

Teacher Labor Market Policy and the Theory of the Second Best

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Policy question

- ▶ Policymakers argue that interventions in the teacher labor market are necessary to deliver high-quality teachers to disadvantaged students.
 - ▶ Teachers are influential and their effectiveness varies widely. Chetty et al. (2014)
 - ▶ Academic achievement gaps between economically advantaged and disadvantaged students are large, widespread, and impactful. Reardon (2011); Neal and Johnson (1996)
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 - ▶ Creates a two-sided market, which is common but often not explicitly modeled.

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Question: Which policies get effective teachers to economically disadvantaged students in a two-sided market?

Literature's answers

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Why is there more to learn?

1. One-sided market conclusions may not hold in two-sided equilibrium.
2. Identifying prefs. from equilibrium allocation requires assuming one side's prefs.

What we do to analyze the two-sided market

New insights using novel data on actions on both sides and a two-sided matching model

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- ▶ We use the concept of stability to form matches based on preferences estimated from unilateral decisions given the choice sets on both sides of the market.
- ▶ We use our model to evaluate counterfactual policies, with a direct measure of productivity (teachers' value-added), which we allow to differ by student type.

Theme of results: theory of the second best

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2. Teacher bonuses for teaching economically disadvantaged students are ineffective, especially when large. Principal hiring based on value-added would considerably reduce equity.
 - ▶ Fixing just one deviation is ineffective or harmful.
3. **The combination of teacher bonuses and principal bonuses is very effective even though either in isolation is ineffective.**
 - ▶ Important interactions demonstrate the need for modeling both sides of the market.

Outline of the talk

1. Model of decentralized equilibrium
2. Data and market timing
3. Student achievement: production model
4. Teacher preferences
5. Principal hiring rules
6. Current allocation
7. Counterfactuals

Overview of the environment

Setting has several important features.

- ▶ **Timing:** the market clears on a rolling basis from April through August
- ▶ **Decentralized:** teachers apply, principals decide on interviews and offers
- ▶ **Wages:** do not vary with assignment

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We model how teacher-school assignments are formed and the value of each assignment.

- ▶ **Agent types:** school principals and teachers
- ▶ **Output measure:** student test scores
- ▶ **Equilibrium restriction:** pairwise stability

Utilities and output

For student i , teacher j , school k , and student type m :

► Teacher utility:

$$u_{jk} = \underbrace{\tilde{u}_{jk}}_{\text{non-wage utility}} + \underbrace{w_{jk}}_{\text{wage}}$$

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► Student output:

$$VA_{jkm}$$

First best and decentralized equilibrium

First best

Let $\phi : \mathcal{J} \rightarrow \mathcal{K}$ be a one-to-one assignment.

The district maximizes the output of disadvantaged students ($m = 1$):

$$\max_{\phi \in \Phi} \sum_{j \in \mathcal{J}} \underbrace{n_{\phi(j)1}}_{\text{num disadv}} \overbrace{VA_{j\phi(j)1}}^{\text{VA with disadv}}$$

Equilibrium forces and policy tools

Allocation depends on:

1. Teacher non-wage preferences ($\tilde{u}_{jk}$)
2. Principal non-wage preferences ($\tilde{v}_{jk}$)
3. Timing
4. Equilibrium selection

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Suggested policy:

1. Teacher “hard-to-staff school” bonuses
2. Principal VA bonuses
3. Market coordination
4. Change market clearing protocol

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Data

Application and vacancy data (**more novel**):

- ▶ Large urban and suburban school district in NC, starting in 2010
- ▶ Flags for principal “positive assessment,” interviewed, hired
- ▶ Include records of timing of (some) events

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Student-level data (**more standard**):

- ▶ For the whole state of NC back to 2000
- ▶ Student demographics
- ▶ Test scores (we focus on math)

Teacher-level data (**more standard**):

- ▶ Experience, demographics, classroom assignment

Focus on elementary schools

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Measuring value-added

$$A_{it}^* = \beta_s X_{it} + f(Z_{jt}; \alpha) + \mu_{jmt} + \mu_k + \mu_t + \theta_{cmt} + \tilde{\epsilon}_{it}$$

- ▶ Student i , type m (“adv”, “disadv”), school k , classroom c , teacher j , year t
- ▶ Z_{jt} : teacher experience
- ▶ μ_{jmt} : teacher-student type-year effects

Forecasting:

- ▶ Allow for drift (Chetty, Friedman, and Rockoff (2014))
- ▶ Allow heterogeneous effectiveness by student type and allow correlation across types (Condie, Lefgren, & Sims (2014); Delgado (2022); Ahn, Aucejo, James (2021); Biasi, Fu, Stromme (2022); Eastmond et al. (2022),)
- ▶ Use prior data only
- ▶ Shrink estimates

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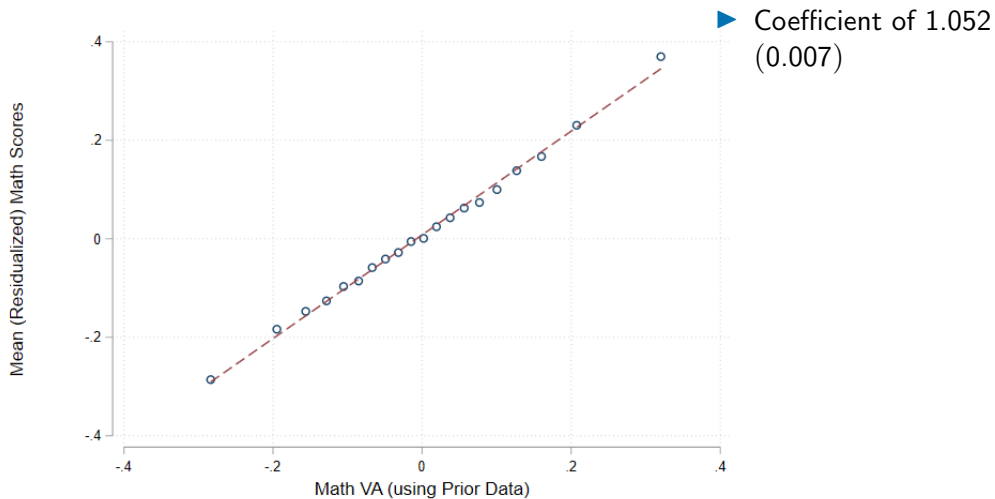
VA of teacher j with type m students in year t : $\hat{\mu}_{jmt}$

VA of teacher j , in school k and year t is:

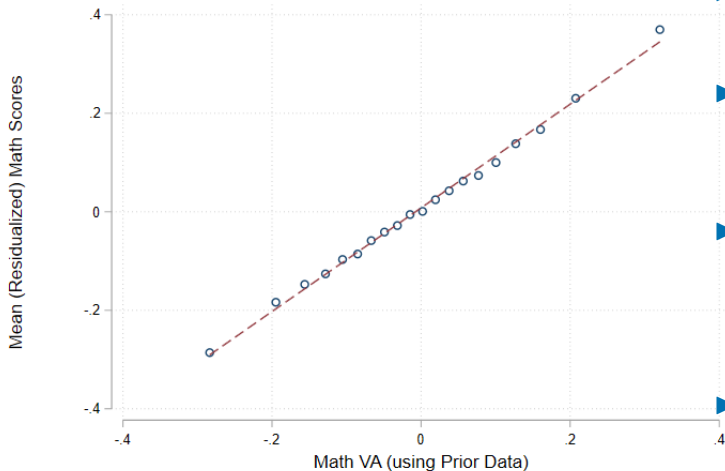
$$\widehat{VA}_{jkt} = n_{k0t} \hat{\mu}_{j0t} + n_{k1t} \hat{\mu}_{j1t} + f(Z_{jt}; \hat{\alpha})$$

▶ Details ▶ Other match specifications ▶ Experience ▶ Structural ▶ Drift ▶ Joint distribution ▶ LR test

Validation of value-added



Validation of value-added



- ▶ Coefficient of 1.052 (0.007)
- ▶ Predictive for transfers: 0.972 ▶ [Link](#)
 - ▶ [Event study](#)
- ▶ Similar across large composition changes
 - ▶ [Link](#) and class size changes ▶ [Link](#)
- ▶ Reject homogeneous model ▶ [Structural](#)
 - ▶ CA ▶ [Differential predictions](#)
 - ▶ [LR test](#)

Properties of the current allocation: student achievement

	Adv	Disadv
<i>Student performance (level scores)</i>		
Math	0.70	-0.16
<i>Student performance (gain scores)</i>		
Math	0.07	0.07
<i>Mean math value-added</i>		
Baseline, Adv	0.01	0.02
Baseline, Dis	0.02	0.02
Homogeneous	0.02	0.01
<i>Mean behavioral value-added</i>		
Behavioral index	0.01	0.02

- ▶ Large absolute gaps
- ▶ No difference in test score growth
- ▶ No/small difference in VA of teachers
- ▶ No sorting on comparative advantage
- ▶ Observables poorly predict VA (R^2 is less than 0.025)

