

# Realized Illiquidity<sup>\*</sup>

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## Abstract

Realized illiquidity is the ratio between realized volatility and trading volume refining the popular price impact measure proposed by Amihud (2002). We provide its theoretical foundation in which market liquidity,  $\ell(t)$ , follows a stochastic process in continuous time. We prove that the realized illiquidity is a precise measurement of the inverse of *integrated liquidity*, that is, the integral of  $\ell(t)$  over periods of unit length (e.g., a day). We show that the realized illiquidity measures are robust to a number of features such as stochastic volatility and microstructure noise. A comprehensive econometric analysis highlights the main distributional and dynamic properties of the realized illiquidity, including jumps, clustering, and leverage effects, and demonstrate that they help explain the time series of stock and currency returns.

**Keywords:** Liquidity, Stochastic Volatility, Trading Volume, Amihud, Jumps.

*J.E.L. classification:* C15, F31, G12, G15

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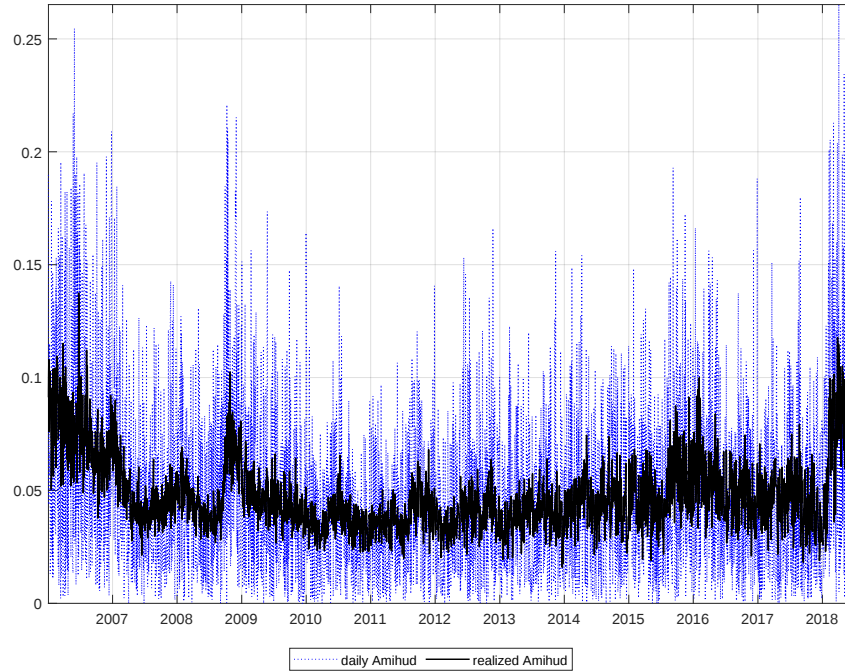
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# 1 Introduction

In addition to being crucial to the quality of financial markets and resilience of the financial system, market liquidity is important for capital market efficiency because “liquidity of investment markets often facilitates, though it sometimes impedes, the course of new investment” (Keynes, 1936, p.102). However, market liquidity is an elusive concept because it includes everything that determines “the degree to which an order can be executed within a short time frame at a price close to the security’s consensus value” (Foucault et al., 2013, p.2). Thus, liquidity manifests itself in at least two important ways: the transaction cost and the impact of transaction volume on security prices, which, in turn, depends on the market depth and price elasticity. The transaction cost is often gauged by the bid-ask spread, while a popular measure for price impact is the Amihud illiquidity measure (Amihud, 2002), which is the ratio between the daily absolute returns and trading volume.<sup>1</sup>



**Figure 1:** Realized Amihud (black solid line) vs classic daily Amihud (blue dotted line, see Amihud, 2002) of SPY. Realized Amihud is computed as the ratio between realized power variation and trading volume. Sample period: January 3, 2006 – June 29, 2018. Both series are scaled by a factor of E+09.

In this paper, we study the theoretical and empirical properties of a refinement of the classic *daily Amihud*, namely the *realized illiquidity*, which is defined as the ratio between a realized volatility measure and the daily trading volume.<sup>2</sup> We propose two measures of realized illiquidity that we call *realized Amihud* and *high-low Amihud*. The former is computed as the ratio between realized power variation (Barndorff-Nielsen and Shephard, 2003) using intraday data and daily trading volume. The latter is the ratio between the daily high-low range (Parkinson, 1980) and the daily volume. Both

<sup>1</sup>To date, the ILLIQ measure proposed by Amihud (2002) has been cited more than 12,000 times according to Google Scholar. Many of these papers were published in top-tier academic journals in finance.

<sup>2</sup>Although the empirical analysis focuses on the realized illiquidity computed at daily frequency, our theory and metrics can be applied to shorter (intraday) or longer (e.g., weekly or monthly) horizons.

numerators are realized measures of volatility but the former requires intraday data while the latter relies on daily data often accessible for many financial securities. Figure 1 provides a simple illustration of the precision of the realized Amihud (black solid line) in relation to the daily Amihud (blue solid line) using the SPDR S&P 500 ETF (ticker SPY) as an indication of the U.S. stock market behavior. Although both series follow similar dynamic patterns, the classic daily Amihud is much noisier than the realized Amihud.

This paper contributes to the literature in several ways. Our first contribution is outlining a simple theoretical framework based on a trading mechanism that fits suitably into the theory of realized volatility, specifically here in the context of liquidity measurement. In developing the theory of realized illiquidity, we consider a setting where the instantaneous liquidity parameter,  $\ell(t)$ , follows a stochastic process in continuous-time. We prove that the realized illiquidity provides a measurement of the *integrated illiquidity*, which is defined as the reciprocal of  $\int_0^1 \ell(s)ds$  over periods of unit length (e.g., a day, a week, or a month). This allows us to establish *how precisely* the realized illiquidity measures the integrated illiquidity vis-a-vis the classic daily Amihud, which is a special case of the realized Amihud when only one observation per period (i.e. daily return) is available. Thanks to its intrinsic non-parametric nature, the realized illiquidity represents a simple (inverse) measure of market resiliency, that is, the elasticity of asset prices to trading volume, where the latter depends on the degree of disagreement in the beliefs among traders.

A set of Monte Carlo simulations illustrates the finite sample accuracy of the realized illiquidity as a nonparametric measurement of integrated illiquidity. In the Monte Carlo simulations, we explore the finite-sample properties of the realized and high-low Amihud measures compared to the classic Amihud proxy. The numerical analysis clearly shows two results: First, the realized Amihud provides a very precise measurement of illiquidity even after considering microstructure frictions such as bid-ask spread and price discreteness. Second, although less precise than the realized Amihud, the high-low Amihud provides estimates of the underlying integrated illiquidity process that are several times more precise than those obtained by the classic daily Amihud. For these reasons, the realized Amihud can be considered the most efficient measure of realized illiquidity while the high-low Amihud is a more efficient *low-frequency* measure than the classic daily Amihud proxy.

Our second contribution is to demonstrate that the same theoretical framework employed to characterize the properties of the realized Amihud can be exploited to derive a non-parametric estimator of *spot illiquidity*. Specifically, we establish that within a context where both liquidity and price volatility follow (possibly correlated) continuous-time stochastic processes, the *localized* version of the realized Amihud is a consistent estimator of the spot (or instantaneous) illiquidity. This measure represents the degree of market illiquidity at a specific time point. We call this estimator *spot Amihud*, while its reciprocal is referred to as *spot Amivest*, where the latter is an efficient estimator of instantaneous liquidity, that is  $\ell(t)$ .

Our third contribution is to derive a simple theory for *jumps* in illiquidity, which are generated by impactful news common to all traders such as earnings press releases for stocks or central bank announcements for currencies. The natural interpretation of such jumps is the consensual price change induced by new fundamental information for which little or no transaction volume is needed. We refer to them as *information jumps*. These events induce large volatility but little to no volume, thus

increasing the observed market illiquidity due to the predominant information component. Building on the results of [Barndorff-Nielsen and Shephard \(2003, 2004, 2006\)](#), we develop a formal way to carry out nonparametric statistical inference on information jumps and disentangle the illiquidity generated by information jumps from the baseline illiquidity associated with disagreement among traders, which is the main driver of trading volume. In particular, we define a test statistic to detect significant jumps, and we construct the jump-robust version of the realized illiquidity estimator.

Our fourth contribution is to empirically study the properties of the realized illiquidity. To do so, we consider a comprehensive set of econometric specifications with the goal of characterizing the main dynamic and distributional properties of the realized Amihud. Inspired by the techniques usually adopted in the context of volatility modeling, we consider a multiplicative error model (MEM) specification ([Engle and Gallo, 2006](#)) that allows for a direct prediction of illiquidity without resorting to nonlinear transformations to preserve positivity. Furthermore, we investigate whether the slow decaying rate of the autocorrelation function of the realized Amihud is well characterized by the heterogeneous autoregressive specification (HAR) model of [Corsi \(2009\)](#) in both a linear and nonlinear context. All this should bring to light the predictive factors of market liquidity and evidence to support heterogeneity of trading activity of market participants operating with different investment horizons, i.e. daily, weekly, or monthly. Another well-known feature of volatility is that it follows asymmetric patterns with respect to positive and negative returns – that is – bad news (negative return shocks) increases volatility by more than good news (positive return shocks). We study whether a similar asymmetric mechanism characterizes illiquidity to find whether liquidity systematically evaporates more when bad news hits, as predicted by theoretical models of binding capital constraints leading to sudden liquidity dry-ups.<sup>3</sup> Reminiscent of two key terms used in the volatility literature, we call the first and second features *heterogeneous illiquidity clustering* and *leverage effect*, respectively.

We consider two categories of financial securities in our empirical analysis: stocks and currencies. Specifically, the time series analysis of the daily realized illiquidity of the main exchange traded fund (ETF) tracking the S&P 500 index (SPY) reveals four main findings shedding new light on the stochastic features of illiquidity. First, the empirical evidence strongly suggests that the latent illiquidity process is a very persistent one that is characterized by long periods of high illiquidity followed by long periods of low illiquidity (clustering). This persistence runs along heterogeneous horizons in that today's illiquidity is predicted not only by that of yesterday, but also by that of the previous week and month with greater intensity (heterogeneous clustering). All this suggests that there are distinct groups of traders with different behaviors, trading schedules, and time horizons, and this heterogeneity impacts the illiquidity timing with important asset pricing implications ([Amihud and Mendelson, 1986](#)).<sup>4</sup> Second, we find strong evidence that liquidity tends to dry up more in market downturns (leverage effect) and we demonstrate that ceteris paribus, a liquidity shock has more persistent effects when combined with negative returns. Third, when regressing excess returns on the realized Amihud, we find a significant negative coefficient. After breaking down illiquidity in its expected and unexpected components, we find that it is rather the unexpected part of illiquidity that negatively predicts returns consistent with the idea that the persistent effect of an illiquidity shock increases expected future illiquidity thus

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<sup>3</sup>[Hameed et al. \(2010\)](#) provide empirical support and comprehensive discussion of this literature.

<sup>4</sup>In reference to this idea, previous literature has used the broad concept of Heterogeneous Market Hypothesis ([Müller et al., 1993](#)).

decreasing asset prices. Fourth, all these findings (i.e. heterogeneous clustering, leverage effect, and how illiquidity is associated with stock returns) are much less evident when the noisy classic daily Amihud is employed, hence reiterating the importance of using a more precise illiquidity measure.

To conclude our empirical analysis, we examine the currency market by focusing on EUR/CHF and USD/CHF exchange rates surrounding the cap removal of the Swiss franc by the Swiss National Bank (SNB) on January 15, 2015. Despite its simplicity, our theory lends itself to several extensions that succeed in explaining and measuring illiquidity in reaction to major events such as a currency regime change. Through the lenses of our theory, the SNB announcement represents an ideal framework for testing the empirical prediction that the observed illiquidity should increase after the cap removal because the Swiss National Bank ceased to supply Swiss francs in unlimited quantities, thus reducing the supply of extra trading volume to the market. By carrying out the break analysis of [Bai and Perron \(1998\)](#), we find strong evidence that illiquidity significantly increased after January 15, 2015, as a joint consequence of an increase in EUR/CHF volatility and a reduction in trading volume of the same currency pair. Notably, we find an analogous pattern on the USD/CHF rate, suggesting that a liquidity shock in one market (or FX rate) immediately spills over into another one.

Our paper adds to prior research on market microstructure and liquidity. The popular [Amihud \(2002\)](#) measure is part of a large literature going back half a century that includes a wide range of illiquidity measures based on price changes and volume (e.g. [Silber, 1975](#)). For instance, [Dubofsky and Groth \(1984\)](#) use the Amivest measure, that is, the ratio of dollar volume to price change. Ideally, to measure the volume price impact, one would need access to data on all orders submitted by traders in a centralized market. However, many markets are decentralized and opaque, such as all the over-the-counter (OTC) markets, including the FX market that we also study here. In this context, the classic daily Amihud measure is extremely useful for approximating the order flow impact on security prices because (a) it is an observable and non-parametric quantity and (b) it is based on daily price and volume data, much more accessible than tick-by-tick orders and transactions.<sup>5</sup> This is why the Amihud measure is so widely used and it has been loosely associated with the [Kyle \(1985\)](#) model that provides an insightful theoretical framework to explain how orders impact asset prices, as captured by his  $\lambda$  factor.<sup>6</sup> Our theoretical contribution is to provide a general theory for realized illiquidity of which the Amihud measure is a specific case. Our empirical contribution is to demonstrate the superior accuracy of the realized Amihud measure, its characteristics, and explanatory power on asset returns.

[Rinaldo and Santucci de Magistris \(2022\)](#) adopt the realized Amihud to empirically investigate the price impact in the global FX market.<sup>7</sup> We extend their work in three ways. First, we provide a theoretical foundation to the Amihud measure, which includes the notions of integrated illiquidity and jumps. Second, we study the appropriate econometric modeling that highlights the main empirical features of the realized Amihud measures, including illiquidity jumps, heterogeneous clustering, and leverage effects, and how they explain asset returns. Finally, we propose two methods, i.e. the high-low Amihud and the realized one. Both metrics share the advantages of the Amihud measure – that is –

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<sup>5</sup>The accuracy of the classic Amihud measure has been documented for stocks ([Hasbrouck, 2009](#)) and currencies ([Rinaldo and Santucci de Magistris, 2022](#)). [Fong et al. \(2018\)](#) analyze global and U.S. stocks applying various liquidity proxies based on volatility over volume.

<sup>6</sup>[Collin-Dufresne and Fos \(2016\)](#) propose an elegant extension of Kyle’s model, with which our model shares the stochastic nature of market liquidity.

<sup>7</sup>Alternatively, [Barardehi et al. \(2018\)](#) revisit the Amihud measure by developing a trade-time liquidity measure.

they are easy-to-compute and rely on the ratio between two observable quantities based on transaction data, namely volume and (realized) volatility (measured either as high-low range or realized power variation). In doing so, the high-low and realized measures extend the previous literature on low- and high-frequency measurement of market liquidity, which has been predominantly based on the estimation of transaction costs rather than price impact.<sup>8</sup>

The manuscript is organized as follows. Section 2 outlines the theoretical setting. Section 2.1 reports the results of several Monte Carlo simulations. Section 3.3 introduces the notion of information jumps and of the jump-robust realized Amihud. Section 4 presents the empirical analysis, while Section 5 concludes the paper. Proof and additional empirical results are in the Appendix.

## 2 A Simple Theory of Illiquidity Measurement

We present here a simple reduced-form model of market volatility, volume and liquidity that is the basis for a theory of non-parametric illiquidity measurement inspired by Amihud (2002). Let us consider an asset traded on a financial market consisting of a finite number  $J \geq 2$  of active participants. Within a given trading period of a certain unit length (e.g., an hour, a day, a week), the market passes through a sequence of  $i = 1, \dots, I$  equilibria. The evolution of the equilibrium price is motivated by the arrival of new information to the market that changes the reservation prices of the traders. At intra-period  $i$ , the exchanged quantity by the  $j$ -th trader is given by

$$q_{i,j} = \ell_i(\Delta p_{i,j}^* - \Delta p_i), \quad \ell_i > 0, \quad j = 1, \dots, J, \quad (1)$$

where  $\Delta p_{i,j}^*$  is the variation in the reservation log-price of the  $j$ -th trader occurring between period  $i-1$  and period  $i$ . Similarly,  $\Delta p_i$  is the variation in the market log-price occurring in the same interval.

The term  $\ell_i$  is a positive coefficient capturing the market depth at time  $i$ : The larger the  $\ell_i$ , the larger quantities of the asset can be exchanged for a given difference  $\Delta p_{i,j}^* - \Delta p_i$ . The equilibrium function in (1) is analogous to the one outlined in Tauchen and Pitts (1983), which provides a stylized representation of the supply-demand mechanism on the market.<sup>9</sup> The reservation price of each trader might reflect some of the following aspects: individual preferences, liquidity issues, asymmetries in information sets, and different expectations about the fundamental values of the asset. In general, the reservation price can deviate from the market price because of idiosyncratic reasons, inducing the  $j$ -th trader to trade.

The quantity  $q_{i,j}$  exchanged for a unit change of  $\Delta p_{i,j}^* - \Delta p_i$  is given by the slope  $\ell_i$ . In other words,  $\ell_i$  measures the capacity of the market to allow large quantities to be exchanged at the intersection between demand and supply, thus recalling the concept of market depth and resilience that reduces the price impact of trading. The baseline assumptions of the model (linearity of the trading function and constant number of active traders) are inevitably stylized. As for the form of the equilibrium function in (1), note that the trades take place on short intra-period intervals of length  $\delta = 1/I$  and

<sup>8</sup>Starting from the seminal work of Roll (1984), several papers propose other measures that estimate the bid-ask spread, including Hasbrouck (2009), Corwin and Schultz (2012), and Abdi and Ranaldo (2017).

<sup>9</sup>See also Clark (1973), Epps and Epps (1976), the survey in Karpoff (1987) and the empirical analysis in Andersen (1996).

are generally associated with small price variations. Therefore, it is not restrictive to assume the equilibrium function is linear on small price changes and for a fixed number of traders during such a short period.

By market clearing, that is  $\sum_j q_{i,j} = 0$ , we have that the average of the variations in the reservation prices clears the market, that is,  $\Delta p_i = \frac{1}{J} \sum_{j=1}^J \Delta p_{i,j}^*$ . As new information arrives, traders adjust their reservation prices,  $\Delta p_{i,j}^*$ , resulting in a change in the market price ( $\Delta p_i$ ), which is given by the average of the increments of the reservation prices. The generated trading volume in each  $i$ -th sub-interval  $v_i = \frac{1}{2} \sum_{j=1}^J |q_{i,j}|$  is as follows

$$v_i = \frac{\ell_i}{2} \sum_{j=1}^J |\Delta p_{i,j}^* - \Delta p_i|, \quad (2)$$

We assume that the reservation price of each trader evolves in continuous-time according to the following process

$$dp_j^*(t) = \mu_j(t)dt + \sigma_j dW_j(t), \quad j = 1, \dots, J, \quad t \geq 0, \quad (3)$$

where  $\{W_j(t), j = 1, \dots, J\}$  is a collection of independent Wiener processes. The drift term  $\mu_j(t)$  might represent the long-term expectation of the  $j$ -th trader about the asset and could be seen as a function of fundamental quantities, such as interest rates and macroeconomic variables. The term  $\sigma_j$  is the volatility parameter of the  $j$ -th trader. By letting  $\sigma_j$  to change across traders, we introduce *heterogeneity* among them.<sup>10</sup> On the  $i$ -th discrete sub-interval of length  $\delta = \frac{1}{I}$ , the reservation prices for the  $j$ -th trader is therefore

$$\Delta p_{i,j}^* = \int_{\delta(i-1)}^{\delta i} \mu_j(s)ds + \int_{\delta(i-1)}^{\delta i} \sigma_j dW_j(s). \quad (4)$$

In a reduced-form model of trading like the current one, the liquidity coefficient  $\ell(t)$  can evolve due several reasons that are not explicitly modeled, such as tightening of funding constraints, or higher risk aversion of the market makers, or inventory rebalancing decisions. In particular, we assume that the liquidity process  $\ell(t)$  is driven by a continuous-time stochastic process fulfilling very mild regularity conditions, such as it has to be strictly positive and càdlàg. For instance,  $\ell(t)$  can display deterministic diurnal effects, jumps, long memory, no unconditional mean or to be non-stationary. Examples of stochastic differential equation (SDE) fulfilling these conditions are those common in the volatility literature, such as the Ornstein–Uhlenbeck process of [Vasicek \(1977\)](#), or the long-memory process of [Comte and Renault \(1998\)](#), see [Ghysels et al. \(1996\)](#) for a review.

We define the *realized Amihud* as follows

$$\mathcal{A} := \frac{RPV}{v}, \quad (5)$$

where the numerator is the *realized power variation* of order one (or realized absolute variation),  $RPV = \sum_{i=1}^I |r_i|$ , where  $r_i = \Delta p_i$  is the log-return, see [Barndorff-Nielsen and Shephard \(2003\)](#). The daily version by [Amihud \(2002\)](#) corresponds to the specific case with  $I = 1$ , i.e. the numerator reduces to the daily

<sup>10</sup>Note that volatility is assumed for now to be constant in the interval of unit length (e.g. a day, a week or a month), but it can change from period to period. In the following analysis (see Section 3.1 below), we will relax the assumption of constant volatility, and present an estimator of illiquidity that is robust to stochastic volatility.

absolute return. Then, it should be evident that the classic daily Amihud is a special case of a more general metrics based on a richer set of intra-period information. Specifically, the realized Amihud gauges the price impact of trading, that is, the amount of volatility on a unit interval (as measured by RPV) associated with the trading “dollar” volume  $v = \sum_{i=1}^I v_i$  generated in the same period. By resorting to the continuous-time framework outlined in [Barndorff-Nielsen and Shephard \(2002a,b\)](#), we can precisely measure the variability of the asset price by computing RPV over intervals of any length (e.g., hours, days, weeks, and months). Hence,  $\mathcal{A}$  measures the amount of volatility associated with a unit of trading volume.<sup>11</sup> The following proposition highlights the main determinants of realized Amihud as an illiquidity measure.

**Proposition 1.** *Consider the illiquidity measure defined in (5), the equilibrium relation in (1), and the diffusive process for reservation prices in (3). Assume that  $\ell(t)$  is a strictly positive càdlàg process. Assume  $J = 2$  active traders, as representative of the two aggregated sides of the market. As  $I \rightarrow \infty$  (i.e.,  $\delta \rightarrow 0$ )*

$$p \lim_{I \rightarrow \infty} \mathcal{A} = \frac{1}{\mathcal{L}}, \quad (6)$$

where  $\mathcal{L} = \int_0^1 \ell(s)ds$  is the integrated liquidity. Furthermore, as  $I \rightarrow \infty$

$$\frac{\log(\mathcal{A}) - \log\left(\frac{1}{\mathcal{L}}\right)}{\sqrt{\frac{2\delta(\pi/2-1)RV}{(\pi\delta/2)RPV^2}}} \xrightarrow{d} N(0, 1), \quad (7)$$

where  $RV = \sum_{i=1}^I r_i^2$  is the realized variance.

The Proof of Proposition 1 is in Appendix A.1. Proposition 1 shows that the realized Amihud is a measure of the inverse of the integrated liquidity  $\mathcal{L}$ , namely it is a measurement of the *integrated illiquidity*, which represents the price impact of trading volume cumulated over periods of unit length. The precision of the measurement increases with  $I$  and the asymptotic distribution in (7) can be used to construct a confidence interval for  $\mathcal{L}$ . By taking the ratio between volatility and volume, we decouple the information about market illiquidity from that of the information flow, which generates price volatility. Hence, by computing  $\mathcal{A}$  over disjoint periods of unit length (e.g. on daily horizons), one can obtain a time-series of measurements of  $\frac{1}{\mathcal{L}}$ , and use them to explore the evolution of illiquidity over time (see Section 4).

The result in Proposition 1 is obtained under the assumption that there are two representative traders (so that  $J = 2$ ), stemming from the demand and supply sides of the market in each intradaily period. Analogous results are obtained if  $J \geq 2$  assuming homogeneity of traders, that is,  $\sigma_j = \sigma \quad \forall j = 1, 2, \dots, J$ . In particular, the following proposition highlights the main determinants of the realized Amihud as an illiquidity measure under homogeneity of traders when  $J \geq 2$ .

**Proposition 2.** *Consider the illiquidity measure defined in (5), the equilibrium relation in (1), and the diffusive process for reservation prices in (3). Assume that  $\ell(t)$  is a strictly positive càdlàg process with*

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<sup>11</sup>By analogy with the realized Amihud, one could employ the *realized Amivest*, which is defined as the reciprocal of the realized Amihud (the ratio of volume over volatility) that gives the volume of trading associated with a one standard deviation change in the price of a security.

$\sigma_j = \sigma \forall j = 1, \dots, J$ . As  $I \rightarrow \infty$  (i.e.,  $\delta \rightarrow 0$ )

$$p \lim_{I \rightarrow \infty} \mathcal{A} = \frac{2}{J\sqrt{J-1}\mathcal{L}}, \quad (8)$$

Furthermore, as  $I \rightarrow \infty$

$$\frac{\log(\mathcal{A}) - \log\left(\frac{2}{J\sqrt{J-1}\mathcal{L}}\right)}{\sqrt{\frac{J\delta(\pi/2-1)RV}{(J-1)(\pi\delta/2)RPV^2}}} \xrightarrow{d} N(0, 1), \quad (9)$$

where  $RV = \sum_{i=1}^I (r_i)^2$  is the realized variance.

In this case, it follows that in the limit for  $I \rightarrow \infty$ , the realized Amihud is inversely proportional to the integrated illiquidity and to the number of active traders in the market. In other words, the more active traders are present on the market, the more the market is liquid (ceteris paribus the level of  $\mathcal{L}$ ).<sup>12</sup> In the special case with  $J = 2$ , we obtain the same limit of  $\mathcal{A}$  as in Proposition 1. In the next section, we run Monte Carlo simulations to assess the finite sample performance of the realized Amihud in measuring illiquidity.

## 2.1 Numerical Analysis

We assess the quality of the measurement of illiquidity based on the realized Amihud by Monte Carlo simulations, in which the daily horizon is taken as the reference unit interval. The liquidity process,  $\ell(t)$ , is generated according to a Ornstein–Uhlenbeck process on  $\log \ell(t)$ , similar to the one used in Melino and Turnbull (1990) and Comte and Renault (1998) in the stochastic volatility context,

$$d \log \ell(t) = \kappa_\ell (\log \ell(t) - \ell_0) dt + \eta_\ell dW_\ell(t), \quad t > 0, \quad (10)$$

where  $\exp(\ell_0) = 50, 200, 500$  represents the long-term mean in the low-, medium-, and high-liquidity scenarios, respectively. The parameters  $\kappa_\ell$  and  $\eta_\ell$  determine the speed of the mean-reversion and the volatility of liquidity, respectively.<sup>13</sup> The reservation prices (and, hence, equilibrium prices) are generated according to the diffusive process in (3) with  $J = 2$ . The variance coefficients are set to  $\sigma_1^2 = 2$  and  $\sigma_2^2 = 1$ , respectively, while  $\mu_j = 0$ .<sup>14</sup>

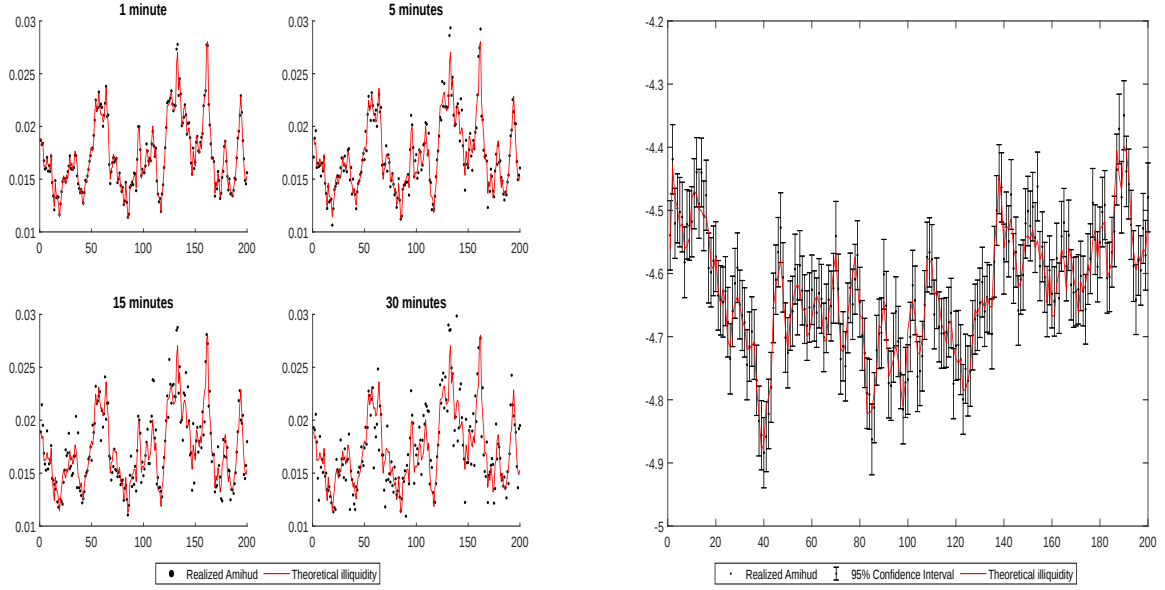
In the simulation of price and volume trajectories, we consider  $M = 1,000$  Monte Carlo replications for  $T = 1,000$  transaction days and  $I^* = 5,760$  intra-daily intervals corresponding to a 15-second frequency over 24 hours. RPV is then computed aggregating at daily horizon the absolute returns sampled at different frequencies: 1 hour; 30, 15, 10, 5 minutes; and 30 and 15 seconds, that is  $I = [24, 48, 96, 144, 288, 1440, 2880, 5760]$  equally-spaced sub-intervals, respectively. Trading volume is obtained aggregating at daily horizon the volume generated in each sub-period according to equation

<sup>12</sup>Furthermore, the number of active traders could be assumed to follow a stochastic process for counts, for instance a Poisson process, to allow for random entry and exit of traders. This would surely add more flexibility to the model, and it is left to future research. In the current context, the variation in illiquidity conditions only depend on changes of  $\ell(t)$ .

<sup>13</sup>In line with the empirical evidence displayed in Section 4, the simulated liquidity process  $\ell(t)$  also displays a persistent behavior.

<sup>14</sup>A number of alternative parametric settings for the Monte Carlo have been considered and the results are reported in Appendix B. In all cases, the results are analogous to those reported below.

