Time Trumps Quantity in the Market for Lemons

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Background & Related Literature

- Leland & Pyle (1977) (and several related papers): owners of a DIVISIBLE asset/firm **signal to the market the quality of their asset by retaining ownership**.
  - important implicit assumption: there is no possibility to trade the remaining fraction of the asset at a later point in time
- Extenting Akerlof (1970): in Janssen and Roy (2002), Fuchs Oery and Skrzypacz (2016) and Fuchs and Skrzypacz (2019) (and several related papers), the owner of NON-DIVISIBLE asset which can trade at any point in time decides when she wants to trade the asset.
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- **What if you have a divisible asset and many opportunities to trade?**

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Background & Related Literature

- Role of commitment and information structure.
  - In the non-divisible case: Spence (1973), Noldeke and van Damme (1990), Swinkels (1999), Horner and Vieille (2009), FOS (2016) and Lee (2021).
- Also related are the search models such as those by Guerrieri, Shimer & Wright (2010) and Auster, Gottardi & Wolhoff (2021).
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Results

- Benchmark: In a linear setting with uniform supply and **commitment**, there is an "equivalence" between: TIME, PROBABILITIES and QUANTITIES.
  - Furthermore, there is an indeterminacy as to which is used.

- With **observable** past trades we obtain the same payoffs but, the equilibrium path is uniquely pinned down for any frequency and horizon. **Time** (delay) is used to signal.

- With **unobservable** past trades equilibria differ and pooling arises when not allowed to trade continuously. With continuous trading we restore full separation and **Time** (delay) used to signal.

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Model

- A seller has a unit of a fully divisible asset it values (per unit) at $c \in [0, 1]$. Seller knows c buyers only $F(c)$ with $f(c) > 0$.
- There is a large mass of potential buyers every period. They value the asset at $v(c)$ where $v(c) > c$ for $c \in [0, 1)$, $v(\cdot)$ is continuously differentiable, $v'(c) > 0$ and $v(1) = 1$.
- $\varsigma \subset [0, \infty)$ denote the set of times in which the market is open for trade. $\varsigma = [0, \infty)$ or $\varsigma = \{0, \Delta, 2\Delta, ..., T\}$
- When the market is open the seller decides how much of its residual supply to offer for sale. The buyers then make competitive bids.

Model

- Players are risk neutral and future payoffs are discounted at rate $r > 0$.
- For sellers with quality c that means that, if they expect to sell an amount q_t at time t with per unit price p_t, their utility *from that trade*, in present expected terms, is:

$$e^{-rt} q_t \left(p_t - c\right).$$

- Note, that the seller's payoffs satisfy a single crossing property, the worse your type, the more eager you are to sell.
- Similarly, for buyers it would be:

$$e^{-rt} q_t E\left[v(c) - p_t\right].$$

Three Cases

1) Commitment benchmark: sellers can commit to long term contracts.

2) Spot contracts with public trades: prices can be contingent on past trades.

3) Spot contracts with private trades.

Contracts are exclusive for (1) and exclusive within the period for (2) & (3).

Multiplicity and Refinements

- As is common in signalling games we can have many equilibria if we don't impose any discipline on off-equilibrium path beliefs.
- We will use "D1" , this close in spirit to the Intuitive Criterion (which we could use if we discretized the type space).
- We also have a "competitive equilibrium" approach in which we price all non-traded contracts by assuming that the type that looses the least would be making this mistake. Same results attain.

Commitment Benchmark

Suppose that the initial contract ω, signed at $t = 0$, specified the complete sequence of trades and transfers for all future periods. For all $t \in \varsigma$ let $q_t(\omega)$ be the quantity to be delivered at each date t under ω.

We say the contract ω is *feasible* if *:*

$$q_t(\omega) \geq 0 \quad \textit{and} \quad \int_{t \in \varsigma} dq(t;\omega) \leq 1.$$

Given the linearity in the preferences, all that really matters is the total discounted amount to be delivered by the seller $Q(\omega)$ and the per unit transfer at $t = 0$, $P(\omega)$.

Commitment Benchmark

The utility of trading contract ω for a seller of type c is then:

$$W(c, \omega) := Q(\omega)(P(\omega) - c),$$

where

$$Q(\omega) = \int_{t \in \varsigma} e^{-rt} dq(t; \omega).$$

Let the maximal payoff attainable by a type c be denoted by

$$U(c) = \max_{\omega \in \Omega} W(c, \omega).$$

Commitment Case

Theorem

When sellers can commit to long term contracts, for all ς, all sequential equilibria that satisfy D1 are perfectly separating. The utility level attained by each seller type c, $U^*(c)$, is the same across all these equilibria, and so is the total discounted quantity traded by this type, $Q^*(\omega^*(c))$. More specifically, we have:

$$Q^*(\omega^*(c)) = \exp\left[-\int_0^c \frac{v'(x)}{v(x)-x}dx\right]$$

strictly decreasing in c, and the per-unit equilibrium price is $P^*(\omega(c)) = v(c)$, for all c.

