. Under a myopic policy regime (low β), policymakers accommodate using regulatory forbearance, and allow some zombie lending at any date t, in spite of the potential long-term costs on the productivity of healthy firms.

Combining the mass of zombies at date-t and (15) yields a first-order Markov process for z

$$z_{t+1} = \alpha \lambda H \left(\theta^b y_t^b - R_{opt}^f(z_t) + p^m(z_t, e_t)(1 - e_t)\left(1 - \theta^b\right) \right).$$

[28] Recall that we abstract from the aggregate demand channel of monetary policy, by which a lower rate and higher aggregate demand could dampen congestion externalities, thus making α lower in states such that the ELB is not binding. Accordingly, the parameter α should be interpreted as the strength of congestion externalities *conditional on the ELB binding*, which is necessary for zombie lending to emerge and thus for α to be relevant. The binding ELB also prevents monetary policy from dampening congestion through the aggregate demand channel.

In particular, since $z_0 > \bar{z}$ the date-0 mass of zombies m_0^b will be positive, which hurts the productivity of good firms at date-1 through $z_1 > 0$, and so on. The myopic policy creates an endogenous "reverse hysteresis" channel: current accommodation leads to endogenous persistence of the initial shock, that worsens when congestion externalities α are larger. If α is high enough, the myopic policy response to a sufficiently severe transitory shock z_0 pushes the economy to a steady state with *permanently* lower output, defined as follows:

Definition 2 (Sclerosis steady state)**.** A sclerosis steady state is a steady state equilibrium with the interest rate at the ELB ($R^f = R^f_{\min}$), permanent forbearance ($p > 0$) and potential output permanently depressed ($z > 0$).

Unlike in standard macroeconomic models, the natural rate becomes an endogenous variable. Sclerosis is associated with a *policy trap*: present policies aimed at minimizing short-term losses tie the hands of future policymakers through their effect on future productivity. As a result, the economy may be stuck at the ELB forever even though the natural interest rate would recover to a positive level under a different policy rule. We can now express our main dynamic result:

Proposition 4 (Myopic policy and sclerosis)**.** *Suppose that congestion externalities are large enough, $\alpha \geq \bar{\alpha}$, for some positive $\bar{\alpha}$ (given in Appendix D) and the technical condition (A.4) in the Appendix holds. Then,*

1. *There exists a unique stable sclerosis steady state. It features maximal forbearance $p = 1$ and permanent output losses $z_\infty > 0$ such that*

$$z_\infty = \alpha\lambda H\left(\theta^b \bar{y}^b - R^f_{\min} + (1 - e_\infty)\left(1 - \theta^b\right)\right)$$

where $e_\infty = \frac{\iota}{1-(1-\rho)R^f_{\min}} < e_0$ denotes steady state bank equity.

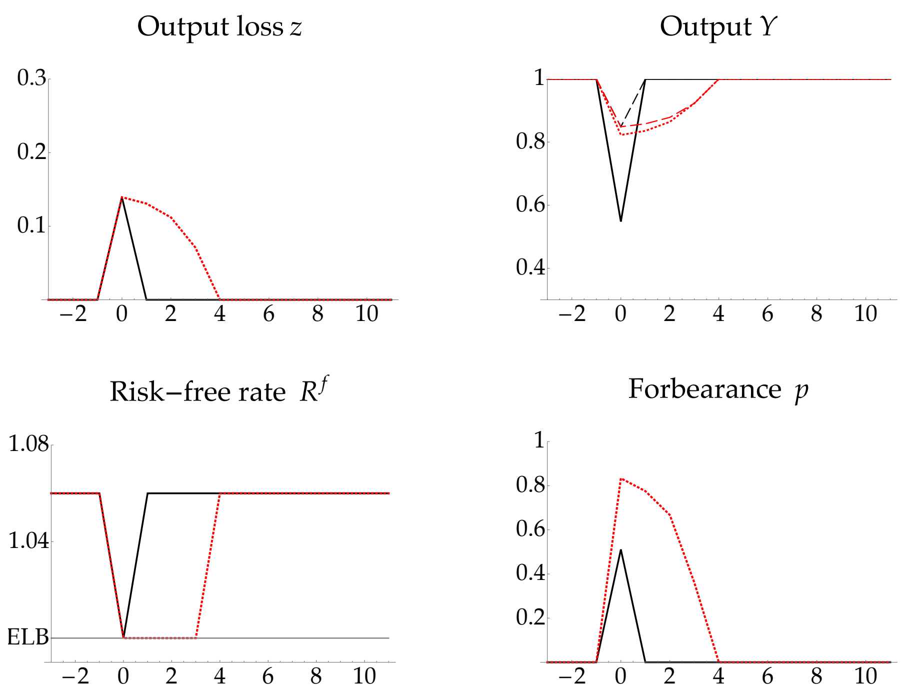
2. *There exists a threshold $z^*(\alpha)$ increasing in α such that for initial shocks $z_0 < z^*(\alpha)$, the economy converges to the no-zombie steady state, while for initial shocks $z_0 > z^*(\alpha)$ the economy converges to the stable sclerosis steady state with $z_t > 0, p_t > 0$ and a binding ELB $R^f_t = R^f_{\min}$ for all t along the transition.*

Figure 6 displays the impulse responses of output losses z_t, aggregate output Y_t, and the optimal policies R^f_t and p_t under the two policy regimes (NZ policy, in black, and myopic policy, in red). Panel A shows equilibrium paths following a shock z_0 that is above $\bar{z}$ but below the threshold $z^*(\alpha)$ defined in Proposition 2. The ELB binds at the time of the shock under both policy regimes. Forbearance also increases in both cases, but by much more under the myopic policy.

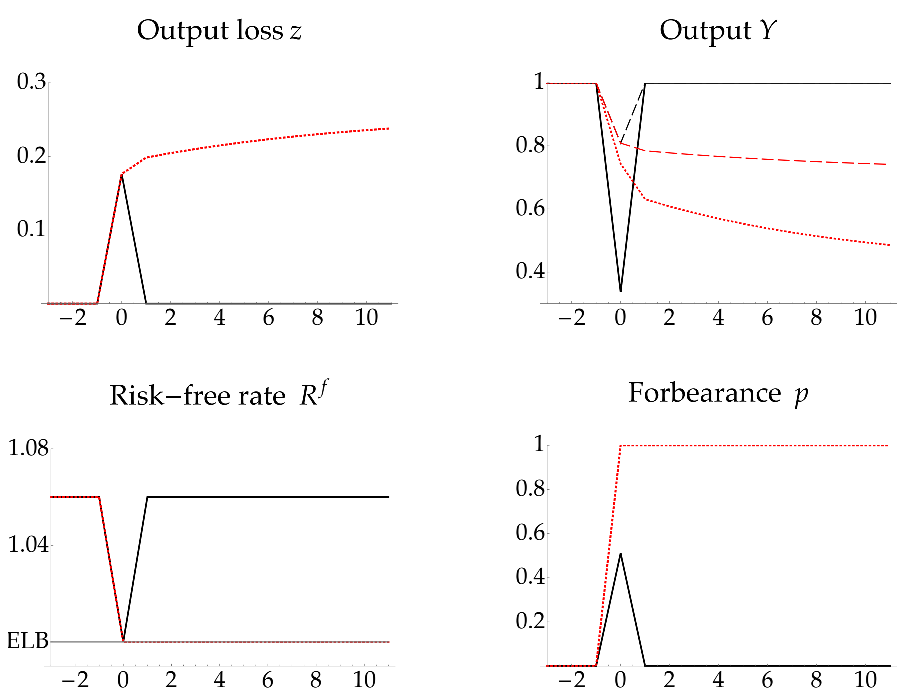
As a result, output drops sharply under the NZ policy, but recovers immediately to its pre-shock level at $t = 1$. The interest rate also recovers after the initial shock. By contrast, the

Figure 6: Impulse responses under the NZ policy (black) and the myopic policy (red dotted).

Panel A: Small initial shock $z_0 < z^* (\alpha)$



Panel B: Large initial shock $z_0 > z^* (\alpha)$



Note: Output losses z_t, aggregate output Y_t and potential output Y_t^* (dashed lines), and the optimal policies R_t^f and p_t under the two policy regimes (No Zombie lending, in solid black lines, and the myopic policy, in red dotted lines). Panel A: small initial shock $z_0 < z^* (\alpha)$. Panel B: large initial shock $z_0 > z^* (\alpha)$.

myopic policy succeeds in stabilizing date-0 output at a higher level thanks to the more generous forbearance policy that keeps some zombie firms alive. However, the stabilization of short-term output comes at the cost of a protracted output loss for multiple periods, with interest rates stuck at the ELB and forbearance p at a high level. While this path features endogenous persistence of the initial shock, the economy eventually converges back to its pre-shock steady state.

Panel B shows the equilibrium paths following a large initial shock $z_0 > z^*(\alpha)$. While initially the paths under the two policy regimes are similar to the ones following a smaller initial shock, they soon start diverging from each other. Like before, the economy experiences a sharp but short-lived output loss under the No Zombie lending regime. But under the myopic policy, the date-1 output loss z_1 stemming from congestion externalities is even larger than the initial shock z_0. This puts the economy on a dangerous path: at $t = 1$, the endogenously weaker fundamentals induce myopic policymakers to accommodate even further, by keeping interest rates as low as possible and allowing even higher forbearance ($p_1^m > p_0^m$), which, in turn, hurts date-2 productivity, and so on. For a while, this myopic policy manages to stabilize output Y_t close to potential output Y_t^*, albeit with a major side effect: potential output Y_t^* itself (dashed red line) starts falling because the presence of zombie firms reduces the productivity other firms in the economy. Moreover, once zombie lending becomes a permanent feature of the economy, all policymakers can do is exert maximal accommodation to stimulate output ($R^f = R^f_{\min}$, $p^m = 1$), which however is not sufficient to prevent a large gap between output and its potential. The economy snowballs towards sclerosis and monetary policy is trapped.

Remark 2. Our specification (15) assumes that the date-t zombie share only affects productivity at date $t + 1$. This is the simplest configuration that leads to policy traps, and highlights the most relevant economic forces. More generally, the key aspect is that zombie lending has *persistent* congestion externalities. Clearly, the risk of falling into a policy trap would be reinforced if zombie lending at t also undermined productivity at $t+2, t+3, \ldots$ as myopic policymakers would put an even lower welfare weight on these future periods. But more interestingly, zombie lending at t could also hurt productivity in the current period t. Adding such contemporaneous congestion externalities would not change the logic behind our results, as long as the effects are not purely contemporaneous. Contemporaneous congestion externalities lower the immediate net benefit of using forbearance p, by hurting aggregate output in a similar way as the contemporaneous misallocation between G and B firms that is already included in our analysis. As long as zombie lending also has an effect on *future* productivity, there remains a trade-off between current and future output.

5.4 Exiting the Policy Trap

Proposition 4 characterizes the steady state for given fundamentals. Can an economy exit a policy trap and recover from sclerosis? An obvious way to exit the trap is to appoint a more conservative or long-term policymaker, as in the literature on inflation bias (Rogoff, 1985). In our context this would correspond, for instance, to switching from a myopic policy regime to a No Zombie lending policy regime. This is isomorphic to our earlier example; the only difference is that the initial shock z_0 is not exogenous but caused by the congestion externalities in the sclerosis steady state (that is, $z_0 = \alpha m_\infty^b$ where m_∞^b is the steady state mass of zombie firms). At date-0, the NZ policy reduces forbearance p sufficiently to induce all zombie firms to exit. This causes a sharp but transitory recession, and allows a clean start at $t = 1$.