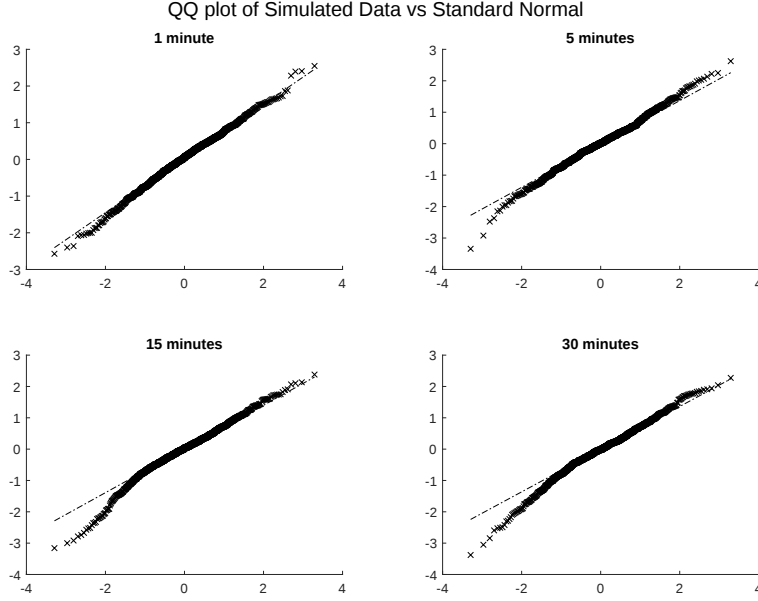
(2). Hence, for each Monte Carlo simulation and for each sampling frequency, we obtain a time-series of realized Amihud  $\{\mathcal{A}_t\}_{t=1}^T$  that we compare with the simulated time-series of integrated illiquidity,  $\{\frac{1}{\mathcal{L}_t^*}\}_{t=1}^T$ , where  $\mathcal{L}_t^* = \sum_{i=1}^{I^*} \ell_{i,t}$ ,  $t = 1, \dots, T$ .



**Figure 2:** The realized Amihud at different frequencies and confidence intervals. The figure on the left reports the true illiquidity limit  $\frac{1}{\mathcal{L}}$  (red solid line), and the realized Amihud (dots) obtained by sampling returns at different frequencies of 1 minute, 5, 15 and 30 minutes for a subset of 200 days. The figure on the right reports the true illiquidity signal ( $\log \mathcal{L}$ , red) and 95% confidence bands based on the realized Amihud computed with 1-minute returns.

As a graphical illustration of the ability of the realized Amihud to precisely measure the latent illiquidity process, the left panel of Figure 2 reports the time-series of the daily true illiquidity signal (in red) together with its estimates (blue dots) that are the realized Amihud obtained by sampling at different frequencies (1 minute, 5, 15 and 30 minutes). As the sampling frequency increases, the variability around the true illiquidity signal decreases, and it becomes negligible at the 1-minute frequency. In other words, by taking the ratio between an increasingly refined (in  $I$ ) measurement of daily volatility and daily trading volume, we obtain a very precise measurement of the day-by-day variations of the trade impact, which is a relevant dimension of illiquidity. The graph to the right of Figure 2 exploits the asymptotic distribution result in Proposition 1 and reports the 95% confidence bands around the true (log) illiquidity signal. The true illiquidity series lies within the confidence bands, thus confirming the goodness of the asymptotic approximation in (7). As a further illustration of asymptotic distribution of the realized Amihud, we construct quantile-quantile (QQ-) plots based on equation (9). Figure 3 reports the QQ-plots based on the simulation experiment described above, where  $I = 24, 76, 288, 1440$ . As  $I$  increases, the fit to the Gaussian distribution improves, being already remarkable at 5-minute frequencies.

The summary of the results of the Monte Carlo simulations are presented in Table 1. The realized Amihud provides accurate measurements of the true integrated illiquidity process (i.e.,  $\frac{1}{\mathcal{L}}$ ) in all scenarios (low-, medium-, and high-illiquidity levels) with a bias (relative to the illiquidity signal) below



**Figure 3:** QQ-plot. Figure reports the QQ-plots and illustrates the approximation to a standard Gaussian random variable as in (7). We consider different sampling frequencies, namely 1 minute, 5 and 15 minutes, and 1 hour.

0.5% in absolute value and a small RMSE, even for relatively moderate sampling frequencies (e.g., 5-10 minutes). As expected from Theorem (1), the RMSE decreases as  $I$  increases (i.e.,  $\delta$  decreases). In general the RMSE of the realized Amihud is much smaller compared with the RMSE achieved by the daily Amihud. This result confirms that the classic daily Amihud represents the limiting case of an illiquidity measurement obtained with only one observation per trading period, that is,  $I = \delta = 1$ . In this case, the illiquidity measure in (5) reduces to  $\mathcal{A}^D = \frac{|r|}{v}$ .

We consider an alternative low-frequency version of the daily Amihud, that is the *high-low Amihud*. We obtain it by exploiting the high-low price range as a proxy of volatility, see Parkinson (1980), among others. In particular, the high-low Amihud is defined as  $\mathcal{A}^{HL} = \frac{range}{v}$ , where  $range = \sqrt{\frac{1}{4 \log(2)} (p^H - p^L)^2}$  with  $p^H$  and  $p^L$  being the daily high and low log-prices on a given unit interval (a day in this case). The Monte Carlo simulations show that the high-low Amihud displays remarkable properties. It is characterized by a small negative bias (around 5%) and is approximately three times more efficient than the traditional daily Amihud. This suggests that the high-low Amihud constitutes a viable alternative to the daily Amihud when intra-day prices are not available.

### 3 Robust Illiquidity Measurement

It should be stressed that the asymptotic results (in the limit for  $I \rightarrow \infty$ ) behind Proposition 1 are derived by abstracting from a number of realistic features that characterize the price formation of financial assets. In particular, in this section we consider alternative and refined realized Amihud measurements that are robust to financial features, such as stochastic volatility, market microstructure noise and information jumps.

| Low Liquidity, $\ell_0 = \log(50)$     |                          |                    |                 |               |                    |                 |
|----------------------------------------|--------------------------|--------------------|-----------------|---------------|--------------------|-----------------|
| Sampling frequency                     | Percentage Relative Bias |                    |                 | Relative RMSE |                    |                 |
|                                        | $\mathcal{A}$            | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 1h                                     | 2.9927                   | -6.4529            | -4.6579         | 0.2347        | 0.2952             | 0.7661          |
| 30min                                  | 1.5529                   | -6.4529            | -4.6579         | 0.1636        | 0.2952             | 0.7661          |
| 15min                                  | 0.8419                   | -6.4529            | -4.6579         | 0.1112        | 0.2952             | 0.7661          |
| 10min                                  | 0.6854                   | -6.4529            | -4.6579         | 0.0912        | 0.2952             | 0.7661          |
| 5min                                   | 0.2782                   | -6.4529            | -4.6579         | 0.0636        | 0.2952             | 0.7661          |
| 1min                                   | 0.0092                   | -6.4529            | -4.6579         | 0.0280        | 0.2952             | 0.7661          |
| 30sec                                  | 0.0589                   | -6.4529            | -4.6579         | 0.0202        | 0.2952             | 0.7661          |
| 15sec                                  | 0.0515                   | -6.4529            | -4.6579         | <b>0.0149</b> | 0.2952             | 0.7661          |
| Medium Liquidity, $\ell_0 = \log(200)$ |                          |                    |                 |               |                    |                 |
| Sampling frequency                     | Percentage Relative Bias |                    |                 | Relative RMSE |                    |                 |
|                                        | $\mathcal{A}$            | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 1h                                     | 3.5953                   | -3.9504            | 2.4229          | 0.2324        | 0.2937             | 0.7551          |
| 30min                                  | 2.2109                   | -3.9504            | 2.4229          | 0.1580        | 0.2937             | 0.7551          |
| 15min                                  | 0.5277                   | -3.9504            | 2.4229          | 0.1085        | 0.2937             | 0.7551          |
| 10min                                  | 0.6932                   | -3.9504            | 2.4229          | 0.0873        | 0.2937             | 0.7551          |
| 5min                                   | 0.2987                   | -3.9504            | 2.4229          | 0.0615        | 0.2937             | 0.7551          |
| 1min                                   | 0.0233                   | -3.9504            | 2.4229          | 0.0276        | 0.2937             | 0.7551          |
| 30sec                                  | -0.0439                  | -3.9504            | 2.4229          | 0.0194        | 0.2937             | 0.7551          |
| 15sec                                  | 0.0062                   | -3.9504            | 2.4229          | <b>0.0139</b> | 0.2937             | 0.7551          |
| High Liquidity, $\ell_0 = \log(500)$   |                          |                    |                 |               |                    |                 |
| Sampling frequency                     | Percentage Relative Bias |                    |                 | Relative RMSE |                    |                 |
|                                        | $\mathcal{A}$            | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 1h                                     | 1.4027                   | -5.2041            | 2.1640          | 0.2330        | 0.2905             | 0.7635          |
| 30min                                  | 0.9288                   | -5.2041            | 2.1640          | 0.1609        | 0.2905             | 0.7635          |
| 15min                                  | 0.2518                   | -5.2041            | 2.1640          | 0.1100        | 0.2905             | 0.7635          |
| 10min                                  | 0.0380                   | -5.2041            | 2.1640          | 0.0884        | 0.2905             | 0.7635          |
| 5min                                   | -0.1005                  | -5.2041            | 2.1640          | 0.0635        | 0.2905             | 0.7635          |
| 1min                                   | 0.0265                   | -5.2041            | 2.1640          | 0.0279        | 0.2905             | 0.7635          |
| 30sec                                  | 0.0287                   | -5.2041            | 2.1640          | 0.0199        | 0.2905             | 0.7635          |
| 15sec                                  | -0.0078                  | -5.2041            | 2.1640          | <b>0.0139</b> | 0.2905             | 0.7635          |

**Table 1:** Illiquidity measurement. The table reports the Monte Carlo relative percentage bias and RMSE (both relative to  $\frac{1}{\mathcal{L}}$ ) for several illiquidity estimators: the realized Amihud  $\mathcal{A} = \frac{RPV}{v}$ ; the daily Amihud measure, that is,  $\mathcal{A}^D = \frac{|r|}{v}$ ; the high-low Amihud,  $\mathcal{A}^{HL} = \frac{range}{v}$ , where *range* is scaled by  $\sqrt{2/\pi}$  to be comparable with the RPV. The series  $\ell(t)$  is generated according to the log Ornstein–Uhlenbeck process in (10), with parameters  $\kappa_\ell = 0.05$  and  $\eta_\ell = 0.1$ . The smallest RMSE is in bold.

### 3.1 Spot Volatility and Illiquidity

We now extend the setting outlined before and allow the reservation prices (and, hence, equilibrium prices) to be generated according to the diffusive process in (3) where both the drift,  $\mu_j(t)$ , and the variance,  $\sigma_j^2(t)$ , can be time-varying processes. In particular, the reservation prices are assumed to

vary in continuous time according to the following law of motion

$$dp_j^*(t) = \mu_j(t)dt + \sigma_j(t)dW_j(t), \quad j = 1, \dots, J, \quad t > 0, \quad (11)$$

where  $\mu_j(t)$  and  $\sigma_j(t)$  are both independent of  $W_j(t)$  and, together with  $\ell(t)$ , they must satisfy mild regularity conditions (see [Kristensen, 2010](#)), that is, for  $(i-1)\delta \leq s \leq q \leq i\delta$

$$\lim_{\delta \rightarrow 0} \delta \sum_{i=1}^I |\mu_s^2 - \mu_q^2| = 0, \quad \lim_{\delta \rightarrow 0} \delta \sum_{i=1}^I |\sigma_{j,s}^4 - \sigma_{j,q}^4| = 0, \quad \lim_{\delta \rightarrow 0} \delta \sum_{i=1}^I |\ell_s^2 - \ell_q^2| = 0, \quad (12)$$

This condition imposes restrictions on the local behavior of the volatility and liquidity processes, and it allows for standard diffusion processes. It allows for deterministic patterns, jumps, and nonstationarity, and is automatically satisfied when the mean, the volatility and liquidity processes have continuous trajectories. The fact that the variance can change across traders reconciles with many realistic features, including the evidence of long-memory in volatility, as obtained by the superposition of traders operating at different frequencies, which, for instance, can be seen with the heterogeneous autoregressive models of [Müller et al. \(1997\)](#) and [Corsi \(2009\)](#).

We propose the following estimator of instantaneous illiquidity, that we call *spot Amihud*, given by

$$\mathcal{A}(\tau) = \sqrt{\frac{RV(\tau)}{v^2(\tau)}}, \quad \tau \in (0, 1), \quad (13)$$

where  $RV(\tau) = \sum_{|s-\tau|<b} \mathcal{K}\left(\frac{s-\tau}{b}\right) r_s^2$ , and  $v^2(\tau) = \sum_{|s-\tau|<b} \mathcal{K}\left(\frac{s-\tau}{b}\right) v_s^2$ . The term  $b$  denotes the bandwidth and  $\mathcal{K}$  denotes the kernel function. For instance, if  $\mathcal{K} = 1$ , then the estimator results into a rolling average. Alternatively, the one side kernel  $\mathcal{K}$  with support on  $[-1, 0]$  yields an estimator that uses the immediate past data, and one side exponential kernel  $\mathcal{K}(x) = e^x \mathbb{I}(x \leq 0)$  results in the *riskmetrics* exponential smoothing, see [Fan and Wang \(2008\)](#).<sup>15</sup> Analogously, we can define a measure of instantaneous liquidity, that we call *spot Amivest*, given by

$$\mathcal{A}^{-1}(\tau) = \sqrt{\frac{v^2(\tau)}{RV(\tau)}}, \quad \tau \in (0, 1). \quad (14)$$

**Proposition 3.** *Consider the illiquidity measure defined in (5), the equilibrium relation in (1), and the diffusive process for reservation prices in (3). Assume that  $\sigma_j^2(t)$  and  $\ell(t)$  are a strictly positive càdlàg processes  $\forall j = 1, \dots, J$ . Assume  $J = 2$  active traders, as representative of the two aggregated sides of the market. As  $I \rightarrow \infty$  (i.e.,  $\delta \rightarrow 0$ ),  $b \rightarrow 0$  and  $Ib \rightarrow \infty$ , we get*

$$\mathcal{A}^{-1}(\tau) \xrightarrow{p} \ell(\tau), \quad \text{and} \quad \mathcal{A}(\tau) \xrightarrow{p} \frac{1}{\ell(\tau)}. \quad (15)$$

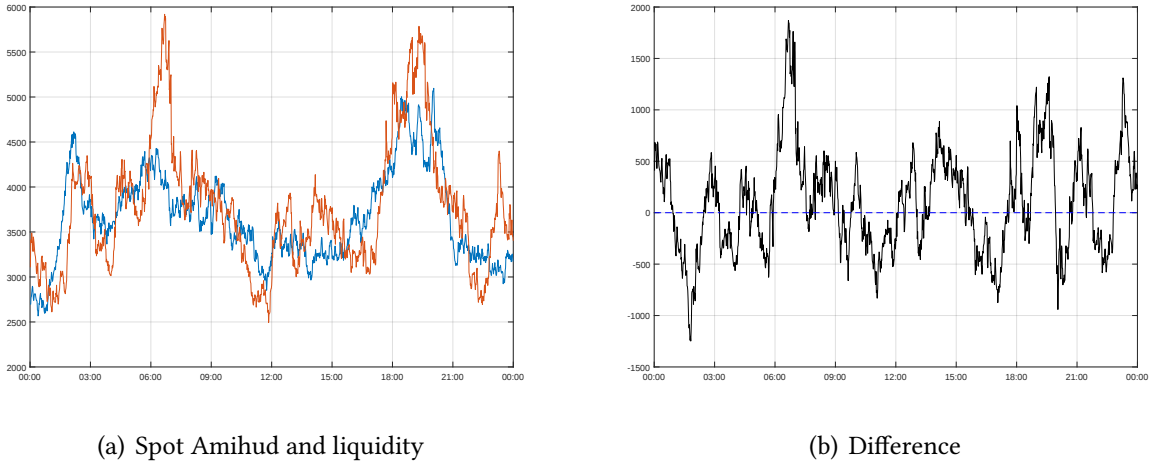
<sup>15</sup>A vast and growing literature addresses the issue of consistently estimating spot volatility using high-frequency returns, see among many others the contributions of [Foster and Nelson \(1996\)](#), [Mykland and Zhang \(2009\)](#), [Jacod and Rosenbaum \(2013\)](#), [Kanaya and Kristensen \(2016\)](#), [Bandi and Renò \(2018\)](#), and [Figueroa-López and Li \(2020\)](#). In this Section, we follow the notation and the set of assumptions outlined in [Kristensen \(2010\)](#).

Furthermore,  $a \rightarrow 0$ ,  $b \rightarrow 0$  (with  $a/b \rightarrow 0$ ) and  $I \rightarrow \infty$ ,

$$\int_a^{1-a} \mathcal{A}^{-1}(s) ds \xrightarrow{p} \mathcal{L}, \quad (16)$$

where the integral  $\int_a^{1-a} \mathcal{A}^{-1}(s) ds$  is approximated by the Riemann sum  $\overline{\mathcal{A}^{-1}} = \frac{1}{I} \sum_{i=1}^I \mathcal{A}^{-1}(\tau_i)$ .

Proposition 3 shows that, when both liquidity and volatility follow (possibly correlated) stochastic processes, we can precisely measure spot liquidity (or spot illiquidity) by resorting to local estimators such as the spot Amivest (or the spot Amihud). Panel a) of Figure 4 reports an intradaily (24 hours) simulated trajectory of the spot liquidity (blue line) and the spot Amivest (red line) computed based on a 1-minute sampling frequency ( $I = 1440$ ) and a bandwidth  $b$  of 1 hour. Spot Amivest timely approximates the underlying spot liquidity process, where deviations from the target are short-lived and they tend to revert to zero (see Panel b).



**Figure 4:** Spot liquidity. Panel a) reports an intradaily (24 hours) simulated trajectory of the spot liquidity (blue line),  $\ell(\tau)$ , and the spot Amivest (red line),  $\mathcal{A}^{-1}(\tau)$ , computed based on a 1-minute sampling frequency ( $I = 1440$ ) and a bandwidth  $b$  of 1 hour. The series  $\ell(t)$  is generated according to the log Ornstein–Uhlenbeck process in (10), with parameters  $\kappa_\ell = 0.1$  and  $\eta_\ell = 0.6$ . The variances evolve following the dynamics of the CIR process, that is,  $d\sigma_j^2(t) = \kappa_\sigma(\sigma^2(t) - \sigma_0^2)dt + \eta_\sigma\sigma_j(t)dW_{\sigma,j}(t)$  with the parameters  $\kappa_\sigma = 0.05$ ,  $\sigma_0^2 = 6$ ,  $\eta_\sigma = 0.05$ . Panel b) reports the difference  $D(\tau) = \mathcal{A}^{-1}(\tau) - \ell(\tau)$ .

Table 2 reports the results of the Monte Carlo simulations in a context with both stochastic liquidity and volatility. The setting of the simulations is very similar to that with constant volatility reported in Table 1, with the exception that the variances,  $\sigma_j^2(t)$   $j = 1, \dots, J$ , now follow the dynamics of the CIR process, that is,  $d\sigma_j^2(t) = \kappa_\sigma(\sigma^2(t) - \sigma_0^2)dt + \eta_\sigma\sigma_j(t)dW_{\sigma,j}(t)$  with parameters  $\kappa_\sigma = 0.05$ ,  $\sigma_0^2 = 6$ , and  $\eta_\sigma = 0.05$ . Table 2 reports relative bias and RMSE for the realized Amihud, the HL Amihud and daily Amihud. Furthermore, it reports relative bias and RMSE of two new estimators, based on the inverse of the daily aggregates of two spot Amivest measures: the *smoothed Amivest*,  $\overline{\mathcal{A}_s^{-1}}(\tau)$  (based on a two-sided window around  $\tau$ ), and the *filtered Amivest*,  $\mathcal{A}_f^{-1}(\tau)$  (which only uses observations in a window before  $\tau$ ). In both cases, the bandwidth (i.e. the length of the window for the spot estimators) is one hour. For all the estimators, the target quantity is the daily integrated illiquidity, that is  $\frac{1}{\mathcal{L}}$ . For all liquidity levels, the estimator with the lowest RMSE is  $\overline{\mathcal{A}_s^{-1}}$ , that is the inverse of the daily average of the

smoothed Amivest computed with returns and volume sampled at 15-second frequency. Interestingly, the realized Amihud,  $\mathcal{A}$ , although not being theoretically robust to stochastic volatility, displays levels of RMSE very close to those of  $\overline{\mathcal{A}}_s$ . This is the consequence of the fact that empirically the amount of intradaily volatility of volatility is not particularly pronounced,<sup>16</sup> so that the integral  $\int_0^1 \ell(s) \bar{\sigma}(s) ds$  that arises in the limit of trading volume for  $I \rightarrow \infty$  is empirically very close to the product of integrals  $\int_0^1 \ell(s) ds \int_0^1 \bar{\sigma}(s) ds$ . Instead, the high-low Amihud and the daily Amihud are much less efficient than the realized Amihud in measuring daily illiquidity.

| Low Liquidity, $\ell_0 = \log(50)$     |                            |                            |               |                    |                 |                            |                            |               |                    |                 |
|----------------------------------------|----------------------------|----------------------------|---------------|--------------------|-----------------|----------------------------|----------------------------|---------------|--------------------|-----------------|
| Sampling frequency                     | $\overline{\mathcal{A}}_s$ | Percentage Relative Bias   |               |                    |                 | $\overline{\mathcal{A}}_s$ | Relative RMSE              |               |                    |                 |
|                                        |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 15min                                  | -10.1547                   | -13.4362                   | 0.7472        | -3.4042            | 4.1120          | 0.2210                     | 0.2596                     | 0.1794        | 0.4618             | 1.2736          |
| 10min                                  | -6.6812                    | -7.9857                    | 0.7285        | -3.4042            | 4.1120          | 0.1596                     | 0.1766                     | 0.1447        | 0.4618             | 1.2736          |
| 5min                                   | -3.7582                    | -4.0519                    | 0.1472        | -3.4042            | 4.1120          | 0.1066                     | 0.1118                     | 0.0978        | 0.4618             | 1.2736          |
| 1min                                   | -0.9260                    | -0.8705                    | 0.0274        | -3.4042            | 4.1120          | 0.0426                     | 0.0448                     | 0.0446        | 0.4618             | 1.2736          |
| 30sec                                  | -0.4573                    | -0.4119                    | 0.0798        | -3.4042            | 4.1120          | 0.0290                     | 0.0334                     | 0.0299        | 0.4618             | 1.2736          |
| 15sec                                  | -0.3011                    | -0.2649                    | 0.0422        | -3.4042            | 4.1120          | <b>0.0221</b>              | 0.0292                     | 0.0226        | 0.4618             | 1.2736          |
| Medium Liquidity, $\ell_0 = \log(200)$ |                            |                            |               |                    |                 |                            |                            |               |                    |                 |
| Sampling frequency                     | $\overline{\mathcal{A}}_s$ | Percentage Relative Bias   |               |                    |                 | $\overline{\mathcal{A}}_s$ | Relative RMSE              |               |                    |                 |
|                                        |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 15min                                  | -11.2163                   | -14.6809                   | 0.1810        | -3.6646            | -3.4094         | 0.3924                     | 0.4746                     | 0.2240        | 0.6180             | 1.9773          |
| 10min                                  | -6.6702                    | -8.1643                    | 0.8248        | -3.6646            | -3.4094         | 0.2236                     | 0.2596                     | 0.1442        | 0.6180             | 1.9773          |
| 5min                                   | -3.9566                    | -4.3750                    | -0.1258       | -3.6646            | -3.4094         | 0.1344                     | 0.1381                     | 0.0946        | 0.6180             | 1.9773          |
| 1min                                   | -0.8942                    | -0.9205                    | 0.0391        | -3.6646            | -3.4094         | 0.0551                     | 0.0496                     | 0.0638        | 0.6180             | 1.9773          |
| 30sec                                  | -0.6679                    | -0.6790                    | -0.2349       | -3.6646            | -3.4094         | 0.0642                     | 0.0716                     | 0.0759        | 0.6180             | 1.9773          |
| 15sec                                  | -0.3623                    | -0.3806                    | -0.0009       | -3.6646            | -3.4094         | <b>0.0280</b>              | 0.0449                     | 0.0358        | 0.6180             | 1.9773          |
| High Liquidity, $\ell_0 = \log(500)$   |                            |                            |               |                    |                 |                            |                            |               |                    |                 |
| Sampling frequency                     | $\overline{\mathcal{A}}_s$ | Percentage Relative Bias   |               |                    |                 | $\overline{\mathcal{A}}_s$ | Relative RMSE              |               |                    |                 |
|                                        |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 15min                                  | -10.9951                   | -14.6120                   | 0.1625        | -4.2758            | -2.8638         | 0.3674                     | 0.4313                     | 0.2040        | 0.4882             | 1.3056          |
| 10min                                  | -7.0540                    | -8.5025                    | 0.5101        | -4.2758            | -2.8638         | 0.2116                     | 0.2264                     | 0.1715        | 0.4882             | 1.3056          |
| 5min                                   | -3.8084                    | -4.1203                    | 0.0517        | -4.2758            | -2.8638         | 0.1283                     | 0.1316                     | 0.1108        | 0.4882             | 1.3056          |
| 1min                                   | -0.7831                    | -0.7803                    | 0.2180        | -4.2758            | -2.8638         | 0.0566                     | 0.0550                     | 0.0519        | 0.4882             | 1.3056          |
| 30sec                                  | -0.3909                    | -0.3945                    | 0.1734        | -4.2758            | -2.8638         | 0.0309                     | 0.0407                     | 0.0322        | 0.4882             | 1.3056          |
| 15sec                                  | -0.2582                    | -0.2513                    | 0.0588        | -4.2758            | -2.8638         | <b>0.0210</b>              | 0.0343                     | 0.0237        | 0.4882             | 1.3056          |

**Table 2:** Integrated spot illiquidity. The table reports the Monte Carlo relative percentage bias and RMSE (both relative to  $\frac{1}{\ell}$ ) for several illiquidity estimators: the smoothed integrated illiquidity, denoted as  $\overline{\mathcal{A}}_s = \left( \frac{1}{I} \sum_{i=1}^I \mathcal{A}_s^{-1}(\tau_i) \right)^{-1}$ , where  $\mathcal{A}_s^{-1}(\tau_i)$  is the spot Amivest based on a (equally weighted) two-sided kernel function; the filtered integrated illiquidity, denoted as  $\overline{\mathcal{A}}_f = \left( \frac{1}{I} \sum_{i=1}^I \mathcal{A}_f^{-1}(\tau_i) \right)^{-1}$ , where  $\mathcal{A}_f^{-1}(\tau_i)$  is the spot Amivest based on an (equally weighted) backward kernel function; the realized Amihud  $\mathcal{A} = \frac{RPV}{v}$ ; the daily Amihud measure, that is,  $\mathcal{A}^D = \frac{|r|}{v}$ ; the high-low Amihud,  $\mathcal{A}^{HL} = \frac{range}{v}$ , where  $range$  is scaled by  $\sqrt{2/\pi}$  to be comparable with the RPV. The series  $\ell(t)$  is generated according to the log Ornstein–Uhlenbeck process in (10), with parameters  $\kappa_\ell = 0.1$  and  $\eta_\ell = 0.6$ . The variances evolve following the dynamics of the CIR process, that is,  $d\sigma_j^2(t) = \kappa_\sigma(\sigma^2(t) - \sigma_0^2)dt + \eta_\sigma\sigma(t)dW_{\sigma,j}(t)$  with the parameters  $\kappa_\sigma = 0.05$ ,  $\sigma_0^2 = 6$ ,  $\eta_\sigma = 0.05$ . The spot Amivest uses a bandwidth length of one hour. The smallest RMSE is in bold.

Finally, we also consider the measurement of illiquidity over a longer time span, that is the monthly horizon.<sup>17</sup> Table 18 in Appendix C.5 shows that at monthly horizon the aggregated realized Amihud

<sup>16</sup>This result remains valid with other more extreme parametric configurations, reported in Appendix C.

<sup>17</sup>While the realized Amihud is conceptually similar to the IDVOL measure of Lou and Shu (2017), the monthly averages of spot volatility estimates could be perceived as a refined version of the monthly ILLIQ measure of Amihud (2002), see the discussion in Amihud and Noh (2021, p.2103-2014). The fact that our results are based on infill asymptotic arguments

provides a remarkably precise measurement of monthly illiquidity, with a relative RMSE comparable to that obtained aggregating spot illiquidity measurements. For this reason, in the subsequent numerical and empirical analyses in this paper we will only consider the realized Amihud estimator of integrated illiquidity and compare it with daily and high-low Amihud. Notably, averaging the daily Amihud over monthly horizons greatly reduces the variability of the estimator (the RMSE is much lower than when measuring daily illiquidity) but the estimator remains several times more inefficient than the realized counterpart.

### 3.2 Microstructure Frictions

In this section, we study the robustness of the realized Amihud to microstructural frictions (namely *microstructure noise*), like transaction costs in the form of bid-ask spread, clearing fees, or price discreteness, which are intimately related and endogenous to the trading process. Although in the simplified context provided by the model outlined in Section 2 these microstructural features are not explicitly included, they pose a relevant issue to be addressed at least in the empirical analysis. From a statistical point of view, the microstructure noise dominates the volatility signal as  $I \rightarrow \infty$ , thus leading to distorted measurements of the variance. In the literature on realized variance it is common practice to resort to moderate sampling frequencies, for example, 5-minute intervals, to reduce the impact of the microstructure noise contamination on the volatility measurement, see [Aït-Sahalia and Jacod \(2014\)](#) and [Liu et al. \(2015\)](#) for a discussion.<sup>18</sup> This approach can be carried over to the illiquidity measurement, and its effectiveness in reducing the impact of microstructure noise contamination is confirmed by the evidence reported in Table 3. In particular, we repeat the analysis of Table 1, but this time, we add the bid-ask spread to the equilibrium prices and consider a rounding mechanism that generates price discreteness (rounding to cents or decimalization effect) and zero returns; for more, see [Bandi et al. \(2020\)](#), among others.<sup>19</sup>

The results in Table 3 deliver two main messages. First, it suggests that, in this setting, we should avoid computing the realized Amihud by sampling at the highest possible frequencies (e.g., from 15 to 30 seconds) because this leads to large estimation biases. Instead, sampling at moderate frequencies (e.g., from 5 to 10 minutes) drastically reduces the bias while also leading to a low RMSE, especially if compared with the daily Amihud, which is about 10 times larger (0.069 compared with 0.733 for the medium liquidity scenario). Second, the realized Amihud measure also performs well in a low liquidity setting suggesting that its accuracy remains high for less frequently traded financial securities.

In summary, the realized Amihud based on 5-minute returns provides accurate measurements of the illiquidity associated with the price impact even in settings characterized by other dimensions of

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allows to avoid the need to expected values of the daily Amihud ratio to characterize its properties, and it provides an ideal bridge between the theoretical properties of the IDVOL and the ILLIQ.

<sup>18</sup>Alternative techniques designed to directly tackle the microstructural features in the measurement of volatility based on intra-daily returns are: the two-scales estimator of [Zhang et al. \(2005\)](#) based on the idea of subsampling (see also [Zu and Boswijk, 2014](#) for an adaptation of the two-scales estimator in the spot volatility context), the optimal sampling frequency method of [Bandi and Russell \(2008\)](#), the realized kernel by [Barndorff-Nielsen et al. \(2008\)](#), the pre-averaging estimator of [Podolskij and Vetter \(2009\)](#). In a large scale empirical analysis, [Liu et al. \(2015\)](#) find little evidence that the realized variance estimator based on returns sampled at 5-minutes frequencies is outperformed by the other measures.