Commitment Case

Remark

Note that the equilibrium only pins down the value of $Q(c)$, but any contract specifying a sequence of trades such that the total discounted trade is equal to $Q^*(c)$ constitutes a contract that may be traded by type c sellers in equilibrium.

If we also allowed for contracts with stochastic deliveries, probabilities would also be isomorphic. Thus, in search models, market tightness plays a similar role to delay or partial trade.

Commitment Case

Example

Suppose $\varsigma = [0, \infty)$ and $v(c) = 1/2 + c/2$ then:

$$Q(c) = 1 - c, \; P(c) = 1/2 + c/2.$$

Leland and Pyle retained ownership implementation:

$$q_0(c) = 1 - c \;, \quad q_t(c) = 0 \; for \; t > 0.$$

FS pure delay implementation:

$$q_t(c) = \begin{cases} 1 & if \; c = 1 - e^{-rt} \\ 0 & otherwise \end{cases}$$

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Spot Contracts with Observable Trades

- We restrict ς to be $\varsigma = \{0, \Delta, 2\Delta, .., T\}$ where $T \leq \infty$.
- At any time $t > 0$ the bids for the current offered quantity q_t will also be indexed by the sequence of past trades $q^{t-\Delta}$.
- There could be many different prices for the same q_t given different histories of trade.

No Commitment and Observable Trades

Sellers cannot commit to a sequence of trades and thus their choice of q_t must be optimal at every t. The seller's problem can be written recursive as follows:

$$U\left(c, q^{t-\Delta}\right) = \max_{q_t} q_t \left(p\left(q^t\right) - c\right) + e^{-r\Delta} U\left(c, q^t\right)$$

The off-equilibrium beliefs are set similarly to the commitment case which type would loose the least from the off-path sequence.

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Spot Contracts with Observable Trades

We show that **every individual trade is separating** implying that the average per unit price of the trades entered by c is still $P(c) = v(c)$:

Proposition (Separation in q)

With uniform supply, for any $T \leq \infty$ and $\Delta \geq 0$, all trades with $q > 0$ must be fully separating.

Spot Contracts with Observable Trades

Proof Idea.

Show that the highest type c'' that ever participates in a pooling trade could be better off by postponing its trades as much as possible and trading ε less than what it does in the candidate equilibrium.

$$\begin{aligned} e^{-r\tau} q_\tau + e^{-r(\tau+\Delta)} (1 - q_\tau) &= Q\left(c''\right) - \varepsilon \quad \textit{if } \tau < T - \Delta \\ e^{-rT} q_T &= Q\left(c''\right) - \varepsilon \quad \textit{otherwise} \end{aligned}$$

Postponing is key because it guarantees that you cannot imitate the trade at τ and have a larger Q. Thus, lower types would not want to imitate this path, implying higher prices and making it profitable for c''. □

If trading were continuous the seller could just find a unique time to delay the trade to. Important for the unobservable case.

Observable Case: Unique Pattern of Trade

Lemma (Separation in Q)

For every $\Delta > 0$ and T, in all observable trade equilibria, the total discounted quantity $Q(c)$ traded by any type c must be strictly decreasing in c.

Implies always Q(c)=Q*(c) trading frequency is irrelevant

Proposition (Exhaustive Trade in Consecutive Periods)

For all $t < T$ if any type c trades $0 < q_t(c) < 1$ it must trade its residual supply the next period.

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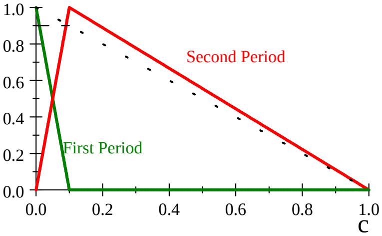
Observable Trades: Two Period Example

Example

Suppose $v(c) = 1/2 + c/2$. Recall that $Q^*(c) = 1 - c$

$$q_0(c) = \begin{cases} 1 - \frac{c}{1-e^{-r\Delta}} & c \leq 1 - e^{-r\Delta} \\ 0 & c > 1 - e^{-r\Delta} \end{cases} \quad q_1(c) = \begin{cases} \frac{c}{1-e^{-r\Delta}} & c \leq 1 - e^{-r\Delta} \\ \frac{1-c}{e^{-r\Delta}} & c > 1 - e^{-r\Delta} \end{cases}$$



NOTE: endogenously get to non-uniform supply after $t = 0$.