More interestingly, suppose we maintain the myopic policy regime but change the initial conditions. We consider two experiments: an improvement in fundamentals and a bank recapitalization. In each case the economy starts from a policy trap with $R^f = R^f_{\min}$ and $p = 1$, hence from the associated sclerosis steady state with output losses z_∞.

Improvement in fundamentals $\theta^g \bar{y}^g$. Fundamentals such as the productivity of good firms affect the threshold $z^*(\alpha)$ in Proposition 2. For instance, z^* is increasing in $\theta^g \bar{y}^g$ and decreasing in the churn parameter λ. A low growth environment is thus particularly dangerous: not only is potential output already low, but the economy is also more fragile and output is more susceptible to fall below potential due to zombie lending. Conversely, an improvement in $\theta^g \bar{y}^g$ can help the economy exit the policy trap and sclerosis; but once the economy is in a trap it needs a large shift in fundamentals. Figure A.2, panel A, shows an example with a sufficiently large increase in $\bar{y}^g$. Lending to good firms becomes relatively more attractive, which again sets the economy on a virtuous path towards a good steady state.

Bank recapitalization. Suppose next that at $t = 0$ the government recapitalizes the banking sector. In our model this corresponds to an exogenous increase of bank equity from e_∞ to a higher level e_0. A small intervention will only have a transitory effect. But a large recapitalization can help the economy exit the policy trap. Figure A.2, panel B, shows such an example. Output falls at the time of the recapitalization: z_0 is still high initially, hence lending opportunities are still weak and the higher equity induces a subset of banks to invest in safe assets. However, a better capitalized banking sector implies that risk-shifting incentives and zombie lending fall, which triggers a virtuous feedback loop: congestion externalities are lower in the next period, which makes lending to good firms more attractive, and so on. Over time, the economy can recover and converge back to the "good" steady state with no zombie lending, high interest rate, no forbearance, and high productivity ($z = 0$).

Historically, recapitalizations of the banking sector by the government—either directly through capital injection or indirectly at times through the establishment of "bad banks"—have been the most effective antidote to the proliferation of zombie lending.[29] Despite their efficacy, decisive interventions have been more the exception than the norm. In both Japan and southern Europe, for example, despite policymakers' recapitalization efforts the capitalization of the banking system effectively shrunk (see Table 1) or did not increase enough to cope with the aggregate shocks hitting the economy. Furthermore, as shown empirically by Peek and Rosengren (2005) and Giannetti and Simonov (2013) in Japan, and Acharya et al. (2019) in Europe, the timid recapitalization measures put in place were unable to prevent the spread of zombie lending, as they were unable to effectively recapitalize the weakest financial institutions.[30]

6 Equity Issuance, Relationship Lending, and Capital Requirements

Our framework highlights how an undercapitalized banking sector constrains policymakers, thereby making the economy more fragile in response to fundamental shocks. In the baseline model, we made this point taking the distribution of bank equity as given. We next consider how the distribution of bank equity itself responds to monetary policy, forbearance, and capital requirement regulation. How do the conclusions change when banks can choose their capital structure? And if capital is endogenous, can regulators solve the misallocation of credit by forcing banks to raise more capital?

6.1 Bank Equity Issuance

We first extend the static environment described in Section 3 by allowing banks to issue equity. We find that conventional monetary policy accommodation can increase zombie lending, by reducing banks' incentives to issue equity relative to debt.

Suppose banks start with a pre-issuance equity level e before deciding jointly how much equity they want to issue ($\Delta \geq 0$) and in which asset to invest (type G loans, type B loans, or safe

[29] Noteworthy cases of successful recapitalization efforts through asset purchases by a "bad bank" are the establishment of the Korea Asset Management Corporation (KAMCO) in South Korea following the 1997–1998 financial crisis and the establishment of the Resolution Trust Corporation (RTC) in the U.S. following the Savings and Loans crisis in the 1980s.

[30] The government may lack the ability or the willingness to recapitalize the banks sufficiently in the short run, due to fiscal constraints or from the fact that just like the instruments we already considered, bank recapitalizations are subject to policy myopia. Hence a government may optimally delay injecting equity if the costs of doing so (e.g., political backlash, heightened sovereign credit risk) are borne immediately while the benefits only materialize over time. Studying the optimal mix of policies as a function of the government's fiscal capacity is an important extension for future research.

assets). Bank e solves:

$$\max_{j\in\{g,b,f\},\Delta} \theta^j \left(R^j - \tilde{R}^j (1-e-\Delta)\right) - \kappa(\Delta)$$

where κ is an increasing, convex, differentiable equity issuance cost function. Conditional on choosing project j, the optimal equity issuance is

$$\Delta^j = (\kappa')^{-1} \left(\theta^j \tilde{R}^j\right) \tag{16}$$

Accounting for their optimal equity issuance decisions, banks sort themselves into projects j. The optimal equity issuance policy does not depend directly on a bank's pre-issuance equity e because the cost κ is additive. Yet, in equilibrium, the amount of issuance issued by different banks varies with e. Intuitively, e determines banks' asset choices, which in turn affect the optimal equity issuance. Hence risk-shifting acts as a "double whammy": banks with a lower initial level of capitalization also issue less equity, anticipating that they will be the ones lending to relatively riskier borrowers. By contrast, banks that start with high capital internalize that they will be the ones lending to safer borrower or even investing in safe assets, and thus have incentives to issue more equity.

As in the baseline model, there is a diabolical sorting: poorly-capitalized banks engage in risk-shifting and zombie lending. But now the equity thresholds e^* and e^{**} depend on the equity issuance margin. In order to focus on e^*, suppose that $e^{**} > e_{\max}$. We have the following result:

Proposition 5. *An decrease in R^f raises e^* and thus zombie lending. An increase in p raises e^* and zombie lending more than without equity issuance.*

Proposition 5 uncovers a new relationship between zombie lending and conventional monetary policy when bank equity is endogenous. As previously discussed, when banks cannot choose their leverage—or, equivalently, when equity issuance costs are infinitely high—the level of R^f has no bite on banks' relative returns from lending to good versus bad types of firms. Once equity issuance costs are introduced, however, a reduction in the monetary policy rate R^f increases the threshold e^*, thereby increasing zombie lending.

A higher interest rate increases the returns on all assets and therefore encourages banks to issue more equity to take advantage of these higher returns. Our reduced-form formulation in which equity is limited by an issuance cost function κ makes this point particularly stark and simple. More generally, higher interest rates will increase equity issuance if the required return on bank equity does not adjust fully with the risk-free rate, as is the case empirically, so that higher interest rates make the cost of equity relatively lower.[31]

[31] An important implication of this result is that the endogenous response of banks' capital structure imposes an

6.2 Relationship Lending and Evergreening

Empirically, an important source of zombie lending stems from weak banks willing to "extend and pretend", by rolling over loans at subsidized rates to *legacy* borrowers that should be declared non-performing.[32] We can incorporate this element by breaking the symmetry between old and new borrowers in a parsimonious way. Banks start the model matched with a legacy borrower. A random fraction λ of banks have an outstanding B borrower, and the remaining $(1-\lambda)$ banks have an outstanding G borrower.

Assumption 6. *If a bank switches from its legacy B borrower to a new borrower, its equity falls from e to $(e-\delta)$, for some switching cost $\delta \geq 0$.*

The presence of a positive switching cost will prolong some borrower-lender relationships. The switching cost δ captures first and foremost the loss provisions that banks must put aside when declaring loans as non-performing; but δ is also meant to include the screening effort that the bank must spend when creating a relationship with a new borrower.[33] Indeed, banks will never want to switch from a legacy B borrower to a new B borrower, so the only switches that could be observed in equilibrium are towards a new G borrower. This presumes some costly information gathering to learn which borrowers are indeed good. Our results extend to a more general switching cost structure, with costs δ_{ij} depending on both the legacy match i and the new match j.

The distinction between legacy and new borrowers requires us to model lending relationships. First, we need to determine which outstanding borrower-lender pairs are continued, and which of them are broken so that the bank can lend to a new borrower. Second, we must specify the loan rates offered to legacy borrowers, as those can differ from the rates offered to new borrowers due to the hold-up problem. We assume that the borrower-lender pair separates if and only if the joint surplus of remaining matched is lower than the joint surplus outside the relationship, in which

additional constraint on monetary policy. Moderate interest rates are needed to prevent banks from investing in safe assets instead of lending, as in the baseline model with exogenous equity. But there is a new force: lowering interest rates "too much" makes zombie lending more likely, by deterring equity issuance. Hence achieving potential output Y^* requires, as in Proposition 1, to set p and R^f low enough, together with a novel restriction that the risk-free rate R^f cannot be set too low either. Proposition 7 in the Appendix formalizes this result.

[32] The empirical literature documents that the credit extended by under-capitalized banks to poorly-performing firms is granted at rates lower than justified by the credit risk of these borrowers. The subsidized nature of these credit transactions is one of the quintessential features of zombie lending. For this reason, Caballero et al. (2008) and most of the following literature use subsidized bank credit as a criterion to empirically identify zombie firms, and finds that their borrowing rates are often as low as those charged to the safest borrowers (Acharya et al. 2019; Schivardi et al. 2021). For incentives to postpone loss recognition, see also Blattner et al. (2023).

[33] The efficiency of the debt resolution system affects the cost of insolvencies and the magnitudes of loan loss provisions. Therefore, bankruptcy reforms may alleviate the incidence of zombie lending (Becker and Ivashina, 2022). However, the benefits of such reforms depend on the level of bank capitalization, which determines the strength of banks' zombie-lending incentives (Kulkarni et al., 2021).

case the bank lends to a new borrower and the borrower seeks to borrow from a new bank. In the case of continuation, the bank makes a take-it-or-leave-it offer to the firm, hence legacy and new B borrowers pay the same rate R^b. Positive bargaining power for the firm would decrease the rate to legacy borrowers to $\bar{R}^b < R^b$.[34]

Lemma 3 in the Appendix extends our sorting result (Lemma 1) to show that relationship lending induces a second source of sorting, in addition to risk-shifting: a positive switching cost δ increases zombie lending at the bottom of the bank equity distribution. Some banks with capitalization between e^* and $(e^* + \nu\delta)$ choose to roll over the loan to their legacy B borrower in order to economize the cost δ, even though given their capital they would lend to a new G borrower absent this preexisting lending relationship. The most interesting implications of this "evergreening" channel, in the next section, arise when we consider how it interacts with capital requirements.

