# Automation, Polarization, and the Dynamics of Otimum Tax Progressivity 

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#### Abstract

This paper presents a calibrated U.S. economy in which employment and wage polarization and their nuances as documented by Autor and Dorn (2013) and Autor (2015) arise from automation. In the face of polarization, we ask: how should tax progressivity imposed on earnings be set in response? We quantitatively characterize the dynamics of optimum tax progressivity.


Keywords: Automation, Polarization, Optimal tax progressivity, Service occupations

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## 1 Introduction

Polarization has two dimensions: employment and wages. Autor et al. (2008), Acemoglu and Autor (2011), and Autor and Dorn (2013) documented the phenomenon, showing that the middle of the earnings distribution experienced a relative decline in employment while the tails gained (employment polarization), and that the wage growth at the upper and lower deciles of the earnings distribution was faster than at the median (wage polarization). ${ }^{1}$

Papers including Katz and Murphy (1992) and Krusell et al. (2000) examined the evolution of the skill premium (the relative wage of skilled workers to unskilled workers) in the face of technological changes. ${ }^{2}$ However, this line of studies failed to account for the phenomenon of polarization. Acemoglu and Autor (2011) highlighted the issue. In light of the data, they proposed the need of a more nuanced approach (p. 1119): "To enable a study of polarization and changes in different parts of the earnings distribution during different periods, it should incorporate at least three different skill groups."

Autor et al. (2003) suggested that occupational tasks (abstract, routine, and manual) are a useful lens into wage inequality dynamics. Heathcote et al. (2023) corroborated the view from their study of empirical data. In an important paper, Autor and Dorn (2013) envisioned an economy in which there are three types of labor (task) inputs (abstract, routine and manual) featuring different degrees of substituability/complementarity with computer capital (the accumulation of which is associated with automation of routine tasks). Built on their work, we present a model economy in which a continuum of heterogeneous households (in terms of their innated talent) assign to three types of tasks (abstract, routine and manual). The first part of our paper is to show that, as computer capital accumulates or advances, our model after calibrated to match the U.S. economy can generate the phenomenon of polarization as documented by Autor and Dorn (2013). ${ }^{3}$ It also shows that our calibrated U.S. economy can accommodate nuanced aspects of polarization as found by Autor (2015). ${ }^{4}$

Given that our model economy is capable of generating and accommodating impor-

[^1]tant features of polarization, the second part of the paper turns to tax policy issues. We consider the imposition of a log-linear in income tax-and-transfer scheme as in Benabou (2002) and Heathcote et al. (2017). This tax scheme is characterized by two parameters: one represents the level of taxation, while the other represents the progressivity of taxation. Heathcote et al. (2017) showed that the tax scheme approximated the actual tax and transfer system of the U.S. economy pretty well. The central question of the second part of our paper is: in the face of polarization, how should tax progressivity imposed on earnings be set in response?

Related literature
The literature on automation and polarization is vast. Here we focus on a limited subset of the studies most closely related to our paper.

Acemoglu and Restrepo (2018) addressed technologically non-automated tasks versus automated tasks. The former type of tasks have to be produced by labor, while the latter type can be produced by either labor or capital since capital and labor are perfect substitutes. They modeled automation as an expansion of the set of technologically automated tasks. This modeling features the displacement effect of automation - capital takes over tasks previously performed by labor. ${ }^{5}$ Following Autor and Dorn (2013), we focus on non-neutral technological progress, which reduces the cost of computerizing routine tasks. We interpret this technological progress as automation of routine tasks. This modeling features the substitution effect of automation - capital substitutes for labor in accomplishing the routine task in which capital and labor need not be perfect substitutes. Acemoglu and Loebbing (2022) characterize conditions for interior automation, whereby tasks of intermediate complexity are assigned to capital. Starting with interior automation, they showed that a reduction in the cost of capital (or an increase in capital productivity) causes employment and wage polarization, and that large enough increases in capital productivity ultimately induce a transition to low-skill automation and qualitatively alter the effects of automation.

Acemoglu and Restrepo (2020) studied the effects of industrial robots on US labor markets and estimated robust negative effects of robots on employment and wages across commuting zones. Acemoglu and Restrepo (2022) documented that between 50\% and $70 \%$ of changes in the U.S. wage structure over the last four decades are accounted for

[^2]by relative wage declines of worker groups specialized in routine tasks in industries experiencing rapid automation. They derived a simple equation linking wage changes of a demographic group to the task displacement it experiences, showing that regression models incorporating task displacement explain much of the changes in education wage differentials between 1980 and 2016. ${ }^{6}$

Jaimovich et al. (2021) addressed the macroeconomics of automation. They developed a emprically relevant model, focusing on general equilibrium effects such as complementarities between the various factors of production, displacement effects of retraining programs, and the effects on labor supply of changes in distortionary taxation required to fund such programs.

The focus of the above papers is not on taxation. Guerreiro et al. (2021) and Thuemmel (2023) addressed the question: should robots be taxed and how should they be taxed? Acemoglu and Restrepo (2020) investigated if the US tax system has promoted inefficiently high levels of automation, and they introduced an automation tax as an additional policy tool to improve efficiency. The focus of our paper is not on capital taxation but on labor taxation, and we ask: how should tax progressivity imposed on earnings be set dynamically as automation of rountine tasks advances? Both Ales et al. (2015) and Tsai et al. (2022) considered a talent-to-task assignment model to address optimal taxation in response to technical change. However, neither of them targets specifically at polarization.

The rest of the paper is organized as follows. Section 2 introduces the model economy. Section 3 conducts the positive analysis, showing the arise of polarization in our economy. Section 4 conducts the normative analysis and addresses optimal tax progressivity in response to polarization. Section 5 concludes.

## 2 Model economy

The model economy builds on Autor and Dorn (2013), in which the skill of workers is either high or low. While high-skill workers (with mass one) always choose abstract tasks, low-skill workers choose between manual and routine tasks. ${ }^{7}$ The skill level of workers is continuous in our setting. We call it "talent," which is presumably linked to people's innate ability. There is an assignment of talent to three tasks: abstract, routine, and manual. As far as the U.S. data are concerned, the fraction of workers who choose

[^3]high-skill, abstract tasks increases significantly over time. ${ }^{8}$ We consider a more general talent-to-task assignment to account for this phenomenon.

The economy consists of production, households, and the government. We describe each of them successively.

### 2.1 Production

Consider an economy with two sectors that produce "goods" $g$ and "services" $s$. We let $g$ be the numeraire and denote the price of $s$ by $p$.

As in Autor and Dorn (2013), goods $g$ are produced by combining abstract and routine tasks and its production technology is specified as

$$
\begin{equation*}
Y_{g}=A L_{a}^{1-\alpha}\left(L_{r}^{\rho}+K^{\rho}\right)^{\alpha / \rho}, \alpha \in(0,1), 0<\rho \leq 1, \tag{1}
\end{equation*}
$$

where $A$ denotes total factor productivity, $K$ is computer capital (say, information and communication technology), and $L_{a}$ and $L_{r}$ are the amount of labor employed in the abstract and the routine task, respectively. The abstract task only requires one input -abstract labor $L_{a}$, while both rountine labor $L_{r}$ and capital $K$ are used to serve the routine task, represented by $\left(L_{r}^{\rho}+K^{\rho}\right)^{1 / \rho}$. A key feature of (1) is that $K$ is a relative complement to $L_{a}$ but a relative substitute for $L_{r}$. Specifically, the elasticity of substitution between $K$ and $L_{a}$ is 1 while the elasticity of substitution between $K$ and $L_{r}$ is $\frac{1}{1-\rho}>1$. Output $Y_{g}$ can be consumed by households or used by the government.