Time Trumps Quantity in the Observable Case

Theorem

For $T = \infty$, in the limit, as $\Delta \to 0$, the sequence of unique equilibria converges to a separating outcome where each type c trades all its quantity at a unique moment in time $\tau(c)$ where $\tau(c)$

$$\exp(-r\tau(c)) = Q^*(c)$$

Remark

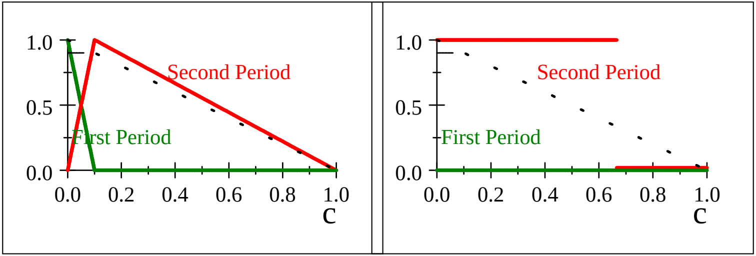
For $T < \infty$ there will be a subset of types that only trade at T and separate via quantity.
We can also work directly in continuous time. No discountinuity.

No-Commitment and Unobservable Past Trades

- The history of trades is **unobservable**.
- We assume that buyers are only active once so there is no scope for private learning.
- At every t buyer bids can only be contingent on t and $q_t \in [0, 1]$.
- We maintain the rest of the equilibrium conditions.

Unobservable Case: Non-Equivalence

- For $\Delta > 0$ the public equilibrium is not an equilibrium with private offers.
- **Welfare is not ranked.**
- Returning to our example we have:



Public Private

Continuous Trading in the Unobservable Case

We will work with $\varsigma = [0, \infty)$ and restrict sellers to choose a countable set of times in which to sell some positive quantity: $\{R\left(c\right), q\left(c\right)\}$.

Since prices are not history dependent the seller's c strategy $\{R\left(c\right), q\left(c\right)\}$ is optimal if for all feasible $\left\{\tilde{R}, \tilde{q}\right\}$:

$$\sum_{t \in R(c)} e^{-rt} q\left(t, c\right)\left(p\left(q, t\right) - c\right) \geq \sum_{t \in \tilde{R}} e^{-rt} \tilde{q}\left(t\right)\left(p\left(\tilde{q}(t), t\right) - c\right)$$

Continuous Trading in the Unobservable Case

When the set of trading times is $\varsigma = [0, \infty)$ by similar argument than in the observable case we can show:

- All trades must be separating.
- There must be separation in $Q(c)$.
- In any equilibrium $Q(c) = Q^*(c)$.

Time Trumps Quantity in the Unobservable Case

Theorem

When $\varsigma = [0, \infty)$ there is an equilibrium in which each type trades its full unit at time $\tau(c)$, where $\tau(c)$ solves:

$$\exp(-r\tau(c)) = Q^*(c)$$

When $\lambda \to 0$, $p(1, \tau(c)) = p(q, \tau(c)) = v(c)$.

Intuition: With continuous trading, given that all types are selling their full supply in a given instant, there is no need to have contracts that depend on past traded amounts, thus the additional restrictions on the contracting space imposed by unobservability do not have any bite in the limit. Therefore, the observable equilibrium continues to be an equilibrium in this case.

Conclusions

- With commitment and uniform supply there is an equivalence between time and quantity (and probabilities) and the exact path of trades is not pinned down.
- With observability and no commitment we see that **time** is the way that separation takes place in equilibrium.
- Without observability we have a really hard problem when $\Delta > 0$ but when $\varsigma = [0, \infty)$ we still get that **time** drives separation.
- Important to understand what might happen with non-uniform supply since it might arise endogenously in equilibrium.

THANK YOU!

Non Uniform Supply and Non-Equivalence

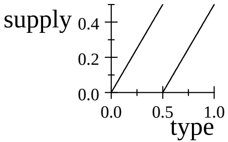
Proposition

If we allow for different types to start the game with different (unobserved) supplies $s(c)$ of the asset then, even with observable trading, it is no longer the case that for all T and Δ the equilibrium payoffs and $Q(c)$ are constant.

Non-Uniform Supply and Non-Equivalence

Proof.

By example: let $v(c) = 1/2 + c/2$ and $s(c)$ as graphed below



- with *only* one trading date types $c \in [0, 1/2]$ trade their full supply and types $c > 1/2$ do not trade at all.
- with $T = \infty$ and Δ small there will be an additional time t^* such that after the initial round of trade there will be another round of trade where types $c \in [1/2, 1]$ trade their full supply; t^* must be such that the low types prefer to trade at $t = 0$ at low prices than wait until t^* to trade at higher prices.

Non-Uniform Supply and Non-Equivalence

Proposition

When $s(c) \neq 1$, observable and unobservable trading is not generally welfare ranked. Unobservable trading can have a higher expected welfare than observable trading or vice-versa.