Fiscal Policy
and the Balance Sheet of the Private Sector

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Introduction

Remember Barro (1974): Can government debt as a financial asset increase overall social welfare?

Our questions:

- What is the role of fiscal policy in general equilibrium with incomplete markets - debt *and* taxes?
- What is optimal? In fact, what *is* the social welfare function with heterogeneous agents?
- How does optimal policy depend on political preferences and public spending needs?

Here: Theory of private and public debt, if firms are heteroge– neous, risky, and markets incomplete. Public debt valuable as a savings vehicle and a risk buffer on private balance sheets.

The Masters (I)

- Borrowing or liquidity constraints:
  Woodford (1990): Liquid public debt can increase the flexibility of liquidity-constrained private agents.
  Aiyagari (1994): Heterogenous households self-insure by buying shares of aggregate capital.
  Angeletos et al. (2016): Public debt as collateral or buffer to ease financial frictions.

- Finance:
  Asset pricing: Merton (1971), Lucas (1978), Dumas (1989): Valuation of risky assets in dynamically complete markets.
  Corporate Finance: Dynamic capital structure design: Leland (1994), Hart-Moore (1989), Bolton et al. (2022)

The Masters (II)

- Reis (2020), Brunnermeier, Merkel, and Sannikov (2020, 2021): Public debt provides collective insurance in incomplete markets.
  $g > r$ and $g < r$ emerge as equilibrium phenomena. Pricing and mining of the financial bubble are possible. But why are these markets incomplete and what are optimal fiscal instruments?

- The Challenge: How do you discount? What about $\frac{1}{r-g}$?
  Over the last 50 years, the real growth rate has exceeded the real safe interest rate: $g > r$ (Barro, 2023)
  So, can governments perpetually finance large deficits if they roll over public debt perpetually? Should they?

Our Contribution

- Basic policy channel: Public debt $\longrightarrow$ private leverage $\longrightarrow$ investment and consumption $\longrightarrow$ growth and interest: no Ricardian equivalence
- Full welfare analysis: Derive a social welfare function from first principles $\longrightarrow$ Dynamic 2nd Welfare Theorem
- Interest and growth: Different optima yield different values (and signs!) of $r - g$, favored by different groups
- Role of redistributionary fiscal policy in implementing optima as General Equilibria
- New rationale for public debt: aggregate balance sheet effect when private assets are insufficient. Do not **complete** missing markets, **extend** existing markets.

Our Insights

- If preferences for public goods are exogenous, public debt as a complement to private debt is always optimal.
- But the optimal debt-GDP ratio depends on the weight of social groups in the economy
- If preferences for public goods are endogenous, public debt may optimally be negative
- $g > r$ can be Pareto optimal: public policy optimally runs a Ponzi Game.
- No crowding out of private investment
- Large public spending shocks: debt or taxes?

A Macro Model of Imperfect Financial Markets

Simple AK model without labor, time continuous, $t \in [0, \infty)$
One perishable (real) good, consumed or invested, one capital good
One financial asset: risk-free debt, issued by firms, and possibly the government

Actors: “Households" (identical), “firms" (heterogeneous), government.

Continuum of firms $i \in [0, 1]$, with initial endowments (equity) e^i.
Aggregate equity: $E = \int_0^1 e^i di$
Representative household with initial endowment H.
Households don't work, simply save and consume.

At each date, firm i has productive assets k_t^i financed by equity e_t^i and debt d_t^i.
Balance sheet of firm i at time t: $k_t^i = d_t^i + e_t^i$ (BS)

Note: Debt d_t^i can be negative.

Instantaneous output:

$$dy_t^i = k_t^i[\mu dt + \sigma dz_t^i],$$

where
$\mu > 0$: instantaneous mean return net of depreciation,
$\sigma \geq 0$: volatility of the instantaneous return,
z_t^i: independent Brownian motions, wash out in the aggregate.

Hence, aggregate production is deterministic: $Y_t = \mu K_t$.

Market imperfection:

Firms are subject to idiosyncratic production risk: output dy_t^i is private information.
Hence, equity cannot be traded. There only is inside equity.