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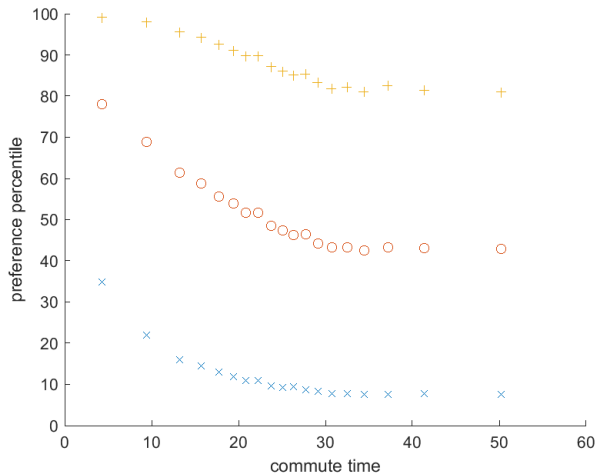
Parameterization of utilities

$$\tilde{u}_{jpt} = -\gamma d_{jpt} + \pi_j VA_{jpt} + \beta_j X_{pt} + \eta_{jt} + \epsilon_{jpt}$$

- ▶ d_{jpt} : commute time (minutes)
- ▶ $VA_{jpt} = n_{k0t}\hat{\mu}_{j0t} + n_{k1t}\hat{\mu}_{j1t}$
- ▶ π_j includes a mean coefficient and a normal RC
- ▶ X_{pt} : frac econ. disadv., frac Black, frac Hispanic, frac above median lagged ach
- ▶ β_j : mean coefficients, interactions with mean VA, normal RCs, same-race
- ▶ η_{jt} : teacher char. (gender, race, mean VA), Mundlak-Chamberlain device, RE
- ▶ ϵ_{jpt} : iid Type I EV error

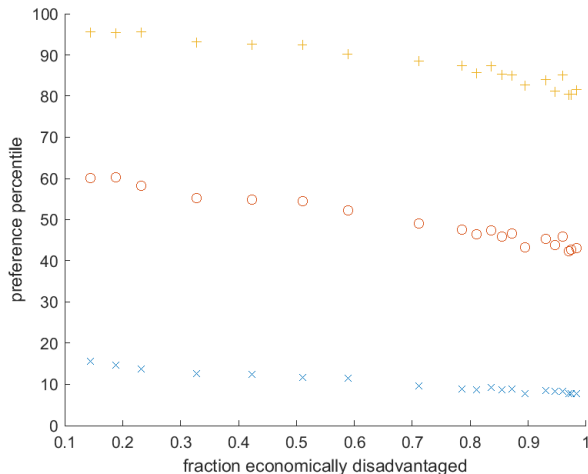
▶ Commute time ▶ Title I gap

Teacher preferences: commute time



- ▶ Pointwise 90th, mean, and 10th percentile
- ▶ Teachers prefer schools with a shorter commute
- ▶ Massive heterogeneity
- ▶ Doesn't show causal effect of commute time

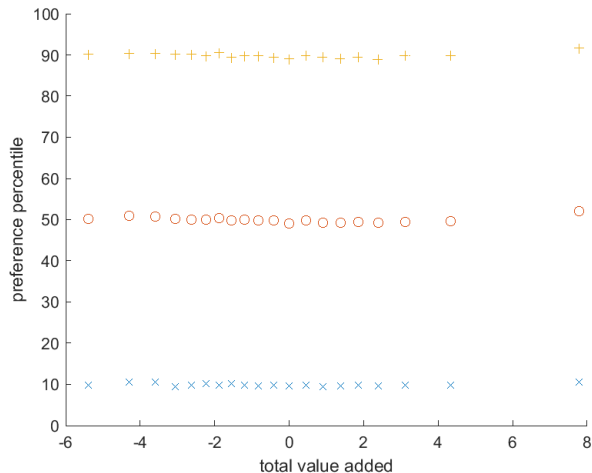
Teacher preferences: fraction disadvantaged



► First best ► Coefficients ► Teacher characteristics coefficients

- Doesn't show causal effect
- Teachers prefer schools with smaller share of disadvantaged students
- Substantial heterogeneity, possibility for marginal $\neq$ average
- Might infer this will lead to equity issues

Teacher preferences: value-added



- ▶ Teachers have weak preferences over output (coefficient/se: 0.081 (0.008))
- ▶ Large amounts of heterogeneity
- ▶ Doesn't show causal effect of output

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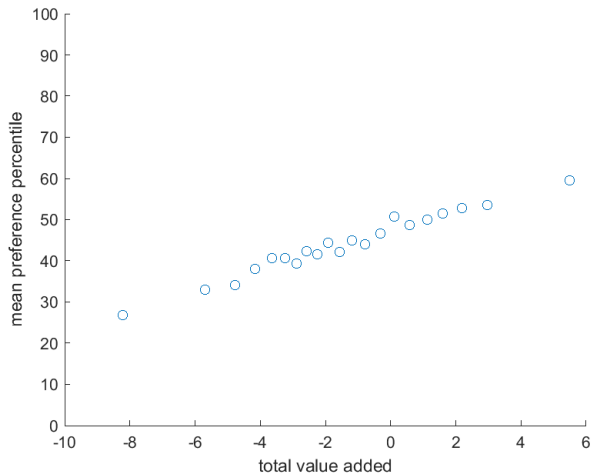
Principal ratings

The principal for position p gives teacher j a positive rating if $\tilde{v}_{jpt} > 0$.

$$\tilde{v}_{jpt} = \alpha_p W_{jpt} + \underbrace{\kappa_{pt}}_{\text{random effect}} + \underbrace{v_{jpt}}_{\text{iid Type I EV}}$$

- ▶ W_{jpt} includes...
 - ▶ teacher's value-added
 - ▶ experience, Masters, Black, Hispanic, female
 - ▶ Black $\times$ fraction Black, Hispanic $\times$ fraction Hispanic
- ▶ α_p : separate coefficients by p 's Title I status

Principal preferences: value-added



- ▶ Principals value output
- ▶ Doesn't perfectly predict ranking
- ▶ Doesn't show causal effect of output
- ▶ $\Rightarrow$ hard to evaluate magnitudes, equilibrium model quantifies

Value-added explains little of principal's ratings

Change in pseudo- R^2 from adding:		
	Non-Title I	Title I
Demographics (DG)	0.008	0.004
Teacher Characteristics (TC)	0.031	0.018
Value Added (VA)	0.006	0.003
Behavioral Value Added	0.005	0.003

- ▶ Demographics: race and gender, interacted with school demo.
- ▶ Teacher char.: experience, licensing, certification, and Praxis scores
- ▶ Recall, observables predict VA poorly
- ▶ Teacher char. go furthest

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Putting the pieces together

Sample:

- ▶ Teachers who apply in 2015-2016, and for whom we have prior data to estimate value-added (both in district, and from the rest of the state)
- ▶ Vacancies posted in 2015-2016. Drop randomly until market is balanced [Ashlagi et al. (2017)]

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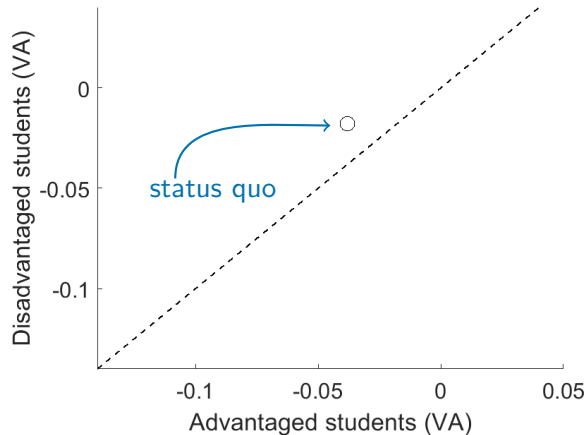
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Teacher and principal preferences:

- ▶ Use estimated preferences plus “best guess” of random coefficients given choices and choice set
- ▶ Simulate error draws

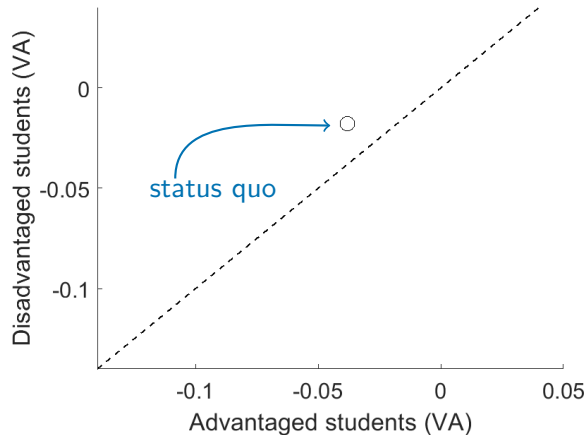
▶ Fit – AA ▶ Fit – CA ▶ Fit – Black ▶ Fit – Experience

Status quo



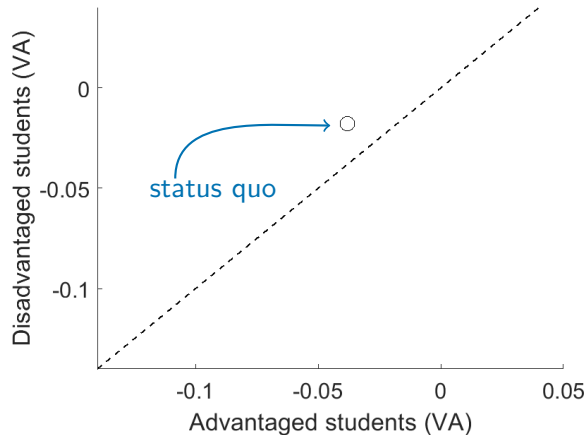
- Status quo: estimated teacher and principal preferences, empirical timing, teacher propose

