Market Design of College-Major Switches for Reducing Student-Major Mismatch*

Job Market Paper

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Abstract

It is not unusual for students to feel that their major is not a good fit for them. When such a mismatch between students and majors occurs, demand for major switching emerges. In practice, the number of major switches is usually constrained by the total number of initially vacant seats at each major. This constraint limits the number of successful major switches, preventing potential welfare gains for students. We propose new class of mechanisms to address the problem and demonstrate how these mechanisms could increase the success rate of major switching using data provided by the National Taiwan University (NTU). With the help of our early findings, in 2022, NTU adopted one of our proposed mechanisms.

JEL classification: C78, D61, D78, I21, I23

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1 Introduction

College education plays a key role in individuals' careers. Choosing the right major affects whether an individual can attain all possible benefits of a college education. Due to various reasons, including choosing a misfit major, some college students drop out, which causes majors to have vacant seats.[1,2] To utilize these vacant seats, colleges allow students from other majors to switch to the majors with vacant seats. Across the world, colleges usually run a *major switching process* through a centralized or semi-centralized procedure at least once a year. For example, the conventional process for major switching in Taiwan is a centralized matching between students and majors within each college.[3,4] Whereas, in the United States, many colleges run their major switching process in a semi-centralized manner.[5] Under both centralized and semi-centralized procedures in practice, the number of major switches is usually restricted to the number of initially vacant seats. By using a simple example, we illustrate the consequences of such a restriction.

Example 1. *College c has two majors, Economics and Math. Two students, Eric and Elizabeth, from Economics, would like to switch to Math. Only one student, Matt, from Math, would like to switch to Economics. Currently, there is only one vacant seat at Math and no vacant seat at Economics. Math ranks Eric over Elizabeth. Then, if the number of switches is restricted to the number of initially vacant seats, only Eric can switch to a new major under both centralized and semi-centralized procedures. Elizabeth and Matt stay*

[1] In the United States, approximately 19% of freshmen enrolled full time in a 4-year institution drop out each year. The most commonly stated reasons are financial pressure, family, and poor fit to the major/college (Hanson, 2022).

[2] In 2020, 13.38% of all college students in Taiwan dropped out, marking an all-time high. The most commonly cited reasons for those who dropped out of college in Taiwan are job needs, lack of interest in their major, poor grades, and financial difficulties (Lin et al., 2020).

[3] In a centralized procedure, every participating student submits a rank order list over the majors to the corresponding office in the college. Every major ranks the students who have listed them mostly based on their academic success. Then, the rankings are used to match students and majors together.

[4] Many colleges in Canada also have major switching procedures that operate in a centralized manner. For example, McGill University and The University of British Columbia (see https://www.mcgill.ca/students/courses/plan/changing and https://vancouver.calendar.ubc.ca/admissions/change-degree-program).

[5] In a semi-centralized procedure, every participating student submits her application to the corresponding majors. Then, majors rank their applicants and send an offer to the best ones. Finally, students pick their favorite offer. Examples of colleges running semi-centralized major switching process include Purdue University and North Carolina State University (see https://www.purdue.edu/science/Current_Students/codo/index.html and https://studentservices.ncsu.edu/academics/degrees/coda/coda-change-or-add-a-major/cnr/)

in their current majors. As a result, after the major switching procedure, there is a vacant seat at Economics. However, it is possible for all three students to switch their majors without violating capacity constraints. That is, Eric and Elizabeth can switch to Math, Matt can switch to Economics, and the total number of students at the end of major switching procedure at each major will not exceed the sum of the number of vacant seats and initially assigned students.

Notice that, in Example 1, we can achieve the outcome in which all students switch majors by allowing initially occupied seats to be used in the major switching procedure. Restricting the number of major switches to the number of initially vacant seats may prevent us from achieving all potential gains for students from the major switching process. Then, an easy solution to improve the major switching process is to allow both occupied and initially vacant seats to be used and ask the majors to include their current students in their rank order list. However, such an easy solution might have unintended consequences: some students might end up in worse majors than their initial majors and some majors might end up with worse cohorts compared to the case in which their occupied seats are not used for major switching.[6]

In this paper, we consider *major switching* as a market design problem and propose solutions to recover welfare losses due to the restrictions used in practice while taking the aforementioned two possible unintended consequences into consideration. In particular, our solutions guarantee no student will be assigned to a major worse than her initial major. Moreover, under our solutions, majors are allowed to choose whether they would like to allow their occupied seats to be used. We consider major switching at elite colleges in Taiwan as our motivating example and demonstrate possible welfare gains (especially an increase in the number of major switches) under our proposed solutions using data provided by National Taiwan University (NTU), which is perceived as the best university in Taiwan (Chen and Kao, 2023). More importantly, our counterfactual analysis illustrates that allowing the occupied seats to be used for major switching is an equilibrium strategy for majors.

[6] To illustrate the second issue, if Economics prefers Elizabeth over Matt, then it will be worse off when the occupied seats are used for major switching.

Using this study's early findings, we collaborated with NTU to change their existing major switching procedure with one of our proposed solutions in which majors are free to allow or not their occupied seats to be included in their capacity. Although almost half of the majors agreed to allow their occupied seats to be used, the short ranking lists submitted by the majors cancel out the possible welfare improvement we expect. Our conversations with the administrators at NTU indicate that some majors are not listing all acceptable students out of fear of possible negative effects on their incoming cohort. To address this concern, which is theoretically valid, we conduct an equilibrium analysis, and our results indicate that it is an equilibrium strategy for all majors to list all of their acceptable students.

First, we provide some brief background information about college admission and major switching in Taiwan.[7] In Taiwan, students have to choose college-major pairs jointly in centralized college admissions.[8] Moreover, each college-major enrollment is subject to an admission quota, which is regulated by the government. A relevant issue in recent years is the oversupply of colleges and their competition to attract students (Kao and Lin, 2017). The government reports the registration ratios of each major and college every year.[9] Surprisingly, the registration ratios display a tail-drop pattern, i.e., the registration ratios decrease very quickly at less selective (i.e., demanded) colleges over time. Since some students avoid applying for majors at a college with a low registration ratio and may be concerned with the prestige of the college that they attend, there exists a mismatch between students and majors, in particular, at selective colleges. That is, when applying to college-major pairs some students prepare their ranking lists based on colleges' prestige and the registration ratio of the majors rather than their actual preference over majors. Hence, many students' assigned majors might not be a good fit (Lin, 2010). As a result, those students are more likely to drop out.[10]

Usually, students who are not happy with their majors first seek to switch majors instead

[7] For more detailed information, see Section 3.

[8] Similar practices can be found in college admissions in Chile, Japan, Spain, and Turkey (Bordon and Fu, 2015).

[9] The registration ratio is defined as the number of students attending a major to its admission quota. The registration ratio was also reported at the college level. Please see Section 3.2 for more details.

[10] Per a government survey conducted in 2019, 24.6% of Taiwanese students who dropped out of college were due to the mismatch between students and majors (Ho and Chen, 2020).

of dropping out. Therefore, there exists a demand for major switching in Taiwan. The conventional major-switching mechanism in Taiwan is a centralized matching between students and majors within a college. However, the number of major switches is usually constrained by the vacant seats. As a result, the success rate of major switching, which is the ratio of the number of major switches to the number of applicants, is usually low in Taiwan, especially at selective colleges.[11]

In this paper, we conduct both theoretical and counterfactual analyses. In our theoretical analysis, we first focus on the mechanism used for major-switching at NTU until 2022, which we call the NTU mechanism, and show it is equivalent to the major proposing deferred acceptance (DA) algorithm (Gale and Shapley, 1962) in which the capacity of each major is the number of initially vacant seats at that major. Motivated by the minimalist market design paradigm of Sönmez (2023), we propose two (classes of) alternative major-switching mechanisms to increase the success rate. These proposed mechanisms are based on the student and major proposing DA mechanisms. In each of these mechanisms, a major can choose not to limit the number of available seats to its initially vacant seats.

In order to provide freedom for majors to choose whether they want to allow occupied seats to be used, we first define a choice function using seat-specific priorities (Kominers and Sönmez, 2016) in which each seat in a major may rank applicants in a different order. We show that this choice function satisfies the desirable properties in the literature, namely, substitutes, law of aggregate demand, and irrelevance of rejected alternatives (Hatfield and Milgrom, 2005; Aygün and Sönmez, 2012).[12] As a result, student proposing and major proposing DA mechanisms both select an outcome such that rankings of the majors over the incoming students are respected for any problem and any set of majors restricting only initially vacant seats to be available for major switches. Moreover, student proposing DA yields a (weakly) better outcome for students in comparison to major proposing DA. This also leads to a (weakly) higher number of major switches under student proposing DA.[13]

[11]For instance, the success rate of major switching at National Tsing Hua University, a selective college in Taiwan, was approximately 50% in 2020. The success rate at NTU in 2020 was approximately 44%.

[12]See Section 4.2 for the definitions of these properties.

[13]It is worth mentioning that this does not contradict the rural hospital theorem (Roth, 1986)

Additionally, we show that whenever a major begins to allow occupied seats to be available during the process, this (weakly) improves student outcomes. Further, while we find that majors can sometimes be assigned lower-ranked students when allowing occupied seats to be available, we show that majors will never be assigned fewer students as a result of allowing occupied seats to be available. Hence, if the primary concern for majors is the number of seats that are filled, in equilibrium, each major would like to allow all of their seats to be available for transfer students. Those features together offer a solid reason to replace the conventional major-switching mechanism with our proposed mechanisms.

By using the data provided by NTU for the years 2019 and 2020, we conduct a counterfactual analysis to demonstrate possible gains predicted by the theory under our proposed solutions. We start by considering a change from major proposing DA to student proposing DA under the case that only the vacant seats are used for major switching. Recall that the theory predicts that students would gain from such a change, and the number of switches would increase. However, we find that such a change does not have an impact as predicted mainly due to the short ranking lists submitted by the majors and students. In fact, the major proposing DA and student proposing DA select the same outcome for every case considered. Then, we consider the impact of majors allowing occupied seats to be available. We find that while this does have a positive impact on the success rate, a 2.4% and 1.4% increase in 2019 and 2020, respectively, the gains are limited by majors not ranking all of their acceptable applicants.[14] Hence, we next evaluate the impact of majors extending their rankings and ultimately find that when majors allow occupied seats to be available and rank all acceptable applicants, there are considerable potential gains to be achieved in the success rate for major switchers. Specifically, we find that in both 2019 and 2020, the success rate for major switchers could be increased by approximately 20%.

Finally, we analyze the implementation of our proposed changes at NTU in August 2022 and find that due to few majors ranking all acceptable students, the success rate increase is not as large as we would expect based on our counterfactual analysis for 2019 and 2020.

[14] Just 12 and 8 majors ranked more than their number of vacant seats in 2019 and 2020, respectively. In 2022, almost half of the majors (23) allow occupied seats to be available.

Our conversations with the administrators at NTU highlighted that such a behavior might be caused by the fear of ending a worse set of students, as has already been discussed in the literature (Roth and Sotomayor, 1990). We are curious if such a strategy is beneficial for majors. Hence, we conduct an equilibrium analysis to evaluate the incentives of majors. We find that it is an equilibrium outcome each year for all majors to allow occupied seats to be available and that majors are never assigned fewer students due to allowing occupied seats to be available. Further, we find just one case where a major would have been assigned students of lower preference due to extending their ranking and that majors are never assigned fewer students when they extend their ranking.[15]

It has been shown in the literature that students with a better major fit have higher academic performance (Porter and Umbach, 2006; Wiswall and Zafar, 2015) and that major switchers have higher graduation rates (Venit, 2016). Hence, student learning outcomes and graduation rates might be improved by carefully designing the major switching procedure, which assigns students to better choices. While we do not have the data from NTU to evaluate if this is indeed the case, we consider this via empirical analysis provided in Appendix A using data from another selective college in Taiwan. There, we find that the higher a major switcher ranked their new major in their preferences in the major switching procedure, the higher their grades are in their new major going forward. This result can be interpreted as follows: If students switch to a more preferred major, then their academic performance is better.

The rest of this paper is organized as follows. Section 2 discusses the related literature. Section 3 describes the college admissions and major switching in Taiwan. Sections 4 and 5 introduce the model and the new mechanism, respectively. Section 6 discusses the implementation of the new mechanisms at NTU. Section 7 concludes.

[15] In Appendix C, we show that majors will never be assigned fewer students as a result of extending their rankings.

2 Related Literature

This paper contributes to the following strands of literature: matching with an initial assignment, matching under seat-specific priorities, comparative statics under matching mechanisms, strategic behaviors in matching markets, the effects of major switching, and the design of matching markets in practice. Below, we provide a literature review on these strands by discussing the most relevant papers.

Matching with an initial assignment:

In some matching markets, given the initial assignments, rematching is demanded afterward. Such matching markets include, but are not limited to, on campus-housing (Abdulkadiroğlu and Sönmez, 1999; Guillen and Kesten, 2012) student and employee exchange programs (Dur and Ünver, 2019), employee reassignment programs (Combe, Tercieux and Terrier, 2022), and major switching programs at colleges. To the best of our knowledge, major switching programs have not been studied in the market design literature.

The on-campus housing problem includes both existing tenants, i.e., upper-level students, and newcomers, i.e., freshman students, where some housing is initially vacant and some housing is initially occupied. In practice, many colleges implement housing assignments through a two-step procedure. First, existing students decide whether or not to participate in the housing assignment. Houses initially occupied by existing tenants who have decided to participate are added to the pool of available houses. Then, all participants are assigned to the available housing through a lottery. Notice that, it is possible that some of the existing tenants may end up with housing of a lower preference. Abdulkadiroğlu and Sönmez (1999) state this problem and suggest a top trading cycles (TTC) (Shapley and Scarf, 1974) based mechanism to assign students to houses. In their mechanism, existing tenants are given exclusive right to their current houses, and the priority order for the rest of the houses and students are determined through a random lottery. Guillen and Kesten (2012) provide examples from practice for the implementation of on-campus housing. They formalize the mechanism used by the Massachusetts Institute of Technology (MIT) as a version of student-proposing DA in which the highest priority at each house is given to its current occupant. Different from these papers, we study a many-to-one assignment problem in which the priority ranking of the students is determined by the preferences and rules of majors. Moreover, we allow majors to decide whether the occupied seats are able to be used for major switching and this allows us to have a more general framework.

Dur and Ünver (2019) study the tuition and worker exchange programs, which include initial assignments at each college or firm. Further, such programs require that exchanges need to occur in a balanced manner. That is, each student (employee) who is transferred to another college (firm) must be replaced by another student (employee) at the initial college (firm). To satisfy such a requirement and maintain an efficient outcome, Dur and Ünver (2019) propose a modification of the TTC. In our problem, maintaining a balance between incoming and outgoing students is not required as we can allow a student to switch her major without her initial seat being filled by another student because our problem includes initially vacant seats. Further, using a TTC-based mechanism would place less emphasis on the priority rankings of majors than might be desirable for our problem. Moreover, we allow majors to have initially vacant seats.

Combe, Tercieux, and Terrier (2022) study the centralized teacher assignment procedure in France.[16] The teacher assignment process includes teachers who have an existing position and wish to be reassigned. Further, tenured teachers have the right to keep their current position if they do not get assigned to another position of higher preference. This is similar to our problem, as students have a right to keep their current major if they are unable to switch. In their model, they do not include vacant seats. Hence, teacher assignment necessitates balancedness between the incoming and outgoing teachers for each school. Combe, Tercieux, and Terrier (2022) show that the existing DA-based mechanism fails to be fair and efficient for both teachers and schools. Instead, they propose a TTC-based mechanism by giving the foremost importance to strategy-proofness and efficiency. Differently, by following the minimalist market design paradigm and considering the fact that respecting major

[16] Similarly, teachers are (re)assigned to public schools in Turkey in a centralized manner (Dur and Kesten, 2019).

priorities is important in our context, our proposal is based on DA by including seat-specific priorities into account. Combe, Dur, Tercieux, Terrier, and Ünver (2022) consider a centralized teacher assignment by including vacant positions and unassigned teachers, but with a different objective for schools than we consider for majors in this paper. Specifically, they aim to improve the matching based on types of teachers assigned to schools, whereas we consider the actual priorities of majors.

Matching under seat-specific priorities:

In the design of our proposed mechanisms, we utilize seat-specific priorities introduced by Kominers and Sönmez (2016). Seat-specific priorities have also been used in modeling affirmative action policies in the school choice and college admissions context (Dur et al., 2018; Dur et al., 2020; Aygün and Bó, 2021). Further, pandemic resource allocations (Pathak et al., 2020) have utilized seat-specific priorities to balance competing ethical objectives. In all these papers, different seats at a given institution may have different priority orders over the individuals. As a result, the sequential order in which the seats are filled might have an impact on the outcome. However, in our case, we show that the processing order does not affect the outcome. We mainly use seat-specific priorities in order to allow majors to be free to choose whether their occupied seats are used in major switching or not while still allowing the current students to be assigned to those seats.

Comparative statics under matching mechanisms:

Within the literature on the theory of stable matchings, there exists a classic result referred to as the entry comparative static. That is, under DA, adding a new agent (or a new position/seat) to one side of the market makes the other side of the market (weakly) better off. Further, Kojima and Manea (2010) characterize DA using resource monotonicity. Some examples of similar results include, but are not limited to, additional vacancies in senior-level labor markets (Blum et al, 1997) and additional agents in the roommate market (Birò et al., 2008). Moreover, Balinski and Sönmez (1999) study the effects of the improvements on student test scores under DA-based mechanisms.

Kominers (2020) shows that the entry comparative static extends to matching markets with seat-specific priorities. As an example, Kominers shows that extending capacity at a firm with seat-specific priorities (weakly) improves the outcome for all employees. In doing so, Kominers treats adding a new seat as moving some students above the unassigned option for a slot that previously was included in the problem, but had no acceptable employees. These results are similar to our comparative static analysis as we also consider the impact of extending the set of acceptable agents for some seats. However, in our analysis, all seats initially have some acceptable students (at the minimum, students initially assigned to the major are acceptable), and we evaluate the impact of increasing the number of acceptable students for such seats. Further, we consider the application under both major and student proposing DA, whereas Kominers only considers employee (student) proposing DA. Moreover, in addition to the effect of such changes on students' welfare, we focus on the effect of such changes on the number of students assigned to the majors.