Result: Equity holders cannot fully diversify production risk, households do not own equity, save by means of debt.

Debt can be continuously traded. Result: Debt is safe.

Firm decisions: Given equity e_t^i,
firm i adjusts capital stock k_t^i and chooses dividend payout c_t^i
pays a wealth tax $\tau_t^E e_t^i$ on its equity.
Reason: Profits and consumption unobservable

Household decision: Given savings H_t,
household chooses consumption c_t^H
pays wealth tax $\tau_t^H H_t$ on its savings.

Tax rates can be negative ($\longrightarrow$ subsidies).

Objectives:

$$\mathbb{E} \int_t^\infty e^{-\rho s} \log c_s^k ds, \quad k = i, H$$

Government:

At $t = 0$, government can redistribute initial endowments and issue debt $B_0 \longrightarrow$ net wealth: H_0, e_0^i, and $E_0 = \int_0^1 e_0^i di$.
Aggregate wealth of private sector: $E_0 + H_0 = E + H + B_0$

Note: Government changes net wealth of private sector by issuing debt, but the aggregate wealth of the economy is still $E + H$.

Dynamics of government debt: $\dot{B}_t = \gamma K_t + r_t B_t - T_t$

where
γK_t: flow of government expenditures (public good),
base model: γ exogenous
r_t: instantaneous risk-free interest rate,
T_t: net aggregate instantaneous tax revenue (taxes – subsidies)

Optimality:
Why should the government use straight public debt and linear wealth taxes?

Preview:
Biais, Gersbach, Rochet, von Thadden, Villeneuve (2023):
Consider the government's choice of fiscal policy as a dynamic multi-agent Principal-Agent problem.

We prove: The optimal mechanism can be implemented by straight public debt and linear wealth taxes, accompanied by one round of initial redistribution.

Individual Decisions

Households

Initial net worth $H_0 > 0$ at time $t = 0$, no further income later, chooses consumption path c^H to

$$\max_{c^H} \int_0^\infty e^{-\rho t} \log c_t^H dt,$$

$$\text{subject to } \dot{H}_t = (r_t - \tau_t^H)H_t - c_t^H$$

Standard optimization: $c_t^H = \rho H_t$
Value function:

$$\rho V^H(t, H) = e^{-\rho t} \log(\rho H) + \int_t^\infty e^{-\rho s} \left(r_s - \tau_s^H - \rho \right) ds.$$

Individual Decisions

Firms

With initial equity $e_0^i > 0$, the firm's flow of funds is given by

$$k_t^i[\mu dt + \sigma dz_t^i] = r_t d_t^i dt + (\tau_t^E e_t^i + c_t^i)dt + de_t^i, \qquad \text{(FoF)}$$

Note: This is simple corporate accounting:

EBIT $=$ interest $+$ taxes/subsidies $+$ dividends $+$ retained earnings

Firm chooses a path $k_t^i, d_t^i, c_t^i,\ t \geq 0$ subject to law of motion (FoF) and balance sheet constraint (BS):

$$\begin{aligned} c_t^i &= \rho e_t^i \\ k_t^i &= \frac{\mu - r_t}{\sigma^2} e_t^i \end{aligned}$$

Hence, stochastic law of motion for optimal equity:

$$de_t^i = \left[\left(\frac{\mu - r_t}{\sigma} \right)^2 + r_t - \tau_t^E - \rho \right] e_t^i dt + \frac{\mu - r_s}{\sigma} e_t^i dz_t^i$$

Hence, no default. Private debt is safe.
Economic reason: Firms adjust leverage continuously.
Formal proof: e_t^i is Geometric Brownian Motion.

Note: $x_t \equiv \frac{k_t^i}{e_t^i} = \frac{\mu - r_t}{\sigma^2}$ is the same for all firms.

By (BS), $x_t = 1 + \frac{d_i}{e_i}$: (a measure of) corporate leverage.

Logic: All firms have the same leverage policy x_t, adjust leverage continuously: after a positive productivity shock they invest more and issue more debt; after a negative shock the opposite.