The routine task in (1) belongs to the so-called technologically automated tasks in Acemoglu and Restrepo (2018). They set $\rho=1$ (i.e., capital and rountine labor being perfect substitutes) for this type of tasks in their modeling automation. We let the advance of $K$ represent automation of routine tasks but allow for the possibility of $\rho<1$. The aggregate production function (1) could be microfounded via the continuum task-based framework of Acemoglu and Restrepo (2018); see the derivation of equation (36) in Guerreiro et al. (2021). ${ }^{9}$ The setup of (1) embodies the emphasis of Autor (2015) that improved technology complements some types of labor (winners $L_{a}$ ) but substitutes for othe types (losers $\left.L_{r}\right)$ is the main characteristic of automation. ${ }^{10}$

Following Autor and Dorn (2013), services s are produced by a single task - manual. The manual task only requires one input - manual labor. Its production technology is

[^4]specified as
\[

$$
\begin{equation*}
Y_{s}=\mu L_{m}, \mu>0, \tag{2}
\end{equation*}
$$

\]

where $\mu$ is an efficiency parameter and $L_{m}$ denotes the amount of labor employed in the manual task. As in Autor and Dorn (2013), we let $\mu \equiv 1$ as a normalization. ${ }^{11}$ Output $Y_{s}$ can only be consumed by households.

The setup of (2) embodies the emphasis of Autor and Dorn (2013) that $K$ neither directly substitutes for nor directly complements $L_{m}$. Services $s$ in (2) are referred to as service occupations in Autor and Dorn (2013). ${ }^{12}$

Corresponding to three tasks: abstract, routine and manual, there are three occupations: $a, r$ and $m$.

All markets are competitive and factors of production are paid according to their marginal product:

$$
\begin{align*}
w_{a} & \equiv \frac{\partial Y_{g}}{\partial L_{a}}=A(1-\alpha)\left[\left(\frac{L_{r}}{L_{a}}\right)^{\rho}+\left(\frac{K}{L_{a}}\right)^{\rho}\right]^{\alpha / \rho}, \\
w_{r} & \equiv \frac{\partial Y_{g}}{\partial L_{r}}=A \alpha\left(\frac{L_{r}}{L_{a}}\right)^{\rho-1}\left[\left(\frac{L_{r}}{L_{a}}\right)^{\rho}+\left(\frac{K}{L_{a}}\right)^{\rho}\right]^{\alpha / \rho-1}, \\
w_{m} & \equiv p \frac{\partial Y_{s}}{\partial L_{m}}=p \\
r & \equiv \frac{\partial Y_{g}}{\partial K}=A \alpha\left(\frac{K}{L_{a}}\right)^{\rho-1}\left[\left(\frac{L_{r}}{L_{a}}\right)^{\rho}+\left(\frac{K}{L_{a}}\right)^{\rho}\right]^{\alpha / \rho-1} \tag{3}
\end{align*}
$$

From (1) and (3), we have

$$
\frac{w_{a} L_{a}}{Y_{g}}=1-\alpha, \frac{w_{r} L_{r}}{Y_{g}}=\alpha \frac{L_{r}^{\rho}}{L_{r}^{\rho}+K^{\rho}}
$$

which implies that, as $K$ advances, the income share of $L_{a}$ in $Y_{g}$ remains constant at $1-\alpha$, whereas the income share of $L_{r}$ in $Y_{g}$ will be "crowded out" by $K$ because of the substitution of $K$ for $L_{r}$. Indeed, we have $\frac{w_{r} L_{r}}{Y_{g}} \rightarrow 0$ as $K \rightarrow \infty$. This constitutes the main mechanism to cause polarization at the upper tail of the earnings distribution in our framework.

Heathcote et al. (2017) considered an economy where agents face uninsurable permanent and insurable transitory productivity shocks. They provided an analytical framework to address optimal tax progressivty on earnings in response to the shocks. This

[^5]paper considers an economy where agents face polarization due to the accumulation or advance of computer capital. We provide a framework to address optimal tax progressivty on earnings in response to polarization. The analysis of Heathcote et al. (2017) abstracts from capital. We follow their framework but with a twist. Specifically, we assume that capital is supplied exogenously by "absentee capitalists" and analyze optimal tax progressivity imosed on earnings in partial equilibrium. This simplifies the analysis by avoiding the endogenous determination of capital, which is not our main concern. ${ }^{13}$ However, by including capital (even though exogenous), we can examine how increasing $K$ (automation of routine tasks) will alter optimal tax progressivity imposed on earnings over time. ${ }^{14}$

### 2.2 Households

The economy is habitated by a continuum of households (consumers/workers) with mass 1 . Households are heterogeneous in terms of their innate talent, $\theta \in[1, \bar{\theta}] \equiv \Theta$. It is assumed that the density function of talent, denoted by $f(\theta)$, is distributed according to a truncated Pareto distribution on $\Theta$ with $f(\theta)=\frac{\eta \theta^{-\eta-1}}{1-\left(1 / \overline{)^{\eta}}\right.}, \eta>0 .{ }^{15}$

The utility function of households is given by

$$
U=u\left(c_{g}, c_{s}\right)-v(h)+\ln \mathcal{E}_{i} \cdot I_{i}(i \in\{a, r, m\})
$$

which is defined over the consumption of goods $g\left(c_{g}\right)$ and of services $s\left(c_{s}\right)$, and hours worked $h ; I_{i}($.$) is an indicator function. Autor and Dorn (2013) considered a setup with$ $v(h) \equiv 0$. We allow for variation in $h$ as in Heathcote et al. (2017).

Normalizing $\mathcal{E}_{r}=1, \mathcal{E}_{i}(i=a, m)$ in $U$ represents the non-pecuniary monetary value of task $i$ relative to task $r$ which is not embodied in the pecuniary monetary reward of working in task $i$ (say, the high stress of working in task $a$ relative to in task $r$; a less rigid

[^6]rule of life of working in task $m$ relative to in task $r$ ). It brings about a fixed utility gain $\ln \mathcal{E}_{i}$ in $U$ to households who choose task $i(i=a, m)$. $\mathcal{E}_{i}(i=a, m)$ could be positive or negative. We take an agnostic view about the value of $\mathcal{E}_{i}(i=a, m)$ and let data determine it through the lens of our model. It is worth noting that the introduction of $\ln \mathcal{E}_{i} \cdot I_{i}$ into $U$ is in line with the work of Lockwood et al. (2017), which highlights the role of nonpecuniary payoffs in the allocation of talent across occupations.