Konishi and Ünver (2006) study the hospital-intern problem and show that anytime a hospital reduces capacity, interns are not made better off under both intern and hospital proposing DA. Although their results have similarities with our students' welfare results, instead of reducing capacity at some majors, we focus on shrinking or enlarging the set of acceptable students for certain seats.

Strategic behavior in college assignment:

Within the major switching framework we consider, majors may have some incentive to behave strategically. Sönmez (1997) considers the manipulation in the hospital-intern problem and shows that there can be incentives for hospitals to reduce their capacity. Abdulkadiroğlu et al. (2005) document a case where schools in New York City withheld the capacity to match with students they preferred. Similarly, in our problem, there may be cases where majors could benefit from restricting the seats available to outside students to just those that are initially vacant acting as a form of capacity manipulation.[17] Konishi and Ünver (2006) study the hospital-intern problem further as a game of capacity manipulation. They find

[17]See Examples 2 and 3.

that there may not be any pure strategy equilibrium of capacity reporting in this game.[18] Additionally, Romm (2014) shows that hospitals can in some cases be certain to get a better outcome by reporting a lower capacity in the hospital-intern problem. Through our counterfactual analysis, we show that allowing all occupied seats for major switching constitutes an equilibrium.

Konishi and Ünver (2006) find that if every hospital prefers more interns to less, then truthful capacity revelation is a weakly dominant strategy for hospitals under intern-proposing DA and is an equilibrium outcome under hospital-proposing DA. Further, they find that if hospitals all have the same preferences for interns, then truthful capacity revelation is also a weakly dominant strategy under both hospital-proposing DA and intern-proposing DA. These results have important ties to our problem, as it typically would be the case that majors prefer more students enrolled to less and it could be the case that majors rank students the same if they all only consider student performance (i.e., GPA) in ranking students. Our theoretical results show similar findings when we consider a game for majors under both student proposing and major proposing DA in which the strategies are allowing all occupied seats for major switching or not. It is worth emphasizing that, a major's capacity is fixed in our analysis. Allowing occupied seats for major switching corresponds to enlarging the set of acceptable students for those seats. This makes our problem different from the rest of the papers discussed here.

Furthermore, Kojima and Pathak (2009) find that the incentive to manipulate DA via capacities goes to zero when markets are large, and Roth and Peranson (1999) find that very few hospitals could have benefited from manipulating capacities in the National Resident Matching Program. Kesten (2012) studies DA in the school choice problem and finds that DA is vulnerable to manipulation via capacity. Further, Kesten finds that DA is immune to capacity manipulation if and only if priority structures satisfy an acyclicity condition proposed by Ergin (2002).

In this paper, we mainly focus on manipulation by allowing the set of acceptable students

[18]Kojima (2006) shows that there does exist a mixed-strategy Nash Equilibrium in such a game and such reporting is preferred by hospitals to truth-telling.

for the occupied seats which resembles manipulation via capacities. The major switching problem focuses on students' choices within a college. In addition to such strategic behaviors, the literature focuses on other types of manipulations. For example, some colleges may choose the same entrance examination date (Avery, Lee, and Roth, 2014; Chen and Kao, 2014; Kao and Lin, 2017; Chen, Chen, and Kao, 2018) or provide the early decision program (Avery, Fairbanks, and Zeckhauser, 2003; Lee, 2009; Avery and Levin, 2010; Kim, 2010; Chen and Kao, 2023) to construct an applicant pool where the number of desired students increases under these strategies. Moreover, colleges may adjust admission standards as a tool to resemble that of a market-clearing price in competition (Chade, Lewis, and Smith, 2014) or to increase their yield rate (Che and Koh, 2016).

Effects of major switching:

Although we do not provide any empirical analysis of the effects of major switching programs, our market design approach will increase the number of students benefiting from these programs. As a result, any effect of major switching can be multiplied by using our proposed mechanism. Here, we discuss some of the papers studying the effects of major switching programs.

The choice of a major of study is one of the most important decisions that a student will make. However, students may not always know which career and major is best suited for them at the time of entry into college (Orndorff and Herr, 1996; Wiswall and Zafar, 2015; Liu et al., 2021). This uncertainty can be costly, Fouarge and Heß (2023) find that students who select majors that do not match their occupational preferences before enrolling in university are more likely to drop out and the primary reasons are indecisiveness and preference changes. When this is the case, major switching can play a role in improving the fit between students and their majors.

There is some literature evaluating the importance of student and major fit as well as the impact of major switching on student outcomes. Some of this research finds that there can be negative consequences of major switching as doing so can delay or impede college completion when credits earned in the original field do not count toward the new major's degree requirements (Fink et al. 2018; Zeidenberg, 2012). Additionally, Oliveira et al. (2022) find that in an environment where major switching is not allowed (Brazil), students who benefit from affirmative action (i.e., are placed into a major they would not have if not for the policy) do not have higher dropout rates than in the United States where students are allowed to switch majors and that these students catch up to their peers in their final years of study if they do not drop out due to early struggles.

On the other hand, Feldman et al. (1999) and Trapmann et al. (2007) find that a better academic match can lead to a higher probability of completion, and Foraker (2012) finds this to be especially true for students who switch majors within the first two years of study. Further, Liu et al. (2021), Venit (2016), and Yue and Fu (2017) find that major switchers have higher graduation rates in comparison to those who persist in their initial major[19] and Venit (2016) finds that time taken to graduate does not increase for those who switch prior to the third year. Additionally, students with a higher interest in their major or a better major fit have increased motivation and self-efficacy, thereby leading to higher major persistence and academic achievement (Porter and Umbach, 2006; Wiswall and Zafar, 2015). It follows that major switching, by improving the student-major fit, should have positive effects on student outcomes.

Meyer et al. (2021) find that person-major fit is one of the primary aspects that drive major switching along with individual achievement and social expectations. Further, Astorne-Figari and Speer (2019) find that major switchers tend to move in the direction of fields that "look like them", i.e., females switch to female-heavy majors, and so on, suggesting that major switching may sometimes be utilized to help students match with their preferred peer group as well as a preferred area of study. Finally, when affirmative action is utilized in college admissions, it is possible that it could lead to some mismatch. In such a case, major switching could increase the realized benefits of affirmative action via improved fit for beneficiaries.

Design of matching markets in practice:

[19] Venit (2016) finds that the graduation rate for major switchers is 83% compared to 79% for those who persist in their initially chosen major

As mentioned in the introduction, by using our initial theoretical and empirical analysis, we were fortunate to play a role in the adoption of a new mechanism in practice that provides improvements for students. There are other key papers in matching theory that have helped policymakers to improve their processes. For example, in school choice, the seminal paper of Abdulkadiroğlu and Sönmez (2003) played an important role for many cities in the US to change their deficient mechanisms with student proposing DA. Roth et al.(2004) were the first to propose a market design approach to the organ exchange, and they played a key role in the improvement and implementation of the centralized organ exchange registries. Sönmez and Switzer (2013) consider the cadet-to-branch assignment in a paper that played a role in the redesign of branching for the US Army. Pathak et al. (2020) played a role in assisting policymakers in the allocation of medical resources in response to the COVID-19 pandemic.

3 College Admissions and Major Switching in Taiwan

In this section, we first provide general information about college admissions in Taiwan. Then, we discuss the trend in the registration ratio and its effects on major-student mismatches and dropouts. Finally, we explain the major switching procedure at NTU, which represents major switching procedures in Taiwanese colleges.

3.1 College Admissions

There are two main channels of college admissions in Taiwan: the examination and application channels.[20,21] On the examination channel, students have to take a standardized test implemented by the college entrance examination center (CEEC) and submit their rank-

[20] As in Li, Lee, and Lien (2016) and Luoh (2018), we do not discuss the mechanisms for vocational colleges.

[21] In addition to these two main channels, students can be admitted by a centralized college admissions process, namely, the *multi-star recommendation*, in which each high school can recommend at most two qualified students to apply for a group of related majors within a college.

ordered preference lists of college-major pairs.[22] Then, according to the test scores and the preference lists, the University Assignment Council (UAC), a centralized clearing house in college admissions, will match students and majors via the student proposing DA algorithm.[23] Each student can apply to at most 100 college-major pairs through this channel.

The application channel typically consists of three steps. In the first step, colleges screen students based on their application documents as well as the scores from another CEEC standardized test. In the second step, the qualified students have to attend the interviews or the individual exams implemented by the colleges. In the third step, acceptable students have to submit their rank-ordered preference lists of college-major pairs, and a centralized clearing house will match students and majors via the student proposing DA algorithm.[24] Each student can apply to at most six college-major pairs through this channel.

Students are admitted to the colleges via first the application channel and then the examination channel. Once a student accepts an admission offer in the application channel, she cannot apply through the examination channel.

3.2 Admission Quota and Registration Ratio

For every major, the Taiwanese government regulates the admission quota for local students. Usually, the maximal enrollment in a class for a new major is 50 local students.[25] One major at a college may have many classes, but the total enrollment of local students of that college is fixed over time. If a college desires to create a new major or to add classes to a major, the college should reallocate the seats among majors and submit the plan to the Ministry of Education (MOE) for approval.[26] According to the admission quota for local students, the

[22] In Taiwan, a major is called a department or a program, and a major admits first-year students directly in college admissions. Following the convention in the literature, we use majors to indicate departments or programs and colleges to indicate universities in this paper.

[23] The test score is a weighted score. Majors can choose different weights over subjects. Thus, a student can have different scores for different weights. The description of the mechanism can be found at the website of UAC, https://www2.uac.edu.tw/.

[24] The official illustration of the mechanism can be found at https://www.cac.edu.tw/.

[25] Some old majors may have 60 local students in a class since they are subject to an old quota regulation.

[26] The regulation on the admission quota can be found at https://jongliang.stust.edu.tw.

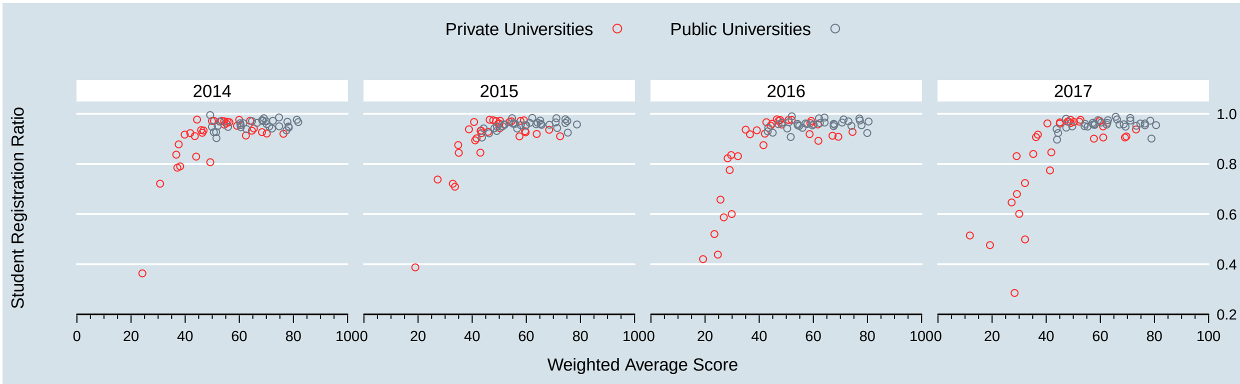

Figure 1: Student registration ratios and weighted average scores of colleges in Taiwan

MOE has reported the annual student registration ratios for all majors and colleges since 2014. For instance, if a college has an admission quota of 1,000 local students in total and eventually 600 local students attend the college, the college's student registration ratio is 60% in that year.[27]

Figure 1 depicts the student registration ratios and the student-weighted average scores for colleges in Taiwan between 2014 and 2017. Here, a weighted average score is the subject-weighted average of a major's cutoff score from the CEEC standardized test for local students. [28] In order to calculate the college-level weighted average scores in Figure 1, we collect the data of all majors' subject weights and their cutoff scores from the UAC during this period. We then compute the mean of the weighted average scores among all majors within each college for each year.[29]

The weighted average scores can serve as a proxy for a college's prestige from the perspective of students. As shown in Figure 1, public institutions on average have a higher score in comparison to private institutions in Taiwan. Moreover, the student registration ratios and the weighted average scores are positively correlated, but the pattern displays a nonlinear relationship.

[27]The student registration ratio can be downloaded at https://udb.moe.edu.tw. Note that after 2018, the student registration ratio definition includes overseas students. Therefore, the actual ratio can be larger than 100%, but the MOE will only report 100% in this case.

[28]For example, a major sets the relative weights of 1.5, 1, and 1.5 in Chinese, English, and mathematics, respectively. After the test, the CEEC will announce the weighted sum of scores for the lowest-scoring student enrolled in the major. If the weighted sum is 320, then the weighted average score is 80 (=320/(1.5+1+1.5)).

[29]The raw data can be downloaded at https://www.uac.edu.tw/downloads.htm.

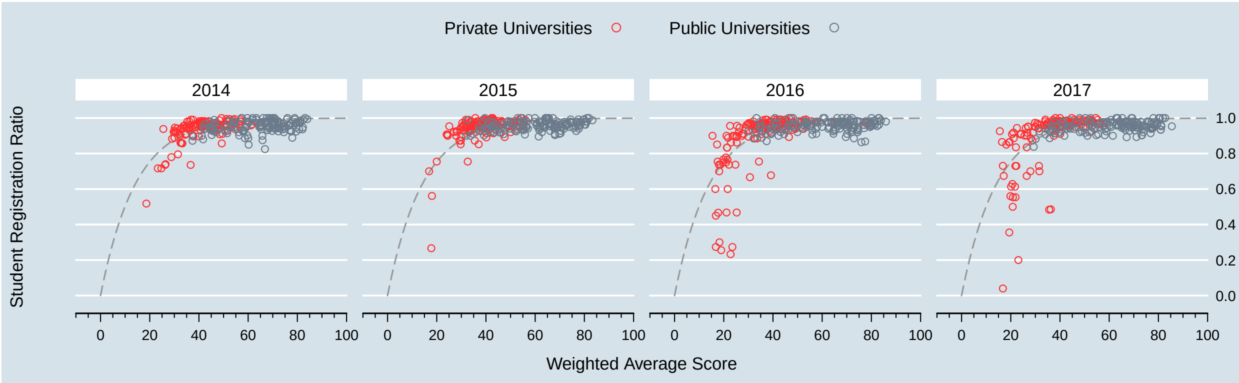

Figure 2: Registration ratios and weighted average scores in the engineering field

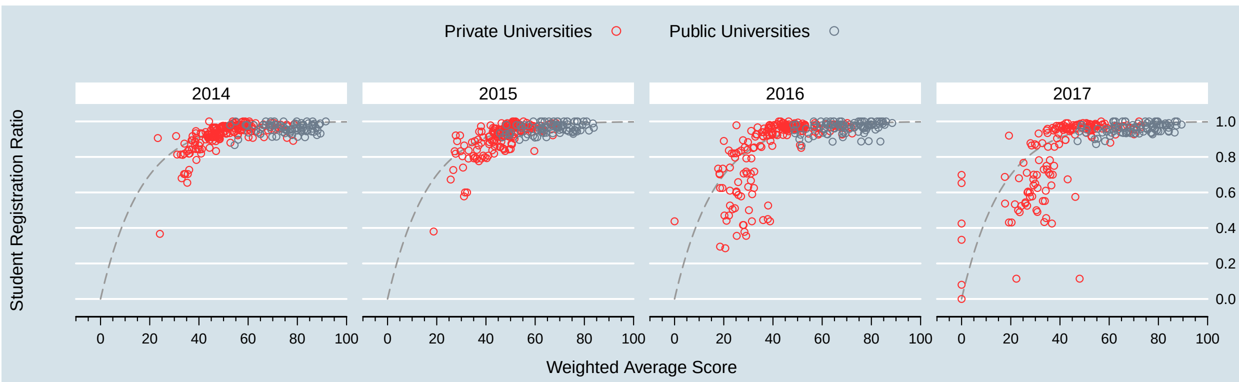

Figure 3: The tail-drop pattern in the business and management field

Since the combination of majors can be quite different between colleges, the college-level weighted average score may be influenced by the combination of majors among colleges. For example, majors in medicine schools usually have the highest weighted average scores in Taiwan. To overcome this problem, we can compare majors in the same field. In Figure 2, we calculate the major-level weighted average scores and the major registration ratios in the engineering field as an example. In addition, we use an exponential function of $y = 1 - e^{-\lambda x}$ to fit the data. Here, y, x, and λ are the major registration ratio, the major-level weighted average score, and the fitted parameter, respectively. In 2014, the exponential model with $\lambda = 0.071$ fits the data well. However, after 2015, we see many majors with lower weighted average scores drop more quickly than the model prediction in the registration ratio. We call this phenomenon the *tail-drop pattern*. As shown in Figure 3, we can observe the tail-drop pattern in other popular fields, such as majors in the business and management fields.

The tail-drop pattern implies that some students' choices may depend on a college's prestige rather than their true preference over majors. This causes a mismatch between students and majors in college admissions. In fact, a survey by Lin (2010) shows that, when applying to college-major pairs, some students propose their ordered lists based on the prestige of colleges rather than their true preference over majors. As a result, those students may end up in a major they do not like and are more likely to drop out. According to a government survey, 24.6% of Taiwanese students dropped out of college in 2019 due to the mismatch between students and majors (Ho and Chen, 2020). As mentioned in the introduction, the dropout ratio can be lowered through the major switching procedure. Next, we explain the current major switching procedure in Taiwan by focusing on the NTU mechanism.

3.3 Major Switching

By the end of the first semester, some students drop out of college and leave empty seats in their majors. The demand for major switching occurs because those empty seats may be preferred by the remaining students in other majors. Most majors set their own standards to screen major-switching applicants. Usually, the number of major switches is limited to the initially vacant seats across majors within the college. We take the NTU mechanism as our motivating example. The steps of this procedure can be summarized as follows:

- Each major reports the number of vacant seats and this information is shared with students.
- Each student can apply to at most two majors with an ordered preference ranking.
- Each applicant is evaluated by major(s), and the evaluation outcome can be "on the primary list", "on the secondary list" or "rejected".
- For each major, the length of the primary list cannot be larger than its quota, i.e., the number of vacant seats. Majors do not rank the students on the primary list; however, majors rank (strictly) any students on the secondary list. The major determines the length of the secondary list.