The Macroeconomy

Aggregates

Households balance sheet: $B_t^H + D_t^H = H_t.$ (ABSH)

Firms: Individual balance sheets random, but aggregate balance sheet of firms is deterministic (LLN):

Assets	Liabilities
K_t	$D_t^G + D_t^H$
B_t^E	E_t

(ABSF)

Hence, aggregate balance sheet of the private sector:

Assets	Liabilities
K_t	H_t
B_t	E_t

(ABPS)

The Macroeconomy

Dynamics

In equilibrium, (ABPS) holds at all times:

$$\dot{K}_t + \dot{B}_t = \dot{H}_t + \dot{E}_t$$

Aggregating individual dynamics yields the aggregate law of motion (IS equation):

$$\begin{aligned} \dot{K}_t &= \dot{H}_t + \dot{E}_t - \dot{B}_t \\ &= (\mu - \gamma)K_t - \rho(H_t + E_t) \end{aligned} \qquad \text{(IS)}$$

The Macroeconomy

Normalization

Homogeneity: Only two variables determine the dynamics of the aggregate balance sheet (ABS). Let's use:

- corporate leverage: $x_t = \frac{K_t}{E_t}$ (the same for all firms),
- household wealth relative to corporate wealth: $h_t = \frac{H_t}{E_t}$.

This transforms the aggregate balance sheet

$$K_t + B_t = H_t + E_t$$

into

$$\frac{B_t}{E_t} = 1 + h_t - x_t$$

Advantage:
(x_t, h_t) has a simple law of motion, 2 ODEs:

$$\dot{h}_t = (\tau_t^E - \tau_t^H - \sigma^2 x_t^2) h_t \quad \text{(LMh)}$$

$$\dot{x}_t = (\sigma^2 x_t^2 - \rho)(1 - x_t) + (\tau_t^E - \gamma) x_t - \rho h_t \quad \text{(LMx)}$$

Initial values:

$$h_0 = \frac{H_0}{E_0} = \frac{H - L^H}{E - L^E}$$

$$x_0 = \frac{K_0}{E_0} = \frac{H + E}{E - L^E}$$

where L^H and L^H are initial transfers by the government.

Proposition

For any choice of fiscal policy, general equilibrium is fully characterized by the trajectory of the two state variables x_t and h_t. If the solution to the associated law of motion (LMh)-(LMx) exists in the interior of $\mathbb{R}^2_+$ for all $t \geq 0$, then the equilibrium exists and is unique.

Furthermore, any differentiable path (x_t, h_t) in $\mathbb{R}^2_{++}$ can be implemented as the unique solution to (LMh)-(LMx) for some choice of taxes (τ^E_t, τ^H_t) and initial transfers (L^E, L^H).

This means: general equilibria and trajectories of (LMh)-(LMx) in $\mathbb{R}^2_{++}$ are the same thing.

Welfare

Specific problem: Define preferences over fiscal policies.

General problem: Evaluate **all** possible redistributionary policies between **all** agents

General approach: Multi-agent mechanism design
$\longrightarrow$ Mean-Field Control Theory

General solution:
Biais, Gersbach, Rochet, von Thadden, Villeneuve (2023)

Welfare

The Second-Best

- Individual random shocks dz_t^i and thus $dy_t^i = k_t^i[\mu dt + \sigma dz_t^i]$ are private information of the firms' owners.
- Hence: equityholders can divert some of their firm's output, consume it secretly, and claim to have incurred a negative productivity shock.
- Planner: determine, for each $t \geq 0$ and firm $i \in [0,1]$, instantaneous consumption c_t^i, new capital stock k_t^i, and aggregate household consumption C_t^H.
- The optimal mechanism controls the distribution of c_t^i, k_t^i at each point in time: mean-field control problem.
- At the optimum, the ratio $y_t \equiv k_t^i / c_t^i$ is independent of firm i's history and of i.

Implementation:

- Define $x_t = \rho y_t$

  $$h_t = y_t \frac{C_t^H}{K_t}$$

- Define tax rates $(\tau_t^E, \tau_t^H) \in \mathbb{R}^2$, $t \geq 0$, by (LMh)-(LMx).
- Define an index $E_t = K_t / x_t$, $t \geq 0$, and call it "aggregate equity".
  Define "household wealth" by $H_t = C_t^H / \rho$ and the value of "government bonds" as $B_t = H_t + E_t - K_t$.
- Transfer households' initial endowment H to the firms, and define the firms' "initial equity" as $e_0^i = (H + E)/x_0$ for all i.