To facilitate analytical exposition and prepare for quantitative results later, we work with ${ }^{16}$

Assumption $1 u\left(c_{g}, c_{s}\right)=(1-\chi(K)) \ln c_{g}+\chi(K) \ln c_{s}(t), 0<\chi(K)<1, \chi^{\prime}(K)>0$; $v(h)=\frac{h^{1+\sigma}}{1+\sigma}, \sigma>0$.

The functional form for $v(h)$ is commonly used in the literature. The functinal form for $u\left(c_{g}, c_{s}\right)$ is also common, except that the weight $\chi$ in $u\left(c_{g}, c_{s}\right)$ depends positively on $K$. This positive dependence gives rise to an extra kick of household demand for services $s$ relative to goods $g$ as $K$ advances. We explain the main mechanism (associated with Assumption 1) that causes polarization at the lower tail of the earnings distribution later. ${ }^{17}$

There is an assignment of talent $\theta$ to task $i \in\{a=$ abstract,$r=$ routine, $m=$ manual $\}$. Following Ales et al. (2015) and Tsai et al. (2022), which build on the work of Teulings (1995), Cummins and Violante (2002) and Acemoglu and Autor (2011), we pose: ${ }^{18}$

1. The match of talent $\theta$ to task $i \in\{a, r, m\}$ gives rise to labor productivity $z_{i}(\theta)=$ $\exp \left(\gamma_{i} \theta\right)$. We let $\gamma_{a}>\gamma_{r}>\gamma_{m}$, which implies that if $\theta^{\prime}>\theta$, then

$$
\log \left(\frac{z_{a}\left(\theta^{\prime}\right)}{z_{a}(\theta)}\right)>\log \left(\frac{z_{r}\left(\theta^{\prime}\right)}{z_{r}(\theta)}\right)>\log \left(\frac{z_{m}\left(\theta^{\prime}\right)}{z_{m}(\theta)}\right)
$$

that is, higher talents have a comparative advantage in performing more "complex" tasks.
${ }^{16}$ An alternative to $u\left(c_{g}, c_{s}\right)$ in Assumption 1 is:

$$
u\left(c_{g}, c_{s}\right)=(1-\chi) \ln c_{g}+\chi \ln \left(c_{s}+\bar{c}\right), \bar{c}>0
$$

which is a familar setup in the literature on growth and structural transformation; see Herrendorf et al. (2014). However, unlike Assumption 1, there is no closed-form solution for $h(\theta)$ under this alternative setup.
${ }^{17}$ Autor and Dorn (2013) worked with $u\left(c_{g}, c_{s}\right)=\left(c_{g}^{\epsilon}+c_{s}^{\epsilon}\right)^{1 / \epsilon}, \epsilon<1$. They showed that, depending on the relationship between $\epsilon$ and $(\alpha, \rho)$ in (1), all of low-skill workers choose either the routine task or the manual task in the asymptotic allocation as $K \rightarrow \infty$.
${ }^{18}$ Acemoglu and Autor (2011) p. 1118 highlighted: "Workers apply their skill endowments to tasks in exchange for wages. Thus, the task-based approaches emphasize that skills are applied to tasks to produce output-skills do not directly produce output."
2. Households choose one of $i \in\{a, r, m\}$ to work in and they cannot work in multiple tasks.

The match of talent $\theta$ to task $i$ gives rise to earnings $y_{i}(\theta)$, which are the product of three components:

$$
\begin{equation*}
y_{i}(\theta)=\underbrace{w_{i}}_{\text {wage rate }} \times \underbrace{z_{i}(\theta)}_{\text {labor productivity }} \times \underbrace{h(\theta)}_{\text {hours worked }}, i \in\{a, r, m\} . \tag{4}
\end{equation*}
$$

Given prices $\left\{w_{a}, w_{r}, w_{m}=p\right\}$ and the tax scheme $T(y)$ imposed by the government, the maximization problem in the face of a household with talent $\theta$ can be represented as

$$
\begin{equation*}
i(\theta) \in \arg \max _{i \in\{a, r, m\}} U(\theta, i) \tag{5}
\end{equation*}
$$

where

$$
\begin{equation*}
U(\theta, i)=\max _{c_{g}, c_{s}, h} u\left(c_{g}(\theta), c_{s}(\theta)\right)-v(h(\theta))+\left(\ln \mathcal{E}_{i}\right) \cdot I_{i}, \tag{6}
\end{equation*}
$$

subject to

$$
\begin{equation*}
c_{g}(\theta)+p c_{s}(\theta)=y_{i}(\theta)-T\left(y_{i}(\theta)\right) \tag{7}
\end{equation*}
$$

### 2.3 Government

The government imposes a log-linear in income tax scheme as in Heathcote et al. (2017) on earings $y$ :

$$
\begin{equation*}
T(y)=y-\tau_{0} y^{1-\tau_{1}}, \tau_{1}<1 \tag{8}
\end{equation*}
$$

where the parameter $\tau_{1}$ determines the degree of tax progressivity and the parameter $\tau_{0}$ controls the average level of taxation. Note that $y-T(y)=\tau_{0} y^{1-\tau_{1}}$, i.e. $\tau_{0} y^{1-\tau_{1}}$ represents posttax earnings for pretax earnings $y$. Since $\tau_{1}=-\frac{\partial \ln \left[1-T^{\prime}(y)\right]}{\partial \ln y}=\frac{y T^{\prime \prime}(y)}{1-T^{\prime}(y)}$ for all $y>0$, the tax scheme $T(y)$ is an CRP (constant rate of progressivity) tax scheme and it is progressive, regressive, or proportional, depending on whether $\tau_{1}$ is positive, negative, or zero. Note that if $\tau_{1}=1$ were true, there would be a complete redistribution since posttax income (i.e. $\tau_{0} y^{1-\tau_{1}}$ ) becomes the same for all households. Because $T(y)$ according to (8) could become negative if $y$ is low, the CRP tax scheme (8) is best seen as a tax and transfer scheme. The tax scheme (8) gains popularity since the work of Benabou (2002) and Heathcote et al. (2017). The later paper shows that $T(y)$ of (8) with $\tau_{1}=0.181$ approximates the actual tax and transfer system of the U.S. economy pretty well. We shall let $\tau_{1}=0.181$ represent the degree of tax progressivity in competitive equilibrium. The CRP tax scheme (8) is often
labeled the HSV tax scheme in the literature.
The government is required to purchase the $G$ amount of goods $g$ for public consumption, and obey its budget constraint

$$
\begin{equation*}
G=\int_{\Theta} T\left(y_{i}(\theta)\right) f(\theta) d \theta \tag{9}
\end{equation*}
$$

Define $\bar{g}=G / Y_{g}$. We let $\bar{g}$ be exogenously given.