Then, all the information collected is taken into the centralized matching mechanism, which we call **NTU mechanism**. The NTU mechanism finds its outcome by following the steps described below in a sequence.

NTU mechanism:

- Step (a) A student is assigned to her first choice if she is on the primary list of that major.
- Step (b) A student is tentatively assigned to her second choice if (1) she is not assigned to her first choice and (2) she is on the primary list of the second-choice major.
- Step (c) For a major having vacant seats after Steps (a) and (b), the students on its secondary list who take it as the first or second choice are sequentially (by the rankings) offered a vacant seat at this major. Each student keeps the most preferred offer she received (among the new offers and the ones received in Step (a) and Step (b)) and rejects the rest. Step (c) is repeated until no offer is rejected by a student. In particular,

Any student who is not assigned through this procedure to a new major stays in her current major.

We would like to provide a further explanation for Step (c) of the NTU mechanism.

As shown in Proposition 1, the NTU mechanism is equivalent to major proposing DA. A student tentatively assigned to her second choice in Step (b) can be assigned to her first choice in Step (c); if this occurs, she will leave an empty seat in her second-choice major and the vacant seat can be used in the next repetition of Step (c). Similarly, a student tentatively assigned to her second choice in Step (c) can be assigned to her first choice in a future repetition, and she leaves a vacant seat in her second-choice major that can be used in the next repetition of Step (c).

In practice, many applicants are rejected by popular majors, and hence the percentage of applicants who successfully switch to a new major (the success rate) is usually low. For example, in 2020, there were 634 applicants at NTU and the success rate was 43.62%. When we look at the 2020 major-switching data at NTU, we see room for improvement. Since each applicant at most could apply to two majors, there were 865 applications in total. The total number of vacant seats (major-switching quota) was 587. Only 277 students successfully switched to a new major, leaving 310 seats unfilled. Some details are summarized in Table 1.

We take the Economics major (indexed by major 20) as an example to illustrate the problem. In 2020, the Economics major is the most popular major at NTU for potential major switchers. As we can see in Table 1, the number of applications to Economics is 118.[30] However, since its major-switching quota was only 24, the success rate was approximately 20%. In fact, there were twelve students from the Economics major who applied to other majors, and five of them succeeded in changing their major in 2020. Their departure left 5 empty seats in the major. If we can incorporate these empty seats in the matching process, then the success rate in the economic major in 2020 can be increased to 24.57% ($= (24+5)/118$).

A higher success rate is desirable for students who want to change their major. Moreover, the evidence from NTU indicates that major switchers tend to have a higher GPA than their peers in the new major. To illustrate the evidence, we collect the data of GPA for 2017-2019 graduates at NTU to examine student performance after major switching. As shown in Figure 4, the distributions of GPA are quite different between majors, and hence the data needs to be standardized for comparison. We calculate students' percentile rank (PR) of GPA in their major. For example, PR = 50 means that the student's GPA is the median of the major, and PR = 100 indicates the best GPA. According to this measure, we summarize student performance after major switching in Figure 5. The Wilcoxon rank sum test suggests that major switchers tend to have a higher PR than their peers in the new major.[31] In addition, Figure 6 summarizes student performance after major switching by major. The first row of the figure depicts the distributions of the PR in majors that have more major-switching students. In most cases, major switchers in those popular majors have a higher mean in PR during this period.[32] Thus, we would like to increase the success rate

[30]Moreover, 89 of those 118 students ranked the Economics major as the first choice.

[31]With p-value $< 1\%$ to reject the null hypothesis of equal PR between two groups each year.

[32]Moreover, since we do not have detailed data about the NTU student grades before and after major

Major index	Major (Department)	Enrollment number	Switching quota	Applications to the major	Success rate (%)
1	Department of Chinese Literature	60	7	8	88
2	Program in Chinese Literature for International Students	11	5	4	50
3	Department of Foreign Languages and Literatures	121	5	18	28
4	Department of History	55	11	8	75
5	Department of Philosophy	46	9	10	30
6	Department of Anthropology	43	8	5	20
7	Department of Library and Information Science	52	10	1	100
8	Department of Japanese Language and Literature	58	11	6	17
9	Department of Drama and Theatre	35	7	5	40
10	Department of Mathematics	47	8	7	14
11	Department of Physics	65	11	4	75
12	Department of Chemistry	62	7	9	56
13	Department of Geosciences	42	9	2	50
14	Department of Psychology	67	14	29	48
15	Department of Geography	40	7	7	86
16	Department of Atmospheric Sciences	40	7	1	0
17	Department of Political Science-Political Theory Major	54	11	19	21
18	Department of Political Science-International Relations Major	60	11	35	31
19	Department of Political Science-Public Administration Major	55	11	11	36
20	Department of Economics	128	24	118	20
21	Department of Sociology	51	9	7	57
22	Department of Social Work	53	9	4	50
23	Department of Medicine	148	2	5	0
24	Department of Dentistry	32	1	2	50
25	Department of Pharmacy	49	8	18	44
26	Department of Clinical Laboratory Sciences and Medical Biotechnology	45	8	9	22
27	Department of Nursing	49	5	4	25
28	Department of Physical Therapy	42	9	6	50
29	Department of Occupational Therapy	44	6	10	40
30	Department of Civil Engineering	115	22	13	38
31	Department of Mechanical Engineering	150	20	4	50
32	Department of Chemical Engineering	112	15	6	67
33	Department of Engineering Science and Ocean Engineering	47	10	11	9
34	Department of Materials Science and Engineering	53	2	1	100
35	Department of Biomedical Engineering	24	4	0	–
36	Department of Agronomy	41	8	0	–
37	Department of Bioenvironmental Systems Engineering	55	5	0	–
38	Department of Agricultural Chemistry	44	10	3	67
39	Department of Forestry and Resource Conservation	73	17	4	75
40	Department of Animal Science and Technology	33	8	3	33
41	Department of Agricultural Economics	58	10	5	40
42	Department of Horticultural Science	57	12	6	50
43	Department of Veterinary Medicine	68	13	29	28
44	Department of Bio-Industry Communication and Development	57	10	22	45
45	Department of Bio-industrial Mechatronics Engineering	44	9	2	100
46	Department of Entomology	32	8	0	–
47	Department of Plant Pathology and Microbiology	32	8	2	100
48	Department of Business Administration-Business Administration Major	64	9	29	31
49	Department of Business Administration-Technology Management Major	41	5	18	28
50	Department of Accounting	121	22	38	58
51	Department of Finance	122	10	10	50
52	Department of International Business	96	10	1	0
53	Department of Information Management	56	5	27	19
54	Department of Public Health	44	5	4	25
55	Department of Electrical Engineering	181	20	50	40
56	Department of Computer Science and Information Engineering	132	7	42	17
57	Department of Law-Legal Science	63	11	61	8
58	Department of Law-Judicial Administration	59	11	59	19
59	Department of Law-Financial Law	63	11	39	28
60	Department of Life Science	64	10	3	100
61	Department of Biochemical Science and Technology	49	10	1	100
62	Bachelor Program of International Sports Affairs	0	0	0	–
63	Trans-disciplinary Bachelor Degree Program	0	0	0	–
Total		3904	587	865	43.62

Note: Majors 62 and 63 are new programs in which there is no quota for major switching before 2022. Enrollment number is the number of first-year students in the department in 2020. The success rate for a major is defined as the percentage of applicants of that major who successfully switch to it. Note that the success rate is not necessarily 100%, even if the number of applicants to the major is smaller than the major-switching quota. This is because some of the applicants may be rejected by the major or they have been assigned to another major.

Table 1: Major index, major-switching quota, and success rates at NTU in 2020

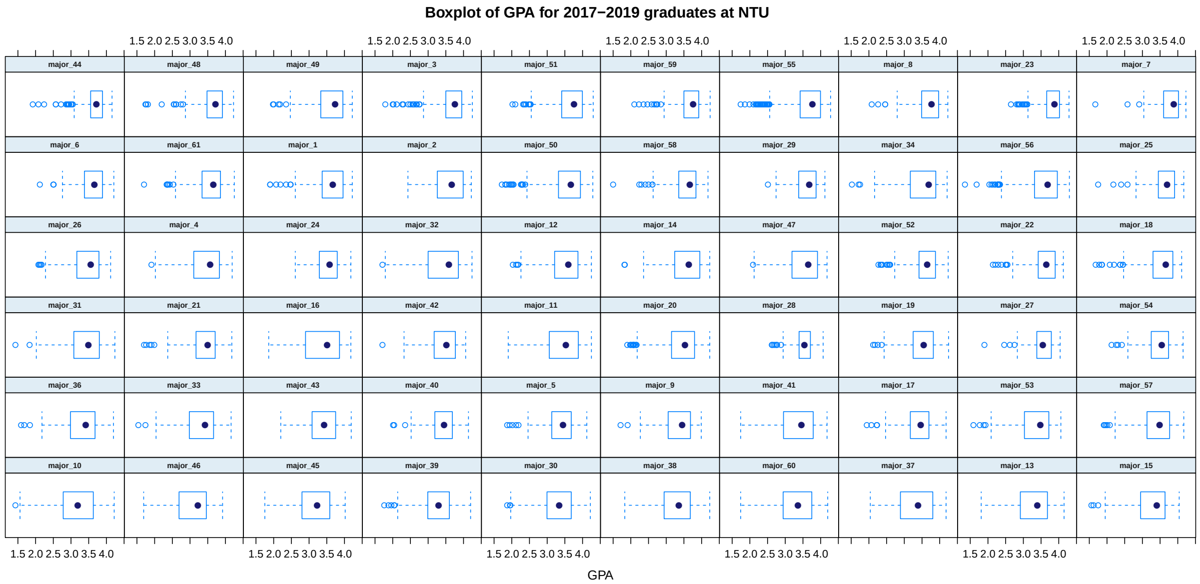

Figure 4: GPA for 2017-2019 graduates at NTU

by introducing a new mechanism.

4 Model

In this section, we first present the basic elements of the major switching problem. Then, we define a general choice function for majors and introduce the desired properties in this setting.

4.1 Basics

We consider a matching market composed of a set of majors, denoted by M, and a set of students, denoted by I.[33] Each student i is currently enrolled in a major. Let c_i denote the current major of student i. Let I_m be the set of students currently enrolled in major m.

switching, we collect the data from another selective university in Taiwan and do the analysis for student-grade changes in Appendix A. The result indicates that the student's grades before major switching and the order of matched choice are the significant variables contributing to the student's grades after major switching.

[33]Here, I excludes the students who prefer to stay at their current majors.

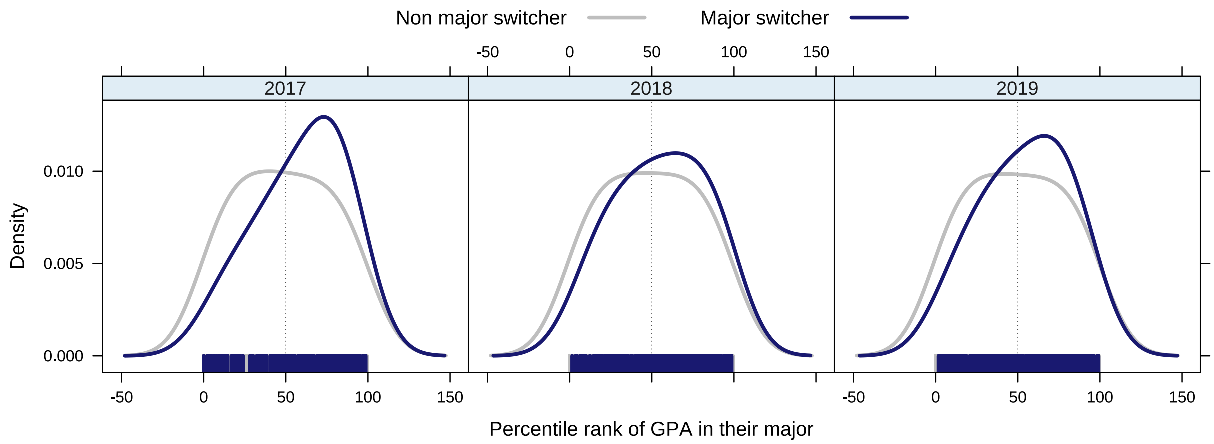

Figure 5: Student performance after major switching

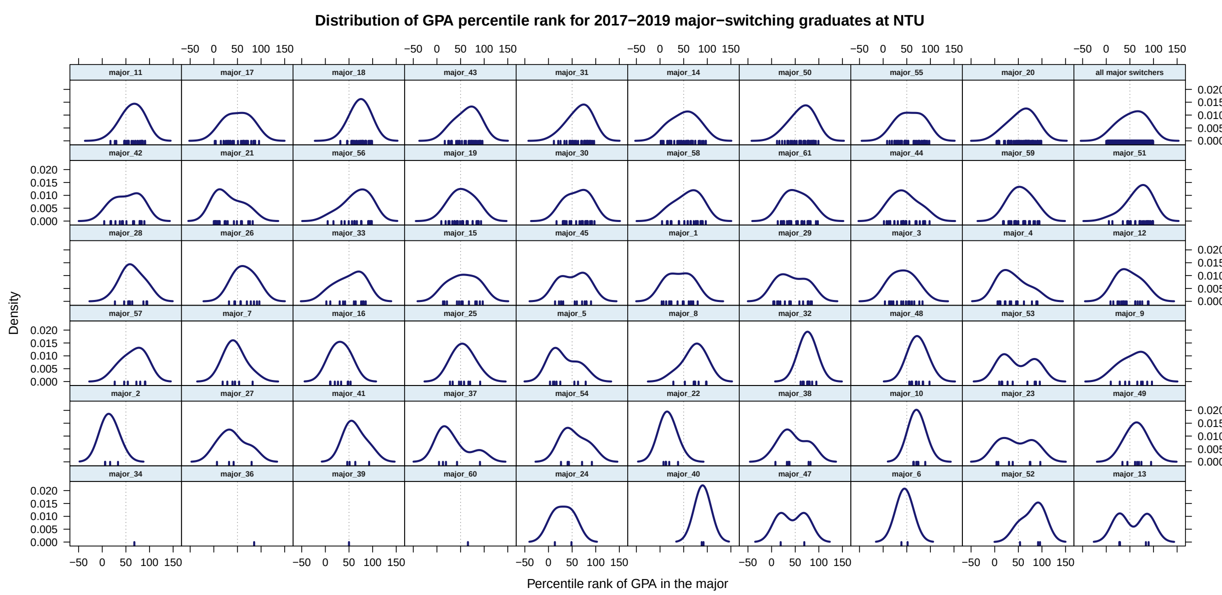

Figure 6: Student performance after major switching (by major)

That is, $I_m = \{i \in I \ : \ c_i = m\}$. Each student has a strict preference order over majors. Let P_i denote the strict preference order of student i over majors. Let R_i be the associated at least as good as relation with P_i. That is, mR_im' implies either mP_im' or $m = m'$. Each major has a priority order over students. Let $\succsim_m$ denote the priority order of major m over students. Here, $\succsim_m$ can be a weak priority order.[34] Let $\succ_m$ and $\sim_m$ be the asymmetric and symmetric parts of $\succsim_m$, respectively. Let q_m be the capacity of major m.[35] Since all students in I_m are currently enrolled to major m, we assume $q_m \geq |I_m|$. Let S_m be the set of seats at major m and $|S_m| = q_m$. The seats at major m can be initially occupied or vacant. Let S^o_m and S^v_m be the set of initially occupied and vacant seats at major m, respectively. Let $q^o_m = |S^o_m|$ and $q^v_m = |S^v_m|$ for all $m \in M$.

Following our main application, we assume that if a major m is indifferent between i and j, then they are ranked among the top q_m under $\succsim_m$

We represent a problem with $(M, I, P, \succsim, q)$. Notice that, the current majors of each student and the number of vacant and occupied seats at each major are embedded in $(M, I, P, \succsim, q)$. Whenever it is convenient, we use P instead of $(M, I, P, \succsim, q)$.

4.2 Choice Function and Desired Properties

Since a current student at some major m might be ranked below an applicant from another major under the priority order, deciding the selected students just based on the priority order might lead to some problems. That is, a student could be replaced by another student and be left without a major. Moreover, allowing the majors to decide to use their occupied seats can lead us to use more complex selection procedures. To this end, we define a general choice function for the majors in this section. In the following section, we provide the description of the exact choice function we would like to use for this setup.

[34] In the current practice, majors do not report ranking over their current students. Even though the relative ranking over the current students does not affect the outcome of the mechanisms in our proposed classes, we allow majors to rank their current students.

[35] Here, we focus on the physical capacity of the major excluding the seats of students who do not participate in the major switching process.

Given a subset of students I', a **choice function** of major m, denoted with C_m, prescribes the selected students in I', i.e., $C_m(I') \subseteq I'$. Next, we define the properties of a choice function that are deemed desirable in the literature.

A choice function C_m satisfies **substitutes** condition if for any $I' \subset I$ and $i, j \notin I'$ the following holds

$$i \notin C_m(I' \cup \{i\}) \implies i \notin C_m(I' \cup \{i, j\}).$$

That is, under a substitutable choice function, a rejected student, from a given subset of students, will not be accepted when additional students are considered.

A choice function C_m satisfies **law of aggregate demand** condition if

$$I' \subset \bar{I} \implies |C_m(I')| \leq |C_m(\bar{I})|.$$

That is, when a choice function satisfies the law of aggregate demand, the number of selected students (weakly) increases when additional students are considered. Notice that, the law of aggregate demand does not imply a set inclusion relation.

A choice function C_m satisfies **irrelevance of rejected alternatives** condition if

$$i \notin C_m(\bar{I}) \implies C_m(\bar{I}) = C_m(\bar{I} \setminus \{i\}).$$

That is, when a choice function satisfies the irrelevance of rejected alternatives, removing any unselected student from consideration by a major does not impact the selection of students.

Now, we are ready to define a matching and a mechanism and their desired properties.