Status quo



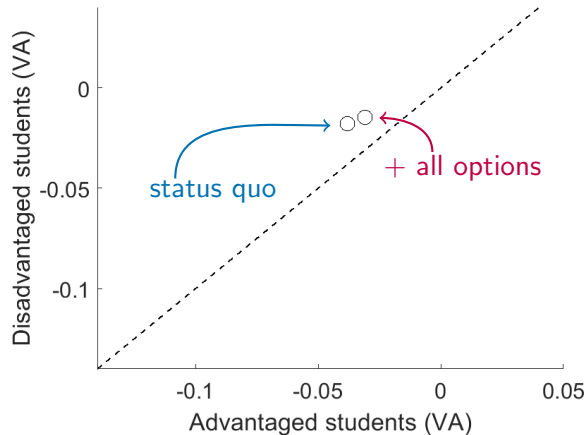
- ▶ Status quo: estimated teacher and principal preferences, empirical timing, teacher propose
- ▶ Model fits well: favors econ disadv. by 0.02
- ▶ Data: econ disadv. have 0.01 higher teacher VA
 - ▶ Data
 - ▶ Other

Status quo + equilibrium selection



- Change to school propose: no change
- $\Rightarrow$ equilibrium selection not an explanation

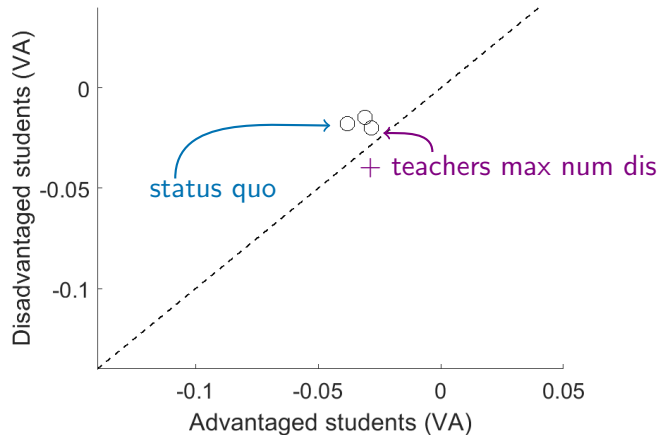
Remove timing constraints



► + all options: little change

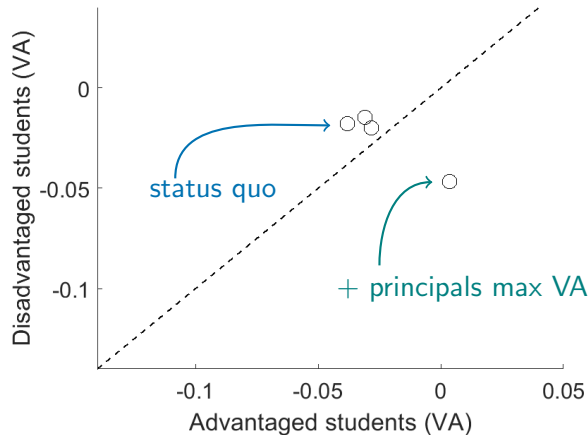
► $\Rightarrow$ timing not an explanation

Teachers max number disadvantaged



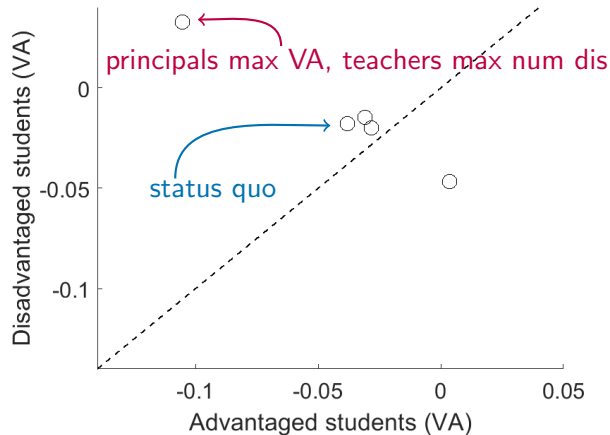
- ▶ + teachers max num dis: slight shift toward econ. adv.
- ▶ “Fixing” one deviation ineffective

Principals max VA



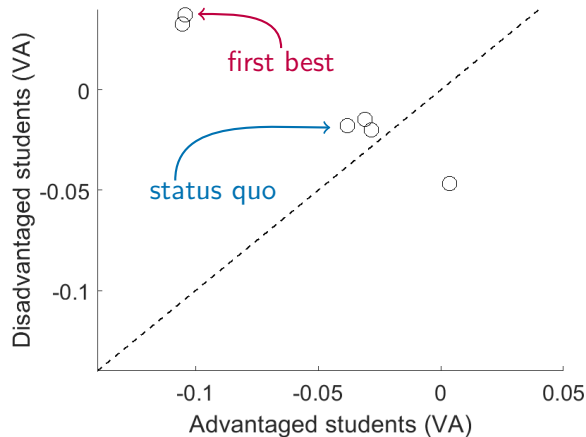
- ▶ + principals max VA: strongly favors econ. adv.
- ▶ Allocation you might expect based on teacher pref
- ▶ Principal preferences push back on teacher prefs ▶ Visually
- ▶ Second deviation interacts with first

Principals max VA, teachers max num disadv



- ▶ + principals max VA, teachers max num disadv: large gains for disadv students

First best

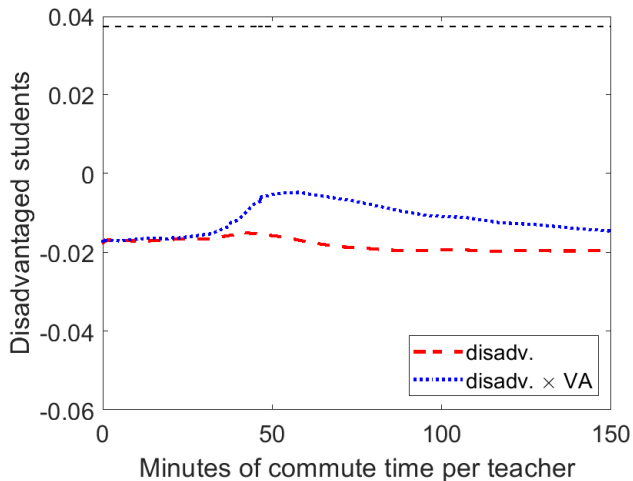


- ▶ **first best:** gains of 0.055 for econ disadv
- ▶ Closes 1/15 of achievement gap annually (and overall achievement $\uparrow$)
- ▶ Principals max VA, teachers max num disadv achieve most of first best
 - ▶ Efficiency
 - ▶ Behavioral VA

Outline of the talk

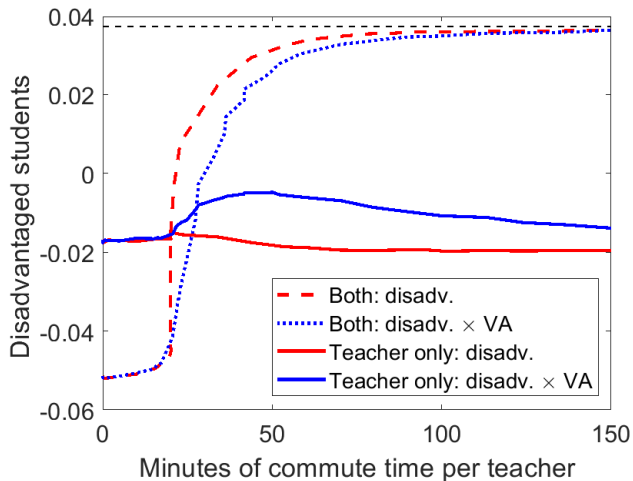
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Achievement with teacher bonuses



- ▶ Untargeted bonuses ineffective: increase applicant pool and worsen principal selection
- ▶ Targeted bonuses: address both deviations
- ▶ Larger bonuses become counterproductive

Achievement with teacher and principal bonuses



- ▶ At low levels of teacher bonuses, optimal to not give principals bonuses
- ▶ At high levels of teacher bonuses, optimal to give principals bonuses
- ▶ Targeting / better selection are substitutes

Summary

We study the two-sided teacher labor market.

- ▶ **Current allocation:** Disadvantaged students do not have worse teachers than advantaged students... because principals hire based on non-quality factors.
- ▶ **Policy:** Teacher and principal bonuses are individually ineffective but jointly effective.

These insights result from two-sided data and reasoning.

- ▶ Construct choice sets without assuming preferences.
- ▶ Link to estimates of teacher quality.
- ▶ Worse data $\Rightarrow$ stronger assumptions $\Rightarrow$ incorrect conclusions.

In other work, we explore efficiency (total learning).

- ▶ Noisy principal hiring *also increases* efficiency (theory of the second best).
- ▶ Bonus policies most effective for equity also tend to increase efficiency.

