Disagreement in Collateral Valuation*

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Abstract

We develop a model of secured lending in which borrowers and lenders agree to disagree about the value of collateral. Lenders' beliefs distort equilibrium prices of collateral assets. Asset prices are more reflective of lenders' beliefs when borrowers are riskier and more reflective of borrowers' beliefs when borrowers are safer. In a dynamic setting, disagreement leads to momentum (or reversal) in the returns of collateral assets. The model yields associations between borrower riskiness, return predictability, and return volatility. Using data on U.S. residential mortgages, we document a series of novel empirical facts consistent with the model's predictions.

Keywords: Disagreement, collateral, momentum, volatility, real estate
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1 Introduction

Much of the literature on disagreement in finance focuses on agents who "agree to disagree" about the value of an asset and then buy or sell the asset according to their beliefs (see Hong and Stein, 2007 for a review). In these models, only the beliefs of buyers and sellers are important for equilibrium outcomes. However, in practice, buyers often finance their asset purchases with debt secured by the asset itself, and the lenders providing such financing have their own beliefs about the asset's value. Since the assets used to secure such loans (e.g., accounts receivable, buildings, equipment, inventory, land, vehicles) are notoriously difficult to value, they may be the subject of valuation disagreement between borrowers and lenders. In this paper, we explore how disagreement in collateral valuation between borrowers and lenders distorts prices and affects returns of assets that serve as collateral.

We first develop a static model in which a borrower finances the purchase of an asset with debt and pledges the asset as collateral. The debt repayment and asset price are determined via bargaining. In equilibrium, the asset price is more reflective of the lender's belief when the borrower is riskier and more reflective of the borrower's belief when the borrower is safer. We then extend the model to a dynamic setting in which transactions occur continuously. The dynamic model includes several realistic features, such as estimates of value based on historical data, uncertainty about the arrival time of default, and recovery values based on market prices for the asset. Here, disagreement between borrowers and lenders generates return autocorrelation. We investigate how return autocorrelation and return volatility vary with borrower riskiness. We end the paper by documenting several novel empirical facts about the U.S. residential real estate market that are consistent with the theoretical predictions of the static and dynamic models.

To understand how disagreement affects the price of an asset that serves as collateral, consider a borrower who wishes to purchase an asset from a seller and who finances the purchase of the asset by borrowing from a lender. To obtain financing from the lender, the borrower pledges the asset as collateral. If the borrower remains solvent, she consumes the asset and makes a repayment to the lender. If the borrower defaults, she obtains nothing, and the lender seizes the asset. The debt repayment and asset price are determined via bargaining between the borrower, the lender, and the seller. Importantly, agents hold heterogeneous beliefs about the value of collateral.

In contrast to existing models of disagreement, in which wealth shares or risk-aversion determine whose beliefs are more reflected in prices (e.g., Atmaz and Basak, 2018), our model highlights the role of borrower riskiness in mediating whose beliefs are more reflected in prices. By borrower riskiness, we mean the propensity of a borrower to default, regardless of the value of collateral. For example, two different shipping companies pledging identical container ships as collateral may have vastly different credit ratings. In this example, there is variation in borrower risk but not in asset risk.

We show that when the borrower is optimistic about the value of the asset relative to the lender, the asset price decreases with borrower riskiness. Conversely, when the lender is optimistic relative to the borrower, the asset price increases with borrower riskiness. In other words, we find a negative relation between borrower riskiness and the distance between the asset price and the lender's value. This finding is robust to changing the order of bargaining, adjusting for repossession costs, partially endogenizing default, endogenizing the leverage choice, imposing loan-to-value (LTV) constraints, allowing the lender to rebate surplus in the event of default, and allowing the lender to sell the loan on a secondary market.

To develop some intuition for this finding, it is helpful to decompose the lender's value from a loan into two parts. The first is the interest and principal repaid over the life of the loan, which the lender obtains while the borrower is solvent. The more optimistic the borrower is about the asset's value, the more she is willing to pay the lender. The second is the value of collateral, which the lender obtains when the borrower defaults. The more optimistic the lender is about the asset's value, the more the lender values the asset as collateral.

