Bias-Corrected Nonlinear Investment-q Relation in the Cross Section of Firms*

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September 3, 2023

ABSTRACT

We study a nonlinear relationship between firm investment and Tobin's q in the cross section of firms. After correcting for nonseparable measurement error in q with the aid of a repeated measurement, we find evidence of heterogeneity in the investment-q relation. The investment-q sensitivity is little for firms with low and high q. On the other hand, the relationship is most sensitive to intermediate values of q. In the cross section, the true investment-q relation is thus not thoroughly linear, and investments are rather clustered at both ends of q.

JEL Classification: C21, C26, E22, G31

Keywords: Investment, Tobin's q, Nonlinearity, Measurement Error

*We are grateful for comments and suggestions from David Bates, James Brown, Mohammad Ghaderi, Foti Grigoris, Preetesh Kantak, Hyung Joo Kim, Wei Li, Erik Lie, Felix Meschke, and Tong Yao. We also thank seminar participants at 2022 Silicon Prairie Finance Conference, 2021 FMA Annual Meeting, and the University of Iowa.

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1. Introduction

After Hayashi (1982) formalized that the neoclassical theory of investment originated by Jorgenson (1963) and the q theory suggested by Tobin (1969) are equivalent, many researchers have empirically tested the idea that the optimal rate of investment is solely determined by Tobin's marginal q (i.e., expected return to investment) using Tobin's average q (i.e., average value of capital) as a proxy for the unobservable true q. While most of the literature after Hayashi estimated linear relations between investment and q, only a few studies paid attention to the possibility of nonlinearity that may reside in the investment-q relation.

Although investment is indeed a function of marginal q, the linear functional form has been used for convenience. However, several theoretical and empirical pieces of evidence in the literature suggest nonlinearity in the investment-q relation. For example, Abel and Eberly (1994) suggest a regime of zero investment in which investment does not respond to q. Recently, Lee, Shin, and Stulz (2021) find that investment does not increase in the cross section with q for large firms. Our paper identifies a nonlinear relationship between investment and q in the cross section of firms by estimating the classic investment regression model of Fazzari, Hubbard, and Petersen (1988) augmented with higher-order terms in q. Specifically, a model that fits a nonlinear relation to the cross section is needed because firms with different q are likely to have different sensitivities of investment to q due to non-homogeneous investment goods, for example. We find that high and low q firms are the least sensitive while intermediate q firms are the most sensitive in adjusting their investments in response to a change in q.

When estimating a nonlinear errors-in-variables model, correcting measurement error bias is more complicated due to multiplicative measurement errors. Because of this nonseparability between mismeasured regressors and measurement errors, the existence of instrumental variables is not sufficient to control for measurement errors. We propose a consistent estimator that corrects for nonseparable measurement errors in higher-order q terms in polynomials.

Our framework is built on the recent literature on nonlinear measurement error. In particular, we adopt the identification strategy proposed by Hu and Schennach (2008) who provide sufficient conditions under which the conditional density of the true unobserved variable can be identified when the true variable is measured with error. We make use of an analyst-based measure of average q as our repeated measurement, which is carefully constructed to satisfy the sufficient conditions for the identification. The conditions are related to conditional independence assumptions associated with mismeasured average q. The assumptions are not violated when measurement errors in average q and the repeated measurement of average q do not have informational content about investment and are not correlated with each other. Additionally, either monotonicity of investment in the true q or conditional heteroskedasticity of investment is needed. Lastly, since the true q is not observed, we need a normalization assumption such as classical measurement error (i.e., conditional mean zero of measurement error given the true q) that is in line with the investment literature.

We find that our augmented regression model outperforms the classic model in fitting the investment-q relation, as evidenced by significantly higher log-likelihood values. Our model reveals an S-shaped investment-q relation in the cross section of firms. This finding predicts that the sensitivity of investment to q (i.e., elasticity of investment with respect to q) first increases gradually as q improves and then rapidly decreases when q exceeds an intermediate value. This pattern can be observed regardless of what proxy is used for q. This result implies that a positive change in q does not always lead a firm with high or low q to increase its investment.

Our paper contributes to the investment literature by revealing how the investment-q sensitivity evolves in the cross section with q. To the best of our knowledge, this is the first attempt to derive a bias-corrected cross-sectional relationship between investment and q in the presence of nonlinear measurement error. Ordinary least squares (OLS) gives biased results showing that investment and q are not closely related (Erickson and Whited, 2000). After correcting for measurement error, the classic investment regression with a linear q can still be misleading because it always predicts that all firms change their investments at the same rate regardless of their q characteristics. On the other hand, our augmented model provides evidence that the true investment-q relation is nonlinear. The augmented model showing an S-shaped relationship between investment and q predicts that investments of firms with low q are clustered at a low level. Similarly, investments of firms with high q are predicted to be clustered at a high level. In a very large q region, firms with higher q are predicted to invest less due to the continuously decreasing response of investment to intermediate-high q. These results are consistent with the theoretical predictions and empirical results in Abel and Eberly (1994), Abel and Eberly (2002), Barnett and Sakellaris (1998), Eberly (1997), and Lee et al. (2021).

Abel and Eberly (1994) extend the standard q theory of investment by adopting an augmented adjustment cost function, which incorporates fixed costs and irreversibility of investment. Investment is a non-decreasing function of marginal q and is in one of three regimes (positive, zero, or negative gross investment) in their model. The regime of zero gross investment makes the response of investment to q nonlinear. Abel and Eberly (2002) and Barnett and Sakellaris (1998) empirically test this idea and find nonlinear S-shapes with no regime of zero gross investment. Eberly (1997) also finds a nonlinear but convex relation from international data. In a recent study, Hoberg and Maksimovic (2022) develop a four-stage 10-K text-based model of product life cycles, which includes q interacted with proxies of firm-year product life cycle exposures. They find that firms initially focus on research and development (R&D). Capital expenditures (CAPX), acquisitions, and divestitures then emerge in sequence as firms mature in their life cycles. These papers estimate how the elasticity of investment develops as q changes within a firm. On the other hand, our paper estimates cross-sectional variation in the investment-q sensitivity. In another recent paper, Lee et al. (2021) show that investment does not increase in q in the cross section of large (old) firms and that the sensitivity of investment to q falls as firms become older and larger. They instead find that repurchases of large (old) firms increase with q in the cross-section, arguing that q is a proxy for rents from the past investment instead of investment opportunities for those firms. We also find that net payout consistently increases in q, which can be one reason for the decreasing response of investment to intermediate-high q.

One early strand of the literature examined the empirical failure of the neoclassical theory of investment. The empirical formulation of the theory performs poorly with real data compared to augmented models with proxies of financial constraints such as cash flow. Fazzari et al. (1988) find that when firms are financially constrained, investment spending also varies with the availability of internal funds. This finding may reflect the existence of asymmetric information in the financial markets and thus contradict the assumption of perfect capital markets. However, Fazzari et al. and the subsequent literature (e.g., Gilchrist and Himmelberg, 1995; Kaplan and Zingales, 1997; Cleary, 1999) are questioned by Erickson and Whited (2000), arguing that the neoclassical model cannot perform well if q is mismeasured.

Since marginal q is unobservable, average q is used instead. This is where a measurement error problem arises since marginal q and average q are unequal in the real world.[1] Using a measurement-error-robust generalized method of moments estimator on balanced panel data, Erickson and Whited (2000) find that cash flow is not relevant to investment decisions even for financially constrained firms, which corroborates good predictive power of the q theory in the absence of measurement error. Cummins, Hassett, and Oliner (2006) also cast doubt on the appropriateness of the existing measure of q, which uses stock prices to proxy for intrinsic value, pointing out its uninformativeness and thus persistent measurement error in it. Using financial analysts' earnings forecasts instead of stock prices, they construct an analyst-based measure of q and find that investment is not sensitive to cash flow. Although positive coefficients on cash flow are observed in a later work by Erickson and Whited (2012) using a high-order moment estimator on unbalanced panel data, they emphasize that

[1] Assuming that the profit function and the adjustment cost function are linearly homogeneous, Hayashi (1982) and Abel and Eberly (1994) show that firm value is equal to marginal q times capital stock and that marginal q and average q are thus essentially the same. It is well known that if a firm is a price taker in output and factor markets with constant returns to scale in production, the profit function is linearly homogeneous.

the estimates are much smaller in magnitude and significance than the OLS counterparts. Erickson, Jiang, and Whited (2014) then apply a new closed-form cumulant estimator to the classic investment regression and find results consistent with Erickson and Whited (2000).

On the other hand, Almeida, Campello, and Weisbach (2004) find that constrained firms save cash out of cash flows, which suggests that the influence of financial constraints on corporate policies is manifested in the form of firms' demand for liquidity. Almeida and Campello (2007) also find that the investment-cash flow sensitivity increases in the tangibility of a firm's assets when a firm is financially constrained, which indicates that the influence of financing frictions on investment decisions is multiplied by asset tangibility. Furthermore, Almeida, Campello, and Galvao (2010) assess the performance of methods dealing with measurement error in q and find that investment decisions are consistently affected by cash flow. Lewellen and Lewellen (2016) find results that cash flow still matters for investment decisions after considering its correlation with q, and Ağca and Mozumdar (2017) also conclude that cash flow is a significant predictor of investment after employing different estimators and alternative proxies of q. Firpo, Galvao, and Song (2017) use quantile regression and find that the investment-cash flow sensitivity is relatively stronger at the lower part of the conditional distribution of investment. With their own efforts to resolve the measurement error problem, these studies provide evidence that internal funds consistently matter for investment decisions. The interpretation of the investment-cash flow sensitivity is, however, controversial. Even in the absence of financing constraints, investment is predicted to be responsive to the availability of internal funds in recent theories (e.g., Abel and Eberly, 2011; Gourio and Rudanko, 2014; Abel, 2018).

Corporate investment is still an actively studied topic, and we have three important recent studies to introduce. Peters and Taylor (2017) find that q explains investment better when intangible capital is taken into account. They show that the so-called "total q" is superior to the standard measure of q in that it can proxy for both physical and intangible investment opportunities. Woeppel (2022) develops another new measure of Tobin's q. "Patent q" that incorporates the replacement cost of patent capital is shown to strengthen the historically weak investment-q relation. Andrei, Mann, and Moyen (2019) find that the investment-q relation itself has also become tighter in recent years than earlier times and claim that the reason is on growing empirical dispersion in q both in the cross section and the time series.

This paper proceeds as follows. Section 2 describes the data. Section 3 explains the identification strategy. Section 4 presents the results. Section 5 discusses the results. Section 6 concludes.

2. Data

We use data from 1982 to 2017. Our sample includes all manufacturing firms on Compustat except utilities (Standard Industrial Classification codes 4900 – 4999), financial firms (6000 – 6999), and public administration firms (9000 – 9999). We require firms to be covered by IBES to construct a repeated measurement $\hat{q}$ using analysts' forecasts. We further discuss our repeated measurement in Sections 2.2 and 3.1. We exclude firm-year observations with missing or non-positive book value of assets or sales and observations with missing or less than $5 million real 1990 dollars in gross property, plant, and equipment (PP&E). We also exclude firm-years with negative $\hat{q}$, which implies a negative firm value, or $\hat{q}$ in excess of 50, which is an unrealistically large value.[2]

Some studies in the literature use balanced panel data with relatively short sample periods (e.g., Whited, 1992; Himmelberg and Petersen, 1994; Gilchrist and Himmelberg, 1995). We instead use unbalanced panel data, so survivorship bias is not our concern.[3] We drop observations missing any of the regressors. To remove extreme outliers, we winsorize the

[2]Cummins et al. (2006) and Almeida and Campello (2007) delete firm-years with negative average q. Researchers sometimes discard observations with unrealistically large average q as a crude attempt to limit the impact of measurement error. For example, Eberly (1997) drops observations with average q in excess of 15. Gilchrist and Himmelberg (1995) and Almeida and Campello (2007) eliminate observations with average q exceeding 10. Abel and Eberly (2002) restrict average q to be less than 5.

[3]Barnett and Sakellaris (1998) and Almeida and Campello (2007) also use unbalanced panel data but require at least five and three consecutive years of data for each firm, respectively. The minimum number of years required in Almeida and Campello is based on the lag structure of their regression models and instrumental variables approach.

regression variables over the entire panel at once at the 1st and 99th percentiles. Our final sample consists of 58,796 firm-year observations. The first year is 1983 because the model specifications include lagged values.

2.1. Tobin's q, Investment, and Cash Flow Measures

Peters and Taylor (2017) show that their new measure of Tobin's q, which is total q, can explain both physical and intangible investments better by accounting for intangible capital. We thus employ both the existing physical measure of Tobin's q, say physical q, and total q. In the literature, physical q denoted by q^{phy} is measured as firm value scaled by the replacement cost of physical assets:

$$q_{it}^{phy} = \frac{V_{it}}{K_{it}^{phy}}. \tag{1}$$

We measure firm value V as the market value of equity (Compustat items *prcc_f* $\times$ *csho*) plus the book value of debt (*dltt*+*dlc*) minus the book value of current assets (*act*). The replacement cost of physical capital denoted by K^{phy} is measured as the book value of gross PP&E (*ppegt*). Total q denoted by q^{tot} is measured as firm value scaled by the replacement cost of total capital K^{tot}, which is the sum of physical and intangible capital:

$$q_{it}^{tot} = \frac{V_{it}}{K_{it}^{tot}} = \frac{V_{it}}{K_{it}^{phy} + K_{it}^{int}}. \tag{2}$$

The replacement cost of intangible capital denoted by K^{int} is Peters and Taylor's (2017) estimate of intangible capital.