Structural parameters

	Estimates	Standard Errors
σ_{ϵ_1}	0.450	0.000
σ_{ϵ_2}	0.470	0.000
σ_{θ_1}	0.110	0.007
σ_{θ_2}	0.088	0.015
$\text{correlation}(\theta_{c0t}, \theta_{c1t})$	0.657	0.162
σ_{μ_1}	0.249	0.007
σ_{μ_2}	0.243	0.015
$\text{correlation}(\mu_{j0t}, \mu_{j1t})$	0.859	0.035

Experience returns

	1	2	3	4	5	6	7+
Estimate	0.056	0.077	0.083	0.088	0.088	0.091	0.070
Standard Error	0.004	0.004	0.005	0.005	0.005	0.005	0.005

[▶ Back](#)

Posting early and late

Posts in April	Posts in July		Total
	No	Yes	
No	8	15	23
Yes	10	88	98
Total	18	103	121

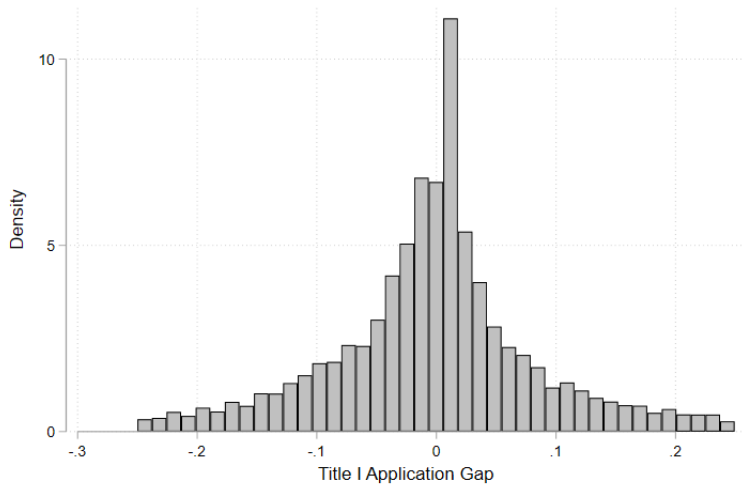
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Application evaluations, outcomes, and timing

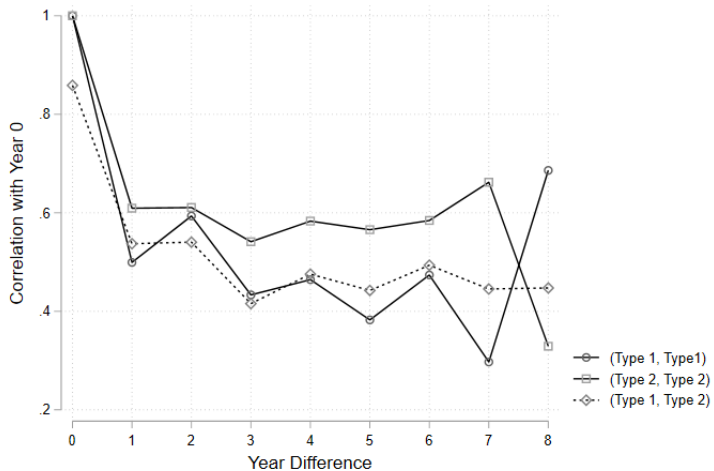
Panel A: Outcomes within job postings								
	Hired	Declined	Interview	Positive	Middle	Negative	Withdrew	No comment
Share with outcome	0.824	0.114	0.002	0.080	0.017	0.075	0.048	0.983
Number with outcome	2,046	283	4	199	41	187	118	2,442
Panel B: Day of application relative to hired application date								
	Obs	Mean	10th	25th	50th	75th	90th	Std. dev.
All applications	434,095	0.0	-18.0	-6.8	-0.9	5.3	18.2	16.78
No evaluation record	423,021	0.1	-17.6	-6.6	-0.9	5.3	17.9	16.53
Evaluation or outcome	11,074	-2.1	-32.1	-15.0	-4.1	7.6	31.2	24.54

[▶ Back](#)

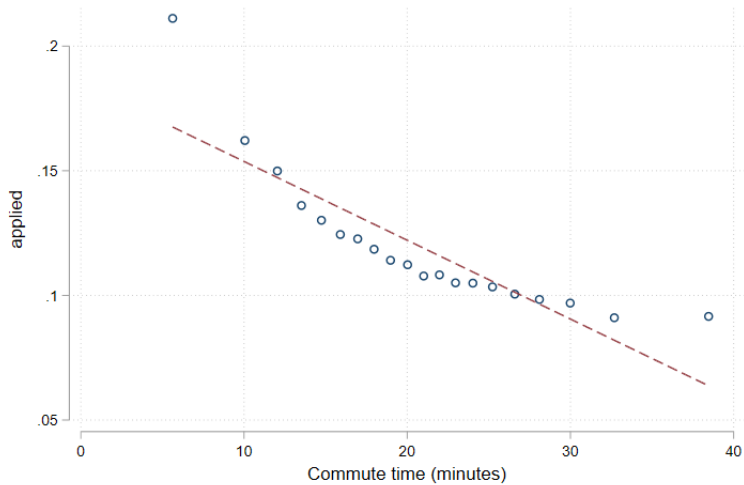
Application gap by Title I status



Drift correlations



Applications by commute time



Assumptions (to generate moment restrictions)

1. Exogeneity and stationarity of classroom and student level shocks

- ▶ $\mathbb{E}[\theta_{c0lt}|t] = \mathbb{E}[\theta_{c1lt}] = 0$; $\text{Var}(\theta_{c0lt}) = \sigma_{\theta_{0l}}^2$, $\text{Var}(\theta_{c1lt}) = \sigma_{\theta_{1l}}^2$, $\text{Cov}(\theta_{c0lt}, \theta_{c1lt}) = \sigma_{\theta_{0l}\theta_{1l}}$ for all t
- ▶ $\mathbb{E}[\tilde{\epsilon}_{ilt}|t] = 0$; $\text{Var}(\tilde{\epsilon}_{ilt}) = \sigma_{\epsilon_m}^2$ for $m = 0, 1$ for all t

2. Joint stationarity of teacher effects

- ▶ $\mathbb{E}[\mu_{j0lt}|t] = \mathbb{E}[\mu_{j1st}|t] = 0$;
 $\text{Var}(\mu_{j0lt}) = \sigma_{\mu_{0l}}^2$, $\text{Var}(\mu_{j1lt}) = \sigma_{\mu_{1l}}^2$, $\text{Cov}(\mu_{j0lt}, \mu_{j1lt}) = \sigma_{\mu_{0l}\mu_{1l}}$
- ▶ $\text{Cov}(\mu_{j0lt}, \mu_{j0l,t+s}) = \sigma_{\mu_{0l}s}$, $\text{Cov}(\mu_{j1lt}, \mu_{j1l,t+s}) = \sigma_{\mu_{1l}s}$; $\text{Cov}(\mu_{j0lt}, \mu_{j1l,t+s}) = \sigma_{\mu_{0l}\mu_{1l}s}$

3. Independence of drift and school effects

- ▶ Let $\bar{\mu}_{jml}$ be teacher j 's mean value-added for student type m in subject l . Let k be j 's assigned school in year t . Then: $(\mu_{jmlt} - \bar{\mu}_{jml}) \perp \mu_{kl}$ for $m = 0, 1$.

Implementation details

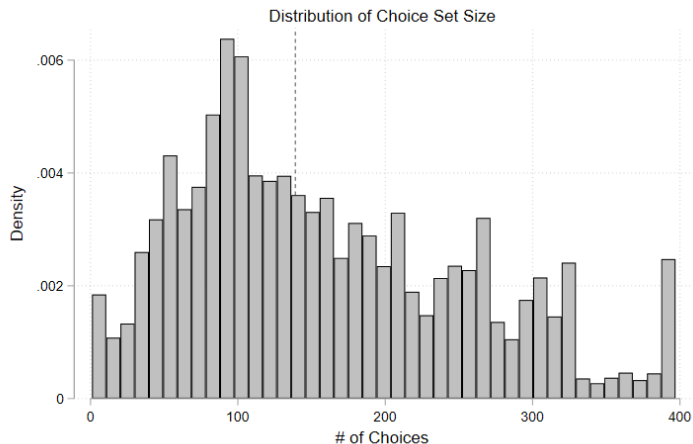
- ▶ Estimate coefficients on X (student char.) using within classroom student-type variation. Form residuals.
- ▶ Estimate experience effects (dummied out 0 to 6, and single category after 7) and school effects, controlling for teacher effects on these residuals. Form residuals.
- ▶ Construct teacher-year-student type value added as best linear predictors of these residuals

Teacher j , in subject l , school k and year t is:

$$\hat{\mu}_{jkl t} = n_{k0t} \hat{\mu}_{j0lt} + n_{k1t} \hat{\mu}_{j1lt} + n_{kt} f(Z_{jt}; \hat{\alpha}_l)$$

- ▶ Type-specific VA, weighted by number of students of each type in school k in year t

Choice set sizes



► Teachers have large sets of vacancies to apply to

► Early/late ► Events

Teacher preference estimates – school characteristics

	Estimate	Standard Error
Constant	2.032	4.453
Commute Time	-0.073	0.001
Commute Time Missing	-1.660	0.223
Value Added	0.081	0.008
St Dev Value Added RC	0.128	0.007
School Characteristics and Interactions		
Fraction Disadvantaged	-1.188	0.136
Fraction Black	-0.452	0.132
Fraction Hispanic	0.441	0.144
Fraction Above Median Achievement	0.163	0.149
Abs Adv x Fraction Disadvantaged	-0.797	1.029
Abs Adv x Fraction Black	-1.635	1.025
Abs Adv x Fraction Hispanic	2.487	1.074
Abs Adv x Fraction Above Median Achievement	-1.997	1.185
Black x Fraction Black	1.072	0.130
Hispanic x Fraction Hispanic	0.491	0.771
St Dev Fraction Disadvantaged RC	1.591	0.034
St Dev Fraction Black RC	1.296	0.054
St Dev Fraction Hispanic RC	0.637	0.065
St Dev Fraction Above Median Achievement RC	1.397	0.045