<sup>19</sup>The bid-ask mechanism on the observed log-price is generated as  $\tilde{p}_i = p_i + \zeta_i \cdot \text{BAS}/2$ , where  $\text{BAS}$  is the bid-ask spread and  $\zeta_i$  is an i.i.d. random variable (independent of  $p_i$ ) taking value 1 or -1 with equal probability. The decimalization mechanism is then imposed on  $\tilde{p}_i$  by rounding  $\tilde{P}_i = \exp(\tilde{p}_i)$  to the closest cent.

| <b>Low Liquidity, <math>\ell_0 = \log(50)</math></b>     |                          |                    |                 |               |                    |                 |
|----------------------------------------------------------|--------------------------|--------------------|-----------------|---------------|--------------------|-----------------|
| Sampling frequency                                       | Percentage Relative Bias |                    |                 | Relative RMSE |                    |                 |
|                                                          | $\mathcal{A}$            | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 1hour                                                    | 1.8020                   | -4.8748            | -0.7818         | 0.2176        | 0.3054             | 0.7711          |
| 30min                                                    | 1.8298                   | -4.8748            | -0.7818         | 0.1569        | 0.3054             | 0.7711          |
| 15min                                                    | 1.6781                   | -4.8748            | -0.7818         | 0.1151        | 0.3054             | 0.7711          |
| 10min                                                    | 1.7789                   | -4.8748            | -0.7818         | 0.0921        | 0.3054             | 0.7711          |
| 5min                                                     | 2.9765                   | -4.8748            | -0.7818         | <b>0.0725</b> | 0.3054             | 0.7711          |
| 1min                                                     | 13.3029                  | -4.8748            | -0.7818         | 0.1476        | 0.3054             | 0.7711          |
| 30sec                                                    | 25.1862                  | -4.8748            | -0.7818         | 0.2720        | 0.3054             | 0.7711          |
| 15sec                                                    | 28.4629                  | -4.8748            | -0.7818         | 0.3149        | 0.3054             | 0.7711          |
| <b>Medium Liquidity, <math>\ell_0 = \log(200)</math></b> |                          |                    |                 |               |                    |                 |
| Sampling frequency                                       | Percentage Relative Bias |                    |                 | Relative RMSE |                    |                 |
|                                                          | $\mathcal{A}$            | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 1hour                                                    | 2.9646                   | -4.8349            | -1.9522         | 0.2258        | 0.2812             | 0.7334          |
| 30min                                                    | 1.5911                   | -4.8349            | -1.9522         | 0.1587        | 0.2812             | 0.7334          |
| 15min                                                    | 1.5683                   | -4.8349            | -1.9522         | 0.1110        | 0.2812             | 0.7334          |
| 10min                                                    | 1.8408                   | -4.8349            | -1.9522         | 0.0930        | 0.2812             | 0.7334          |
| 5min                                                     | 2.6054                   | -4.8349            | -1.9522         | <b>0.0690</b> | 0.2812             | 0.7334          |
| 1min                                                     | 12.0901                  | -4.8349            | -1.9522         | 0.1276        | 0.2812             | 0.7334          |
| 30sec                                                    | 22.9034                  | -4.8349            | -1.9522         | 0.2355        | 0.2812             | 0.7334          |
| 15sec                                                    | 25.5046                  | -4.8349            | -1.9522         | 0.2640        | 0.2812             | 0.7334          |
| <b>High Liquidity, <math>\ell_0 = 500</math></b>         |                          |                    |                 |               |                    |                 |
| Sampling frequency                                       | Percentage Relative Bias |                    |                 | Relative RMSE |                    |                 |
|                                                          | $\mathcal{A}$            | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 1hour                                                    | 1.8504                   | -5.9370            | -1.3766         | 0.2463        | 0.2818             | 0.7248          |
| 30min                                                    | 0.9157                   | -5.9370            | -1.3766         | 0.1640        | 0.2818             | 0.7248          |
| 15min                                                    | 0.9431                   | -5.9370            | -1.3766         | 0.1097        | 0.2818             | 0.7248          |
| 10min                                                    | 1.1633                   | -5.9370            | -1.3766         | 0.0892        | 0.2818             | 0.7248          |
| 5min                                                     | 2.2907                   | -5.9370            | -1.3766         | <b>0.0676</b> | 0.2818             | 0.7248          |
| 1min                                                     | 8.9761                   | -5.9370            | -1.3766         | 0.0964        | 0.2818             | 0.7248          |
| 30sec                                                    | 16.9324                  | -5.9370            | -1.3766         | 0.1736        | 0.2818             | 0.7248          |
| 15sec                                                    | 17.0739                  | -5.9370            | -1.3766         | 0.1764        | 0.2818             | 0.7248          |

**Table 3:** Illiquidity measurement with microstructure noise. The table reports Monte Carlo simulations for assessing the quality of illiquidity through the realized Amihud. The bid-ask spread is set at 0.15% relative to the price level, which is in line with the observed relative bid-ask spread of SPY (see Section 4). Table reports the Monte Carlo relative percentage bias and RMSE (both relative to  $\frac{1}{\ell}$ ) for the realized Amihud reported in (5). The smallest RMSE is in bold. As a reference, we also consider the daily Amihud measure, that is,  $\mathcal{A}^D = \frac{|r|}{v}$ , and the high-low Amihud,  $\mathcal{A}^{HL} = \frac{range}{v}$ , where  $range$  is scaled by  $\sqrt{2/\pi}$  to be comparable with the RPV.

illiquidity: the transaction costs such as the bid-ask spread, and additional frictions such as staleness due to inherent price discreteness and absence of new information. In the empirical analysis below, we work assuming that sampling asset returns at 5-minute intervals is sufficient to achieve a precise measurement of the asset illiquidity.

### 3.3 Information Jumps

Now, we further explore the relationship between liquidity and volatility, focusing on the arrival of large news common to all traders. Note that the increments of the reservation log-prices can be disentangled as follows

$$\Delta p_{i,j}^* = \phi_i + \psi_{i,j}, \quad \text{with } j = 1, \dots, J,$$

where  $\phi_i$  represents a fundamental information component common to all traders, stemming from public information events, such as those associated with earnings press releases or central banks' announcements. This could be related to events that trigger common directional beliefs among practitioners. The term  $\psi_{i,j}$  is the change in the discretized diffusive process outlined in (2), which represents the investor's specific component of the reservation price variation. Furthermore, the assumption of independence between  $\phi_i$  and  $\psi_{i,j}$  across time and traders does not allow for reversal or spillover effects, such as those studied in Grossman and Miller (1988) to investigate the mechanics of liquidity provision. The same type of sequential trading behavior has been recently proven to be responsible for crash episodes, as shown in Christensen et al. (2022), and associated with changes in the level of investors' disagreement around important news announcements, see, e.g., Bollerslev et al., 2018.<sup>20</sup> The detection of such informational events needs an accurate identification econometric technique and granular (intra-daily) data.

The recent advances in the literature on *jump* processes help in this analysis. Similarly to Bollerslev et al. (2018), we rely on a simple setup for the common news component, to separately identify it from the component of price variation due to the disagreement among traders, which is responsible for generating the trading volume.<sup>21</sup> For this reason, we refer to the term  $\phi_i$  as *information jumps*. In other words, when a common large news hits the market, the reservation prices all change in the same direction, thus leading to a new equilibrium price. This generates price volatility with little or no associated trading volume, so the realized Amihud is expected to be subject to a large positive shock. Ideally, we would like to disentangle this spike from the measurement of illiquidity related with the disagreement among traders.

We rely upon the theory of multipower variation developed in Barndorff-Nielsen and Shephard (2003, 2004, 2006) to carry out inference on information jumps in illiquidity. Following Barndorff-Nielsen and Shephard (2006), we assume that  $\phi_i$  is a compound Poisson process, namely  $\phi_i = \sum_{j=1}^{\mathcal{N}_i} c_j$ , where  $\mathcal{N}_i$  is a simple process (e.g., a Poisson) that counts the number of jump arrivals in the interval  $[i-1, i]$ , and it is finite for all intervals  $i$ . The terms  $c_j$  are nonzero random variables that determine the size of the jumps. When  $\mathcal{N}_i = 0$ , then  $\Delta p_{i,j}^*$  reduces to the stochastic volatility plus drift model, whose dynamics are described in (3). To carry out inference on jumps, we consider the multipower variation of order 1/2, namely

$$MPV = \frac{I}{I-1} \sum_{i=2}^I |r_i|^{1/2} |r_{i-1}|^{1/2}. \quad (17)$$

<sup>20</sup>Perraudin and Vitale (1996) also consider jump times as moments at which significant information becomes public knowledge.

<sup>21</sup>Other studies associating large price jumps with news announcements can be found in Andersen et al. (2007), Chaboud et al. (2008), Lee (2011), Jiang et al. (2011).

Following [Barndorff-Nielsen and Shephard \(2004, 2006\)](#), MPV converges to  $S$ , even when  $\phi_i \neq 0$ , that is

$$p \lim_{I \rightarrow \infty} \delta^{1/2} \mu_{1/2}^{-2} MPV = S, \quad (18)$$

where  $\mu_{1/2} = E[|X|]^{1/2}$  is a normalizing constant. Therefore, we can devise the jump test statistic,  $\mathcal{J}$ , which is given by

$$\mathcal{J} = \frac{\sqrt{\pi/2} \mu_{1/2}^2 RPV - MPV}{\widetilde{BPV}}, \quad (19)$$

where  $\widetilde{BPV} = \sqrt{\xi \sum_{i=2}^I |r_i| |r_{i-1}|}$  and  $\xi = \theta(\mu_1^2 + 2\mu_{1/2}^2 \mu_1 - 3\mu_{1/2}^4) / \mu_1^2$  is a scaling constant with  $\theta = 0.1032$ . Under the null hypothesis that  $\phi_i = 0$ ,  $\mathcal{J} \xrightarrow{d} N(0, 1)$ , we can use this result for testing the presence of a significant information jump in a given day. With the goal of wiping out the impact of information jumps on  $\mathcal{A}$ , we can construct the *jump-robust* realized Amihud as

$$\mathcal{A}^C = \frac{RPV_C}{\nu}, \quad (20)$$

where  $RPV_C$  is the jump-robust realized power variation, given by

$$RPV_C = \mathbb{I}(\mathcal{J} \leq q_{1-\alpha}) RPV + \mathbb{I}(\mathcal{J} > q_{1-\alpha}) \widetilde{MPV}, \quad (21)$$

where  $\mathbb{I}(\cdot)$  is the indicator function,  $q_{1-\alpha}$  denotes the  $(1 - \alpha)$ -th quantile of a standard Gaussian distribution,  $\alpha$  is the significance level of the jump test, and  $\widetilde{MPV} = \sqrt{2/\pi} \mu_{1/2}^{-2} MPV$ .

In the following, we study by Monte Carlo simulations the finite-sample size and power properties of the  $\mathcal{J}$ -test. In the left panel of Table 4, we consider a frictionless setting and three levels of nominal size, namely  $\alpha = 5\%, 1\%, 0.5\%$ . The best performance in terms of empirical size and power is achieved sampling at the highest possible frequency of 15-seconds. Furthermore, considering a setting where  $\phi_i \neq 0$  allows us to study the power of the test, that is, the ability to identify a jump if it occurs. In particular, we assume that  $\phi_i$  follows a compound Poisson process with an intensity equal to 5% (one jump every twenty days on average). Notably, the test correctly identifies jumps, that is, rejects the null hypothesis of no jumps, in more than 90% of the cases when sampling at the highest frequencies.

The results are quite different when considering microstructure noise, in the form of bid-ask spread and rounding effects, as shown in the right panel of Table 4. In particular, rounding to the closest cent of the dollar generates a large number of zeros in the returns sampled at the highest frequencies, as shown by [Bandi et al. \(2020\)](#). [Kolokolov and Renò \(2021\)](#) illustrate the detrimental effect of zeros on the quality of the jump-robust volatility estimates, especially when power variation measures are adopted. In circumstances where zeros are the dominant feature of the high-frequency returns (like for very illiquid stocks), the effect of the rounding on MPV is detrimental. In this case, MPV is associated with a large negative bias, and the jump test rejects the null hypothesis very frequently. In this case, sampling at 10 to 15 minutes frequencies drastically reduces the bias in MPV, thus leading to empirical sizes of the jump test closer to the theoretical ones. At these frequencies, the power remains high and above 85% in all cases.

| No noise         |  |        |        |        |        |       |       |         |        |        |         |         |         | Microstructure noise |    |      |  |  |  |      |  |  |  |  |  |  |  |
|------------------|--|--------|--------|--------|--------|-------|-------|---------|--------|--------|---------|---------|---------|----------------------|----|------|--|--|--|------|--|--|--|--|--|--|--|
| Low Liquidity    |  | Power  |        |        |        |       |       | Size    |        |        |         |         |         | Power                |    |      |  |  |  | Size |  |  |  |  |  |  |  |
|                  |  | 5%     | 1%     | 0.5%   | 5%     | 1%    | 0.5%  | 5%      | 1%     | 0.5%   | 5%      | 1%      | 0.5%    | 5%                   | 1% | 0.5% |  |  |  |      |  |  |  |  |  |  |  |
| 1h               |  | 79.83% | 75.35% | 73.80% | 10.31% | 4.69% | 3.52% | 80.99%  | 76.34% | 74.43% | 10.98%  | 5.08%   | 3.98%   |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |
| 30min            |  | 84.71% | 80.85% | 79.55% | 8.48%  | 3.16% | 2.18% | 84.94%  | 81.39% | 79.92% | 10.78%  | 4.70%   | 3.32%   |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |
| 15min            |  | 86.97% | 83.54% | 82.56% | 7.68%  | 2.51% | 1.56% | 88.59%  | 85.77% | 84.44% | 11.04%  | 4.64%   | 3.28%   |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |
| 10min            |  | 88.83% | 85.83% | 84.40% | 6.65%  | 2.02% | 1.29% | 90.49%  | 87.41% | 86.32% | 12.26%  | 4.74%   | 3.16%   |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |
| 5min             |  | 90.55% | 87.55% | 86.48% | 6.25%  | 1.77% | 1.10% | 93.07%  | 90.78% | 89.83% | 20.64%  | 8.82%   | 6.44%   |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |
| 1min             |  | 92.67% | 90.19% | 89.16% | 5.63%  | 1.35% | 0.77% | 98.96%  | 98.22% | 97.87% | 97.62%  | 93.02%  | 90.92%  |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |
| 30sec            |  | 93.35% | 90.71% | 89.77% | 5.72%  | 1.33% | 0.69% | 99.70%  | 99.45% | 99.32% | 100.00% | 100.00% | 100.00% |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |
| 15sec            |  | 93.52% | 91.45% | 90.67% | 5.44%  | 1.18% | 0.63% | 99.96%  | 99.96% | 99.94% | 100.00% | 100.00% | 100.00% |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |
| Medium Liquidity |  | Power  |        |        |        |       |       | Size    |        |        |         |         |         | Power                |    |      |  |  |  | Size |  |  |  |  |  |  |  |
|                  |  | 5%     | 1%     | 0.5%   | 5%     | 1%    | 0.5%  | 5%      | 1%     | 0.5%   | 5%      | 1%      | 0.5%    |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |
| 1h               |  | 79.87% | 75.46% | 73.96% | 10.70% | 4.89% | 3.69% | 80.04%  | 75.75% | 73.89% | 10.96%  | 5.00%   | 3.88%   |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |
| 30min            |  | 85.06% | 81.18% | 79.60% | 8.72%  | 3.42% | 2.31% | 84.32%  | 80.17% | 78.93% | 8.94%   | 3.50%   | 2.44%   |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |
| 15min            |  | 87.82% | 84.48% | 83.05% | 7.07%  | 2.42% | 1.54% | 88.67%  | 85.07% | 83.44% | 8.84%   | 3.06%   | 1.90%   |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |
| 10min            |  | 89.16% | 86.04% | 84.94% | 6.98%  | 2.29% | 1.30% | 89.99%  | 86.88% | 85.93% | 8.54%   | 2.72%   | 1.64%   |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |
| 5min             |  | 90.47% | 87.56% | 86.65% | 6.53%  | 1.92% | 1.05% | 93.05%  | 90.22% | 89.17% | 10.76%  | 4.00%   | 2.56%   |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |
| 1min             |  | 92.86% | 90.29% | 89.55% | 5.66%  | 1.24% | 0.68% | 99.27%  | 98.59% | 98.34% | 56.15%  | 36.61%  | 30.07%  |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |
| 30sec            |  | 93.13% | 90.97% | 90.05% | 5.47%  | 1.38% | 0.81% | 99.70%  | 99.62% | 99.54% | 97.88%  | 92.80%  | 89.72%  |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |
| 15sec            |  | 93.59% | 91.15% | 90.48% | 5.77%  | 1.35% | 0.73% | 100.00% | 99.92% | 99.92% | 100.00% | 100.00% | 100.00% |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |
| High Liquidity   |  | Power  |        |        |        |       |       | Size    |        |        |         |         |         | Power                |    |      |  |  |  | Size |  |  |  |  |  |  |  |
|                  |  | 5%     | 1%     | 0.5%   | 5%     | 1%    | 0.5%  | 5%      | 1%     | 0.5%   | 5%      | 1%      | 0.5%    |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |
| 1h               |  | 80.24% | 76.05% | 74.22% | 10.48% | 5.10% | 4.10% | 81.21%  | 76.91% | 75.55% | 10.72%  | 5.16%   | 3.90%   |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |
| 30min            |  | 84.04% | 80.22% | 78.79% | 8.53%  | 3.32% | 2.31% | 86.53%  | 82.76% | 81.48% | 10.12%  | 4.48%   | 3.22%   |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |
| 15min            |  | 87.96% | 84.13% | 82.89% | 7.51%  | 2.54% | 1.64% | 89.81%  | 86.61% | 85.37% | 12.22%  | 4.96%   | 3.48%   |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |
| 10min            |  | 89.04% | 85.32% | 84.49% | 6.86%  | 2.23% | 1.43% | 90.74%  | 87.90% | 86.92% | 13.52%  | 5.52%   | 3.76%   |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |
| 5min             |  | 90.59% | 88.03% | 86.67% | 6.46%  | 1.87% | 1.12% | 94.07%  | 91.53% | 90.66% | 24.08%  | 11.30%  | 8.14%   |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |
| 1min             |  | 92.40% | 90.14% | 89.25% | 5.67%  | 1.29% | 0.67% | 99.74%  | 99.27% | 99.13% | 97.84%  | 94.36%  | 92.30%  |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |
| 30sec            |  | 93.22% | 91.00% | 90.02% | 5.45%  | 1.35% | 0.71% | 100.00% | 99.94% | 99.92% | 100.00% | 100.00% | 99.98%  |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |
| 15sec            |  | 93.69% | 91.52% | 90.64% | 5.35%  | 1.21% | 0.69% | 99.98%  | 99.98% | 99.98% | 100.00% | 100.00% | 100.00% |                      |    |      |  |  |  |      |  |  |  |  |  |  |  |

**Table 4:** Jump test: power and size. The table reports Monte Carlo simulations for the assessment of the power and size of the jump test without microstructure noise (left panel) and with microstructure noise (right panel). In the configuration with microstructure noise, the bid-ask spread is set at 0.15% relative to the price level. The empirical size and power of the jump test are computed at the 5%, 1%, and 0.5% theoretical size levels, respectively.

## 4 Empirical Analysis

We now illustrate how changes in the liquidity conditions of financial markets can be studied by looking at the temporal evolution of the realized illiquidity measures.<sup>22</sup> In particular, we consider the time series of the realized Amihud of SPDR S&P 500 ETF (ticker SPY), which is the ETF tracking the S&P 500 index, and of the EUR/CHF and USD/CHF foreign exchange (FX) rates. With a daily volume of 82.45 million shares in the period 2016–2021, SPY is the ETF with the largest trading volume in the world. For this reason, the realized Amihud of SPY can be considered a good proxy for the overall liquidity condition on the equity market. As for the EUR/CHF and USD/CHF rates, our intent is to consider another financial security with different characteristics, for example, the OTC and dealership structure as well as a price setting for which it is not obvious to expect the (illiquidity) leverage effect. The Swiss franc also provides us with an interesting laboratory, given the peculiar behavior of both volume and volatility after the cap removal by the Swiss national bank in January 2015. This circumstance makes it an interesting case study for assessing the evolution of price impact measures around policy events that change the currency policy and trading environment. The analysis of the stock market illiquidity is presented in Section 4.1, while Section 4.2 presents the results of the analysis conducted on the currency market.

### 4.1 Stock Market Illiquidity

We consider the daily time series of the realized Amihud of SPY, which is computed as the ratio between the daily RPV obtained from intra-daily returns sampled at different frequencies (source TAQ database) and the daily volume (expressed in dollars, source Bloomberg) on SPY for the period January 3, 2006, to June 29, 2018.

#### 4.1.1 Sample Statistics

Table 5 reports the sample statistics of RPV, volume,  $\mathcal{A}$  (realized Amihud),  $\mathcal{A}^C$  (the jump-robust version), and the jump-component of the realized Amihud, computed as  $\mathcal{A}_t^J = \frac{\mathbb{I}(J > q_{1-\alpha})(RPV - \widehat{MPV})}{v}$ . The table also contains the sample statistics for the classic daily Amihud ( $\mathcal{A}^D$ ) and of the high–low Amihud ( $\mathcal{A}^{HL}$ ).<sup>23</sup> The series are computed using returns sampled at several frequencies (1, 5, 10, 15, and 30 minutes). In the last row, the table reports the number of significant jumps identified with the test in (19). As expected, the variability of the realized Amihud is much smaller than that of the daily Amihud (by about 9 or 10 times), and this holds irrespective of the sampling frequency used to compute the estimator. In turn, the variability of the  $\mathcal{A}^{HL}$  is approximately three or four times lower than that of  $\mathcal{A}^D$ , reiterating that the range-based illiquidity measure is more desirable than daily Amihud when low-frequency illiquidity proxies are to be used. All series display positive skewness and excess

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<sup>22</sup>Another way to assess the accuracy of realized Amihud is to compare it with liquidity measures of price impact originating from order flow as done in [Rinaldo and Santucci de Magistris \(2022\)](#) showing that the realized Amihud is an accurate measure of FX volume price impact.

<sup>23</sup>To interpret these measures as the amount of daily price volatility associated with one dollar of trading volume, in the empirical application, we multiply  $\mathcal{A}^D$  by the constant  $\sqrt{\pi/2}$  and  $\mathcal{A}$  by the constant  $\delta^{1/2}\sqrt{\pi/2}$ . This guarantees that  $\mathcal{A}$ ,  $\mathcal{A}^D$ , and  $\mathcal{A}^{HL}$  are on the same scale.

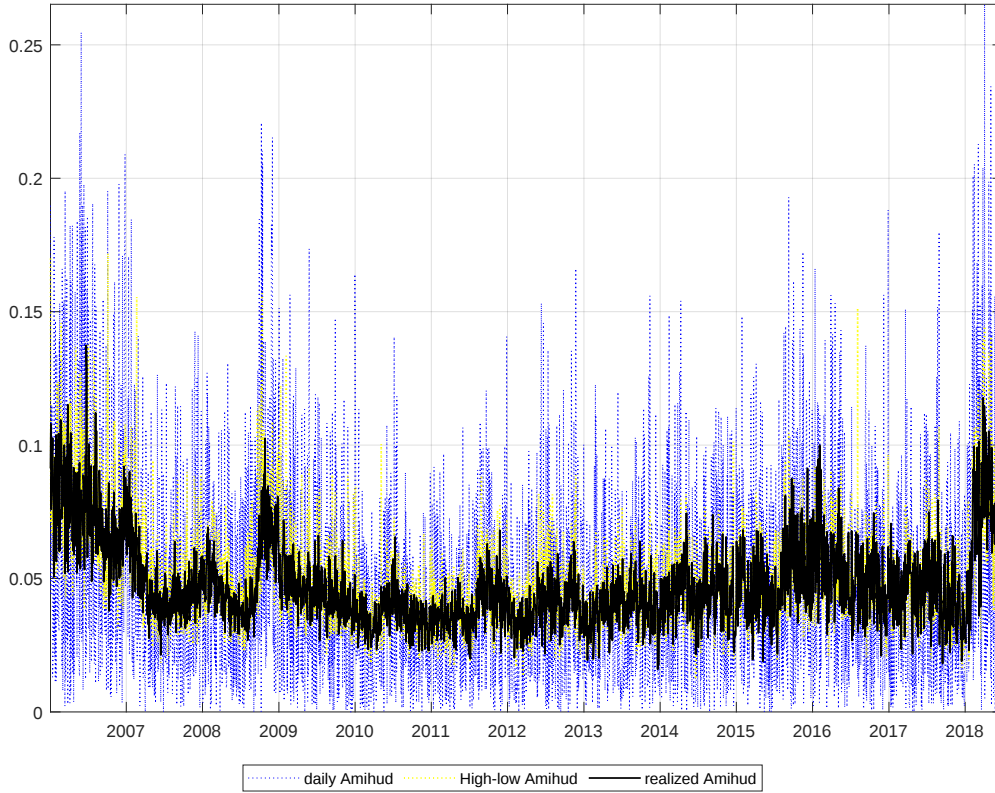
|                        |                             | Variance           |         |               |                   |                             |                 |                    |
|------------------------|-----------------------------|--------------------|---------|---------------|-------------------|-----------------------------|-----------------|--------------------|
|                        |                             | 1min               | 5min    | 10min         | 15min             | 30min                       | Daily           |                    |
|                        | RPV                         | 0.311              | 0.312   | 0.314         | 0.317             | 0.335                       | 0.547           |                    |
|                        | Volume                      | –                  | –       | –             | –                 | –                           | 1.029           |                    |
|                        | $\mathcal{A}$               | 2.433              | 2.420   | 2.427         | 2.491             | 2.722                       | –               |                    |
|                        | $\mathcal{A}^C$             | 2.262              | 2.419   | 2.456         | 2.553             | 2.804                       | –               |                    |
|                        | $\mathcal{A}^{\mathcal{I}}$ | 0.053              | 0.030   | 0.042         | 0.063             | 0.149                       | –               |                    |
|                        | $\mathcal{A}^D$             | –                  | –       | –             | –                 | –                           | 9.541           |                    |
|                        | $\mathcal{A}^{HL}$          | –                  | –       | –             | –                 | –                           | 3.755           |                    |
|                        |                             | Skewness           |         |               |                   |                             |                 |                    |
|                        | RPV                         | 3.871              | 3.650   | 3.700         | 3.820             | 3.824                       | 3.476           |                    |
|                        | Volume                      | –                  | –       | –             | –                 | –                           | 1.984           |                    |
|                        | $\mathcal{A}$               | 1.306              | 1.294   | 1.200         | 1.152             | 1.038                       | 1.245           |                    |
|                        | $\mathcal{A}^C$             | 1.203              | 1.277   | 1.184         | 1.139             | 0.963                       | –               |                    |
|                        | $\mathcal{A}^{\mathcal{I}}$ | 1.765              | 3.160   | 3.720         | 3.569             | 3.342                       | –               |                    |
|                        | $\mathcal{A}^{HL}$          | –                  | –       | –             | –                 | –                           | 1.453           |                    |
|                        |                             | Kurtosis           |         |               |                   |                             |                 |                    |
|                        | RPV                         | 28.728             | 24.426  | 25.352        | 26.867            | 26.072                      | 22.381          |                    |
|                        | Volume                      | –                  | –       | –             | –                 | –                           | 8.839           |                    |
|                        | $\mathcal{A}$               | 5.322              | 5.210   | 4.855         | 4.669             | 4.419                       | 5.015           |                    |
|                        | $\mathcal{A}^C$             | 4.942              | 5.210   | 4.889         | 4.696             | 4.115                       | –               |                    |
|                        | $\mathcal{A}^{\mathcal{I}}$ | 7.741              | 13.490  | 18.639        | 18.161            | 16.162                      | –               |                    |
|                        | $\mathcal{A}^{HL}$          | –                  | –       | –             | –                 | –                           | 6.497           |                    |
| # of significant jumps |                             | 1896               | 386     | 314           | 343               | 413                         | –               |                    |
|                        |                             | Correlation Matrix |         |               |                   |                             |                 |                    |
|                        |                             | RPV                | Volume  | $\mathcal{A}$ | $\mathcal{A}_t^C$ | $\mathcal{A}^{\mathcal{I}}$ | $\mathcal{A}^D$ | $\mathcal{A}^{HL}$ |
|                        | RPV                         | –                  | 0.8860  | 0.2264        | 0.2377            | -0.1005                     | 0.1171          | 0.1991             |
|                        | Volume                      | 0.8470             | –       | -0.1498       | -0.1356           | -0.1272                     | 0.0374          | -0.0316            |
|                        | $\mathcal{A}$               | 0.2200             | -0.2788 | –             | 0.9937            | 0.0575                      | 0.2250          | 0.6328             |
|                        | $\mathcal{A}^C$             | 0.2375             | -0.2582 | 0.9906        | –                 | -0.0544                     | 0.2198          | 0.6215             |
|                        | $\mathcal{A}^{\mathcal{I}}$ | -0.1568            | -0.1267 | -0.0470       | -0.1680           | –                           | 0.0472          | 0.1015             |
|                        | $\mathcal{A}^D$             | 0.1109             | 0.0239  | 0.1510        | 0.1443            | 0.0328                      | –               | 0.6662             |
|                        | $\mathcal{A}^{HL}$          | 0.1830             | -0.1340 | 0.5997        | 0.5892            | 0.0171                      | 0.5917          | –                  |

**Table 5:** Descriptive statistics. The table reports the sample statistics for the RPV, daily volume, the realized Amihud ( $\mathcal{A}$ ), the Amihud’s jump-robust version ( $\mathcal{A}^C$ ), the jump component ( $\mathcal{A}^{\mathcal{J}}$ ), the classic daily Amihud ( $\mathcal{A}^D$ ), the high–low Amihud ( $\mathcal{A}^{HL}$ ), as well as the number of significant jumps detected by the  $\mathcal{J}$  test at the 1% significance level. The variance is scaled by a factor of E+04, while for the volume is scaled by a factor of E-16. In the bottom panel, the table reports the Pearson–Spearman (upper/lower triangular matrix) correlations.

kurtosis compared with the reference value of the Gaussian distribution. Interestingly, the kurtosis of RPV is much larger than that of the realized Amihud. This suggests that RPV and trading volume tend to spike on the same days, so that dividing RPV by trading volume drastically reduces the observed

kurtosis. Finally, volatility and volume are strongly correlated (around 85%); a somewhat expected result. However, the correlations of  $\mathcal{A}$  with RPV and trading volume are smaller, about 22% and -28% (Pearson), signaling that liquidity interrelates with volatility and trading volume, but still differs from them. The realized Amihud (computed with returns sampled at the 5-minute frequency) is positively correlated with both the daily and high-low Amihud. In particular, the correlation is much stronger with the latter, suggesting that  $\mathcal{A}^{HL}$  is a more precise (or less noisy) measure of the illiquidity signal compared with  $\mathcal{A}^D$ .

