Partial Identification Under Iterated Strict Dominance

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Section 1

Motivation

Structural Estimation of Static Games

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- Theory provides little guidance on what an Equilibrium Selection Mechanism should look like.
- In complicated environments, the "equilibrium play" assumption itself may be suspect.

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- Apply to games with any informational structure.
- For binary choice games, the identified set is sharp.

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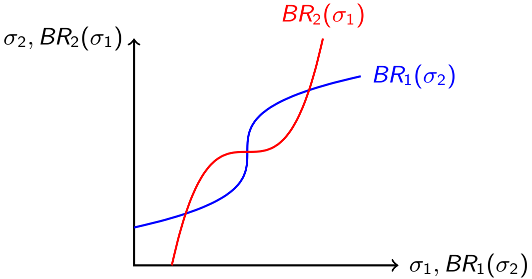

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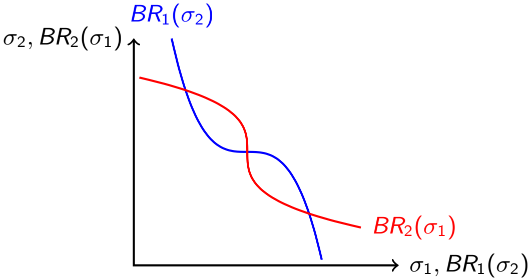

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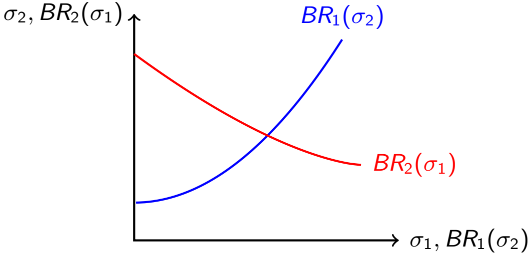

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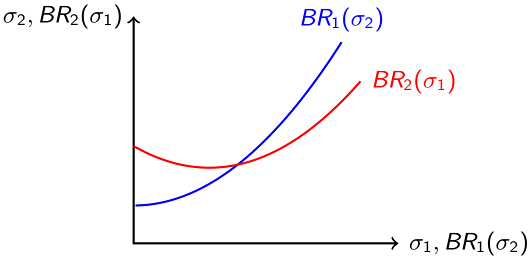

(Very Incomplete) Literature

Set identification in games:

- Bounds: Tamer '03; Ciliberto, Tamer '09; Ciliberto, Murry, Tamer '21; Fan, Yang '22; Aradillas-Lopez, Tamer '08; Aradillas-Lopez '10; Pakes, Porter, Ho, Ishii '15; Aradillas-Lopez, Rosen '22, Magnolfi, Roncoroni '22.

Supermodular Games:

- Theory: Milgrom, Roberts '90; Van Zandt, Vives '07.
- Empirics: Ackerberg, Gowrinsankaran '06; Molinari, Rosen '08; Jia '08; Uetake, Watanabe '13, Uetake, Watanabe '20.

Static games:

- Berry '92; Mazzeo '02; Jia '06; Ellickson, Houghton, Timmins '18; Wollman '18, Ciliberto, Jäkel '21, Schaumans, Verboven '08, Verboven, Schaumans '15; Seim '06; Draganska et al '08; Molinari, Rosen '08; Uetake, Watanabe '13, Uetake, Watanabe '20.

Outline

Section 2

Entry Game Example

2×2 Incomplete Information Entry Game

Two firms simultaneously decide whether to enter a market.

- $y_f \in \{0 \text{ (No Entry)}, 1 \text{ (Entry)}\}$.
- ϵ_f: *iid* private shock. $V[\epsilon_f] = 1$.
- R: variable profit, $R^{mon} > R^{duo}$

$$\pi_f = \begin{cases} R^{mon}(x_f) - \theta_{ec} + \theta_{sc}\epsilon_f & \text{if} \quad y_f = 1 \text{ and } y_{-f} = 0 \\ R^{duo}(x_f) - \theta_{ec} + \theta_{sc}\epsilon_f & \text{if} \quad y_f = 1 \text{ and } y_{-f} = 1 \\ 0 & \text{if} \quad y_f = 0 \end{cases}$$

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- $\theta = (\theta_{ec}, \theta_{sc})$: parameters of interest.