By Proposition 1, the decentralized economy thus defined has a unique equilibrium that generates the same consumption and production allocation as the planning allocation.

The Welfare Function

Have shown: It is sufficient to consider linear tax implementation. Hence, define preferences over trajectories (x_t, h_t). Use the value functions from the individual optimizations to define group preferences.

Define welfare as the weighted average:

$$
\begin{aligned}
W &\equiv \alpha V^E(0, E_0) + (1-\alpha) V^H(0, H_0) \\
&= \frac{1}{\rho}\left(\log(\rho(N+E)) + \frac{\mu-\gamma}{\rho}\right) \\
&\quad - \int_0^\infty e^{-\rho t}\left[\frac{h_t+1}{x_t} + \log x_t - (1-\alpha)h_t + \frac{\sigma^2}{2\rho}x_t^2\right] dt
\end{aligned}
$$

where $0 < \alpha < 1$ is the weight given to equity holders.

Welfare Optimum

Proposition

There is a unique welfare optimum, and this is stationary. It is given by the first-order conditions

$$\begin{aligned} (1-\alpha)x^* &= h^* \\ \frac{\sigma^2}{\rho}x^{*3} + x^* - \frac{1}{\alpha} &= 0. \end{aligned}$$

Since $\sigma > 0$, optimal government debt is positive:

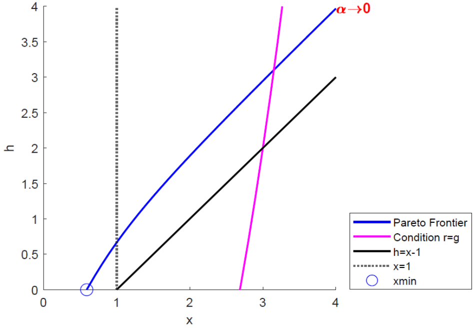
$$1 + h^* > x^*.$$

Welfare

The Pareto Frontier

Pareto frontier traces the welfare optima for varying weights α:

$$h^P(x) = \frac{\sigma^2 x^3 + \rho x - \rho}{\rho + \sigma^2 x^2}$$



Results

Laisser Faire

A passive government does not engage in fiscal policy or redistribution. Laisser-Faire therefore means

- $B_t = 0$ for all t (balanced budget)
- $L^H = L^B = 0$ (no lump-sum redistribution)
- $\tau_t^H = \tau_t^B = \tau_t$ (equal taxation).

Proposition

There is a range $\alpha \in [\underline{\alpha}, \overline{\alpha}]$, $0 < \underline{\alpha} < \overline{\alpha} < 1$, where the welfare optimum is a Pareto improvement over Laisser-Faire. If $\alpha < \underline{\alpha}$, equity holders strictly prefer Laisser-Faire; if $\alpha > \overline{\alpha}$, households strictly prefer Laisser-Faire.

Results

Taxes

The instantaneous tax/subsidy rates that implement the second best allocations are

$$\tau^E = \gamma + \sigma^2 x^2 - \frac{\sigma^4 x^3}{\rho + \sigma^2 x^2}$$

$$\tau^H = \gamma - \frac{\sigma^4 x^3}{\rho + \sigma^2 x^2}$$

Note: A shock of public spending requirements γ implies a 1:1 increase in taxes.

Proposition

At the optimum, households are continuously subsidized.

What does this mean?

- $\tau^H H_t < (1-\alpha)\gamma K_t$ for all γ.
- $\tau^H < 0$ iff $\gamma < \overline{\gamma} \leq \mu$

Reason: Direct public stimulus goes to firms. Resulting wealth increase of firms is taxed away as a function of α.