### 2.4 Competitive equilibrium

The following definition of competitive equilibrium is standard.
Definition 1 Given $K$, a competitive equilibrium is a tax scheme $T(y)$ of (8) and public consumption $G=\bar{g} Y_{g}$, market prices $\left\{w_{a}, w_{r}, w_{m}=p, r\right\}$, and non-negative quantities $\left\{c_{g}(\theta), c_{s}(\theta), h(\theta)\right\}_{\theta \in \Theta}$ such that:

1. Household behavior satisfies (5) and (6) subject to (7).
2. Factors of production are paid according to (3).
3. Labor markets clear:

$$
\begin{aligned}
L_{a} & =\int_{\Theta_{a}} z_{a}(\theta) h(\theta) f(\theta) d \theta \\
L_{r} & =\int_{\Theta_{r}} z_{r}(\theta) h(\theta) f(\theta) d \theta \\
L_{m} & =\int_{\Theta_{m}} z_{m}(\theta) h(\theta) f(\theta) d \theta
\end{aligned}
$$

where $\Theta_{i} \subseteq \Theta, i \in\{a, r, m\}$, denotes the set of households choosing task $i$ to work in.
4. The markets for goods $g$ and services $s$ clear:

$$
\begin{align*}
Y_{g}-r K & =\int_{\Theta} c_{g}(\theta) f(\theta) d \theta+G  \tag{10}\\
Y_{s} & =\int_{\Theta} c_{s}(\theta) f(\theta) d \theta \tag{11}
\end{align*}
$$

where the amount of $g$ available to the economy is given by $Y_{g}-r K$, due to the "extraction" of absentee capitalists.
5. The government's budget is balanced:

$$
G=\bar{g} Y_{g}=\int_{\Theta} T\left(y_{i}(\theta)\right) f(\theta) d \theta
$$

This completes the description of the model economy.

## 3 Positive analysis

We first provide a preliminary exposition of the model economy.
Under Assumption 1, solving for the problem (6)-(7) yields

$$
\begin{equation*}
c_{g}(\theta)=(1-\chi) \tau_{0} y_{i}(\theta)^{1-\tau_{1}}, c_{s}(\theta)=\chi \frac{\tau_{0} y_{i}(\theta)^{1-\tau_{1}}}{p}, h(\theta)=\left(1-\tau_{1}\right)^{\frac{1}{1+\sigma}} \tag{12}
\end{equation*}
$$

Note that $h(\theta)$ is independent of $\theta$, a result similar to that in Heathcote et al. (2017).
We have the indirect utility function:

$$
\begin{align*}
U(\theta, i) & =\ln \tau_{0}+\left(1-\tau_{1}\right) \ln y_{i}(\theta)-\chi \ln p-\frac{1-\tau_{1}}{1+\sigma}+\ln \mathcal{E}_{i}  \tag{13}\\
+ \text { constant, } i & \in\{a, r, m\}, \mathcal{E}_{r}=1
\end{align*}
$$

The following lemma characterizes $\left\{\Theta_{i}\right\}_{i \in\{a, r, m\}}$.
Lemma 1 Impose Assumption 1 and suppose $\Theta_{r}$ is non-empty. ${ }^{19}$ There exist $\theta_{m}$ and $\theta_{a}$ such that

$$
\Theta_{m}=\left[0, \theta_{m}\right], \Theta_{r}=\left[\theta_{m}, \theta_{a}\right], \Theta_{a}=\left[\theta_{a}, \infty\right)
$$

Proof: See Appendix ??.
From (13), $i(\theta) \equiv \arg \max _{i \in\{a, r, m\}} U(\theta, i)=\arg \max _{i \in\{a, r, m\}}\left\{\left(1-\tau_{1}\right) \ln y_{i}(\theta)+\ln \mathcal{E}_{i}\right\}$. Thus, given $h(\theta)=\left(1-\tau_{1}\right)^{\frac{1}{1+\sigma}}$ and $\mathcal{E}_{r}=1$, Lemma 1 implies the following equalities for household $\theta_{a}$ and $\theta_{r}$ :

$$
\begin{aligned}
w_{a} z_{a}\left(\theta_{a}\right) \mathcal{E}_{a}^{\frac{1}{1-\tau_{1}}} & =w_{r} z_{r}\left(\theta_{a}\right) \\
w_{m} z_{m}\left(\theta_{m}\right) \mathcal{E}_{m}^{\frac{1}{1-\tau_{1}}} & =w_{r} z_{r}\left(\theta_{m}\right)
\end{aligned}
$$

[^7]which lead to
\[

$$
\begin{align*}
\frac{w_{a}}{w_{r}} \mathcal{E}_{a}^{\frac{1}{1-\tau_{1}}} & =\frac{z_{r}\left(\theta_{a}\right)}{z_{a}\left(\theta_{a}\right)}=\exp \left[\left(\gamma_{r}-\gamma_{a}\right) \theta_{a}\right]  \tag{14}\\
\frac{w_{m}}{w_{r}} \mathcal{E}_{m}^{\frac{1}{1-\tau_{1}}} & =\frac{z_{r}\left(\theta_{m}\right)}{z_{m}\left(\theta_{m}\right)}=\exp \left[\left(\gamma_{r}-\gamma_{m}\right) \theta_{m}\right]
\end{align*}
$$
\]

The two equalities simply state that household $\theta_{a}$ (resp. $\theta_{m}$ ) is indifferent between working in task $a$ and task $r$ (resp. task $m$ and task $r$ ).

Given $f(\theta)=\frac{\eta \theta^{-\eta-1}}{1-(1 / \bar{\theta})^{\eta}}, \eta>0, \theta \in[1, \bar{\theta}], h(\theta)=\left(1-\tau_{1}\right)^{\frac{1}{1+\sigma}}$ in (12), and $L_{i}=\int_{\Theta_{i}} z_{i}(\theta) h(\theta) f(\theta) d \theta$, we obtain

$$
\begin{align*}
L_{a} & =l \int_{\theta_{a}}^{\bar{\theta}} \frac{\exp \left(\gamma_{a} \theta\right)}{\theta^{\eta+1}} d \theta=l\left[\frac{-\exp \left(\gamma_{a} \theta\right)}{\eta \theta^{\eta}}+\frac{\gamma_{a}}{\eta} \int_{\theta_{a}}^{\bar{\theta}} \frac{\exp \left(\gamma_{a} \theta\right)}{\theta^{\eta}} d \theta\right] \\
L_{r} & =l \int_{\theta_{m}}^{\theta_{a}} \frac{\exp \left(\gamma_{r} \theta\right)}{\theta^{\eta+1}} d \theta=l\left[\frac{-\exp \left(\gamma_{r} \theta\right)}{\eta \theta^{\eta}}+\frac{\gamma_{r}}{\eta} \int_{\theta_{m}}^{\theta_{a}} \frac{\exp \left(\gamma_{r} \theta\right)}{\theta^{\eta}} d \theta\right],  \tag{15}\\
L_{m} & =l \int_{1}^{\theta_{m}} \frac{\exp \left(\gamma_{m} \theta\right)}{\theta^{\eta+1}} d \theta=l\left[\frac{-\exp \left(\gamma_{m} \theta\right)}{\eta \theta^{\eta}}+\frac{\gamma_{m}}{\eta} \int_{1}^{\theta_{m}} \frac{\exp \left(\gamma_{m} \theta\right)}{\theta^{\eta}} d \theta\right],
\end{align*}
$$

where $l=\frac{\eta\left(1-\tau_{1}\right)^{\frac{1}{1+\sigma}}}{1-(1 / \bar{\theta})^{\eta}}$.
From (3), we derive

$$
\begin{equation*}
\frac{w_{a} L_{a}}{w_{r} L_{r}}=\frac{1-\alpha}{\alpha}\left[1+\left(\frac{K}{L_{r}}\right)^{\rho}\right] . \tag{16}
\end{equation*}
$$