Physical investment denoted by i^{phy} is measured as CAPX (*capx*) divided by the replacement cost of lagged physical capital:

$$i_{it}^{phy} = \frac{CAPX_{it}}{K_{it-1}^{phy}}. \tag{3}$$

Total investment denoted by i^{tot} is measured as the sum of CAPX and intangible investment (i.e., R&D (*xrd*) plus 30% of Selling, General and Administrative (SG&A) expenses (*xsga*–*xrd*–*rdip*)) divided by the replacement cost of lagged total capital following Peters and Taylor (2017):

$$i_{it}^{tot} = \frac{CAPX_{it} + R\&D_{it} + 0.3 \times SG\&A_{it}}{K_{it-1}^{tot}}. \tag{4}$$

Standard cash flow denoted by c^{phy} is measured as income before extraordinary items (*ib*) plus depreciation (*dp*) divided by lagged physical capital. We also construct Peters and Taylor's (2017) measure of cash flow, which is total cash flow c^{tot}, by adding tax-adjusted intangible investment back to the free cash flow in the numerator and dividing it by lagged total capital:

$$c_{it}^{phy} = \frac{ib_{it} + dp_{it}}{K_{it-1}^{phy}}, \quad c_{it}^{tot} = \frac{ib_{it} + dp_{it} + (CAPX_{it} + R\&D_{it} + 0.3 \times SG\&A_{it})(1-\tau)}{K_{it-1}^{phy} + K_{it-1}^{int}}, \tag{5}$$

where τ is the marginal tax rate. We use the simulated marginal tax rates based on income before interest expense from Graham (1996). When a simulated marginal tax rate is not available, it is assumed to be 30%, which is close to the average marginal tax rate in the sample.

2.2. *Analyst-Based Measure of Tobin's q*

Our identification strategy relies on the availability of a repeated measurement. Motivated by Cummins et al. (2006), we construct an analyst-based measure of average q (henceforth, analyst q) and use it as our repeated measure of average q. Compared to the existing market-based measure of q, analyst q adopts a different numerator in which equity value is measured based on analysts' earnings forecasts on IBES.

We focus on the means (IBES item *meanest*) of analysts' forecasts of earnings per share (*measure*=EPS) over the current (*fiscalp*=ANN with *fpi*=1) and the next fiscal years (*fiscalp*=ANN with *fpi*=2) and the mean (*meanest*) of long-term growth forecasts (*fiscalp*=LTG with *fpi*=0), which in general represents analysts' consensus on the average annual growth of earnings over three years after the next fiscal year. In the neoclassical theory of investment, firms make investment decisions based on the expected long-term returns to capital. A long-term growth forecast can thus be a strong predictor of investment. Due to the limited availability of long-term growth forecasts on IBES, our sample starts from 1983.

Shortly after the beginning of a firm's fiscal year, analysts send IBES initial forecasts of earnings for that year and the next few fiscal years. We use the first forecasts for the current and the next fiscal years as well as the first long-term growth forecast to reduce the risk of using more information than a firm actually has when deciding its investment spending for the current fiscal year. We then measure the analyst-based measure of equity value $\hat{E}$ using those forecasts and construct $\hat{q}$, which is the analyst-based measure of q:

$$\hat{q}^{phy}_{it-1} = \frac{\hat{E}_{it} + dltt_{it-1} + dlc_{it-1} - act_{it-1}}{K^{phy}_{it-1}}, \quad \hat{q}^{tot}_{it-1} = \frac{\hat{E}_{it} + dltt_{it-1} + dlc_{it-1} - act_{it-1}}{K^{tot}_{it-1}}, \tag{6}$$

where $\hat{E}_{it} = \Pi_{it} + \frac{1}{1+r_t}\Pi_{it+1} + \frac{1}{(1+r_t)^2}\overline{\Pi}_{it}(1 + \text{LTG}_{it}) + \frac{1}{(1+r_t)^3}\overline{\Pi}_{it}(1 + \text{LTG}_{it})^2 + \frac{1}{(1+r_t)^4}\overline{\Pi}_{it}(1 + \text{LTG}_{it})^3 + \frac{1}{(1+r_t)^4}\frac{1}{\bar{r}-\bar{g}}\overline{\Pi}_{it}(1+\text{LTG}_{it})^4$. Π_{it} and Π_{it+1} represent the averages of earnings forecasts per share over the current and the next fiscal years multiplied by the number of shares outstanding (IBES item *shout*). $\overline{\Pi}_{it}$ is the average of these two annual forecasts. LTG_{it} is the mean of analysts' long-term growth forecasts. We use the one-year Treasury bill rate in year t plus an assumed equity risk premium of 8% as a discount factor r_t, which proxies for the annual nominal equity return expected by investors in year t. $\bar{r}$ is the mean nominal equity return (12%) during the sample period. $\bar{g}$ is the mean growth rate of nominal GDP (5%) during the sample period. Other non-IBES items are lagged by one year to align them with the time when $\hat{E}_{it}$ becomes available and construct lagged $\hat{q}$.

2.3. Summary Statistics

Table 1 provides summary statistics for the regression variables that are mainly investment, lagged Tobin's q, and contemporaneous cash flow. Total investment shows a different distribution, which is slightly right to the distribution of physical investment. Considering that total investment is scaled by the larger denominator K^{tot}, this indicates that the amount of intangible investment itself is large. Our consideration of intangible capital and intangible investment can thus help reveal the true relationship between investment and q.

[Table 1 here]

We also construct other versions of total q. Using Woeppel's (2022) estimate of intangible capital (i.e., patent capital plus on-balance sheet intangible capital), we construct patent q denoted by q^{pat}. Using Ewens, Peters, and Wang's (2022) estimate of intangible capital, we construct "EPW q" denoted by q^{EPW}. Overall, patent q is higher than total q and EPW q. Since patents are only considered as internally created intangible capital in patent q, its denominator is smaller than those of the other two proxies. Nevertheless, patent q still gives a more reasonable range of q compared to physical q.

The analyst-based measures of q are consistently higher than the corresponding market-based measures of q in every statistic of the distributions. This reflects the finding that analysts' forecasts are over-optimistic (e.g., De Bondt and Thaler, 1990; Easterwood and Nutt, 1999). In other words, analysts tend to believe that firms have better growth opportunities than the financial markets perceive. Nevertheless, both types of q are highly correlated. The correlation coefficients $\rho(q^{phy}, \hat{q}^{phy})$ and $\rho(q^{tot}, \hat{q}^{tot})$ are 0.79 and 0.75, and $\rho(q^{pat}, \hat{q}^{pat})$ and $\rho(q^{EPW}, \hat{q}^{EPW})$ are 0.79 and 0.74, respectively. Overall, total cash flow has lower values than standard cash flow. This is due to the bigger denominator K^{tot}, although tax-adjusted intangible investment is added back to the numerator.

To examine how payout differs across firms with different values of q, we additionally construct a payout variable. Since firms often issue and repurchase equity simultaneously, a positive gross payout does not always indicate distribution of funds to shareholders (Grullon, Paye, Underwood, and Weston, 2011). Net payout is rather computed as cash dividends (Compustat item *dv*) plus the purchase of common and preferred stock (*prstkc*) minus the sale of common and preferred stock (*sstk*), scaled by lagged total capital. Net payout is set to zero for missing or negative values.

3. Identification and Implementation

The investment literature has been interested in the true investment-q relation, and most studies estimate linear relations in various ways such as the higher-order moment estimator (Erickson and Whited, 2000), instrumental variables approaches (e.g., Almeida et al., 2004; Almeida and Campello, 2007), and repeated measures approaches (e.g., Firpo et al., 2017) to correct for measurement error in q. Considering potential nonlinearity in the investment-q relation, we fit a nonlinear relation to the data to capture heterogeneous investment-q sensitivities across firms. Polynomial regressions with mismeasured regressors such as the classic investment regression augmented with higher-order terms in q inevitably contain measurement errors nonseparable from the true regressors. Unfortunately, the conventional instrumental variables approach cannot control for nonseparable measurement error. Instead, nonlinear errors-in-variables models are identifiable through an eigenvalue-eigenfunction decomposition.

3.1. Identification of the True Model

The true investment model is defined by the distribution of investment conditional on marginal q. Since marginal q is unobservable, its error-contaminated counterpart, which is average q, is used instead in empirical research. Using the framework in Hu and Schennach (2008), the conditional distribution of investment given marginal q is identifiable from the distribution of observed variables. Also, their treatment of measurement error models can be extended to allow for a vector of additional correctly measured regressors by conditioning all relevant densities on that vector (Song, 2015). Cash flow is thus included in the estimation considering its explanatory power in predicting investment. Given the availability of a repeated measurement of average q, we propose an estimator built on the nonlinear measurement error literature and derive the true investment-q relation by correcting for nonseparable measurement error in q.

We use investment (denoted by i), mismeasured average q (denoted by q^A), cash flow (denoted by c), and a repeated measure (denoted by q^H) for the identification of the conditional distribution of investment given unobserved marginal q (denoted by q^M) and cash flow. As long as we correctly identify the density of interest, we can estimate the true investment model expressed in terms of the unobserved q^M and the observed c:

$$f_{i|q^M c}(i|q^M, c). \tag{7}$$

Let $\mathcal{I}$, $\mathcal{Q}^{\mathcal{A}}$, $\mathcal{Q}^{\mathcal{M}}$, $\mathcal{C}$, and $\mathcal{Q}^{\mathcal{H}}$ denote the supports of the distributions of the random variables i, q^A, q^M, c, and q^H. Assumptions 1–4 in the Appendix lead to a theorem that enables the identification of unknown densities associated with the unobserved q^M. Theorem 1 in the Appendix states that under Assumptions 1–4, the equation

$$f_{i\,q^A|q^H c}(i, q^A|q^H, c) = \int_{\mathcal{Q}^{\mathcal{M}}} f_{i|q^M c}(i|q^M, c) f_{q^A|q^M c}(q^A|q^M, c) f_{q^M|q^H c}(q^M|q^H, c)\, dq^M \tag{8}$$

admits a unique solution $(f_{i|q^M c}, f_{q^A|q^M c}, f_{q^M|q^H c})$ for all $i \in \mathcal{I}$, $q^A \in \mathcal{Q}^{\mathcal{A}}$, $c \in \mathcal{C}$, $q^H \in \mathcal{Q}^{\mathcal{H}}$, given the observed density $f_{i\,q^A|q^H c}(i, q^A|q^H, c)$. Note that the true q^M is integrated out on the right-hand side of the equation.

Here, we provide a brief proof of the theorem (see the Appendix for details). Note that

Assumption 1 implies

$$\begin{aligned}
f_{i\,q^A|q^H c}(i, q^A|q^H, c) &= \int_{\mathcal{Q}^{\mathcal{M}}} f_{i\,q^M q^A|q^H c}(i, q^M, q^A|q^H, c)\, dq^M \\
&= \int_{\mathcal{Q}^{\mathcal{M}}} f_{i|q^M c}(i|q^M, c) f_{q^A|q^M c}(q^A|q^M, c) f_{q^M|q^H c}(q^M|q^H, c)\, dq^M.
\end{aligned}$$

By the definitions of the operators in the Appendix, we have the following operator equivalence:

$$L_{i\,q^A|q^H c} = L_{q^A|q^M c} D_{i|q^M c} L_{q^M|q^H c}. \tag{9}$$

Assumption 2 implies that the relevant operators are one-to-one, so we get

$$L_{i\,q^A|q^H c} L^{-1}_{q^A|q^H c} = L_{q^A|q^M c} D_{i|q^M c} L^{-1}_{q^A|q^M c}.$$

It shows that the operator on the left-hand side defined in terms of the observed variables admits an eigenvalue-eigenfunction decomposition on the right-hand side. Assumptions 3–4 ensure the uniqueness of the spectral decomposition following Theorem XV.4.3.5 in Dunford and Schwartz (1971). As a result, the eigenvalues of the operator $D_{i|q^M c}$ provide the unobserved density of interest $f_{i|q^M c}(i|q^M, c)$.

A model satisfying Assumptions 1–4 can be easily constructed because they are not mutually contradictory. Specifically, the existence of a repeated measurement (or an instrument) satisfying the assumptions facilitates the identification of a model of interest under the least restrictive possible conditions. Analyst q can serve as a valid repeated measurement for the identification of the true investment-q relation. The forward-looking nature of forecasts makes analyst q relevant with the corresponding market-based measure of q satisfying Assumption 2. Since analyst q is not a determinant of investment, Assumption 1 (i) is satisfied. More importantly, analyst q is unlikely to provide further information on the mismeasured q given the true q and cash flow, which satisfies Assumption 1 (ii). First, the forecast database has its own source and data collection method, which makes measurement error in analyst

q unlikely to be correlated with measurement error in the market-based measure of q given the true q and cash flow (Abel and Eberly, 2002). Second, we use mean forecasts instead of medians. Mean forecasts are more affected by outliers, and we exploit this feature on purpose for our estimation. By using mean estimates, analyst q is still relevant, and at the same time, we can introduce more forecast errors to the repeated measure, making it much less likely to be correlated with measurement error in the existing market-based measure.