Given (x, θ) the goal is to build bounds around $Pr(y_f = 1 | x, \theta)$.

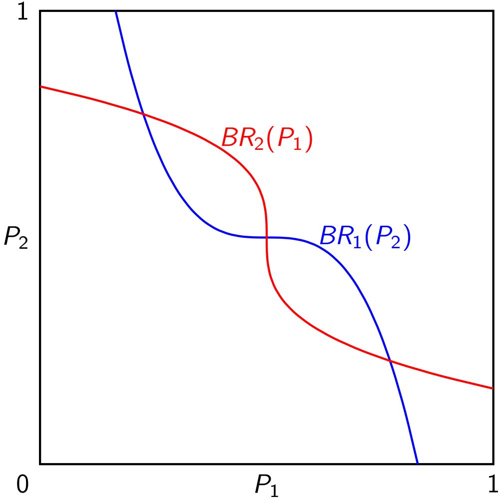
Best Responses in Probabilities

Fix a game, (x, θ), and let ρ_2 be 1's belief about $Pr(y_2 = 2)$.

Given ρ_2, firm 1's optimal entry probability is:

$$BR_1(\rho_2) = Pr\left(\rho_2 R^{duo} + (1 - \rho_2) R^{mon} - \theta_{ec} + \theta_{sc}\epsilon_1 > 0\right)$$

Applying ISD (in probability space)



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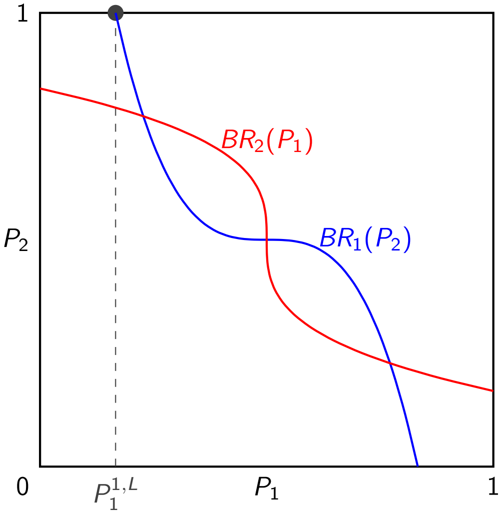

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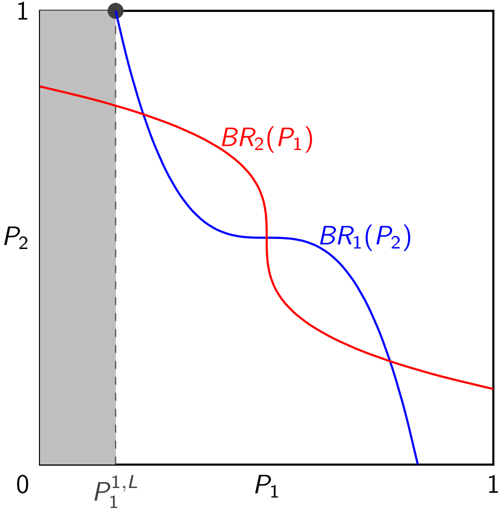

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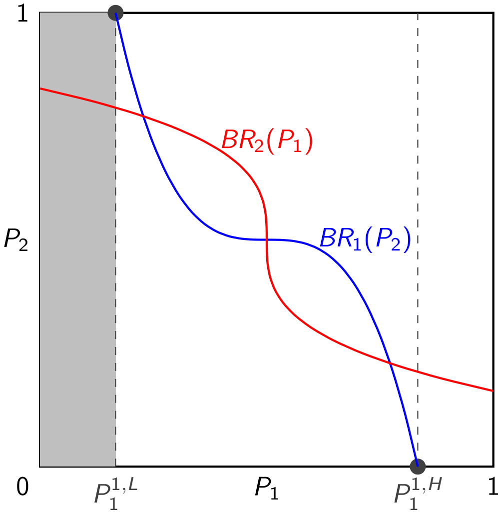