Results

Debt/GDP

Optimal public debt-to-GDP ratio:

$$\delta^* = \frac{B_t^*}{Y_t^*} = \frac{1 - \alpha x^*}{\mu x^*} = \frac{\sigma^2 x^*}{\mu(\rho + \sigma^2 x^{*2})}$$

Note: Increasing public debt, correctly distributed, has counterveiling effects

- Balance Sheet: $\searrow$ firm leverage $\rightarrow$ $\nearrow$ risky investment
- Interest: Effect on households and on firms: interest rate effect ambiguous
- Growth: $\nearrow$ consumption $\rightarrow$ $\searrow$ growth
- Distribution: fiscal support firms $\rightarrow$ subsidies to households

Results

Growth and Interest

Proposition

At the welfare optimum, $g^* > r^*$ if and only if

$$2\alpha(\rho+\gamma) + (\rho+\gamma+\alpha)\sqrt{\alpha\left(1+\frac{\gamma}{\rho}\right)} < \sigma^2$$

In particular, the growth rate will optimally exceed the interest rate when

- the private propensity to consume ρ is small,
- the amount of public spending γ is small,
- idiosyncratic production risk σ is large,
- the political weight of firm interests α is low.

Results

The Government's Budget Constraint

Budget Constraint: $\dot{B}_t = \gamma K_t + r_t B_t - T_t = rB_t - S_t$,
where S_t is the primary surplus.
Integrating between dates 0 and some later date T yields

$$B_0 = \int_0^T S_t e^{-rt} dt + B_T e^{-rT}$$

Government balance sheet identity:

Assets	Liabilities
$X_0 = \lim_{T\to\infty} \int_0^T S_t e^{-rt} dt$	B_0
$Y_0 = \lim_{T\to\infty} B_T e^{-rT}$	

where – X_0 are "tangible assets" (future primary surpluses)
– Y_0 are "intangible assets" (power to borrow)

Proposition

There are two types of Pareto Optima, which arise under different parameter constellations:

- $r > g$, $Y_0 = 0$, *and* $X_0 = B_0$
- $r < g$, $X_0 = -\infty$, $Y_0 = +\infty$, *and an "aggregate transversality condition" does not hold.*

Result: If α is small, the government plays a "Ponzi Scheme" with exponentially increasing public deficits. If α is large, there are no deficits, and "the market value of government debt equals the present discounted value of primary surpluses" (Cochrane, 2019).

Results

Expenditure Shocks

Thought experiment: Big unexpected events require a large increase of γ for the foreseeable future.
Observation: At the optimum, x and h remain unchanged, while the tax rates increase one to one.

Proposition

At the optimum, a positive permanent public spending shock increases taxes and reduces growth, but does not affect public debt or the debt-to-GDP ratio.

Note: Private investments are crowded out by higher values of γ, thus the growth rate declines (and so does the rate of growth of public debt).

Results

Generalized Model: Preference Shocks

Explicit utility derived from the consumption of private and public goods. Agents' consumption bundle is $(P_t)^{\beta}\left(c_t^k\right)^{1-\beta}$, $k = i, H$, where $\beta \in [0, 1)$ and $P_t = \gamma_t K_t$. Hence, objective:

$$\int_t^{\infty} e^{-\rho s} \log (P_s)^{\beta} \left(c_s^k\right)^{1-\beta} ds = \beta \int_t^{\infty} e^{-\rho s} \log P_s ds + (1-\beta) \int_t^{\infty} e^{-\rho s} \log c_s^k$$

Result: At the optimum, γ becomes endogenous, x^* and h^* adjust:

$$\begin{aligned} \frac{\sigma^2}{\rho} x^{*3} + x^* - \frac{1}{(1-\beta)\alpha} &= 0 \\ (1-\beta)(1-\alpha)x^* &= h^* \\ \gamma^* &= \beta\rho \end{aligned}$$

Thought experiment: Big unexpected events increase β.

Proposition

At the optimum, a positive permanent preference shock for public goods increases taxes and decreases public debt.

What happens:
Stronger preference for public goods consumption requires higher ongoing government expediture and shifts government budgetary funding from the future (debt) to the present (taxes). If β becomes large, public debt is optimally negative: the government creates a “sovereign wealth fund" in order to invest in the private sector and uses the proceeds to pay for public goods over time.