Utilizing the solution for $c_{g}(\theta)$ and $c_{s}(\theta)$ in (12), one can combine (10) and (11) into a single constraint,

$$
Y_{g}-r K-G=\frac{1-\chi(K)}{\chi(K)} p Y_{s}
$$

which, using $Y_{g}=w_{a} L_{a}+w_{r} L_{r}+r K$ and $Y_{s}=L_{m}$, gives

$$
\begin{equation*}
w_{m}=p=\frac{\chi(K)}{1-\chi(K)} \frac{w_{a} L_{a}+w_{r} L_{r}-G}{L_{m}} . \tag{17}
\end{equation*}
$$

With $G=\bar{g} Y_{g}=\frac{\bar{g}}{1-\alpha} w_{a} L_{a}$, we obtain from (16)-(17)

$$
\begin{equation*}
\frac{w_{m} L_{m}}{w_{r} L_{r}}=\frac{\chi(K)}{1-\chi(K)}\left[\left(1-\frac{\bar{g}}{1-\alpha}\right) \frac{w_{a} L_{a}}{w_{r} L_{r}}+1\right] \tag{18}
\end{equation*}
$$

Increasing $K$ will drive up $\frac{w_{a} L_{a}}{w_{r} L_{r}}$ according to (16). ${ }^{20}$ Through the term $\frac{w_{a} L_{a}}{w_{r} L_{r}}$ shown up

[^8]in (18), $\frac{w_{m} L_{m}}{w_{r} L_{r}}$ inherits the property of $\frac{w_{a} L_{a}}{w_{r} L_{r}}$ given by (16). As long as $1-\alpha>\bar{g}$ (which holds in our calibration), increasing $K$ will indirectly drive up $\frac{w_{m} L_{m}}{w_{r} L_{r}}$ via driving up $\frac{w_{a} L_{a}}{w_{r} L_{r}}$ according to (18). This is true even if $\chi^{\prime}(K)=0$ instead of $\chi^{\prime}(K)>0$ under Assumption 1. The indirect effect is related to the so-called Baumol (1967) effect. It dictates in our context that, even though the production of services $s$ (see (2)) benefits little from the advance of $K$ directly, the sector benefits indirectly, since there is complementarity between $c_{g}$ and $c_{s}$ according to our utility function $u\left(c_{g}, c_{s}\right)$. Mazzolari and Ragusa (2013) empirically found that there are spillovers from high-skill consumption to low-skill labor markets as relative skilled wages increase. This evidence supports the presence of the Baumol effect. The imposition of $\chi^{\prime}(K)>0$ under Assumption 1 gives rise to an extra kick of demand for services $s$ relative to goods $g$, in addition to the Baumol effect.

Given $\left\{w_{a}, w_{r}, w_{m}\right\}$, one can use (14) to solve for $\left\{\theta_{a}, \theta_{m}\right\}$. With $\left\{\theta_{a}, \theta_{m}\right\}$ in hand, one can use (15) to calculate $\left\{L_{a}, L_{r}, L_{m}\right\}$. With $\left\{L_{a}, L_{r}, L_{m}\right\}$ in hand, one can use (3) and (17) to update $\left\{w_{a}, w_{r}, w_{m}\right\}$. This is basically the way of solving the equilibrium of our model economy.

The impact of taxes in our model economy is as follows. More tax progrssivity (a higher $\tau_{1}$ ) will cause lower hours worked $h(\theta)$ according to (12). This effect will alter $\left\{L_{a}, L_{r}, L_{m}\right\}$ according to (15) and hence $\left\{w_{a}, w_{r}, w_{m}\right\}$ according to (3) (see also (16) and (18)). Changes in $\left\{w_{a}, w_{r}, w_{m}\right\}$ will cause changes in $\left\{\theta_{a}, \theta_{m}\right\}$ accordingly by (14).

### 3.1 Automation and polarization

Automation of routine tasks is inherently redistributive, complementing some type of labor $\left(L_{a}\right)$ but substituting for other type $\left(L_{r}\right)$. We qualitatively address how employment and wage polarization and their nuances arise in our model economy.

### 3.1.1 Employment polarization

The size of households choosing to work in task $i$, denoted by $\left|\Theta_{i}\right|$, is given by

$$
\begin{aligned}
\left|\Theta_{a}\right| & =\frac{1}{1-(1 / \bar{\theta})^{\eta}} \int_{\theta_{a}}^{\bar{\theta}} \eta \theta^{-\eta-1} d \theta=\frac{1}{1-(1 / \bar{\theta})^{\eta}}\left(\theta_{a}^{-\eta}-\bar{\theta}^{-\eta}\right) \\
\left|\Theta_{r}\right| & =\frac{1}{1-(1 / \bar{\theta})^{\eta}} \int_{\theta_{m}}^{\theta_{a}} \eta \theta^{-\eta-1} d \theta=\frac{1}{1-(1 / \bar{\theta})^{\eta}}\left(\theta_{m}^{-\eta}-\theta_{a}^{-\eta}\right) \\
\left|\Theta_{m}\right| & =\frac{1}{1-(1 / \bar{\theta})^{\eta}} \int_{1}^{\theta_{m}} \eta \theta^{-\eta-1} d \theta=\frac{1}{1-(1 / \bar{\theta})^{\eta}}\left(1-\theta_{m}^{-\eta}\right)
\end{aligned}
$$

where we have utilized $f(\theta)=\frac{\eta \theta^{-\eta-1}}{1-(1 / \bar{\theta})^{\eta}}, \eta>0, \theta \in[1, \bar{\theta}]$. Let $H_{i}, i \in\{a, r, m\}$, denote the total work hours in task $i$. Since $h(\theta)=\left(1-\tau_{1}\right)^{\frac{1}{1+\sigma}}$ for all $\theta$ according to (12), we have

$$
\begin{align*}
\frac{H_{a}}{H_{r}} & =\frac{\left|\Theta_{a}\right|}{\left|\Theta_{r}\right|}=\frac{\theta_{a}^{-\eta}-\bar{\theta}^{-\eta}}{\theta_{m}^{-\eta}-\theta_{a}^{-\eta}} \\
\frac{H_{m}}{H_{r}} & =\frac{\left|\Theta_{m}\right|}{\left|\Theta_{r}\right|}=\frac{1-\theta_{m}^{-\eta}}{\theta_{m}^{-\eta}-\theta_{a}^{-\eta}} . \tag{19}
\end{align*}
$$

Define employment share as share in total work hours as in Autor and Dorn (2013). We then see that $\frac{\left|\Theta_{a}\right|}{\left|\Theta_{r}\right|}$ and $\frac{\left|\Theta_{m}\right|}{\left|\Theta_{r}\right|}$ of (19) represent the employment share of task $a$ relative to task $r$ and that of task $m$ relative to task $r$, respectively. Both $\frac{\left|\Theta_{a}\right|}{\left|\Theta_{r}\right|}$ and $\frac{\left|\Theta_{m}\right|}{\left|\Theta_{r}\right|}$ hinge on $\theta_{a}$ and $\theta_{m}$. It is clear from (19) that if $\frac{d \theta_{a}}{d K}<0$ and $\frac{d \theta_{m}}{d K}>0$, then, as $K$ advances, we have employment polarization as documented by Autor and Dorn (2013) (i.e. both $\frac{\left|\Theta_{a}\right|}{\left|\Theta_{r}\right|}$ and $\frac{\left|\Theta_{m}\right|}{\left|\Theta_{r}\right|}$ and hence both $\frac{H_{a}}{H_{r}}$ and $\frac{H_{m}}{H_{r}}$ are increasing in $K$ ).