Given the true investment model in Equation (7), Equation (8) suggests the following measurement-error-robust maximum likelihood estimator:

$$(\hat{\theta}_0, \hat{\theta}_1, \hat{\theta}_2) =$$
$$\underset{(\theta_0,\theta_1,\theta_2)\in\Theta}{\arg\max} \frac{1}{n}\sum_{j=1}^{n} \ln \int_{\mathcal{Q}^{\mathcal{M}}} f_{i|q^M c}(i_j|q^M, c_j; \theta_0) f_{q^A|q^M c}(q_j^A|q^M, c_j; \theta_1) f_{q^M|q^H c}(q^M|q_j^H, c_j; \theta_2)\, dq^M, \tag{10}$$

where Θ is the parameter space. The analyst-based measure of q is used as q^H. When the dependent variable is physical investment i^{phy}, the physical measures q^{phy}, c^{phy}, and $\hat{q}^{phy}$ are used as q^A, c, and q^H, respectively. Similarly, when the dependent variable is total investment i^{tot}, the total measures q^{tot}, c^{tot}, and $\hat{q}^{tot}$ are employed.

3.2. Practical Implementation

We propose the parametric version of the estimator.[4] We approximate the unobservable conditional densities by normal distributions with the assumption of conditional het-

[4] It is straightforward to extend the estimator to a nonparametric or semiparametric counterpart. However, it is accompanied by a considerably higher computational burden.

eroskedasticity of investment (i.e., Assumption 3):

$$(i)\;\; f_{i|q^M c}(i|q^M, c; \theta_0) \dot{\sim} N(\alpha_0 + \alpha_1 q^M + \alpha_2 (q^M)^2 + \cdots + \alpha_m (q^M)^m + \alpha_{m+1} c,\;\; e^{\sigma_{01} + \sigma_{02} q^M + \sigma_{03} c}),$$

$$(ii)\;\; f_{q^A|q^M c}(q^A|q^M, c; \theta_1) \dot{\sim} N(\beta_0 + \beta_1 q^M + \beta_2 c,\; \sigma_1^2),$$

$$(iii)\;\; f_{q^M|q^H c}(q^M|q^H, c; \theta_2) \dot{\sim} N(\gamma_0 + \gamma_1 q^H + \gamma_2 c,\; \sigma_2^2),$$

where $\theta_0 = (\alpha_0, \alpha_1, \ldots, \alpha_{m+1}, \sigma_{01}, \sigma_{02}, \sigma_{03})^{\mathrm{T}}$, $\theta_1 = (\beta_0, \beta_1, \beta_2, \sigma_1)^{\mathrm{T}}$, and $\theta_2 = (\gamma_0, \gamma_1, \gamma_2, \sigma_2)^{\mathrm{T}}$. We test whether augmented regression models with higher-order terms in q perform better than the classic investment regression by comparing log-likelihood values.

The virtue of the estimator lies in its wide applicability, encompassing measurement error models ranging from classical linear errors-in-variables models to nonclassical nonlinear errors-in-variables models. We impose Assumption 4 on the conditional density $f_{q^A|q^M c}(q^A|q^M, c; \theta_1)$ in the form of classical measurement error following the literature (e.g., Erickson and Whited, 2000; Erickson et al., 2014; Abel, 2018). Abel and Panageas (2022) show that nonclassical measurement error in average q is theoretically possible when there is a stark financial constraint that precludes raising external funds. Given that analysts tend to follow firms of investors' interest and typically provide coverage to larger firms within an industry (Bhushan, 1989), it is unlikely that firms in our sample, covered by IBES, face such an extreme financial constraint. Classical measurement error has zero mean conditional on marginal q and cash flow: $M[f_{q^A|q^M c}(\cdot|q^M, c)] = E[q^A|q^M, c] = E[q^M + \varepsilon|q^M, c] = q^M + E[\varepsilon|q^M, c] = q^M$ where $M[f] = \int_{\mathcal{X}} x f(x) dx$. This leads to the following restrictions on the coefficients of the conditional density: $\beta_0 = 0$, $\beta_1 = 1$, and $\beta_2 = 0$.

q^{phy} ranges from -0.49 to 37.69, and q^{tot} ranges from -0.18 to 10.70, while $\hat{q}^{phy}$ ranges from 0.12 to 38.72, and $\hat{q}^{tot}$ ranges from 0.06 to 12.70. When the dependent variable is i^{phy}, q^M is assumed to range from 0 to 16.87, which is the 95th percentile of q^{phy}. Likewise, when i^{tot} is the dependent variable, we assume that q^M ranges from 0 to 4.69, which is the 95th percentile of q^{tot}. We then conduct the numerical integral in Equation (10) through the

Gaussian quadrature.

The optimization procedure is repeated with plausible initial parameter guesses to minimize the risk of not reaching the global optimum. We use 100 random initial values around the coefficients estimated by OLS. One convenient and thus widely used empirical approach to control for potential functional form misspecification is to include squared and cubed versions of variables of interest. To choose the degree of a polynomial m, we instead continue to include higher-order q terms until the next higher-order term does not increase the log-likelihood significantly.

4. Estimation Results

In this section, we test whether nonlinearity exists in the investment-q relation in the cross section of firms. We first estimate a linear relation and then fit a nonlinear relation using higher-order terms in q while correcting for measurement errors in q terms using the proposed estimator above.

4.1. Measurement-Error-Corrected MLE Results

When estimating a regression model with mismeasured regressors, OLS gives biased results. Specifically, in the case of investment regressions, coefficients estimated by OLS are biased due to measurement error in average q. Several seminal studies in the literature address this issue. For example, Erickson and Whited (2000, 2002) develop the higher-order moment estimator, and Erickson et al. (2014) develop the higher-order cumulant estimator to correct for measurement error in the observed q. These methods estimate linear errors-in-variables models such as the classic investment regression. In the classic model, measurement error is separable from the true q. In contrast, we estimate polynomials to capture potential nonlinearity, and measurement errors are nonseparable from the true q in higher-order terms. Accordingly, we use a different estimator to correct measurement error bias in polynomial regressions. Moreover, the measurement error issue is not a special case of endogeneity anymore in our estimation due to the nonseparability (Song, 2015; Song, Schennach, and White, 2015). As a result, the existence of a conventional instrument satisfying the relevance condition and the exclusion restriction is not sufficient to correct measurement error bias in the augmented investment regression.

As described in Section 3, we use measurement-error-robust maximum likelihood estimation (MLE). The MLE approach can consistently estimate the parameters of interest from the joint distribution of the observed variables after correcting for measurement errors residing in a polynomial in a nonseparable fashion. We find the optimal model specification by adding higher-order terms in q until the log-likelihood does not increase. We then test whether the higher-order terms bring a significant improvement in log-likelihood to the classic investment regression through the likelihood-ratio test.

Table 2 contains the estimation results. Columns (1) and (2) show results from regressions of physical investment on lagged physical q terms and contemporaneous standard cash flow. Columns (3) and (4) report results from regressions using the total measures of investment, q, and cash flow. We report block bootstrapped standard errors by correcting standard errors using 500 bootstrap replications over firms. Specifically, we treat a specific firm's time-series data as a cluster (i.e., block) to account for possible correlation within a cluster.[5] We consistently find that a polynomial of degree 3 is optimal when fitting a nonlinear relation. The difference in log-likelihood for each pair of the classic and augmented models is statistically significant at the 1% level. The test statistics of the likelihood-ratio test for the null hypothesis of the classic model specification are extremely high and thus easily exceed the critical value of the chi-squared distribution with two degrees of freedom. The number of degrees of freedom is determined by the difference in the number of parameters between the nested models. The results show that q^2 and q^3 contribute to detecting different investment-q sensitivities across firms, which cannot be captured by the linear q

[5] Bootstrapping is used to estimate standard errors after MLE when the objective function is complicated. Petersen (2009) find that bootstrapped standard errors are almost identical to clustered standard errors.

alone. Since there is no unambiguous guideline on how to choose the degree of a polynomial m, Stock and Watson (2015) point out that it is appropriate to choose small orders like 2, 3, or 4 because economic data is often smooth. Our findings are also consistent with their argument, and the estimated nonlinear relations are shown to provide significantly better fits to the data compared to the linear relations.

[Table 2 here]

Notably, we can observe that total q performs better at explaining the cross section of firm investments than physical q, as it delivers greater log-likelihood values regardless of the model specification. This corroborates the previous finding in Peters and Taylor (2017) that total q is a superior proxy for investment opportunities.

Table 3 reports estimation results from both OLS and our proposed MLE estimators. The OLS counterparts of our estimation are provided in Columns (1) and (3) of Table 3. The results are from OLS regressions of investment on lagged Tobin's q terms, contemporaneous cash flow, and year fixed effects. The MLE results in Table 2 are restated in Columns (2) and (4) to compare the results.

[Table 3 here]

As in the MLE results, we find that the augmented model estimated by OLS fits the data better, as evidenced by higher adjusted R^2 values. Also in the OLS results, the total measures in Panel B outperform the physical measures in Panel A, as they deliver higher adjusted R^2s. Compared to the OLS results, the linear relations estimated by the proposed MLE estimator show much more sensitive investment-q relations in Column (2). The influence of cash flow on investment consistently becomes smaller after correcting for measurement error. Furthermore, the slope coefficients on cash flow are the smallest in the error-corrected polynomials in Column (4). These results reveal the effect of correcting attenuation bias in the OLS estimates of the slope coefficient on q. For easier interpretation of the results from the different estimators, we visualize them in Figure 1.

[Figure 1 here]

We plot each result in a way that investment at $q = 0$ equals the estimated intercept plus the slope coefficient on cash flow multiplied by its mean. Panels A and C plot the classic investment regression estimated by OLS and the measurement-error-robust MLE in Columns (1) and (2) of Table 3. The MLE estimate of the slope coefficient on physical q indicates a closer relationship between physical investment and physical q in Panel A compared to the OLS estimate. Specifically, physical investment shows sufficient variation in the assumed support of the true q, which is from 0 to the 95th percentile of the observed physical q, after measurement error in physical q is corrected. The same pattern of bias correction is observed in Panel C after correcting for measurement error in total q.

The goal of our paper is to estimate varying elasticities of investment with respect to q in the cross section of firms. Panels B and D plot the augmented model estimated by OLS and MLE in Columns (3) and (4) of Table 3. The polynomial of degree 3 estimated by OLS does not bring notable dynamics to the cross-sectional variation in investment. The plotted nonlinear relations are seemingly identical to the linear relations in Panels A and C, although they are actually concave. On the other hand, we can observe more dynamics from the MLE results, indicating that the true relationship between investment and q is actually tighter and more dynamic in the cross section. We can observe how the investment-q sensitivity evolves from the augmented regression. The estimated nonlinear investment-q relations from the proposed estimator are consistently S-shaped for both types of investment and q.

Figure 2 plots the error-corrected polynomials in Panels A and C and their first derivatives with respect to q in Panels B and D. The estimated nonlinear and linear physical investment-physical q relations in Panel A predict similar levels of investment for physical q below about 6. Physical investment is then predicted to be higher than in the linear relation for intermediate values of physical q up to about 12. The S-shape finally suggests that firms with higher q do not invest more or invest even less in the high q region where physical q is at least 12. We find a similar pattern from total investment and total q in Panel C. Compared to the classic investment regression, the augmented regression predicts similar levels of total investment for total q below about 2. Total investment is then predicted to be higher than in the linear relation for intermediate values of total q up to about 4. We can also observe non-increasing investment in the high q region where total q is at least 4, although it is not as conspicuous as in Panel A.

[Figure 2 here]

Panels B and D show that the sensitivity of investment first increases and then decreases in q. In Panel B, the physical investment-physical q sensitivity is around zero for low physical q below about 2 and becomes zero again for physical q around 12. It then becomes negative, meaning that the elasticity of physical investment with respect to physical q is non-positive for high physical q above 12. Similarly in Panel D, the sensitivity of total investment stays around zero again for high total q around 4 and then becomes negative for higher total q. To summarize, when q is small (i.e., $q^{phy} < 3$ and $q^{tot} < 1$), the response of investment to an improvement in q is not as instantaneous as predicted by the linear relations, as evidenced by the lower investment-q sensitivities than those derived from the classic investment regression. As q improves, the elasticity of investment gradually increases with respect to q up to some intermediate value (i.e., $q^{phy} = 6$ and $q^{tot} = 2$) in the corresponding support. After q exceeds an intermediate value, it starts to decrease in q. Moreover, the sensitivity of investment to very large q is predicted to be negative, indicating that firms with very high q are predicted to invest less than firms with less high q.