► Main Coefficients

Teacher preference estimates – teacher characteristics

	Estimate	Standard Error
Teacher Characteristics		
VA Non-Disadvantaged Students	0.746	0.307
VA Disadvantaged Students	0.937	0.331
In District	-0.509	0.061
Black	-0.095	1.043
Hispanic	6.017	3.762
Female	0.284	0.064
Experience 2-3	0.070	0.083
Experience 4-6	-0.268	0.082
Experience 7+	-0.141	0.074
St Dev Random Effect	1.687	0.030
Chamberlain-Mundlak Device		
Fraction Disadvantaged Mean	-1.903	3.182
Commute Time Mean	0.032	0.004
Commute Time Missing Mean	1.231	0.249
Value Added Mean	-0.489	0.295
Fraction Black Mean	-2.786	2.707
Fraction Hispanic Mean	0.041	2.457
Fraction Above Median Achievement Mean	-0.986	4.718
Abs Adv x Fraction Disadvantaged Mean	-37.628	19.086
Abs Adv x Fraction Black Mean	36.183	18.362
Abs Adv x Fraction Hispanic Mean	15.838	19.942
Abs Adv x Fraction Above Median Achievement Mean	-16.346	6.488
Black x Fraction Black Mean	-2.200	2.412
Hispanic x Fraction Hispanic Mean	-20.462	14.686
Number of Students Mean	0.009	0.023

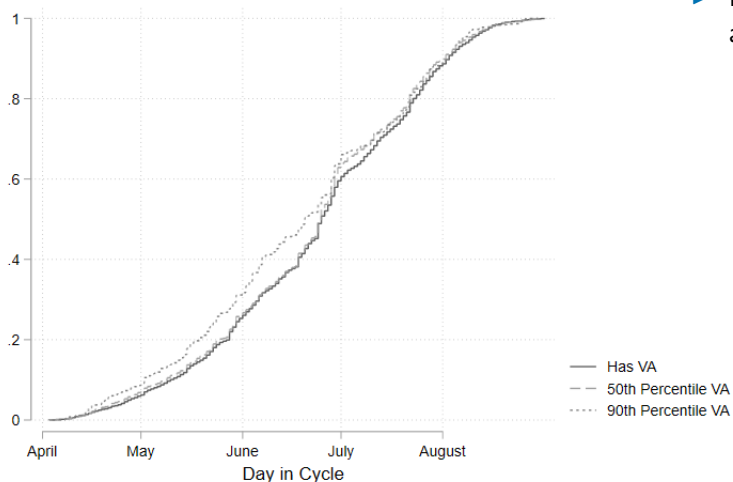
Principal preference estimates – part 1

	Estimate	Standard Error
Constant	-4.363	0.127
St Dev Random Effect	1.531	0.022
Title I	0.521	0.156
Value Added	0.092	0.026
Value Added × Title I	0.038	0.034
Experience 2-3	0.351	0.128
Experience 2-3 × Title I	-0.005	0.163
Experience 4-6	0.271	0.117
Experience 4-6 × Title I	0.035	0.160
Experience 7+	0.097	0.089
Experience 7+ × Title I	-0.344	0.120
Experience Missing	-0.342	0.060
Experience Missing × Title I	0.371	0.086
Masters	0.188	0.098
Masters × Title I	0.124	0.125

Principal preference estimates – part 2

	Estimate	Standard Error
Black	-1.035	0.227
Black x Title I	1.722	0.453
Black x Fraction Black	0.396	0.267
Black x Fraction Black x Title I	-0.253	0.511
Hispanic	-0.690	0.454
Hispanic x Title I	0.450	0.561
Hispanic x Fraction Hispanic	2.259	2.219
Hispanic x Fraction Hispanic x Title I	-1.833	2.345
Female	0.053	0.106
Female x Title I	0.031	0.129
Gender Missing	-0.327	0.230
Gender Missing x Title I	-0.197	0.277
Race Missing	-0.530	0.210
Race Missing x Title I	0.374	0.247
VA Missing	0.490	0.089
VA Missing x Title I	-0.230	0.124

Market timing



► Higher VA teachers arrive slightly earlier

► [Back](#)

Validation of value-added forecasts

	Mean Res	Mean Res	Mean Res
VA (Heterog)	1.052*** (0.00650)		1.060*** (0.00681)
Post Transfer		-0.00243 (0.00367)	0.00576** (0.00280)
VA * Post Transfer			-0.0885*** (0.0212)
Constant	0.00810*** (0.000835)	0.00779*** (0.00174)	0.00745*** (0.000883)
Subject	Math	Math	Math
Mean DV	0.00764	0.00754	0.00764
Clusters	21514	21834	21514
N	74552	75459	74552

► [Back](#)

Same-race and same-gender match effects

	Student Res
Black Teacher - Black Student	0.00225 (0.00164)
Hispanic Teacher - Hispanic Student	-0.00556 (0.00549)
Female Teacher - Female Student	0.00478*** (0.000550)
Fixed Effects	Teacher, School
Mean DV	0.0000115
Clusters	37940
N	5158740

Match effects across transfers

	Student res	Student res
Share disadvantaged	-0.0549 (0.0251)	-0.0409 (0.0202)
Share disadvantaged $\times$ CA in disadvantaged	0.0820 (0.0356)	0.0697 (0.0283)
Num teachers	3214	3214
Num students	157671	157671
Mean CA	-0.00805	-0.00805
SD CA	0.0624	0.0624
Controls	No	Yes

Validation across big composition changes

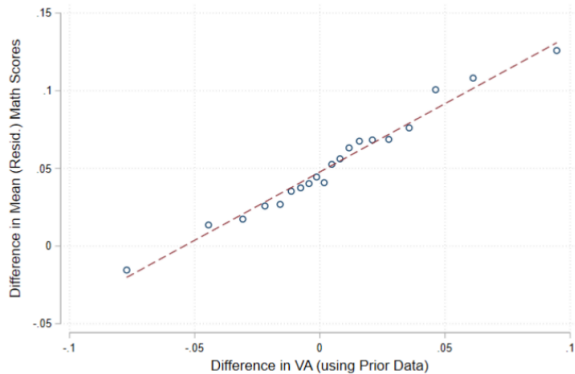
	Mean Res	Mean Res
VA – below 25th (disadv)	1.030*** (0.0127)	
VA – 25th-75th (disadv)	1.054*** (0.00877)	
VA – above 75th (disadv)	1.075*** (0.0121)	
VA – below 10th (disadv)		0.990*** (0.0223)
VA – 10th-90th (disadv)		1.058*** (0.00698)
VA – above 90th (disadv)		1.066*** (0.0228)
Constant	0.00810*** (0.000835)	0.00810*** (0.000835)
Subject	Math	Math
Mean DV	0.00764	0.00764
p-value on Equal Coeff	.035	.012
Clusters	21514	21514
N	74552	74552

Validation across big class size changes

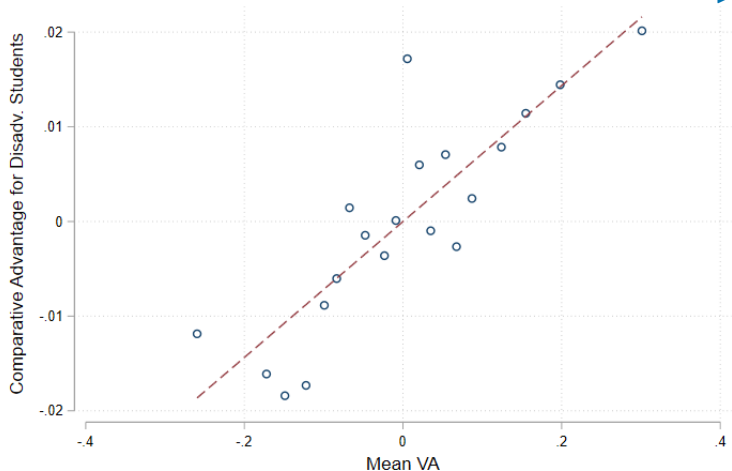
	Mean Res	Mean Res
VA – below 25th (size)	1.052*** (0.0128)	
VA – 25th-75th (size)	1.057*** (0.00920)	
VA – above 75th (size)	0.978*** (0.0115)	
VA – below 10th (size)		1.011*** (0.0224)
VA – 10th-90th (size)		1.066*** (0.00713)
VA – above 90th (size)		0.961*** (0.0188)
Constant	0.00819*** (0.000845)	0.00800*** (0.000843)
Subject	Math	Math
Mean DV	0.00764	0.00764
p-value on Equal Coeff	0	0
Clusters	21514	21514
N	74552	74552

Forecast unbiasedness of comparative advantage

Figure A1: Math Comparative Advantage Forecast Unbiasedness

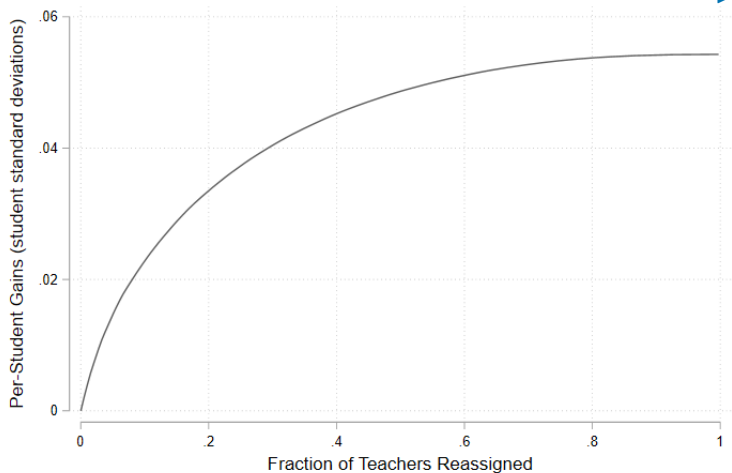


Relationship between absolute and comparative advantage



- High absolute advantage teachers tend to have higher comparative advantage with disadvantaged students