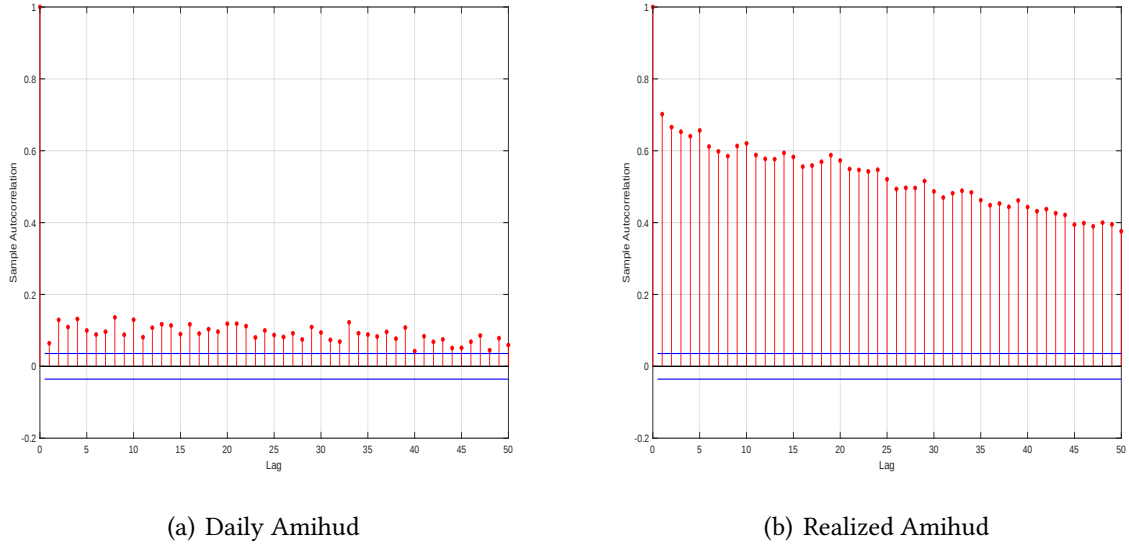
Figure 5 displays the time series of the realized Amihud computed by employing returns sampled at the 5-minute frequency (black solid line), the daily Amihud (blue dotted line), and the high-low Amihud (yellow dashed line). Notably, the three series share very similar dynamic patterns, signaling that they are all nonparametric measurements of the same underlying stochastic quantity, that is the integrated illiquidity. However, the daily Amihud displays much more variability than the realized Amihud. Interestingly, the high-low Amihud, although being based on low-frequency data available at the daily horizon, displays a range of variability of an order close to that of the realized Amihud.



**Figure 5:** Illiquidity measurements. Realized Amihud (black solid line), daily Amihud (blue dotted line), and high-low Amihud (yellow dashed line) of SPY. Sample period: January 3, 2006 – June 29, 2018. Both series are scaled by a factor of E+09.

From a visual inspection of Figure 5 it also emerges that long periods with low illiquidity are followed by protracted periods of high illiquidity. Within the volatility literature, this is a well-established phenomenon known as *volatility clustering*. Because a similar pattern applies to the realized Amihud, we refer to it as *illiquidity clustering*. Figure 6 reports the empirical autocorrelation function (ACF) of

the daily and realized Amihud. The realized Amihud displays strong persistence, as measured by the slow decay rate of the ACFs that remain very high even after 50 lags, whereas the autocorrelation of the daily Amihud is much less persistent. Concerning the daily Amihud, this is the typical behavior of persistent time series contaminated by additive noise, as indicated, for instance, by [Hurvich and Ray \(2003\)](#) and [Hurvich et al. \(2005\)](#) in the context of stochastic volatility estimation. Altogether, the evidence outlined in Figures 5 and 6 suggest two points: First, it justifies the use of well-established volatility models for predicting illiquidity. Second, the strong persistence of an illiquidity shock has important implications in asset pricing.

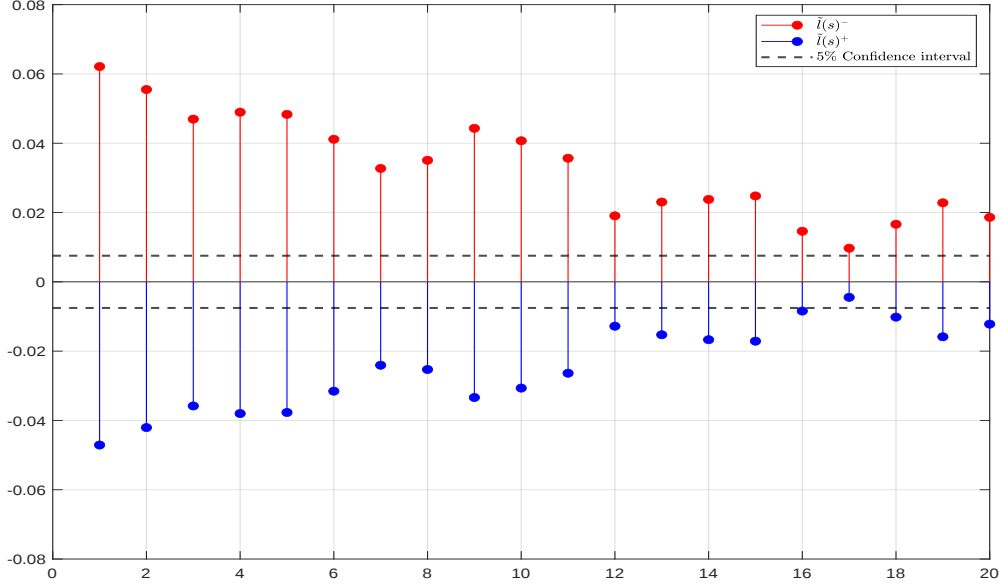


**Figure 6:** Autocorrelation. Empirical autocorrelation function of the daily Amihud (left panel) and of the realized Amihud (right panel).

Finally, Figure 7 reports prima-facie evidence on the relationship between negative returns and illiquidity. In particular, Figure 7 displays the increase (in relative terms) in the realized Amihud following a negative shock to returns for different time horizons,  $s = 1, \dots, 20$ . Analogously, a positive shock to returns improves liquidity (or reduces illiquidity) for many periods. The main stylized fact emerging from Figure 7 is that past negative returns are associated with an increase in illiquidity in the subsequent periods. Furthermore, this effect is rather persistent, remaining significant after 15 periods. In particular, a negative return generates on average an increase in the realized Amihud of about 6% in the next day and a persistent increase of about 2% after two weeks.

#### 4.1.2 Modeling Realized Illiquidity

We consider both linear and nonlinear dynamic specifications for the realized Amihud to uncover the distributional characteristics of illiquidity and ascertain its superiority compared to traditional Amihud measure in capturing them. In particular, we consider a parametric model pertaining to the class of multiplicative error models (MEM) – as introduced by [Engle \(2002\)](#) and [Engle and Gallo \(2006\)](#). Inspired by the heterogenous autoregressive (HAR) model of [Corsi \(2009\)](#), we consider the



**Figure 7:** Illiquidity and negative/positive return shock propagation. Figure reports the estimated propagation of the effect of a negative (red bars) and positive (blue bars) shock on returns on the realized Amihud. Following Catania (2020), these quantities are computed as the empirical counterpart of  $l^+(s) = E[\mathcal{A}_{t+s}|r_t > 0] - E[\mathcal{A}_{t+s}]$ ,  $l^-(s) = E[\mathcal{A}_{t+s}|r_t < 0] - E[\mathcal{A}_{t+s}]$  for  $s = 1, \dots, 20$ . The estimators  $\hat{l}^+(s)$  and  $\hat{l}^-(s)$  are reported relative to the average illiquidity, i.e.  $\tilde{l}^+(s) = l^+(s)/E[\mathcal{A}_{t+s}]$  and  $\tilde{l}^-(s) = l^-(s)/E[\mathcal{A}_{t+s}]$ . The black-dashed horizontal lines denote the 95% interval around 0.

MEM-AHAR model,<sup>24</sup> as follows

$$\mathcal{A}_t = \mu_t \epsilon_t, \quad (22)$$

where  $\mu_t$  is the conditional mean of the process, and it follows asymmetric HAR dynamics as

$$\mu_t = \omega + \alpha_d \mathcal{A}_{t-1} + \alpha_w \bar{\mathcal{A}}_{w,t-1} + \alpha_m \bar{\mathcal{A}}_{m,t-1} + \gamma D_{t-1} \mathcal{A}_{t-1}, \quad (23)$$

where  $\bar{\mathcal{A}}_{w,t-1} = \frac{1}{5} \sum_{i=1}^5 \mathcal{A}_{t-i}$ ,  $\bar{\mathcal{A}}_{m,t-1} = \frac{1}{22} \sum_{i=1}^{22} \mathcal{A}_{t-i}$ , and  $D_{t-1}$  is a dummy variable taking value of 1 if the return is negative and 0 otherwise; this accounts for an asymmetric response of illiquidity to positive or negative returns. Although with a different interpretation, this setting is reminiscent of the GJR-GARCH(1,1) model by Glosten et al. (1993) and it is supposed to capture the illiquidity *leverage* effect. The asymmetric mechanism would be consistent with the stronger illiquidity persistence originated from negative returns (Figure 7), empirical evidence provided in Hameed et al. (2010), and theoretical models of binding capital constraints of financial intermediaries (e.g. Brunnermeier and Pedersen, 2009; Gromb and Vayanos, 2002) as well as limits-to-arbitrage models (e.g. Kyle and Xiong, 2001). The term  $\epsilon_t$  denotes the innovation term, which is a non-negative random variable whose density is Gamma with a unit mean and variance equal to  $\frac{1}{\beta}$ . A sufficient condition for positivity of the conditional mean,  $\mu_t$ , is that all coefficients in (23) are positive, while imposing  $\alpha_d + \alpha_w + \alpha_m + \gamma/2 < 1$  ensures stationarity. Estimations are carried out using the maximum likelihood (ML), and we impose

<sup>24</sup>Appendix D reports a complete description of various alternative MEM and linear specifications, as well as their estimates on the sample under investigation.

stationarity and positivity conditions upon estimating the models on the data.

| Panel a)    | Estimation results              |                                  |                                  |                                  |
|-------------|---------------------------------|----------------------------------|----------------------------------|----------------------------------|
|             | Daily Amihud                    | High-Low Amihud                  | Jump-robust Amihud               | Realized Amihud                  |
| $\omega$    | 0.0160 <sup>a</sup><br>(0.0022) | 0.0052 <sup>a</sup><br>(0.0011)  | 0.0030 <sup>a</sup><br>(0.0007)  | 0.0029 <sup>a</sup><br>(0.0007)  |
| $\alpha_d$  | 0.0000<br>(0.0234)              | 0.0000<br>(0.0227)               | 0.1265 <sup>a</sup><br>(0.0225)  | 0.1358 <sup>a</sup><br>(0.0225)  |
| $\alpha_w$  | 0.0011<br>(0.0494)              | 0.3146 <sup>a</sup><br>(0.0476)  | 0.3808 <sup>a</sup><br>(0.0401)  | 0.3865 <sup>a</sup><br>(0.0396)  |
| $\alpha_m$  | 0.6595 <sup>a</sup><br>(0.0661) | 0.5549 <sup>a</sup><br>(0.0466)  | 0.4087 <sup>a</sup><br>(0.0381)  | 0.3983 <sup>a</sup><br>(0.0373)  |
| $\gamma$    | 0.0297<br>(0.0235)              | 0.0546 <sup>a</sup><br>(0.0103)  | 0.0399 <sup>a</sup><br>(0.0071)  | 0.0371 <sup>a</sup><br>(0.0069)  |
| $\vartheta$ | 1.4608 <sup>a</sup><br>(0.0361) | 12.0371 <sup>a</sup><br>(0.3188) | 24.7189 <sup>a</sup><br>(0.5895) | 26.5139 <sup>a</sup><br>(0.5591) |
| <hr/>       |                                 |                                  |                                  |                                  |
| Panel b)    | Ljung-Box statistics (p-value)  |                                  |                                  |                                  |
| LB(1)       | 0.0045                          | 0.2788                           | 0.6561                           | 0.7265                           |
| LB(5)       | 0.0453                          | 0.0213                           | 0.1608                           | 0.2813                           |
| LB(10)      | 0.0714                          | 0.0980                           | 0.0001                           | 0.0000                           |

**Table 6:** MEM-AHAR estimated coefficients with robust standard errors (in parenthesis). The bottom panel reports the  $p$ -value of the Ljung-Box statistics. Superscript  $a$ ,  $b$  and  $c$  denote the 1%, 5%, and 10% significance levels, respectively. Sample period: January 3, 2006 – June 29, 2018.

Table 6 reports the estimation results of the MEM-AHAR model on the daily Amihud, high–low Amihud, and on the realized Amihud based on 5-minute returns (including the jump-robust version). Focusing on the last column of Panel a), the coefficients governing the dynamics of  $\mu_t$  for the realized Amihud are all positive and strongly significant at a 1% significance level, with a value of persistence around 93.6%, as measured by  $\hat{\alpha}_d + \hat{\alpha}_w + \hat{\alpha}_m + \hat{\gamma}/2$ . In the HAR framework, the coefficients  $\alpha_d$ ,  $\alpha_w$  and  $\alpha_m$  summarize the trade impact at different timeframes, here induced by heterogeneity among market participants on market illiquidity. The estimated coefficients indicate that today's illiquidity is predicted by that of yesterday, the previous week and month with a greater influence of the latter two. This heterogeneous persistence of illiquidity is a novel result that could have asset pricing implications. The results do not change when adopting the jump-robust realized Amihud.

On the other hand, when noisier proxies of illiquidity are employed, such as in the daily Amihud or the high–low Amihud, the parameter associated with daily past illiquidity,  $\alpha_d$ , is estimated on the lower bound, while the parameter  $\alpha_w$  is significant only for the high–low Amihud. The only significant parameter is  $\alpha_m$ , indicating that a substantial smoothing is required on the daily Amihud to disentangle the expected illiquidity signal from its noisy ex-post measurement. Analogous evidence is found in the volatility literature when employing the realized GARCH model of [Hansen et al. \(2012\)](#) instead of the classic GARCH model on squared returns. In other words, the model assigns a small weight to the

innovation term when the latter contains a substantial degree of measurement error. Furthermore, the implied persistence obtained by the estimates of the MEM-AHAR coefficients on the daily and high-low Amihud is 67.5% and 89.7%, respectively. These values are much lower than those obtained on the realized Amihud, which is in line with the evidence of the ACFs reported in Figure 6. Finally, the estimate of  $\vartheta$  (which is reciprocal of the variance of  $\varepsilon$ ) for the daily Amihud is found to be around 16 times smaller than that of the realized Amihud. This implies that the variability of the innovation term is 16 times larger when employing the daily Amihud rather than the realized Amihud. This proportion is drastically reduced (approximately to half) when employing the high-low Amihud.

Overall, the results presented in Table 6 remain valid, even when alternative model specifications are employed; for more, see Tables 19 and 20 in Appendix D. In particular, the distribution of the innovation term seems to be well described by a Gamma distribution, while more sophisticated distributional choices do not provide a remarkable improve in the fit.<sup>25</sup> This is due to the fact that the realized Amihud does not display extreme distributional features because the ratio between volatility and volume drastically reduces the observed kurtosis (see Table 5). Finally, Panel b) of Table 6 reports the  $p$ -value for the Ljung-Box statistics. All model specifications correctly capture the persistent features of the series at hand because of the HAR-type specification for  $\mu_t$ ; for instance, see Caporin et al. (2016, 2017).

#### 4.1.3 Illiquidity and Stock Market Excess Returns

The economic rationale for analyzing the relationship between returns and illiquidity is twofold. First, higher expected market illiquidity leads to higher ex ante asset (excess) return. Second, as illustrate in Figure 6 and captured by the clustering illiquidity effect, illiquidity shocks are very persistent. Therefore, an illiquidity shock that happens now will raise expected illiquidity for the future, which in turn causes ex ante asset returns to rise and asset prices to fall.<sup>26</sup> Taken together, these mechanisms point to two hypotheses to be tested: a positive coefficient relating *expected* illiquidity and asset return; a negative one relating *unexpected* illiquidity and contemporaneous asset return. Following Amihud (2002), we carry out an analysis on the impact of illiquidity on stock market excess returns but here enhanced by more accurate estimates of realized illiquidity.

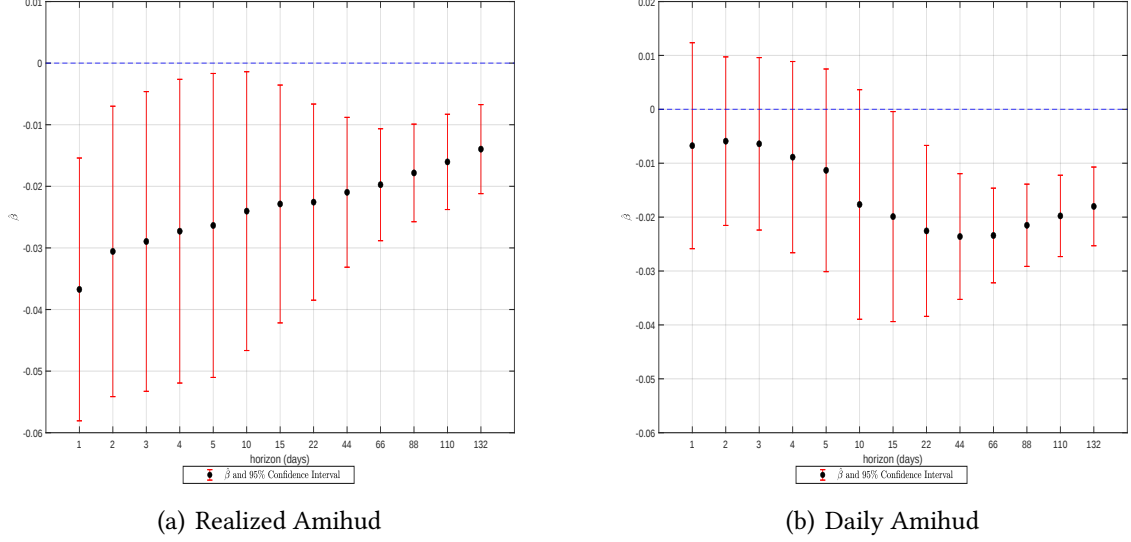
Figure 8 reports the slope coefficient ( $\beta_1$ ) of a simple regression of the stock market excess returns (as measured by the difference between the returns of SPY and the risk free rate the U.S. 3-month treasury bills in our case) on illiquidity at different aggregation frequencies, that is

$$\bar{r}_t^e = \beta_0 + \beta_1 \bar{\mathcal{A}}_t + u_t, \quad (24)$$

where  $\bar{r}_t^e = \frac{1}{K} \sum_{i=0}^{K-1} r_{t-i}^e$ , where  $r_t^e = r_t - r_t^f$ , and  $\bar{\mathcal{A}}_t = \frac{1}{K} \sum_{i=0}^{K-1} \mathcal{A}_{t-i}$ . The integer  $K$  denotes the aggregation horizon, that we set equal to one day ( $K = 1$ ), two days ( $K = 2$ ), three days ( $K = 3$ ), four days ( $K = 4$ ), one week ( $K = 5$ ), two weeks ( $K = 10$ ), three weeks ( $K = 15$ ), one month ( $K = 22$ ), two months ( $K = 44$ ), three months ( $K = 66$ ), 4 months ( $K = 88$ ), 5 months ( $K = 110$ ), 1 semester ( $K = 132$ ).

<sup>25</sup>Figure 12 in the Appendix reports the probability-integral transform based on various MEM model specifications (including the mixture MEM of Caporin et al., 2017 presented in Appendix D.1). Even the simple MEM specifications with Gamma distributed innovations display a remarkable fit of the empirical distribution.

<sup>26</sup>Concerning stocks, this assumes that corporate cash flows are unaffected by market illiquidity.



**Figure 8:** Illiquidity vs Excess Returns. The figure reports the estimates of  $\beta_1$  (black dots) in (24) with a 95% interval (red bars) as a function of the aggregation horizon,  $K$ , that is reported on the x-axis. The following values for  $K$  are used: one day ( $K = 1$ ), two days ( $K = 2$ ), three days ( $K = 3$ ), four days ( $K = 4$ ), one week ( $K = 5$ ), two weeks ( $K = 10$ ), three weeks ( $K = 15$ ), one month ( $K = 22$ ), two months ( $K = 44$ ), three months ( $K = 66$ ), 4 months ( $K = 88$ ), 5 months ( $K = 110$ ), 1 semester ( $K = 132$ ). Panel a) and b) reports estimates of  $\beta_1$  obtained with the realized and daily Amihud, respectively.

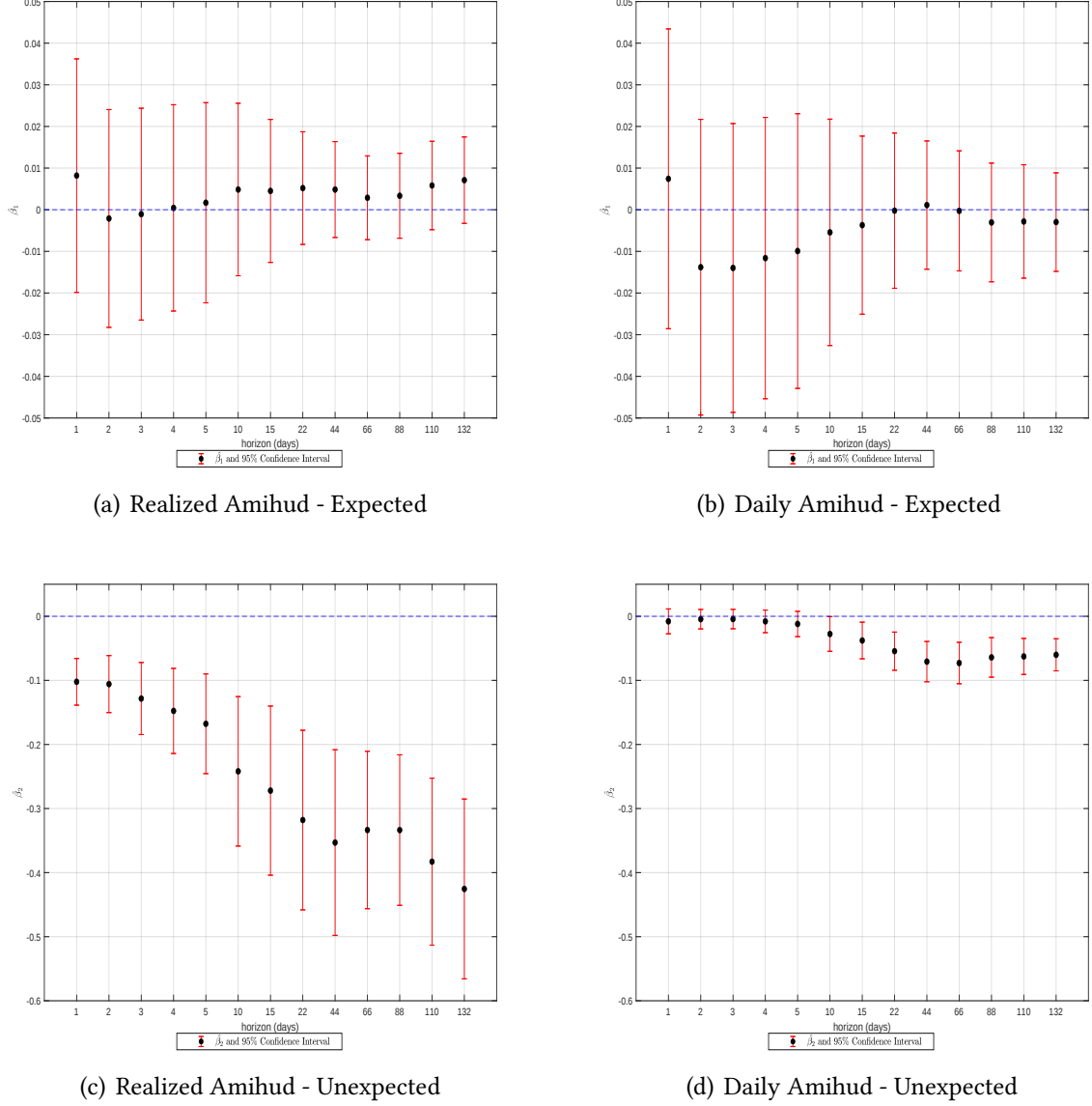
Panels a) and b) of Figure 8 reports the estimates of  $\beta_1$  (black dots) with a 95% interval as a function of  $K$  that is reported on the x-axis. The estimates of  $\beta_1$  are negative and significant for all aggregation horizons,  $K$ , when employing the realized Amihud estimator. In particular, the effect is particularly pronounced for  $K = 1$ . On the other hand, the estimates of  $\beta_1$  obtained when employing the daily Amihud become significant only if aggregating over two weeks horizons, meaning that a substantial degree of smoothing is needed for daily Amihud to extract a liquidity signal that correlates with excess returns. Notably, at long horizons the scale of  $\beta_1$  is between -0.01 and -0.02 for both daily and realized Amihud, while for  $K = 1$  the point estimate of  $\hat{\beta}_1$  is -0.0378 for the realized Amihud and around -0.008 (and not significant) for the daily Amihud.

The illiquidity measures can be disentangled into two components: the *expected* illiquidity, as measured by the estimated conditional mean of the MEM-AHAR model,  $\hat{\mu}_t$ , and the *unexpected* illiquidity, as measured by the residual component  $\hat{\varepsilon}_t$ . Therefore, we consider the following regression

$$r_t^e = \beta_0 + \beta_1 \hat{\mu}_t + \beta_2 \hat{\varepsilon}_t + u_t. \quad (25)$$

and report the estimates of  $\beta_1$  and  $\beta_2$  for different aggregation frequencies in Figure 9. From the figure, it clearly emerges that the expected part of illiquidity,  $\mu_t$ , is not related with the excess returns irrespective of the illiquidity measure used and/or the aggregation frequency. On the contrary, the unexpected part of illiquidity, denoted by  $\hat{\varepsilon}_t$ , is negatively related with the actual observed excess returns. In particular, when  $\beta_2$  is estimated using the residuals of the realized Amihud,  $\hat{\varepsilon}_t = \mathcal{A}_t - \hat{\mu}_t$ , Panel c) of Figure 9 clearly highlights the fact that it is the unexpected component of illiquidity that is significantly and negatively associated with excess returns. Notably, this holds true irrespective of the

sampling frequency used in the estimation. On the contrary, the estimates of  $\beta_2$  based on the residuals of the daily Amihud are not significant, unless aggregated at 3 weeks horizons or more. Furthermore, the magnitude of the point estimate of  $\beta_2$  is, in absolute terms, much larger when realized Amihud residuals are used instead of daily Amihud residuals.



**Figure 9:** Illiquidity Decomposition vs Excess Returns. Panels a) and b) report the estimates of  $\beta_1$  (black dots) in (25) obtained with the realized and daily Amihud, respectively, with a 95% interval (red bars) as a function of the aggregation horizon,  $K$ , that is reported on the x-axis. Panels c) and d) report the estimates of  $\beta_2$  (black dots) in (25) obtained with the realized and daily Amihud, respectively, with a 95% interval (red bars) as a function of the aggregation horizon,  $K$ , that is reported on the x-axis. The following values for  $K$  are used: one day ( $K = 1$ ), two days ( $K = 2$ ), three days ( $K = 3$ ), four days ( $K = 4$ ), one week ( $K = 5$ ), two weeks ( $K = 10$ ), three weeks ( $K = 15$ ), one month ( $K = 22$ ), two months ( $K = 44$ ), three months ( $K = 66$ ), 4 months ( $K = 88$ ), 5 months ( $K = 110$ ), 1 semester ( $K = 132$ ).

Table 7 reports a summary of the estimates of the parameters of regressions (24) and (25) for daily, weekly and monthly frequencies. In Panel a), when employing the realized Amihud as the explanatory variable, the estimate of  $\beta_1$  suggests that illiquidity is in a significantly negative relationship with the

excess return. This holds true when aggregating at weekly and monthly (nonoverlapping) horizons. Notably, the  $R^2$  turns out to be particularly large at the monthly horizon, being above 16%. Contrary to this, the relationship is less pronounced when employing the classic daily Amihud as the regressor. In particular,  $\beta_1$  is not significant at the daily and weekly frequencies, but it is significant when the aggregation at monthly level is considered (in this case, the  $R^2$  is around 5%). Again, this signals the fact that a less accurate measurement of illiquidity obtained via the daily Amihud masks the fundamental relationship between returns and illiquidity.

The results most closely related to testing the hypotheses discussed above are shown in Panel b) of Table 7. The estimated coefficients support the two hypotheses. Although not statistically significant, expected illiquidity is positively related to asset return. On the contrary, unexpected illiquidity is negatively and significantly associated with asset return. More specifically, the expected illiquidity weighs positively on returns but it is never significant at the daily and weekly frequencies (irrespective of the measure adopted), while the *unexpected* illiquidity (as measured by the residuals of the MEM-AHAR model) is significantly and negatively related with the excess returns when both the realized illiquidity measures (i.e. the realized and high–low Amihud metrics) are employed. Instead, the unexpected liquidity for the classic daily Amihud is only significant at the monthly horizon. The results are unchanged when including the Fama-French factors (SMB and HML) to the regression. This result squares well with the idea that the residual term of the MEM-AHAR model estimated on the daily Amihud is made of two components: the prediction error and the measurement error, where the latter seems to dominate at the daily and weekly frequencies.

## 4.2 Currency Market Illiquidity

Another way to assess the general validity of the realized illiquidity and its higher accuracy in measuring high-frequency market liquidity is by considering another financial instrument and using a meaningful episode, which is a sort of natural experiment. Through the lenses of the theory developed in Section 2, the announcement of the cap removal of the Swiss franc by the Swiss National Bank (SNB) on January 15, 2015, represents an ideal framework for conducting further testing. Starting from September 6, 2011, the SNB set a minimum exchange rate of 1.20 francs to the euro (capping the franc’s appreciation), stating that “the value of the franc is a threat to the economy” and that it was “prepared to buy foreign currency in unlimited quantities.” This means that the SNB had a declared binding *cap* on the transaction price that was removed on January 15, 2015.