6.3 Capital Requirements

A key policy question in the face of prevalent zombie lending is whether tightening capital requirements is a good remedy. In light of our sorting result, improving the distribution of bank capital appears to be a natural solution to tilt credit allocation towards safer and more productive lending. The counterargument is that tighter regulation may backfire, by generating incentives for banks to extend and pretend out of fear of having to recapitalize to satisfy the requirement. We now allow for both equity issuance and positive switching costs $\delta > 0$ as described in the two previous subsections. In addition, the regulator can impose a capital requirement: post-issuance equity e' must remain above a floor $\hat{e}$. Consistent with Assumption 3, the capital requirement does not depend on banks' asset risk. Our main result is that if switching costs δ are high enough, and capital requirements are already strict, then tightening regulation further (increasing $\hat{e}$) can *increase* zombie lending through the evergreening channel.

Throughout this section we keep other policies R^f and p fixed (for instance, because the economy has already fallen into a dynamic policy trap) to focus on the effect of capital requirements. It is convenient to define

$$\sigma(e') = \theta^g \left[R^g - \tilde{R}^g (1 - e')\right] - \theta^b \left[R^b - \tilde{R}^b (1 - e')\right]$$

which represents the payoff difference between lending to a G firm and a B firm (ignoring any equity issuance costs) for a bank with post-issuance equity e'. We restrict attention to parameters

[34] Low loan rates are also a way to ensure repayment of the zombie loans when the default probability depends on loan rates, as in the literature on credit rationing (Stiglitz and Weiss, 1981). Faria E Castro et al. (2021) focus on this mechanism and find empirical support in U.S. data.

such that if the regulator sets a capital requirement low enough that it does not bind even for the bank with the lowest capital $e = e_{\min}$ then that bank prefers to lend to a type-B firm. Formally,

$$\sigma(\hat{e}) < \kappa(\hat{e} - e_{\min} + \delta) - \kappa(\hat{e} - e_{\min}). \tag{17}$$

for all $\hat{e} \leq \min\left\{e_{\min} + \Delta^b, e_{\min} + \Delta^g - \delta\right\}$. Condition (17) means that there is indeed some zombie lending absent capital requirements. This is the only interesting case to consider, as otherwise capital requirements would be irrelevant for credit allocation and aggregate output, and introducing them would only create a deadweight loss in terms of equity issuance costs.[35]

In the absence of any switching costs ($\delta = 0$), it is straightforward to deter zombie lending completely: the regulator can just impose a capital requirement $\hat{e}$ that is sufficiently high, and more precisely, above the equity threshold e^* in an equilibrium without zombie lending. Intuitively, the case of low enough switching costs must be similar to when there are no switching costs at all. Indeed, we find that for low enough δ, there always exists a sufficiently tight capital requirement $\hat{e}^{NZ}$ (where NZ stands for No Zombie lending) that suppresses zombie lending altogether ($m^b = 0$). Does it mean that we can always solve the zombie lending problem using capital regulation? We find that the answer is no. Surprisingly, when the switching cost δ is high enough, no capital requirement can deter zombie lending completely: some positive equilibrium zombie lending is inevitable. In fact, the stronger result is that increasing capital requirements beyond some level can even backfire, by further encouraging zombie lending:

Proposition 6. *Let $\hat{e}^{NZ}$ solve $\sigma\left(\hat{e}^{NZ}\right) = \kappa\left(\hat{e}^{NZ} - e_{\min} + \delta\right) - \kappa\left(\hat{e}^{NZ} - e_{\min}\right)$.*

- *If $\delta < \Delta^g - \Delta^b$, then any capital requirement above $\hat{e}^{NZ}$ suppresses zombie lending.*
- *If $\delta > \Delta^g - \Delta^b$, then zombie lending is minimized by setting the capital requirement*

$$\hat{e} = 1 - \frac{\theta^g R^g - \theta^b R^b}{p\Delta\theta}$$

and increasing capital requirements above that level strictly increases zombie lending. No capital requirement can suppress zombie lending altogether.

The case of a high δ captures the evergreening motive of zombie lending. The intuition is as follows. A bank compares two options: recognizing the loss at a cost δ, which allows a fresh start with a new G borrower, or rolling over the loan to the legacy B borrower. The second option allows to economize the switching cost δ, and becomes especially attractive with a high δ. Switching to a new borrower brings an additional cost if the bank is already poorly-capitalized:

[35] In a dynamic setting, capital requirements could matter for future credit allocation even if they do not bind in the present. This is one rationale behind precautionary cyclical capital requirements.

its equity will drop to $e - \delta$, which forces the bank to undertake a costly recapitalization to satisfy the requirement $\hat{e}$. Thus there is a set of banks for which the cost of recapitalization acts as an additional motive to roll over the zombie loan, and the set of such banks expands as the capital requirement $\hat{e}$ increases.[36]

Proposition 6 highlights a subtle link between capital requirements and zombie lending. In particular, both cases are likely to be relevant because the switching cost δ and the threshold $\bar{\delta}$ depend on the country and industry of the borrower, and the history of the lending relationship. For instance, δ will be higher when there is more asymmetric information between banks and potential new borrowers, and when zombie debt has been accumulating for a longer time (as this increases the losses that banks would eventually recognize). Just like in the dynamic model, the longer policymakers wait before tackling the zombie lending problem, the harder it becomes to solve it. The case of high switching costs δ is consistent with some of the empirical evidence on the unintended consequences of capital requirements, for instance following the increase in capital requirements by the European Banking Authority in 2011, as documented by Blattner, Farinha and Rebelo (2023). Relatedly, Chopra, Subramanian and Tantri (2020) show that other regulatory actions such as ex-post bank cleanups can also trigger zombie lending if they are not accompanied by ex-ante bank recapitalization.

7 Conclusion

In this paper we develop a theoretical framework with heterogeneous firms and banks to study the complex feedback loop between bank under-capitalization, credit misallocation due to zombie lending, accommodative monetary policy and regulatory forbearance, and adverse aggregate outcomes such as permanent losses in growth and productivity. Our model generates linkages that are consistent with several features of aggregate and banking sector data characterizing the "lost decade" of Japan following its real estate crisis, and more recently, the aftermath of the sovereign debt crisis in southern Europe. Viewed through the lens of our model, policymakers should avoid excessively "pushing on a string" of forbearance towards banks precisely when economies are hit by large shocks, as this can lead to delayed recoveries and persistent output losses.

Our results have salient policy implications and suggest several directions for further research. A focal point of our model is the interaction between monetary and banking policy with the

[36] In this analysis we are holding p fixed. Proposition 6 shows that the mapping between an increase in p and forbearance in the form of lower capital requirements $\hat{e}$, discussed in Section 3, is more complex once we introduce relationship lending. If δ is low, then both a higher p and a lower $\hat{e}$ increase zombie lending. If δ and the initial capital requirement $\hat{e}$ are sufficiently high, however, lowering $\hat{e}$ decreases zombie lending, hence it becomes more important to distinguish between between government guarantees and forbearance in capital regulation.

fundamental of firms and bank in the economy, potentially converting transitory shocks into accommodative policy traps and lost decades. This risk is receiving increasing attention in the aftermath of the recent pandemic, especially in the case of China, where the adoption of lenient regulatory stances toward financial intermediaries has inevitably raised the specter of long-term economic stagnation from a zombification of the economy. Further empirical work is needed to better inform policy makers coping with large shocks on how to optimally resolve the trade-off between short-term versus long-term losses.

Finally, our study also suggests how properly designed capital injections in the banking sector can effectively tackle the incentives problems at the root of zombie lending. However, it is assumed in our framework, as in the real world, that governments lack the willingness or the ability to recapitalize the banks sufficiently in a timely fashion. This can be due to policy myopia or to binding fiscal constraints. Studying the optimal mix of macro-financial policies as a function of the government's fiscal capacity also remains an open question and an important extension for future research.

References

Abadi, Joseph, Markus Brunnermeier, and Yann Koby (2023), "The Reversal Interest Rate", *American Economic Review*, Vol. 113, pp. 2084–2120.

Acharya, Viral V., Lea Borchert, Maximilian Jager, and Sascha Steffen (2021a), "Kicking the Can Down the Road: Government Interventions in the European Banking Sector", *The Review of Financial Studies*, Vol. 34, pp. 4090–4131.

Acharya, Viral v., Matteo Crosignani, Tim Eisert, and Christian Eufinger (2024), "Zombie Credit and (Dis-)Inflation: Evidence from Europe", *The Journal of Finance*, Vol. 79, pp. 1883–1929.

Acharya, Viral V., Matteo Crosignani, Tim Eisert, and Sascha Steffen (2021b), "Zombie Lending: Theoretical, International, and Historical Perspectives", Working Paper 29904, National Bureau of Economic Research.

Acharya, Viral V., Tim Eisert, Christian Eufinger, and Christian Hirsch (2019), "Whatever It Takes: The Real Effects of Unconventional Monetary Policy", *The Review of Financial Studies*, Vol. 32, pp. 3366–3411.

Acharya, Viral V., Björn Imbierowicz, Sascha Steffen, and Daniel Teichmann (2020), "Does the Lack of Financial Stability Impair the Transmission of Monetary Policy?", *Journal of Financial Economics*, Vol. 138, pp. 342–365.

Alesina, Alberto and Guido Tabellini (1990), "A Positive Theory of Fiscal Deficits and Government Debt", *The Review of Economic Studies*, Vol. 57, pp. 403–414.

Asriyan, Vladimir, Luc Laeven, Alberto Martin, Alejandro Van der Ghote, and Victoria Vanasco (2024), "Falling Interest Rates and Credit Reallocation: Lessons from General Equilibrium", *The Review of Economic Studies*, p. rdae065.

Banerjee, Ryan and Boris Hofmann (2018), "The Rise of Zombie Firms: Causes and Consequences", *BIS Quarterly Review.*

Barlevy, Gadi (2002), "The Sullying Effect of Recessions", *The Review of Economic Studies*, Vol. 69, pp. 65–96.

Becker, Bo and Victoria Ivashina (2022), "Weak Corporate Insolvency Rules: The Missing Driver of Zombie Lending", *AEA Papers and Proceedings*, Vol. 112, pp. 516–20.

Begenau, Juliane, Saki Bigio, Jeremy Majerovitz, and Matias Vieyra (2021), "A Q-theory of Banks", Working Paper 27935, National Bureau of Economic Research.

Benigno, Gianluca and Luca Fornaro (2018), "Stagnation Traps", *Review of Economic Studies*, Vol. 85, pp. 1425–1470.

Berger, Allen N., Sally M. Davies, and Mark J. Flannery (2000), "Comparing Market and Supervisory Assessments of Bank Performance: Who Knows What When?", *Journal of Money, Credit and Banking*, Vol. 32, pp. 641–667.

Berger, Philip G, Michael Minnis, and Andrew Sutherland (2017), "Commercial Lending Concentration and Bank Expertise: Evidence from Borrower Financial Statements", *Journal of Accounting and Economics*, Vol. 64, pp. 253–277.

Bernanke, Ben S. and Mark Gertler (1995), "Inside the Black Box: The Credit Channel of Monetary Policy Transmission", *Journal of Economic Perspectives*, Vol. 9, pp. 27–48.

Bertola, Giuseppe and Ricardo J. Caballero (1994), "Cross-Sectional Efficiency and Labour Hoarding in a Matching Model of Unemployment", *The Review of Economic Studies*, Vol. 61, pp. 435–456.

Blattner, Laura, Luisa Farinha, and Francisca Rebelo (2023), "When Losses Turn into Loans: The Cost of Weak Banks", *American Economic Review*, Vol. forthcoming.