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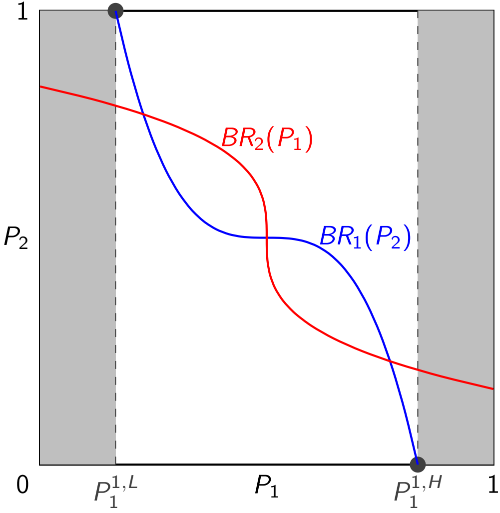

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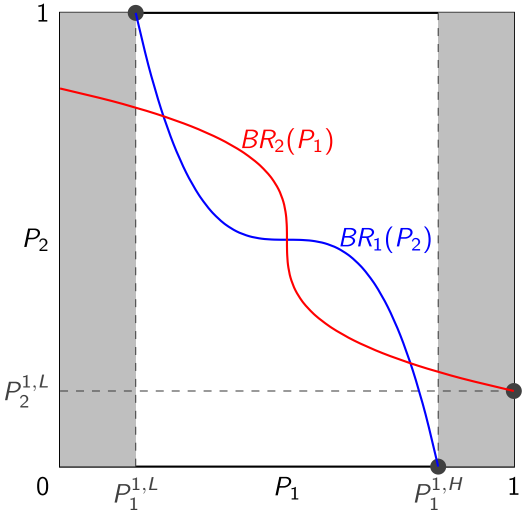

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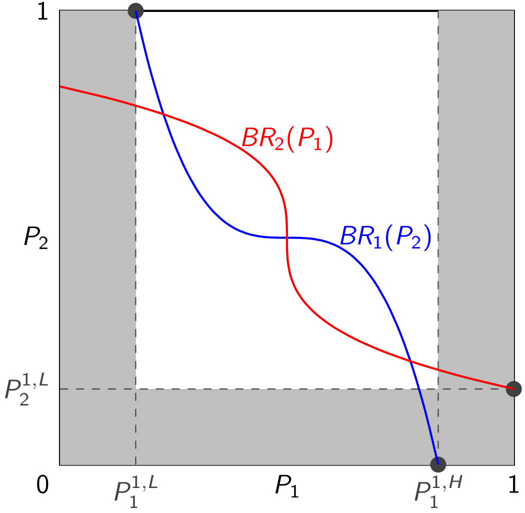

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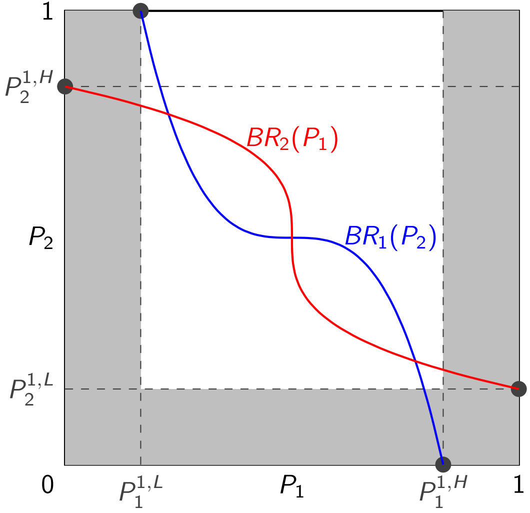

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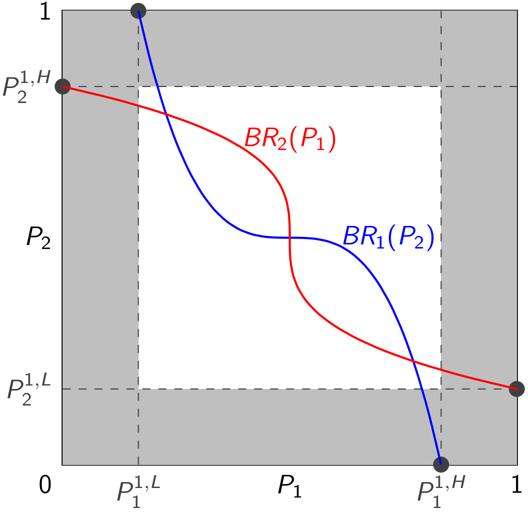