In terms of the trading model presented in Section 2, the SNB can be considered the special  $(J + 1)$ -th trader. The SNB intervention strategy of selling CHF for EUR in potentially unlimited quantities would be implemented if the average of the reservation prices of the remaining  $J$  traders ever fell below the cap, that is if  $\frac{1}{J} \sum_{j=1}^J p_{i,j}^* < \log(1.2)$ .<sup>27</sup> Indeed, despite the cap on the transaction price, the reservation prices of individual traders might well be below the 1.20 threshold. For instance, a trader with a reservation price of 1.12 (as the agents with the actual lowest forecast in Thomson Reuters survey before the SNB cap removal) is inclined to sell EUR for CHF. In other words, the SNB buys

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<sup>27</sup>See Breedon et al. (2022) for a discussion of how the SNB surprised the market and how it actually implemented the intervention strategy by setting a huge ask volume at 1.20. Furthermore, the Thomson Reuters survey indicates a wide dispersion of the beliefs of professional market participants around 1.20 for most of the capping period.

| TOTAL ILLIQUIDITY   |                                 |                                 |                                  |                                 |                                 |                                  |                                  |                                  |                                  |                                  |
|---------------------|---------------------------------|---------------------------------|----------------------------------|---------------------------------|---------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| Frequency           | Daily Amihud                    |                                 |                                  | High-Low Amihud                 |                                 |                                  | Realized Amihud                  |                                  |                                  | Monthly                          |
|                     | Daily                           | Weekly                          | Monthly                          | Daily                           | Weekly                          | Monthly                          | Daily                            | Weekly                           | Monthly                          |                                  |
| Constant            | 0.0363<br>(0.0359)              | 0.0409<br>(0.0338)              | 0.1515 <sup>a</sup><br>(0.0492)  | 0.0853<br>(0.0605)              | 0.7038<br>(0.0588)              | 0.1016 <sup>e</sup><br>(0.0530)  | 0.1411 <sup>b</sup><br>(0.0579)  | 0.1055 <sup>b</sup><br>(0.0511)  | 0.1337 <sup>a</sup><br>(0.0510)  | 0.1073 <sup>b</sup><br>(0.0464)  |
| Illiquidity         | -0.6747<br>(0.9147)             | -0.7538<br>(0.8472)             | -3.0133 <sup>a</sup><br>(1.1137) | -1.6627<br>(1.3896)             | -1.4155<br>(1.3383)             | -1.9886 <sup>c</sup><br>(1.1566) | -2.3331 <sup>b</sup><br>(1.2980) | -2.1905 <sup>c</sup><br>(1.1414) | -2.7951 <sup>b</sup><br>(1.2282) | -2.2436 <sup>b</sup><br>(1.0599) |
| SMB                 | 0.3355 <sup>a</sup><br>(0.0635) | 0.3172 <sup>a</sup><br>(0.0908) | 0.5500 <sup>a</sup><br>(0.1903)  | 0.3341 <sup>a</sup><br>(0.0634) | 0.3160 <sup>a</sup><br>(0.0905) | 0.3160 <sup>a</sup><br>(0.0905)  | 0.5668 <sup>a</sup><br>(0.1935)  | 0.3143 <sup>a</sup><br>(0.0896)  | 0.5643 <sup>a</sup><br>(0.1943)  | 0.5643 <sup>a</sup><br>(0.1943)  |
| HML                 | 0.5072 <sup>a</sup><br>(0.0497) | 0.4277 <sup>a</sup><br>(0.0974) | 0.3890 <sup>a</sup><br>(0.1569)  | 0.5068 <sup>a</sup><br>(0.0497) | 0.4239 <sup>a</sup><br>(0.0971) | 0.4239 <sup>a</sup><br>(0.0971)  | 0.3801 <sup>b</sup><br>(0.1631)  | 0.4238 <sup>a</sup><br>(0.0976)  | 0.3808 <sup>b</sup><br>(0.1657)  | 0.3808 <sup>b</sup><br>(0.1657)  |
| Diagnostic          |                                 |                                 |                                  |                                 |                                 |                                  |                                  |                                  |                                  |                                  |
| R <sup>2</sup>      | 0.0007                          | 0.1555                          | 0.0044                           | 0.0011                          | 0.1554                          | 0.0056                           | 0.0341                           | 0.0056                           | 0.1396                           | 0.2298                           |
| R <sup>2</sup> adj. | 0.0004                          | 0.1547                          | 0.0028                           | 0.0008                          | 0.1546                          | 0.0040                           | 0.0271                           | 0.0040                           | 0.1354                           | 0.2128                           |
| F-test (p-value)    | 0.2628                          | 0.0000                          | 0.1865                           | 0.1195                          | 0.0000                          | 0.1169                           | 0.0521                           | 0.1165                           | 0.0000                           | 0.0000                           |

| ILLIQUIDITY DECOMPOSITION |                                 |                                 |                                  |                                  |                                 |                                  |                                   |                                   |                                   |                                    |
|---------------------------|---------------------------------|---------------------------------|----------------------------------|----------------------------------|---------------------------------|----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|------------------------------------|
| Frequency                 | Daily Amihud                    |                                 |                                  | High-Low Amihud                  |                                 |                                  | Realized Amihud                   |                                   |                                   | Monthly                            |
|                           | Daily                           | Weekly                          | Monthly                          | Daily                            | Weekly                          | Monthly                          | Daily                             | Weekly                            | Monthly                           |                                    |
| Constant                  | -0.0334<br>(0.0877)             | -0.0135<br>(0.0846)             | -0.0184<br>(0.0720)              | -0.0373<br>(0.0726)              | -0.0290<br>(0.0707)             | 0.0471<br>(0.0550)               | 0.0285<br>(0.0577)                | -0.0529<br>(0.0598)               | -0.0035<br>(0.0504)               | -0.0208<br>(0.0649)                |
| Expected Illiquidity      | 0.7428<br>(1.8686)              | 0.3531<br>(1.8086)              | 0.4537<br>(1.5292)               | 0.8171<br>(1.5508)               | 0.6635<br>(1.5121)              | -0.8852<br>(1.1404)              | -0.4951<br>(1.2079)               | 1.2410<br>(1.2600)                | 0.1513<br>(1.0595)                | 3.6338 <sup>c</sup><br>(1.8563)    |
| Unexpected Illiquidity    | -0.7860<br>(0.9224)             | -0.8409<br>(0.8447)             | -7.9535 <sup>a</sup><br>(2.5456) | -3.1602 <sup>c</sup><br>(1.7577) | -2.6779<br>(1.6555)             | -4.6013 <sup>c</sup><br>(2.5736) | -17.0426 <sup>a</sup><br>(6.0024) | -14.0262 <sup>a</sup><br>(5.0226) | -12.3487 <sup>b</sup><br>(4.8460) | -30.9824 <sup>b</sup><br>(12.4300) |
| SMB                       | 0.3351 <sup>a</sup><br>(0.0635) | 0.3166 <sup>a</sup><br>(0.0914) | 0.5142 <sup>a</sup><br>(0.1818)  | 0.3313 <sup>a</sup><br>(0.0636)  | 0.3092 <sup>a</sup><br>(0.0923) | 0.3092 <sup>a</sup><br>(0.0923)  | 0.5037 <sup>a</sup><br>(0.1762)   | 0.2917 <sup>a</sup><br>(0.0644)   | 0.2917 <sup>a</sup><br>(0.0644)   | 0.3950 <sup>a</sup><br>(0.1319)    |
| HML                       | 0.5072 <sup>a</sup><br>(0.0497) | 0.4284 <sup>a</sup><br>(0.0980) | 0.4001 <sup>a</sup><br>(0.1500)  | 0.5071 <sup>a</sup><br>(0.0496)  | 0.5071 <sup>a</sup><br>(0.0496) | 0.4008 <sup>b</sup><br>(0.1571)  | 0.5031 <sup>a</sup><br>(0.0495)   | 0.4261 <sup>a</sup><br>(0.0992)   | 0.4261 <sup>a</sup><br>(0.0992)   | 0.4036 <sup>b</sup><br>(0.1664)    |
| Diagnostic                |                                 |                                 |                                  |                                  |                                 |                                  |                                   |                                   |                                   |                                    |
| R <sup>2</sup>            | 0.0010                          | 0.1557                          | 0.0044                           | 0.0025                           | 0.1564                          | 0.0084                           | 0.0748                            | 0.0254                            | 0.1553                            | 0.3177                             |
| R <sup>2</sup> adj.       | 0.0003                          | 0.1546                          | 0.0012                           | 0.0019                           | 0.1553                          | 0.0052                           | 0.0613                            | 0.0223                            | 0.1498                            | 0.2975                             |
| F-test (p-value)          | 0.6132                          | 0.0000                          | 0.6884                           | 0.0536                           | 0.0000                          | 0.2004                           | 0.0132                            | 0.0010                            | 0.0000                            | 0.0000                             |

**Table 7:** Analysis regressing stock market excess returns on illiquidity measures (the top of the table) and on illiquidity decomposed into expected and unexpected components (the bottom of the table). The coefficients and Newey and West (1987) robust standard errors (in parenthesis) are multiplied by 100. Superscript *a*, *b* and *c* denote the 1%, 5%, and 10% levels of significance, respectively.

(sells) foreign (domestic) currency to guarantee that the transaction price is above the threshold, that is

$$p_i^{EUR|CHF} = \frac{1}{J+1} \sum_{j=1}^{J+1} p_{i,j}^{EUR|CHF,*} \geq \log(1.2), \quad (26)$$

where  $p_{i,J+1}^{EUR|CHF,*} = \left( \log(1.2) - \frac{1}{J} \sum_{j=1}^J p_{i,j}^{EUR|CHF,*} \right) \times \mathbb{I} \left( \sum_{j=1}^J p_{i,j}^{EUR|CHF,*} < \log(1.2) \right)$ , and  $\mathbb{I}(\cdot)$  is the indicator function. The enforcement of the capping regime by the SNB generates extra trading volume. In particular, the trading volume is as follows

$$v_i^{EUR|CHF} = \frac{\ell_i}{2} \sum_{j=1}^J |\Delta p_{i,j}^{EUR|CHF,*}| - \frac{1}{J} \sum_{j=1}^J \Delta p_{i,j}^{EUR|CHF,*} + v_i^{SNB}, \quad (27)$$

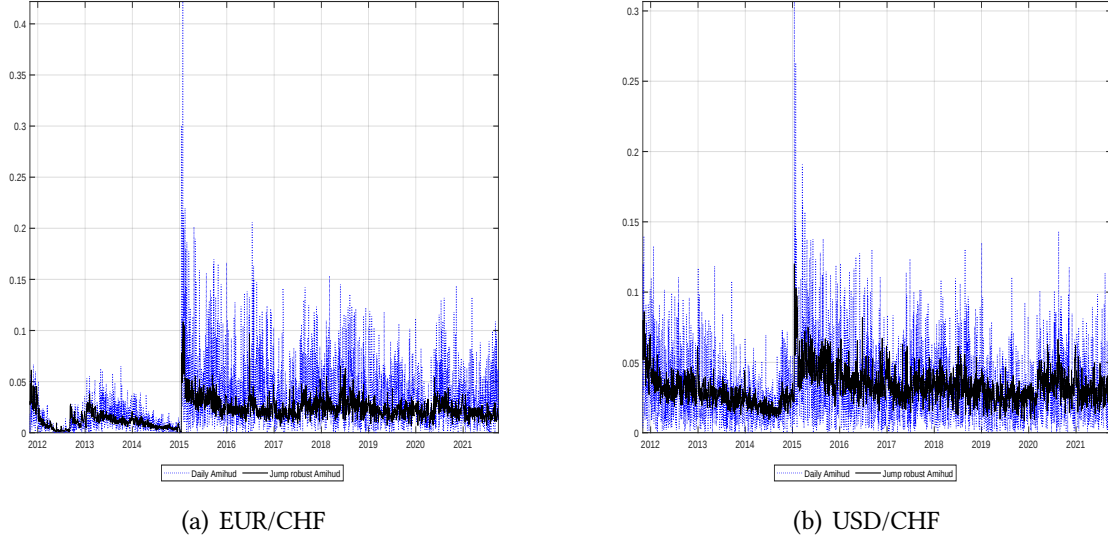
where  $v_i^{SNB}$  is the trading volume generated by the central bank to maintain the cap on the FX rate. Hence, the model prescribes a low volatility of the observed returns because of the implicit constraint given by the capping and a larger volume because of the SNB interventions. This implies that the realized illiquidity is lower (higher) before (after) the removal of the FX capping regime.

To empirically explore these model implications, we analyze at the daily time series of realized Amihud of the EUR/CHF and the USD/CHF FX spot rates covering the period November 1, 2011, to September 30, 2021. The time series are constructed from two distinct data sets. First, the CLS Group – the largest payment system for the settlement of foreign exchange transactions – provides us hourly data on the traded volume representative for the FX global activity on both the EUR/CHF and USD/CHF rates.<sup>28</sup> Second, 1-minute EUR/CHF and USD/CHF spot rates (bid, ask, high, low, and mid-quotes) are obtained from Olsen Financial Technologies. Given that trading on FX rates is active 24 hours a day but concentrated during the so-called London hours (Ranaldo and Somogyi, 2021), we consider the intra-daily returns and volume solely between 8 a.m. and 8 p.m. (GMT time), thus limiting the influence of possible noisy observations associated with overnight hours in which there is minimal trading activity. Our final data set consists of 1,943,760 intra-daily 1-minute returns and 32,396 hourly trading volume for a total of 2,492 trading days. Once again, we consider the returns sampled at both 5-minute frequencies to compute RPV and RPV<sub>C</sub>, while the daily volume is computed simply by aggregating the hourly volume.

Figure 10 displays the time series of the realized Amihud (black solid lines) and daily Amihud (blue dashed lines) of the EUR/USD (Panel a) and USD/CHF (Panel b) rates. As for SPY, the illiquidity clustering phenomenon is evident. High illiquidity characterizes the years after the onset of the sovereign debt crisis (in the period between 2010 and 2012). This period of stress greatly characterized the Swiss franc being considered a safe haven currency and giving it strong appreciation pressures (Jordan, 2020). On January 15, 2015, after the announcement by the SNB, we observe the highest peak of illiquidity, which remains very high for several months after this event. For both rates, Figure 10 reconfirms that the classic daily Amihud is much noisier than the corresponding realized version.

Figure 11 reports the result of Bai and Perron (1998)'s testing procedure to identify the date of a level break (if any) in a time series of RPV, volume, daily Amihud, and realized Amihud of EUR/CHF.

<sup>28</sup>See Ranaldo and Somogyi (2021) and Cespa et al. (2021) for a discussion and empirical evidence on the global representativeness of CLS data.



**Figure 10:** Daily Amihud (blue dotted lines, see [Amihud \(2002\)](#)) vs. the realized Amihud (black solid lines) of EUR/USD (Panel a) and USD/CHF (Panel b) exchange rates. Sample period: November 1, 2011 – September 30, 2021. Both series are scaled by a factor of E+11.

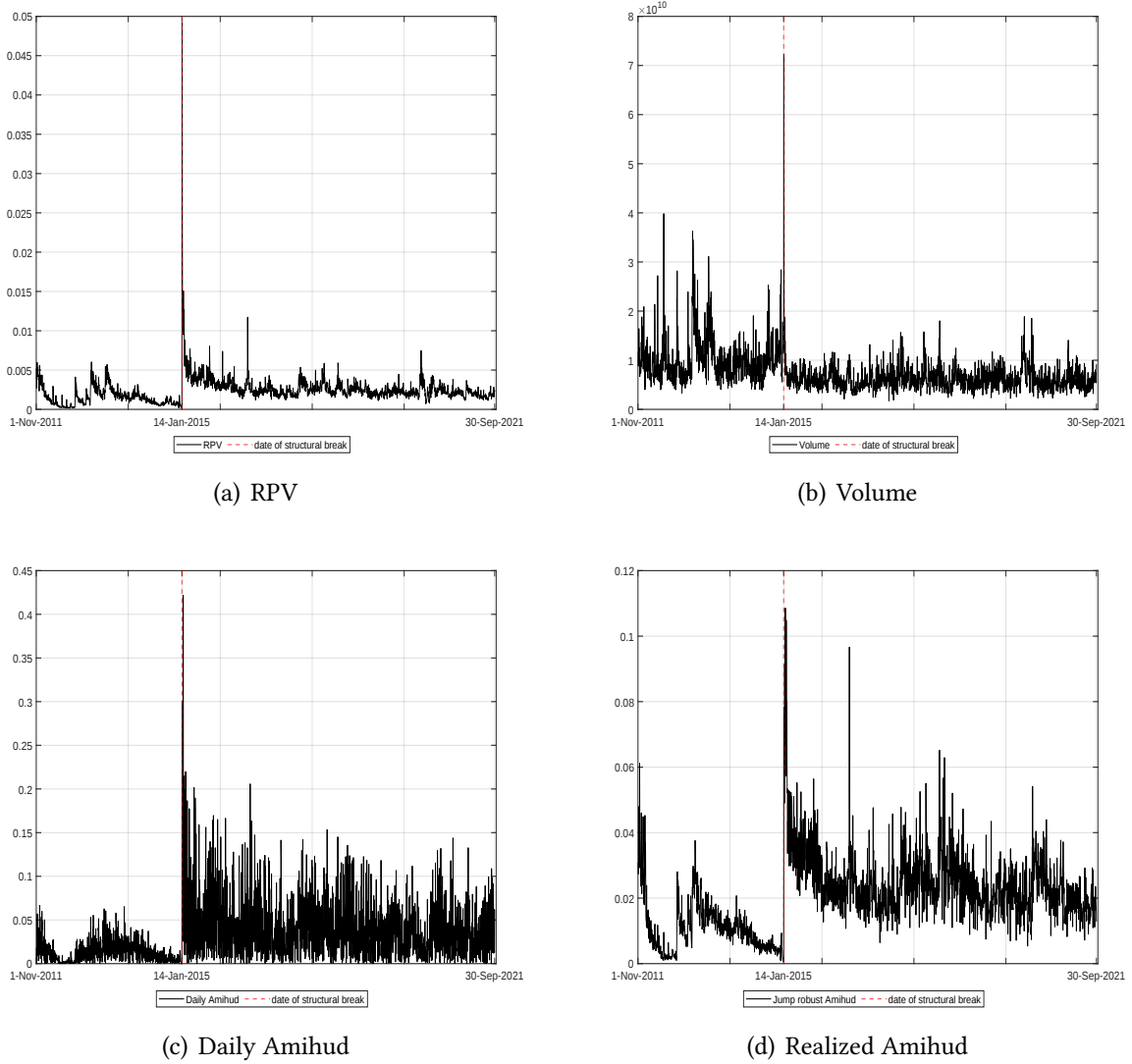
The test considers the possible presence of one or more structural changes occurring at unknown dates: in particular, [Bai and Perron \(1998\)](#) propose a procedure designed to estimate the break dates, while testing for the presence of structural changes at the same time. We apply the [Bai and Perron \(1998\)](#) test with 15% trimming at the beginning and the end of the sample period and between the break dates (see also [Casini and Perron, 2019](#), for a review and up-to-date recommendations about the procedure).<sup>29</sup> The series are characterized by strong persistence, and the procedure of [Bai and Perron \(1998\)](#) is robust to autocorrelation and heteroskedasticity. In all cases, the test identifies a significant break date on January 14, 2015, thus suggesting that the level of the series changes before and after this break date. The results for USD/CHF are reported in the Appendix and are analogous to those reported for EUR/CHF. In addition to significantly impacting a single currency pair, this result suggests that important events such as currency regime changes can affect commonality in the liquidity of financial securities.

Table 8 reports the estimates of the mean of the series under investigation before and after the break date, which has been found on January 14, 2015. The results reported in the table strongly support the prescriptions of the theory. More specifically, the level of volatility (volume) significantly increases (decreases), thus leading to an increase of illiquidity (as measured by both daily and realized Amihud) on EUR/CHF. In a similar way, this shock also affects the other currency pair USD/CHF, which was not directly exposed to the capping on the EUR/CHF rate.

## 5 Conclusion

Liquidity is crucial to the well-functioning of financial markets and depends on how trading volume impacts asset prices. We develop the theory of realized volatility in the context of measuring market

<sup>29</sup>We are grateful to Alessandro Casini for having shared with us the MATLAB codes for the break test.



**Figure 11: Break dates.** The figures report the time series of RPV, volume, daily Amihud, and realized Amihud for the EUR/CHF rate (black solid line). The red vertical line denotes the break date, which is estimated by means of the [Bai and Perron \(1998\)](#) test for unknown break points. The test is performed with 15% trimming at the beginning and the end of the sample period and between break dates.

liquidity. Similar to the spot volatility, the instantaneous liquidity,  $\ell(t)$ , is assumed to vary over time based on a dynamic process in continuous time; that is, liquidity is stochastic. We propose two realized illiquidity measures that we call *realized Amihud* and *high-low Amihud*. The former is defined as the ratio between the realized power variation computed using intraday data and trading volume while the latter is the ratio between the high-low range based on daily data and volume. We demonstrate theoretically and numerically that the realized Amihud is a very accurate measure and that both realized illiquidity metrics refine the widely used measure of volume price impact proposed in [Amihud \(2002\)](#). Furthermore, by employing the theory of multipower variation in [Barndorff-Nielsen and Shephard \(2003, 2004, 2006\)](#) we develop a simple theory for jumps that capture the information component of market illiquidity.

Five main results stand out from our econometric analysis based on more than a decade of representative data for the stock and currency markets. First, the realized Amihud provides a very precise

| Estimation Results       |                                 |                                 |                                 |                                 |                                 |                                 |                                 |                                 |
|--------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
|                          | <i>Realized Amihud</i>          | EUR/CHF                         |                                 |                                 | <i>Realized Amihud</i>          | USD/CHF                         |                                 |                                 |
|                          |                                 | <i>RPV</i>                      | <i>Volume</i>                   | <i>Daily Amihud</i>             |                                 | <i>RPV</i>                      | <i>Volume</i>                   | <i>Daily Amihud</i>             |
| <i>Mean</i> <sub>1</sub> | 0.0109 <sup>a</sup><br>(0.0015) | 0.0016 <sup>a</sup><br>(0.0002) | 1.0159 <sup>a</sup><br>(0.0390) | 0.0129 <sup>a</sup><br>(0.0008) | 0.0279 <sup>a</sup><br>(0.0013) | 0.0039 <sup>a</sup><br>(0.0002) | 1.4617 <sup>a</sup><br>(0.0384) | 0.0300 <sup>a</sup><br>(0.0010) |
| <i>Mean</i> <sub>2</sub> | 0.0248 <sup>a</sup><br>(0.0008) | 0.0027 <sup>a</sup><br>(0.0001) | 0.6205 <sup>a</sup><br>(0.0120) | 0.0422 <sup>a</sup><br>(0.0010) | 0.0345 <sup>a</sup><br>(0.0008) | 0.0038 <sup>a</sup><br>(0.0001) | 1.1315 <sup>a</sup><br>(0.0153) | 0.0352 <sup>a</sup><br>(0.0007) |
| Diagnostic               |                                 |                                 |                                 |                                 |                                 |                                 |                                 |                                 |
| <i>R</i> <sup>2</sup>    | 0.3127                          | 0.1009                          | 0.2168                          | 0.1638                          | 0.0714                          | 0.0004                          | 0.1112                          | 0.0076                          |
| D-W statistics           | 0.6384                          | 0.6558                          | 1.0136                          | 1.7513                          | 0.7912                          | 0.9394                          | 1.2655                          | 1.8778                          |

**Table 8:** Level estimation (robust standard errors in parenthesis) before and after the date January 14, 2015, for the EUR/CHF and USD/CHF FX rates. Superscripts *a*, *b*, and *c* denote the 1%, 5%, and 10% levels of significance, respectively. To account for heteroskedasticity and autocorrelation, the standard errors are computed based on the HAC estimator by following the automatic method in Andrews (1991) for the selection of the number of lagged autocovariances. Sample period: November 1, 2011 – September 30, 2021.

measurement of the inverse of integrated liquidity, namely the integral  $\mathcal{L} = \int_0^1 \ell(s)ds$ , over periods of fixed length (e.g., a day, a week, or a month). If intraday data are not accessible, then the high-low Amihud ensures a more precise illiquidity measurement than the classic daily Amihud proxy. Second, the proposed test statistic is effective in detecting significant information jumps – that is – illiquidity reaction to impactful news common to all traders. Based on this, we also propose a jump-robust version of the realized illiquidity estimator. Third, using heterogeneous autoregressive specifications (HAR) and multiplicative error models (MEM) we show that illiquidity is highly and heterogeneously persistence in the sense that an illiquidity shock triggers prolonged effects and today’s illiquidity is predicted by that of yesterday, of one week and one month ago. Fourth, illiquidity undergoes leverage effects in the sense that it increases more in market downturns and illiquidity shocks materialize and are more persistent when associated with negative returns. Finally, we find that it is rather the unexpected part of illiquidity that negatively predicts returns consistent with the idea that the persistent effect of an illiquidity shock increases expected future illiquidity thus decreasing asset prices.

Prior research extensively used the traditional Amihud measure in asset pricing (among the most recent papers, see, e.g., Lou and Shu, 2017 and Amihud and Noh, 2021). In this paper, we have proposed more accurate measures of realized illiquidity and we have highlighted their characteristics such as (illiquidity) jumps, heterogeneous clustering, and leverage effects. We have demonstrated that all this helps explain the temporal variation in asset returns. Future research should highlight whether and how these features predict the cross-section of asset prices and can apply what we have proposed on many other aspects of financial economics.

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## A Proofs

### A.1 Proof of Proposition 1

To simplify the exposition of the proof and without loss of generality, let assume that  $\mu_j = 0 \forall j = 1, \dots, J$ . The results of [Barndorff-Nielsen and Shephard \(2003\)](#) carry over in the limit of  $I \rightarrow \infty$  under mild regularity conditions on the *smoothness* of the drift, even when  $\mu_j$  is not constant. Let's assume that  $J = 2$ . The proof of Proposition 1 proceeds as follows. By the properties of the super-position of independent processes and by Theorem 2.5 of [Kabanov et al. \(2006\)](#), the limit for  $\delta \rightarrow 0$  (or  $I \rightarrow \infty$ ) of RPV is as follows:

$$p \lim_{I \rightarrow \infty} \delta^{1/2} RPV = \sqrt{\frac{2}{\pi}} \bar{\sigma}, \quad (28)$$

where  $\bar{\sigma} = \frac{1}{2}\sqrt{\sigma_1^2 + \sigma_2^2}$ . Indeed, similar to [Barndorff-Nielsen and Shephard \(2002b\)](#),  $\Delta p_i = \frac{1}{2} \sum_{j=1}^2 \Delta p_{i,j}^*$  is equivalent to  $\int_{\delta(i-1)}^{\delta i} \bar{\sigma} dW^*(t)$ . The aggregated volume on a unit (daily) interval is  $\nu = \sum_{i=1}^I \nu_i$ , so that

$$\begin{aligned} \nu &= \sum_{i=1}^I \frac{\ell_i}{2} \sum_{j=1}^2 |\Delta p_{i,j}^* - \Delta p_i| \\ &= \sum_{i=1}^I \frac{\ell_i}{2} \left| \frac{1}{2} \int_{\delta(i-1)}^{\delta i} \sigma_1 dW_1(s) - \frac{1}{2} \int_{\delta(i-1)}^{\delta i} \sigma_2 dW_2(s) + \sum_{i=1}^I \frac{\ell_i}{2} \left| \frac{1}{2} \int_{\delta(i-1)}^{\delta i} \sigma_2 dW_2(s) - \frac{1}{2} \int_{\delta(i-1)}^{\delta i} \sigma_1 dW_1(s) \right| \right. \\ &= \sum_{i=1}^I \frac{\ell_i}{2} \zeta_i + \sum_{i=1}^I \frac{\ell_i}{2} \zeta_i = \sum_{i=1}^I \ell_i \zeta_i, \end{aligned}$$

with  $\zeta_i = \left| \frac{1}{2} \int_{\delta(i-1)}^{\delta i} \sigma_2 dW_2(s) - \frac{1}{2} \int_{\delta(i-1)}^{\delta i} \sigma_1 dW_1(s) \right| = \left| \frac{1}{2} \int_{\delta(i-1)}^{\delta i} \sigma_1 dW_1(s) - \frac{1}{2} \int_{\delta(i-1)}^{\delta i} \sigma_2 dW_2(s) \right|$ . By superposition of independent processes, see [Barndorff-Nielsen and Shephard \(2002a, p.262\)](#) and [Barndorff-Nielsen and Shephard \(2001\)](#), by [Barndorff-Nielsen and Shephard \(2003\)](#) we have that

$$p \lim_{I \rightarrow \infty} \delta^{1/2} \sum_{i=1}^I \zeta_i = \sqrt{\frac{2}{\pi}} \bar{S}^*,$$

where  $\bar{S}^* = \frac{1}{J} \sqrt{(J-1)^2 \sigma_j^2 + \sum_{s \neq j} \sigma_s^2} = \frac{1}{2} \sqrt{\sigma_1^2 + \sigma_2^2} = \bar{\sigma}$ , so that

$$p \lim_{I \rightarrow \infty} \delta^{1/2} \nu = p \lim_{I \rightarrow \infty} \delta^{1/2} \left[ \sum_{i=1}^I \ell_i \zeta_i \right] = \sqrt{\frac{2}{\pi}} \bar{\sigma} \mathcal{L},$$

where  $\mathcal{L} = \int_0^1 \ell(s) ds$  denotes the *integrated liquidity*. Hence, we get

$$p \lim_{I \rightarrow \infty} \mathcal{A} = \frac{\bar{\sigma}}{\mathcal{L} \bar{\sigma}}, \quad (29)$$

so that equation (6) in Proposition 1 follows directly, that is,  $p \lim_{I \rightarrow \infty} \mathcal{A} = \frac{1}{\mathcal{L}}$ .

Furthermore, by straightforward application of [Barndorff-Nielsen and Shephard \(2003, p.260\)](#), we get

$$\frac{\log \left( \sqrt{\pi \delta / 2} \cdot \nu \right) - \log (\bar{\sigma} \mathcal{L})}{\sqrt{\frac{\delta(\pi/2-1)RV}{(\pi\delta/2)RPV^2}}} \xrightarrow{d} N(0, 1), \quad (30)$$

where  $RV$  is the realized variance and is defined as  $RV = \sum_{i=1}^I (r_i)^2$ ; for more, see, among others, [Andersen and Bollerslev \(1998\)](#). Following [Barndorff-Nielsen and Shephard \(2002a,b\)](#) and taking the limit for  $\delta \rightarrow 0$  (i.e.,  $I \rightarrow \infty$ ), we get  $p \lim_{I \rightarrow \infty} RV = \frac{1}{j^2} \mathcal{V}$ , where  $\mathcal{V} = \sum_{j=1}^J \mathcal{V}_j$  is the variation of the asset price on the unit interval generated by the aggregated individual components of  $r$ . The term  $\mathcal{V}_j = \int_0^1 \sigma_j^2 ds$  is the *integrated variance* associated with the  $j$ -th trader's specific component. By noticing that  $\log(\mathcal{A}) = \log \left( \sqrt{\pi \delta / 2} \cdot RPV \right) - \log \left( \sqrt{\pi \delta / 2} \cdot \nu \right)$ , the result in (7) follows. ■

## A.2 Proof of Proposition 2

The proof of Proposition 2 follows the same steps as the proof of Proposition 1 in Section A.1. In this case,

$$p \lim_{I \rightarrow \infty} \delta^{1/2} RPV = \sqrt{\frac{2}{\pi}} \bar{\sigma}, \quad (31)$$

where  $\bar{\sigma} = \frac{1}{\sqrt{J}} \sigma$ , and

$$p \lim_{I \rightarrow \infty} \delta^{1/2} \nu = p \lim_{I \rightarrow \infty} \delta^{1/2} \left[ \sum_{i=1}^I \frac{\ell_i}{2} |\Delta p_{i,j}^* - \Delta p_i| \right] = \sqrt{\frac{2}{\pi}} \frac{\mathcal{L}}{2} \bar{S},$$

where  $\bar{S} = \frac{1}{J} \sum_{j=1}^J \sqrt{(J-1)^2 \sigma_j^2 + \sum_{s \neq j} \sigma_s^2} = \sqrt{J(J-1)} \sigma$ , so that

$$p \lim_{I \rightarrow \infty} \mathcal{A} = \frac{2}{J\sqrt{J-1}\mathcal{L}}. \quad (32)$$

Furthermore,

$$\frac{\log(\sqrt{\pi\delta/2} \cdot v) - \log(\sigma J \sqrt{J-1} \mathcal{L}/2)}{\sqrt{\frac{\delta(\pi/2-1)RV}{((J-1)\pi\delta/2)RPV^2}}} \xrightarrow{d} N(0, 1), \quad (33)$$

so that

$$\frac{\log(\mathcal{A}) - \log\left(\frac{2}{\mathcal{L}J\sqrt{J-1}}\right)}{\sqrt{\frac{J\delta(\pi/2-1)RV}{(J-1)(\pi\delta/2)RPV^2}}} \xrightarrow{d} N(0, 1). \quad (34)$$

■

## A.3 Proof of Proposition 3

The proof of Proposition 3 follows similar steps as the proof of Proposition 1 in Section A.1. However, this time we focus on an estimator of spot volatility, as studied in [Kristensen \(2010\)](#) among many others.<sup>30</sup> In particular, by considering again the case of  $J = 2$  traders, we have that the spot variance estimator, denoted as

$$RV(\tau) = \sum_{|s-\tau|<b} \mathcal{K}\left(\frac{s-\tau}{b}\right) r_s^2, \quad (35)$$

by Theorem 3 in [Kristensen \(2010\)](#) and under mild regularity conditions on the kernel function<sup>31</sup> is such that

$$p \lim_{b \rightarrow 0} RV(\tau) = \mathcal{V}(\tau), \quad (36)$$

<sup>30</sup>Several contributions in the literature on spot volatility estimation follow from the seminal paper by [Foster and Nelson \(1996\)](#). The most notable are [Fan and Wang \(2008\)](#), [Mykland and Zhang \(2008, 2009\)](#), [Zu and Boswijk \(2014\)](#), [Kanaya and Kristensen \(2016\)](#), [Bandi and Renò \(2018\)](#), and [Figueroa-López and Li \(2020\)](#).