4.2. Other Proxies of q

Although total q is a superior proxy of the true q, it is not the only measure that can proxy for both physical and intangible investment opportunities. We further test nonlinearity with other versions of total q. Woeppel (2022) focuses on the market value of patents to measure internally created intangible capital that does not appear on the balance sheet and constructs patent q that incorporates the replacement cost of patent capital. Using Ewens, Peters, and Wang's (2022) estimate of intangible capital, we construct another q proxy and call it EPW q. Ewens et al. (2022) re-estimate parameters used in Peters and Taylor (2017) to construct intangible capital using market prices of intangibles of firms that exit publicly traded markets due to acquisitions or bankruptcies. Using patent q and EPW q, we again identify nonlinearity in the data, and the results are reported in Table 4.

[Table 4 here]

The true q is assumed to range from 0 to the 95th percentile of each proxy. Patent q and the corresponding total investment and total cash flow are used in Columns (1) and (2). EPW q and its corresponding total investment and total cash flow based on Ewens, Peters, and Wang's (2022) estimate of the fraction of SG&A invested in organizational capital are employed in Columns (3) and (4). We consistently find a polynomial of degree 3, and the augmented model consistently outperforms the classic model, as evidenced by significantly higher log-likelihoods. The slope coefficients on cash flow are also consistently smaller in the augmented model. Notably, EPW q outperforms patent q in explaining the cross-sectional variation in firm investment as shown in the higher log-likelihood values. The estimated models are plotted in Figure 3.

[Figure 3 here]

We can observe S-shapes from both proxies of q. Panels A and B show that when patent q is below 1, the investment-q sensitivity is little. It then starts to increase until $q^{pat} = 4$. After that, it decreases and stays around zero for patent q around 8. A similar pattern can be observed from EPW q in Panels C and D. The elasticity of investment is lower than predicted by the linear relation for EPW q below 1. It only increases until $q^{EPW} = 2$ and then decreases. The estimated elasticity is close to zero for EPW q around 4 and significantly negative for very large EPW q at least 4.5.

Overall, the estimation results from the measurement-error-robust MLE consistently suggest that the elasticity of investment with respect to q is low when q is small, meaning that investment does not increase instantly unlike in a linear relation when q starts to increase from 0. The sensitivity of investment to q increases gradually as q improves and then decreases in q above an intermediate value, resulting in firms with larger q do not invest more in a high q region. This phenomenon completes the observed S-shape. We can make two predictions from the S-shaped investment-q relation. First, when firms have similarly low or high q, ones with relatively larger q do not invest noticeably more than others. The levels of investment are rather clustered at certain rates (e.g., $i^{tot} = 0.1$ and $i^{tot} = 0.4$ in Panel C of Figure 2) for low and high q (e.g., $q^{tot} < 1$ and $q^{tot} \approx 4$). Moreover, if there is a firm with q larger than high values of q for which investments are clustered (e.g., $q^{phy} > 12$ in Panel A of Figure 2), the firm is predicted to invest less. Second, among firms with intermediate values of q (e.g., $1 \leq q^{EPW} \leq 4$ in Panel C of Figure 3), firms with higher q invest more than predicted in a linear relation in most cases. In other words, the true investment-q relation is in fact more sensitive than estimated by a linear relation for intermediate q.

We can also figure out which q proxy is the best at explaining the investment-q relation based on the log-likelihood values. Although all three new measures of Tobin's q can proxy for both physical and intangible investment opportunities and do outperform physical q, total q is shown to be the best. One possible reason is that total q provides a better reference for the support of the true q, which is important in our estimation. Considering that the 95th percentile of total q provides the lowest upper bound for the unobserved q, the range of the true q is expected to be much more limited than depicted by average q. We now employ total q and its corresponding measures of investment and cash flow for further analyses.

4.3. Investment-q Relation in Different Periods

We find that the classic investment regression augmented with q^2 and q^3 performs better and suggests an S-shape. Nevertheless, since the proposed estimator cannot accommodate fixed effects internally, we re-estimate polynomials of degrees 1 and 3 in different periods to see whether we can consistently observe a higher log-likelihood and an S-shape from the augmented model in each subperiod. We fit the models to four different periods, which are 1983 − 1991, 1992 − 1999, 2000 − 2008, and 2009 − 2017. The assumed range of the true q is from 0 to the 95th percentile of total q in each subsample, and the results are reported in Table 5.

[Table 5 here]

The augmented model consistently outperforms the classic model in all the subperiods, indicating that nonlinearity exists throughout the entire period. The visualized results are presented in Figure 4. We can observe S-shapes in all the subperiods. Non-increasing investment in a high q region is more evident in the first two periods, especially in the second period from 1992 to 1999. On the other hand, similar levels of investment are predicted by both the classic and augmented models in the last two periods. Nevertheless, when it comes to the investment-q sensitivity, the augmented model provides substantially different predictions for the recent periods as well.

[Figure 4 here]

Figure 5 plots the first derivatives of the classic and augmented models estimated by the proposed estimator with respect to q in different periods. Low and non-positive investment-q sensitivities in low and high q regions are found in all the subperiods. During the earliest period from 1983 to 1991, the response of investment to q is little for low q below about 0.7. It then increases but soon decreases and becomes non-positive around $q^{tot} = 2.5$. The investment-q sensitivity is predicted to be significantly negative for q around 3. In that region, firms with higher q are predicted to invest less as presented in Panel A of Figure 4. We find a similar pattern in the next period. In the original function (Panel B of Figure 4), we cannot find enough variation in investment in the low q region in which q is below 1 because of the low investment-q sensitivity depicted in Panel B of Figure 5. A negative response of investment to q is more evident in this period, which spans q from 4 to 5.44. In the recent periods, we can still find a first convex and then concave relationship between investment and q. However, a significantly negative elasticity of investment with respect to very large q is not observed in these periods. Instead, the estimated elasticity is not significantly different from zero for q around 5 in Panel C, indicating that firms with higher q do not invest more in that region. Moreover, investment is not sensitive to q at both ends of the support in the latest period. We find that investment is only sensitive to intermediate values of q in Panel D. The estimated sensitivity is not significantly different from zero for q below 1 and above 2.5.

[Figure 5 here]

These results confirm that firm investment in the cross section does not increase immediately in response to a change in q when q is low and does not further increase when q is high. In contrast, the response of investment is more sensitive than predicted by a linear relation to intermediate values of q in all the subperiods. We find similar results using the other proxies of q (untabulated).

The evidence so far supports nonlinearity between investment and q in the cross section of firms, which indicates that the true investment-q relation can be depicted more precisely by higher-order terms in q with the existing regressors. Specifically, an S-shaped relationship is found regardless of the q proxy, and the S-shape persists throughout the entire period. The S-shape suggests that firms with low and high q are not always predicted to increase their investments even if q improves. More importantly, this phenomenon cannot be observed from the OLS results that suffer from measurement error bias. We further discuss the MLE results in Section 5.

4.4. Cumulant Estimator Results

The classic investment regression can also be correctly estimated by the cumulant estimator of Erickson et al. (2014). The cumulant estimator estimates classical linear errors-in-variables models using the information in the higher-order cumulants of observable variables to identify coefficients of interest. We compare results from the proposed estimator with those from the cumulant estimator to see whether our estimator corrects measurement error bias in the classic investment regression in a way consistent with the cumulant estimator. The OLS and cumulant estimator results using the total measures of investment, q, and cash flow are reported in Columns (1) and (2) of Table 6.[6] The original data is within-year transformed to control for year fixed effects.

[Table 6 here]

Similar to the estimates from our estimator in Column (2) of Panel B of Table 3, the cumulant estimator produces a higher coefficient on q and a lower coefficient on cash flow in Column (2) of Table 6 after correcting attenuation bias. Those estimates are slightly different for the q slope (0.0718 vs. 0.0820) and largely different for the cash flow slope (0.3574 vs. 0.0313). Possible reasons for the discrepancies could be primarily due to the use of a different estimator and then the consideration of year fixed effects. Nevertheless, conditional on cash flow, our estimate of the q slope gives a similar investment-q sensitivity, suggesting the validity of our proposed estimator in addressing measurement error in average q. The same pattern of correcting attenuation bias can be observed from both estimators in the subperiods in Table 7. Compared to the OLS estimates (Columns (1) and (3) of Table 7), both the proposed estimator (Columns (1) and (3) of Table 5) and the cumulant estimator (Columns (2) and (4) of Table 7) produce larger slop coefficients on q and smaller slope coefficients on cash flow. Especially after 2000, the investment-q sensitivities estimated by

[6] Following Peters and Taylor (2017) and Woeppel (2022), we use the third-order cumulant estimator. All Sargan-Hansen J statistics associated with the third- and higher-order cumulants (untabulated) reject the null hypothesis that the overidentifying restrictions are valid.

these estimators are very similar (0.0652 vs. 0.0671 and 0.0867 vs. 0.0810) in Panel B of Tables 5 and 7, which corroborates the usefulness of our estimator in estimating the classic investment regression. More importantly, the necessity of our estimator is related to the goal of estimating a bias-corrected nonlinear relation with the aid of a repeated measurement.

[Table 7 here]

As discussed in Section 3.1, analyst q is a valid repeated measure for the identification of the true investment-q relation. We also check whether analyst q is a relevant measure of the true q. To this end, we investigate analyst q itself as a sole proxy of q using the cumulant estimator. Columns (3) and (4) of Table 6 report results from the cumulant estimator when using the analyst-based measure of total q in the classic investment regression. Compared to the market-based measure of total q, analyst total q produces a higher R^2 and a higher ρ^2, which is the coefficient of determination in the hypothetical regression of investment on the true q and cash flow. Moreover, the analyst-based measure of total q also produces a higher τ^2, which is the coefficient of determination in the hypothetical regression of the true q on a q proxy. These results indicate that the forward-looking nature of earnings forecasts makes the analyst-based measure of total q itself a good proxy of q, although it also contains measurement error. Similar results are found when using the other analyst-based measures of q and also when using the fourth- and higher-order cumulant estimators (untabulated).

5. Discussion

We find evidence that heterogeneous investment-q sensitivities exist in the cross section of firms. According to the estimated nonlinear investment-q relation, firms with low q invest similarly low. In other words, there is little variation in firm investment for low q. As q improves, the investment-q sensitivity gradually increases. It then starts to decrease in intermediate-high q, although investment keeps growing until q reaches a high value in an assumed support. The elasticity of investment with respect to large q is non-positive, and investment is predicted to decrease as q increases beyond a large value. In other words, firms with higher q are not predicted to invest more in a high q region. In this section, we discuss these results from both theoretical and empirical perspectives.

5.1. Capital Heterogeneity

We can find a theoretical basis for the S-shaped investment-q relation from Abel and Eberly (1994), Barnett and Sakellaris (1998), Abel and Eberly (2002), and Eberly (1997). The idea of a nonlinear relationship between investment and q originates from Abel and Eberly (1994) who extend the standard q theory of investment in Hayashi (1982) by incorporating fixed costs and irreversibility of investment (i.e., different purchase and resale prices of capital goods) in the adjustment cost function. They show that if the augmented adjustment cost function is non-differentiable at the level of investment equal to zero due to fixed costs and irreversibility of investment, there can be a region in which the optimal investment stays at zero but does not satisfy the first-order condition for some range of q. As a result, investment behavior in response to q alternates between responsive (i.e., positive and negative gross investments) and irresponsive (i.e., zero gross investment) regimes, which implies nonlinearity in the investment-q relation. Investment is still a nondecreasing function of q, but its sensitivity to q can vary across regions around the regime of irresponsive investment in the extended framework.

Barnett and Sakellaris (1998) empirically test this idea by allowing the relationship to vary across regimes defined by the level of average q. They emphasize that a direct test of Abel and Eberly's (1994) theory is difficult in practice because there are few observations of zero and negative investments when using the Compustat item *capx* as a gross investment variable. They instead show that the model can be further extended by embracing another regime. Suppose that an additional proportional cost incurs when a firm invests beyond a threshold rate of investment, say $I/K = \theta$. If θ equals the depreciation rate of capital stock, say δ, investment beyond δK is for expansion, and the price of capital for expansion can be higher than that for replacement. If θ is the usual rate of investment and a firm invests beyond that, an additional proportional cost can incur in the form of overtime wage for installing additional capital goods. As a result, the augmented adjustment cost function has a kink at $I/K = \theta$, and there comes another region of insensitivity to q, resulting in three regimes for positive investment.

Barnett and Sakellaris (1998) then estimate a nonlinear specification in each regime that includes up to a cubic term in average q or a cubic term in the log of average q to take account of the effect of higher-order terms in q.[7] The investment-q relation is shown to be better explained under the three regimes. The estimated nonlinear relation is convex for low q and concave for intermediate and high q with no regime of insensitivity, which to some extent resembles the S-shape in our results. However, the estimated relationship is overall concave because investment is convex only in a very limited range of small q.

Abel and Eberly (2002) and Eberly (1997) specify the parametric form of the augmented adjustment cost function and show that a firm starts to invest when its q exceeds an upper threshold, say $\bar{q}$. In other words, a firm decides to invest when the total return on investment is high enough to cover at least the total costs. Abel and Eberly find significant nonlinearity by allowing for either a nonquadratic adjustment cost function or capital heterogeneity.[8] A concave relationship between investment and q is estimated in their homogeneous capital model with nonquadratic adjustment costs. On the other hand, their heterogeneous capital model, which enables a firm to invest in heterogeneous capital goods, provides some rationale for the S-shape found in our paper. In the heterogeneous capital model, firms increase their investments both in the scale (intensive margin) and in the number of types (extensive margin) as q grows. When q is low, firms choose to invest little and in few types of capital.