An increase in borrower riskiness affects the two parts of the loan's value differently. On the one hand, it decreases the expected present value of interest and principal payments. On the other hand, it increases the likelihood that the lender repossesses the asset. The dominant effect is determined by whether the borrower or the lender is more optimistic about the asset's value. If the lender is more optimistic than the borrower, the increase in value from possible repossession is larger than the decrease in value from a smaller repayment. Therefore, the lender's total value from the loan increases with borrower riskiness. Since the total value increases, the asset's price increases as well, which keeps the lender indifferent between lending and not. However, the opposite is true if the borrower is more optimistic than the lender: The effect of the smaller repayment dominates, so the lender's value – and hence the asset price – decreases with borrower riskiness.

Up to this point, we have not relied on disagreement per se. Borrowers and lenders could simply have different tastes for the asset and the aforementioned results would still obtain. In this case, a safer borrower's tastes (rather than beliefs) would be more reflected in the asset price. Relatedly, some agents in our model may have a strategic incentive to misrepresent their beliefs. An interested buyer, for example, may wish to convince the seller that she values the asset less than she actually does, a phenomenon that arises in models of bargaining under asymmetric information (e.g., Myerson and Satterthwaite, 1983). In this case, a safer borrower's reported (rather than actual) value would be more reflected in the asset price.

We focus on disagreement because of its implications for the dynamics of returns. We therefore extend the model to a dynamic setting in which the asset's value, which is assumed to be publicly observable, evolves with an unobserved growth rate. Whereas the static model takes agents' beliefs as given, the dynamic model specifies how agents arrive at different beliefs. Specifically, borrowers and lenders update their beliefs about the unobserved growth rate and "agree to disagree" about how much weight to place on new information. For example, borrowers and lenders might place weights of 25% and 15%, respectively, on new information while agreeing to disagree about which weight is actually correct. Neither borrowers' weights nor lenders' weights need be Bayesian. Each borrower makes a fixed repayment to a lender until the arrival of default, at which time the lender repossesses the asset and sells it for the current market price.

We show that disagreement between borrowers and lenders can result in unconditional autocorrelation in the returns of assets that serve as collateral. Depending on how borrowers and lenders incorporate new information, return autocorrelation may be positive (momentum) or negative (reversal). Furthermore, return autocorrelation and return volatility may increase or decrease with borrower riskiness. However, to prepare for our empirical work, we focus on the case in which lenders update more slowly than borrowers and borrowers update like Bayesians. In this case, returns are positively autocorrelated, and the return autocorrelation is stronger when borrowers are riskier, which is precisely when default and repossession are more likely. Intuitively, prices are more reflective of lenders' beliefs when borrowers are riskier, and since lenders' beliefs incorporate new information more slowly, prices also incorporate new information more slowly. Relatedly, we find that the variance of returns decreases with borrower riskiness.

Empirically, lenders' beliefs have been shown to affect loan terms (Murfin, 2012; Koudijs and Voth, 2016; Carvalho et al., 2023), but less is known about how they affect the prices and returns of the assets that secure such loans. We therefore provide an empirical application of our analysis using data on U.S. residential mortgages. These data allow us to establish a number of novel empirical facts that are associative, rather than causal, in nature. Nonetheless, these facts demonstrate the empirical usefulness of our theory.

We document both micro- and macro-level associations consistent with the model's predictions. At the micro level, we investigate the model's prediction that sale prices should be closer to lenders' values when borrowers are riskier. We proxy for lenders' values with appraisals. These appraisals help lenders obtain an independent opinion of the market value of the property and are often required by law (Eriksen et al., 2019, 2020). We proxy for borrower riskiness with three different variables: initial home equity, initial LTV ratio, and FICO score at time of origination. A meta-analysis reveals that home equity and FICO are negatively associated with default risk, and initial LTV ratio is positively associated with default risk (Jones and Sirmans, 2015). We find that in both ordinary least squares and Poisson regressions, the coefficients on each of the three proxies, as well as the coefficient on their first principal component, are of the expected sign and highly significant. This result is robust to accounting for lender repossession costs, focusing on loans that are more likely to be sold on the secondary market, and considering appraisal bias.

At the macro level, we investigate the model's predictions regarding return autocorrelation and return volatility. Guided by the well-known phenomenon of appraisal-smoothing (Clayton et al., 2001), we explore the case in which lenders update more slowly than borrowers and borrowers update like Bayesians. In this parameterization of the model, return autocorrelation is positive, and during episodes of higher borrower riskiness, return autocorrelation is stronger and return volatility is weaker. We first confirm the longstanding empirical finding of positive return autocorrelation (i.e., momentum) in the real estate market (Case and Shiller, 1989; Beracha and Skiba, 2011; Ghysels et al., 2013). Using the same three proxies for borrower riskiness (i.e., initial home equity, LTV ratio, and FICO) and their first principal component, we then show that return autocorrelation is stronger and return volatility is weaker when borrower riskiness is higher.