<sup>31</sup>We refer to a assumption K.1 in [Kristensen \(2010, p.69\)](#), which imposes regularity conditions on the kernel function, which are satisfied by most standard kernels. As noted by [Kristensen \(2010\)](#) the asymptotic results only go through for  $\tau \in (0, 1)$ , for general kernels satisfying assumption K.1. In order to obtain results for  $\tau \in [0, 1]$  (i.e. including the boundaries), we would need to use boundary kernels or local polynomial estimators, see the discussion in Section 4 in [Kristensen \(2010\)](#).

where  $\mathcal{V}(\tau) = \frac{\sigma_1^2(\tau) + \sigma_2^2(\tau)}{4}$ . As for the denominator of the spot Amihud, we have that

$$v^2(\tau) = \sum_{|s-\tau|<b} \mathcal{K}\left(\frac{s-\tau}{b}\right) v_s^2, \quad (37)$$

is such that

$$p \lim_{b \rightarrow 0} v^2(\tau) = \ell^2(\tau) \mathcal{V}(\tau). \quad (38)$$

Hence, given that the square root is a continuous operator, by the continuous mapping theorem it follows that  $p \lim_{b \rightarrow 0} \mathcal{A}(\tau) = \frac{1}{\ell(\tau)}$ .

Finally, following Theorem 4 of [Kristensen \(2010\)](#) as  $a \rightarrow 0$ ,  $b \rightarrow 0$  (with  $a/b \rightarrow 0$ ) and  $I \rightarrow \infty$ , we have that

$$\int_a^{1-a} \mathcal{A}(s) ds \xrightarrow{p} \frac{1}{\mathcal{L}},$$

where the integral  $\int_a^{1-a} \mathcal{A}(s) ds$  is approximated by the Riemann sum  $\overline{\mathcal{A}} = \frac{1}{I} \sum_{i=1}^{I-b+1} \mathcal{A}(\tau_i)$ , see Theorem 9.2.1 of [Jacod and Protter \(2011\)](#) and [Jacod and Rosenbaum \(2013\)](#).

■

## B Monte Carlo simulations

### B.1 Setting 1

| Low Liquidity, $\ell_0 = \log(50)$     |                          |                    |                 |               |                    |                 |
|----------------------------------------|--------------------------|--------------------|-----------------|---------------|--------------------|-----------------|
| Sampling frequency                     | Percentage Relative Bias |                    |                 | Relative RMSE |                    |                 |
|                                        | $\mathcal{A}$            | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 1h                                     | -0.2555                  | -6.3853            | 1.4113          | 0.3447        | 0.4520             | 1.0798          |
| 30min                                  | -0.7071                  | -6.3853            | 1.4113          | 0.2560        | 0.4520             | 1.0798          |
| 15min                                  | -1.0859                  | -6.3853            | 1.4113          | 0.1979        | 0.4520             | 1.0798          |
| 10min                                  | 0.2521                   | -6.3853            | 1.4113          | 0.1385        | 0.4520             | 1.0798          |
| 5min                                   | -0.2316                  | -6.3853            | 1.4113          | 0.0902        | 0.4520             | 1.0798          |
| 1min                                   | -0.1161                  | -6.3853            | 1.4113          | 0.0494        | 0.4520             | 1.0798          |
| 30sec                                  | -0.0926                  | -6.3853            | 1.4113          | 0.0358        | 0.4520             | 1.0798          |
| 15sec                                  | -0.0491                  | -6.3853            | 1.4113          | <b>0.0249</b> | 0.4520             | 1.0798          |
| Medium Liquidity, $\ell_0 = \log(200)$ |                          |                    |                 |               |                    |                 |
| Sampling frequency                     | Percentage Relative Bias |                    |                 | Relative RMSE |                    |                 |
|                                        | $\mathcal{A}$            | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 1h                                     | 3.4030                   | -3.8845            | 2.7552          | 0.4548        | 0.4400             | 1.1898          |
| 30min                                  | 1.1264                   | -3.8845            | 2.7552          | 0.2773        | 0.4400             | 1.1898          |
| 15min                                  | 0.3307                   | -3.8845            | 2.7552          | 0.1956        | 0.4400             | 1.1898          |
| 10min                                  | 0.1412                   | -3.8845            | 2.7552          | 0.1354        | 0.4400             | 1.1898          |
| 5min                                   | 0.2525                   | -3.8845            | 2.7552          | 0.0896        | 0.4400             | 1.1898          |
| 1min                                   | -0.0719                  | -3.8845            | 2.7552          | 0.0506        | 0.4400             | 1.1898          |
| 30sec                                  | 0.0972                   | -3.8845            | 2.7552          | 0.0351        | 0.4400             | 1.1898          |
| 15sec                                  | -0.0165                  | -3.8845            | 2.7552          | <b>0.0264</b> | 0.4400             | 1.1898          |
| High Liquidity, $\ell_0 = \log(500)$   |                          |                    |                 |               |                    |                 |
| Sampling frequency                     | Percentage Relative Bias |                    |                 | Relative RMSE |                    |                 |
|                                        | $\mathcal{A}$            | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 1h                                     | 2.7000                   | -4.1219            | 3.2622          | 0.3800        | 0.4979             | 1.2103          |
| 30min                                  | 1.6175                   | -4.1219            | 3.2622          | 0.2361        | 0.4979             | 1.2103          |
| 15min                                  | 0.8570                   | -4.1219            | 3.2622          | 0.1852        | 0.4979             | 1.2103          |
| 10min                                  | 0.8488                   | -4.1219            | 3.2622          | 0.1462        | 0.4979             | 1.2103          |
| 5min                                   | 0.4213                   | -4.1219            | 3.2622          | 0.1020        | 0.4979             | 1.2103          |
| 1min                                   | 0.1024                   | -4.1219            | 3.2622          | 0.0494        | 0.4979             | 1.2103          |
| 30sec                                  | 0.1397                   | -4.1219            | 3.2622          | 0.0398        | 0.4979             | 1.2103          |
| 15sec                                  | -0.0631                  | -4.1219            | 3.2622          | <b>0.0280</b> | 0.4979             | 1.2103          |

**Table 9:** Illiquidity measurement. The table reports the Monte Carlo relative percentage bias and RMSE (both relative to  $\frac{1}{\mathcal{L}}$ ) for several illiquidity estimators: the realized Amihud  $\mathcal{A} = \frac{RPV}{v}$ ; the daily Amihud measure, that is,  $\mathcal{A}^D = \frac{|r|}{v}$ ; the high-low Amihud,  $\mathcal{A}^{HL} = \frac{range}{v}$ , where  $range$  is scaled by  $\sqrt{2/\pi}$  to be comparable with the RPV. The series  $\ell(t)$  is generated according to the log Ornstein–Uhlenbeck process in (10), with parameters  $\kappa_\ell = 0.1$  and  $\eta_\ell = 0.5$ . The spot Amivest uses a bandwidth length of one hour. The smallest RMSE is in bold.

## B.2 Setting 2

| Low Liquidity, $\ell_0 = \log(50)$     |                          |                    |                 |               |                    |                 |
|----------------------------------------|--------------------------|--------------------|-----------------|---------------|--------------------|-----------------|
| Sampling frequency                     | Percentage Relative Bias |                    |                 | Relative RMSE |                    |                 |
|                                        | $\mathcal{A}$            | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 1h                                     | 3.0929                   | -4.4342            | -1.6361         | 0.3155        | 0.4050             | 0.9730          |
| 30min                                  | 1.6145                   | -4.4342            | -1.6361         | 0.2112        | 0.4050             | 0.9730          |
| 15min                                  | 0.7817                   | -4.4342            | -1.6361         | 0.1567        | 0.4050             | 0.9730          |
| 10min                                  | 0.5719                   | -4.4342            | -1.6361         | 0.1299        | 0.4050             | 0.9730          |
| 5min                                   | 0.2458                   | -4.4342            | -1.6361         | 0.0874        | 0.4050             | 0.9730          |
| 1min                                   | 0.2185                   | -4.4342            | -1.6361         | 0.0416        | 0.4050             | 0.9730          |
| 30sec                                  | 0.1525                   | -4.4342            | -1.6361         | 0.0264        | 0.4050             | 0.9730          |
| 15sec                                  | 0.1104                   | -4.4342            | -1.6361         | <b>0.0199</b> | 0.4050             | 0.9730          |
| Medium Liquidity, $\ell_0 = \log(200)$ |                          |                    |                 |               |                    |                 |
| Sampling frequency                     | Percentage Relative Bias |                    |                 | Relative RMSE |                    |                 |
|                                        | $\mathcal{A}$            | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 1h                                     | 2.1351                   | -4.7548            | -0.4336         | 0.3300        | 0.3808             | 0.9607          |
| 30min                                  | 0.7496                   | -4.7548            | -0.4336         | 0.2122        | 0.3808             | 0.9607          |
| 15min                                  | 0.9511                   | -4.7548            | -0.4336         | 0.1660        | 0.3808             | 0.9607          |
| 10min                                  | 0.1907                   | -4.7548            | -0.4336         | 0.1318        | 0.3808             | 0.9607          |
| 5min                                   | 0.3255                   | -4.7548            | -0.4336         | 0.0985        | 0.3808             | 0.9607          |
| 1min                                   | 0.1149                   | -4.7548            | -0.4336         | 0.0374        | 0.3808             | 0.9607          |
| 30sec                                  | 0.0738                   | -4.7548            | -0.4336         | 0.0297        | 0.3808             | 0.9607          |
| 15sec                                  | 0.0209                   | -4.7548            | -0.4336         | <b>0.0208</b> | 0.3808             | 0.9607          |
| High Liquidity, $\ell_0 = \log(500)$   |                          |                    |                 |               |                    |                 |
| Sampling frequency                     | Percentage Relative Bias |                    |                 | Relative RMSE |                    |                 |
|                                        | $\mathcal{A}$            | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 1h                                     | 3.1770                   | -6.0210            | -0.9337         | 0.2979        | 0.3626             | 1.0276          |
| 30min                                  | 0.8770                   | -6.0210            | -0.9337         | 0.2285        | 0.3626             | 1.0276          |
| 15min                                  | 0.6909                   | -6.0210            | -0.9337         | 0.1635        | 0.3626             | 1.0276          |
| 10min                                  | 0.3240                   | -6.0210            | -0.9337         | 0.1248        | 0.3626             | 1.0276          |
| 5min                                   | 0.4126                   | -6.0210            | -0.9337         | 0.0781        | 0.3626             | 1.0276          |
| 1min                                   | 0.2478                   | -6.0210            | -0.9337         | 0.0383        | 0.3626             | 1.0276          |
| 30sec                                  | 0.1383                   | -6.0210            | -0.9337         | 0.0274        | 0.3626             | 1.0276          |
| 15sec                                  | 0.0456                   | -6.0210            | -0.9337         | <b>0.0176</b> | 0.3626             | 1.0276          |

**Table 10:** Illiquidity measurement. The table reports the Monte Carlo relative percentage bias and RMSE (both relative to  $\frac{1}{\mathcal{L}}$ ) for several illiquidity estimators: the realized Amihud  $\mathcal{A} = \frac{RPV}{v}$ ; the daily Amihud measure, that is,  $\mathcal{A}^D = \frac{|r|}{v}$ ; the high-low Amihud,  $\mathcal{A}^{HL} = \frac{range}{v}$ , where  $range$  is scaled by  $\sqrt{2/\pi}$  to be comparable with the RPV. The series  $\ell(t)$  is generated according to the log Ornstein–Uhlenbeck process in (10), with parameters  $\kappa_\ell = 0.2$  and  $\eta_\ell = 0.6$ . The spot Amivest uses a bandwidth length of one hour. The smallest RMSE is in bold.

### B.3 Setting 3

| Low Liquidity, $\ell_0 = \log(50)$     |                          |                    |                 |               |                    |                 |
|----------------------------------------|--------------------------|--------------------|-----------------|---------------|--------------------|-----------------|
| Sampling frequency                     | Percentage Relative Bias |                    |                 | Relative RMSE |                    |                 |
|                                        | $\mathcal{A}$            | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 1h                                     | 3.6173                   | -4.5770            | 1.1549          | 0.3849        | 0.3726             | 0.9825          |
| 30min                                  | 0.7457                   | -4.5770            | 1.1549          | 0.2280        | 0.3726             | 0.9825          |
| 15min                                  | -0.0678                  | -4.5770            | 1.1549          | 0.1431        | 0.3726             | 0.9825          |
| 10min                                  | 0.0640                   | -4.5770            | 1.1549          | 0.1195        | 0.3726             | 0.9825          |
| 5min                                   | 0.0290                   | -4.5770            | 1.1549          | 0.0883        | 0.3726             | 0.9825          |
| 1min                                   | -0.1105                  | -4.5770            | 1.1549          | 0.0358        | 0.3726             | 0.9825          |
| 30sec                                  | 0.0081                   | -4.5770            | 1.1549          | 0.0239        | 0.3726             | 0.9825          |
| 15sec                                  | 0.0328                   | -4.5770            | 1.1549          | <b>0.0174</b> | 0.3726             | 0.9825          |
| Medium Liquidity, $\ell_0 = \log(200)$ |                          |                    |                 |               |                    |                 |
| Sampling frequency                     | Percentage Relative Bias |                    |                 | Relative RMSE |                    |                 |
|                                        | $\mathcal{A}$            | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 1h                                     | 2.6574                   | -5.7905            | -1.1114         | 0.2916        | 0.3552             | 0.9040          |
| 30min                                  | 2.6892                   | -5.7905            | -1.1114         | 0.2083        | 0.3552             | 0.9040          |
| 15min                                  | 1.0359                   | -5.7905            | -1.1114         | 0.1534        | 0.3552             | 0.9040          |
| 10min                                  | 0.6878                   | -5.7905            | -1.1114         | 0.1184        | 0.3552             | 0.9040          |
| 5min                                   | 0.1346                   | -5.7905            | -1.1114         | 0.0831        | 0.3552             | 0.9040          |
| 1min                                   | -0.0029                  | -5.7905            | -1.1114         | 0.0400        | 0.3552             | 0.9040          |
| 30sec                                  | 0.1446                   | -5.7905            | -1.1114         | 0.0250        | 0.3552             | 0.9040          |
| 15sec                                  | 0.0799                   | -5.7905            | -1.1114         | <b>0.0190</b> | 0.3552             | 0.9040          |
| High Liquidity, $\ell_0 = \log(500)$   |                          |                    |                 |               |                    |                 |
| Sampling frequency                     | Percentage Relative Bias |                    |                 | Relative RMSE |                    |                 |
|                                        | $\mathcal{A}$            | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 1h                                     | 3.9148                   | -4.7822            | -0.0812         | 0.3137        | 0.4217             | 1.0350          |
| 30min                                  | 1.4832                   | -4.7822            | -0.0812         | 0.1998        | 0.4217             | 1.0350          |
| 15min                                  | 0.5369                   | -4.7822            | -0.0812         | 0.1397        | 0.4217             | 1.0350          |
| 10min                                  | 0.5818                   | -4.7822            | -0.0812         | 0.1238        | 0.4217             | 1.0350          |
| 5min                                   | 0.4215                   | -4.7822            | -0.0812         | 0.0899        | 0.4217             | 1.0350          |
| 1min                                   | -0.0800                  | -4.7822            | -0.0812         | 0.0381        | 0.4217             | 1.0350          |
| 30sec                                  | -0.0051                  | -4.7822            | -0.0812         | 0.0279        | 0.4217             | 1.0350          |
| 15sec                                  | -0.0080                  | -4.7822            | -0.0812         | <b>0.0215</b> | 0.4217             | 1.0350          |

**Table 11:** Illiquidity measurement. The table reports the Monte Carlo relative percentage bias and RMSE (both relative to  $\frac{1}{\mathcal{L}}$ ) for several illiquidity estimators: the realized Amihud  $\mathcal{A} = \frac{RPV}{v}$ ; the daily Amihud measure, that is,  $\mathcal{A}^D = \frac{|r|}{v}$ ; the high-low Amihud,  $\mathcal{A}^{HL} = \frac{range}{v}$ , where  $range$  is scaled by  $\sqrt{2/\pi}$  to be comparable with the RPV. The series  $\ell(t)$  is generated according to the log Ornstein–Uhlenbeck process in (10), with parameters  $\kappa_\ell = 0.3$  and  $\eta_\ell = 0.7$ . The spot Amivest uses a bandwidth length of one hour. The smallest RMSE is in bold.

## B.4 Setting 4

| Low Liquidity, $\ell_0 = \log(50)$     |                          |                    |                 |               |                    |                 |
|----------------------------------------|--------------------------|--------------------|-----------------|---------------|--------------------|-----------------|
| Sampling frequency                     | Percentage Relative Bias |                    |                 | Relative RMSE |                    |                 |
|                                        | $\mathcal{A}$            | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 1h                                     | 3.5798                   | -2.9887            | 3.7235          | 0.2966        | 0.3392             | 0.9399          |
| 30min                                  | 1.5970                   | -2.9887            | 3.7235          | 0.1919        | 0.3392             | 0.9399          |
| 15min                                  | 1.1358                   | -2.9887            | 3.7235          | 0.1474        | 0.3392             | 0.9399          |
| 10min                                  | 0.6631                   | -2.9887            | 3.7235          | 0.1127        | 0.3392             | 0.9399          |
| 5min                                   | 0.2320                   | -2.9887            | 3.7235          | 0.0785        | 0.3392             | 0.9399          |
| 1min                                   | 0.0110                   | -2.9887            | 3.7235          | 0.0375        | 0.3392             | 0.9399          |
| 30sec                                  | 0.0358                   | -2.9887            | 3.7235          | 0.0252        | 0.3392             | 0.9399          |
| 15sec                                  | -0.0540                  | -2.9887            | 3.7235          | <b>0.0184</b> | 0.3392             | 0.9399          |
| Medium Liquidity, $\ell_0 = \log(200)$ |                          |                    |                 |               |                    |                 |
| Sampling frequency                     | Percentage Relative Bias |                    |                 | Relative RMSE |                    |                 |
|                                        | $\mathcal{A}$            | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 1h                                     | 3.1562                   | -5.5008            | -4.2529         | 0.3215        | 0.3831             | 0.8858          |
| 30min                                  | 1.8243                   | -5.5008            | -4.2529         | 0.2009        | 0.3831             | 0.8858          |
| 15min                                  | 1.5570                   | -5.5008            | -4.2529         | 0.1518        | 0.3831             | 0.8858          |
| 10min                                  | 0.6421                   | -5.5008            | -4.2529         | 0.1186        | 0.3831             | 0.8858          |
| 5min                                   | 0.4454                   | -5.5008            | -4.2529         | 0.0862        | 0.3831             | 0.8858          |
| 1min                                   | -0.1061                  | -5.5008            | -4.2529         | 0.0387        | 0.3831             | 0.8858          |
| 30sec                                  | -0.0380                  | -5.5008            | -4.2529         | 0.0252        | 0.3831             | 0.8858          |
| 15sec                                  | 0.0024                   | -5.5008            | -4.2529         | <b>0.0182</b> | 0.3831             | 0.8858          |
| High Liquidity, $\ell_0 = \log(500)$   |                          |                    |                 |               |                    |                 |
| Sampling frequency                     | Percentage Relative Bias |                    |                 | Relative RMSE |                    |                 |
|                                        | $\mathcal{A}$            | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 1h                                     | 2.5510                   | -5.0968            | -3.3069         | 0.3180        | 0.3521             | 0.9510          |
| 30min                                  | 2.2379                   | -5.0968            | -3.3069         | 0.2078        | 0.3521             | 0.9510          |
| 15min                                  | 1.1307                   | -5.0968            | -3.3069         | 0.1399        | 0.3521             | 0.9510          |
| 10min                                  | 0.6598                   | -5.0968            | -3.3069         | 0.1121        | 0.3521             | 0.9510          |
| 5min                                   | 0.2552                   | -5.0968            | -3.3069         | 0.0817        | 0.3521             | 0.9510          |
| 1min                                   | -0.0496                  | -5.0968            | -3.3069         | 0.0353        | 0.3521             | 0.9510          |
| 30sec                                  | -0.0391                  | -5.0968            | -3.3069         | 0.0250        | 0.3521             | 0.9510          |
| 15sec                                  | -0.0299                  | -5.0968            | -3.3069         | <b>0.0175</b> | 0.3521             | 0.9510          |

**Table 12:** Illiquidity measurement. The table reports the Monte Carlo relative percentage bias and RMSE (both relative to  $\frac{1}{\mathcal{L}}$ ) for several illiquidity estimators: the realized Amihud  $\mathcal{A} = \frac{RPV}{v}$ ; the daily Amihud measure, that is,  $\mathcal{A}^D = \frac{|r|}{v}$ ; the high-low Amihud,  $\mathcal{A}^{HL} = \frac{range}{v}$ , where  $range$  is scaled by  $\sqrt{2/\pi}$  to be comparable with the RPV. The series  $\ell(t)$  is generated according to the log Ornstein–Uhlenbeck process in (10), with parameters  $\kappa_\ell = 0.4$  and  $\eta_\ell = 0.8$ . The spot Amivest uses a bandwidth length of one hour. The smallest RMSE is in bold.

## B.5 Setting 5

| Low Liquidity, $\ell_0 = \log(50)$     |                          |                    |                 |               |                    |                 |
|----------------------------------------|--------------------------|--------------------|-----------------|---------------|--------------------|-----------------|
| Sampling frequency                     | Percentage Relative Bias |                    |                 | Relative RMSE |                    |                 |
|                                        | $\mathcal{A}$            | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 1h                                     | 3.9235                   | -5.3437            | 0.9339          | 0.2782        | 0.3607             | 0.9267          |
| 30min                                  | 2.3883                   | -5.3437            | 0.9339          | 0.1858        | 0.3607             | 0.9267          |
| 15min                                  | 1.1108                   | -5.3437            | 0.9339          | 0.1377        | 0.3607             | 0.9267          |
| 10min                                  | 1.1285                   | -5.3437            | 0.9339          | 0.1127        | 0.3607             | 0.9267          |
| 5min                                   | 0.2850                   | -5.3437            | 0.9339          | 0.0826        | 0.3607             | 0.9267          |
| 1min                                   | 0.0027                   | -5.3437            | 0.9339          | 0.0346        | 0.3607             | 0.9267          |
| 30sec                                  | -0.0410                  | -5.3437            | 0.9339          | 0.0237        | 0.3607             | 0.9267          |
| 15sec                                  | 0.0365                   | -5.3437            | 0.9339          | <b>0.0171</b> | 0.3607             | 0.9267          |
| Medium Liquidity, $\ell_0 = \log(200)$ |                          |                    |                 |               |                    |                 |
| Sampling frequency                     | Percentage Relative Bias |                    |                 | Relative RMSE |                    |                 |
|                                        | $\mathcal{A}$            | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 1h                                     | 4.4418                   | -5.3832            | 1.2452          | 0.3312        | 0.3530             | 1.0286          |
| 30min                                  | 1.9177                   | -5.3832            | 1.2452          | 0.2299        | 0.3530             | 1.0286          |
| 15min                                  | 0.6341                   | -5.3832            | 1.2452          | 0.1557        | 0.3530             | 1.0286          |
| 10min                                  | 0.1306                   | -5.3832            | 1.2452          | 0.1233        | 0.3530             | 1.0286          |
| 5min                                   | 0.2943                   | -5.3832            | 1.2452          | 0.0840        | 0.3530             | 1.0286          |
| 1min                                   | -0.0065                  | -5.3832            | 1.2452          | 0.0363        | 0.3530             | 1.0286          |
| 30sec                                  | -0.0490                  | -5.3832            | 1.2452          | 0.0246        | 0.3530             | 1.0286          |
| 15sec                                  | 0.0372                   | -5.3832            | 1.2452          | <b>0.0169</b> | 0.3530             | 1.0286          |
| High Liquidity, $\ell_0 = \log(500)$   |                          |                    |                 |               |                    |                 |
| Sampling frequency                     | Percentage Relative Bias |                    |                 | Relative RMSE |                    |                 |
|                                        | $\mathcal{A}$            | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 1h                                     | 3.0090                   | -5.3856            | 1.9886          | 0.2924        | 0.3494             | 0.8825          |
| 30min                                  | 1.6147                   | -5.3856            | 1.9886          | 0.2019        | 0.3494             | 0.8825          |
| 15min                                  | 1.0477                   | -5.3856            | 1.9886          | 0.1312        | 0.3494             | 0.8825          |
| 10min                                  | 0.3926                   | -5.3856            | 1.9886          | 0.1046        | 0.3494             | 0.8825          |
| 5min                                   | 0.4766                   | -5.3856            | 1.9886          | 0.0778        | 0.3494             | 0.8825          |
| 1min                                   | -0.0158                  | -5.3856            | 1.9886          | 0.0342        | 0.3494             | 0.8825          |
| 30sec                                  | -0.0088                  | -5.3856            | 1.9886          | 0.0247        | 0.3494             | 0.8825          |
| 15sec                                  | 0.1167                   | -5.3856            | 1.9886          | <b>0.0176</b> | 0.3494             | 0.8825          |

**Table 13:** Illiquidity measurement. The table reports the Monte Carlo relative percentage bias and RMSE (both relative to  $\frac{1}{\mathcal{L}}$ ) for several illiquidity estimators: the realized Amihud  $\mathcal{A} = \frac{RPV}{v}$ ; the daily Amihud measure, that is,  $\mathcal{A}^D = \frac{|r|}{v}$ ; the high-low Amihud,  $\mathcal{A}^{HL} = \frac{range}{v}$ , where  $range$  is scaled by  $\sqrt{2/\pi}$  to be comparable with the RPV. The series  $\ell(t)$  is generated according to the log Ornstein–Uhlenbeck process in (10), with parameters  $\kappa_\ell = 0.5$  and  $\eta_\ell = 0.9$ . The spot Amivest uses a bandwidth length of one hour. The smallest RMSE is in bold.

## C Monte Carlo simulations with stochastic volatility

### C.1 Setting 1

Spot liquidity,  $\ell(t)$ , evolves according to the following Ornstein–Uhlenbeck process

$$d \log \ell(t) = 0.2(\log \ell(t) - \ell_0)dt + 0.7dW_t(t), \quad t > 0. \quad (39)$$

while the variances (on the scale of percentage returns) evolve following the dynamics of the CIR process

$$d\sigma_j^2(t) = 0.05\sigma(t)(\sigma^2(t) - 6)dt + 0.05\sigma(t)dW_{\sigma,j}(t).$$

| Low Liquidity, $\ell_0 = \log(50)$     |                            |                            |               |                    |                            |                 |                            |               |                    |                 |  |
|----------------------------------------|----------------------------|----------------------------|---------------|--------------------|----------------------------|-----------------|----------------------------|---------------|--------------------|-----------------|--|
| Sampling frequency                     | $\overline{\mathcal{A}}_s$ | Percentage Relative Bias   |               |                    |                            | $\mathcal{A}^D$ | $\overline{\mathcal{A}}_s$ | Relative RMSE |                    |                 |  |
|                                        |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\overline{\mathcal{A}}_f$ |                 |                            | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |  |
| 15min                                  | -10.3690                   | -14.0067                   | 0.9407        | -0.8847            | 5.0471                     | 0.2993          | 0.4027                     | 0.1952        | 0.5455             | 1.3514          |  |
| 10min                                  | -6.7148                    | -8.1868                    | 0.6604        | -0.8847            | 5.0471                     | 0.1972          | 0.2212                     | 0.1627        | 0.5455             | 1.3514          |  |
| 5min                                   | -3.4769                    | -3.9317                    | 0.5177        | -0.8847            | 5.0471                     | 0.1204          | 0.1273                     | 0.1081        | 0.5455             | 1.3514          |  |
| 1min                                   | -0.8901                    | -0.9487                    | 0.1791        | -0.8847            | 5.0471                     | 0.0521          | 0.0567                     | 0.0531        | 0.5455             | 1.3514          |  |
| 30sec                                  | -0.7221                    | -0.7707                    | -0.1839       | -0.8847            | 5.0471                     | 0.0412          | 0.0469                     | 0.0406        | 0.5455             | 1.3514          |  |
| 15sec                                  | -0.4976                    | -0.5575                    | -0.1886       | -0.8847            | 5.0471                     | 0.0288          | 0.0405                     | <b>0.0287</b> | 0.5455             | 1.3514          |  |
| Medium Liquidity, $\ell_0 = \log(200)$ |                            |                            |               |                    |                            |                 |                            |               |                    |                 |  |
| Sampling frequency                     | $\overline{\mathcal{A}}_s$ | Percentage Relative Bias   |               |                    |                            | $\mathcal{A}^D$ | $\overline{\mathcal{A}}_s$ | Relative RMSE |                    |                 |  |
|                                        |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\overline{\mathcal{A}}_f$ |                 |                            | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |  |
| 15min                                  | -11.4926                   | -14.9020                   | -0.9072       | -6.1235            | 1.1195                     | 0.3368          | 0.4117                     | 0.2414        | 0.6821             | 1.5500          |  |
| 10min                                  | -7.5927                    | -9.0495                    | 0.2383        | -6.1235            | 1.1195                     | 0.2809          | 0.3142                     | 0.1872        | 0.6821             | 1.5500          |  |
| 5min                                   | -4.2517                    | -4.5411                    | -0.4191       | -6.1235            | 1.1195                     | 0.1424          | 0.1501                     | 0.1143        | 0.6821             | 1.5500          |  |
| 1min                                   | -1.1461                    | -1.1875                    | -0.2050       | -6.1235            | 1.1195                     | 0.0716          | 0.0807                     | 0.0676        | 0.6821             | 1.5500          |  |
| 30sec                                  | -0.6679                    | -0.7235                    | -0.1307       | -6.1235            | 1.1195                     | 0.0356          | 0.0529                     | 0.0328        | 0.6821             | 1.5500          |  |
| 15sec                                  | -0.3706                    | -0.4287                    | -0.0016       | -6.1235            | 1.1195                     | 0.0237          | 0.0404                     | <b>0.0219</b> | 0.6821             | 1.5500          |  |
| High Liquidity, $\ell_0 = \log(500)$   |                            |                            |               |                    |                            |                 |                            |               |                    |                 |  |
| Sampling frequency                     | $\overline{\mathcal{A}}_s$ | Percentage Relative Bias   |               |                    |                            | $\mathcal{A}^D$ | $\overline{\mathcal{A}}_s$ | Relative RMSE |                    |                 |  |
|                                        |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\overline{\mathcal{A}}_f$ |                 |                            | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |  |
| 15min                                  | -10.5214                   | -14.0662                   | 0.5619        | -4.1779            | -1.6551                    | 0.2885          | 0.3373                     | 0.1813        | 0.4522             | 1.1350          |  |
| 10min                                  | -6.8335                    | -8.2686                    | 0.7493        | -4.1779            | -1.6551                    | 0.1813          | 0.1931                     | 0.1725        | 0.4522             | 1.1350          |  |
| 5min                                   | -3.7256                    | -4.0240                    | 0.1943        | -4.1779            | -1.6551                    | 0.1123          | 0.1160                     | 0.1036        | 0.4522             | 1.1350          |  |
| 1min                                   | -0.7930                    | -0.8017                    | 0.2074        | -4.1779            | -1.6551                    | 0.0495          | 0.0512                     | 0.0468        | 0.4522             | 1.1350          |  |
| 30sec                                  | -0.4416                    | -0.4553                    | 0.1335        | -4.1779            | -1.6551                    | 0.0295          | 0.0376                     | 0.0297        | 0.4522             | 1.1350          |  |
| 15sec                                  | -0.3036                    | -0.3059                    | 0.0559        | -4.1779            | -1.6551                    | <b>0.0202</b>   | 0.0320                     | 0.0231        | 0.4522             | 1.1350          |  |

**Table 14:** Integrated spot illiquidity. The table reports the Monte Carlo relative percentage bias and RMSE (both relative to  $\frac{1}{L}$ ) for several illiquidity estimators: the smoothed integrated illiquidity, denoted as  $\overline{\mathcal{A}}_s = \left( \frac{1}{L} \sum_{i=1}^L \mathcal{A}_s^{-1}(\tau_i) \right)^{-1}$ , where  $\mathcal{A}_s^{-1}(\tau_i)$  is the spot Amivest based on a (equally weighted) two-sided kernel function; the filtered integrated illiquidity, denoted as  $\overline{\mathcal{A}}_f = \left( \frac{1}{L} \sum_{i=1}^L \mathcal{A}_f^{-1}(\tau_i) \right)^{-1}$ , where  $\mathcal{A}_f^{-1}(\tau_i)$  is the spot Amivest based on an (equally weighted) backward kernel function; the realized Amihud  $\mathcal{A} = \frac{RPV}{v}$ ; the daily Amihud measure, that is,  $\mathcal{A}^D = \frac{|r|}{v}$ ; the high-low Amihud,  $\mathcal{A}^{HL} = \frac{range}{v}$ , where  $range$  is scaled by  $\sqrt{2/\pi}$  to be comparable with the RPV. The series  $\ell(t)$  is generated according to the log Ornstein–Uhlenbeck process in (10), with parameters  $\kappa_\ell = 0.2$  and  $\eta_\ell = 0.7$ . The variances evolve following the dynamics of the CIR process, that is,  $d\sigma_j^2(t) = \kappa_\sigma(\sigma^2(t) - \sigma_0^2)dt + \eta_\sigma\sigma(t)dW_{\sigma,j}(t)$  with the parameters  $\kappa_\sigma = 0.05$ ,  $\sigma_0^2 = 6$ ,  $\eta_\sigma = 0.05$ . The spot Amivest uses a bandwidth length of one hour. The smallest RMSE is in bold.