A **matching** $\mu : I \to M$ is a function such that $|\mu(i)| = 1$ and $|\mu^{-1}(m)| \leq q_m$ for all $i \in I$ and $m \in M$. With a slight abuse of notation, we use μ_i and μ_m instead of $\mu(i)$ and $\mu^{-1}(m)$, respectively. A **mechanism** ψ is a procedure that selects a matching for any problem. Let $\psi(P)$ be the matching selected by ψ for problem P.

A matching μ is **nonwasteful** if whenever a student is reassigned, the vacated seat is filled if desired by an acceptable student. That is, whenever there exists $i \in I_m \setminus \mu_m$, then it is not the case that (i) $|\mu_m^{-1}| < q_m$ and (ii) there exists $j \notin I_m$ such that $m \; P_j \; \mu_j$ and $j \succ_m \emptyset$. A matching μ is **individually rational for students** if no student is matched with a major worse than her current major, i.e., $\mu_i \; R_i \; c_i$ for all $i \in I$.

A matching μ is **stable** if (i) it is individually rational for students and $C_m(\mu^{-1}(m)) = \mu^{-1}(m)$ for all $m \in M$ and (ii) there does not exist a student-major pair (i, m) such that $m \; P_i \; \mu(i)$ and $i \in C_m(\mu^{-1}(m) \cup \{i\})$.

A matching μ **Pareto dominates** matching ν (for students) if $\mu_i \; R_i \; \nu_i$ for all $i \in I$ and $\mu_j \; P_j \; \nu_j$ for some $j \in I$.

A mechanism ψ is individually rational for students , and stable if $\psi(P)$ is individually rational for students, and stable for any problem P, respectively.

A mechanism ψ Pareto dominates mechanism ϕ (for students), if either $\psi(P) = \phi(P)$ or $\psi(P)$ Pareto dominates $\phi(P)$ for students for any problem P.

A mechanism ψ is **strategy-proof for students**, if there does not exist a problem P, a student i, and preferences P_i' such that $\psi(P_i', P_{-i}) \; P_i \; \psi(P)$. Here P_{-i} denotes truthful preferences for all students except i.

5 The Proposed Mechanisms and the Results

In this section, our goal is to introduce a class of mechanisms that are based on both the major and student proposing DA mechanisms. Our choice of introducing mechanisms based on DA is due to two points. Firstly, in Proposition 1 we show that the NTU mechanism is equivalent to major proposing DA. Following the minimalist paradigm of Sönmez (2023) we prefer to introduce mechanisms based on DA. Secondly, respecting majors' ranking is a desideratum in practice, and DA-based mechanisms do that. In each of our proposed classes of mechanisms, different sets of majors allow their occupied seats to be used by the students from other majors.

Before introducing our proposed class of mechanisms, we focus on the NTU mechanism. We show that the NTU mechanism is based on the major proposing DA mechanism where the capacity of each major m is q_m^v.

Proposition 1. *Let $(M, I, P, \succsim, q)$ be a problem and $\hat{P}_i$ be a preference order in which at most two majors other than c_i are ranked acceptable for all $i \in I$.*[36] *Let μ and ν be the outcome of the NTU mechanism and major proposing DA mechanism under $(M, I, P, \succsim, q)$ and $(M, I, P, \succsim, (q_m^v)_{m\in M})$, respectively. Then, for every $i \in I$ the following are true:*

- *if $\nu(i) \in M$, then $\nu(i) = \mu(i)$, and*
- *if $\nu(i) \notin M$, then $\mu(i) = c_i$.*

Proof. In this proof, we show that the main part of the NTU mechanism in which students are reassigned to a new major is equivalent to major proposing DA. Then, by assigning unmatched students under both mechanisms to their current majors, we achieve the stated result.

We first formally define major proposing DA (mpDA) as follows:

Major Proposing DA:

Step 1: Each major m proposes to the highest ranked students under $\succsim_m$ up to its capacity (q_m^v in this case). Each student i tentatively accepts the best acceptable offer she receives and rejects the rest.

In general,

Step $k > 1$: Each major m proposes to the highest ranked students under $\succsim_m$ who have not rejected m in a prior step up to its capacity (q_m^v in this case). Each student i tentatively accepts the best acceptable offer she receives and rejects the rest.

The mechanism terminates when no offer is rejected.

[36]Recall that, students do not rank their current major in the major-switching practice in NTU and they can rank at most two majors. See Section 3.3 for details.

Recall that, under the NTU mechanism, in the first step all students who rank among the number of vacant seats at their first choice are assigned. Then in the second step, students not assigned in the first step are tentatively assigned to their second choice if they rank among the number of vacant seats at their second choice. In the last step students are offered remaining unfilled seats sequentially following the majors' priority orders until no offer is rejected.

First, note that under mpDA whenever a student receives an offer from her top choice, she keeps it in each of the following steps and is assigned to her top choice. Moreover, a student ranked k^{th} by major m will receive an offer only if all students ranked higher (i.e., ranked l^{th} where $l < k$) have received an offer from m.

Now we are ready to show the outcome equivalence between mpDA and NTU. First of all, the major tentatively kept by a student in Step 1 of mpDA, say m, ranks her among the top q_m students under $\succsim_m$. As a result, m will be her permanent or temporary assignment at the end of Steps a and b under NTU. Moreover, if the major held at the end of Step 1 is the top choice as explained above, then that would be the permanent assignment under mpDA.

If a student receives a new offer in the second step of mpDA, then she receives the same offer in the first repetition of Step c of NTU. Then the major tentatively held by each student at the end of Step 2 of mpDA and the first repetition of NTU are the same. By the definition of both mechanisms, this observation holds for each of the following steps of mpDA and repetitions of NTU. As a result, students are assigned to the same majors, possibly their initial major, under NTU and mpDA. □

In Proposition 1, we show that the NTU mechanism and a two-step procedure in which vacant seats are first assigned via mpDA and then the unmatched students are assigned to their current majors are outcome equivalent. Next, we will introduce two classes of mechanisms which are based on student and major proposing DA. Each member of these classes of mechanisms selects its outcome utilizing a one-stage assignment procedure.

Under both classes of mechanisms, each major can allow either only initially vacant seats or all seats to be available for transfer students. To do this, we equip each major with a choice function. We let M^a and M^v be the sets of majors allowing all and only vacant seats to be available, respectively. In the construction of the choice function for both types of majors, we use seat-specific priorities (Kominers and Sönmez, 2016). In the rest of the paper, without loss of generality, we assume each major to have a strict priority order over students, i.e., $i \succsim_m i'$ if and only if $i \succ_m i'$ for all $i, i' \in I$ and $m \in M$.[37] We denote the priority order of seat $s \in S_m$ with $\succ_m^s$ and construct is as follows:

Construction of seat specific priority order:

- If $s \in S_m^v$, then
  - $i \succ_m^s j$ if and only if $i \succ_m j$ for any $i, j \notin I_m$ $(i, j \in I_m)$.
  - $\emptyset \succ_m^s i$ if either $\emptyset \succ_m i$ or $i \in I_m$.
- If $s \in S_m^o$ and $m \in M^a$, then
  - $i \succ_m^s j$ for any $i \in I_m$ and $j \notin I_m$,
  - $i \succ_m^s j$ if and only if $i \succ_m j$ for any $i, j \in I_m$ $(i, j \notin I_m)$.
  - $i \succ_m^s \emptyset$ if and only if $i \succ_m \emptyset$ for any $i \in I$.
- If $s \in S_m^o$ and $m \in M^v$, then
  - $i \succ_m^s j$ if and only if $i \succ_m j$ for any $i, j \in I_m$ $(i, j \notin I_m)$.
  - $i \succ_m^s \emptyset$ if and only if $i \in I_m$.

First notice that, for any major m, if a seat is vacant, then only the students of the other majors are acceptable for that seat, and the relative ranking of the acceptable student is consistent with the major m's ranking over the students. Moreover, for any major m, all occupied seats rank current students over the students of the other majors and the acceptability of other majors' students depends on whether the major m is in M^a or not. Let $\succ_m^S = (\succ_m^s)_{s \in S_m}$ and $\succ^S = (\succ_m^S)_{m \in M}$.

Next, we define a choice function for major $m \in M$ which calculates the selected students for each seat at m according to $\succ_m^S$ in a sequential manner following a precedence order (i.e., a

[37] Recall that in practice, a major m is indifferent between two students i and j only if both are ranked within the top q_m.

processing order) over the seats in S_m. Let $\rhd_m$ denote the precedence order that determines the sequence the seats are filled. Here, $s \rhd_m s'$ means seat s is filled before s', whenever possible. Let $S_m = \{s_1, \ldots, s_{|S_m|}\}$ and $s_k \rhd_m s_{k+1}$ for all $k \in \{1, \ldots, |S_m| - 1\}$. Given a set of students $I' \subseteq I$, we denote the chosen students by major m with $C_m(I'; \succ_m^S, \rhd_m)$ and it is calculated as follows[38]:

The choice function of major m:

Step 0: Set $C_m(I'; \succ_m^S, \rhd_m) = \emptyset$.

Step 1: If there is an acceptable student in $I' \setminus C_m(I'; \succ_m^S, \rhd_m)$ under $\succ_m^{s_1}$, then add the highest ranked one to $C_m(I'; \succ_m^S, \rhd_m)$ and continue with the next step. Otherwise, continue with the next step.

In general;

Step $k > 1$: If there is an acceptable student in $I' \setminus C_m(I'; \succ_m^S, \rhd_m)$ under $\succ_m^{s_k}$, then add the highest ranked one to $C_m(I'; \succ_m^S, \rhd_m)$ and continue with the next step. Otherwise, continue with the next step.

The procedure terminates at the end of Step $|S_m|$.

Here, our choice function processes the seats one by one according to the precedence order and selects the highest priority acceptable student for the corresponding seat. Since a seat may be considered by some applicants unacceptable, it is possible that some seats might be left unfilled.

Next, we show that our choice function satisfies the substitutes, the law of aggregate demand, and the irrelevance of rejected alternative conditions.

Proposition 2. *C_m satisfies substitutes, the law of aggregate demand, and the independence of rejected alternatives conditions.*

[38]Notice that, in addition to the set of students considered we add seat specific priorities and precedence order to the notation of the choice function. Later on, we explain the outcome of this choice function does not depend on the precedence order (see Proposition 3).

Proof. First, notice that the construction of $\succ_m^S$ depends on the actual priority order of major m, $\succ_m$, and whether m allows all or only vacant seats to be available for transfers. Hence, we consider an arbitrary $\succ_m$ and $\rhd_m$.

Let I' be the set of students considered. Then, independent of whether $m \in M^a$ or $m \in M^v$, all students in I'_m are selected by the choice function C_m and these students are assigned to only the seats in S^o_m. Suppose there exists $i \in I'$ such that $i \notin C_m(I'; \succ_m^S, \rhd_m)$. Then, we consider the following two cases:

Case 1: If $m \in M^a$, then there are $q_m - |I'_m|$ students in $I' \setminus I'_m$ who have higher priority than i for all seats.

Case 2: If $m \in M^v$, then there are $|S^v_m|$ students in $I' \setminus I'_m$ who have higher priority than i for all seats. And these students cannot be assigned to seats in S^o_m.

Hence, when we consider $I' \cup \{j\}$ for any $j \notin I'$ Case 1 and Case 2 continue to hold. The selection of i by the choice function C_m requires rejection of another student $k \in I' \setminus I'_m$ with higher priority under $\succ_m$. Hence, each seat in which students in $I' \setminus I'_m$ are acceptable ranks k over i. Then, the rejection of k contradicts the definition of the choice function. Hence C_m satisfies *substitutes* condition.

Also notice that, when we consider additional students, independent of the precedence order and priorities, the number of acceptable students for each step of the choice function weakly increases. As a result, the number of selected students increases as we consider a larger set of students. Hence, C_m satisfies *law of aggregate demand.*

Additionally, notice that in each Step k, any student added to $C_m(\bar{I}; \succ_m, \rhd_m)$ is the highest ranked student under $\succ_m^{s_k}$ within $\bar{I} \setminus C_m(\bar{I}; \succ_m^S, \rhd_m)$. Therefore, in any Step k, either i is the highest ranked acceptable student under $\succ_m^{s_k}$ within $\bar{I} \setminus C_m(\bar{I}; \succ_m^S, \rhd_m)$ or i is not added to $C_m(\bar{I}; \succ_m^S, \rhd_m)$ in that step. Hence, it follows that if $i \notin C_m(\bar{I}; \succ_m^S, \rhd_m)$, then i is never the highest ranked acceptable student remaining under $\succ_m^{s_k}$ for any Step k. Therefore, when $C_m(\bar{I} \setminus \{i\}; \succ_m^S, \rhd_m)$ is calculated, the procedure will terminate prior to any step where i would have been considered for addition to C_m. As a result, the same students will be considered for addition to C_m as were considered when i was included in the set of students. Therefore, if $i \notin C_m(\bar{I}; \succ_m^S, \rhd_m)$, then $C_m(\bar{I}; \succ_m^S, \rhd_m) = C_m(\bar{I} \setminus \{i\}; \succ_m^S, \rhd_m)$. Hence, C_m satisfies *irrelevance of rejected alternatives.* □

Notice that, the precedence order of a major m, $\rhd_m$, is not an integrant of the problem and it is only used in the calculation of the chosen students. One can wonder whether the choice of the precedence order plays a crucial role in the selection of the students based on the procedure defined above. In the following proposition, we show that for any precedence order, the set of selected students is the same under the choice function, C_m.

Proposition 3. *Let $I' \subseteq I$ and $\succ_m^S$ be the seat specific priorities.*[39] *Then, for any precedence orders $\rhd_m$ and $\rhd'_m$ we have $C_m(I'; \succ_m^S, \rhd_m) = C_m(I'; \succ_m^S, \rhd'_m)$.*

Proof. We show that for any precedence order, the outcome of the choice function can be found by first assigning students in $I_m \cap I'$ to occupied seats, S_m^o, and then considering other students for the remaining seats. First notice that, whether $m \in M^a$ or $m \in M^v$, students in $I_m \cap I'$ are only acceptable for seats in S_m^o. Moreover, each seat $s \in S_m$ ranks any student pair $i, i' \in I_m$ $(i, i' \in I \setminus I_m)$ in the same order they are ranked under $\succ_m^s$. Finally, by our construction under our choice function a student $i \notin I_m$ is assigned to seats in S_m^o if all students in $I' \cap I_m$ are assigned to some seats. Hence, for any precedence order, we can determine the selected students by first considering the students in $I_m \cup I'$ and then assigning the remaining students to the seats they are selected for one by one following $succ_m$. Then, the desired result follows. □

Proposition 3 shows that precedence order does not affect the chosen set by our choice function. In other words, precedence order is more a mechanical component of the choice function. In order to emphasize its inclusion in the choice function, we keep it in our formulation.

Next, we define a class of algorithms that is based on student proposing DA and each member differs from the others due to the set of majors allowing all seats to be available for transfers.

[39]Recall that, $\succ_m^S$ is constructed based on $\succ_m$ and whether m is in M^a or M^v

In particular, we use the same procedure for each member but the choice function of each major depends on whether they allow all seats to be available for the transfer students through the seat-specific priority orders. Let $\succ^{S,\bar{M}^a}$ be the seat priority profile induced when the set of majors allowing all seats to be used by transfer is $\bar{M}^a \subseteq M$.[40]

Student Proposing DA (DA^{sp}) under $\bar{M}^a$:

Step 1: Each student i proposes to her best choice. Let $I^{1,m}$ be the set of students proposing to major $m \in M$. Each major m tentatively holds students in $C_m(I^{1,m}; \succ_m^{S,\bar{M}^a}, \rhd_m)$ and rejects the rest of the applicants in $I^{1,m}$.

In general;

Step $k > 1$: Each student i proposes to her best choice which has not rejected her yet. Let $I^{k,m}$ be the set of students proposing to major $m \in M$. Each major m tentatively holds students in $C_m(I^{k,m}; \succ_m^{S,\bar{M}^a}, \rhd_m)$ and rejects the rest of the applicants in $I^{k,m}$.

The algorithm terminates when no student is rejected.

Every student is assigned to the major she has proposed to in the terminal step. We denote the matching selected by student proposing DA given P and $\bar{M}^a$ with $DA^{sp}(\bar{M}^a, P)$.

We can also define a class of mechanisms that are based on the major proposing version of DA similarly. Given $\bar{M}^a$, let $\succ^{S,\bar{M}^a}$ be the seat priority profile induced by the set of majors $\bar{M}^a$ and $I_m^1 = I$.

Major Proposing DA (DA^{mp}) under $\bar{M}^a$:

Step 1: Each major m offers seats to students in $C_m(I_m^1; \succ_m^{S,\bar{M}^a}, \rhd_m)$. Let M_i^1 be the set of majors offering a seat to student i in Step 1. Each student i tentatively holds the best acceptable proposal and rejects the rest. Let I_m^2 be the set of students who have not rejected major $m \in M$.

[40]Note that, when $M^a = M$, the mechanisms are equivalent to the traditionally defined DA where current students at majors have the highest priority.

In general;

Step $k > 1$**:** Each major m offers seats to students in $C_m(I_m^k; \succ_m^{S,\bar{M}^a}, \rhd_m)$. Let M_i^k be the set of majors offering a seat to student i in Step k. Each student i tentatively holds the best acceptable proposal and rejects the rest. Let I_m^{k+1} be the set of students who have not rejected major $m \in M$ so far.

The algorithm terminates when no offer of a major is rejected by some student.

Every student is assigned to the major she is holding in the terminal step. We denote the matching selected by major proposing DA given P and $\bar{M}^a$ with $DA^{mp}(\bar{M}^a, P)$.

Next, we state that any member of these two classes selects a stable outcome and that there is a Pareto dominance relation between the classes. Moreover, DA^{sp} is strategy-proof for students.

Proposition 4. *For any $(\bar{M}^a, P)$, DA^{sp} and DA^{mp} select stable outcome, DA^{sp}'s outcome (weakly) Pareto dominates DA^{mp}'s outcome, and DA^{sp} is strategy-proof for students.*

Proof. First, recall that our stability notion is composed of individual rationality for students according to initial assignments and the no-blocking pair condition. This differs from the standard stability notion since individual rationality is defined according to initial assignments as opposed to the unassigned option. Nevertheless, under our choice function, the no blocking pair condition implies individual rationality according to initial assignments. Moreover, no student or major receives an unacceptable assignment under the mechanisms we consider.

