Online Price Discrimination: Online sellers.
- Preferential treatment.
- They claim that these tags can be used to improve their services, such as accurately recommending the goods.

Regulations on Data Protection:
- General Data Protection Regulation (GDPR), EU, 2018: Grants consumers the right to rectify their data. (Art.16)
- Grants consumers the right to erase their data. (Art.17)
- The Right to be Forgotten.
- Internet Information Service Algorithmic Recommendation Management Provisions, China, 2022: Allows consumers to delete their tags after registration. Once allows consumers to change their tags after registration.

Research Questions:
- A monopolist producer can charge different prices in different markets that are divided by tags. [3rd Price Discrimination]
- Strategic consumers can manipulate their tags in an online environment.
  - Non-cooperative GT + Collision → Stable Market Segmentation
- The producer sets an optimal price in each market.
  - Can consumers fight against a monopolist producer?
  - Is it necessary for consumers to stand united?
- What are the market equilibria like? (Preliminary)
- What are the possible welfare consequences? (Core)
  - The limits of price discrimination (BBM2015@AER)
  - Non-cooperative GT + Collusion
  - Uniform Monopoly
  - Strategic Discrimination
- Further Analysis
  - Robustness:
    - Relaxing marginal pricing rule, i.e. any ex-post rational pricing rule.
    - Producer surplus is fixed at the uniform monopoly level.
    - Consumer surplus is at least at the uniform monopoly level (point A) but the upper bound may shrink.
    - No consumer is worse off compared with uniform monopoly.
  - Policy Implications
    - Release prohibitions on price discrimination.
    - Enabling price discrimination is Pareto-improving.
    - Empower consumers with more freedom to edit their tags.
    - Free circulation is desirable.
    - The Right to be partially Forgotten should be mandated.
    - First-degree price discrimination outcome is stable if only the Right to be entirely Forgotten is enforced.
    - Promoting frictionless second-hand markets may be harmful.
    - Prevent ex-post arbitrages, which originally benefits consumers.
    - Data brokers help solve the equilibrium selection problem.
    - A mediator is helpful in selecting the best equilibrium.

Basic model: The producer sells homogeneous products to a continuum of consumers.
- Homogeneous products.
- Unit demand.
- Consumers’ reservation price can take values from a finite set \( \{v_1, v_2, \ldots, v_K\} \) with \( 0 < v_1 < v_2 < \cdots < v_K \).
- Constant marginal cost, which can be normalized to zero.
- The producer and all consumers can learn value distributions in each market.

Market Segmentation:
- A market (segment) can be represented by a vector \( x = (x_1, \ldots, x_K) \), where \( x_k \geq 0 \) is the proportion of consumers with reservation price \( v_k \).
- There is an aggregate market \( x^* = (x_1^*, \ldots, x_K^*) \), where \( |x^*| \) is normalized to 1.
- A segmentation of the aggregate market, denoted by \( \sigma(x^*) \), is a collection (possibly not a set) of segments \( \{x_1, \ldots, x_l\} \) such that \( \sigma(x^*) = x^* \).

Pricing & Surplus (Baseline):
- Third-degree Price Discrimination (i) In each market segment, the producer offers a take-it-or-leave-it price. (ii) Each consumer will buy the product if the price is NO LARGER THAN his reservation price.
- The price \( v_i \) is optimal for a given market \( x \) iff \( v_i \geq v_j \) for all \( j \neq i \).

Welfare Consequences
- The surplus of the producer and consumers (\( \pi, u \)) can be achieved by a stable segmentation iff \( \pi = \pi^* \) and \( u \in [\omega^*, \omega^* - \pi^*] \).

Model
- If there are multiple optimal prices in one segment, the producer will take the minimum optimal one, which favors consumers the most. \( \phi^{\text{min}}(x) \) denotes the minimum optimal price for market segment \( x \).
- The surplus of producer is defined as \( \pi(x) = \sum_{k=1}^{K} p_k x_k \).
- The surplus of consumers is defined as \( u(x) = \sum_{k=1}^{K} (p_{k} - v_{k}) x_k \).
- The social welfare is defined as \( \pi(x) + u(x) \).
- Uniform Monopoly: The producer sets an optimal price in each market. The producer sells homogeneous products to a continuum of consumers.
- Constant marginal cost, which can be normalized to zero.
- Non-cooperative GT + Collusion: Alternatively, a producer/mediator designs a market segmentation, which should be robust to strategic consumers.
- The social welfare is defined as \( \pi(x) + u(x) \).
- Uniform Monopoly: The producer can charge different prices in different markets such that all consumers in \( y \) have strictly higher utility in market \( x \) than before.

Tag-editable Framework
- Strategic consumers choose their tags simultaneously. Alternatively, a producer/mediator designs a market segmentation, which should be robust to strategic consumers.
- The producer chooses the minimum optimal price. (Robust)

Stable Segmentation
- A segmentation \( \sigma(x^*) = (x_1, \ldots, x_l) \) is stable if for any group of consumers \( y \), there is no decomposition \( y = y_1 + y_2 \) such that all consumers in \( y \) have strictly higher utility in the segmentation \( \{x_1, y_1 + y_2, \ldots, y_l, \ldots, x_{l-1}, x_l - y_1 + y_2\} \) than in \( \sigma(x^*) \).
- We require each group to have a positive measure.
- We assume any group of consumers cannot build a new market.
- This assumption is not a loss of generality since the consumer with the lowest valuation within the group must have zero utility by establishing a new market.

Weak-Stable Segmentation: A segmentation \( \sigma(x^*) = (x_1, \ldots, x_l) \) is weak-stable if for a small group of consumers with the same valuation in market \( x_k \), it is not profitable for them to deviate to any other market.
- “Small” indicates the measure of them is positive but arbitrarily close to zero.
- A relaxed concept that facilitates our analysis.
- Manifest individual deviation scenario. Since individual in real world has small but non-negligible market share.

Verification Condition
- Weak-stable verification: If \( \phi^{\text{min}}(x_k) < \phi^{\text{min}}(x_{k'}) \), \( v \in \text{supp}(x_k) \setminus x_{k'} \) and \( \phi^{\text{min}}(x_k), \phi^{\text{min}}(x_{k'}) \), \( v_k \) should be optimal in market \( x_k \).
- Stable: no-inflow condition: The segmentation \( \sigma(x^*) \) is stable iff the following no-inflow condition holds: For any market \( x_i \neq x_{k'} \), there is not a group of consumers \( y \neq 0 \) from other markets such that all consumers in \( y \) have strictly higher utility in market \( x_k \) than before.

Further Analysis
- Stable at the buyer-optimal outcome:
  - Stable and weak-stable are equivalent definitions.
  - Full characterization of all stable, social-optimal, and direct (SSD) segmentations with geometrical characterization.
- For a SSD segmentation, \( \sigma(x^*) = \{x_1, \ldots, x_K\} \).
  - Price profile: \( \phi^{\text{min}}(x_1), \ldots, \phi^{\text{min}}(x_K) \).
  - Revenue profile: \( \{v_1, \ldots, v_K\} \) is the revenue of market \( x_k \).
  - For SSD segmentation, revenue profile and price profile are identical which are characterized as following figures.

Main Theorem
The surplus of the producer and consumers (\( \pi, u \)) can be achieved by a stable segmentation iff \( \pi = \pi^* \) and \( u \in [\omega^*, \omega^* - \pi^*] \). No consumer is worse off compared with uniform monopoly.