## C.2 Setting 2

Spot liquidity,  $\ell(t)$ , evolves according to the following Ornstein–Uhlenbeck process

$$d \log \ell(t) = 0.3(\log \ell(t) - \ell_0)dt + 0.8dW_t(t), \quad t > 0. \quad (40)$$

while the variances (on the scale of percentage returns) evolve following the dynamics of the CIR process

$$d\sigma_j^2(t) = 0.05\sigma(\sigma^2(t) - 6)dt + 0.05\sigma(t)dW_{\sigma,j}(t).$$

| Low Liquidity, $\ell_0 = \log(50)$     |                            |                            |               |                    |                 |                            |                            |               |                    |                 |
|----------------------------------------|----------------------------|----------------------------|---------------|--------------------|-----------------|----------------------------|----------------------------|---------------|--------------------|-----------------|
| Sampling frequency                     | $\overline{\mathcal{A}}_s$ | Percentage Relative Bias   |               |                    |                 | $\overline{\mathcal{A}}_s$ | Relative RMSE              |               |                    |                 |
|                                        |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 15min                                  | -10.9543                   | -14.3990                   | 0.0028        | -5.2463            | -3.7320         | 0.2620                     | 0.3100                     | 0.1600        | 0.3980             | 0.9806          |
| 10min                                  | -8.0131                    | -9.5391                    | -0.1452       | -5.2463            | -3.7320         | 0.1995                     | 0.2245                     | 0.1251        | 0.3980             | 0.9806          |
| 5min                                   | -4.4389                    | -4.8404                    | -0.3232       | -5.2463            | -3.7320         | 0.1192                     | 0.1275                     | 0.0908        | 0.3980             | 0.9806          |
| 1min                                   | -1.2381                    | -1.2955                    | -0.2103       | -5.2463            | -3.7320         | 0.0514                     | 0.0530                     | 0.0463        | 0.3980             | 0.9806          |
| 30sec                                  | -0.7859                    | -0.8326                    | -0.1550       | -5.2463            | -3.7320         | 0.0316                     | 0.0366                     | 0.0309        | 0.3980             | 0.9806          |
| 15sec                                  | -0.5242                    | -0.5603                    | -0.1002       | -5.2463            | -3.7320         | 0.0269                     | 0.0329                     | <b>0.0247</b> | 0.3980             | 0.9806          |
| Medium Liquidity, $\ell_0 = \log(200)$ |                            |                            |               |                    |                 |                            |                            |               |                    |                 |
| Sampling frequency                     | $\overline{\mathcal{A}}_s$ | Percentage Relative Bias   |               |                    |                 | $\overline{\mathcal{A}}_s$ | Relative RMSE              |               |                    |                 |
|                                        |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 15min                                  | -11.0170                   | -14.4757                   | 0.3255        | -3.5767            | -5.2678         | 0.2684                     | 0.3229                     | 0.1771        | 0.4552             | 1.4657          |
| 10min                                  | -6.7394                    | -8.2255                    | 0.9990        | -3.5767            | -5.2678         | 0.1755                     | 0.1890                     | 0.1361        | 0.4552             | 1.4657          |
| 5min                                   | -4.0039                    | -4.4114                    | 0.0608        | -3.5767            | -5.2678         | 0.1152                     | 0.1164                     | 0.0845        | 0.4552             | 1.4657          |
| 1min                                   | -0.9261                    | -0.9679                    | 0.1351        | -3.5767            | -5.2678         | 0.0475                     | 0.0461                     | 0.0550        | 0.4552             | 1.4657          |
| 30sec                                  | -0.6351                    | -0.6782                    | -0.0201       | -3.5767            | -5.2678         | 0.0353                     | 0.0431                     | 0.0431        | 0.4552             | 1.4657          |
| 15sec                                  | -0.4573                    | -0.4994                    | 0.0249        | -3.5767            | -5.2678         | <b>0.0257</b>              | 0.0455                     | 0.0300        | 0.4552             | 1.4657          |
| High Liquidity, $\ell_0 = \log(500)$   |                            |                            |               |                    |                 |                            |                            |               |                    |                 |
| Sampling frequency                     | $\overline{\mathcal{A}}_s$ | Percentage Relative Bias   |               |                    |                 | $\overline{\mathcal{A}}_s$ | Relative RMSE              |               |                    |                 |
|                                        |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 15min                                  | -10.5643                   | -13.7643                   | 0.4408        | -4.5164            | 1.2396          | 0.2107                     | 0.2460                     | 0.1619        | 0.4386             | 1.0868          |
| 10min                                  | -7.0428                    | -8.2826                    | 0.4448        | -4.5164            | 1.2396          | 0.1556                     | 0.1731                     | 0.1362        | 0.4386             | 1.0868          |
| 5min                                   | -4.0531                    | -4.2903                    | -0.0521       | -4.5164            | 1.2396          | 0.1062                     | 0.1118                     | 0.0929        | 0.4386             | 1.0868          |
| 1min                                   | -1.0248                    | -0.9458                    | 0.0018        | -4.5164            | 1.2396          | 0.0398                     | 0.0451                     | 0.0398        | 0.4386             | 1.0868          |
| 30sec                                  | -0.5306                    | -0.4602                    | 0.0921        | -4.5164            | 1.2396          | 0.0261                     | 0.0348                     | 0.0268        | 0.4386             | 1.0868          |
| 15sec                                  | -0.4022                    | -0.3415                    | 0.0218        | -4.5164            | 1.2396          | <b>0.0196</b>              | 0.0309                     | 0.0201        | 0.4386             | 1.0868          |

**Table 15:** Integrated spot illiquidity. The table reports the Monte Carlo relative percentage bias and RMSE (both relative to  $\frac{1}{\mathcal{L}}$ ) for several illiquidity estimators: the smoothed integrated illiquidity, denoted as  $\overline{\mathcal{A}}_s = \left(\frac{1}{I} \sum_{i=1}^I \mathcal{A}_s^{-1}(\tau_i)\right)^{-1}$ , where  $\mathcal{A}_s^{-1}(\tau_i)$  is the spot Amivest based on a (equally weighted) two-sided kernel function; the filtered integrated illiquidity, denoted as  $\overline{\mathcal{A}}_f = \left(\frac{1}{I} \sum_{i=1}^I \mathcal{A}_f^{-1}(\tau_i)\right)^{-1}$ , where  $\mathcal{A}_f^{-1}(\tau_i)$  is the spot Amivest based on an (equally weighted) backward kernel function; the realized Amihud  $\mathcal{A} = \frac{RPV}{v}$ ; the daily Amihud measure, that is,  $\mathcal{A}^D = \frac{|r|}{v}$ ; the high-low Amihud,  $\mathcal{A}^{HL} = \frac{range}{v}$ , where *range* is scaled by  $\sqrt{2/\pi}$  to be comparable with the RPV. The series  $\ell(t)$  is generated according to the log Ornstein–Uhlenbeck process in (10), with parameters  $\kappa_\ell = 0.3$  and  $\eta_\ell = 0.8$ . The variances evolve following the dynamics of the CIR process, that is,  $d\sigma_j^2(t) = \kappa_\sigma(\sigma^2(t) - \sigma_0^2)dt + \eta_\sigma\sigma(t)dW_{\sigma,j}(t)$  with the parameters  $\kappa_\sigma = 0.05$ ,  $\sigma_0^2 = 6$ ,  $\eta_\sigma = 0.05$ . The spot Amivest uses a bandwidth length of one hour. The smallest RMSE is in bold.

### C.3 Setting 3

Spot liquidity,  $\ell(t)$ , evolves according to the following Ornstein–Uhlenbeck process

$$d \log \ell(t) = 0.4(\log \ell(t) - \ell_0)dt + 0.9dW_t(t), \quad t > 0. \quad (41)$$

while the variances (on the scale of percentage returns) evolve following the dynamics of the CIR process

$$d\sigma_j^2(t) = 0.05\sigma(\sigma^2(t) - 6)dt + 0.05\sigma(t)dW_{\sigma,j}(t).$$

| Low Liquidity, $\ell_0 = \log(50)$     |                            |                            |               |                    |                 |                            |                            |               |                    |                 |
|----------------------------------------|----------------------------|----------------------------|---------------|--------------------|-----------------|----------------------------|----------------------------|---------------|--------------------|-----------------|
| Sampling frequency                     | $\overline{\mathcal{A}}_s$ | Percentage Relative Bias   |               |                    |                 | $\overline{\mathcal{A}}_s$ | Relative RMSE              |               |                    |                 |
|                                        |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 15min                                  | -9.9093                    | -13.1826                   | 1.4126        | -4.3538            | 1.9622          | 0.2066                     | 0.2478                     | 0.1645        | 0.4169             | 1.1097          |
| 10min                                  | -7.1656                    | -8.6914                    | 1.0259        | -4.3538            | 1.9622          | 0.1671                     | 0.1816                     | 0.1514        | 0.4169             | 1.1097          |
| 5min                                   | -3.7826                    | -4.1840                    | 0.5438        | -4.3538            | 1.9622          | 0.1028                     | 0.1102                     | 0.0901        | 0.4169             | 1.1097          |
| 1min                                   | -1.2659                    | -1.3044                    | -0.1810       | -4.3538            | 1.9622          | 0.0433                     | 0.0502                     | 0.0364        | 0.4169             | 1.1097          |
| 30sec                                  | -0.7793                    | -0.8053                    | -0.1095       | -4.3538            | 1.9622          | 0.0323                     | 0.0431                     | 0.0323        | 0.4169             | 1.1097          |
| 15sec                                  | -0.5304                    | -0.5497                    | -0.0244       | -4.3538            | 1.9622          | 0.0223                     | 0.0365                     | <b>0.0217</b> | 0.4169             | 1.1097          |
| Medium Liquidity, $\ell_0 = \log(200)$ |                            |                            |               |                    |                 |                            |                            |               |                    |                 |
| Sampling frequency                     | $\overline{\mathcal{A}}_s$ | Percentage Relative Bias   |               |                    |                 | $\overline{\mathcal{A}}_s$ | Relative RMSE              |               |                    |                 |
|                                        |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 15min                                  | -10.1964                   | -13.4666                   | 0.7938        | -5.3177            | -1.3106         | 0.2232                     | 0.2605                     | 0.176         | 0.4218             | 1.0368          |
| 10min                                  | -7.5081                    | -8.8915                    | 0.4772        | -5.3177            | -1.3106         | 0.1667                     | 0.1801                     | 0.1258        | 0.4218             | 1.0368          |
| 5min                                   | -3.9159                    | -4.2415                    | 0.1162        | -5.3177            | -1.3106         | 0.1061                     | 0.1107                     | 0.0934        | 0.4218             | 1.0368          |
| 1min                                   | -0.9013                    | -0.8721                    | 0.2117        | -5.3177            | -1.3106         | 0.0429                     | 0.0471                     | 0.0428        | 0.4218             | 1.0368          |
| 30sec                                  | -0.5691                    | -0.5229                    | 0.1215        | -5.3177            | -1.3106         | 0.0300                     | 0.0367                     | 0.031         | 0.4218             | 1.0368          |
| 15sec                                  | -0.5428                    | -0.4938                    | -0.0907       | -5.3177            | -1.3106         | 0.0219                     | 0.0301                     | <b>0.0197</b> | 0.4218             | 1.0368          |
| High Liquidity, $\ell_0 = \log(500)$   |                            |                            |               |                    |                 |                            |                            |               |                    |                 |
| Sampling frequency                     | $\overline{\mathcal{A}}_s$ | Percentage Relative Bias   |               |                    |                 | $\overline{\mathcal{A}}_s$ | Relative RMSE              |               |                    |                 |
|                                        |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 15min                                  | -11.6163                   | -14.8600                   | -0.2372       | -3.3199            | 3.3217          | 0.2618                     | 0.3029                     | 0.2013        | 0.4355             | 1.1204          |
| 10min                                  | -8.0050                    | -9.2383                    | -0.0224       | -3.3199            | 3.3217          | 0.1954                     | 0.2085                     | 0.1455        | 0.4355             | 1.1204          |
| 5min                                   | -4.2153                    | -4.5348                    | 0.1545        | -3.3199            | 3.3217          | 0.1162                     | 0.1216                     | 0.0963        | 0.4355             | 1.1204          |
| 1min                                   | -1.0101                    | -1.0338                    | 0.0997        | -3.3199            | 3.3217          | 0.0462                     | 0.0554                     | 0.0445        | 0.4355             | 1.1204          |
| 30sec                                  | -0.6197                    | -0.6406                    | 0.0612        | -3.3199            | 3.3217          | 0.0307                     | 0.0406                     | 0.0289        | 0.4355             | 1.1204          |
| 15sec                                  | -0.5260                    | -0.5506                    | -0.0051       | -3.3199            | 3.3217          | 0.0232                     | 0.0359                     | <b>0.0201</b> | 0.4355             | 1.1204          |

**Table 16:** Integrated spot illiquidity. The table reports the Monte Carlo relative percentage bias and RMSE (both relative to  $\frac{1}{\mathcal{L}}$ ) for several illiquidity estimators: the smoothed integrated illiquidity, denoted as  $\overline{\mathcal{A}}_s = \left(\frac{1}{I} \sum_{i=1}^I \mathcal{A}_s^{-1}(\tau_i)\right)^{-1}$ , where  $\mathcal{A}_s^{-1}(\tau_i)$  is the spot Amivest based on a (equally weighted) two-sided kernel function; the filtered integrated illiquidity, denoted as  $\overline{\mathcal{A}}_f = \left(\frac{1}{I} \sum_{i=1}^I \mathcal{A}_f^{-1}(\tau_i)\right)^{-1}$ , where  $\mathcal{A}_f^{-1}(\tau_i)$  is the spot Amivest based on an (equally weighted) backward kernel function; the realized Amihud  $\mathcal{A} = \frac{RPV}{v}$ ; the daily Amihud measure, that is,  $\mathcal{A}^D = \frac{|r|}{v}$ ; the high-low Amihud,  $\mathcal{A}^{HL} = \frac{range}{v}$ , where  $range$  is scaled by  $\sqrt{2/\pi}$  to be comparable with the RPV. The series  $\ell(t)$  is generated according to the log Ornstein–Uhlenbeck process in (10), with parameters  $\kappa_\ell = 0.4$  and  $\eta_\ell = 0.9$ . The variances evolve following the dynamics of the CIR process, that is,  $d\sigma_j^2(t) = \kappa_\sigma(\sigma^2(t) - \sigma_0^2)dt + \eta_\sigma\sigma(t)dW_{\sigma,j}(t)$  with the parameters  $\kappa_\sigma = 0.05$ ,  $\sigma_0^2 = 6$ ,  $\eta_\sigma = 0.05$ . The spot Amivest uses a bandwidth length of one hour. The smallest RMSE is in bold.

## C.4 Setting 4

Spot liquidity,  $\ell(t)$ , evolves according to the following Ornstein–Uhlenbeck process

$$d \log \ell(t) = 0.5(\log \ell(t) - \ell_0)dt + 0.95dW_t(t), \quad t > 0. \quad (42)$$

while the variances (on the scale of percentage returns) evolve following the dynamics of the CIR process

$$d\sigma_j^2(t) = 0.05\sigma(\sigma^2(t) - 6)dt + 0.05\sigma(t)dW_{\sigma,j}(t).$$

| Low Liquidity, $\ell_0 = \log(50)$     |                            |                            |               |                    |                 |                            |                            |               |                    |                 |
|----------------------------------------|----------------------------|----------------------------|---------------|--------------------|-----------------|----------------------------|----------------------------|---------------|--------------------|-----------------|
| Sampling frequency                     | $\overline{\mathcal{A}}_s$ | Percentage Relative Bias   |               |                    |                 | $\overline{\mathcal{A}}_s$ | Relative RMSE              |               |                    |                 |
|                                        |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 15min                                  | -11.1210                   | -14.3058                   | 0.3738        | -6.2492            | -3.4513         | 0.2372                     | 0.2707                     | 0.1694        | 0.3779             | 0.9756          |
| 10min                                  | -7.4584                    | -8.9105                    | 0.4397        | -6.2492            | -3.4513         | 0.1726                     | 0.1873                     | 0.1291        | 0.3779             | 0.9756          |
| 5min                                   | -3.9760                    | -4.3026                    | 0.2290        | -6.2492            | -3.4513         | 0.0992                     | 0.1068                     | 0.0881        | 0.3779             | 0.9756          |
| 1min                                   | -1.0419                    | -1.0411                    | 0.1006        | -6.2492            | -3.4513         | 0.0420                     | 0.0520                     | 0.0389        | 0.3779             | 0.9756          |
| 30sec                                  | -0.7074                    | -0.6916                    | -0.0765       | -6.2492            | -3.4513         | 0.0293                     | 0.0405                     | 0.0292        | 0.3779             | 0.9756          |
| 15sec                                  | -0.5029                    | -0.5013                    | -0.0402       | -6.2492            | -3.4513         | 0.0215                     | 0.0357                     | <b>0.0203</b> | 0.3779             | 0.9756          |
| Medium Liquidity, $\ell_0 = \log(200)$ |                            |                            |               |                    |                 |                            |                            |               |                    |                 |
| Sampling frequency                     | $\overline{\mathcal{A}}_s$ | Percentage Relative Bias   |               |                    |                 | $\overline{\mathcal{A}}_s$ | Relative RMSE              |               |                    |                 |
|                                        |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 15min                                  | -10.3185                   | -13.5161                   | 1.2050        | -4.5914            | 1.1992          | 0.2193                     | 0.2542                     | 0.1678        | 0.4527             | 1.2352          |
| 10min                                  | -7.4065                    | -8.8283                    | 0.7228        | -4.5914            | 1.1992          | 0.1540                     | 0.1713                     | 0.1226        | 0.4527             | 1.2352          |
| 5min                                   | -3.7758                    | -4.1055                    | 0.4690        | -4.5914            | 1.1992          | 0.1007                     | 0.1047                     | 0.1067        | 0.4527             | 1.2352          |
| 1min                                   | -0.8839                    | -0.8802                    | 0.1304        | -4.5914            | 1.1992          | 0.0415                     | 0.0474                     | 0.0385        | 0.4527             | 1.2352          |
| 30sec                                  | -0.5094                    | -0.5047                    | 0.1075        | -4.5914            | 1.1992          | 0.0278                     | 0.0368                     | 0.0290        | 0.4527             | 1.2352          |
| 15sec                                  | -0.3054                    | -0.3026                    | 0.1861        | -4.5914            | 1.1992          | <b>0.0211</b>              | 0.0323                     | 0.0242        | 0.4527             | 1.2352          |
| High Liquidity, $\ell_0 = \log(500)$   |                            |                            |               |                    |                 |                            |                            |               |                    |                 |
| Sampling frequency                     | $\overline{\mathcal{A}}_s$ | Percentage Relative Bias   |               |                    |                 | $\overline{\mathcal{A}}_s$ | Relative RMSE              |               |                    |                 |
|                                        |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 15min                                  | -10.0610                   | -13.3596                   | 0.9613        | -4.6665            | -1.6260         | 0.2050                     | 0.2445                     | 0.1640        | 0.4081             | 1.0814          |
| 10min                                  | -7.2696                    | -8.7110                    | 0.4951        | -4.6665            | -1.6260         | 0.1714                     | 0.1910                     | 0.1334        | 0.4081             | 1.0814          |
| 5min                                   | -3.7378                    | -4.0772                    | 0.3036        | -4.6665            | -1.6260         | 0.1018                     | 0.1093                     | 0.0941        | 0.4081             | 1.0814          |
| 1min                                   | -1.1144                    | -1.1042                    | 0.0130        | -4.6665            | -1.6260         | 0.0458                     | 0.0516                     | 0.0511        | 0.4081             | 1.0814          |
| 30sec                                  | -0.7596                    | -0.7455                    | -0.0491       | -4.6665            | -1.6260         | 0.0299                     | 0.0388                     | 0.0334        | 0.4081             | 1.0814          |
| 15sec                                  | -0.4989                    | -0.4864                    | 0.0428        | -4.6665            | -1.6260         | <b>0.0210</b>              | 0.0340                     | 0.0211        | 0.4081             | 1.0814          |

**Table 17:** Integrated spot illiquidity. The table reports the Monte Carlo relative percentage bias and RMSE (both relative to  $\frac{1}{\ell}$ ) for several illiquidity estimators: the smoothed integrated illiquidity, denoted as  $\overline{\mathcal{A}}_s = \left(\frac{1}{I} \sum_{i=1}^I \mathcal{A}_s^{-1}(\tau_i)\right)^{-1}$ , where  $\mathcal{A}_s^{-1}(\tau_i)$  is the spot Amivest based on a (equally weighted) two-sided kernel function; the filtered integrated illiquidity, denoted as  $\overline{\mathcal{A}}_f = \left(\frac{1}{I} \sum_{i=1}^I \mathcal{A}_f^{-1}(\tau_i)\right)^{-1}$ , where  $\mathcal{A}_f^{-1}(\tau_i)$  is the spot Amivest based on an (equally weighted) backward kernel function; the realized Amihud  $\mathcal{A} = \frac{RPV}{v}$ ; the daily Amihud measure, that is,  $\mathcal{A}^D = \frac{|r|}{v}$ ; the high-low Amihud,  $\mathcal{A}^{HL} = \frac{range}{v}$ , where  $range$  is scaled by  $\sqrt{2/\pi}$  to be comparable with the RPV. The series  $\ell(t)$  is generated according to the log Ornstein–Uhlenbeck process in (10), with parameters  $\kappa_\ell = 0.5$  and  $\eta_\ell = 0.95$ . The variances evolve following the dynamics of the CIR process, that is,  $d\sigma_j^2(t) = \kappa_\sigma(\sigma^2(t) - \sigma_0^2)dt + \eta_\sigma\sigma(t)dW_{\sigma,j}(t)$  with the parameters  $\kappa_\sigma = 0.05$ ,  $\sigma_0^2 = 6$ ,  $\eta_\sigma = 0.05$ . The spot Amivest uses a bandwidth length of one hour. The smallest RMSE is in bold.

## C.5 Monthly Illiquidity

Spot liquidity,  $\ell(t)$ , evolves according to the following Ornstein–Uhlenbeck process

$$d \log \ell(t) = 0.5(\log \ell(t) - \ell_0)dt + 0.95dW_t(t), \quad t > 0. \quad (43)$$

while the variances (on the scale of percentage returns) evolve following the dynamics of the CIR process

$$d\sigma_j^2(t) = 0.05\sigma(\sigma^2(t) - 6)dt + 0.05\sigma(t)dW_{\sigma,j}(t).$$

| Low Liquidity, $\ell_0 = \log(50)$     |                            |                            |               |                    |                 |                            |                            |               |                    |                 |
|----------------------------------------|----------------------------|----------------------------|---------------|--------------------|-----------------|----------------------------|----------------------------|---------------|--------------------|-----------------|
| Sampling frequency                     | $\overline{\mathcal{A}}_s$ | Percentage Relative Bias   |               |                    |                 | $\overline{\mathcal{A}}_s$ | Relative RMSE              |               |                    |                 |
|                                        |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 15min                                  | -10.5724                   | -13.8903                   | 0.6419        | -2.7094            | 4.4172          | 0.1490                     | 0.1925                     | 0.0355        | 0.1174             | 0.3227          |
| 10min                                  | -6.9913                    | -8.4575                    | 0.7600        | -3.1010            | 3.8676          | 0.1002                     | 0.1194                     | 0.0341        | 0.1219             | 0.3151          |
| 5min                                   | -3.5485                    | -3.9353                    | 0.5628        | -3.0956            | 4.0122          | 0.0555                     | 0.0603                     | 0.0261        | 0.1172             | 0.3120          |
| 1min                                   | -0.7657                    | -0.7905                    | 0.1891        | -3.3569            | 3.6409          | 0.0140                     | 0.0146                     | 0.0107        | 0.1202             | 0.3097          |
| 30sec                                  | -0.4444                    | -0.4588                    | 0.1246        | -3.4315            | 3.5718          | 0.0091                     | 0.0096                     | 0.0087        | 0.1190             | 0.3074          |
| 15sec                                  | -0.2637                    | -0.2764                    | 0.1208        | -3.4864            | 3.5201          | 0.0062                     | 0.0073                     | <b>0.0056</b> | 0.1178             | 0.3052          |
| Medium Liquidity, $\ell_0 = \log(200)$ |                            |                            |               |                    |                 |                            |                            |               |                    |                 |
| Sampling frequency                     | $\overline{\mathcal{A}}_s$ | Percentage Relative Bias   |               |                    |                 | $\overline{\mathcal{A}}_s$ | Relative RMSE              |               |                    |                 |
|                                        |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 15min                                  | -10.1277                   | -13.4621                   | 1.0054        | -3.5146            | 2.0781          | 0.1361                     | 0.1797                     | 0.0483        | 0.1384             | 0.2938          |
| 10min                                  | -6.8135                    | -8.2566                    | 0.9884        | -3.5146            | 2.0781          | 0.0953                     | 0.1140                     | 0.0348        | 0.1384             | 0.2938          |
| 5min                                   | -3.3891                    | -3.7382                    | 0.7197        | -3.5146            | 2.0781          | 0.0509                     | 0.0552                     | 0.0206        | 0.1384             | 0.2938          |
| 1min                                   | -0.7459                    | -0.7301                    | 0.1677        | -3.5146            | 2.0781          | 0.0153                     | 0.0145                     | 0.0115        | 0.1384             | 0.2938          |
| 30sec                                  | -0.4202                    | -0.3958                    | 0.1204        | -3.5146            | 2.0781          | 0.0098                     | 0.0091                     | 0.0076        | 0.1384             | 0.2938          |
| 15sec                                  | -0.2735                    | -0.2536                    | 0.0425        | -3.5146            | 2.0781          | 0.0052                     | 0.0053                     | <b>0.0040</b> | 0.1384             | 0.2938          |
| High Liquidity, $\ell_0 = \log(500)$   |                            |                            |               |                    |                 |                            |                            |               |                    |                 |
| Sampling frequency                     | $\overline{\mathcal{A}}_s$ | Percentage Relative Bias   |               |                    |                 | $\overline{\mathcal{A}}_s$ | Relative RMSE              |               |                    |                 |
|                                        |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |                            | $\overline{\mathcal{A}}_f$ | $\mathcal{A}$ | $\mathcal{A}^{HL}$ | $\mathcal{A}^D$ |
| 15min                                  | -10.0334                   | -13.5023                   | 0.8992        | -4.4136            | -2.7195         | 0.1358                     | 0.1857                     | 0.0435        | 0.1246             | 0.4341          |
| 10min                                  | -7.4173                    | -8.8670                    | 0.0961        | -4.4136            | -2.7195         | 0.1074                     | 0.1283                     | 0.0276        | 0.1246             | 0.4341          |
| 5min                                   | -3.6598                    | -4.0057                    | 0.2617        | -4.4136            | -2.7195         | 0.0545                     | 0.0596                     | 0.0249        | 0.1246             | 0.4341          |
| 1min                                   | -0.9282                    | -0.9041                    | -0.0993       | -4.4136            | -2.7195         | 0.0174                     | 0.0173                     | 0.0101        | 0.1246             | 0.4341          |
| 30sec                                  | -0.5329                    | -0.4949                    | -0.0802       | -4.4136            | -2.7195         | 0.0137                     | 0.0130                     | 0.0121        | 0.1246             | 0.4341          |
| 15sec                                  | -0.3791                    | -0.3393                    | -0.0449       | -4.4136            | -2.7195         | 0.0109                     | 0.0102                     | <b>0.0094</b> | 0.1246             | 0.4341          |

**Table 18:** Monthly Integrated spot illiquidity. The table reports the Monte Carlo relative percentage bias and RMSE (both relative to  $\frac{1}{\mathcal{I}}$ ) for several illiquidity estimators aggregated at a monthly level: the smoothed integrated illiquidity, denoted as  $\overline{\mathcal{A}}_s = \left(\frac{1}{I} \sum_{i=1}^I \mathcal{A}_s^{-1}(\tau_i)\right)^{-1}$ , where  $\mathcal{A}_s^{-1}(\tau_i)$  is the spot Amivest based on a (equally weighted) two-sided kernel function; the filtered integrated illiquidity, denoted as  $\overline{\mathcal{A}}_f = \left(\frac{1}{I} \sum_{i=1}^I \mathcal{A}_f^{-1}(\tau_i)\right)^{-1}$ , where  $\mathcal{A}_f^{-1}(\tau_i)$  is the spot Amivest based on an (equally weighted) backward kernel function; the realized Amihud  $\mathcal{A} = \frac{RPV}{v}$ ; the daily Amihud measure, that is,  $\mathcal{A}^D = \frac{|r|}{v}$ ; the high-low Amihud,  $\mathcal{A}^{HL} = \frac{range}{v}$ , where  $range$  is scaled by  $\sqrt{2/\pi}$  to be comparable with the RPV. The series  $\ell(t)$  is generated according to the log Ornstein–Uhlenbeck process in (10), with parameters  $\kappa_\ell = 0.5$  and  $\eta_\ell = 0.95$ . The variances evolve following the dynamics of the CIR process, that is,  $d\sigma_j^2(t) = \kappa_\sigma(\sigma^2(t) - \sigma_0^2)dt + \eta_\sigma\sigma(t)dW_{\sigma,j}(t)$  with the parameters  $\kappa_\sigma = 0.05$ ,  $\sigma_0^2 = 6$ ,  $\eta_\sigma = 0.05$ . The spot Amivest uses a bandwidth length of one hour. The smallest RMSE is in bold.