Recall that in Proposition 2 , we have shown that the choice function used under both classes of mechanism satisfies the sufficiency conditions for stability (Hatfield and Milgrom, 2005; Hatfield and Kominers, 2012; Aygün and Sönmez, 2012). In fact, since the choice function satisfies the properties stated in Proposition 2, the outcomes of $DA^{sp}(\bar{M}^a, P)$ and $DA^{mp}(\bar{M}^a, P)$ are stable, $DA^{sp}(\bar{M}^a, P)$ (weakly) Pareto dominates $DA^{mp}(\bar{M}^a, P)$, and $DA^{sp}(\bar{M}^a, P)$ is strategy-proof for students.

For interested readers, we provide detailed proof of the dominance relation between DA^{sp} and DA^{mp} below.

We prove the dominance relation by showing that DA^{sp} and DA^{mp} under our constructed choice function are outcome equivalent to standard mpDA and spDA under the copy economy constructed as follows:

- For each major m, create two copies: m^o and m^v.
- Copies m^o and m^v have capacity of $|S_m^o|$ and $|S_m^v|$, respectively.
- The priority order of the copies is defined as the seat-specific priorities. Let $\succ_{m^v}^{co}$ and $\succ_{m^o}^{co}$ be the priority orders of copies m^o and m^v, respectively.
- Each student $i's$ preference order over copies, denoted with P_i^{co}, is constructed as follows:
  - $m^o \ P_i^{co} \ m^v$ for every $m \in M$.
  - $m \ P_i \ \bar{m}$ implies $m^v \ P_i^{co} \ \bar{m}^o$ for every $m, \bar{m} \in M$.

Notice that, under P_i^{co}, the copies of each major are ranked consecutively.

Let $P^{co} = (P_i^{co})_{i\in I}$, $M^{co} = \bigcup\limits_{m\in M} \{m^v, m^o\}$, $\succ^{co} = (\succ_{\bar{m}^{co}})_{\bar{m}\in M^{co}}$, $q^{co} = (q_{\bar{m}}^{co})_{\bar{m}\in M^{co}}$ where $q_{m^o}^{co} = |S_m^o|$ and $q_{m^v}^{co} = |S_m^v|$. Let $(I, M^{co}, q^{co}, P^{co}, \succ^{co})$ be the associated copy problem of $(I, M, M^a, P, \succ, q)$.

In the copy problem, we consider the following variants of standard spDA and mpDA:[41]

spDA: In each step, if a student would be rejected from the occupied copy of a major m (i.e., m^o), then that student skips applying to m^o and instead applies to m^v. This change does not affect the outcome. One can see this by considering the sequential implementation of spDA.

mpDA: In some step, if both m^v and m^o would propose to some student i, then m^v skips proposing to that student. If m^o has been rejected by some student i in Step k, then m^v skips proposing to i in any further step. This change does not affect the outcome. Since in

[41]The definition of spDA and mpDA can be found in Appendix B and the proof of Proposition 1, respectively.

the construction of P_i^{co}, m^v is ranked below m^o and m will be rejected by i.

Recall that our constructed choice function's outcome does not depend on the precedence order. Without loss of generality, we consider the precedence order in which occupied seats are processed first.

For any given problem and its corresponding copy problem, we compare DA^{sp} under our choice function and standard spDA step-by-step. In Step 1 of DA^{sp} if a student i proposes to major m and is rejected, then she will propose to m^o and m^v and is rejected by both copy schools in the first step of spDA. If a student i proposes to major m and is tentatively assigned to an occupied seat under DA^{sp}, then she will propose to m^o and will be tentatively held by it under spDA. Finally, if a student i proposes to major m and is tentatively assigned to a vacant seat under DA^{sp}, then she will propose to m^o and be rejected and then propose to m^v and will be tentatively held.

Since each subsequent step works exactly the same as Step 1, the same argument works as a result, if a student i is assigned to major m under DA^{sp} then she is either assigned to m^v or m^o under spDA.

Similarly, we compare DA^{mp} under our choice function and standard mpDA step by step. In Step 1 of DA^{mp} if a major m offers to some student i and is rejected, then under mpDA either m^o offers to i and is rejected while m^v skips i or m^o does not offer to i while m^v offers to i and is rejected. If major m offers to i under DA^{mp} and is tentatively held by i then under mpDA either m^o offers to i and is tentatively held while m^v skips i or m^o does not offer to i and m^v offers to i and is tentatively held.

Since all the following steps work exactly the same, the same argument works as a result. Hence, if an offer from major m is held by i when DA^{mp} terminates, then i holds an offer from either m^o or m^v when mpDA terminates.

Therefore, DA^{sp} and spDA are outcome equivalent and DA^{mp} and mpDA are outcome equivalent. Given that spDA Pareto dominates mpDA, we have the desired result. □

Our stability notion is defined using choice functions rather than just using majors' priority orders. Hence, one can wonder whether a higher-priority student could envy a lower-priority student's assignment. The answer is yes, however, such a situation can occur for the sake of preserving individual rationality. Due to the construction of our choice function, under a stable matching, it is not possible to have two students $i, j \notin I_m$ such that i is assigned to m, $j \succ_m i$, and j prefers m to her assignment.

Recall that, thanks to our choice function formulation, the two classes of mechanisms allow any set of majors to allow their occupied seats to be used by transfer students. Then, a natural question arises: "what is the effect of one more major allowing all seats to be used by transfer students?" We answer this question by showing that whenever one more major allows all seats to be available to transfers, all students become weakly better off and the number of students matched to that major weakly increases under both DA^{sp} and DA^{mp}.

Theorem 1. *Let $\bar{M}^a = \hat{M}^a \cup \{m\}$ and $m \notin \hat{M}^a$. Then, for any P,*

- $DA_i^{sp}(\bar{M}^a, P) \; R_i \; DA_i^{sp}(\hat{M}^a, P)$ *and* $|DA_m^{sp}(\bar{M}^a, P)| \geq |DA_m^{sp}(\hat{M}^a, P)|$
- $DA_i^{mp}(\bar{M}^a, P) \; R_i \; DA_i^{mp}(\hat{M}^a, P)$ *and* $|DA_m^{mp}(\bar{M}^a, P)| \geq |DA_m^{mp}(\hat{M}^a, P)|$

.

Proof. Consider first DA^{sp} for the cases where $\bar{M}^a = \hat{M}^a \cup \{m\}$ and $\hat{M}^a$ denote the sets of majors allowing all seats to be available to transfer students.

We consider a sequential version of DA^{sp} in which in each step only one student who is not tentatively held by a major applies. Specifically, we do the following for both cases:

- In each step, if the student in turn applies to some major $m' \neq m$, then we apply the choice function of m', and the new applicant under both cases will have the same result since the choice function of m' is the same in each case.
- If the student in turn applies to m, then she is added to the applicant pool for m, denoted by A_m. If $|A_m| > q_m$, then we run the choice function of m for the case where m allows all seats to be available to transfer students. Notice that among students in A_m, only one student who is not in I_m will be rejected and that student is ranked lowest

among applicants in $A_m \setminus I_m$. It is easy to verify that this student will be rejected under the choice function when m does not allow its initially occupied seats to be used by transfer students.

- If an unacceptable student applies to m, reject her under both cases.

Due to the finite number of students and majors, after some steps, each student is either tentatively held by a major $m' \neq m$ or is in A_m and $A_m \leq q_m$. First, we consider the case for $\bar{M}^a$. Since all students in A_m are acceptable under the case in which m allows occupied seats to be available to transfer students, they all will be accepted and the procedure ends.

If the procedure ends in the other case, i.e., all students in A_m are accepted, then students get the same outcome for both cases. Otherwise, we will continue and students will become weakly worse off in each further step.

If x students are rejected from A_m under the case in which occupied seats are not allowed to be used by transfer students at m, then in each further step the number of tentatively held students by other majors will not decrease. As a result, the number of students accepted by major m under this case cannot exceed A_m.

Hence $DA_i^{sp}(\bar{M}^a, P)\ R_i\ DA_i^{sp}(\hat{M}^a, P)$ and $|DA_m^{sp}(\bar{M}^a, P)| \geq |DA_m^{sp}(\hat{M}^a, P)|$.

Now, consider DA^{mp} for the cases where $\bar{M}^a = \hat{M}^a \cup \{m\}$ and $\hat{M}^a$ denote the sets of majors allowing all seats to be available to transfer students.

Let O_i^{k,M^a} denote the set of majors from which student i has received an offer in steps up to and including Step k given some set of majors allowing all seats to be available to transfers, denoted M^a. Let H_m^{k,M^a} denote the set of students who hold an offer from major m after Step k given M^a.

It is easy to see that, $O_i^{1,\hat{M}^a} = O_i^{1,\bar{M}^a}$ and $H_m^{1,\hat{M}^a} = H_m^{1,\bar{M}^a}$ for all $i \in I$ and $m \in M$.

In Step 2, either $O_i^{2,\hat{M}^a} = O_i^{2,\bar{M}^a}$ or $O_i^{2,\hat{M}^a} \cup \{m\} = O_i^{2,\bar{M}^a}$ for all $i \in I$. As a result, $H_m^{2,\bar{M}^a} \supseteq H_m^{2,\hat{M}^a}$ and $|H_m^{2,\bar{M}^a}| \geq |H_m^{2,\hat{M}^a}|$. Let n denote the number of students holding an offer from m in case $\bar{M}^a$, but not in case $\hat{M}^a$. That is, $n = |H_m^{2,\bar{M}^a}| - |H_m^{2,\hat{M}^a}| \geq 0$.

If $n = 0$, then $H^{2,\bar{M}^a} = H_m^{2,\hat{M}^a}$ and in Step 3 students will receive the same offers from any majors $m' \neq m$ in either case. If $n > 0$, then in Step 3 some students may receive an offer from majors $m' \neq m$ in the case where occupied seats are available at m that they would not receive under the other case. In either case, some additional students may receive an offer from m in the case where occupied seats are available to transfers at m that they would not receive in the other case. Additionally, any offer received by a student in the case where occupied seats are not available to students will also be made in the case where they are available, if not made in the previous step. Hence, $O_i^{3,\hat{M}^a} \subseteq O_i^{3,\bar{M}^a}$ for all $i \in I$.

Additionally, there can be no more than n students receiving an offer from a major $m' \neq m$ in Step 3 when occupied seats are available to transfers that they would not otherwise receive under the other case. Hence, no more than n students holding m after Step 2 can receive such an offer that they would prefer to m. Furthermore, weakly more students receive a new offer from m in this step. Hence, $|H_m^{3,\bar{M}^a}| \geq |H_m^{3,\hat{M}^a}|$.

Each Step $k > 3$ operates the same as Step 3 including the terminal step. Therefore, at the terminal step, each student has been made every offer that would be made when occupied seats are not available to transfers in either case. Additionally, in the case where occupied seats are available for transfers, some students may have received offers that they would not receive in the other case. Hence, each student holds the same offer or one of higher preference in the case where occupied seats are available for transfers. Further, major m is held by weakly more students in the case where occupied seats are available to transfer students.

Hence, $DA_i^{mp}(\bar{M}^a, P)\ R_i\ DA_i^{mp}(\hat{M}^a, P)$ and $|DA_m^{mp}(\bar{M}^a, P)| \ \geq\ |DA_m^{mp}(\hat{M}^a, P)|$ $\square$

In the following example, under some problem, we show that a major m can be assigned students of lower preference when it starts allowing all of its seats to be available to transfers under DA^{sp}.

Example 2. *Let $I = \{i_1, i_2\}$ and $M = \{m_1, m_2\}$. Let m_1 and m_2 be the current majors of i_1 and i_2, respectively. Let there be no vacant seats at either m_1 or m_2. That is $q_{m_1}^v = q_{m_2}^v = 0$.*

Let student preferences be defined as follows: $P_{i_1} : m_2 - m_1; P_{i_2} : m_1 - m_2$*. Suppose* m_1 *prefers* i_1 *over* i_2*,* m_2 *prefers* i_2 *over* i_1*, and both students are acceptable to both majors. That is, each major prefers their current student. If one major, say* m_1*, does not allow all seats to be considered for transfers, but* m_2 *does allow all seats to be available for transfers, then no major switch will occur. If, instead, both* m_1 *and* m_2 *were to allow all seats to be available to transfers under* DA^{sp}*, then in Step 1* i_1 *will be assigned to* m_2 *and* i_2 *will be assigned to* m_1 *and the algorithm terminates. Both majors are therefore assigned students of lower preference due to allowing all seats to be available for transfers.*

Next, we show via another example that a major m can also be assigned students of lower preference when it starts allowing all of its seats to be available to transfers under DA^{mp}.

Example 3. *Let* $I = \{i_1, i_2, i_3\}$ *and* $M = \{m_1, m_2, m_3, m_4\}$*. Let the current majors for students be defined as follows:* $I_{m_1} = \{i_2\}; I_{m_4} = \{i_1, i_3\}$*. Let there be one vacant seat for all majors except* m_4*. That is* $q^v_{m_1} = q^v_{m_2} = q^v_{m_3} = 1$ *and* $q^v_{m_4} = 0$*. Let student preferences be defined as follows:* $P_{i_1} : m_3 - m_1 - m_4 - m_2$*;* $P_{i_2} : m_2 - m_3 - m_1 - m_4$*;* $P_{i_3} : m_1 - m_2 - m_3 - m_4$*. Suppose that preferences for majors are defined as follows:* $\succ_{m_1}$*:* i_1, i_2, i_3*;* $\succ_{m_2}$*:* i_2, i_1, i_3*;* $\succ_{m_3}$*:* i_3, i_1, i_2*;* $\succ_{m_4}$*:* i_1, i_2, i_3*. Hence, the priority orderings for vacant seats at majors are as follows:* $\succ^v_{m_1}$*:* i_1, i_3*;* $\succ^v_{m_2}$*:* i_2, i_1, i_3*;* $\succ^v_{m_3}$*:* i_3, i_1, i_2*. Assume initially that none of the majors are allowing occupied seats to be available to transfers, i.e.,* $M^v = M$ *and* $M^o = \emptyset$*. Denote the assignments of students to majors under* DA^{mp} *as* μ^v*. Assignments are as follows:* $\mu^v(i_1) = m_1, \mu^v(i_2) = m_2$ *and* $\mu^v(i_3) = m_3$*. Now, consider if* m_1 *begins allowing occupied seats to be available to transfers, i.e.,* $M^v = \{m_2, m_3, m_4\}$ *and* $M^o = \{m_1\}$*. Then the priority ordering for the seat at* m_1 *occupied by* i_2 *is defined as* $\succ^o_{m_1}$*:* i_2, i_1, i_3 *and the priority ordering for the vacant seat remains unchanged. Denote the assignments of students to majors under* DA^{mp} *as* μ^o*. Assignments are as follows:* $\mu^o(i_1) = m_3, \mu^o(i_2) = m_2$ *and* $\mu^o(i_3) = m_1$*. Notice that, since* m_1 *prefers* i_1 *to* i_3*,* m_1 *prefers their assigned student when* $m_1 \in M^v$ *to their assigned student when* $m_1 \in M^o$*.*

Our theoretical analysis concludes that for students it is always beneficial to have more majors allowing occupied seats to be available for transfers and the number of students assigned to a major does not decline when that major allows transfer students to be assigned to occupied seats. In some cases, however, a major can be worse off by allowing occupied seats to be available to transfer students. In the next section, by using data provided by NTU, we measure the potential gains of our proposed mechanisms.

6 Counterfactual Analysis

Throughout the entirety of our relationship with policymakers at NTU, they have been welcoming and open to the improvement of their major switching procedure. In this section, we will discuss the process of implementing our proposed changes with supporting counterfactual analysis, which has led to changes to the NTU mechanism being implemented starting in August of 2022. First, we provide a simple example to illustrate how to implement the proposed classes of mechanisms.

Example 4. *Let* $I = \{i_1, i_2, i_3\}$ *and* $M = \{m_1, m_2, m_3\}$*. Student* i_k *is currently enrolled in major* m_k *for every* $k \in \{1, 2, 3\}$*. Only* m_2 *has an initially vacant seat, i.e.,* $q^v_{m_1} = q^v_{m_3} = 0$ *and* $q^v_{m_2} = 1$*. Hence,* $q_{m_1} = q_{m_3} = 1$ *and* $q_{m_2} = 2$*.*

In practice, students report their ranking over the majors they would like to switch to, and majors do not provide ranking over their current students. Hence, the submitted student preferences and major priorities are:

$$P_{i_1} : m_3 - m_2 \qquad \succ_{m_1}: i_2 - i_3$$

$$P_{i_2} : m_3 - m_1 \qquad \succ_{m_2}: i_1 - i_3$$

$$P_{i_3} : m_1 - m_2 \qquad \succ_{m_3}: i_1 - i_2$$

The NTU mechanism allows only the vacant seat at m_2 *to be used and it assigns* i_1 *to that seat. Students* i_2 *and* i_3 *stay at their current majors. That is, the outcome of the NTU mechanism is:* $\nu(i_1) = m_2$*,* $\nu(i_2) = m_2$*, and* $\nu(i_3) = m_3$*.*

In order to implement the classes of mechanisms we have introduced, we first add the current major of each student to her preference list as her last (acceptable) choice. In order to construct seat-specific priorities, we first define the sets of occupied and vacant seats for each major. Let $S^o_{m_1} = \{s_1\}$, $S^o_{m_2} = \{s^o_2\}$, $S^v_{m_2} = \{s^v_2\}$, *and* $S^o_{m_3} = \{s_3\}$. *The seat-specific priorities are:*

$$\succ^{s_1}_{m_1}: i_1 - i_2 - i_3 - \emptyset$$
$$\succ^{s^o_2}_{m_2}: i_2 - i_1 - i_3 - \emptyset$$
$$\succ^{s^v_2}_{m_2}: i_1 - i_3 - \emptyset$$
$$\succ^{s_3}_{m_3}: i_3 - i_1 - i_2 - \emptyset$$

Now suppose, all majors allow their occupied seats to be used for major switching.

Then, DA^{sp} *and* DA^{mp} *select the same matching:* $\mu(i_1) = m_3$, $\mu(i_2) = m_1$, *and* $\mu(i_3) = m_2$.