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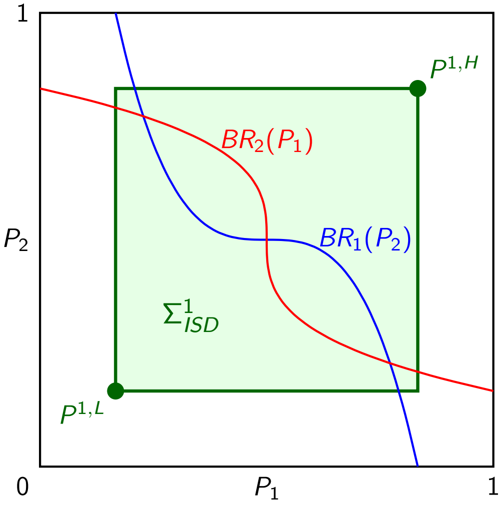

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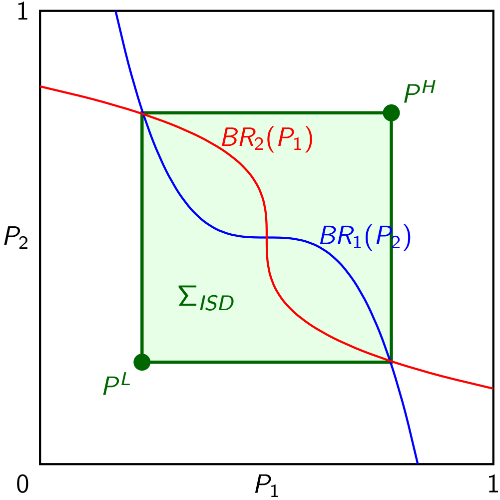
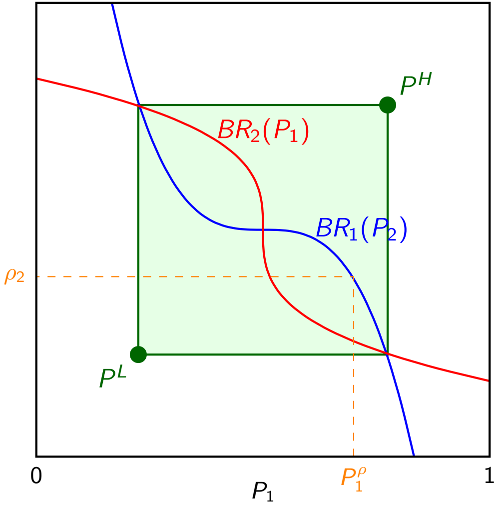


ISD Bounds ▸ ID Case ▸ SM Case



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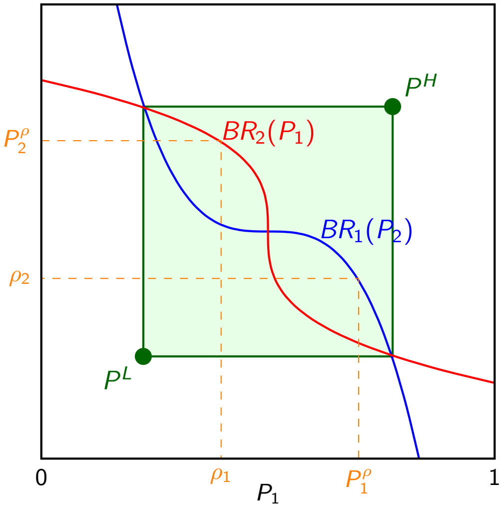

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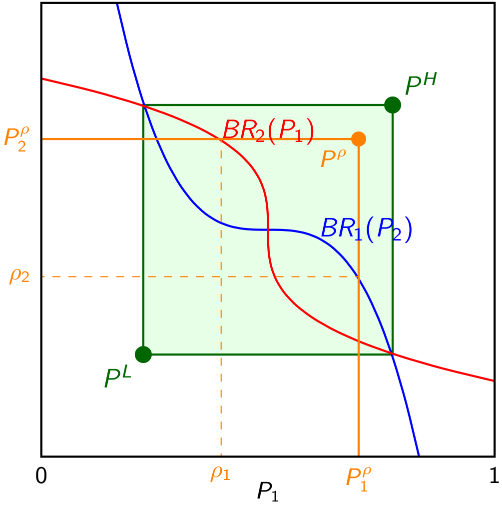

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For any (x, θ), and any ISD Consistent beliefs ρ:

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Remark

This is a specific instance of a general result that applies to games with many firms, multidimensional y_f, and (in)complete information, as long as:

- ***Supermodularity:*** π_f is supermodular in y_f for all f.
- ***Strategic Monotonicity:*** For any all f and all $g \neq f$:
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Section 3

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ISD Identified Set

The ISD Identified Set is:

$$\Theta_{ISD} = \{\theta \in \Theta : P_f^L(x,\theta) \leq P_f^0(x) \leq P_f^H(x,\theta) \text{ for all } x \text{ and } f\}$$

Section 4

Identification Power of ISD Bounds

Trivial ISD Bounds

▸ Identification at Infinity

We say θ generates trivial bounds if for all x and all f:

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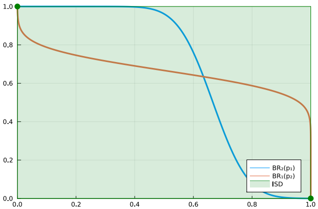

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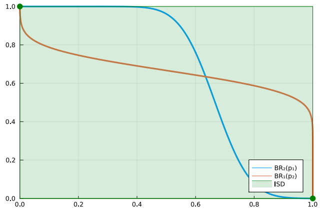

Theorem: Non-Trivial Bounds

No $\theta \in \Theta$ generates trivial bounds *iff* there exist x, x' such that:

$$R^{mon}(x) < R^{duo}(x')$$

What generates trivial bounds? ▸ Details.

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Hence:

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$$R^{duo}(x) - \theta_{ec} < 0 < R^{mon}(x) - \theta_{ec}$$

Hence:

$$P_f^L(x,\theta) = Pr\left(R^{duo}(x) - \theta_{ec} + \theta_{sc}\epsilon_f > 0\right) \to 0 \text{ as } \theta_{sc} \to 0$$

$$P_f^H(x,\theta) = Pr\left(R^{mon}(x) - \theta_{ec} + \theta_{sc}\epsilon_f > 0\right) \to 1 \text{ as } \theta_{sc} \to 0$$

We get, $P_f^L(x,\theta) \to 0$, $P_f^H(x,\theta) \to 1$, for all x, so $\theta \in \Theta_{ISD}$.

What generates trivial bounds? ▸ Details.

Say $R^{duo}(x') < R^{mon}(x), \forall x, x'$. There is a θ_{ec} such that $\forall x$:

$$R^{duo}(x) - \theta_{ec} < 0 < R^{mon}(x) - \theta_{ec}$$

Hence:

$$P_f^L(x,\theta) = Pr\left(R^{duo}(x) - \theta_{ec} + \theta_{sc}\epsilon_f > 0\right) \to 0 \text{ as } \theta_{sc} \to 0$$
$$P_f^H(x,\theta) = Pr\left(R^{mon}(x) - \theta_{ec} + \theta_{sc}\epsilon_f > 0\right) \to 1 \text{ as } \theta_{sc} \to 0$$

We get, $P_f^L(x,\theta) \to 0$, $P_f^H(x,\theta) \to 1$, for all x, so $\theta \in \Theta_{ISD}$.

Take away: Non-trivial ISD Bounds require variables that shift profits uniformly, e.g. *market size*, so that:

$$R^{mon}(x) < R^{duo}(x')$$

for some x, x'

Section 5

Montecarlo

Monte Carlo Set Up

Consider a **imperfect information** entry game with payoffs:

$$\pi_f = y_f \left(\overbrace{\frac{x_f}{(1 + \sum_{f'} y_{f'})^2}}^{R(x)\text{:Cournot Prof.}} - \theta_{ec} + \theta_{sc} \left(\sqrt{0.5}\xi_f + \sqrt{0.5}\epsilon_f \right) \right)$$

with $F \in \{2, 3\}$.

Monte Carlo Set Up

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- $x_f \sim \text{Uniform}\{0.5, 1.5, \ldots, 4.5\}$: observables.

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- $(\theta^0_{ec}, \theta^0_{sc}) = (1, 1)$: parameters.

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with $F \in \{2, 3\}$.

- $x_f \sim \text{Uniform}\{0.5, 1.5, \ldots, 4.5\}$: observables.
- $(\theta^0_{ec}, \theta^0_{sc}) = (1, 1)$: parameters.
- $\xi_f \sim N(0, 1)$: common knowldge unobservables.
- $\epsilon_f \sim N(0, 1)$: private information.