The mechanism of automation that causes employment polarization in our model economy runs as follows. Increasing $K$ raises $w_{a}$ and cuts $w_{r}$ according to (3) ( $\frac{\partial w_{a}}{\partial K}>0$; $\frac{\partial w_{r}}{\partial K}<0$ under $\rho>\alpha$, which holds with our calibration). Higher $w_{a}$ implies that the demand curve for abstract labor shifts outward, while lower $w_{r}$ implies that the demand curve for routine labor shifts inward. From (16), we have

$$
\begin{equation*}
\frac{w_{a}}{w_{r}}=\frac{1-\alpha}{\alpha}\left[1+\left(\frac{K}{L_{r}}\right)^{\rho}\right] \frac{L_{r}}{L_{a}}, \tag{20}
\end{equation*}
$$

which must hold in equilibrium. The initial impact of increasing $K$ on $\frac{w_{a}}{w_{r}}$ is summarized by (20), in that it raises $\frac{w_{a}}{w_{r}}$, given $\frac{L_{r}}{L_{a}}$. Since $\gamma_{a}>\gamma_{r}$, this will then lead to a lower $\theta_{a}$ according to (14). The lower $\theta_{a}$ will induce the response of labor supply with changes in $\frac{L_{r}}{L_{a}}$ according to (15). As long as the initial impact of increasing increasing $K$ on $\frac{w_{a}}{w_{r}}$ dominates the impact of the subsequent induced changes of $\frac{L_{r}}{L_{a}}$ on $\frac{w_{a}}{w_{r}}$, we see from (20) that the result of $\frac{d \theta_{a}}{d K}<0$ will arise in equilibrium.

From (18), we have

$$
\begin{equation*}
\frac{w_{m}}{w_{r}}=\frac{\chi(K)}{1-\chi(K)}\left[\left(1-\frac{\bar{g}}{1-\alpha}\right) \frac{w_{a} L_{a}}{w_{r} L_{r}}+1\right] \frac{L_{r}}{L_{m}} \tag{21}
\end{equation*}
$$

which must hold in equilibrium. Increasing $K$ must raise $\frac{w_{a} L_{a}}{w_{r} L_{r}}$ according to (16). Thus, the initial impact of increasing $K$ on $\frac{w_{m}}{w_{r}}$ (via raising $\frac{w_{a} L_{a}}{w_{r} L_{r}}$ ) is summarized by (21), in that it raises $\frac{w_{m}}{w_{r}}$, given $\frac{L_{r}}{L_{m}}$. Since $\gamma_{r}>\gamma_{m}$, this will then lead to a higher $\theta_{m}$ according to (14). The higher $\theta_{m}$ will induce the response of labor supply with changes in $\frac{L_{r}}{L_{m}}$ according to
(15). As long as the initial impact of increasing increasing $K$ on $\frac{w_{m}}{w_{r}}$ dominates the impact of the subsequent induced changes of $\frac{L_{r}}{L_{m}}$ on $\frac{w_{m}}{w_{r}}$, we see from (21) that the result of $\frac{d \theta_{m}}{d K}>0$ will arise in equilibrium.

Autor (2015) Figure 5 found that the pace of employment gains in low-wage, manual task-intensive jobs relative to other jobs has risen successively across periods from 1979 to 2012. The setup of $\chi(K)$ with $\chi^{\prime}(K)>0$ under Assumption 1 (the extra kick of demand for services $s$ relative to goods $g$ driven by automation) will potentially enable us to account for this nuanced aspect of polarization.

Autor (2015) also found a different nuanced aspect of employment polarization, in that the growth of occupational employment across skill levels looks like a downward ramp rather than a U shape more recently. He himself provided a possible reason (pp. 21-22): "I suspect that the huge falloff in information investment may have dampened innovative activity and demand for high-skilled workers more broadly." According to Giandrea et al. (2022), from 1985 forward, the compound growth rate of capital stock for the industry of computer and electronic products reaches $4.716 \%$ between 1987 and 2000, but reduces to $1.225 \%$ between 2000 and 2009, and further to $0.278 \%$ between 2009 to 2018. The growth rates of $K$ do become significantly smaller after year 2000 relative to those before year 2000.

### 3.1.2 Wage polarization

The mean hourly wage of task $i$ is given by $\frac{w_{i} L_{i}}{H_{i}}(i \in\{a, r, m\})$. Thus, the mean hourly wage of task $i(i \in\{a, m\})$ relative to that of task $r$ is given by

$$
\frac{w_{a} L_{a}}{w_{r} L_{r}} \frac{H_{r}}{H_{a}}=\frac{w_{a} L_{a}}{w_{r} L_{r}} \frac{\left|\Theta_{r}\right|}{\left|\Theta_{a}\right|}, \frac{w_{m} L_{m}}{w_{r} L_{r}} \frac{H_{r}}{H_{m}}=\frac{w_{m} L_{m}}{w_{r} L_{r}} \frac{\left|\Theta_{r}\right|}{\left|\Theta_{m}\right|}
$$

where we have utilized $\frac{H_{r}}{H_{a}}=\frac{\left|\Theta_{r}\right|}{\left|\Theta_{a}\right|}$ and $\frac{H_{r}}{H_{m}}=\frac{\left|\Theta_{r}\right|}{\left|\Theta_{m}\right|}$. If both $\frac{w_{a} L_{a}}{w_{r} L_{r}} \frac{H_{r}}{H_{a}}$ and $\frac{w_{m} L_{m}}{w_{r} L_{r}} \frac{H_{r}}{H_{m}}$ are increasing in $K$, then, as $K$ advances, we have wage polarization as documented by Autor and Dorn (2013). ${ }^{21}$

While employment polarization arises if both $\frac{H_{a}}{H_{r}}$ and $\frac{H_{m}}{H_{r}}$ are increasing in $K$, wage polarization arises if both $\frac{w_{a} L_{a}}{w_{r} L_{r}} \frac{H_{r}}{H_{a}}$ and $\frac{w_{m} L_{m}}{w_{r} L_{r}} \frac{H_{r}}{H_{m}}$ are increasing in $K$. Although polarization has two dimensions, including both employment and wages, Autor (2015) documented that wage polarization is a less robust phenomenon than employment polarization. Our framework provides a mechanism for why this is true. Increasing $K$ will drive up both

[^9]$\frac{w_{a} L_{a}}{w_{r} L_{r}}$ and $\frac{w_{m} L_{m}}{w_{r} L_{r}}$ according to (16) and (18). If $\frac{H_{r}}{H_{a}}$ and $\frac{H_{r}}{H_{m}}$ were fixed, this implies an increase of hourly wages in both task $a$ and $m$. The increase then induces the occupational movement of households from working in task $r$ to working in task $a$ and $m$. This movement leads to employment polarization but, at the same time, it could upset wage polarization through suppressing the increase of hourly wages in both tasks $a$ and $m$. This possible upset of wage polarization due to the response of labor supply is well recognized by Autor (2015). His main argument for why wage polarization is a less robust phenomenon exactly lies in that the elasticity of labor supply can mitigate or even fully offset wage gains that would otherwise have occurred. The phenomenon of wage polarization is more stringent than that of employment polarization, in that it is required that the increasing $\frac{w_{a} L_{a}}{w_{r} L_{r}}\left(\frac{w_{m} L_{m}}{w_{r} L_{r}}\right)$ dominate the decreasing $\frac{H_{r}}{H_{a}}\left(\frac{H_{r}}{H_{m}}\right)$ implied by employment polarization.