Notice that, both DA^{sp} and DA^{mp} select an outcome in Example 4 that improves the match for all students when we compare it to the outcome of the NTU mechanism. In this section, we would like to show that such welfare gains are not limited to specific examples by conducting a counterfactual analysis with the data provided by NTU.

6.1 Data

In this section, we first explain the data provided by NTU to us to run a counterfactual analysis to compare our proposed mechanisms with the NTU mechanism. To do that, we use the data set collected to run the major-switching procedure under the NTU mechanism in the years 2019 and 2020. We provide the summary statistics for both years in Table 2. In 2019 and 2020, 654 and 635 students applied to switch their majors, respectively. The number of majors for both years is 61. For each student, we observe their submitted preferences and their current major. For each major, we observe the number of initially vacant seats, the

	2019	2020
Number of majors	61	61
Number of students	654	635
Total number of applications	886	847
Total number of initially vacant seats	588	587
Majors reporting a secondary list	12	8
Total acceptable applicants not ranked	376	367
Average number of applications by student	1.35	1.33
Average number of applicants from majors	10.72	10.41
Average number of applicants to majors	14.52	13.89
Average number of initially vacant seats	9.64	9.62
Average length of reported list by majors	6.36	5.56
Average length of reported secondary list by majors	1.10	0.61
Average number of acceptable applicants not ranked by majors	6.16	6.02

Table 2: Summary statistics for years 2019 and 2020

unranked primary list, and the ranked secondary lists.[42] Moreover, we can also observe the list of applicants for each major who were not ranked and the list of applicants for each major who are considered unacceptable. Recall that, to run the NTU mechanism, we do not need a strict ranking over the students in the primary list but we need strict ranking over the secondary list.

Our data set includes information about the actual major switches that were made in 2019 and 2020. In order to verify that the major switches were assigned following the described NTU mechanism, we replicated the assignments, and our results matched with the actual major switches. Next, we compare the number of applicants from each major (outflows) and the number of applicants ranking each major (inflows). As shown in Figure 7, imbalances between the inflows and outflows exist at many majors. For example, in 2020, the number of applicants to Economics (Major 20) and the number of applicants from it were 118 and 12, respectively. As one can see, Economics was the most popular major during this period. In total, there were 588 and 587 vacant seats available in 2019 and 2020, respectively. In many cases, the number of students applying to a major exceeds the major's quota, as can be seen in Figure 8. In particular, 46% of majors received more applications than the number

[42] The number of students in the primary list cannot exceed the number of initially vacant seats at each major.

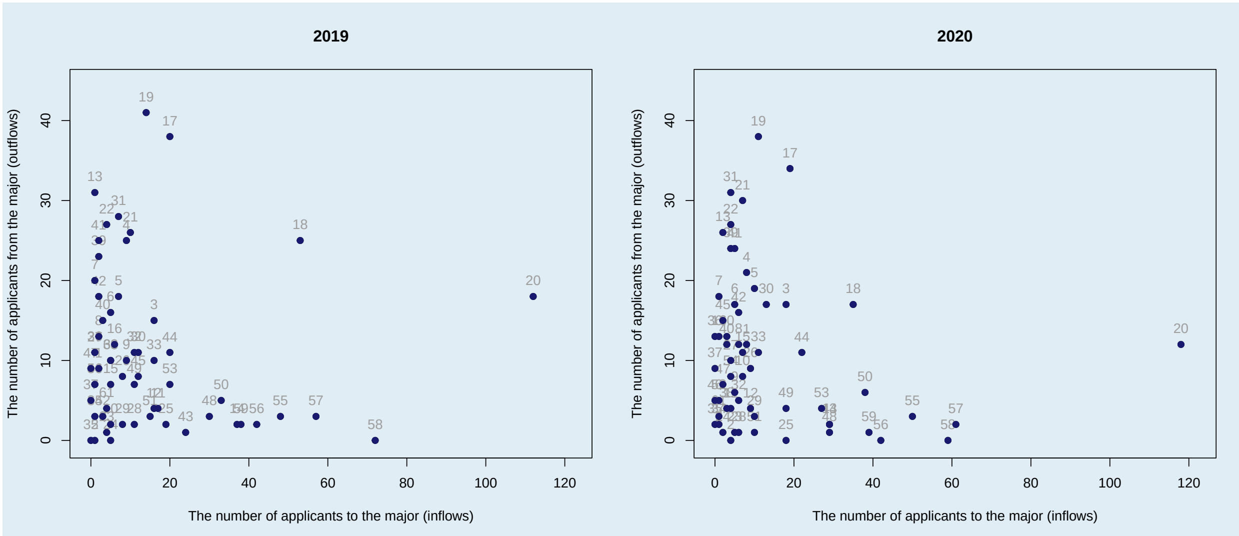

Figure 7: Inflows and outflows of applicants in 2019 and 2020

of vacant seats they had in 2019. Further, 33% of majors received more applications from students who ranked the major as their first choice than the number of vacant seats they had in 2019.[43] Under the NTU mechanism, the maximal number of successful applicants is equal to the major-switching quota if there is a sufficient number of students who are evaluated as acceptable students on the primary or secondary lists. For instance, due to the quota constraint, the major-switching quota for Economics (Major 20) was 24 and the success rate in this major was only 20.34% in 2020.

Under the proposed mechanisms, a major's seats available to transfers could be expanded by adding the number of its students who have applied for major switching. For example, in 2020, there were twelve students from Economics who applied to other majors, and hence, its expanded quota could have been 36 (= 24 + 12). Given the number of students ranking Economics exceeds 36 in 2020, such an expansion would have resulted in a higher success rate even under NTU mechanism. We next aim to show that this success rate could be increased with the proposed mechanisms.

[43]In 2020, 40% of majors received more applications than the number of vacant seats they had and 32% received more first-ranked applications than the number of vacant seats they had.

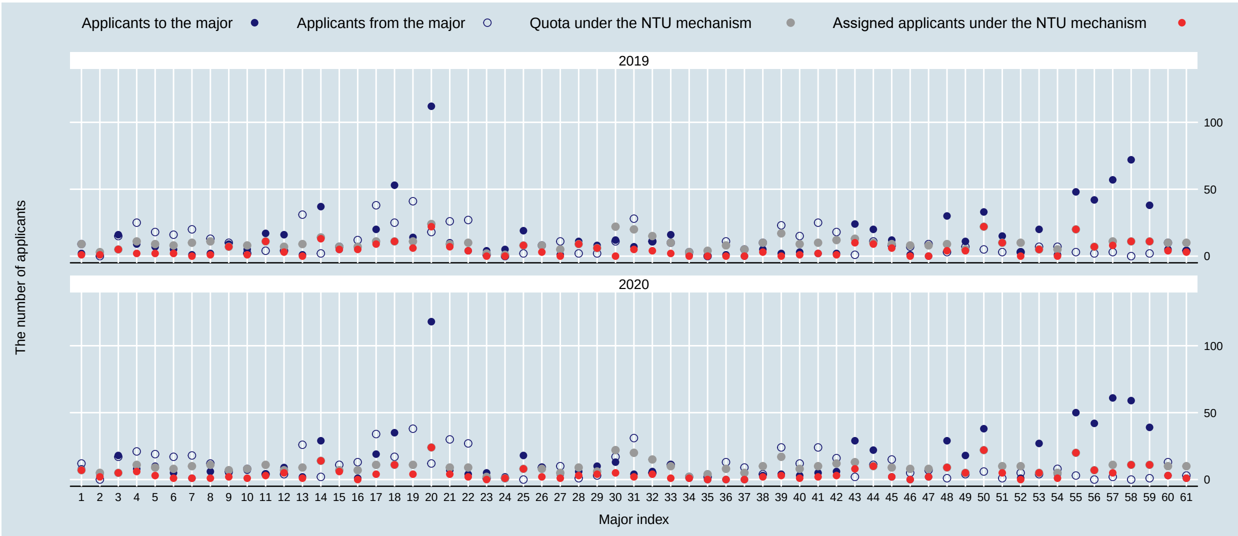

Figure 8: Comparison of the numbers of inflows/outflows, vacant seats, and the assigned applicants in 2019 and 2020

6.2 Impact of Allowing Occupied Seats to be Available

As discussed in our theoretical results, the NTU mechanism is outcome equivalent to DA^{mp} where all majors allow only the initially vacant seats to be available. Since DA^{sp} (weakly) Pareto dominates DA^{mp} (Proposition 4), we first consider we can achieve welfare gains by replacing the NTU mechanism with DA^{sp}. To evaluate if this is the case, we compare the outcome of DA^{mp} to the outcome of DA^{sp} where only initially vacant seats are available using the 2019 and 2020 data. Surprisingly, we find that the outcomes are identical, i.e., each student in each year has the same assignment under both mechanisms. Equivalence between the two versions of the DA mechanisms in two-sided matching markets has been studied for large markets (Roth and Peranson, 1999; Immorlica and Mahdian, 2005; Kojima and Pathak, 2009). However, given the size of this market, there may be alternative explanations for this equivalence. First, just 12 and 8 majors (out of 61 majors) submitted secondary lists in 2019 and 2020, respectively. In other words, many majors rank no more students than the number of initially vacant seats. Further, of the majors who do submit a secondary list, they include just 5.6 and 4.7 students on average in 2019 and 2020, respectively. It is clear to see that in a stable outcome, only a subset of these ranked students can be assigned to these majors and moreover, when the rural hospital theorem (Roth, 1986) is applied, it will be the same set of students in any stable matching. Further, we observe a substantially high correlation between students' preferences and majors' priorities. For example, 81.5% of students who ranked within the number of vacant seats at a major ranked the same major as their first choice.

Given that changing the mechanism to DA^{sp} may not create the intended improvement over the NTU mechanism under the current situation in which many majors are hesitant to submit a secondary list, we focused on the effect of majors allowing all seats to be available and aimed to exhaust potential welfare improvement. To evaluate the impact of majors doing so, we run 100 simulations for each possible number of majors (1 to 61) allowing their occupied seats to be available for major switchers. That is, for the case where the number of major allowing occupied seats is $k \in \{1, \ldots, 61\}$, we randomly select which of the k majors do so in each simulation run. The results of this analysis for both 2019 and 2020 are provided in Figure 9. Note that the outcome where zero majors allow occupied seats to be available corresponds with the NTU mechanism. As mentioned in the previous paragraph, we obtained the same outcomes under DA^{mp} and DA^{sp}, as a result, in both figures, we only include DA^{sp} and NTU.

Additionally, while there is an increase in the success rate for students, a 2.4% and 1.4% increase in the success rate in 2019 and 2020, respectively, the increase is limited.[44] This limitation arises due to the fact that under the current practice majors rarely rank more students than they have initially vacant seats. In fact, if the number of ranked students by a major does not exceed the number of initially vacant seats, allowing occupied seats to be used in major switching does not change the outcome of the mechanism. We next consider the effect of expanding the number of students ranked by all majors to be all acceptable applicants to the major.

[44]In particular, when we increase the number of majors allowing their occupied seats to be used in the major switching, we mainly observe an increase in the percentage of students assigned to their first choice.

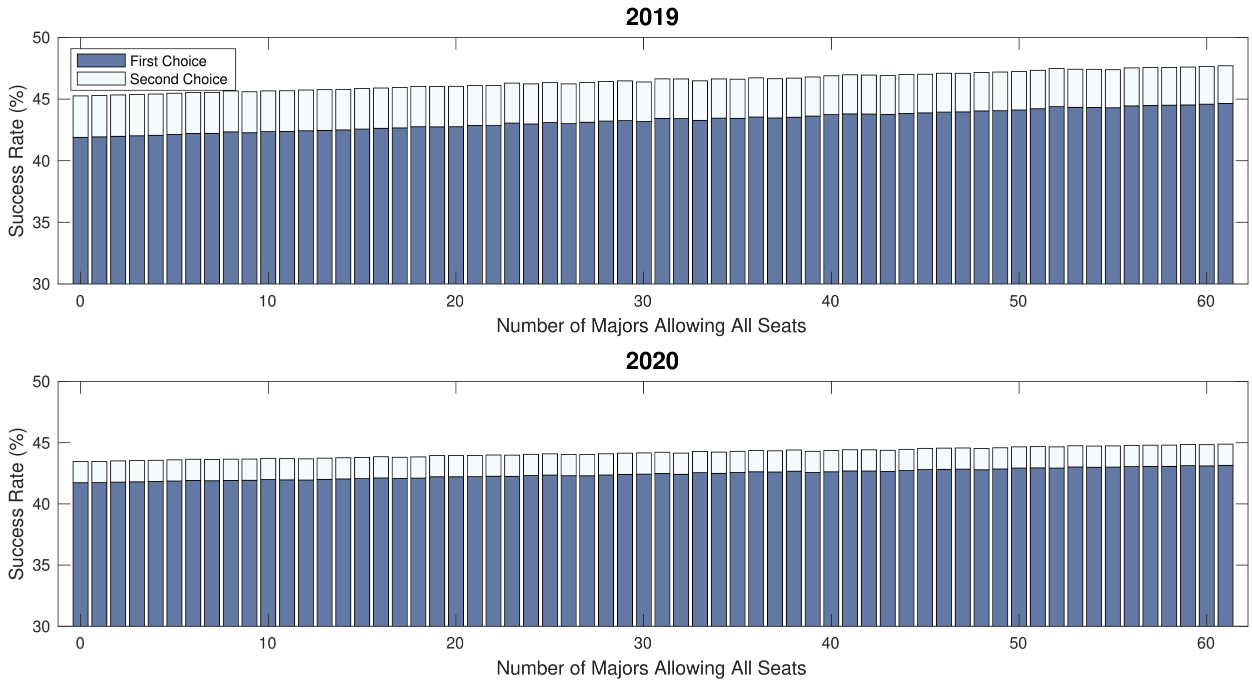

Figure 9: Effects of allowing all seats for major switching on success rates in 2019 and 2020

6.3 Impact of Ranking All Acceptable Students

We next proceed to evaluate the effect of majors ranking all acceptable applicants, i.e., the ones satisfying all requirements, on their priority orders.[45] In our data, we can observe the applicants who satisfy all the requirements but are not included in the submitted rankings by each major. In order to achieve this goal, we run 100 simulations for all possible numbers of majors ranking all acceptable students on their priority orders. Additionally, we conduct this analysis for two cases: when no major allows occupied seats to be available and when all majors allow occupied seats to be available. As we can see in Figure 10, the increase in the success rate is significant when majors include all acceptable students on their priority order when no major allows occupied seats to be available. Specifically, this increases the success rate by 9.1% and 9.2% in 2019 and 2020, respectively. This effect is 91% greater in 2019 and 80% greater in 2020 when all majors allow occupied seats to be available. Hence, if majors both allow occupied seats to be available and rank all acceptable applicants, we

[45]We consider the theoretical impacts of doing so in Appendix C.

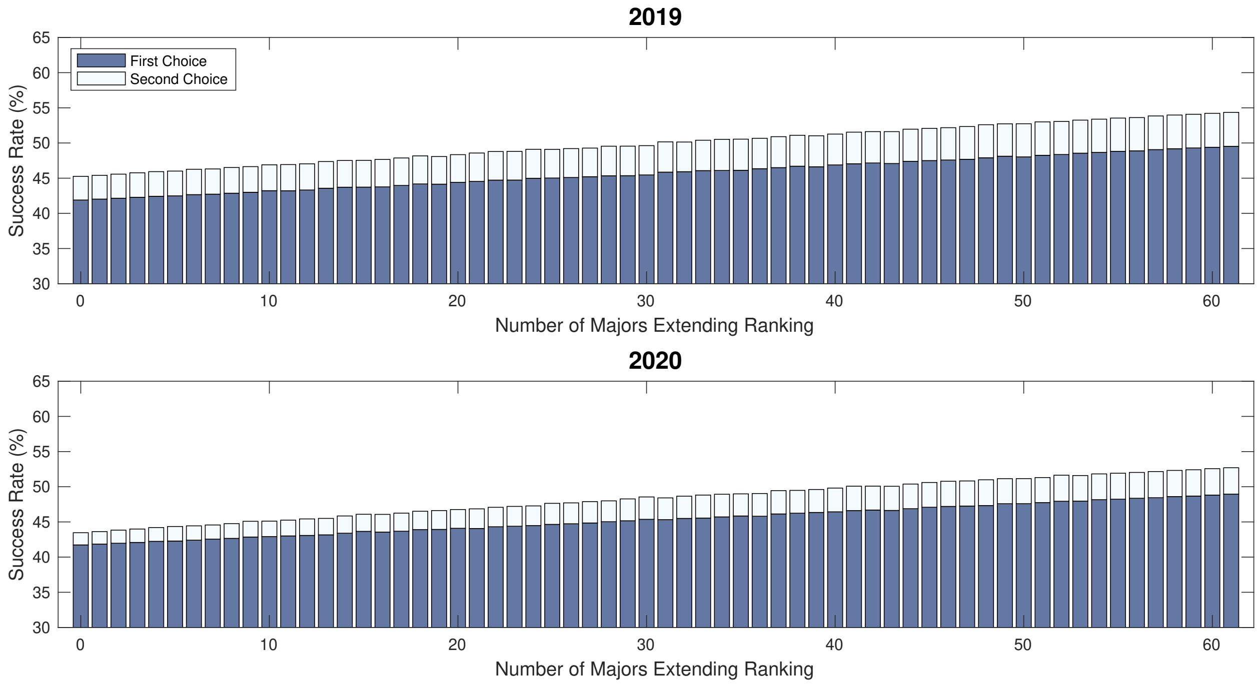

Figure 10: Effects of extending rankings on success rates when no major allows all seats for major switch

can see a large increase in the success rate for major switchers. We summarize these results in Figure 11. Specifically, the success rate of the proposed mechanisms when all majors employ both of these changes increases from 45.3% and 43.6% to 62.7% and 60.0% in 2019 and 2020, respectively. Furthermore, more students are assigned their first or second choices under the proposed mechanisms. We summarize these results in Table 3 for the case when all acceptable students are ranked and all majors allow their occupied seats for major switching.