Blickle, Kristian, Cecilia Parlatore, and Anthony Saunders (2023), "Specialization in Banking", Working Paper 31077, National Bureau of Economic Research.

Bolton, Patrick and Xavier Freixas (2006), "Corporate Finance and the Monetary Transmission Mechanism", *The Review of Financial Studies*, Vol. 19, pp. 829–870.

Boot, Arnoud and Anjan Thakor (1993), "Self-interested Bank Regulation", *American Economic Review P&P*, Vol. 83, pp. 206–12.

Bruche, Max and Gerard Llobet (2013), "Preventing Zombie Lending", *The Review of Financial Studies*, Vol. 27, pp. 923–956.

Buera, Francisco J., Benjamin Moll, and Yongseok Shin (2013), "Well-intended Policies", *Review of Economic Dynamics*, Vol. 16, pp. 216–230, Misallocation and Productivity.

Caballero, Ricardo J, Takeo Hoshi, and Anil K Kashyap (2008), "Zombie Lending and Depressed Restructuring in Japan", *American Economic Review*, Vol. 98, pp. 1943–77.

Carpinelli, Luisa and Matteo Crosignani (2021), "The Design and Transmission of Central Bank Liquidity Provisions", *Journal of Financial Economics*, Vol. 141, pp. 27–47.

Chan, Yuk-Shee, Stuart I. Greenbaum, and Anjan V. Thakor (1992), "Is Fairly Priced Deposit Insurance Possible?", *The Journal of Finance*, Vol. 47, pp. 227–245.

Chopra, Yakshup, Krishnamurthy Subramanian, and Prasanna L Tantri (2020), "Bank Cleanups, Capitalization, and Lending: Evidence from India", *The Review of Financial Studies*, Vol. 34, pp. 4132–4176.

Crouzet, Nicolas and Fabrice Tourre (2021), "Can the Cure Kill the Patient? Corporate Credit Interventions and Debt Overhang", working paper.

Drechsler, Itamar, Thomas Drechsel, David Marques-Ibanez, and Philipp Schnabl (2016), "Who Borrows from the Lender of Last Resort?", *The Journal of Finance*, Vol. 71, pp. 1933–1974.

Faria E Castro, Miguel, Paul Pascal, and Juan M. Sánchez (2021), "Evergreening", Working Paper 2021-12, Federal Reserve Bank of St. Louis.

Flannery, Mark J. (1998), "Using Market Information in Prudential Bank Supervision: A Review of the U.S. Empirical Evidence", *Journal of Money, Credit and Banking*, Vol. 30, pp. 273–305.

Fukao, Mitsuhiro (2003), "Financial Sector Profitability and Double-gearing", in *Structural Impediments to Growth in Japan*: University of Chicago Press, pp. 9–36.

——— (2007), "Financial Crisis and the Lost Decade", *Asian Economic Policy Review*, Vol. 2, pp. 273–297.

Gambacorta, Leonardo and Hyun Song Shin (2018), "Why bank capital matters for monetary policy", *Journal of Financial Intermediation*, Vol. 35, pp. 17–29, Banking and regulation: the next frontier.

Giannetti, Mariassunta and Andrei Simonov (2013), "On the Real Effects of Bank Bailouts: Micro Evidence from Japan", *American Economic Journal: Macroeconomics*, Vol. 5, pp. 135–67.

Gopinath, Gita, Sebnem Kalemli-Ozcan, Loukas Karabarbounis, and Carolina Villegas-Sanchez (2017), "Capital Allocation and Productivity in South Europe", *The Quarterly Journal of Economics*, Vol. 132, pp. 1915–1967.

Gourinchas, Pierre-Olivier, Sebnem Kalemli-Ozcan, Veronika Penciakova, and Nick Sander (2020), "Estimating SME Failures in Real Time: An Application to the Covid-19 Crisis", Working Paper 27877, National Bureau of Economic Research.

Gourio, Francois and Leena Rudanko (2014), "Customer Capital", *The Review of Economic Studies*, Vol. 81, pp. 1102–1136.

Greenwood, Robin, Benjamin Iverson, and David Thesmar (2020), "Sizing up Corporate Restructuring in the Covid Crisis", Working Paper 28104, National Bureau of Economic Research.

Gropp, Reint, Andre Guettler, and Vahid Saadi (2020), "Public Bank Guarantees and Allocative Efficiency", *Journal of Monetary Economics*, Vol. 116, pp. 53–69.

Van den Heuvel, Skander J (2002), "The bank capital channel of monetary policy", *The Wharton School, University of Pennsylvania, mimeo*, pp. 2013–14.

Hoshi, Takeo (2006), “Economics of the Living Dead”, *The Japanese Economic Review*, Vol. 57, pp. 30–49.

Hoshi, Takeo and Anil K Kashyap (2015), “Will the US and Europe Avoid a Lost Decade? Lessons from Japan’s Postcrisis Experience”, *IMF Economic Review*, Vol. 63, pp. 110–163.

Hu, Yunzhi and Felipe Varas (2021), “A Theory of Zombie Lending”, *The Journal of Finance*, Vol. 76, pp. 1813–1867.

Hurst, Erik, Benjamin J. Keys, Amit Seru, and Joseph Vavra (2016), “Regional Redistribution through the US Mortgage Market”, *American Economic Review*, Vol. 106, pp. 2982–3028.

Jafarov, Etibar and Enrico Minnella (2023), “Too Low for Too Long: Could Extended Periods of Ultra Easy Monetary Policy Have Harmful Effects?”, imf working paper.

Kashyap, Anil K. and Jeremy C. Stein (2000), “What Do a Million Observations on Banks Say about the Transmission of Monetary Policy?”, *American Economic Review*, Vol. 90, pp. 407–428.

Klemperer, Paul (1987), “Markets with Consumer Switching Costs”, *The Quarterly Journal of Economics*, Vol. 102, pp. 375–394.

Kulkarni, Nirupama, S. K. Ritadhi, Siddharth Vij, and Katherine Waldock (2021), “Unearthing Zombies”, Working Paper 3495660, Georgetown McDonough School of Business.

Kydland, Finn E. and Edward C. Prescott (1982), “Time to Build and Aggregate Fluctuations”, *Econometrica*, Vol. 50, pp. 1345–1370.

Li, Wenhao and Ye Li (2021), “Firm Quality Dynamics and the Slippery Slope of Credit Intervention”, Working Paper 2020-03-028, USC Marshall School of Business.

Mailath, George J. and Loretta J. Mester (1994), “A Positive Analysis of Bank Closure”, *Journal of Financial Intermediation*, Vol. 3, pp. 272–299.

Paravisini, Daniel, Veronica Rappoport, and Philipp Schnabl (2020), “Specialization in Bank Lending: Evidence from Exporting Firms”, *Journal of Finance*, Forthcoming.

Parlatore, Cecilia and Thomas Philippon (2024), “Designing Stress Scenarios”.

Passalacqua, Andrea, Paolo Angelini, Fracesca Lotti, and Giovanni Soggia (2020), “The Real Effects of Bank Supervision”, working paper.

Peek, Joe and Eric S Rosengren (2005), “Unnatural Selection: Perverse Incentives and the Misallocation of Credit in Japan”, *American Economic Review*, Vol. 95, pp. 1144–1166.

Phelps, Edmund S and Sidney G Winter (1970), “Optimal price policy under atomistic competition”, in Edmund S Phelps et al. eds. *Microeconomic Foundations of Employment and Inflation Theory*, New York: W. W. Norton, pp. 309–337.

Ravn, Morten, Stephanie Schmitt-Grohé, and Martín Uribe (2006), “Deep Habits”, *The Review of Economic Studies*, Vol. 73, pp. 195–218.

Rogoff, Kenneth (1985), “The Optimal Degree of Commitment to an Intermediate Monetary Target”, *Quarterly Journal of Economics*, Vol. 100, pp. 1169–1189.

Saunders, Anthony, Marcia Millon Cornett, and Otgo Erhemjamts (2021), *Financial Institutions Management: A Risk Management Approach*: McGraw-Hill Education, 10th edition.

Schivardi, Fabiano, Enrico Sette, and Guido Tabellini (2021), "Credit Misallocation during the European Financial Crisis", Working Paper 1139, Bank of Italy Temi Di Discussione.

Schmidt, Christian, Yannik Schneider, Sascha Steffen, and Daniel Streitz (2023), "Does Zombie Lending Impair Innovation?", working paper.

Stiglitz, Joseph E and Andrew Weiss (1981), "Credit Rationing in Markets with Imperfect Information", *American Economic Review*, Vol. 71, pp. 393–410.

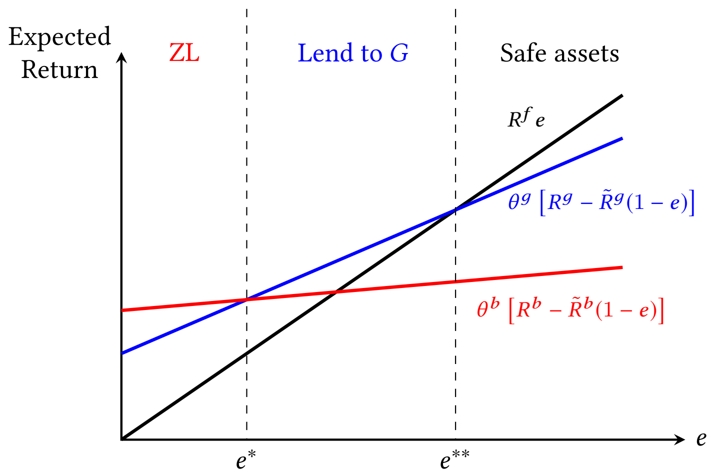
Tracey, Belinda (2021), "The Real Effects of Zombie Lending in Europe", Working Paper 783, Bank of England.

Vasicek, Oldrich (1977), "An Equilibrium Characterization of the Term Structure", *Journal of Financial Economics*, Vol. 5, pp. 177–188.