Confidence Set

CCK Test Details

Simulations:

- $MC = 100$ simulated samples.
- $N = 2000$ sample size.

Inference:

- Follow Chernozukov, Chetverikov, Kato '18.
- $H_0 : \theta = \theta_0$.

Coverage Probability: $F = 2$ (top), $F = 3$ (bottom)

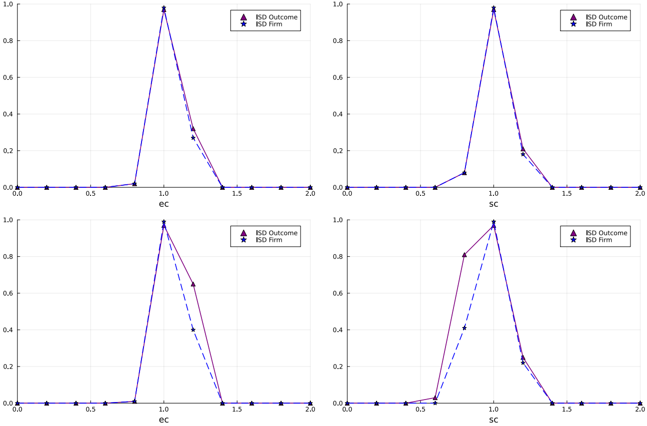


Figure: Monte Carlo simulations for two-firm (top) three-firm (bottom) games.

Section 6

Conclusion

Closing Remarks

I propose ISD bounds for set identification. The bounds:

1. Are robust to multiple equilibria/non-equilibrium play.
2. Allow for discrete/continuous choice variables.
3. Allow for arbitrary informational structures.

For a large class of games, i.e., SMSGs, I argue that:

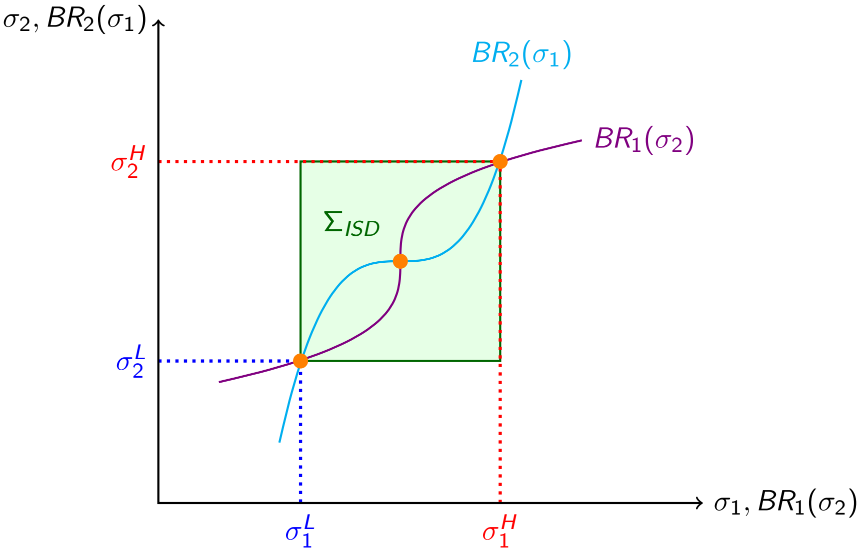
1. Bounds have bite.
2. They are easy to compute.
3. Many (most?) empiral games literature fall under this category!

In the paper:

1. Results on sharpness and point identification for binary games.
2. Application to the airline industry with a focus on the Spirit/JetBlue merger.

Thanks!