Next, we focus on the gains at the major level. Figure 12 compares the number of major-switches to and from each major under NTU to DA^{sp} when all majors allow occupied seats to be used and rank all acceptable applicants. In this figure, the points scattered right to the 45-degree dashed line indicate majors having a net increase of students under the mechanism. We can see that the number of successful applicants increases in many majors under the proposed mechanisms. In fact, 54% of all majors have strictly more successful applicants in both years with an increase of as many as 16 more successful applicants (Political Science-International Relations, Major 18, in 2019) in comparison to the NTU mechanism. Also,

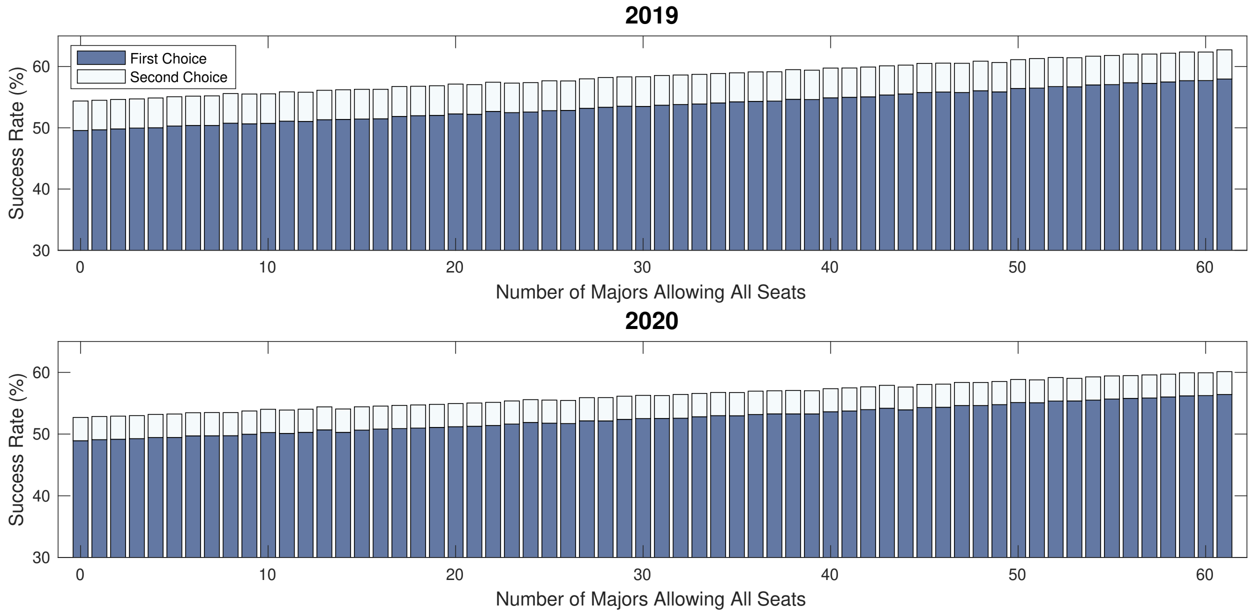

Figure 11: Effects of allowing all seats for major switching on success rates when all majors extend their rankings

	2019			2020		
	Actual	DA^{sp}	DA^{mp}	Actual	DA^{sp}	DA^{mp}
Switching to the first choice	41.59	56.73	56.73	41.26	55.59	55.59
Switching to the second choice	3.67	5.96	5.96	2.36	4.41	4.41
Total success rate	45.26	62.69	62.69	43.62	60.00	60.00

Table 3: Gains under the proposed mechanisms compared to the actual assignments in 2019 and 2020

note that Economics (Major 20) has the largest increase of students in 2019 with a net inflow of 13 ($= 22 - 9$) under the NTU mechanism and a net inflow of 24 (=36-12) under the DA^{sp}.[46] In contrast, Majors 19 and 17 have the largest decrease of students in 2019 and 2020, respectively.[47]

[46]Similar patterns can be seen in 2020. The net inflow number of students at major 20 is from 19 ($= 24-5$) under the NTU mechanism to 24 ($= 30 - 6$) under the new mechanism in 2020.

[47]The net inflow numbers of students at Majors 19 and 17 is from -15 ($= 6 - 21$) and -14 ($= 4 - 18$) under the NTU mechanism to -21 ($= 8 - 29$) and -17 ($= 7 - 24$) under the new mechanism in 2019 and 2020, respectively.

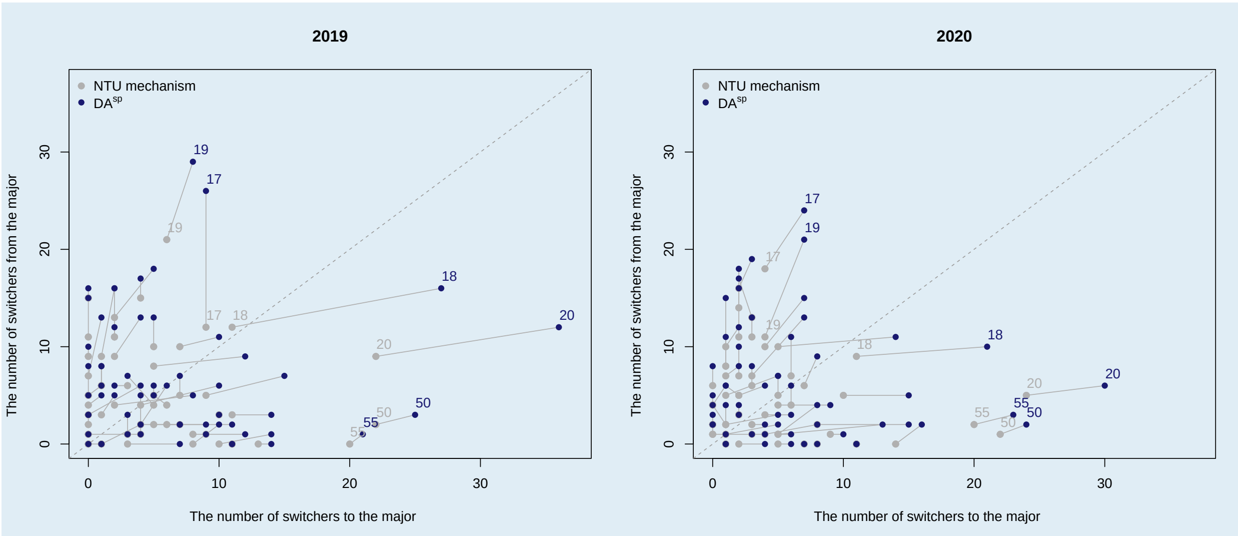

Figure 12: The numbers of incoming and outgoing students

6.4 Implementation at NTU in 2022

After completing our initial analysis, We met with administrators at NTU and explained the potential welfare gains if they were to make our proposed changes to their major switching process. The administrators at NTU were very progressive about making changes, but they still wanted to give freedom to choose to the majors. In particular, they did not want to coerce the majors in their choice of allowing occupied seats and including all acceptable students in their ranking.

In August 2022, NTU implemented the changes proposed in this paper to allow majors to include occupied seats if they choose to and replaced the mechanism with DA^{sp}. Information about the 2022 process is summarized in Table 4.[48] During the 2022 major switching process, 23 of the 61 majors at NTU allowed all seats to be available during the process. We illustrate the number of students listing each major as the top choice from each major in the network flow graph in Figure 13. As one can see, Economics (Major 20) is the most demanded major. In particular, 60 students ranked Economics as their first choice and an additional 25 students ranked the major as their second choice. However, the number of initially vacant seats at Economics is just 36. Hence, there are 24 more students who ranked the major first

[48]We include all years to make it easy to compare the summary statistics.

	2019	2020	2022
Number of majors	61	61	61
Number of students	654	635	661
Total applications	886	847	878
Total vacant seats	588	587	700
Majors allowing occupied seats	0	0	23
Majors reporting a secondary list	12	8	6
Total acceptable applicants not ranked	376	367	431
Average applicants from major	10.72	10.41	10.84
Average applicants to major	14.52	13.89	14.39
Average vacant seats	9.64	9.62	11.48
Average length of reported list	6.36	5.56	5.31
Average length of reported secondary list	1.10	0.61	0.31
Average acceptable applicants not ranked	6.16	6.02	7.07

Table 4: Summary statistics from the NTU major switching process

than there are initially vacant seats. This large imbalance between demand and supply is not specific to Economics. Other majors with such large imbalances are Electrical Engineering, Political Science-International Relations, and Information Management with a difference of 43, 29, and 24, respectively.

One cause for concern in the 2022 major switching process is that just six majors submitted a secondary list. Even more striking, is that of the six majors who did so, 5 of them did not allow occupied seats to be available. The sole major that both allows occupied seats to be available and submits a secondary list is the Economics major. Unfortunately, however, the length of their secondary list submitted by Economics is just one, and 47 acceptable applicants to the Economics major were not on the reported list of the major. In total, 47 majors did not include their acceptable applicants in their reported rankings. Further, there are 431 cases where a student applies to a major and is not included in the ranking of the relevant major.

For these reasons, the observed success rate in 2022 is not as high as it could be and much less than we would have expected, given the counterfactual analysis from 2019 and 2020. In fact, no additional students changed their major due to these 23 majors allowing occupied seats to be available. This is due to the fact that only one student was ranked in a secondary list by these 23 majors. Had the same set of 23 majors allowed occupied seats to be available in 2019 and 2020, with the submitted rankings from those years, 2.2% and 1.5% more students would have switched, respectively. This highlights the consequences of majors submitting short or no secondary lists. Although these improvements are non-negligible, it is clear that we do not attain all possible gains suggested by our theoretical results. To evaluate the gains in success rate that could be achieved with this set of 23 majors allowing occupied seats, we run 100 simulations where different sets of $k \in \{1, \ldots, 61\}$ majors extend their priority rankings to include all acceptable students. Here, we find that if even just ten majors extended their priority rankings, then the success rate would increase by 7.2% and if all majors do so then an additional 114 students would switch their majors compared to the actual result in 2022. These results are summarized in Figure 14.

Based on this, it is apparent that some majors may feel that they will be better off when they either do not allow occupied seats to be available or do not rank all acceptable students. To address this, we next conduct equilibrium analyses evaluating if this is the case using data from the NTU major-switching process in 2019, 2020, and 2022.

6.5 Equilibrium Analysis

In Section 5, we demonstrated that there may be some cases where majors have incentives not to allow occupied seats to be available or not rank all acceptable (see Example 2). In this section, we will conduct an equilibrium analysis to evaluate if this is the case using the data from 2019, 2020, and 2022.

First, we evaluate if it is an equilibrium outcome for majors to allow occupied seats to be available.[49] Specifically, we consider if it would be the best response for each major to allow occupied seats to be available if every other major is already allowing. We do this for two different cases: if majors use actual priority orders and if majors use extended priority orders (i.e., all acceptable applicants are ranked). The answer is affirmative in all three years for

[49]Consistent with the earlier literature in matching theory, we chose Nash equilibrium under complete information as our equilibrium concept.

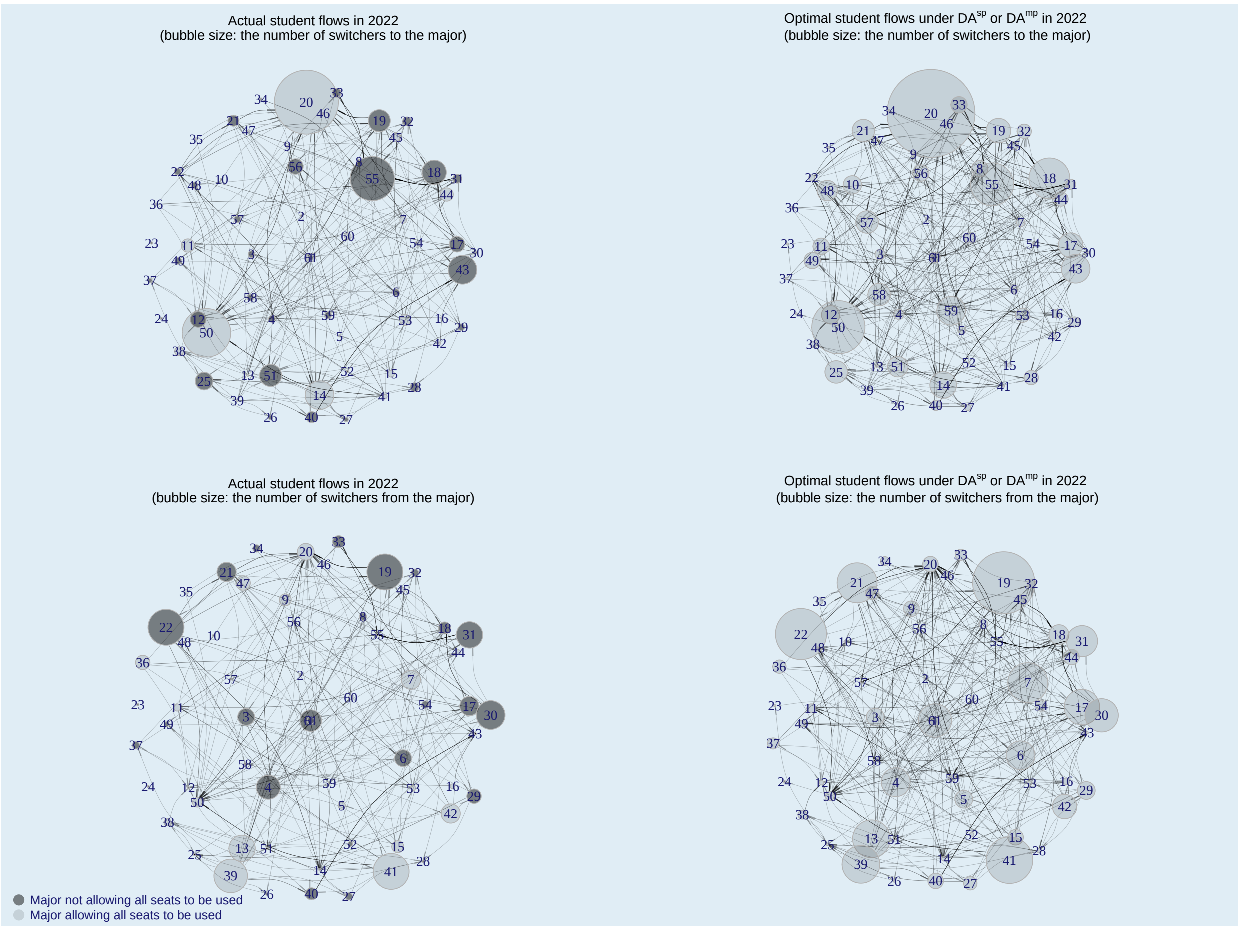

Figure 13: Network of inflows and outflows in 2022 NTU major switching process

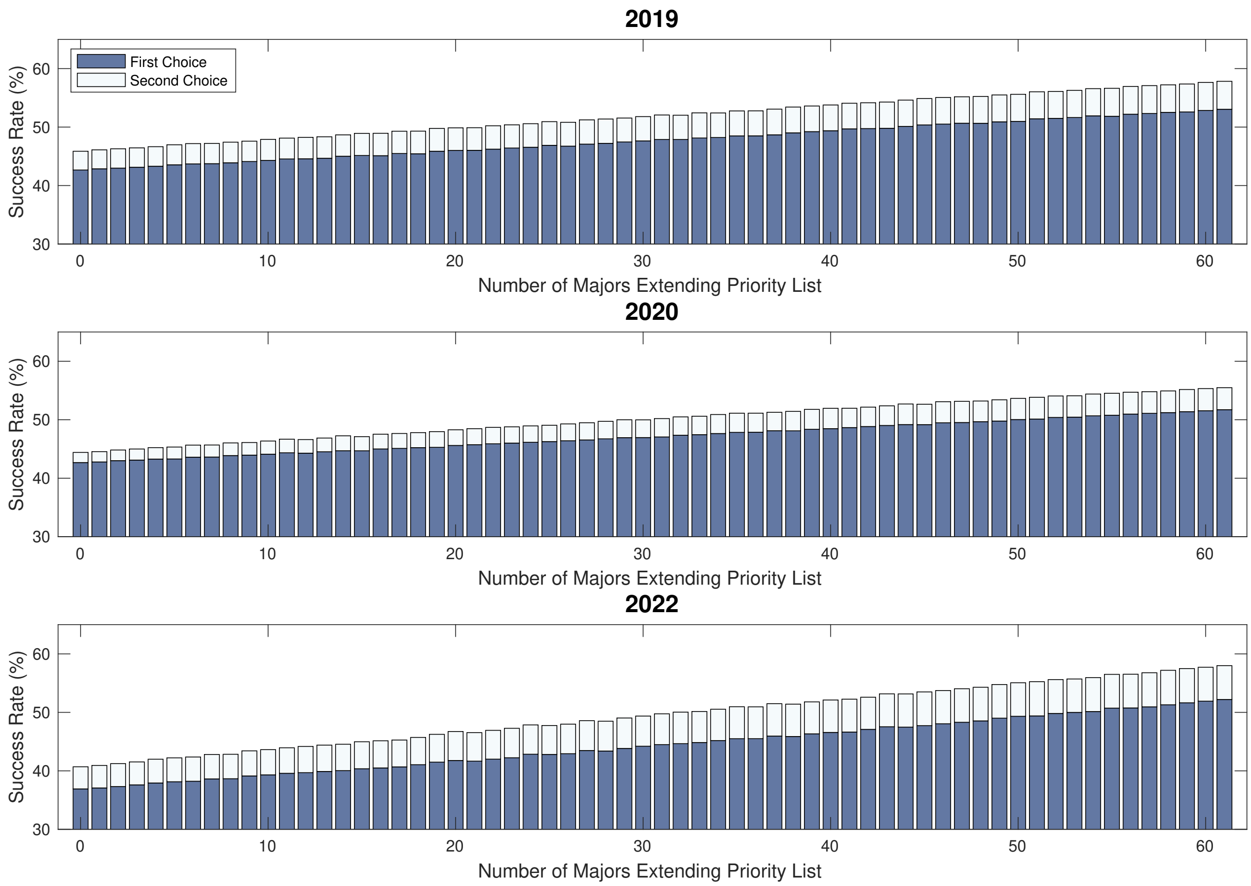

Figure 14: Success rates and the number of majors extending their seats when 23 majors allow all seats to be available

	2019	2020	2022
Number of majors better off allowing occupied seats	18	16	15
Number of majors worse off allowing occupied seats	0	0	0
Total additional students assigned when $M = M^a$	62	54	41

Table 5: Equilibrium analysis results for allowing occupied seats when all majors extended their rankings

both cases. Specifically, six, three, and three majors (out of 61) would have been strictly better off by allowing occupied seats to be available in response to all other majors doing so, respectively, when using actual priority orders. When using extended priority orders, 18, 16, and 15 majors are strictly better off allowing occupied seats to be available. Further, no majors would have been better off not allowing all seats to be available in any of the three years. Hence, it is an equilibrium outcome for majors to allow occupied seats to be available following the submitted rankings for each major in 2019, 2020, and 2022. Furthermore, 62, 54, and 41 additional students are assigned when majors allow occupied seats to be available in 2019, 2020, and 2022, respectively. These results are summarized in Table 5.