Our model is purposefully agnostic on the exact reason why borrowers and lenders systematically disagree (our empirical analysis takes a slightly stronger stand). Our point is instead that systematic disagreement between borrowers and lenders generates useful empirical predictions. We are unaware of a theory that links borrower riskiness to asset prices and returns in the manner suggested by our model. In housing, the source of disagreement is clear: Lenders use an appraisal process that is standardized and is often required by law (a sort of institutional dogmatism). It is not difficult, however, to imagine what our theory might say about other markets where secured lending is prevalent.

2 Related Literature

In this section, we review some of the key papers in the literature most closely related to our paper and highlight our contributions.

2.1 Disagreement

Our model belongs to the class of models in which agents hold dogmatic beliefs regarding some aspect of the economy (e.g., asset values or signal precisions) and agree to disagree with other agents.[1] For example, an agent may be overconfident and place too much weight on her own information and not enough weight on other agents' information. An attractive feature of this approach is its ability to explain empirical puzzles, such as the positive relationship between volume and volatility (e.g., Banerjee and Kremer, 2010) and return momentum (e.g., Kyle et al., 2023). These papers almost always model disagreement arising between traders and focus on explaining puzzles from the equity markets (i.e., return predictability, return volatility, and trading volume).[2] In contrast, we model disagreement between borrowers and lenders to generate new predictions about price distortion and return predictability in the market for collateral assets.

Our application of disagreement to secured lending is closest to that of Simsek (2013), who also considers a static setting in which agents disagree about the value of collateral. In his model, agents hold heterogeneous beliefs about the asset value. Pessimists lend to optimists who wish to purchase an asset that will serve as collateral. Default arises endogenously when the value of the collateral falls below the borrower's promised repayment. Simsek (2013) finds that what agents disagree about (e.g., the probability of good states versus bad states and the recovery values therein), matters more for asset prices than the level of disagreement. Our focus is markedly different. Taking the beliefs of borrowers and lenders as given, we study the role of borrower riskiness (rather than asset riskiness) in mediating disagreement.

[1]See Harrison and Kreps (1978); Harris and Raviv (1993); Kandel and Pearson (1995); Scheinkman and Xiong (2003); Cao and Ou-Yang (2008); Banerjee et al. (2009); Banerjee and Kremer (2010); Banerjee (2011); Atmaz and Basak (2018); Kyle et al. (2023) for examples.

[2]Additionally, Broer (2018) looks at disagreement in the context of securitization, Burnside et al. (2016) examine the role of disagreement in creating booms and busts in the housing market, and Xiong and Yan (2010) explore disagreement in bond markets.

2.2 Collateral Valuation

Our paper is also related to the literature on collateral valuation. Stroebel (2016) shows that lenders with relatively superior information about the value of collateral earn higher returns on their secured loans. Jiang and Zhang (2023) find that mortgages collateralized by houses whose estimated values are more disperse – which the authors argue may be driven by information asymmetries – receive higher interest rates and are smaller in size. Our contribution to this strand of literature is to show that lenders' estimates of collateral values can also affect asset prices and returns, not just loan terms.

2.3 Momentum and Volatility

We contribute to the literature on return momentum (i.e., positive return autocorrelation), which has been documented across asset classes and geographies (Moskowitz et al., 2012; Asness et al., 2013). More specifically, our paper fits into the literature on return momentum in residential real estate, which was first documented by Case and Shiller (1989, 1990) (see Ghysels et al., 2013 for a review). Explanations put forth for this phenomenon include extrapolative expectations (Case and Shiller, 1987; Glaeser and Nathanson, 2017), information frictions (Capozza et al., 2004; Anenberg, 2016), search costs (Head et al., 2014, 2016), and strategic complementarity (Guren, 2018).[3] We contribute to this literature in two ways. First, we propose disagreement between borrowers and lenders (rather than buyers and sellers) as a novel mechanism for momentum. In our model, momentum can arise when borrowers and lenders incorporate information at different rates. This difference in beliefs is complementary to the empirical evidence in Genesove and Hanse (2023), who argue that return momentum in the residential housing market may be driven by sellers updating their values much more slowly than buyers. Second, we document a positive association between borrower riskiness and return autocorrelation in residential real estate. To the best of our knowledge, we are the first to document this stylized fact.