Online Appendix

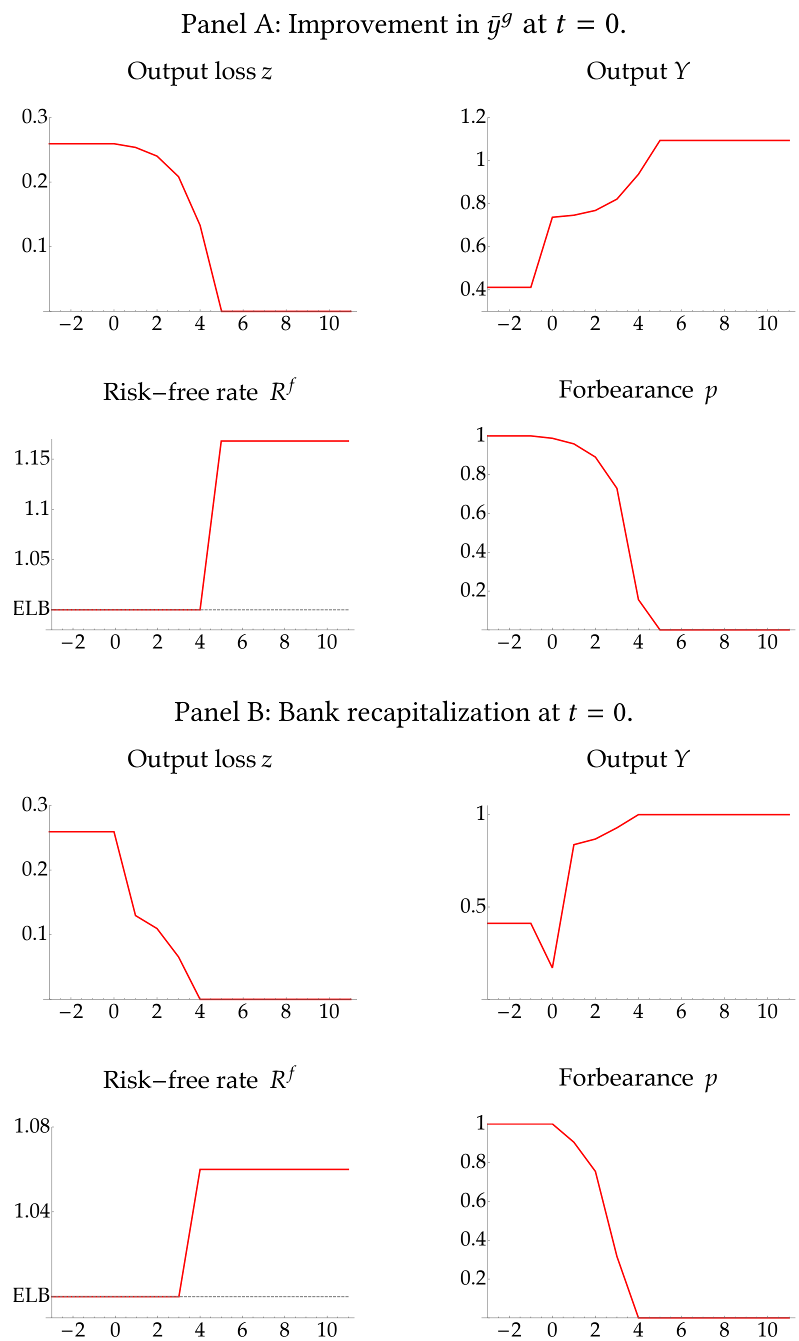
A Additional Figures and Tables

Figure A.1: Optimal asset choice as a function of bank capital e.



Note: Each line shows the expected profit from investing in asset i, $\theta^i \left[R^i - \tilde{R}^i (1-e)\right]$, as a function of e. The red line shows $i = b$ (lending to a type B firm). The blue line shows $i = g$ (lending to a type G firm). The black line shows $i = f$ (investing in safe assets).

Figure A.2: Impulse responses under the myopic policy.



Note: Output losses z_t, aggregate output Y_t, and the optimal policies R^f_t and p_t under the myopic policy regime. Panel A: permanent increase in $\bar{y}^g$ at $t = 0$. Panel B: bank recapitalization at $t = 0$.

B Data Sources

Japan. The share of zombie firms is computed by weighting firms by their assets from Caballero et al. (2008). Real GDP and aggregate TFP is from the Penn World Tables. The capitalization of the banking system is defined as the total adjusted core capital over total assets of Japanese banks. It is an authors' elaboration using data from Fukao (2003, 2007). Adjusted core capital is defined as Core capital + Unrealized capital gains and losses − Estimated under-reserves − Deferred Tax Assets. Core capital is Tier 1 capital. The computation of unrealized capital gains and losses uses the calculations suggested by Fukao (2003, 2007) and is equal to 0.6*(Market Value Shares − Book Value Shares). Estimated under-reserves is loss reserves minus estimated loan losses. The calculation of Deferred Tax Assets follows (2003, 2007). The industry data used to compute the correlation between the percentage change in share of zombie firms in a given industry (1981–1992 average to 1993–2002 average) and the TFP growth (average growth rate between 1990 and 2000) comes from Caballero et al. (2008).

European countries. Real GDP and aggregate TFP is from the Penn World Tables. Bank capital is the Tier 1 ratio (Tier 1 capital over risk-weighted asses) from the ECB statistics warehouse. The country-industry level data used to compute the binned scatter plots in Figure 2 and the liner projection estimates in Figure 3 is from Acharya et al. (2021a).
In Figure 2, the percentage change in share of zombie firms (from 2012 to 2015) is the asset weighted share of zombie firms in a given country $\times$ 4-digit industry industry pair. TFP growth is the asset weighted average change in firm-level TFP (from 2012 to 2015) of firms operating in a given country-industry pair. The percentage change in productivity is the asset weighted average percentage change of EBITDA over total assets (between 2009 and 2011) of firms operating in a given country-industry pair. The weighted average is computed focusing on the subsample of incumbent firms observed in the data both before the crisis (2009) and in the aftermath of the crisis (2010–2011). The productivity gap between good firms and poorly-performing firms is computed as the difference between the percentage change in the average productivity of good firms in a country-industry pair and the percentage change in the average productivity of poorly-performing firms (firms with interest coverage ratio below the industry median and bank leverage above the industry median). The binned scatter plots in Figure 2 are obtained weighting each country-industry pair by its assets in 2009.

In Figure 3 we report the coefficient estimates β_h obtained from the local linear projection $\ln(\text{TFP})_{ic,t+h} - \ln(\text{TFP})_{ic,t} = \alpha_i + \alpha_c + \beta_h \,\text{Zombie Share}_{ic,t} + \gamma_h \boldsymbol{X}_{ic,h} + \epsilon_{ic,t}$, where year $t = 2011$ (i.e., the first year after the burst of the European sovereign debt crisis). $\ln(\text{TFP})_{ic,t}$ is the logarithm of the asset-weighted average productivity (percentage change in EBITDA/Assets) in a given country×4-digit industry pair in year t. The variable Zombie Share$_{ic,t}$ is the (asset-weighted) share of zombie in the country × 4-digit industry pair in year t. We standardize the zombie share to be mean zero and standard deviation one.

C Additional Results

C.1 Asymmetric shocks

We generalize the shocks structure (12) to show that whether zombie lending constrains macroeconomic stabilization depends on the nature of the shocks, and more precisely on their relative incidence on good and bad firms. Let firm outputs be decreasing functions of z:

$$y^i(z), \quad i = g, b.$$

We define two types of shocks:

Definition 3. Shocks z are *gap-reducing* if they affect G firms' expected revenue more than B firms':

$$\theta^g \frac{dy^g}{dz} < \theta^b \frac{dy^b}{dz} \leq 0 \tag{A.1}$$

and *gap-augmenting* otherwise, i.e., if $0 \geq \theta^g \frac{dy^g}{dz} > \theta^b \frac{dy^b}{dz}$.

We call shocks satisfying (A.1) "gap-reducing" to make the economic mechanism transparent in what follows, but this class of shocks is much less restrictive than it sounds: it includes standard specifications of aggregate shocks, including symmetric multiplicative shocks (12) as well as symmetric additive shocks $y^i(z) = \bar{y}^i - z$. Equation (A.1) simply rules out shocks that have a disproportionate impact on zombie firms.

The nature of the shock matters through the no-zombie lending constraint $p \leq \bar{p}(z)$. Our model highlights that gap-reducing shocks are the ones that tighten this constraint:

Lemma 2. *$\bar{p}(z)$ is decreasing in z for gap-reducing shocks and increasing in z for gap-augmenting shocks.*

Gap-reducing shocks make loans to healthy firms less attractive for a given p, hence the constraint $p \leq \bar{p}(z)$ gets tighter for larger shocks z. Conversely, with gap-augmenting shocks, shocks z relax the constraint $p \leq \bar{p}(z)$ and policymakers can safely increase forbearance p in deeper recessions (higher z) to stimulate lending, as zombie lending becomes less of a threat.

Therefore we focus on gap-reducing shocks, that imply a meaningful trade-off between stimulating aggregate bank lending and preventing zombie lending. As discussed in Section 2, empirical studies on the Japanese and European crises document a lower investment and employment growth of healthy firms in sectors with a larger share of zombie firms. This evidence is consistent with a combination of congestion externalities—which is the leading interpretation in the literature—and gap-reducing shocks. For instance, our model predicts that with multiple sectors or regions facing different gap-reducing shocks z_s but subject to a common forbearance policy p, the sectors suffering from larger shocks z_s would display more zombie lending in equilibrium. This is indeed what Section 2, Figure 2 documented.[37]

C.2 Evergreening

C.2.1 Sorting with $\delta > 0$

With $\delta > 0$, Lemma 1 generalizes as follows:

Lemma 3 (Bank-firm sorting with evergreening)**.** *Let $v = \frac{R^f - p(1-\theta^g)}{p\Delta\theta}$ and suppose that $e^* \leq e^{**}$.*[38] *Banks matched with a legacy B borrower invest as follows:*

(i) Banks with equity $e < e^* + \delta v$ lend to a type B borrower at rate R^b.

(ii) Banks with equity $e \in (e^* + \delta v, e^{**})$ lend to a type G borrower at rate R^g.

(iii) Banks with equity $e > e^{**}$ do not lend and invest in safe assets at rate R^f.

Other banks follow the policies (i)-(iii) with a threshold e^* instead of $e^* + \delta v$ as in Lemma 1.

C.2.2 Static equilibrium conditions with $\delta > 0$

The market clearing condition for new B loans

$$(1-\lambda)F(e^*) + \lambda F(e^*)\left[1 - H\left(\theta^b\left(y^b - \bar{R}^b\right)\right)\right] = \lambda H\left(\theta^b\left(y^b - R^b\right)\right)\left[1 - F\left(e^* + z\delta\right)\right]$$

so indeed for $\delta = 0$ we have the simple form

$$F(e^*) = \lambda H\left(\theta^b\left(y^b - R^b\right)\right)$$

as in the main text.

[37]The model suggests important directions for future empirical research. Firstly, whether accommodative policies trigger zombie lending depends on the nature of the recession, e.g., on the capitalization of the banking sector at the time of the shock (see Corollary 1) and how the shock affects the profitability gap between zombie firms and healthy firms. Secondly, it would be valuable in future work to disentangle congestion externalities and gap-reducing shocks.

[38]As before this is the case if condition (A.3) in the Appendix holds.

The market clearing condition for new G loans is

$$\begin{aligned}H\left(\theta^{g}\left(y^{g}-R^{g}\right)\right)=&(1-\lambda)\left[F\left(e^{**}\right)-F\left(e^{*}\right)\right]\\&+\lambda\left[F\left(e^{**}\right)-F\left(e^{*}+z\delta\right)\right]\\&+\lambda\left[1-H\left(\theta^{b}\left(y^{b}-\bar{R}^{b}\right)\right)\right]\left[F\left(e^{*}+z\delta\right)-F\left(e^{*}\right)\right]\end{aligned}$$

which also specializes to the simple form in the main text

$$H\left(\theta^{g}\left(y^{g}-R^{g}\right)\right)=F\left(e^{**}\right)-F\left(e^{*}\right)$$

for $\delta=0$.