[7] The three regimes are defined by threshold parameters, say ω_h and ω_l. These are nuisance parameters that are not identified under the null hypothesis of only one regime. These parameters are identified by estimating an alternative threshold model using the technique developed by Hansen (1996). See Barnett and Sakellaris (1998) for details.

[8] Abel and Eberly (2002) emphasize that even if the adjustment cost function is quadratic, so the investment-q relation is linear for any given type of capital, the aggregation across different types of capital with different investment thresholds can make the overall relationship nonlinear.

The model predicts a stronger response of investment as q improves beyond low values, and an increasing investment-q sensitivity can be interpreted as an outcome of both intensive and extensive margins being reflected in investment decisions. However, the extensive margin is gradually depleted as q keeps growing. This phenomenon predicts a decreasing response of investment, which is also found in our results.

Eberly (1997) estimates the homogeneous capital model and finds convex investment-q relations in 6 of 11 countries including the U.S. In the heterogeneous capital model, Eberly shows that a degenerate extensive margin can be rejected in 9 of 11 countries and that a degenerate intensive margin can also be rejected in 4 of the 9 countries. This means that both margins play significant roles in deciding investment spending in those four countries including the U.S., which predicts a convex relationship between investment and q. The contradictory results between Abel and Eberly (2002) and Eberly (1997) could be due to different samples. Abel and Eberly use an unbalanced panel of U.S. firms, while Eberly uses a balanced panel of international firms. Eberly emphasizes that sample selection is a critical issue in identifying nonlinearity because the distribution of q determines where to be examined, and one's conclusion can be affected accordingly. On the other hand, it is less important in a linear relation because an identical relationship between investment and q is assumed for different values of q under linearity.

The S-shape in the cross section of firms can also be interpreted by the heterogeneous capital model. For firms with low q (e.g., $q^{tot} < 1$ in Panel C of Figure 2), their investments are clustered at a certain low level with little variation (e.g., $i^{tot} = 0.1$ in Panel C of Figure 2). This is consistent with the prediction in the heterogeneous capital model that firms choose to invest little and in few types of capital when q is low. The elasticity of investment with respect to q first increases and then decreases as q keeps improving, resulting in a decreasing response of investment to q above an intermediate value (e.g., $q^{tot} > 2$ in Panel D of Figure 2). This prediction is also consistent with the heterogeneous capital model in which firms invest more both in the scale and in the number of types of capital goods as q grows and then begin to invest mostly in the scale because only a few types of capital remain for investment when q is high.

5.2. Industrial Organization (IO) q

Interestingly, the elasticity of investment with respect to q becomes non-positive for large q (e.g., $q^{tot} \geq 4$ in Panel D of Figure 2) in our results. This phenomenon results in investments clustered at a certain high level (e.g., $i^{tot} = 0.4$ in Panel C of Figure 2) in a high q region, suggesting that the intensive margin can also be depleted. Moreover, firms reduce their investments, although q increases in a very high q region. The response of investment to very large q is predicted to be significantly negative, as evidenced by the plotted confidence bands in Panel D of Figures 2 and 3, for example. Although depletion of the intensive margin is not considered in the heterogeneous capital model, it is plausible in practice because firms cannot invest infinitely in a limited set of capital goods. There are also theoretical and empirical perspectives that view q as a proxy for rents instead of investment opportunities in a certain situation.

In the industrial organization literature, q is mostly used as a measure of monopoly and Ricardian rents (e.g., Stigler, 1964; Lindenberg and Ross, 1981; Montgomery and Wernerfelt, 1988). Lee et al. (2021) predict that if a firm has high q because of rents, it would not increase its size through investment because it wants its unique assets to be in short supply. In this case, Tobin's q is called "IO q", and they find that both the industry-level and firm-level sensitivities of investment to q collapse in a recent period (1997 – 2014). In the meantime, capital flows out of high q industries because high q industries return equity capital to investors. Lee et al. find that in the case of large old firms, q capitalizes rents from their market power, and those with higher q have more cash flows to pay out. As a result, equity capital flows out of high q industries because paying out to investors is optimal when there are only poor projects left.

Decreasing and non-positive investment-q sensitivity is not restricted to the recent period in our results. We can observe it from the entire period, and the observed negative investment-q sensitivity is consistent with the idea of IO q. After the extensive margin is depleted, firms start to focus on the intensive margin. However, the intensive margin is also depleted eventually as firms keep investing in a few types of capital. As a result, high q firms with less margin of investment are predicted to pay out more rents to investors instead of investing in poor projects. We run OLS regressions of net payout on lagged total q, contemporaneous total cash flow, and year fixed effects in Table 8. As shown in Lee et al. (2021), firms with higher q pay out more and repurchase more, indicating that a firm with the largest q pays out the most. Considering that cash for dividends and repurchases mainly comes from internal cash and leads to reductions in capital expenditures and R&D expenses (Wang, Yin, and Yu, 2021), the negative response of investment to very large q in our results can be explained by IO q.

[Table 8 here]

The negative response of investment to very large q and the positive payout-q sensitivity are consistently observed throughout the entire period. The phenomenon that funds flow out of high q firms is more evident in the recent periods, as evidenced by the increasing payout-q sensitivity over the period. We emphasize that firms with large IO q have always existed, but there were not many such firms in the earlier periods. As a result, their presence was not influential and thus could not be detected by Lee et al. (2021). Nevertheless, the recent period (1997 – 2014) in their paper has an overlap with our second early period (1992 – 1999) when non-increasing investment in a high q region is most evident. The fact that firms in our sample are also covered by IBES could be another reason why the phenomenon that is consistent with the concept of IO q is observed throughout the entire period because analysts tend to cover larger firms within an industry (Bhushan, 1989).

6. Conclusion

The neoclassical theory of investment has been widely tested on its performance in explaining corporate investment behavior, and most studies estimate linear relations. However, as suggested by theoretical and empirical evidence in the literature, we pay attention to the possibility that the true investment-q relation is more dynamic than described by a simple linear relation. The optimal investment is in fact an unknown function of marginal q, which indicates that the true investment-q relation is not necessarily linear. Also in the cross section, firms with different q characteristics are likely to show non-homogeneous investment behavior due to heterogeneous capital goods to consider for investment. We thus augment the classic investment regression model with higher-order terms in q and fit the augmented model to the cross section of firms to capture potential nonlinearity while correcting for nonseparable measurement error in q with the aid of a repeated measurement.

We find that the bias-corrected investment-q relation is S-shaped in the cross section. Firms with higher q invest more in most cases. However, the elasticity of investment with respect to q is little for firms with low q. It also decreases in intermediate-high q and is predicted to be negative for very large q. These results imply that an improvement in q caused by a policy change, for example, a change in the corporate tax rate, is unlikely to lead firms with small and large q to unconditionally increase their investments. We consistently find S-shapes throughout the entire period and from all types of q proxies. We suggest two reasons for our findings. First, the set of capital goods for investment differs across firms and changes as q improves. Second, depleting margin of investment as q and investment increase contributes to the decreasing response of investment to intermediate-high q and the negative response to very large q. Firms are thus predicted to pay out more as the investment margin is exhausted, and we find supportive evidence.

Our sample is constructed based on Compustat firms covered by IBES from 1983 to 2017, which is to construct a repeated measurement of average q using analysts' forecasts of earnings and long-term growth. As a result, firms in our sample might not represent the average U.S. public firm, so one must be careful with the extrapolation of our results to the rest of the firms that are not included in the sample. Nevertheless, our results still have important implications because our findings explain investment decisions made by firms that represent about 80% (unreported) of the total market capitalization of the U.S. firms that are suitable for corporate investment studies in general. Consequently, our results describe the investment behavior of firms that mainly drive the U.S. economy.

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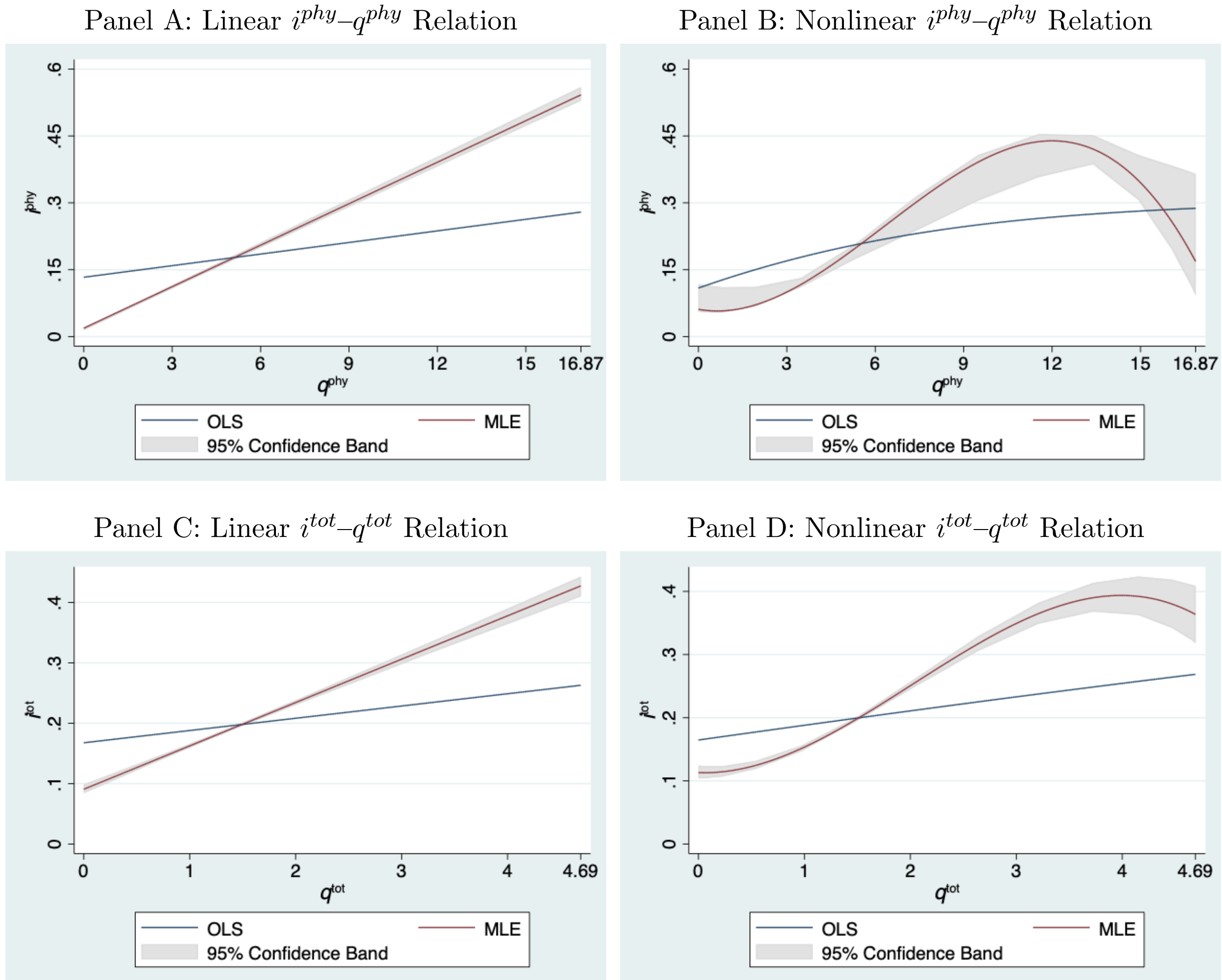

Figure 1. Investment-q Relation

This figure presents investment as a function of q. The classic and augmented investment regressions estimated by OLS and the measurement-error-robust MLE in Table 3 are plotted. Panels A and C (B and D) plot the estimated linear (nonlinear) relationship between investment and q. 95% bootstrapped confidence bands are plotted. Physical (total) investment and physical (total) q are the variables in Panels A and B (C and D).