Lastly, our paper adds to a much smaller literature on return volatility in housing markets. Time-varying housing return volatility is well established in the literature (Dolde and

[3] In a working paper, Martel and Van Wesep (2016) examine the effect of appraisal-based price constraints on prices, returns, and liquidity. In their model, buyers cannot pay significantly more than the average price in recent transactions, which causes sluggish adjustment to changes in fundamental value (i.e., momentum). They show that in a rising market, sellers strategically delay sale, which exacerbates the already sluggish adjustment process. In contrast, estimates of value based on historical data arise endogenously in our model because lenders require an estimate of collateral value. Moreover, we explore the role of borrower riskiness in appraisals' effect on prices. Our framework is therefore complementary to theirs.

Tirtiroglu, 1997; Miller and Peng, 2006). This volatility has been shown to be driven in part by economic policy uncertainty (André et al., 2017) and can lead to changes in income growth (Dolde and Tirtiroglu, 2002; Miller and Peng, 2006). We contribute to this strand of literature by showing that housing return volatility is lower when borrower riskiness is higher.

3 Static Model

In this section, we present a static model to illustrate how disagreement between a borrower and a lender can distort the price of an asset that serves as collateral. We then show that borrower riskiness mediates disagreement between the borrower and the lender. Specifically, a safer borrower will have her beliefs reflected more in the price of an asset than will a riskier borrower. In addition to generating empirical predictions of its own, the static model lays the groundwork for the dynamic model that we develop in the next section.

3.1 Base Model

3.1.1 Environment

There are two dates, $t = 0, 1$, and three agents indexed by $j \in \{\mathcal{B}, \mathcal{L}, \mathcal{S}\}$: borrower/buyer $\mathcal{B}$ (she), lender $\mathcal{L}$ (he), and seller $\mathcal{S}$. Agents are risk-neutral. There is an indivisible asset for which $\mathcal{B}$ has unit demand. We will henceforth refer to $\mathcal{B}$ as the borrower, though it should be understood that she is both "borrower" and "buyer." The date $t = 1$ value of owning the asset is

$$v_1 = v_0 + x, \tag{1}$$

where v_0 is a constant and x is a normally distributed growth rate. The main assumption of the model is that agents agree to disagree about the exact distribution of x. Specifically, agents agree that x is normally distributed with precision $\tau_0 > 0$ but agree to disagree about the mean of x. That is, agent j believes the mean is $\widehat{x}_j$, which need not equal $\widehat{x}_i$ for some other agent i. Let

$$a_j = v_0 + x_j, \tag{2}$$

where a_j represents agent j's estimate of the asset's value.

The borrower wishes to purchase the asset from the seller and seeks a loan from the lender to finance the purchase. To secure the loan, the borrower pledges the asset as collateral. The borrower may default on her loan, in which case the lender repossesses the asset. Let d be a Bernoulli random variable representing default: $d = 1$ (default) with probability $\lambda \in (0, 1)$, and $d = 0$ (solvency) with probability $1-\lambda$. Furthermore, let c be the borrower's repayment in solvency, p be the price paid by the borrower to the seller for the asset, and $\ell \in (0, 1)$ be the fraction of the purchase price financed by the lender (the remaining fraction $1 - \ell$ is financed by the borrower). We take ℓ to be exogenous and solve for the equilibrium debt repayment c and asset price p.

3.1.2 Bargaining Solution

Suppose the three agents agree on a debt repayment c and asset price p. The surpluses of the borrower, the lender, and the seller are as follows:

$$S_{\mathcal{B}}(c, p) = (1 - d)(v_1 - c) - (1 - \ell)p \tag{3}$$

$$S_{\mathcal{L}}(c, p) = dv_1 + (1 - d)c - \ell p \tag{4}$$

$$S_{\mathcal{S}}(c, p) = p - v_1. \tag{5}$$

The borrower obtains the asset value v_1 less the repayment c if she is solvent and nothing if she defaults. She has an initial outlay of $(1 - \ell)p$. The lender obtains the asset value v_1 if the borrower defaults and the repayment c if she remains solvent. He has an initial outlay of ℓp. The seller obtains the price p but gives up the asset, which has value v_1.