Gains as a function of number of new assignments



► Large gains even if only 20% – 60% of teachers are reassigned

Likelihood ratio test

$\bar{A}_{jcm t}$: teacher-classroom-student type mean residuals

$$\begin{pmatrix} \bar{A}_{jc1t} \\ \bar{A}_{jc'2t} \end{pmatrix} \sim \mathcal{N} \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{\mu_1}^2 + \sigma_{\theta_1}^2 + \frac{\sigma_{\epsilon_1}^2}{N_{jc0t}} & \sigma_{\mu_1\mu_2} \\ \sigma_{\mu_1\mu_2} & \sigma_{\mu_2}^2 + \sigma_{\theta_2}^2 + \frac{\sigma_{\epsilon_2}^2}{N_{jc2t}} \end{pmatrix} \right) \quad (1)$$

- ▶ H_0 : baseline model
- ▶ H_1 : $\sigma_{\mu_1}^2 = \sigma_{\mu_2}^2$, $\sigma_{\theta_1}^2 = \sigma_{\theta_2}^2$, $\sigma_{\epsilon_1}^2 = \sigma_{\epsilon_2}^2$, and $\sigma_{\mu_1\mu_2} = 0$

Reject homogeneous model in favor of heterogeneous model (test stat = 610)

		Posting			Applying			Hiring	
	Vacs	Share	Share TI	Apps	Share	Share TI	Apps	Share	Share TI
April	295	16.24	0.62	24799	7.13	0.50	393	13.23	0.69
May	392	21.57	0.52	70248	20.21	0.50	585	19.70	0.63
June	502	27.63	0.52	108776	31.29	0.51	827	27.85	0.60
July	451	24.82	0.42	94171	27.09	0.50	755	25.42	0.50
August	167	9.19	0.46	44673	12.85	0.51	358	12.05	0.57
Total	1807	100		342667	100		2918	2918	

[▶ Back](#)

Output across different allocations – White / non-White VA

0.0002

principals max VA

-0.0012

all options

0.0278

principals and teachers max VA

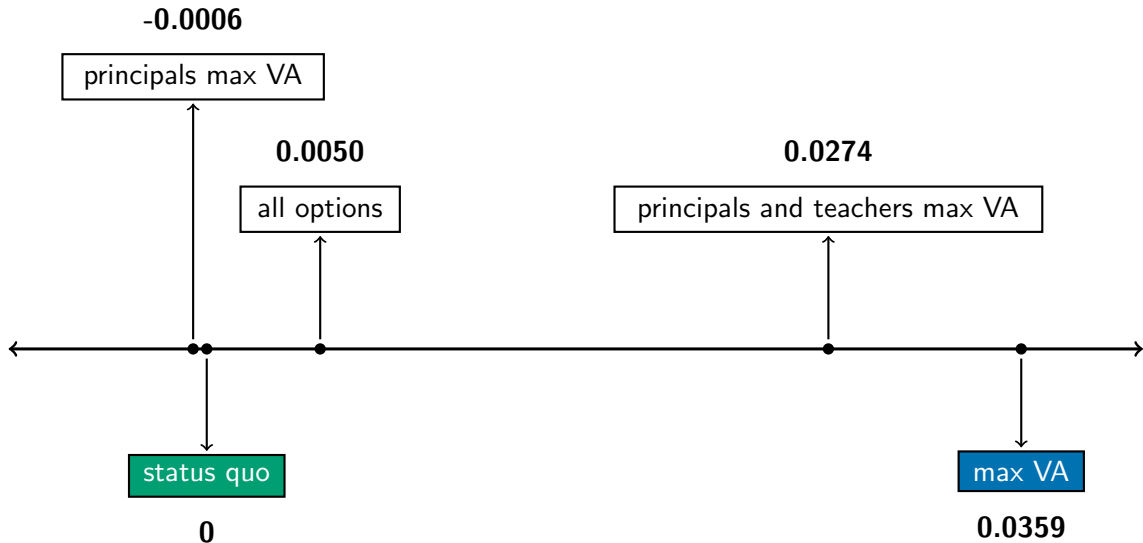
status quo

0

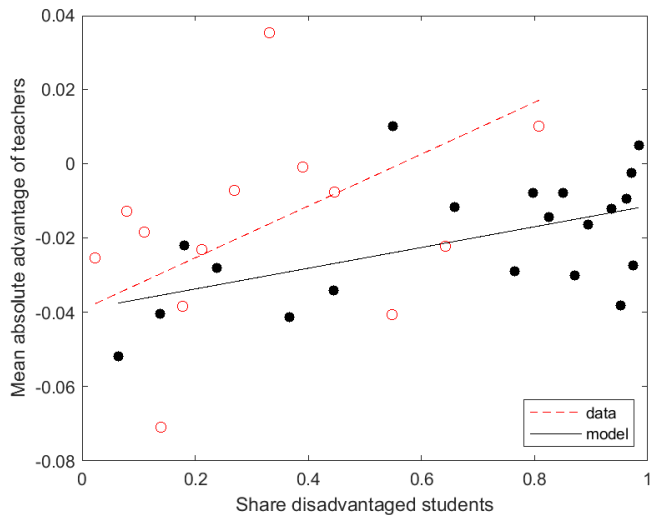
max VA

0.0358

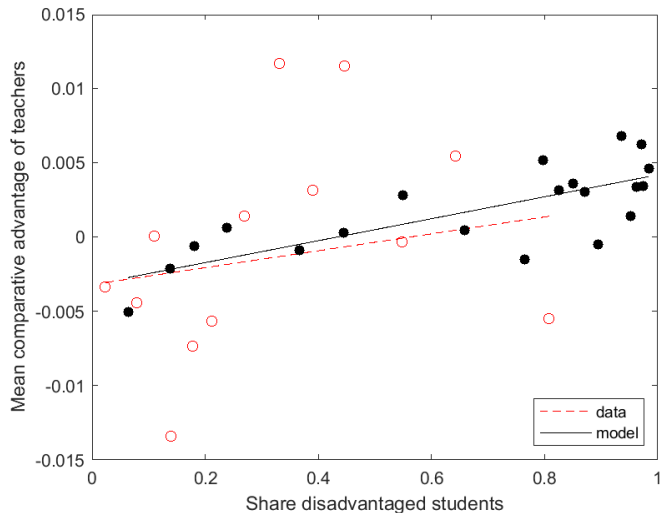
Output across different allocations – Lagged Achievement VA