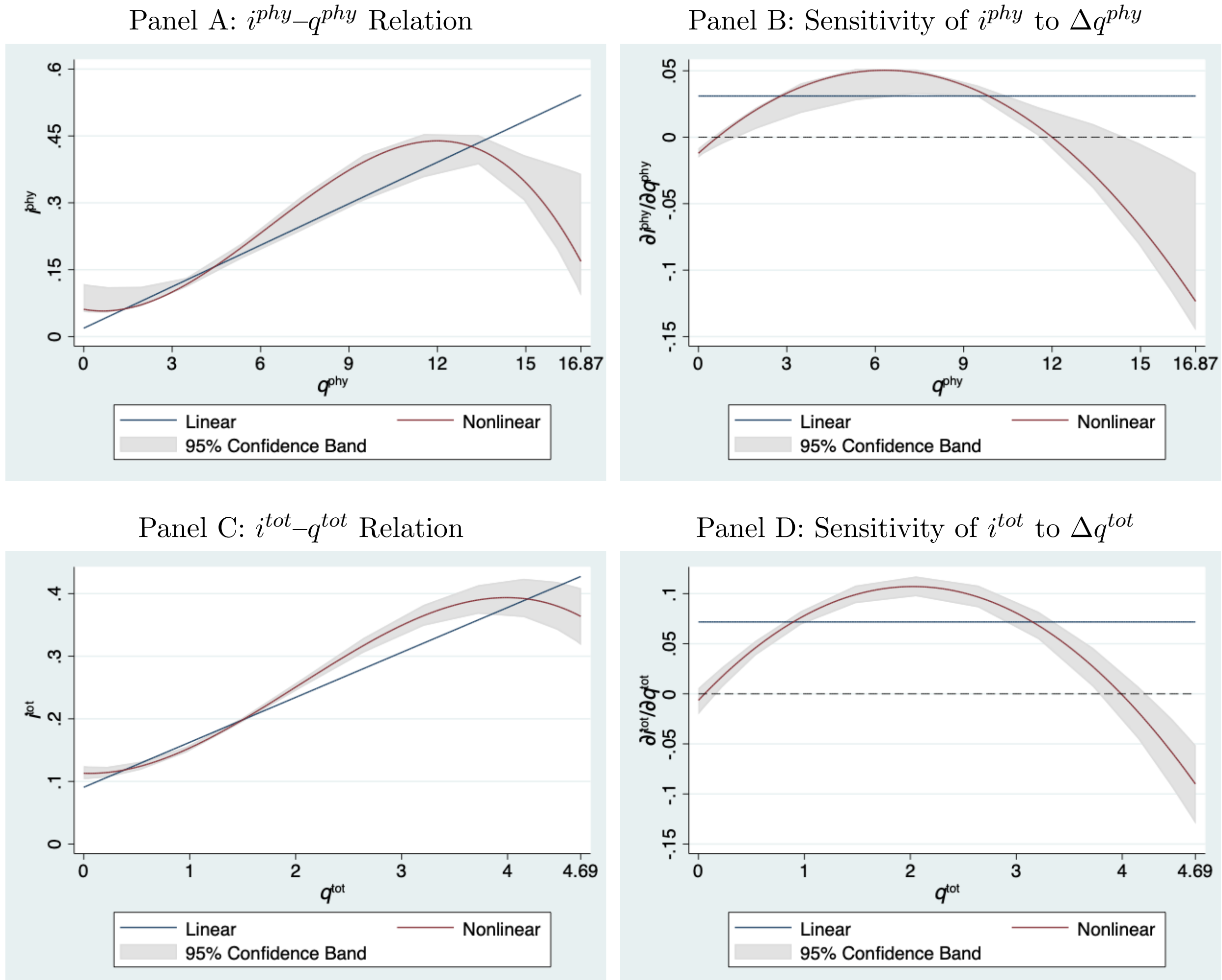

Figure 2. Elasticity of Investment with respect to *q*

This figure presents the relationship between investment and q. The classic and augmented investment regressions estimated by the measurement-error-robust MLE in Table 2 are plotted in Panels A and C. The first derivatives of the estimated regressions with respect to q are plotted in Panels B and D. 95% bootstrapped confidence bands are plotted. Physical (total) investment and physical (total) q are the variables in Panels A and B (C and D).

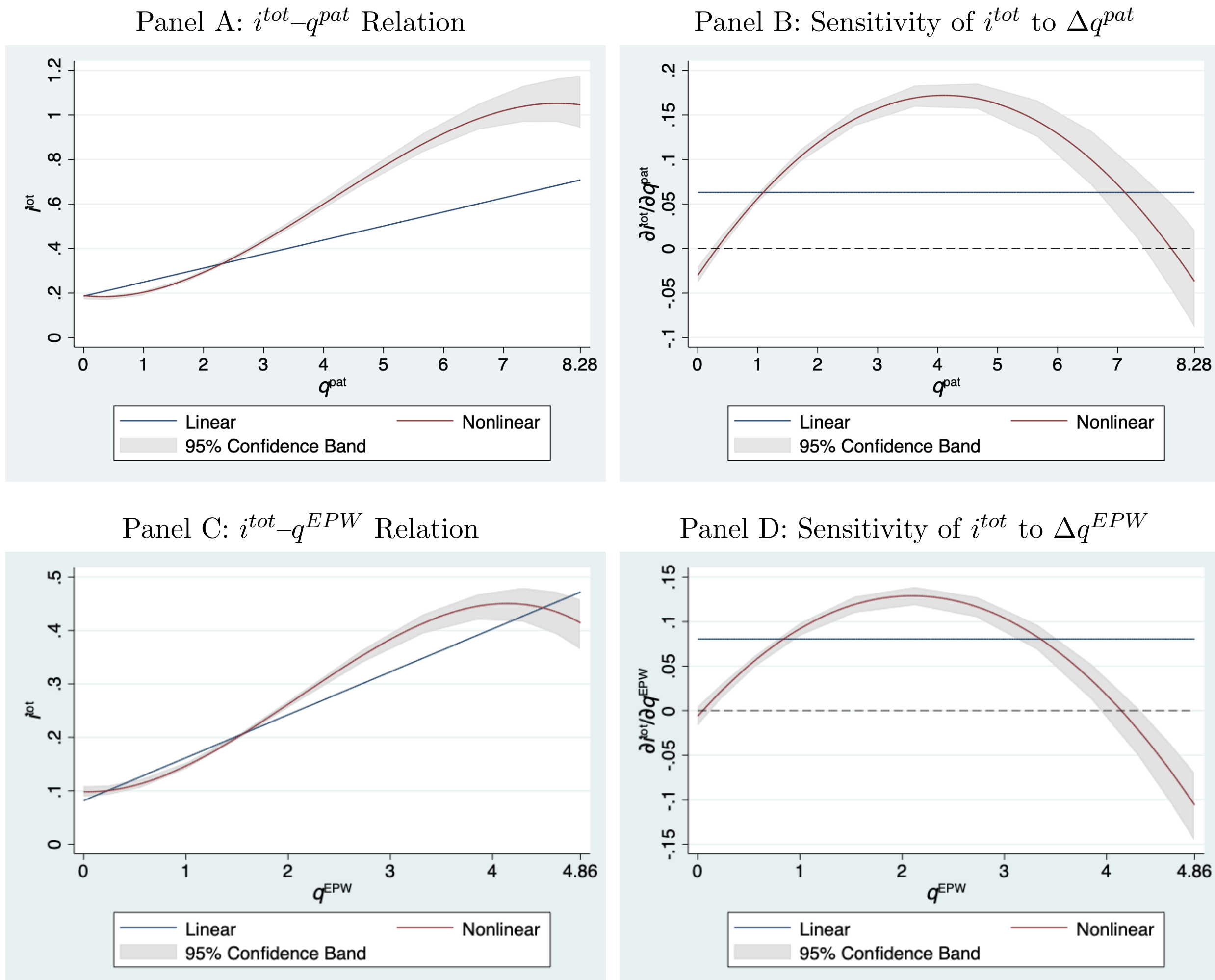

Figure 3. Investment-Patent q/EPW q Relation

This figure presents the relationship between investment and patent q/EPW q. The classic and augmented investment regressions estimated by the measurement-error-robust MLE in Table 4 are plotted in Panels A and C. The first derivatives of the estimated regressions with respect to q are plotted in Panels B and D. 95% bootstrapped confidence bands are plotted. Patent q (EPW q) and its corresponding total investment are the variables in Panels A and B (C and D).

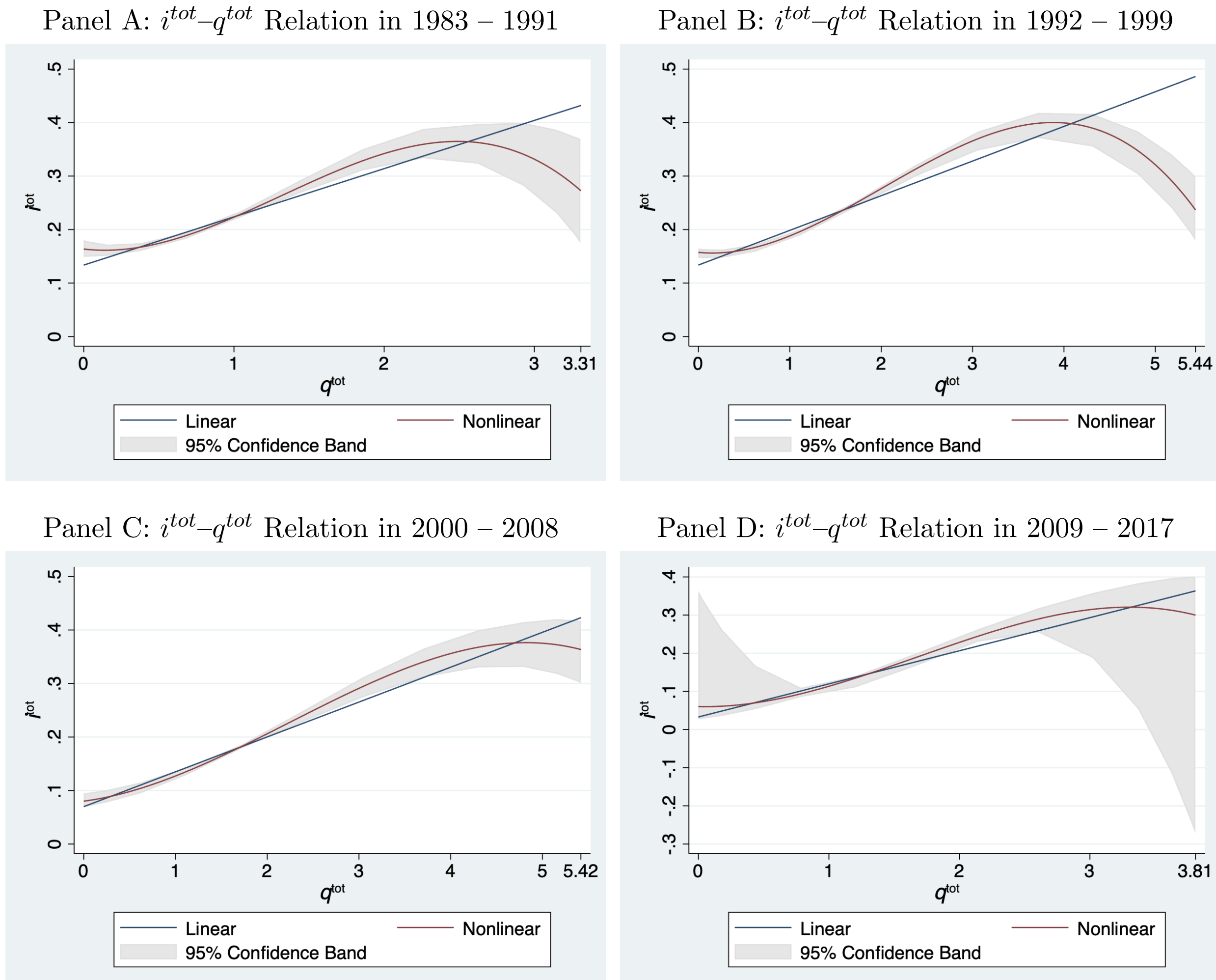

Figure 4. Investment-q Relation in Different Periods
This figure presents investment as a function of q in different periods. The classic and augmented investment regressions estimated by the measurement-error-robust MLE in Table 5 are plotted. 95% bootstrapped confidence bands are plotted. Total investment and total q are the variables.