To determine whether or not there are gains from trade, we need to evaluate the agents' expected surpluses under their respective subjective beliefs:

$$\mathbb{E}^{\mathcal{B}}[S_{\mathcal{B}}(c, p)] = (1 - \lambda)(a_{\mathcal{B}} - c) - (1 - \ell)p \tag{6}$$

$$\mathbb{E}^{\mathcal{L}}[S_{\mathcal{L}}(c, p)] = \lambda a_{\mathcal{L}} + (1 - \lambda)c - \ell p \tag{7}$$

$$\mathbb{E}^{\mathcal{S}}[S_{\mathcal{S}}(c, p)] = p - a_{\mathcal{S}}. \tag{8}$$

The total expected surplus is therefore

$$\sum_j \mathbb{E}^j[S_j(c, p)] = (1 - \lambda)a_{\mathcal{B}} + \lambda a_{\mathcal{L}} - a_{\mathcal{S}}. \tag{9}$$

The total surplus may be defined in other ways when agents hold heterogeneous beliefs. For example, the extent to which an agents' surplus is evaluated under her own beliefs could depend on her bargaining power (e.g., Brunnermeier et al., 2014 develop a measure of welfare under heterogeneous beliefs). Equation 9 serves as a natural starting point for our analysis.

The debt repayment c and asset price p are determined via multilateral bargaining. Specifically, let $\eta_j \geq 0$ be j's bargaining weight where $\sum_j \eta_j = 1$. These weights may be interpreted literally as bargaining weights or more abstractly as parameters that capture agents' relative market power in a competitive market. Our bargaining solution is as follows: c and p are chosen to maximize the product

$$\prod_j \mathbb{E}^j[S_j(c,p)]^{\eta_i}. \tag{10}$$

This is a multilateral generalization of the Nash product (Nash, 1950). Since the total subjective, expected surplus does not depend on c and p, there is a simple solution to equation (10): Agent j obtains a fraction η_j of the total surplus in equation (9). Gains from trade exist whenever equation (9) is positive (i.e., whenever $a_{\mathcal{S}} < (1-\lambda)a_{\mathcal{B}} + \lambda a_{\mathcal{L}}$). Note that we have ruled out the formation of coalitions for parsimony. The following proposition gives the equilibrium debt repayment and asset price under our bargaining solution.

Proposition 1. *The equilibrium repayment and price are*

$$c^*(\lambda) = (1-\lambda)^{-1}((1-\lambda)(\eta_{\mathcal{L}} + \eta_{\mathcal{S}}\ell)a_{\mathcal{B}} - \lambda(1-(\eta_{\mathcal{L}} + \eta_{\mathcal{S}}\ell))a_{\mathcal{L}} - (\eta_{\mathcal{L}} - (1-\eta_{\mathcal{S}})\ell)a_{\mathcal{S}}) \tag{11}$$

$$p^*(\lambda) = \eta_{\mathcal{S}}((1-\lambda)a_{\mathcal{B}} + \lambda a_{\mathcal{L}}) + (1-\eta_{\mathcal{S}})a_{\mathcal{S}}. \tag{12}$$

From equation (12), it is clear that the equilibrium price not only reflects the beliefs of both the borrower and the seller but also the beliefs of the lender. Furthermore, the lender's beliefs are more reflected in the price when the borrower is riskier. It follows from equation (12) that when the borrower is optimistic relative to the lender ($a_{\mathcal{B}} > a_{\mathcal{L}}$), the price decreases with borrower riskiness ($p^{*\prime}(\lambda) < 0$). Conversely, when the borrower is pessimistic relative to the lender ($a_{\mathcal{B}} < a_{\mathcal{L}}$), the price increases with borrower riskiness ($p^{*\prime}(\lambda) > 0$).

To study the role of disagreement between the borrower and the lender, we often focus on the case in which neither agent earns surplus ($\eta_{\mathcal{B}} = \eta_{\mathcal{L}} = 0$ and $\eta_{\mathcal{S}} = 1$). One might imagine, for example, that assets markets are competitive, loan markets are competitive, and that the asset is in sufficiently short supply. In this case, the beliefs of both the borrower and the lender are reflected in the price of the asset, but the beliefs of the seller are not. Focusing on this case allows us to highlight the economic forces behind Proposition 1.