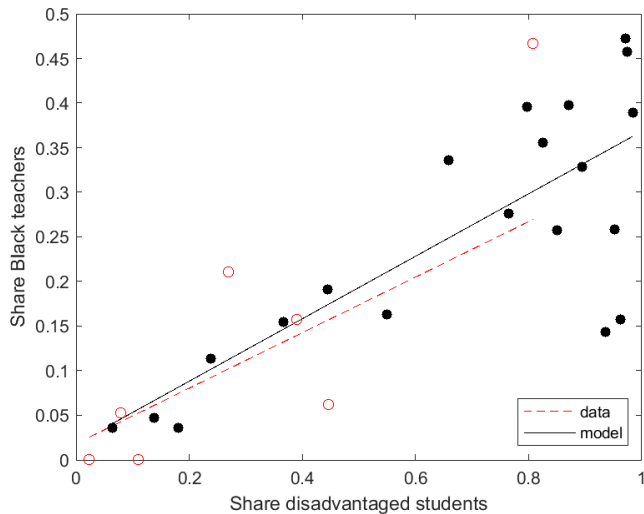
Fit – Absolute Advantage



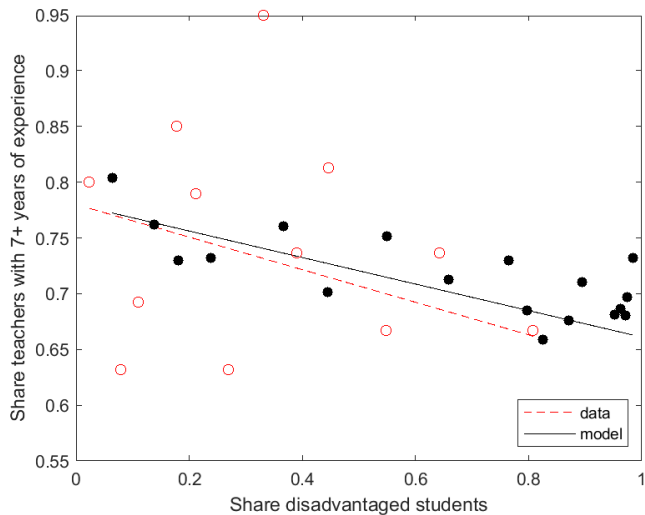
Fit – Comparative Advantage



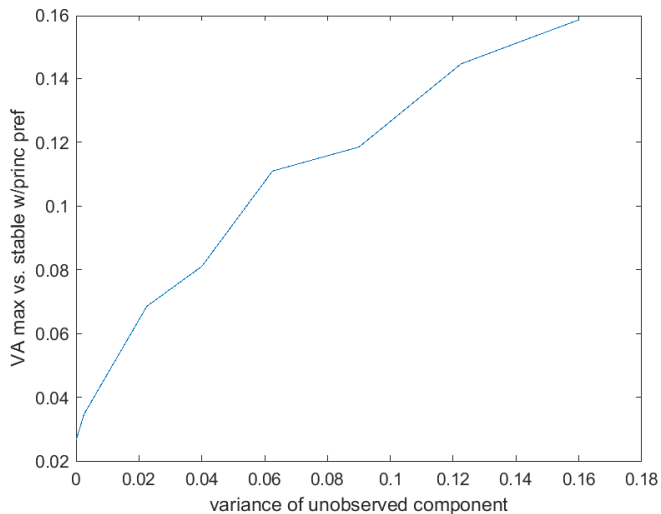
Fit – Fraction black teachers



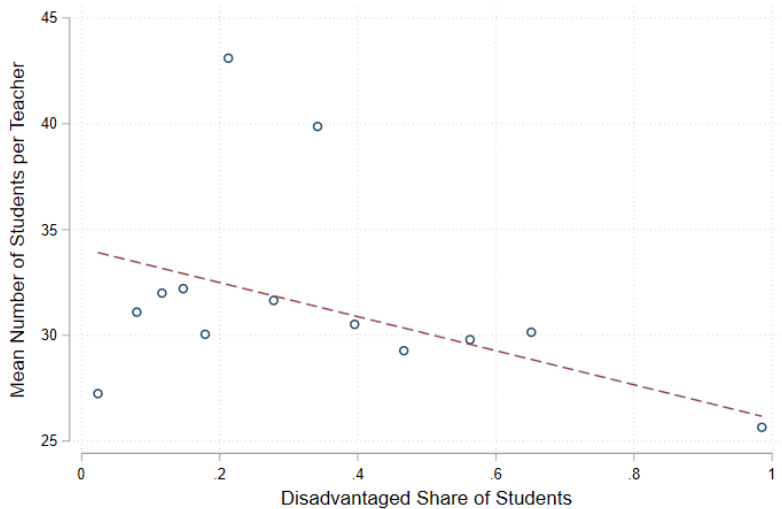
Fit – Experienced teachers



Simulation of additional unobserved value added components



Relationship between student composition and class size



Transfer event study: math residual scores



Autocorrelation in school's mean class size

Table: Cross-correlation table

Variables	Class Size t	Class Size t-1	Class Size t-2	Class Size t-3	Class Size t-4
Class Size t	1.0000				
Class Size t-1	0.7329 (0.0000)	1.0000			
Class Size t-2	0.6248 (0.0000)	0.6966 (0.0000)	1.0000		
Class Size t-3	0.4093 (0.0000)	0.5261 (0.0000)	0.6598 (0.0000)	1.0000	
Class Size t-4	0.3722 (0.0000)	0.3746 (0.0000)	0.4365 (0.0000)	0.5796 (0.0000)	1.0000

Nb. obs. : 247

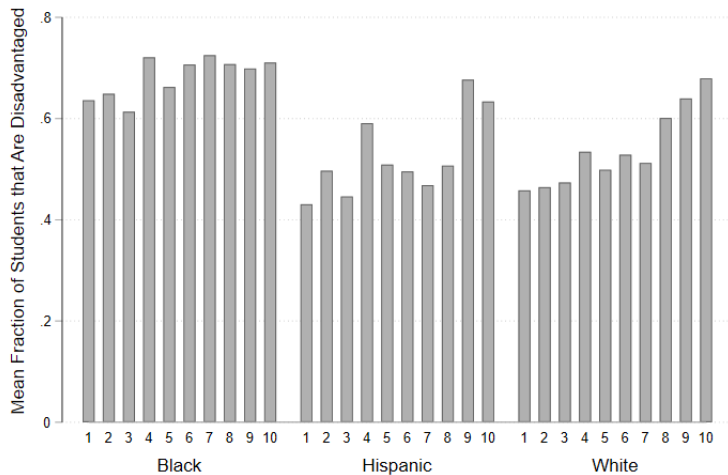
Autocorrelation in teacher's mean class size, controlling for school mean

Table: Cross-correlation table

Variables	(Res.) Size t	(Res.) Size t-1	(Res.) Size t-2	(Res.) Size t-3	(Res.) Size t-4
(Res.) Size t	1.0000				
(Res.) Size t-1	0.3668 (0.0000)	1.0000			
(Res.) Size t-2	0.2688 (0.0000)	0.3717 (0.0000)	1.0000		
(Res.) Size t-3	0.2900 (0.0000)	0.1272 (0.0186)	0.2699 (0.0000)	1.0000	
(Res.) Size t-4	0.1173 (0.0301)	0.1438 (0.0077)	0.0698 (0.1978)	0.3098 (0.0000)	1.0000

Nb. obs. : 342

Student composition by teachers' commute time



Black teachers' closest schools are more likely to have disadvantaged students [▶ Back](#)

Autocorrelation in school's composition

Table: Cross-correlation table

Variables	Frac Disadv t	Frac Disadv t-1	Frac Disadv t-2	Frac Disadv t-3	Frac Disadv t-4
Frac Disadv t	1.0000				
Frac Disadv t-1	0.9602 (0.0000)	1.0000			
Frac Disadv t-2	0.9430 (0.0000)	0.9555 (0.0000)	1.0000		
Frac Disadv t-3	0.9363 (0.0000)	0.9370 (0.0000)	0.9496 (0.0000)	1.0000	
Frac Disadv t-4	0.9435 (0.0000)	0.9467 (0.0000)	0.9554 (0.0000)	0.9775 (0.0000)	1.0000

Nb. obs. : 247

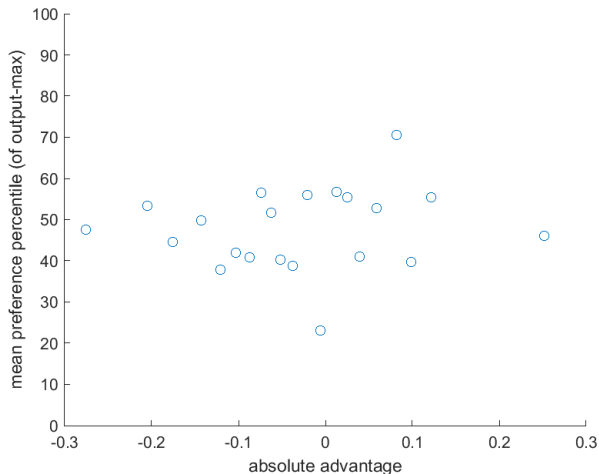
Autocorrelation in teacher's classroom composition, controlling for school mean

Table: Cross-correlation table

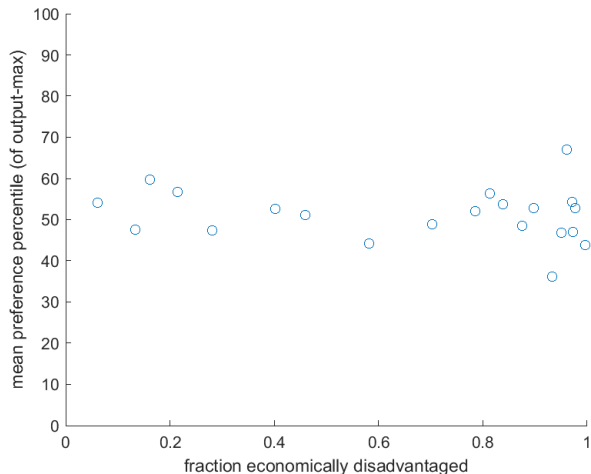
Variables	(Res.) Dis t	(Res.) Dis t-1	(Res.) Dis t-2	(Res.) Dis t-3	(Res.) Dis t-4
(Res.) Dis t	1.0000				
(Res.) Dis t-1	0.3170 (0.0000)	1.0000			
(Res.) Dis t-2	0.2898 (0.0000)	0.3200 (0.0000)	1.0000		
(Res.) Dis t-3	0.1524 (0.0047)	0.2076 (0.0001)	0.3723 (0.0000)	1.0000	
(Res.) Dis t-4	0.0921 (0.0889)	0.0512 (0.3450)	0.2203 (0.0000)	0.3925 (0.0000)	1.0000
Nb. obs. : 342					

Teacher preferences: rank of first-best assignment

- First-best assignment is essentially random in preferences

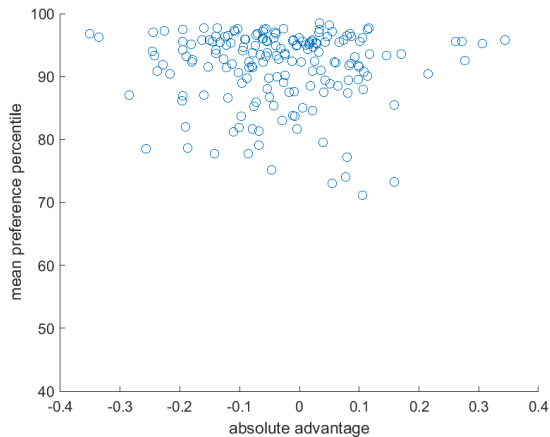


Principal preferences: rank of first-best assignment



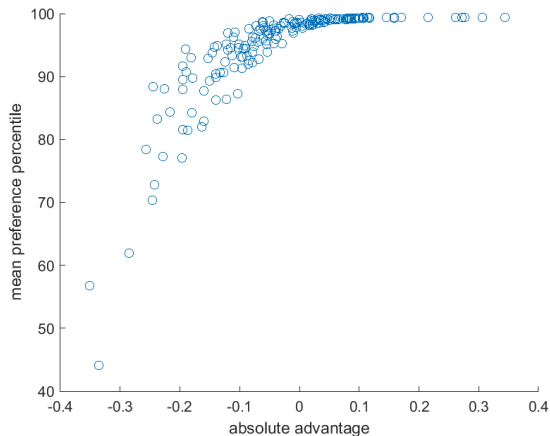
- ▶ First-best assignment is essentially random in preferences
- ▶ Slightly more aligned than for teachers: 52nd percentile, vs. 48th for teachers