C.3 Dynamic Equilibrium

Given a path of policies $\left\{R_t^f, p_t\right\}_{t\geq 0}$ and fundamentals $\left\{y_t^g, y_t^b\right\}_{t\geq 0}$, a dynamic equilibrium is a sequence of masses $\left\{m_t^b, m_t^g, m_t^f\right\}_{t\geq 0}$, equity e_t, and loan rates $\left\{R_t^g, R_t^b\right\}$ such that for all t, banks sort optimally:

$$\begin{aligned}m_t^b>0\Rightarrow e_t\leq e_t^{*}&=1-\frac{\theta^g R_t^g-\theta^b R_t^b}{p_t\left(\theta^g-\theta^b\right)},\\ m_t^g+m_t^b<1\Rightarrow e_t\geq e_t^{**}&=1-\frac{R_t^f-\theta^g R_t^g}{p_t\left(1-\theta^g\right)},\end{aligned}$$

bank equity e_t follows

$$e_t=\iota+(1-\rho)\left[m_{t-1}^f R_{t-1}^f+m_{t-1}^g\theta^g\left[R_{t-1}^g-\tilde{R}_{t-1}^g\left(1-e_{t-1}\right)\right]+m_{t-1}^b\theta^b\left[R_{t-1}^b-\tilde{R}_{t-1}^b\left(1-e_{t-1}\right)\right]\right],\tag{A.2}$$

where m_{t-1}^i is the mass of banks investing in asset class $i\in\{b,g,f\}$ at $t-1$, markets clear

$$\begin{aligned}F\left(e_t^{*}\right)&=m_t^b=\left(m_{t-1}^b+\lambda m_{t-1}^g\right)H\left(\theta^b\left(y_t^b-R_t^b\right)\right),\\ F\left(e_t^{**}\right)-F\left(e_t^{*}\right)&=m_t^g=\left[(1-\lambda)m_{t-1}^g+\lambda\right]H\left(\theta^g\left(y_t^g-R_t^g\right)\right),\\ 1-F\left(e_t^{**}\right)&=m_t^f=1-m_t^b-m_t^g,\end{aligned}$$

and productivity follows (15).

C.4 Forward-looking firm dynamics

Incumbent firms draw a cost shock ϵ in each period. If they do not exit they earn current expected profit

$$\pi_t^i(\epsilon) = \theta^i \left(y_t^i - R_t^i\right) - \epsilon$$

Assume firms exit when their project fails. A forward-looking incumbent firm's value function if it does not exit is

$$\Pi_t^i(\epsilon) = \pi_t^i(\epsilon) + \beta\theta^i \underbrace{\mathbf{E}_t\left[\left(1-\lambda^i\right)\max\left\{\Pi_{t+1}^i(\epsilon_{t+1}), 0\right\} + \lambda^i \max\left\{\Pi_{t+1}^{-i}(\epsilon_{t+1}), 0\right\}\right]}_{=W_{t+1}^i}$$

where with a probability λ^i the firm can change type to $-i$ next period. Then the firm does not exit if and only if

$$\Pi_t^i(\epsilon) \geq 0 \Leftrightarrow \epsilon \leq \bar{\epsilon}_t^i = \theta^i \left(y_t^i + \beta W_{t+1}^i - R_t^i\right)$$

A myopic firm ignores the W_{t+1}^i part, hence does not exit if and only if $\pi_t^i(\epsilon) \geq 0$, i.e., $\epsilon \leq \theta^i \left(y_t^i - R_t^i\right)$.

Potential entrants are all of the $i = g$ type, and have cost $c_t - \gamma - \epsilon$. If they enter they must pay an entry cost κ, hence they earn current expected profit

$$\pi_t^n(\epsilon) = \theta^g \left(y^g - R_t^g\right) + \gamma - \epsilon - \kappa$$

in the first period. After one period they become incumbents and lose their productivity advantage γ (it is straightforward but inconvenient to generalize to γ lasting multiple periods). Thus a potential entrant enters if and only if

$$\epsilon \leq \bar{\epsilon}_t^n = \bar{\epsilon}_t^g + \gamma - \kappa$$

Incumbents' value functions satisfy

$$\Pi_t^i(\epsilon) = \pi_t^i(\epsilon) + \beta\theta^i \left[\left(1-\lambda^i\right)\int_0^{\bar{\epsilon}_{t+1}^i} \Pi_{t+1}^i(\epsilon')\, dH(\epsilon') + \lambda^i \int_0^{\bar{\epsilon}_{t+1}^{-i}} \Pi_{t+1}^{-i}(\epsilon')\, dH(\epsilon')\right]$$

Since ϵ is additive and iid, $\Pi_t^i(\epsilon) = \Pi_t^i(0) - \epsilon$ and by definition (in the case of an interior solution which we will check)

$$\Pi_t^i(0) = \bar{\epsilon}_t^i$$

Thus we need only keep track of the two paths of the two thresholds $\left\{\bar{\epsilon}_t^g, \bar{\epsilon}_t^b\right\}_t$. Rearranging the

Bellman equation, they solve

$$\bar{\epsilon}_t^i = \pi_t^i(0) + \beta\theta^i \left[\left(1-\lambda^i\right) \int_0^{\bar{\epsilon}_{t+1}^i} \left(\bar{\epsilon}_{t+1}^{i,o} - \epsilon'\right) dH(\epsilon') + \lambda^i \int_0^{\bar{\epsilon}_{t+1}^{-i}} \left(\bar{\epsilon}_t^{-i,o} - \epsilon'\right) dH(\epsilon') \right]$$

If H is uniform between 0 and 1, this simplifies to two quadratic equations.

D Proofs

D.1 Proof of Lemma 1

There are two cases to consider:

Case 1. A bank prefers lending to a type G borrower at rate R^g instead of lending to a type B borrower if:

$$\theta^g \left(R^g - \tilde{R}^g(1-e)\right) \geq \theta^b \left(R^b - \tilde{R}^b(1-e)\right).$$

Using the definition of $\tilde{R}^j$, $j = g, b$, this condition is met for banks with level of capitalization above the following threshold:

$$e \geq e^* = 1 - \frac{\theta^g R^g - \theta^b R^b}{p\Delta\theta}.$$

When $\delta > 0$ and a bank has a legacy B borrower, the bank prefers to switch to a new G borrower if

$$\theta^g \left(R^g - \tilde{R}^g(1-e+\delta)\right) \geq \theta^b \left(R^b - \tilde{R}^b(1-e)\right)$$

which is equivalent to

$$e \geq e^* + \delta\nu$$

where $\nu = \frac{\theta^g \tilde{R}^g}{p\Delta\theta} = \frac{R^f - p(1-\theta^g)}{p\Delta\theta}$.

Case 2. A bank prefers investing its capital in safe assets rather than lending to a type G borrower at rate R^g if:

$$R^f - R^d(1-e) > \theta^g \left(R^g - \tilde{R}^g(1-e)\right)$$

Using the definition of $\tilde{R}^g = \frac{R^d - (1-\theta^g)p}{\theta^g}$ and under the assumption that $R^d = R^f$, this condition is met for banks with level of capitalization above the following threshold:

$$e > e^{**} = 1 - \frac{R^f - \theta^g R^g}{p(1-\theta^g)}.$$

As long as $e^{**} > e^*$, a bank that prefers investing in safe assets over lending to type G firms *a fortiori* prefers investing in safe assets over lending to type B firms. The following conditions ensured that $e^* < e^{**}$:

$$\frac{R^f - \theta^g R^g}{1-\theta^g} < \frac{\theta^g R^g - \theta^b R^b}{\theta^g - \theta^b},$$

or, equivalently,

$$R^f \Delta\theta < \theta^g R^g(1-\theta^b) - \theta^b R^b(1-\theta^g). \tag{A.3}$$

D.2 Proof of Proposition 1

$Y = Y^*$ is achieved when all banks lend and there is no zombie lending, hence $m^g = 1$ and $m^b = 0$. Relative to this composition of firms, any substitution towards bonds decreases output, and any increase in zombie lending decreases output by Assumption 1.

In an equilibrium with $Y = Y^*$ loan rates are given by

$$R^b = \bar{y}^b(1-z)$$
$$R^g = \bar{y}^g(1-z) - \frac{\bar{\epsilon}}{\theta^g}$$

Given these equilibrium loan rates, we verify that all banks lend, that is $e^{**} \geq e_{\max}$, and that there is indeed no zombie lending, that is $e^* \leq e_{\min}$.

These conditions can be rewritten respectively as

$$1 - \frac{R^f - \theta^g R^g}{p(1-\theta^g)} = 1 - \frac{R^f + \bar{\epsilon} - \theta^g \bar{y}^g(1-z)}{p(1-\theta^g)} \geq e_{\max} \Leftrightarrow R^f \leq \bar{R}^f(p,z)$$

where $\bar{R}^f(p,z) = \theta^g \bar{y}^g(1-z) - \bar{\epsilon} + (1-e_{\max})(1-\theta^g)p$, and

$$1 - \frac{\theta^g R^g - \theta^b R^b}{p\left(\theta^g - \theta^b\right)} = 1 - \frac{\left(\theta^g \bar{y}^g - \theta^b \bar{y}^b\right)(1-z) - \bar{\epsilon}}{p\left(\theta^g - \theta^b\right)} \leq e_{\min} \Leftrightarrow p \leq \bar{p}(z)$$

where

$$\bar{p}(z) = \frac{(\theta^g \bar{y}^g - \theta^b \bar{y}^b)(1-z) - \bar{\epsilon}}{(1-e_{\min})\left(\theta^g - \theta^b\right)}.$$

If R^f is lower than the type G project with the lowest net present value, i.e. $R^f < \theta^g \bar{y}^g(1-z) - \bar{\epsilon}$, then all banks lend and with $p \leq \bar{p}$ the economy reaches Y^* because there is also no zombie lending. Finally, if $p > \bar{p}$ then there is necessarily zombie lending in equilibrium and $Y < Y^*$, regardless of the level of R^f.

D.3 Proof of Proposition 2

When the shock z is small, an accommodating conventional monetary policy alone can achieve $Y = Y^*$ at no costs ($p = 0$), without violating the ELB constraint. Adapting the results of Proposition 1, the monetary policy rate that achieves $m^g = 1$ with $p = 0$ is

$$R^f(z) = \theta^g \bar{y}^g (1 - z) - \bar{\epsilon}.$$

This interest rate satisfies the ELB constraint if $\theta^g \bar{y}^g (1 - z) - \bar{\epsilon} \geq R^f_{\min}$ or

$$z \leq \underline{z} = 1 - \frac{R^f_{\min} + \bar{\epsilon}}{\theta^g \bar{y}^g}.$$

For moderate shocks, $z_t > \underline{z}$, a combination of conventional and a lax forbearance policy, $p(z)$, can still achieve $Y = Y^*$ even if the ELB binds. Adapting the results of Proposition 1, given the loan rates in an equilibrium without zombie lending, this requires

$$R^f(z) = \theta^g \bar{y}^g (1 - z) - \bar{\epsilon} + (1 - e_{\max})(1 - \theta^g) p(z).$$

Exhausting the stimulus from conventional monetary policy, the optimal policy sets $R^f(z) = R^f_{\min}$, so p must satisfy $R^f_{\min} = \theta^g \bar{y}^g (1 - z) - \bar{\epsilon} + (1 - e_{\max})(1 - \theta^g) p(z)$, or

$$p(z) = \frac{R^f_{\min} + \bar{\epsilon} - \theta^g \bar{y}^g (1 - z)}{(1 - e_{\max})(1 - \theta^g)}$$

which is an increasing function of z. The conjectured equilibrium loan rates are correct as long as $p(z) \leq \bar{p}(z)$ or

$$\frac{R^f_{\min} + \bar{\epsilon} - \theta^g \bar{y}^g (1 - z)}{(1 - e_{\max})(1 - \theta^g)} \leq \frac{\left(\theta^g \bar{y}^g - \theta^b \bar{y}^b\right)(1 - z) - \bar{\epsilon}}{(1 - e_{\min})\left(\theta^g - \theta^b\right)}$$

$$z \leq \bar{z} = 1 - \frac{\frac{(1-e_{\min})\left(\theta^g - \theta^b\right)}{(1-e_{\max})(1-\theta^g)}\left[R^f_{\min} + \bar{\epsilon}\right] + \bar{\epsilon}}{\theta^g \bar{y}^g - \theta^b \bar{y}^b + \theta^g \bar{y}^g \frac{(1-e_{\min})\left(\theta^g - \theta^b\right)}{(1-e_{\max})(1-\theta^g)}}.$$

For large shocks, $z > \bar{z}$, conventional monetary policy is constrained by the lower bound and increasing the level of forbearance induces credit misallocation. There are two cases, depending on the value of the weight β on congestion externalities in social welfare (11):

- If β is high as in (14), the optimal policy response ensures $m^b = 0$ but Y^* is not attainable.