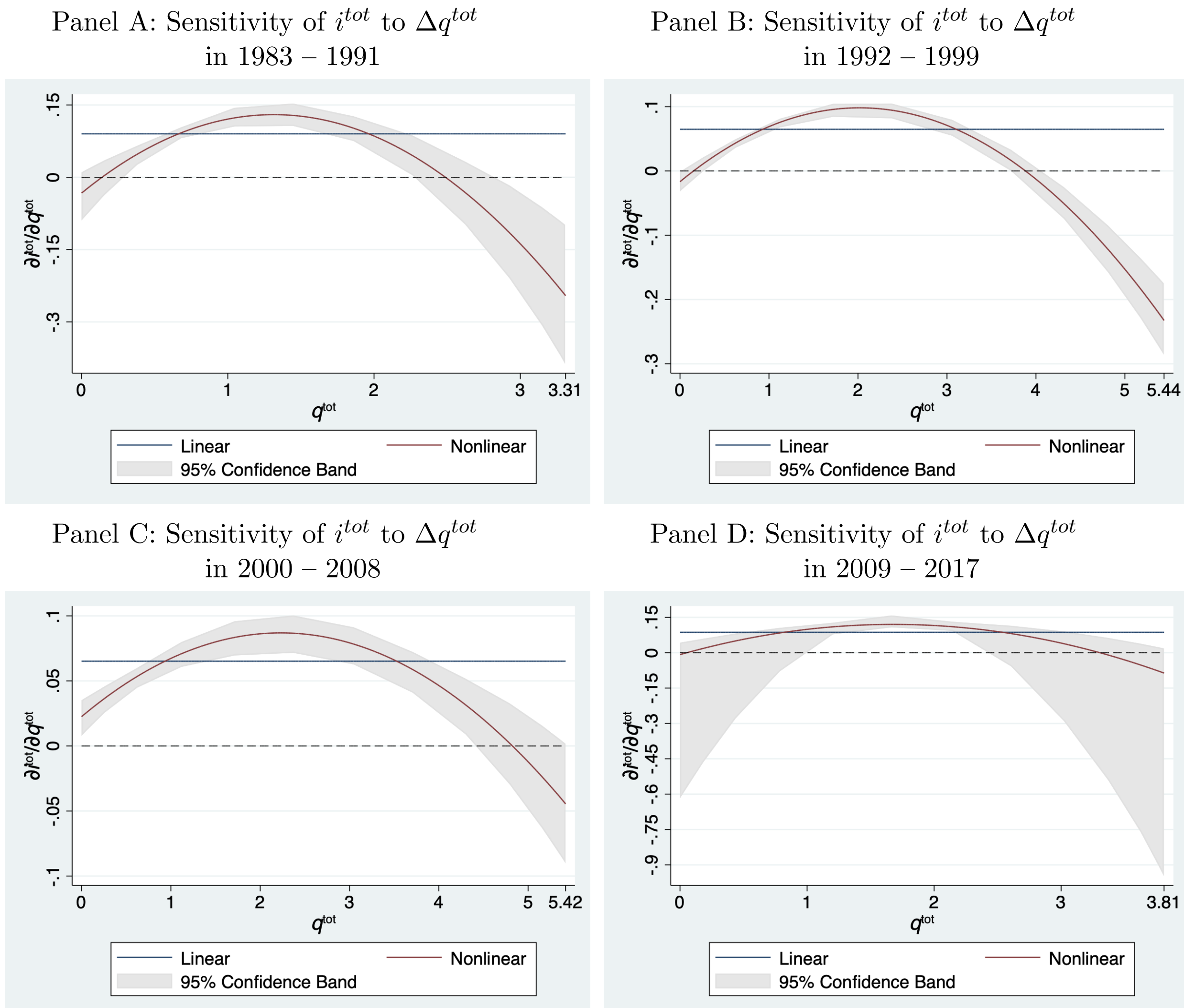

Figure 5. Elasticity of Investment with respect to q in Different Periods

This figure presents the investment-q sensitivity in different periods. The first derivatives of the classic and augmented investment regressions estimated by the measurement-error-robust MLE with respect to q are plotted. 95% bootstrapped confidence bands are plotted. Total investment and total q are the variables.

Table 1. Summary Statistics

This table presents summary statistics for the annual Compustat sample covered by IBES from 1983 to 2017. Physical investment is CAPX divided by lagged physical capital (i.e., gross PP&E). Total investment is CAPX plus intangible investment (i.e., R&D expense plus 30% of SG&A expense), divided by lagged total capital (i.e., the sum of physical capital and Peters and Taylor's (2017) estimate of intangible capital). The numerator of each market-based (i.e., non-analyst-based) measure of q is the market value of equity plus the book value of debt minus current assets. The numerator of each analyst-based measure of q is the analyst value of equity plus the book value of debt minus current assets. The denominators of physical q and analyst physical q are physical capital. The denominators of total q and analyst total q are total capital. The denominators of patent q and analyst patent q are the sum of physical capital and Woeppel's (2022) estimate of intangible capital. The denominators of EPW q and analyst EPW q are the sum of physical capital and Ewens, Peters, and Wang's (2022) estimate of intangible capital. Standard cash flow is income before extraordinary items plus depreciation, scaled by lagged physical capital. Total cash flow is income before extraordinary items plus depreciation plus tax-adjusted intangible investment, scaled by lagged total capital. Net payout is cash dividends plus the purchase of common and preferred stock minus the sale of common and preferred stock, divided by lagged total capital. All ratios are winsorized at the 1st and 99th percentiles.

	Mean	Standard Deviation	P5	Median	P95	Observations
Physical Investment	0.169	0.158	0.035	0.118	0.480	58,796
Total Investment	0.196	0.146	0.049	0.157	0.488	58,796
Physical q	4.119	6.359	0.172	1.748	16.872	58,796
Analyst Physical q	5.685	7.412	0.409	2.832	22.456	58,796
Total q	1.399	1.720	0.087	0.865	4.693	58,796
Analyst Total q	2.020	2.199	0.239	1.312	6.531	58,796
Patent q	2.183	3.375	0.139	1.089	8.282	58,796
Analyst Patent q	3.274	4.573	0.312	1.710	12.411	58,796
EPW q	1.459	1.764	0.097	0.908	4.855	58,796
Analyst EPW q	2.116	2.268	0.254	1.389	6.791	58,796
Standard Cash Flow	0.288	0.415	-0.135	0.205	1.069	58,796
Total Cash Flow	0.190	0.150	0.014	0.163	0.479	58,796
Net Payout	0.029	0.054	0.000	0.006	0.133	58,796

Table 2. Measurement-Error-Corrected MLE Results

This table reports the classic and augmented investment regressions estimated by the measurement-error-robust MLE. Physical investment, lagged physical q (q^{phy}), and contemporaneous standard cash flow (c^{phy}) are used in Columns (1) and (2). Total investment, lagged total q (q^{tot}), and contemporaneous total cash flow (c^{tot}) are used in Columns (3) and (4). Block bootstrapped standard errors treating each firm as one block are computed based on 500 replications and reported in parentheses.

	(1)	(2)	(3)	(4)
	Physical Investment		Total Investment	
q^{phy}	0.0310	-0.0121		
	(0.0005)	(0.0142)		
c^{phy}	-0.0173	0.0013		
	(0.0059)	(0.0043)		
$(q^{phy})^2$		0.0099		
		(0.0035)		
$(q^{phy})^3$		-0.0005		
		(0.0002)		
q^{tot}			0.0718	-0.0065
			(0.0024)	(0.0277)
c^{tot}			0.3574	0.3155
			(0.0169)	(0.0223)
$(q^{tot})^2$				0.0561
				(0.0159)
$(q^{tot})^3$				-0.0092
				(0.0026)
Log-Likelihood	-122814.89	-122057.61	-37838.07	-37581.75
LR Test Statistic		1514.56		512.64
Observations	58,796	58,796	58,796	58,796

Table 3. OLS and Measurement-Error-Corrected MLE Results

This table reports the classic and augmented investment regressions estimated by OLS and the measurement-error-robust MLE. Physical investment, lagged physical q (q^{phy}), and contemporaneous standard cash flow (c^{phy}) are used in Panel A. Total investment, lagged total q (q^{tot}), and contemporaneous total cash flow (c^{tot}) are used in Panel B. OLS results are from regressions of investment on lagged Tobin's q terms, contemporaneous cash flow, and year fixed effects. Measurement-error-corrected MLE results are restated from Table 2. Standard errors clustered by firm from OLS and block bootstrapped standard errors from the MLE are reported in parentheses.

	(1)	(2)	(3)	(4)
Panel A: Physical Investment	OLS	MLE	OLS	MLE
q^{phy}	0.0087	0.0310	0.0233	-0.0121
	(0.0003)	(0.0005)	(0.0009)	(0.0142)
c^{phy}	0.0433	-0.0173	0.0341	0.0013
	(0.0034)	(0.0059)	(0.0034)	(0.0043)
$(q^{phy})^2$			-0.0011	0.0099
			(0.0001)	(0.0035)
$(q^{phy})^3$			0.0000	-0.0005
			(0.0000)	(0.0002)
Adjusted R^2	0.2071		0.2207	
Log-Likelihood		-122814.89		-122057.61
LR Test Statistic				1514.56
Panel B: Total Investment	OLS	MLE	OLS	MLE
q^{tot}	0.0203	0.0718	0.0235	-0.0065
	(0.0010)	(0.0024)	(0.0026)	(0.0277)
c^{tot}	0.4285	0.3574	0.4251	0.3155
	(0.0092)	(0.0169)	(0.0093)	(0.0223)
$(q^{tot})^2$			-0.0002	0.0561
			(0.0008)	(0.0159)
$(q^{tot})^3$			-0.0000	-0.0092
			(0.0001)	(0.0026)
Adjusted R^2	0.4316		0.4319	
Log-Likelihood		-37838.07		-37581.75
LR Test Statistic				512.64
Observations	58,796	58,796	58,796	58,796

Table 4. Other Proxies of q: Patent q and EPW q

This table reports the classic and augmented investment regressions estimated by the measurement-error-robust MLE. Total investment, lagged patent q (q^{pat}), and contemporaneous total cash flow (c^{tot}) are used in Columns (1) and (2). Total investment, lagged EPW q (q^{EPW}), and contemporaneous total cash flow (c^{tot}) based on Ewens, Peters, and Wang's (2022) estimate of the fraction of SG&A invested in organizational capital are used in Columns (3) and (4). Block bootstrapped standard errors treating each firm as one block are computed based on 500 replications and reported in parentheses.

	(1)	(2)	(3)	(4)
	Total Investment			
q^{pat}	0.0630	-0.0301		
	(0.0018)	(0.0046)		
c^{tot}	0.6292	0.4783		
	(0.0112)	(0.0291)		
$(q^{pat})^2$		0.0492		
		(0.0056)		
$(q^{pat})^3$		-0.0040		
		(0.0008)		
q^{EPW}			0.0804	-0.0061
			(0.0024)	(0.0056)
c^{tot}			0.3279	0.2448
			(0.0173)	(0.0225)
$(q^{EPW})^2$				0.0644
				(0.0042)
$(q^{EPW})^3$				-0.0102
				(0.0007)
Log-Likelihood	-104735.87	-103408.08	-43086.53	-42672.65
LR Test Statistic		2655.58		827.76
Observations	58,796	58,796	58,796	58,796

Table 5. Investment-q Relation in Different Periods

This table reports the classic and augmented investment regressions estimated by the measurement-error-robust MLE in different periods. Total investment, lagged total q (q^{tot}), and contemporaneous total cash flow (c^{tot}) are used. Block bootstrapped standard errors treating each firm as one block are computed based on 500 replications and reported in parentheses.

	(1)	(2)	(3)	(4)
Panel A: Before 2000	1983 – 1991		1992 – 1999	
q^{tot}	0.0901	-0.0328	0.0648	-0.0167
	(0.0054)	(0.0583)	(0.0031)	(0.0077)
c^{tot}	0.5149	0.5014	0.4653	0.4306
	(0.0277)	(0.0270)	(0.0224)	(0.0253)
$(q^{tot})^2$		0.1239		0.0569
		(0.0489)		(0.0054)
$(q^{tot})^3$		-0.0314		-0.0094
		(0.0114)		(0.0008)
Log-Likelihood	-3575.47	-3546.82	-13148.40	-13057.39
LR Test Statistic		57.30		182.02
Observations	10,407	10,407	16,988	16,988
Panel B: After 2000	2000 – 2008		2009 – 2017	
q^{tot}	0.0652	0.0225	0.0867	-0.0081
	(0.0033)	(0.0071)	(0.0038)	(0.1899)
c^{tot}	0.2452	0.2026	0.1283	0.1080
	(0.0232)	(0.0326)	(0.0207)	(0.0884)
$(q^{tot})^2$		0.0289		0.0764
		(0.0051)		(0.1165)
$(q^{tot})^3$		-0.0043		-0.0152
		(0.0008)		(0.0226)
Log-Likelihood	-11513.63	-11470.89	-3459.75	-3376.47
LR Test Statistic		85.48		166.56
Observations	17,774	17,774	13,627	13,627

Table 6. Classic Investment Regression Estimated by the Cumulant Estimator

This table reports the classic investment regression estimated by OLS and the cumulant estimator. Total investment, lagged total q (q^{tot} in Columns (1) and (2)) or the analyst-based measure of lagged total q ($\hat{q}^{tot}$ in Columns (3) and (4)), and contemporaneous total cash flow (c^{tot}) are used. ρ^2 is the within-year R^2 from the hypothetical regression of investment on the true q and cash flow. τ^2 is the within-year R^2 from the hypothetical regression of a q proxy on the true q. Standard errors clustered by firm are reported in parentheses.

	(1)	(2)	(3)	(4)
Total Investment	OLS	Cumulant	OLS	Cumulant
q^{tot}	0.0203	0.0820		
	(0.0010)	(0.0023)		
$\hat{q}^{tot}$			0.0186	0.0697
			(0.0007)	(0.0017)
c^{tot}	0.4285	0.0313	0.3833	-0.1004
	(0.0092)	(0.0220)	(0.0095)	(0.0210)
R^2	0.3881		0.3966	
ρ^2		0.5130		0.5329
		(0.0123)		(0.0114)
τ^2		0.4853		0.5669
		(0.0167)		(0.0132)
Observations	58,796	58,796	58,796	58,796

Table 7. Classic Investment Regression Estimated by the Cumulant Estimator in Different Periods

This table reports the classic investment regression estimated by OLS and the cumulant estimator in different periods. Total investment, lagged total q (q^{tot}), and contemporaneous total cash flow (c^{tot}) are used. ρ^2 is the within-year R^2 from the hypothetical regression of investment on the true q and cash flow. τ^2 is the within-year R^2 from the hypothetical regression of a q proxy on the true q. Standard errors clustered by firm are reported in parentheses.