Teacher choice by absolute advantage – Status quo



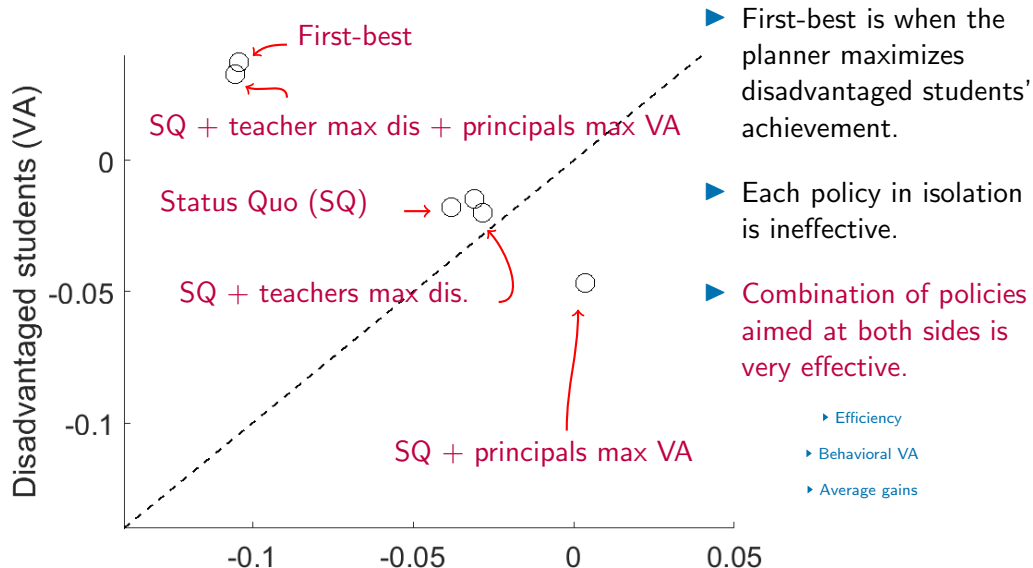
- Higher absolute advantage teachers **don't** get much more ability to choose

Teacher choice by absolute advantage – Principals maximize VA



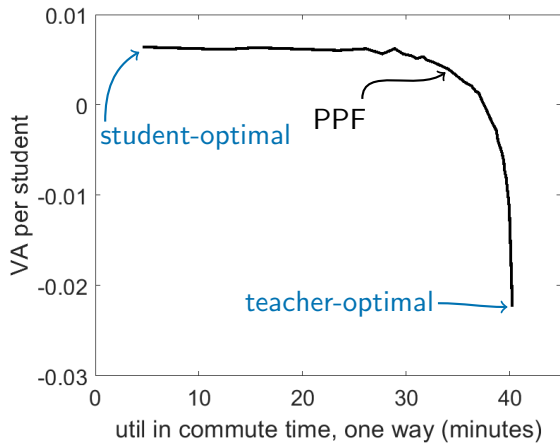
- Higher absolute advantage teachers **now** get much more ability to choose

Distributional: **Combination** of teacher and principal interventions



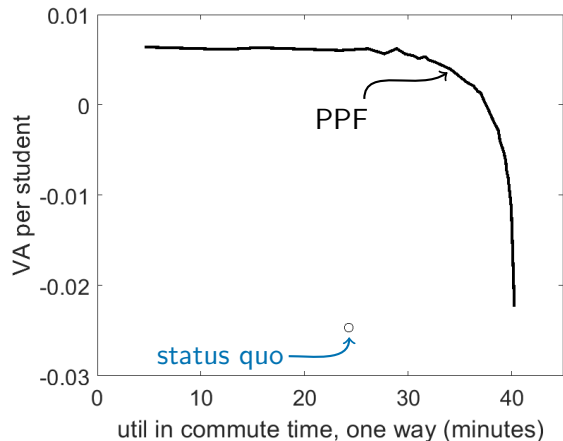
- ▶ First-best is when the planner maximizes disadvantaged students' achievement.
- ▶ Each policy in isolation is ineffective.
- ▶ Combination of policies aimed at both sides is very effective.

PPF



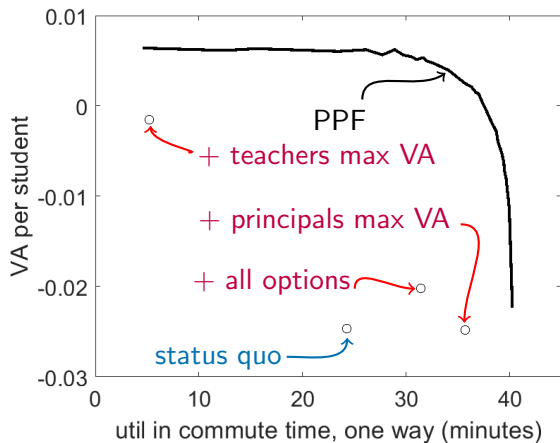
- ▶ PPF is very flat/steep near the corners

PPF and status quo allocation



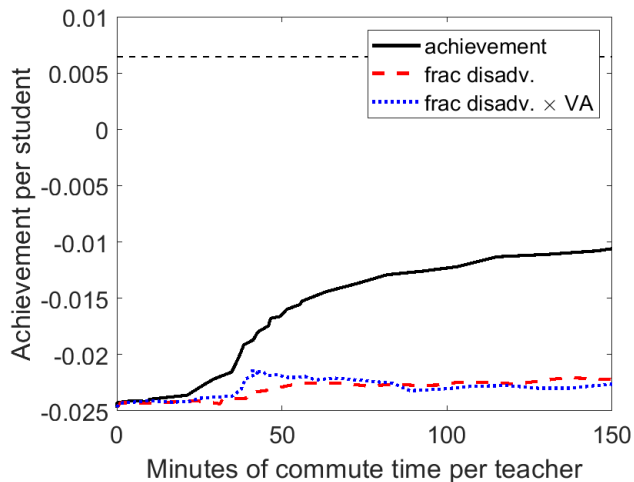
- **Status quo:** teacher and principal preferences, options based on timing, teacher propose

All potential allocations



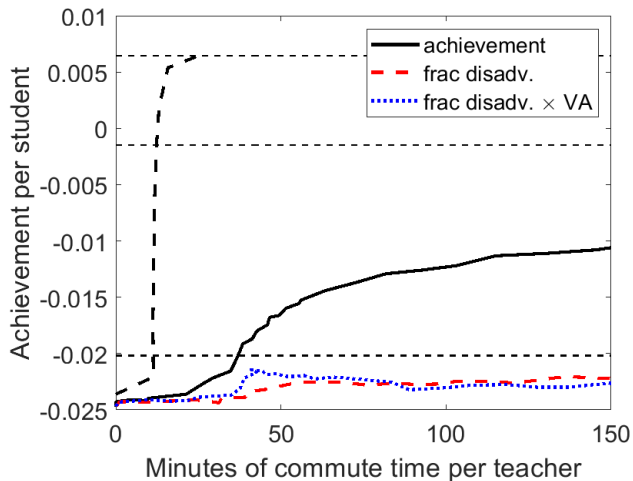
- **Status quo:** teacher and principal preferences, options based on timing, teacher propose
- **+ all options:** small gains
- **+ principals max VA:** small losses
- **+ teachers max VA:** large gains

Student achievement with teacher bonuses



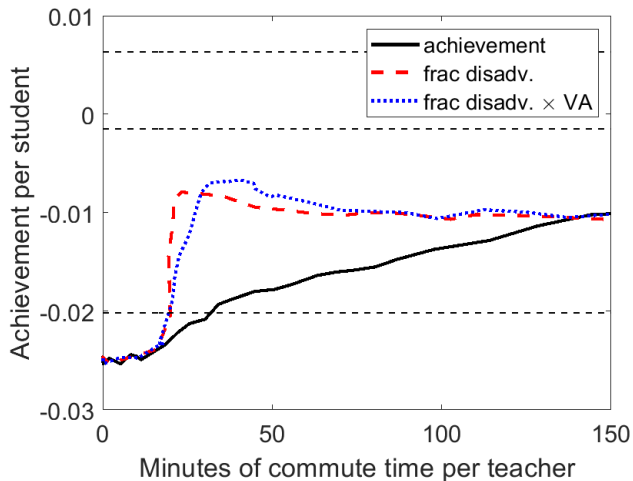
- ▶ Targeting achievement more cost-effective than targeting disadvantaged students
- ▶ Achievement gains are very costly

Student achievement with teacher bonuses



- ▶ Targeting achievement more cost-effective than targeting disadvantaged students
- ▶ Achievement gains are very costly
- ▶ **Bonuses based on preferences are much more cost-effective**

Student achievement with teacher and principal bonuses



- ▶ When principals hire according to VA, targeting disadvantaged students is more cost-effective
- ▶ $\Rightarrow$ optimal teacher bonus depends on whether principals receive bonuses

Distributional: status quo with behavioral VA in preferences

- ▶ **beh VA in prefs:** teacher and princ prefs estimated incl behavioral VA
- ▶ $\Rightarrow$ hardly affects results