### 3.2 A calibrated U.S. economy

The set of parameters in the model economy include (i) production: $\{\alpha, \rho, A\}$ (production technology); (ii) households: $\left\{\chi(K), \sigma, \mathcal{E}_{a}, \mathcal{E}_{m}\right\}$ (preferences), $\left\{\gamma_{i}\right\}_{i \in\{a, r, m\}}$ (labor productivity), and $\{\eta, \bar{\theta}\}$ (talent distribution); and (iii) government: $\left\{\tau_{1}, \bar{g}\right\}$ (tax progressivity and government spending). ${ }^{22}$ This subsection presents a calibrated U.S. economy built on our model economy. Following Autor and Dorn (2013), we choose year 1980 as our benchmark year, focusing on the era of secularly rising wage inequality from year 1980 onward. We quantitatively show that the calibrated U.S. economy can generate employment and wage polarization and their nuances as documented by Autor and Dorn (2013) and Autor (2015). Below we explain our parameterization.

Production Eden and Gaggl (2018) documented data on $L_{a} \cdot L_{r}, L_{m}$ and $K$ (information and communication technology, ICT capital) over years 1950-2013. Our $\alpha(t=1980)$ in (1) is obtained directly from their data. Eden and Gaggl (2018) calculated the elasticity of substitution between routine task and ICT over years 1950-2013. Our $\rho(t=1980)$ in (1) adopts their calculated value.

Eden and Gaggl (2018) documented the U.S. data since the 1950s, finding that there has been substantial reallocation of labor income from occupations relatively substitutable with ICT (routine) to ones relatively complementary (nonroutine). More specifically, they documented from their data that $1-\alpha$ is increasing over time. The increasing $1-\alpha$ implies the increasing income share $\frac{w_{a} L_{a}}{Y_{g}}=1-\alpha$ in (1), which highlights the gaining weight

[^10]of the abstract task in the economy. Along with the advance of $K$, technical progress could bring about a higher $\rho$ to reinforce the force of automation. Eden and Gaggl (2018) empirically confirmed this plausibility, showing according to their data that $\rho$ is increasing over time. In our numerical analysis over time, we adopt $\{\alpha(t)\}_{t=1980}^{2013}$ and $\{\rho(t)\}_{t=1980}^{2013}$ according to the data documented and calculated in Eden and Gaggl (2018). We study their effects on polarization, in addition to the effects of increasing $K$.

We let $A=1$ in (1) without loss of generality. It can be checked that varying the value of $A>0$ will not affect our results qualitatively and quantitatively. The reason behind this result is similar to that of varying the value of $\mu$ in (2). Thus, neutral technical progress has little role to play in the exposition of polarization. This is true at least within our model economy.

Households We follow Heathcote et al. (2017) to set $\sigma=2$. For the talent distribution parameter, we choose $\eta=1$ and $\bar{\theta}=20$ to match the Gini coefficients of the earnings distribution and its Pareto parameters.

Government Heathcote et al. (2017) estimated $\tau_{1}=0.181$ for the U.S. economy. ${ }^{23} \mathrm{We}$ adopt this number in computing competitive equilibrium. We set $\bar{g}=0.189$ following Heathcote et al. (2017).

Calibration The parameters that remain to be determined in their values are $\left\{\gamma_{i}\right\}_{i \in\{a, r, m\}}$, $\left\{\mathcal{E}_{i}\right\}_{i \in\{a, m\}}$, and $\chi(K)$. We calibrate $\left\{\gamma_{i}\right\}_{i \in\{a, r, m\}},\left\{\mathcal{E}_{i}\right\}_{i \in\{a, m\}}$, and $\chi(K)$ to match the four empirical moments in 1980 reported in Table 1 of Autor and Dorn (2013): (i) employment share of $\Theta_{i}, i \in\{a, m\}$, relative to that of $\Theta_{r}$, where employment share is defined as share in total work hours, and (ii) mean hourly wage of $\Theta_{i}, i \in\{a, m\}$, relative to that of $\Theta_{r}$, where hourly wage is defined as yearly wage and salary income divided by total work hours. We seek the minimal $\left\{\mathcal{E}_{i}\right\}_{i \in\{a, m\}}$ consistent with the match. Table 1 reports the results from our calibration. ${ }^{24}$

Note that $\gamma_{m} \approx 0$ from our calibration. This is consistent with the theoretical setup of Autor and Dorn (2013) that low-skill agents who choose manual task have homogeneous productivity in performance.

Finally, we need to specify the evolution of $K$ over time. We first derive a value of $K$ for 1980, which is consistent with our calibration for the model economy of 1980. It is found

[^11]Table 1: Parameters calibrated inside the model

| Parameter | Value | Target moments |
| :---: | :---: | :--- |
|  |  | Hourly wage ratios |
| $\left\{\gamma_{a}, \gamma_{r}, \gamma_{m}\right\}$ | $\{0.348,0.250,0.001\}$ | $\frac{w_{a} L_{a} / H_{a}}{w_{r} L_{r} / H_{r}}=1.31 ; \frac{w_{m} L_{m} / H_{m}}{w_{r} L_{r} / H_{r}}=0.58$ |
| $\chi$ | 0.075 | Employment ratios |
| $\mathcal{E}_{a}$ | 9.00 | $\frac{H_{a}}{H_{r}}=0.54 ; \frac{H_{m}}{H_{r}}=0.17$ |
| $\mathcal{E}_{m}$ | 1.25 | Earnings Gini $=0.408$ |

$K \approx 60$. We then use this value of $K$ in 1980 as a baseline to obtain the values of $K$ after 1980 for our model economy according to the data reported in Giandrea et al. (2022). ${ }^{25}$

We specify $\chi(K(t))=\chi(t=1980) \cdot\left[1+\left(\frac{K(t)}{K(t-1)}-1\right) / 2\right]$.
Given $K$, our algorithm of computing the equilibrium of the economy is as follows:

1. Guess $\left(\frac{w_{a}}{w_{r}}, \frac{w_{m}}{w_{r}}\right)$ and use (14) to solve for $\left(\theta_{m}, \theta_{a}\right)$.
2. Use the obtained $\left(\theta_{m}, \theta_{a}\right)$ to calculate $\left\{L_{i}\right\}_{i \in\{a, r, m\}}$ according to (15).
3. Use $\left\{L_{i}\right\}_{i \in\{a, r, m\}}$ along with $K$ to calculate $\frac{w_{a}}{w_{r}}$ from (16) and $\frac{w_{m}}{w_{r}}$ from (18).
4. Iterate until $\left(\frac{w_{a}}{w_{r}}, \frac{w_{m}}{w_{r}}\right)$ converge.