We also evaluate whether or not it is a best response for majors to extend their rankings in response to all other majors doing so. We consider if this is the case both when no major allows occupied seats to be available and when all majors allow occupied seats to be available. We identify one case where a major would have been assigned a student of lower preference when extending their ranking when all other majors do so. Specifically, in 2020, if the Political Science - Public Administration major extends their ranking, then one more of their initial students is assigned to some other major when this would not be the case if they did not extend their ranking. However, this seat is filled with a student from the major's secondary list, and two additional seats are filled in the case where they extend their ranking that would not be assigned without the extended ranking. That is, by extending their ranking, the major loses one current student and gains three students from their secondary list. There are no other cases identified where a student of lower preference is assigned as a result of a major extending their ranking.

The best responses for majors in the case where all majors allow occupied seats to be available

	2019	2020	2022
Majors assigned students of lower preference with extended list	0	1	0
Majors assigned students of lower preference without extended list	0	0	0
Number of majors assigned more students with extended list	30	35	40
Number of majors assigned more students without extended list	0	0	0
Total additional students assigned with extended lists	104	103	145

Note: When a major is assigned a student of lower preference, this implies that a student on the secondary list is assigned in place of a student either on the primary list or a current student in the alternative case.

Table 6: Summary of equilibrium analysis - extended priority with all allowing occupied seats

are summarized by Table 6. Importantly, majors do not get assigned fewer students when expanding their priority order. In fact, 30, 35, and 40 majors are assigned additional students when they extend their priority rankings in 2019, 2020, and 2022, respectively. Additionally, in total 104, 103, and 145 additional students are assigned when majors extend their rankings in 2019, 2020, and 2022, respectively.

Hence, our equilibrium analysis concludes that it is an equilibrium outcome for all majors to allow occupied seats to be available and that there is only one case where it is a best response not to extend rankings when all other majors do so.

7 Conclusion

The demand for major switching exists because of the mismatch between students and majors. Under the conventional mechanism, the success rate of major switching is low at many universities in Taiwan. To the best of our knowledge, the major-switching problem has not been studied in the market design literature. In this paper, we propose two alternative mechanisms to resolve the problem. Our counterfactual analysis for NTU shows that the success rate can increase and more students are assigned their first-choice major under the proposed mechanisms. Moreover, the evidence from NTU indicates that major switchers tend to have a higher GPA than their peers in the new major.

Our study has provided a policy impact by replacing the NTU mechanism with one of the mechanisms we propose in this paper. Such an impact would not be possible without the support and vision of the policymakers at NTU. Moreover, the stand they took by freeing the majors on their decision can be considered as the best possible approach. In fact, if they had forced the majors, the majors would have taken actions that would have damaged the major switching program. Our initial results affected almost half of the majors, as they chose to allow their occupied seats to be used. Although we did not achieve our expected outcomes due to the short rankings reported by majors, we expect that the additional analysis we conduct as a result will influence the majors to extend their rankings. Moreover, the seat-specific priority framework we used in the paper would allow majors to select the maximum number of acceptable but not initially ranked students to be assigned to themselves. Further, while we do not address it in this paper, our framework could enable majors to achieve other initiatives by giving priority to students of a certain type for some seats. We believe that our experience in the field will help market designers in the process of creating policy impact.

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Appendices

A Student grades before and after major switching

Although the major switching procedure we consider in our motivating example (NTU) is centralized, some colleges in Taiwan run their major switching procedure in a decentralized manner. In order to expand the choice set for students, C University (CU)[50], a selective private university in Taiwan, introduced a centralized matching in 2012 for major switching. In the new major switching procedure, a student can apply to at most three majors with an ordered preference list, and the majors have to determine their priority rankings for students who apply to them.

We measure students' learning outcomes using a standardized academic grade for major switchers with a data set provided by CU for the period of 2013-2015. We collect the grades of all students at the university during this period, and define the standardized grade as follows:

$$s_{imt} = \frac{x_{imt} - \bar{x}_{mt}}{\sigma_{mt}},$$

where s_{imt} is the standardized grade of student i with major m in semester t; x_{imt} is i's average academic grade in semester t; $\bar{x}_{mt}$ and σ_{mt} are the average and the standard deviation of all students' average academic grades within major m in semester t, respectively. We then compute s_{imt} for all major switchers in the semesters before and after they switch their major.

Moreover, the major switchers can be categorized into two types: students from a controlled quota and students from the uncontrolled quota. The admission quota for local students is regulated in Taiwan, and CU has a quota of 3,006 students each year. Those students are defined as students from the controlled quota. However, the government has no restrictions on the number of overseas students, foreign students, and students defined by some preferential policies, such as indigenous students. These students are defined as students from the

[50]Due to confidentiality, we do not provide the name of this college.

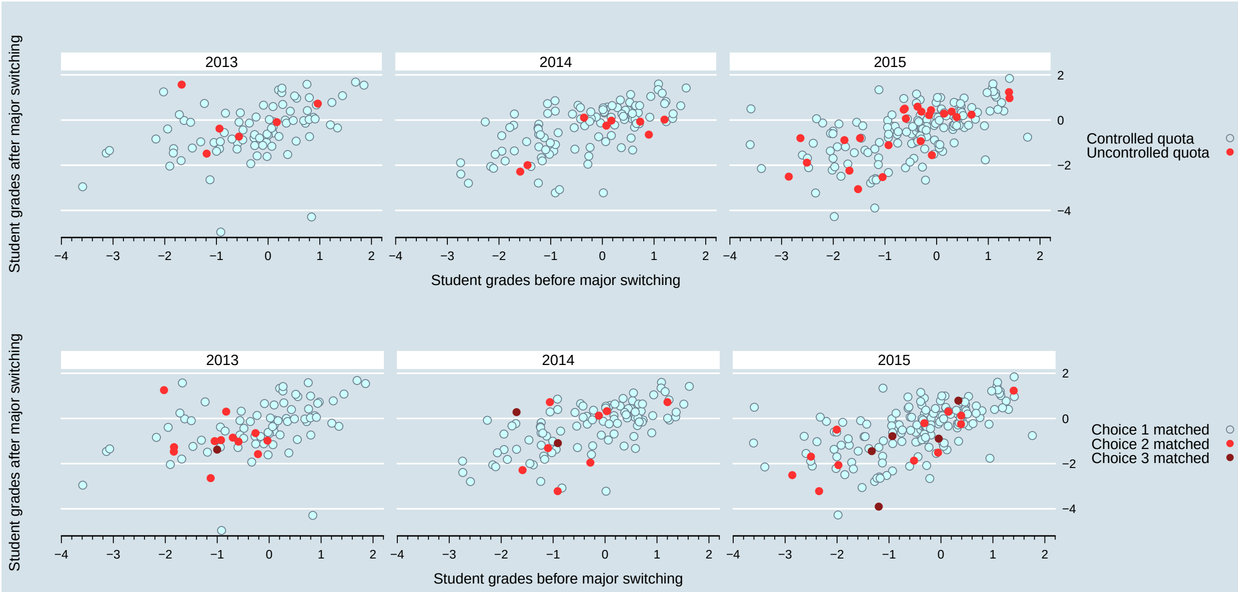

Figure 15: Standardized grades before and after major switching

uncontrolled quota.

Figure 15 depicts the standardized grades of major switchers for the years before and after major switching, which are distinguished by students from the controlled and uncontrolled quota and by whether they are matched with their first, second, or third choice in their submitted preferences. We are interested in the student grades after major switching compared to their grades prior to major switching. Regardless of if students are from controlled quota or uncontrolled quota, there exist positive correlations between grades after major switching and grades before major switching, i.e., students' performance after switching is related to their performance prior to switching. Moreover, students matched with their second or third choice tend to have a lower grade after major switching, compared to those assigned to their first choice. That is, students assigned to majors of higher preference perform better than those assigned to majors of lower preference. In addition, Figure 16 depicts students' grades after major switching and their priority rankings evaluated by the new majors. However, the relationship between the student grades and their priority rankings does not exhibit an obvious pattern like that in Figure 15.

Table 7 summarizes the regression analysis for the standardized student grades after major

Figure 16: Standardized grades and priority rankings in the new major

	Model 1		Model 2	
Quantitative variables:				
Grades before major switching	0.5406***	(0.0445)	0.5409***	(0.0452)
Order of matched choice	-0.2697**	(0.1259)	-0.2664**	(0.1263)
Priority rankings	-0.0010	(0.0133)	-0.0144	(0.0530)
Dummy variables:				
Controlled quota	-0.0325	(0.1678)	-0.0288	(0.1684)
2014	0.0062	(0.1368)	0.0127	(0.2242)
2015	0.0278	(0.1299)	-0.0290	(0.2026)
Interaction terms:				
2014×Priority rankings			0.0035	(0.0596)
2015×Priority rankings			0.0175	(0.0548)
Observations	369		369	
Adjusted R^2	0.3023		0.2989	

Note: Standard errors are shown in parentheses. Significances of estimated coefficients at 10%, 5%, and 1% levels are denoted by *, **, and ***, respectively.

Table 7: Regressions for the student grades after major switching

switching. Model 1 considers all the quantitative and categorical variables depicted in Figures 15 and 16.[51] Since the number of applications and hence the range of priority rankings is increasing during this period, we also include the interaction term between the yearly dummies and the priority rankings in Model 2. The result shows that the student's grades before major switching and the order of matched choice are the significant variables contributing to the student's grades after major switching. The estimated coefficients indicate that, given other factors, a major switcher with a higher grade in the initial major or matched with the first choice, tends to have a better grade in the new major.

The regression analysis for the CU major switching suggests that, when the learning outcome is measured by the standardized grade, previous performance and the ranking of the matched major indeed contain useful information for predicting the learning outcome in the new major. By contrast, the student priority rankings evaluated by the new major do not significantly contribute to the learning outcome. In other words, students with lower priority rankings can still exhibit a better learning outcome than students with higher rankings in their new majors. This result might be due to a limited time for new majors to interview their applicants.

B Definition of Student Proposing DA

Below, we formally define student proposing DA (spDA).[52]

> **Step 1:** Each student i proposes to her highest ranked major under P_i. Each major m tentatively accepts acceptable proposing students up to its capacity and rejects the rest.
>
> In general,
>
> **Step $k > 1$:** Each student i proposes to her highest ranked major under P_i that

[51] In each of these figures, we represent the standardized grades of students, i.e., how many standard deviations that students grades are above or below the average grade in their major that semester.

[52] The definition of major proposing DA is provided in the proof of Proposition 1.

has not rejected her yet. Each major m tentatively accepts acceptable proposing students up to its capacity and rejects the rest.

The mechanism terminates when no offer is rejected.

C Impact of Extended Rankings by Majors

In Theorem 1, we demonstrate that whenever a major m begins to allow occupied seats to be available all students are (weakly) better off as a result and (weakly) more students are assigned to m. We now show that the same results hold whenever a major includes additional acceptable students on their priority order.

First, we define a relationship between two priority orders for a major. We say that priority order $\succ'_m$ is an **extension** of $\succ_m$ if all of the following hold:

- If $i \succ_m j \succ_m \emptyset$ then $i \succ'_m j \succ'_m \emptyset$.
- If $i \succ_m \emptyset \succ_m j$ then $i \succ'_m j$.
- There exists some j such that $\emptyset \succ_m j$ and $j \succ'_m \emptyset$.

That is, a priority order $\succ'_m$ is an extension of priority order $\succ_m$ if the relative priority for any two students is unchanged, any student acceptable student under $\succ_m$ is also acceptable under $\succ'_m$, and at least one additional student is considered acceptable under $\succ'_m$. Next, we show that if a major replaces their priority order with an extension of the priority order, the same results hold as described by Theorem 1 for a major beginning to allow occupied seats to be available.

Theorem 2. *Let $\succ'_m$ be an extension of $\succ_m$. Then, for any P,*

- $DA^{sp}_i(\succ', P)\ R_i\ DA^{sp}_i(\succ, P)$ *and* $|DA^{sp}_m(\succ', P)| \geq |DA^{sp}_m(\succ, P)|$
- $DA^{mp}_i(\succ', P)\ R_i\ DA^{mp}_i(\succ, P)$ *and* $|DA^{mp}_m(\succ', P)| \geq |DA^{mp}_m(\succ, P)|$

Where $\succ = (\succ_m)_{m\in M}$, *and* $\succ' = (\succ_{m'})_{m'\in M\setminus m} \cup \{\succ'_m\}$

Proof. Consider first DA^{sp} under the cases where priority orders are given by $\succ$ and $\succ'$.

Consider the following sequential version of DA^{sp}, similar to the one defined in the proof of Theorem 1, in which each step only one student who is not tentatively held by a major applies. Specifically, we do the following for both cases:

- In each step, if the student in turn applies to some major $m' \neq m$, then we apply the choice function of m' and the new applicant under both cases will have the same result since the choice function of m' is the same in each case.
- If the student in turn applies to m, then she is added to the applicant pool for m, denoted by A_m. If $|A_m| > q_m$, then we run the choice function of m using priority order $\succ'_m$. Notice that among students in A_m, only one student who is not in I_m will be rejected and that student is ranked lowest among applicants in $A_m \setminus I_m$ under $\succ'_m$. It is easy to verify that this student will be rejected under the choice function when priority order $\succ_m$ is used.
- If an unacceptable student under $\succ'_m$ applies to m, reject her under both cases.

Due to the finite number of students and majors, after some steps, each student is either tentatively held by a major $m' \neq m$ or is in A_m and $A_m \leq q_m$. First, we consider the case where $\succ'_m$ is the priority order. Since all students in A_m are acceptable under $\succ'_m$, they all will be accepted and the procedure ends.

If the procedure ends in the other case, i.e., all students in A_m are accepted, then students get the same outcome for both cases. Otherwise, we will continue and students will become weakly worse off in each further step.

If x students are rejected from A_m when priority order $\succ_m$ is used, then in each further step the number of tentatively held students by other majors will not decrease. As a result, the number of students accepted by major m under this case cannot exceed A_m.

Hence $DA^{sp}_i(\succ', P)\ R_i\ DA^{sp}_i(\succ, P)$ and $|DA^{sp}_m(\succ', P)| \geq |DA^{sp}_m(\succ, P)|$.

Next consider DA^{mp} under the cases where priority orders are given by $\succ$ and $\succ'$.

Let $O_i^{k,\succ}$ denote the set of majors from which student i has received an offer in steps up to and including Step k given priority order $\succ$. Let $H_m^{k,\succ}$ denote the set of students who hold an offer from major m after Step k given $\succ$.

It is easy to verify that in Step 1, either $O_i^{1,\succ} = O_i^{1,\succ'}$ or $O_i^{1,\succ'} = O_i^{1,\succ} \cup \{m\}$ for all $i \in I$. As a result, $H_m^{1,\succ'} \supseteq H_m^{1,\succ}$ and $|H_m^{1,\succ'}| \geq |H_m^{1,\succ}|$. Let n denote the number of students who hold an offer from m under $\succ'$ but not under $\succ$. That is, $n = |H_m^{1,\succ'}| - |H_m^{1,\succ}| \geq 0$.

If $n = 0$, then $H_m^{1,\succ'} = H_m^{1,\succ}$ and in Step 2 students receive the same offers from any major $m' \neq m$ in either case. If $n > 0$, then in Step 2 some students may receive an offer from majors $m' \neq m$ under $\succ'$ that they do not receive under $\succ$. In either case, some students may receive offers from m under $\succ'$ that they do not receive under $\succ$. Additionally, any offer received by a student under $\succ$ by Step 2 will also be received under $\succ'$ by Step 2. Hence, $O_i^{2,\succ} \subseteq O_i^{2,\succ'}$ for all $i \in I$.

Further, there can be no more than n students receiving an offer from $m' \neq m$ in Step 2 under $\succ'$ than they would receive under $\succ$. Hence, no more than n students can receive such an offer that they would prefer to m. Furthermore, weakly more students receive a new offer from m in this step. Hence, $H_m^{2,\succ'} \supseteq H_m^{2,\succ}$.

Each Step $k > 2$ operates the same as Step 2 including the terminal step. Let Step K be the terminal step. Then $O_i^{K,\succ'} \supseteq O_i^{K,\succ}$ and $|H_m^{K,\succ'}| \geq |H_m^{K,\succ}|$.

Hence, $DA_i^{mp}(\succ', P)\ R_i\ DA_i^{mp}(\succ, P)$ and $|DA_m^{mp}(\succ', P)| \geq |DA_m^{mp}(\succ, P)|$. □

Note that, similar to when majors allow occupied seats to be available, majors can be assigned students of lower preferences when submitting a priority order that is an extension of another priority order. If we reconsider Example 3 above and let both majors allow all seats to be available, but consider the case where i_2 is unacceptable at m_1, we can see that DA^{sp} yields the same outcome as when m_1 does not allow all seats to be available. Hence, m_1 prefers the outcome where i_2 is unacceptable at m_1 to the outcome where i_2 is considered acceptable by m_1 under DA^{sp}.

Similarly, if we reconsider Example 4 when major m_1 allows occupied seats to be available, but suppose that i_3 is not acceptable at m_1 and otherwise leave the priority orders unchanged, then we can see that in this case the assignment of DA^{mp} would be the same as when m_1 does not allow occupied seats to be available. Hence, m_1 is assigned a student of lower preference when i_3 is acceptable than when i_3 is unacceptable under DA^{mp}.