The optimal forbearance policy $p(z) > 0$ solves:

$$F\left(1 - \frac{(1-e_{\min})\left(\theta^g - \theta^b\right)}{1-\theta^g} - \frac{R^f_{\min} - \theta^b \bar{y}^b(1-z)}{p\,(1-\theta^g)}\right) = H\left(\theta^g \bar{y}^g\,(1-z) - \theta^b y^b - p\,(1-e_{\min})\,\Delta\theta\right),$$

which implies that the optimal $p\,(z)$ is decreasing in the size of the shock.

- If β is low as in (13), the optimal policy response maximizes bank lending and thus output, ignoring congestion externalities. Given z and p the equilibrium loan rates $R^b(z,p)$ and $R^g(z,p)$ must solve

$$\lambda H\left(\theta^b\left(\bar{y}^b(1-z) - R^b\right)\right) = F\Bigg(\overbrace{1 - \frac{\theta^g R^g - \theta^b R^b}{p\Delta\theta}}^{=e^*}\Bigg)$$

$$\lambda H\left(\theta^b\left(\bar{y}^b(1-z) - R^b\right)\right) + H\left(\theta^g\left(\bar{y}^g(1-z) - R^g\right)\right) = F\Bigg(\underbrace{1 - \frac{R^f_{\min} - \theta^g R^g}{p\,(1-\theta^g)}}_{=e^{**}}\Bigg)$$

If e^{**} evaluated when $p = 1$ is strictly below $e_{\max}$, i.e., $1 - \frac{R^f_{\min} - \theta^g R^g(z,1)}{(1-\theta^g)} < e_{\max}$, then the optimal p is the maximal possible forbearance $p = 1$. Otherwise the optimal p is the lowest p ensuring that all banks lend, solving

$$1 - \frac{R^f_{\min} - \theta^g R^g(z,p)}{p\,(1-\theta^g)} = e_{\max}$$

which yields a solution $p(z)$ that is increasing in z. To see this, we rewrite the equilibrium system under the optimal policy as

$$D_b(R^b, z) = S_b(R^b, R^g, p)$$
$$D_b(R^b, z) + D_g(R^g, z) = 1$$
$$(1-e_{\max})(1-\theta^g)p + \theta^g R^g = R^f_{\min}$$

where we define the demand for type B loans $D_b = \lambda H\left(\theta^b\left(\bar{y}^b(1-z) - R^b\right)\right)$, the demand for type G loans $D_g = H\left(\theta^g\left(\bar{y}^g(1-z) - R^g\right)\right)$, and the supply of type B loans $S_b = F\left(1 - \frac{\theta^g R^g - \theta^b R^b}{p\Delta\theta}\right)$. The argument only relies on the monotonicity of these functions and goes through even without differentiability, but the exposition is simpler using derivatives and the implicit function theorem. Suppose that the optimal forbearance is not everywhere non-decreasing with z, i.e., there exists a z such that $\frac{dp}{dz} < 0$. Then from the third line of the system we have

$\frac{dR^g}{dz} > 0$. The second line then implies

$$\frac{dR^b}{dz} = -\underbrace{\frac{1}{\frac{\partial D_b}{\partial R^b}}}_{\leq 0}\left(\overbrace{\frac{\partial D_g}{\partial R^g}}^{\leq 0}\overbrace{\frac{dR^g}{dz}}^{>0} + \overbrace{\frac{\partial D_g}{\partial z} + \frac{\partial D_b}{\partial z}}^{\leq 0}\right) \leq 0.$$

and that locally $D_g(R^g, z)$ falls with z, hence $D_b(R^b, z)$ rises with z. But this is incompatible with the first line since $S_b(R^b, R^g, p)$ would fall with z. Therefore $p'(z) \geq 0$ everywhere.

D.4 Proof of Corollary 1

We have from the expression of $\bar{z}$ in the proof of Proposition 2

$$\bar{z} = 1 - \frac{\frac{(1-e_{\min})(\theta^g-\theta^b)}{(1-e_{\max})(1-\theta^g)}\left[R^f_{\min} + \bar{\epsilon}\right] + \bar{\epsilon}}{\theta^g\bar{y}^g - \theta^b\bar{y}^b + \theta^g\bar{y}^g\frac{(1-e_{\min})(\theta^g-\theta^b)}{(1-e_{\max})(1-\theta^g)}}$$

Therefore $\bar{z}$ increases with $e_{\min}$ if the function $x \mapsto \frac{x\left[R^f_{\min}+\bar{\epsilon}\right]+\bar{\epsilon}}{x\theta^g\bar{y}^g+\theta^g\bar{y}^g-\theta^b\bar{y}^b}$ increases with $x = \frac{(1-e_{\min})(\theta^g-\theta^b)}{(1-e_{\max})(1-\theta^g)}$. The derivative of this function with respect to x is positive if and only if

$$\left[R^f_{\min} + \bar{\epsilon}\right]\left[x\theta^g\bar{y}^g + \theta^g\bar{y}^g - \theta^b\bar{y}^b\right] > \theta^g\bar{y}^g\left[x\left[R^f_{\min} + \bar{\epsilon}\right] + \bar{\epsilon}\right]$$
$$R^f_{\min} > -\frac{\theta^b\bar{y}^b}{\theta^g\bar{y}^g - \theta^b\bar{y}^b}\bar{\epsilon}$$

which is satisfied.

D.5 Proof of Proposition 4

We assume a technical condition on the distribution H of idiosyncratic cost shocks ϵ,

$$\sup_{e\in[0,1]} \frac{h\left(\theta^b y^b - R^f_{\min} + (1-e)\left(1-\theta^b\right)\right)}{h\left(H^{-1}\left(1-\lambda H\left(\theta^b y^b - R^f_{\min} + (1-e)\left(1-\theta^b\right)\right)\right)\right)} \geq 1 - \frac{\Delta\theta}{1-\theta^b} \qquad \text{(A.4)}$$

which is satisfied when H is uniform, for instance.

A stable sclerosis steady state must have

$$p^m\left(z, e_\infty\right) = 1$$

i.e.

$$H\left(\theta^g y^g (1-z) - R^f_{\min} + (1-e_\infty)(1-\theta^g)\right) + \lambda H\left(\theta^b y^b - R^f_{\min} + (1-e_\infty)\left(1-\theta^b\right)\right) < 1$$

This can be written concisely as

$$z > Z(e_\infty)$$

where

$$\zeta(e) = 1 - \frac{R^f_{\min} + c - (1-e)(1-\theta^g) + H^{-1}\left(1 - \lambda H\left(\theta^b y^b - R^f_{\min} + (1-e)\left(1-\theta^b\right)\right)\right)}{\theta^g \bar{y}^g}$$

$$Z(e) = \max\{\bar{z}, \zeta(e)\}$$

are decreasing functions of e by (A.4).

At any t the zero lower bound binds and $p^m(z_t, e_t) > 0$ if and only if $z_t \geq \bar{z}$. Moreover, if $z_t \geq Z(e_t)$ then the optimal myopic policy sets $p^m(z_t, e_t) = 1$ and therefore

$$z_{t+1} = \alpha\lambda H\left(\theta^b y^b_t - R^f_{\min} + (1-e_t)\left(1-\theta^b\right)\right)$$

Thus we have a permanent sclerosis equilibrium (defined below) if for each t, $z_{t+1} \geq Z(e_{t+1})$ or

$$\alpha\lambda H\left(\theta^b y^b_t - R^f_{\min} + (1-e_t)\left(1-\theta^b\right)\right) \geq \max\left\{\bar{z}, \zeta\left(\iota + (1-\rho) R^f_{\min} e_t\right)\right\}$$

that is for all t

$$\alpha \geq \frac{\max\left\{\bar{z}, \zeta\left(\iota + (1-\rho) R^f_{\min} e_t\right)\right\}}{\lambda H\left(\theta^b y^b_t - R^f_{\min} + (1-e_t)(1-\theta^b)\right)}$$

ζ is decreasing in e but the denominator is also decreasing in e_t. We always have

$$\frac{\iota}{1-(1-\rho) R^f_{\min}} = \underline{e}_\infty \leq e_t \leq e_0 = \frac{\iota}{1-(1-\rho)\left[\theta^g \bar{y}^g - \bar{\epsilon}\right]}$$

Therefore an upper bound on the right-hand side is

$$\hat{\alpha} = \frac{\max\left\{\bar{z}, \zeta\left(\iota + (1-\rho) R^f_{\min} \underline{e}_\infty\right)\right\}}{\lambda H\left(\theta^b \bar{y}^b (1-z_0) - R^f_{\min} + (1-e_0)(1-\theta^b)\right)}$$

and a sufficient condition for permanent sclerosis to happen is $\alpha \geq \hat{\alpha}$.

D.6 Proof of Proposition 5

Following the same steps as without equity issuance costs we find:

$$e^* = 1 - \frac{\theta^g R^g - \theta^b R^b}{(\theta^g - \theta^b)p} - \frac{\varphi\left(\theta^g \tilde{R}^g\right) - \varphi\left(\theta^b \tilde{R}^b\right)}{\theta^g \tilde{R}^g - \theta^b \tilde{R}^b}$$
$$e^{**} = 1 - \frac{R^f - \theta^g R^g}{p\left(1 - \theta^g\right)} - \frac{\varphi\left(R^f\right) - \varphi\left(\theta^g \tilde{R}^g\right)}{R^f - \theta^g \tilde{R}^g}$$

where $\varphi(x) = x\left(\kappa'\right)^{-1}(x) - \kappa\left(\left(\kappa'\right)^{-1}(x)\right)$. The function φ inherits the properties of κ, as $\varphi'(x) = \left(\kappa'\right)^{-1}(x)$ and $\varphi''(x) = \frac{1}{\kappa''\left(\left(\kappa'\right)^{-1}(x)\right)}$. Since $\theta^g \tilde{R}^g - \theta^b \tilde{R}^b = (\theta^g - \theta^b)p > 0$, it follows from the convexity of φ that the slope of $\frac{\varphi\left(\theta^g \tilde{R}^g\right) - \varphi\left(\theta^b \tilde{R}^b\right)}{\theta^g \tilde{R}^g - \theta^b \tilde{R}^b}$ is increasing with R^f and (decreasing with p).