The above algorithm enables us to solve for $\left(\frac{w_{a}}{w_{r} r}, \frac{w_{m}}{w_{r}}\right),\left(\theta_{m}, \theta_{a}\right)$ and $\left\{L_{i}\right\}_{i \in\{a, r, m\}}$ in equilibrium. Using $\left\{L_{i}\right\}_{i \in\{a, r, m\}}$, we are able to solve for $\left(w_{a}, w_{r}, r\right)$ from (3) and $w_{m}(=p)$ from (17). Using $h(\theta)$ in (12), we are able to calculate $y_{i}(\theta)$ according to (4). Using (12), we are able to calculate $\left(c_{g}(\theta), c_{S}(\theta)\right)$ and hence $U(\theta)$.

### 3.3 Polarization of the U.S. economy

[^12]Table 2: Benchmark and computer capital $(K)$ induced polarization

|  | Employment ratio |  | Hourly wage ratio |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $H_{a} / H_{r}$ | $H_{m} / H_{r}$ | A-R | M-R | Earnings Gini |
| Benchmark (1980) | 0.54 | 0.17 |  | 1.35 | 0.65 |
| 0.408 |  |  |  |  |  |
| Data (2005) | 0.89 | 0.28 |  | 1.69 | 0.70 |$) 0.470$

Note: K grows at a rate $2.8 \%$ annually from 1980 to 2005 (Giandrea et al., 2021). Technology changes: $\alpha$ decreases from 0.615 in 1980 to 0.570 in 2005; $\rho$ increases from 0.56 in 1980 to 0.67 in 2005 (Eden and Gaggl, 2018)

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[^1]:    ${ }^{1}$ See also Autor et al. (2006), Goos and Manning (2007), Goos et al. (2009), Goos et al. (2014), Autor (2015), Autor (2019), and Heathcote et al. (2023).
    ${ }^{2}$ Maliar et al. (2020) and Ohanian et al. (2021) considered more recent data to revisit the skill-premium issue addressed by Krusell et al. (2000).
    ${ }^{3}$ Autor and Dorn (2013) showed polarization in the asymptotic allocation (the stock of capital approaches infinity as time goes to infinity) with the planner's problem (there are no distortions in their model economy), while we show polarization in competitive equilibrium in the presence of distortionary taxes. Offshoring could cause job polarization as well. However, Goos et al. (2014) found that technology (automation of routine tasks) is much more important than offshoring.
    ${ }^{4} \mathrm{We}$ address these nuanced aspects later.

[^2]:    ${ }^{5}$ Besides the displacement effect, there are the productivity effect (automation inceases aggregate output per worker) and the reinstatement effect (automation creates new tasks in favor of labor). Acemoglu and Restrepo (2019) highlighted that the net impact of automation on labor demand depends on how these three effects weigh together. Aghion et al. (2023) surveyed two contrasting views on the impacts of automation on labor demand: (i) firms that automate reduce employment, and (ii) automating firms become more productive, and the resulting increase in scale translates into higher employment.

[^3]:    ${ }^{6}$ Factors other than automation could also be relevant for the disappearance of routine jobs, see, for example, the work of Cortes et al. (2017). Aghion et al. (2017) provided the "Baumol Cost Disease" effect to expose the impact of automation.
    ${ }^{7}$ Those choose manual task have homogeneous skills in performance, whereas those choose routine task have heterogenous skills.

[^4]:    ${ }^{8}$ See, for example, Autor (2014), Autor (2015), and Tsai et al. (2022) (Figure 10).
    ${ }^{9}$ Given that there are only two types of workers in Guerreiro et al. (2021), it is clear that the focus of their study is not on polarization.
    ${ }^{10}$ Autor et al. (2003) emphasized that the declining price of computer capital is the causal force of automation.

[^5]:    ${ }^{11}$ One can check that a higher $\mu$ will be fully offset by a lower $p$ in equilibrium. Intuitively, a higher $\mu$ implies a larger quantity of $Y_{s}$ and thereby a lower price $p$ in equilibrium.
    ${ }^{12}$ These occupations are "jobs that involve assisting or caring for others, for example, food service workers, security guards, janitors and gardeners, cleaners, home health aides, child care workers, hairdressers and beauticians, and recreation occupations." (p. 1555)

[^6]:    ${ }^{13}$ Hoffmann et al. (2020) Figures 1 and 2 showed that the long-run growth in total income inequality in the U.S. economy is driven primarily by growth in labor income inequality. They also showed that the evolution of total income inequality in large European economies (France, Germany, Italy, and the United Kingdom) is almost entirely driven by changes in the distribution of labor income, and capital income plays a very small role.
    ${ }^{14}$ Autor and Dorn (2013) assumed that capital fully depreciates between periods and consumption equals output. As such, the equilibrium in each time period in their model can be analyzed in isolation. To address polarization, Acemoglu and Loebbing (2022) considered a setup in which the aggregate capital stock is exogenously given.
    ${ }^{15}$ The truncated Pareto distribution will converge to the corresponding Pareto distribution on $[1, \infty)$ as $\bar{\theta} \rightarrow \infty$. Autor and Dorn (2013) assumed that the skill distribution of the low skill workers in performing routine task follows an exponential distribution. We also consider the case where $\theta$ is distributed exponentially, i.e. $f(\theta)=\eta \exp (-\eta \theta), \eta>0, \theta \in[0, \infty)$. However, the Gini coefficients for the earnings distribution resulting from our model is way below a realistic value for the U.S. economy, regardless of the value of $\eta$.

[^7]:    ${ }^{19}$ We verify this presumption numerically in our quantitative study.

[^8]:    ${ }^{20}$ Given the substitution of $K$ for $L_{r}, \frac{K}{L_{r}}$ is increasing in $K$.

[^9]:    ${ }^{21}$ Both Acemoglu and Autor (2011) and Autor and Dorn (2013) employed mean log hourly wages to report wage polarization.

[^10]:    ${ }^{22}$ There is no need to know $\tau_{0}$, since by Walras' law the the government budget constraint will be satisifed once all other markets are clear.

[^11]:    ${ }^{23}$ Guerreiro et al. (2021) re-estimated the CRP tax scheme for the U.S. economy, finding $\tau_{1}=0.18$.
    ${ }^{24}$ See Online Appendix for more details on calibration.

[^12]:    ${ }^{25}$ The Bureau of Economic Analysis (BEA) and the Bureau of Labor Statistics (BLS) use estimates of depreciation rates for structures and equipment to construct estimates of capital stock from data on capital investments. However, the estimates of these depreciation rates are based mainly on research from the early 1980 s and may be out of date. These authors re-estimated BEA capital stock measures and the BLS capital and multifactor productivity measures using some updated estimates of depreciation rates. We choose the asset category of computer and electronic products in their re-estimated capital stock as our values of $K$ in the model.