## D Modeling Realized Amihud

In this section, we propose an econometric specification to characterize the dynamic evolution and distributional features of the realized Amihud. First, we consider parametric models belonging to the class of multiplicative error models (MEM), as introduced by Engle (2002) and Engle and Gallo (2006).

### D.1 A mixture MEM model

Following the same approach adopted by Caporin et al. (2017), it is possible to construct a MEM model featuring a component responsible for generating large and unexpected moves in illiquidity. In particular, following Caporin et al. (2017), we model  $\mathcal{A}$  according to an AMEM with a mixture specification, namely AMEM-Mix. In the AMEM-Mix,  $\mathcal{A}_t$  is given by the product of three elements:

$$\mathcal{A}_t = \mu_t Z_t \epsilon_t, \quad (44)$$

where  $\mu_t$  is the conditional mean, and it follows asymmetric MEM (AMEM) dynamics:

$$\mu_t = \omega + \alpha \mathcal{A}_{t-1} + \beta \mu_{t-1} + \gamma D_{t-1} \mathcal{A}_{t-1}, \quad (45)$$

where  $D_{t-1}$  is a dummy variable taking a value of 1 if the return is negative and 0 otherwise; it accounts for an asymmetric response of illiquidity to good or bad news. The term  $\epsilon_t$  denotes the innovation term, whose density (conditional on the information set  $\mathcal{F}_{t-1}$ ) is

$$f(\epsilon_t | \mathcal{F}_{t-1}) = \frac{1}{\Gamma(\vartheta)} \vartheta^\vartheta \epsilon_t^{\vartheta-1} e^{-\vartheta \epsilon_t}, \quad \epsilon_t > 0, \quad \vartheta > 0. \quad (46)$$

This means that  $\epsilon_t | \mathcal{F}_{t-1}$  is a gamma-distributed random variable with unit mean and variance equal to  $\frac{1}{\vartheta}$ . Finally, the mixing term,  $Z_t$ , is assumed to be independent of  $\epsilon_t$  and distributed as a compound Poisson random variable

$$Z_t = \begin{cases} d_\kappa & N_t = 0, \\ \sum_{j=1}^{N_t} Y_{j,t} & N_t > 0, \end{cases} \quad (47)$$

where the expected number of arrivals at time  $t$  ( $N_t$ ) is governed by a Poisson random variable with a time-varying intensity,  $\kappa_t$ , and  $d_\kappa$  is a positive function of  $\kappa_t$  such that  $E[Z_t \epsilon_t | \mathcal{F}_{t-1}] = 1$  and  $E(\mathcal{A} | \mathcal{F}_{t-1}) = \mu_t$ . Furthermore,  $Y_{j,t} | \mathcal{F}_{t-1} \sim \Gamma(d_\kappa, \zeta)$ . By the Poisson distribution, the probability of observing a  $j \geq 0$ , conditioning  $\mathcal{F}_{t-1}$ , is

$$Pr(N_t = j | \mathcal{F}_{t-1}) = \frac{e^{-\kappa_t} \kappa_t^j}{j!}, \quad j = 0, 1, 2, \dots \quad (48)$$

Similar to Chan and Maheu (2002), we specify the dynamics of  $\kappa_t$  as

$$\kappa_t = \phi_1 + \phi_2(\kappa_{t-1} - \phi_1) + \phi_3 \xi_{t-1}, \quad (49)$$

where the innovations,  $\xi_t$ , are defined as the error in predicting  $N_t$  as new information becomes available in  $\mathcal{F}_t$ . In other words,  $\xi_t = E(N_t|\mathcal{F}_t) - E(N_t|\mathcal{F}_{t-1})$ , and in terms of model parameters

$$\xi_t = \sum_{j=0}^{\infty} j \Pr(N_t = j|\mathcal{F}_t) - \kappa_t. \quad (50)$$

The conditions  $\phi_1 > 0$  and  $1 > \phi_2 > \phi_3 > 0$  are sufficient to ensure positiveness and stationarity of  $\kappa_t$ , see [Chan and Maheu \(2002\)](#). A similar dynamics for the intensity has been adopted by [Maheu and McCurdy \(2004\)](#) and [Maheu et al. \(2013\)](#) for stock returns and by [Caporin et al. \(2016\)](#) for the realized variance. Finally, the filtered probability needed to compute  $E(N_t|\mathcal{F}_t)$  is obtained via Bayes' rule, as follows:

$$\Pr(N_t = j|\mathcal{F}_t) = \frac{f(\mathcal{A}_t|N_t = j, \mathcal{F}_{t-1})\Pr(N_t = j|\mathcal{F}_{t-1})}{f(\mathcal{A}_t|\mathcal{F}_{t-1})}, \quad (51)$$

where the density of  $\mathcal{A}_t$  conditional on  $N_t = j$  and  $\mathcal{F}_{t-1}$  is

$$f(\mathcal{A}_t|N_t = j, \mathcal{F}_{t-1}) = \begin{cases} \frac{1}{\mathcal{A}_t} \left( \frac{\partial \mathcal{A}_t}{\partial \kappa \mu_t} \right)^{\vartheta} \frac{e^{\left( \frac{-\partial \mathcal{A}_t}{\partial \kappa \mu_t} \right)}}{\Gamma(\vartheta)}, & N_t = 0 \\ \frac{2}{\mathcal{A}_t} \left( \frac{\mathcal{A}_t}{\mu_t} \frac{\partial \zeta}{\partial \kappa} \right) \left( \frac{j\zeta + \vartheta}{2} \right) \frac{1}{\Gamma(j\zeta)\Gamma(\vartheta)} \mathbb{K}_{j\zeta - \vartheta} \left( 2\sqrt{\frac{\mathcal{A}_t}{\mu_t} \frac{\partial \zeta}{\partial \kappa}} \right), & N_t = j > 0, \end{cases} \quad (52)$$

where  $\mathbb{K}(\cdot)$  is the modified Bessel function of a second kind, while the denominator in (51) is given by the following:

$$f(\mathcal{A}_t|\mathcal{F}_{t-1}) = e^{-\kappa_t} g_{\mathcal{A}_t} + \sum_{j=1}^{\infty} \frac{e^{-\kappa_t} \kappa_t^j}{j!} w_{\mathcal{A}_t}, \quad (53)$$

where  $g_{\mathcal{A}_t}$  and  $w_{\mathcal{A}_t}$  are  $f(\mathcal{A}_t|N_t = j, \mathcal{F}_{t-1}, \alpha, \beta, \gamma)$  for  $j = 0$  and  $j > 0$ , respectively. Therefore, the conditional density of  $\mathcal{A}_t$  is a mixture of gamma and kappa distributions, whose weights are governed by the Poisson probabilities and by  $\kappa_t$ . The density is available in closed form, thus allowing us to estimate the model parameters by maximum likelihood. In the following analysis, we distinguish between the AMEM-Mix model with time-varying Poisson intensity (i.e., AMEM-Mix $_{\kappa_t}$ ) and with constant intensity (i.e., AMEM-Mix $_{\kappa}$ ), that is,  $\kappa_t = \phi_1$ .

## D.2 Estimation results

Table 19 reports the parameter estimates (and goodness of fit tests) for several AMEM and AMEM-Mix specifications estimated on daily and realized Amihud series of SPY, where the realized Amihud series is computed using returns sampled at 5- and 10-minute frequencies. Panel a) of Table 6 shows the coefficients from the specifications under consideration, which are all estimated using maximum likelihood. As suggested by [Engle and Gallo \(2006\)](#), we allow  $\mu_t$  to follow asymmetric GARCH-type or asymmetric HAR-type process with asymmetric response to negative returns. Panel b) of Table 6 displays the results of a number of diagnostics tests on the residuals and Poisson intensity innovations, that is, the  $p$ -values of the Ljung-Box test (at 1, 5, and 10, lags). Table 20 reports the parameter estimates of the linear and log-linear models. Looking at the estimates of the AMEM models in Table 19, we notice that the parameter  $\alpha$  in the volatility literature is generally found in the range between 0.02

and 0.1 (Bauwens et al., 2012, ch. 1), while we find values in the range between 0.175 and 0.217 for the realized Amihud, signaling that illiquidity is more responsive to news than volatility. Instead, when the AMEM model is estimated on the daily Amihud, the estimates of  $\alpha$  are significantly reduced because the daily Amihud is a more noisy proxy of the signal compared with the realized Amihud, so the model assigns a smaller weight to the parameter governing the news. Analogous evidence is found in the volatility literature when employing the realized GARCH model of Hansen et al. (2012) rather than a classic GARCH model on squared returns.

As for the parameter  $\gamma$ , it enters the model with the expected positive sign and is significant across all models, pointing at a more pronounced reaction of illiquidity against negative returns rather than against positive returns. Altogether, by adopting the general view that uncertainty arises when new information reaches the market, we can state that illiquidity is strongly related to uncertainty among investors (as measured by volatility) and that this is responsible for the illiquidity persistence. Conversely, the estimates of the parameter  $\beta$  is around 0.78 across models. This is in the lower bound of the interval of estimates of  $\beta$  typically found for volatility (according to Bauwens et al., 2012, ch. 1, it is usually close to the upper limit of the interval 0.75–0.98). Finally, it is important to emphasize the behavior of the coefficient  $\vartheta$  in the various AMEM and AMEM-Mix specifications, whose reciprocal is an estimate of the residuals variance. Notably,  $\vartheta$  increases when moving from the simplest AMEM specifications to the sophisticated AMEM-Mix. Indeed, in the models with the mixtures, a significant part of the variability of the illiquidity is explained by mixture component  $Z_t$ . This finding testifies the need for a model that can assign the correct probability to large realizations of the illiquidity measure. As expected, the value of the log-likelihood function is larger for the AMEM-Mix specifications when compared with then simpler AMEM.

As for the Poisson intensity, the process  $\kappa_t$  is very persistent ( $\phi_2$  is around 0.998), signaling that the number of arrivals strongly depend on its past realizations.

| Panel a)                              | Daily Amihud                    |                                 |                                  | High-Low Amihud                  |                                  |                                  | Realized Amihud - 5 minutes      |                                  |                                   | Realized Amihud - 10 minutes     |                                  |                                  |
|---------------------------------------|---------------------------------|---------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|-----------------------------------|----------------------------------|----------------------------------|----------------------------------|
|                                       | AMEM - HAR                      |                                 |                                  | AMEM(1,1)                        |                                  |                                  | AMEM - HAR                       |                                  |                                   | AMEM(1,1)                        |                                  |                                  |
|                                       | AMEM                            | HAR                             | AMEM(1,1)                        | AMEM                             | HAR                              | AMEM(1,1)                        | AMEM(2,1)                        | AMEM(2,1) - Mix <sub>K</sub>     | AMEM - HAR                        | AMEM(2,1)                        | AMEM(2,1) - Mix <sub>K</sub>     | AMEM(2,1) - Mix <sub>K</sub>     |
| $\omega$                              | 0.0160 <sup>a</sup><br>(0.0022) | 0.0008 <sup>a</sup><br>(0.0003) | 0.0006 <sup>a</sup><br>(0.0002)  | 0.0052 <sup>a</sup><br>(0.0011)  | 0.0006 <sup>a</sup><br>(0.0002)  | 0.0008 <sup>a</sup><br>(0.0002)  | 0.0008 <sup>a</sup><br>(0.0002)  | 0.0008 <sup>a</sup><br>(0.0002)  | 0.0031 <sup>a</sup><br>(0.0007)   | 0.0008 <sup>a</sup><br>(0.0002)  | 0.0008 <sup>a</sup><br>(0.0002)  | 0.0007 <sup>a</sup><br>(0.0002)  |
| $\alpha_4$                            | 0.0000<br>(0.0234)              | 0.0176 <sup>a</sup><br>(0.0047) | 0.0759 <sup>a</sup><br>(0.0082)  | 0.0000<br>(0.0227)               | 0.0759 <sup>a</sup><br>(0.0082)  | 0.1752 <sup>a</sup><br>(0.0193)  | 0.2070 <sup>a</sup><br>(0.0180)  | 0.1978 <sup>a</sup><br>(0.0182)  | 0.1252 <sup>a</sup><br>(0.0225)   | 0.1593 <sup>a</sup><br>(0.0183)  | 0.1851 <sup>a</sup><br>(0.0165)  | 0.1848 <sup>a</sup><br>(0.0162)  |
| $\alpha_w$                            | 0.0011<br>(0.0494)              | -                               | -                                | 0.3146 <sup>a</sup><br>(0.0476)  | -                                | -                                | -                                | -                                | 0.3394 <sup>a</sup><br>(0.0417)   | -                                | -                                | -                                |
| $\alpha_m$                            | 0.6595 <sup>a</sup><br>(0.0661) | -                               | -                                | 0.5549 <sup>a</sup><br>(0.0466)  | -                                | -                                | -                                | -                                | 0.4432 <sup>a</sup><br>(0.0388)   | -                                | -                                | -                                |
| $\beta_1$                             | -                               | 0.9490 <sup>a</sup><br>(0.0100) | 0.8938 <sup>a</sup><br>(0.0107)  | -                                | -                                | 0.7962 <sup>a</sup><br>(0.0219)  | 0.4494 <sup>a</sup><br>(0.0803)  | 0.4287 <sup>a</sup><br>(0.0842)  | -                                 | 0.8104 <sup>a</sup><br>(0.0213)  | 0.3958 <sup>a</sup><br>(0.0715)  | 0.3958 <sup>a</sup><br>(0.0709)  |
| $\beta_2$                             | -                               | -                               | -                                | -                                | -                                | -                                | 0.3103 <sup>a</sup><br>(0.0746)  | 0.3393 <sup>a</sup><br>(0.0779)  | -                                 | -                                | 0.3588 <sup>a</sup><br>(0.0673)  | 0.3588 <sup>a</sup><br>(0.0668)  |
| $\gamma$                              | 0.0297<br>(0.0235)              | 0.0406 <sup>a</sup><br>(0.0101) | 0.0392 <sup>a</sup><br>(0.0053)  | 0.0546 <sup>a</sup><br>(0.0103)  | 0.0392 <sup>a</sup><br>(0.0053)  | 0.0259 <sup>a</sup><br>(0.0042)  | 0.0329 <sup>a</sup><br>(0.0051)  | 0.0352 <sup>a</sup><br>(0.0051)  | 0.0501 <sup>a</sup><br>(0.0072)   | 0.0299 <sup>a</sup><br>(0.0044)  | 0.0413 <sup>a</sup><br>(0.0053)  | 0.0408 <sup>a</sup><br>(0.0053)  |
| $\theta$                              | 1.4608 <sup>a</sup><br>(0.0361) | 1.4737 <sup>a</sup><br>(0.0364) | 12.1911 <sup>a</sup><br>(0.3146) | 12.0371 <sup>a</sup><br>(0.3188) | 12.1911 <sup>a</sup><br>(0.3146) | 24.2300 <sup>a</sup><br>(0.4910) | 24.4870 <sup>a</sup><br>(0.5108) | 26.1708 <sup>a</sup><br>(0.7786) | 21.91366 <sup>a</sup><br>(0.5023) | 21.4118 <sup>a</sup><br>(0.5299) | 22.9429 <sup>a</sup><br>(0.5672) | 24.3646 <sup>a</sup><br>(0.5847) |
| $\zeta$                               | -                               | -                               | -                                | -                                | -                                | -                                | -                                | 9.2630 <sup>a</sup><br>(1.0397)  | -                                 | -                                | 7.3968 <sup>a</sup><br>(0.9656)  | 7.7027 <sup>a</sup><br>(0.8691)  |
| $\phi_1$                              | -                               | -                               | -                                | -                                | -                                | -                                | -                                | 0.0242 <sup>b</sup><br>(0.0109)  | -                                 | -                                | 0.0186 <sup>a</sup><br>(0.0071)  | 0.0186 <sup>b</sup><br>(0.0085)  |
| $\phi_2$                              | -                               | -                               | -                                | -                                | -                                | -                                | -                                | -                                | -                                 | -                                | -                                | 0.9986 <sup>a</sup><br>(0.0008)  |
| $\phi_3$                              | -                               | -                               | -                                | -                                | -                                | -                                | -                                | 0.0215 <sup>a</sup><br>(0.0046)  | -                                 | -                                | -                                | 0.0248 <sup>a</sup><br>(0.0072)  |
| LogLik                                | 6465.90                         | 6517.19                         | 9080.08                          | 9007.54                          | 9080.08                          | 10296.32                         | 10308.27                         | 10318.59                         | 10114.51                          | 10134.01                         | 10150.41                         | 10159.28                         |
| LRI                                   | -0.0083                         | -0.0163                         | -0.0920                          | -0.0833                          | -0.0920                          | -0.1528                          | -0.1541                          | -0.1553                          | -0.1385                           | -0.1407                          | -0.1425                          | -0.1435                          |
| Ljung-Box statistics (p-value)        |                                 |                                 |                                  |                                  |                                  |                                  |                                  |                                  |                                   |                                  |                                  |                                  |
| Residuals: $\epsilon_t$               |                                 |                                 |                                  |                                  |                                  |                                  |                                  |                                  |                                   |                                  |                                  |                                  |
| LB 1                                  | 0.0045                          | 0.0031                          | 0.5652                           | 0.2788                           | 0.5652                           | 0.0007                           | 0.2836                           | 0.2580                           | 0.6598                            | 0.0001                           | 0.2147                           | 0.1231                           |
| LB 5                                  | 0.0453                          | 0.0781                          | 0.0698                           | 0.02132                          | 0.0698                           | 0.0034                           | 0.0835                           | 0.0603                           | 0.2707                            | 0.0013                           | 0.1114                           | 0.0706                           |
| LB 10                                 | 0.0714                          | 0.0650                          | 0.0407                           | 0.0980                           | 0.0407                           | 0.0000                           | 0.0000                           | 0.0000                           | 0.0001                            | 0.0000                           | 0.0000                           | 0.0000                           |
| Filtered Poisson innovations: $\xi_t$ |                                 |                                 |                                  |                                  |                                  |                                  |                                  |                                  |                                   |                                  |                                  |                                  |
| LB 1                                  | -                               | -                               | -                                | -                                | -                                | -                                | -                                | 0.1184                           | -                                 | -                                | -                                | 0.3115                           |
| LB 5                                  | -                               | -                               | -                                | -                                | -                                | -                                | -                                | 0.0079                           | -                                 | -                                | -                                | 0.0070                           |
| LB 10                                 | -                               | -                               | -                                | -                                | -                                | -                                | -                                | 0.0362                           | -                                 | -                                | -                                | 0.0678                           |

**Table 19:** AMEM and mixture-AMEM estimated coefficients with robust standard errors (White, 1980) and  $p$ -value of the Ljung-Box statistics. Superscript  $a$ ,  $b$  and  $c$  denote the 1%, 5%, and 10% significance levels, respectively. For the AMEM-Mix specification, we consider the model with constant jump intensity (AMEM - Mix<sub>K</sub>), i.e.,  $\phi_2 = \phi_3 = 0$  and with dynamic jump intensity (AMEM - Mix<sub>K<sub>t</sub></sub>). Sample period: January 3, 2006 - June 29, 2018.

### D.3 Linear specifications

We also consider linear specifications based on the HAR model of Corsi (2009). In particular, the linear asymmetric HAR (linear AHAR) model is given by

$$\mathcal{A}_t = \omega + \alpha_d \mathcal{A}_{t-1} + \alpha_w \bar{\mathcal{A}}_{w,t-1} + \alpha_m \bar{\mathcal{A}}_{m,t-1} + \gamma D_{t-1} \mathcal{A}_{t-1} + \varepsilon_t, \quad (54)$$

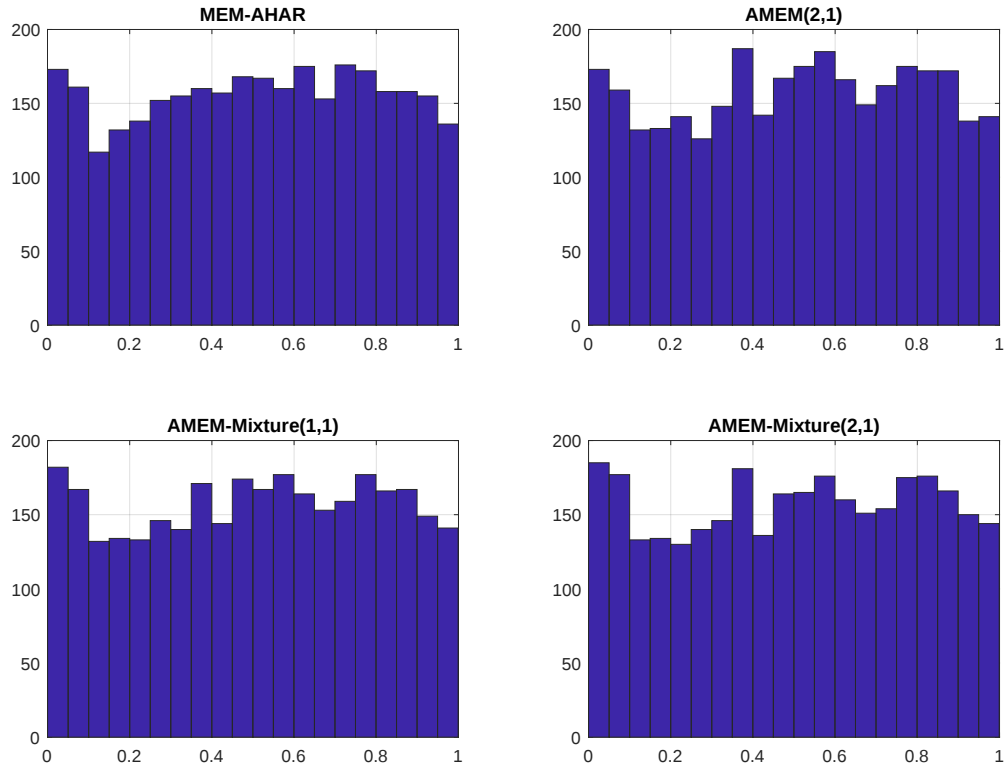
and the log-linear AHAR (log-linear AHAR) specification given by

$$\log \mathcal{A}_t = \omega + \alpha_d \log \mathcal{A}_{t-1} + \alpha_w \log \bar{\mathcal{A}}_{w,t-1} + \alpha_m \log \bar{\mathcal{A}}_{m,t-1} + \gamma D_{t-1} \log \mathcal{A}_{t-1} + \varepsilon_t, \quad (55)$$

| Panel a)   | Estimation results               |                                  |                                  |                                 |                                  |                                    |
|------------|----------------------------------|----------------------------------|----------------------------------|---------------------------------|----------------------------------|------------------------------------|
|            | Daily Amihud                     |                                  | High-Low Amihud                  |                                 | Realized Amihud                  |                                    |
|            | Loglinear AHAR                   | Linear AHAR                      | Loglinear AHAR                   | Linear AHAR                     | Loglinear AHAR                   | Linear AHAR                        |
| $\omega$   | -1.6182 <sup>a</sup><br>(0.2231) | 0.0148 <sup>a</sup><br>(0.0025)  | -0.3237 <sup>a</sup><br>(0.0663) | 0.0057 <sup>a</sup><br>(0.0015) | -0.2024 <sup>a</sup><br>(0.0439) | 0.0027 <sup>a</sup><br>(0.0006)    |
| $\alpha_d$ | -0.0275<br>(0.0227)              | -0.0838 <sup>a</sup><br>(0.0283) | 0.0304<br>(0.0207)               | -0.0152<br>(0.0223)             | 0.1448 <sup>a</sup><br>(0.0259)  | 0.1171 <sup>a</sup><br>(0.0261)    |
| $\alpha_w$ | 0.0406<br>(0.0542)               | 0.0556<br>(0.0575)               | 0.3313 <sup>a</sup><br>(0.0526)  | 0.3534 <sup>a</sup><br>(0.0576) | 0.3711 <sup>a</sup><br>(0.0568)  | 0.4208 <sup>a</sup> \$<br>(0.0730) |
| $\alpha_m$ | 0.5244 <sup>a</sup><br>(0.0845)  | 0.7017 <sup>a</sup><br>(0.0832)  | 0.5428 <sup>a</sup><br>(0.0496)  | 0.5190 <sup>a</sup><br>(0.0570) | 0.4253 <sup>a</sup><br>(0.0513)  | 0.3872 <sup>a</sup><br>(0.0633)    |
| $\gamma$   | -0.0259 <sup>a</sup><br>(0.0100) | 0.0561 <sup>b</sup><br>(0.0214)  | -0.0210 <sup>a</sup><br>(0.0032) | 0.0565 <sup>a</sup><br>(0.0107) | -0.0135 <sup>a</sup><br>(0.0024) | 0.0366 <sup>a</sup><br>(0.0067)    |
| <hr/>      |                                  |                                  |                                  |                                 |                                  |                                    |
| Panel b)   | Ljung-Box statistics (p-value)   |                                  |                                  |                                 |                                  |                                    |
| LB(1)      | 0.9764                           | 0.9260                           | 0.5730                           | 0.5507                          | 0.5923                           | 0.7050                             |
| LB(5)      | 0.8462                           | 0.6273                           | 0.2495                           | 0.7036                          | 0.0827                           | 0.1768                             |
| LB(10)     | 0.4676                           | 0.3781                           | 0.0970                           | 0.2749                          | 0.0000                           | 0.0000                             |

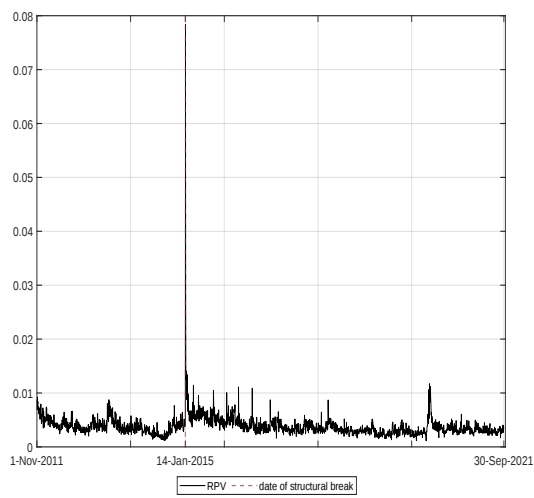
**Table 20:** AHAR estimated coefficients with robust standard errors (White, 1980) and  $p$ -value of the Ljung-Box statistics. Superscript  $a$ ,  $b$  and  $c$  denote the 1%, 5%, and 10% significance levels, respectively. Sample period: January 3, 2006 – June 29, 2018.

### D.3.1 PITs

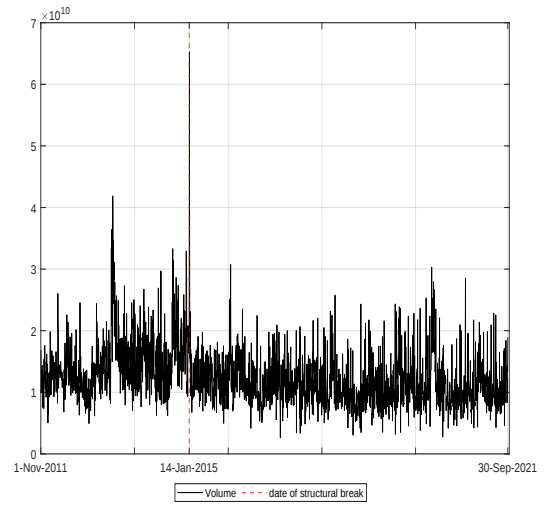


**Figure 12:** Probability integral transform (PIT) from MEM-HAR, AMEM(2,1) and mixture-AMEM . Sample period: January 3, 2006 – June 29, 2018.

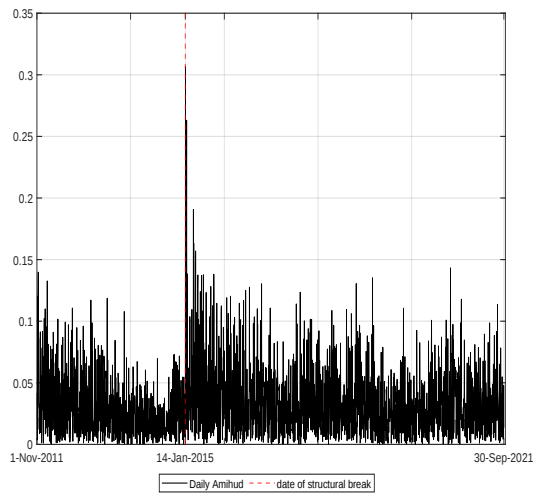
## D.4 The USD/CHF analysis



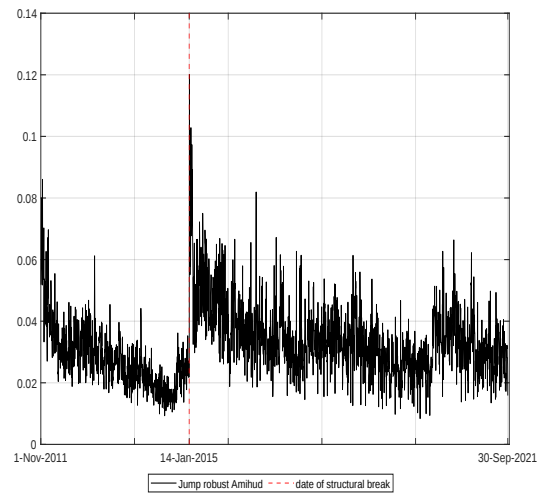
(a) RPV



(b) Volume



(c) Daily Amihud



(d) Realized Amihud

**Figure 13:** Break dates. The figures report the time series of RPV, volume, daily Amihud, and realized Amihud for the USD/CHF rate (black solid line). The red vertical line denotes the break date, which is estimated by means of the [Bai and Perron \(1998\)](#) test for unknown break points. The test is performed with 15% trimming at the beginning and the end of the sample period and between break dates.