D.7 Proof of Proposition 6

When $\delta > \Delta^g - \Delta^b$, there are three relevant regions for banks initially matched with a bad firm. If $e < \hat{e} - \Delta^b$, then the capital requirement is binding even if the bank remains with its legacy B borrower. If $e > \hat{e} - \Delta^g + \delta$, the capital requirement is never binding, whether the bank switches or not. For intermediate equity $e \in \left[\hat{e} - \Delta^b, \hat{e} - \Delta^g + \delta\right]$, the capital requirement is binding only if the bank switches.

We start with the banks matched to a borrower that turns B.

1. Suppose that $\hat{e}$ is high enough that the bank $e = \hat{e} - \Delta^b$ prefers to switch to a new G borrower and thus issue $\hat{e} - e - \delta = \Delta^b + \delta$, that is

$$\sigma(\hat{e}) \geq \kappa\left(\Delta^b + \delta\right) - \kappa\left(\Delta^b\right) \tag{A.5}$$

or

$$\delta \leq \kappa^{-1}\left(\sigma(\hat{e}) + \kappa\left(\Delta^b\right)\right) - \Delta^b$$

Therefore, all the banks above $e = \hat{e} - \Delta^b$ will prefer to switch, and the only potential for zombie lending is for banks below $\hat{e} - \Delta^b$. In that case, banks lending to zombies are those with pre-issuance equity e below the indifference threshold e^* solving

$$\sigma(\hat{e}) = \kappa\left(\hat{e} - e^* + \delta\right) - \kappa\left(\hat{e} - e^*\right)$$

Note that $\sigma(\hat{e}) > 0$ implies $\hat{e} > E^*$. From the implicit function theorem, when $\hat{e}$ increases (holding loan rates fixed in this partial equilibrium first step) we have

$$\frac{\partial e^*}{\partial \hat{e}} = 1 - \frac{\sigma'(\hat{e})}{\kappa'(\hat{e} - e^* + \delta) - \kappa'(\hat{e} - e^*)} = 1 - \frac{\theta^g \tilde{R}^g - \theta^b \tilde{R}^b}{\kappa'(\hat{e} - e^* + \delta) - \kappa'(\hat{e} - e^*)}$$

This can be rewritten as

$$\frac{\partial e^*}{\partial \hat{e}} = 1 - \frac{\kappa'(\Delta^g) - \kappa'\left(\Delta^b\right)}{\kappa'(\hat{e} - e^* + \delta) - \kappa'(\hat{e} - e^*)}$$

Since $\delta > \Delta^g - \Delta^b$ and $\hat{e} - e^* \geq \Delta^b$, we necessarily have

$$\frac{\partial e^*}{\partial \hat{e}} > 0$$

and thus in this region, increasing capital requirements worsens legacy zombie lending.

(a) Suppose then that (A.5) doesn't hold:

$$\delta > \kappa^{-1}\left(\sigma(\hat{e}) + \kappa\left(\Delta^b\right)\right) - \Delta^b$$

which implies that the bank with $e = \hat{e} - \Delta^b$ prefers to stay matched with its legacy B borrower.

i. If the bank with $e = \hat{e} - \Delta^g + \delta$ prefers to switch to a new G borrower, that is

$$\sigma(\hat{e}) > \kappa(\Delta^g) - \kappa\left(\Delta^b\right) + \theta^b \tilde{R}^b\left(\delta - \Delta^g + \Delta^b\right) \tag{A.6}$$

holds, then all banks with even higher e also switch. Thus the indifference threshold e^* is in the intermediate region $\left[\hat{e} - \Delta^b, \hat{e} - \Delta^g + \delta\right]$ and solves

$$\theta^b\left[R^b - \tilde{R}^b\left(1 - e^* - \Delta^b\right)\right] - \kappa\left(\Delta^b\right) = \theta^g\left[R^g - \tilde{R}^g(1 - \hat{e})\right] - \kappa(\hat{e} - e^* + \delta)$$

or

$$\sigma(\hat{e}) = \theta^b \tilde{R}^b\left(e^* - \hat{e} + \Delta^b\right) + \kappa(\hat{e} - e^* + \delta) - \kappa\left(\Delta^b\right)$$

By the implicit function theorem,

$$\frac{\partial e^*}{\partial \hat{e}} = 1 - \frac{\sigma'(\hat{e})}{\kappa'(\hat{e} - e + \delta) - \theta^b \tilde{R}^b} = \frac{\kappa'(\hat{e} - e + \delta) - \theta^g \tilde{R}^g}{\kappa'(\hat{e} - e + \delta) - \theta^b \tilde{R}^b} > 0$$

which follows from $\hat{e} - e + \delta \geq \Delta^g > \Delta^b$. Therefore, in this region as well, increasing capital requirements worsens legacy zombie lending.

ii. The last case is when $\hat{e}$ is so low that even the bank with $e = \hat{e} - \Delta^g + \delta$ prefers to lend to its legacy B borrower, that is

$$\sigma\left(\hat{e}\right) < \kappa\left(\Delta^{g}\right) - \kappa\left(\Delta^{b}\right) + \theta^{b}\tilde{R}^{b}\left(\delta - \Delta^{g} + \Delta^{b}\right) \tag{A.7}$$

holds, and so all the banks with lower equity also rollover the B loan. Then the indifference threshold e^{*} is above $\hat{e} - \Delta^{g} + \delta$ and is the same as in the absence of a capital requirement:

$$e^{*} = \underbrace{1 - \frac{\theta^{g}R^{g} - \theta^{b}R^{b}}{\theta^{g}\tilde{R}^{g} - \theta^{b}\tilde{R}^{b}}}_{E^{*}} - \frac{\varphi\left(\theta^{g}\tilde{R}^{g}\right) - \varphi\left(\theta^{b}\tilde{R}^{b}\right)}{\theta^{g}\tilde{R}^{g} - \theta^{b}\tilde{R}^{b}} + \frac{\theta^{g}\tilde{R}^{g}}{\theta^{g}\tilde{R}^{g} - \theta^{b}\tilde{R}^{b}}\delta$$

so does not vary with $\hat{e}$. Low enough capital requirements become irrelevant for legacy zombie lending.

For banks matched with a good firm, since we abstract from switching costs δ, they will switch to a new zombie borrower if their post-issuance equity is below

$$E^{*} = 1 - \frac{\theta^{g}R^{g} - \theta^{b}R^{b}}{\theta^{g}\tilde{R}^{g} - \theta^{b}\tilde{R}^{b}}$$

hence capital requirements have a knife-edge effect: either $\hat{e} \leq E^{*}$ and the capital requirement is irrelevant, or $\hat{e} \geq E^{*}$ and the capital requirement prevents all these banks (matched with a G firm) from switching to a new B borrower. Since we just showed that increasing $\hat{e}$ can never decrease legacy zombie lending, the only potential benefit is to prevent "new" zombie lending.

Next, note that the point $\hat{e}$ such that (A.7) holds with equality, that is

$$\hat{e} = E^{*} + \Delta^{g} - \frac{\varphi\left(\theta^{g}\tilde{R}^{g}\right) - \varphi\left(\theta^{b}\tilde{R}^{b}\right)}{\theta^{g}\tilde{R}^{g} - \theta^{b}\tilde{R}^{b}} + \frac{\theta^{b}\tilde{R}^{b}}{\theta^{g}\tilde{R}^{g} - \theta^{b}\tilde{R}^{b}}\delta$$

is strictly above E^{*} since $\sigma\left(\hat{e}\right) = \kappa\left(\Delta^{g}\right) - \kappa\left(\Delta^{b}\right) + \theta^{b}\tilde{R}^{b}\left(\delta - \Delta^{g} + \Delta^{b}\right) > 0 = \sigma\left(E^{*}\right)$.

The following result generalizes Proposition 1 and characterizes the optimal policy, in the case of quadratic equity issuance costs $\kappa\left(x\right) = \frac{1}{a}\frac{x^{2}}{2}$ that allow for closed-form solutions:

Proposition 7 (Optimal policy with equity issuance)**.** *Output reaches its potential ($Y = Y^{*}$) if and only if*

$$\underline{R}^{f}\left(p\right) \leq R^{f} \leq \bar{R}^{f}\left(p\right)$$

and

$$p \leq \bar{p}$$

where $\underline{R}^f(p)$ *and* $\bar{R}^f(p)$ *are given in the Appendix.*

The limit case $a \to 0$ recovers the no-issuance benchmark from Proposition 1. Under quadratic issuance costs, the optimal policy is characterized by the thresholds

$$\underline{R}^f(p) = p\left(1 - \frac{\theta^g + \theta^b}{2}\right) - \frac{1}{a}(1 - e_{\min})\left[\frac{\bar{p}}{p} - 1\right]$$
$$\bar{R}^f(p) = \frac{1}{1 + ap(1-\theta^g)}\bar{R}^f_{\text{no issuance}}(p) + \frac{ap^2(1-\theta^g)^2}{2(1+ap(1-\theta^g))}$$

and $\bar{p}$ and $\bar{R}^f_{\text{no issuance}}(p)$ are as defined in Proposition 1.

D.8 Proof of Proposition 7

Banks choose borrower type based on their post-issuance equity $e' = e + \Delta e$. Define the function $\varphi(x) = x(\kappa')^{-1}(x) - \kappa\left((\kappa')^{-1}(x)\right)$. There are two cases to consider:
Case 1. A bank with pre-issuance equity e prefers lending to a type G borrower at rate R^g instead of lending to a type B borrower if:

$$\theta^g\left(R^g - \tilde{R}^g(1 - e - \Delta^g)\right) - \kappa(\Delta^g) \geq \theta^b\left(R^b - \tilde{R}^b\left(1 - e - \Delta^b\right)\right) - \kappa\left(\Delta^b\right)$$

which can be rewritten as

$$e > e^* = 1 - \frac{\theta^g R^g - \theta^b R^b}{\theta^g \tilde{R}^g - \theta^b \tilde{R}^b} - \frac{\varphi\left(\theta^g \tilde{R}^g\right) - \varphi\left(\theta^b \tilde{R}^b\right)}{\theta^g \tilde{R}^g - \theta^b \tilde{R}^b}.$$

Case 2. A bank with pre-issuance equity e prefers investing its capital in safe assets rather than lending to a type G borrower at rate R^g if:

$$R^f\left(e + \Delta^f\right) - \kappa\left(\Delta^f\right) \geq \theta^g\left(R^g - \tilde{R}^g(1 - e - \Delta^g)\right) - \kappa(\Delta^g)$$

which can be rewritten as

$$e > e^{**} = 1 - \frac{R^f - \theta^g R^g}{R^f - \theta^g \tilde{R}^g} - \frac{\varphi\left(R^f\right) - \varphi\left(\theta^g \tilde{R}^g\right)}{R^f - \theta^g \tilde{R}^g}.$$