	(1)	(2)	(3)	(4)
	OLS	Cumulant	OLS	Cumulant
Panel A: Before 2000	1983 – 1991		1992 – 1999	
q^{tot}	0.0183	0.1370	0.0231	0.0905
	(0.0026)	(0.0174)	(0.0016)	(0.0040)
c^{tot}	0.5764	0.0520	0.4739	0.0401
	(0.0213)	(0.0848)	(0.0145)	(0.0340)
R^2	0.4359		0.4257	
ρ^2		0.5445		0.5547
		(0.0326)		(0.0182)
τ^2		0.3717		0.5010
		(0.0355)		(0.0209)
Observations	10,407	10,407	16,988	16,988
Panel B: After 2000	2000 – 2008		2009 – 2017	
q^{tot}	0.0222	0.0671	0.0171	0.0810
	(0.0013)	(0.0021)	(0.0022)	(0.0062)
c^{tot}	0.3178	-0.0169	0.3274	-0.1117
	(0.0136)	(0.0255)	(0.0187)	(0.0541)
R^2	0.3806		0.3017	
ρ^2		0.5341		0.4316
		(0.0189)		(0.0305)
τ^2		0.5449		0.4860
		(0.0236)		(0.0341)
Observations	17,774	17,774	13,627	13,627

Table 8. Payout-q Relation

This table reports the payout-q relation estimated by OLS. Results are from regressions of net payout on lagged total q (q^{tot}), contemporaneous total cash flow (c^{tot}), and year fixed effects. Standard errors clustered by firm are reported in parentheses.

	(1)	(2)	(3)	(4)	(5)
Net Payout	Full	1983 – 1991	1992 – 1999	2000 – 2008	2009 – 2017
q^{tot}	0.0044	0.0018	0.0023	0.0048	0.0083
	(0.0004)	(0.0008)	(0.0005)	(0.0006)	(0.0012)
c^{tot}	0.0410	0.0209	0.0158	0.0598	0.0991
	(0.0032)	(0.0061)	(0.0042)	(0.0052)	(0.0098)
Adjusted R^2	0.0817	0.0164	0.0237	0.1060	0.1478
Observations	58,796	10,407	16,988	17,774	13,627

Appendix: Identification Strategy

Let $\mathcal{I}$, $\mathcal{Q^A}$, $\mathcal{Q^M}$, $\mathcal{C}$, and $\mathcal{Q^H}$ denote the supports of the distributions of the random variables i, q^A, q^M, c, and q^H. We consider all the random variables to be jointly continuously distributed. We assume that the joint density of i and q^A, q^M, c, q^H admits a bounded density with respect to the product measure of some dominating measure μ defined on $\mathcal{I}$ and the Lebesgue measure on $\mathcal{Q^A} \times \mathcal{Q^M} \times \mathcal{C} \times \mathcal{Q^H}$ and that all marginal and conditional densities are also bounded. For the identification of the density of interest $f_{i|q^M c}(i|q^M, c)$, we make similar assumptions as in Hu and Schennach (2008) and Song (2015).

Assumption 1. *(i)* $f_{i|q^A q^M c\, q^H}(i|q^A, q^M, c, q^H) = f_{i|q^M c}(i|q^M, c)$ *for all* $(i, q^A, q^M, c, q^H) \in \mathcal{I} \times \mathcal{Q^A} \times \mathcal{Q^M} \times \mathcal{C} \times \mathcal{Q^H}$ *and (ii)* $f_{q^A|q^M c\, q^H}(q^A|q^M, c, q^H) = f_{q^A|q^M c}(q^A|q^M, c)$ *for all* $(q^A, q^M, c, q^H) \in \mathcal{Q^A} \times \mathcal{Q^M} \times \mathcal{C} \times \mathcal{Q^H}$.

Assumption 1 (i) specifies that the mismeasured q (q^A) and a repeated measurement (q^H), which is the analyst-based measure of q, do not provide any more information about investment i than the true unobserved q (q^M) and the additional observed cash flow c already provide. This is satisfied when measurement errors in q^A and q^H do not have any informational content about i. Similarly, Assumption 1 (ii) indicates that q^H does not provide any further information about q^A given q^M and c. This is satisfied unless measurement error in q^A is correlated with q^{IV} other than q^M and c.

Let a and b denote random variables with respective supports $\mathcal{A}$ and $\mathcal{B}$. Given two corresponding spaces $\mathcal{G}(\mathcal{A})$ and $\mathcal{G}(\mathcal{B})$ of functions with domains $\mathcal{A}$ and $\mathcal{B}$, respectively, let $L_{b|a}$ denote an integral operator mapping $\mathrm{g} \in \mathcal{G}(\mathcal{A})$ to $L_{b|a}\mathrm{g} \in \mathcal{G}(\mathcal{B})$ defined by

$$[L_{b|a}\mathrm{g}](b) \equiv \int_{\mathcal{A}} f_{b|a}(b|a)\, \mathrm{g}(a)\, da.$$

Similarly, let $D_{b|a}$ denote a diagonal operator mapping $\mathrm{g} \in \mathcal{G}(\mathcal{A})$ to $D_{b|a}\mathrm{g} \in \mathcal{G}(\mathcal{B})$ defined by

$$[D_{b|a}\mathrm{g}](b) \equiv f_{b|a}(b|a)\, \mathrm{g}(a).$$

For ease of notation, we omit conditioning variables in the definitions. If the space $\mathcal{G}(\mathcal{A})$ upon which the operator $L_{b|a}$ acts is sufficiently large, the density $f_{b|a}(b|a)$ can be sampled everywhere and thus can be uniquely determined by the operator.

Assumption 2. *The operators $L_{q^A|q^M c}$ and $L_{q^H|q^A c}$ are injective.*

Assumption 2 places restrictions on the relationships between q^A, q^M, c, and q^H. An operator $L_{b|a}$ is injective if there is enough variation in the density of b for different values of a. In this sense, Assumption 2 implies that the mismeasured q has enough information about the true q and that the repeated measure of q has enough information about the mismeasured q, given cash flow c. These conditions are related to the rank condition (i.e., relevance condition) in the conventional instrumental variables approach and do easily hold as long as the amounts of measurement errors in q^A and q^H are reasonable.

Assumption 3. *Given $c \in \mathcal{C}$, the set $\{i : f_{i|q^M c}(i|q_1^M, c) \neq f_{i|q^M c}(i|q_2^M, c)\}$ has a positive probability whenever $q_1^M \neq q_2^M$ for all q_1^M, $q_2^M \in \mathcal{Q}^{\mathcal{M}}$.*

Assumption 4. *Given $c \in \mathcal{C}$, there exists a known functional M such that*
$M[f_{q^A|q^M c}(\cdot|q^M, c)] = q^M$ for all $q^M \in \mathcal{Q}^{\mathcal{M}}$.

Assumptions 3 and 4 ensure a unique decomposition of an integral operator associated with the joint density of the observables $f_{i\, q^A|q^H c}(i, q^A|q^H, c)$. Assumption 3 is only violated if the distribution of i conditional on q^M and c is identical at different values of q^M. The presence of either conditional heteroskedasticity of investment i or monotonicity of investment i in the true q^M conditional on cash flow c is sufficient to satisfy the assumption. Assumption 4 imposes a restriction on some measure of location of the density $f_{q^A|q^M c}(q^A|q^M, c)$. M is a general functional mapping a density to a real number. This assumption allows for measurement error to be either classical or nonclassical. To deal with the case of nonclassical measurement error, Hu and Schennach (2008) exploit the observation that even though measurement error may not have zero mean conditional on the true regressors, some other measure of location (e.g., mode or median) could still be zero.[9] Assumption 4 is invoked by this observation.

Theorem 1. *Under Assumptions 1–4, given the observed density $f_{i\,q^A|q^H c}(i, q^A|q^H, c)$, the equation*

$$f_{i\,q^A|q^H c}(i, q^A|q^H, c) = \int_{\mathcal{Q}^{\mathcal{M}}} f_{i|q^M c}(i|q^M, c) f_{q^A|q^M c}(q^A|q^M, c) f_{q^M|q^H c}(q^M|q^H, c)\, dq^M \qquad (8')$$

admits a unique solution $(f_{i|q^M c}, f_{q^A|q^M c}, f_{q^M|q^H c})$ for all $i \in \mathcal{I}$, $q^A \in \mathcal{Q}^{\mathcal{A}}$, $c \in \mathcal{C}$, $q^H \in \mathcal{Q}^{\mathcal{H}}$.

Assumption 1 facilitates the integral Equation (8′) that relates the joint density of the observables on the left-hand side to the unobservable conditional densities associated with the unobserved q^M on the right-hand side. Specifically, the identification of the density of interest $f_{i|q^M c}(i|q^M, c)$ can be achieved through an eigenvalue-eigenfunction decomposition of the integral operator associated with the joint density of the observables. Assumptions 3 and 4 ensure the uniqueness of the decomposition.

Proof: From Assumption 1, we get

$$\begin{aligned}
&f_{i\,q^A|q^H c}(i, q^A|q^H, c)\\
&= \int_{\mathcal{Q}^{\mathcal{M}}} f_{i\,q^M q^A|q^H c}(i, q^M, q^A|q^H, c)\, dq^M\\
&= \int_{\mathcal{Q}^{\mathcal{M}}} f_{i|q^M q^A q^H c}(i|q^M, q^A, q^H, c) f_{q^M q^A|q^H c}(q^M, q^A|q^H, c)\, dq^M\\
&= \int_{\mathcal{Q}^{\mathcal{M}}} f_{i|q^M q^A q^H c}(i|q^M, q^A, q^H, c) f_{q^A|q^M q^H c}(q^A|q^M, q^H, c) f_{q^M|q^H c}(q^M|q^H, c)\, dq^M\\
&= \int_{\mathcal{Q}^{\mathcal{M}}} f_{i|q^M c}(i|q^M, c) f_{q^A|q^M c}(q^A|q^M, c) f_{q^M|q^H c}(q^M|q^H, c)\, dq^M.
\end{aligned}$$

We now show that the solution to this equation is unique. By the definitions of the operators,

[9]For example, consider M that defines the mode or median of a density: $M[f] = \arg\max_{x \in \mathcal{X}} f(x)$ or $M[f] = \inf\{x^* \in \mathcal{X}^* : \int 1(x \leq x^*) f(x) dx \geq 0.5\}$ where x^* is the unobserved true regressor, and x is the error-contaminated counterpart. These are examples of M that cover nonclassical measurement errors in x of various forms.

we have the operator equivalence such that for an arbitrary $f \in \mathcal{F}(\mathcal{Q}^{\mathcal{M}})$,

$$\begin{aligned}
&\left[L_{i\, q^A|q^H c} f\right](q^A) \\
&= \int_{\mathcal{Q}^{\mathcal{H}}} f_{i\, q^M q^A|q^H c}(i, q^M, q^A|q^H, c)\, \mathrm{g}(q^H)\, dq^H \\
&= \int_{\mathcal{Q}^{\mathcal{H}}} \int_{\mathcal{Q}^{\mathcal{M}}} f_{q^A|q^M c}(q^A|q^M, c) f_{i|q^M c}(i|q^M, c) f_{q^M|q^H c}(q^M|q^H, c)\, dq^M \mathrm{g}(q^H)\, dq^H \\
&= \int_{\mathcal{Q}^{\mathcal{M}}} f_{q^A|q^M c}(q^A|q^M, c) f_{i|q^M c}(i|q^M, c) \left[L_{q^M|q^H c}\, \mathrm{g}\right](q^H)\, dq^M \\
&= \int_{\mathcal{Q}^{\mathcal{M}}} f_{q^A|q^M c}(q^A|q^M, c) \left[D_{i|q^M c} L_{q^M|q^H c}\, \mathrm{g}\right](q^H)\, dq^M \\
&= \left[L_{q^A|q^M c} D_{i|q^M c} L_{q^M|q^H c}\, \mathrm{g}\right](q^H),
\end{aligned}$$

and we obtain

$$L_{i\, q^A|q^H c} = L_{q^A|q^M c} D_{i|q^M c} L_{q^M|q^H c}. \tag{9'}$$

Note that the integration of the diagonal operator becomes the identity operator. By integration of the equation above over all $i \in \mathcal{I}$, we obtain

$$L_{q^A|q^H c} = L_{q^A|q^M c} L_{q^M|q^H c}.$$

Since $L_{q^A|q^M c}$ is invertible by Assumption 2, we get

$$L_{q^M|q^H c} = L^{-1}_{q^A|q^M c} L_{q^A|q^H c}.$$

Plugging $L_{q^M|q^H c}$ into Equation (9′) yields

$$\begin{aligned}
L_{i\, q^A|q^H c} &= L_{q^A|q^M c} D_{i|q^M c} L^{-1}_{q^A|q^M c} L_{q^A|q^H c} \\
L_{i\, q^A|q^H c} L^{-1}_{q^A|q^H c} &= L_{q^A|q^M c} D_{i|q^M c} L^{-1}_{q^A|q^M c},
\end{aligned}$$

since $L_{q^A|q^H c}$ is invertible by Assumption 2. The spectral decomposition on the right-hand side is unique by Assumptions 3–4 following Theorem XV.4.3.5 in Dunford and Schwartz (1971). *Q.E.D.*