Games with Strategic Complements ▸ back



Games with Strategic Substitutes ▸ back

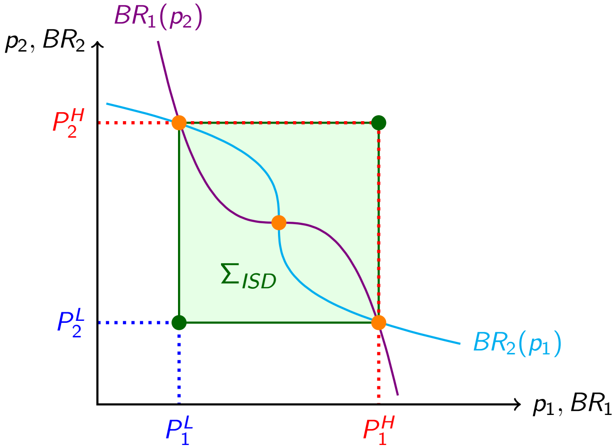


Figure: ISD in Games of Strategic Substitution

Game with General Strategic Monotonicity ▸ back

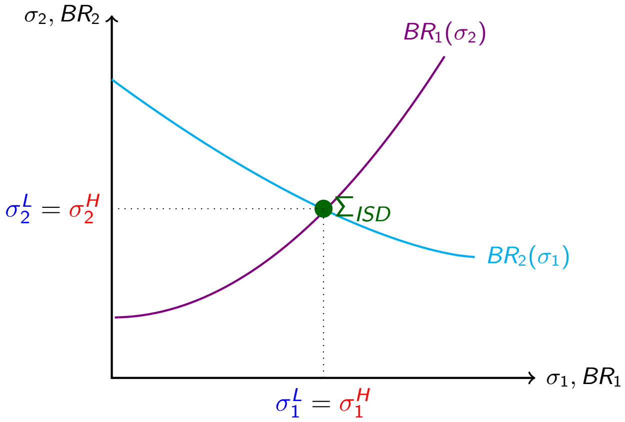


Figure: ISD in Games Strategic Monotonicity

Previous Bounds

back

Ciliberto and Tamer '09 (CT)

For a given (x, θ, ξ) and rational expectations ρ:

- $\{y\} = NE(x, \theta, \xi) \Rightarrow \sigma^\rho = y$
- $\sigma^\rho = y \Rightarrow y \in NE(x, \theta, \xi)$.

$$P(\{y\} = NE(x, \theta, \xi)) \leq P(\sigma^\rho = y) \leq P(y \in NE(x, \theta, \xi))$$

Fan and Yang '22 (FY)

For a given (x, θ, ξ) and any ρ

- $\{y_f\} = \Sigma^1_{f,ISD}(x, \theta, \xi) \Rightarrow \sigma^\rho = y$
- $\sigma^\rho = y \Rightarrow y \in \Sigma^1_{f,ISD}(x, \theta, \xi)$.

$$P(\{y\} = \Sigma^1_{f,ISD}(x, \theta, \xi)) \leq P(\sigma^\rho = y) \leq P(\{y\} \in \Sigma^1_{f,ISD}(x, \theta, \xi))$$

Test for Confidence Set

Back to Monte Carlo

Let $(h_l(X))_l$ be a collection of $l = 1, \ldots, L$ non-negative functions.

Moment inequalities are:

$$\begin{aligned} \overline{\psi}_{fml}(\theta) &\equiv \Big(Y_{fm} - P_f^L(X_m, \theta)\Big)\, h_l(X_m) \leq 0 \\ \underline{\psi}_{fml}(\theta) &\equiv \Big(P_f^H(X_m, \theta) - Y_{fm}\}\Big)\, h_l(X_m) \leq 0 \end{aligned}$$

Letting $\hat{E}$ and $\widehat{SD}$ be the cross-market mean and s.d. operators.

$$T_{CCK}(\theta) = \sqrt{M} \max_{f,k,l} \left\{ \max \left\{ \frac{\hat{E}[\overline{\psi}_{fml}(\theta)]}{\widehat{SD}[\overline{\psi}_{fml}(\theta)]} \frac{\hat{E}[\underline{\psi}_{fml}(\theta)]}{\widehat{SD}[\underline{\psi}_{fml}(\theta)]} \right\} \right\}$$

And test $\theta = \theta_0$ using Chernozhukov, Chetverikov, Kato '18 (CCK):

$$T_{CCK}(\theta; y) \leq CCK(\alpha)$$

Instruments

I choose h_l's to capture the relative relative profitability of a player. Let:

$$\tilde{X}_{fm} \;=\; \frac{X_{fm}}{\left(\prod_{g\neq f} X_{gm}\right)^{1/|\mathcal{F}|}}$$

and

$$\tilde{X}_q \;=\; \{q\text{'th percentile of } (\tilde{X}_{fm})_{fm}\}$$

Letting $l(f,q)$ be the index associated to (f,q) I define:

$$h_{l(f,q)}(X_m) \;=\; \mathbb{1}\left\{\tilde{X}_{fm} \in \left[\tilde{X}_q, \tilde{X}_{20+q}\right]\right\}$$

where $q \in \{0, 20, 40, 60, 80\}$.