To develop the intuition behind Proposition 1, consider the indifference conditions for the borrower and the lender when neither agent obtains surplus:

$$\text{Borrower: } 0 = (1-\lambda)(a_{\mathcal{B}} - c(\lambda)) - (1-\ell)p(\lambda) \tag{13}$$

$$\text{Lender: } 0 = \lambda a_{\mathcal{L}} + (1-\lambda)c(\lambda) - \ell p(\lambda). \tag{14}$$

From the borrower's indifference condition, we see that to keep the borrower indifferent, the unconditional expected repayment $(1-\lambda)c(\lambda)$ must equal the unconditional asset value $(1-\lambda)a_{\mathcal{B}}$ less the amount of the purchase price she finances herself $(1-\ell)p(\lambda)$. Substituting $(1-\lambda)c(\lambda)$ into the lender's indifference condition, we obtain

$$0 = \overbrace{\lambda a_{\mathcal{L}}}^{\text{(A)}} + \overbrace{(1-\lambda)a_{\mathcal{B}} - (1-\ell)p(\lambda)}^{\text{(B)}} - \overbrace{\ell p(\lambda)}^{\text{(C)}}. \tag{15}$$

From equation (15), we see that increasing borrower riskiness increases the unconditional collateral value (A) at a rate of $a_{\mathcal{L}}$ and decreases the unconditional repayment (B) at a rate of $a_{\mathcal{B}} + (1-\ell)p'(\lambda)$. To keep the lender indifferent between lending and not, the loan amount (C) must change to offset the net change of (A) and (B):

$$\ell p'(\lambda) = a_{\mathcal{L}} - (a_{\mathcal{B}} + (1-\ell)p'(\lambda)), \tag{16}$$

from which it follows that $p'(\lambda) > 0$ when $a_{\mathcal{L}} > a_{\mathcal{B}}$ and $p'(\lambda) < 0$ when $a_{\mathcal{L}} < a_{\mathcal{B}}$.

Before concluding this section, we state an important empirical prediction of the model. In many applications (such as our application to housing), one may observe the lender's value of the asset and the price paid by the borrower for the asset, but not the borrower's value of the asset (in fact, one of the main points of this paper is that the price paid for the asset is distorted by the lender's beliefs and does not fully reflect the beliefs of the buyer and the seller).

Corollary 1. *If neither the borrower nor the lender earn surplus ($\eta_{\mathcal{B}} = \eta_{\mathcal{L}} = 0$), then*

$$p^*(\lambda) = (1-\lambda)a_{\mathcal{B}} + \lambda a_{\mathcal{L}} \tag{17}$$

and hence

$$p^{*\prime}(\lambda) = a_{\mathcal{L}} - a_{\mathcal{B}} = (1-\lambda)^{-1}(a_{\mathcal{L}} - p^*(\lambda)). \tag{18}$$

As in Proposition 1, the price decreases with borrower riskiness ($p^{*\prime}(\lambda) < 0$) when the borrower is relatively optimistic ($a_{\mathcal{B}} > a_{\mathcal{L}}$) and increases with borrower riskiness ($p^{*\prime}(\lambda) > 0$)

when the borrower is relatively pessimistic ($a_{\mathcal{B}} < a_{\mathcal{L}}$). However, Corollary 1 enables us to establish that the equilibrium price is greater than the lender's value ($p^*(\lambda) > a_{\mathcal{L}}$) when the borrower is relatively optimistic, and the equilibrium price is less than the lender's value ($p^*(\lambda) < a_{\mathcal{L}}$) when the borrower is relatively pessimistic. Taken together, these results suggest that increasing borrower riskiness pulls the price towards the lender's value.

3.2 Model Extensions

In Section B of the Appendix, we work through several extensions of the base model. We discuss these extensions here.

We first explore the robustness of our results to reordering the bargaining game. We consider two versions of the model: one in which the borrower and the lender bargain first and the borrower and the seller bargain last, and one in which the borrower and the seller bargain first and the borrower and the lender bargain last. Unsurprisingly, whichever pair of agents bargains first extracts more surplus because they enjoy a first-mover advantage. However, regardless of the ordering, the price increases with borrower riskiness if the lender is relatively optimistic, and conversely, the price decreases with borrower riskiness if the lender is relatively pessimistic.

In many applications, collateral repossession is costly; lenders may incur substantial holding costs (such as in the case of equipment repossession) or have to sell at a foreclosure sale discount (Conklin et al., 2023). We extend the model to include repossession costs and show that increasing borrower riskiness still pulls the price of the asset towards the lender's effective value (i.e., his value net of repossession costs).

We then consider the possibility that the probability of default is correlated with the asset value. To be concrete, consider a household that defaults on its mortgage. More likely than not, the household defaults on its mortgage because of financial distress (e.g., unemployment). We consider a version of the model in which borrowers remain solvent in good states but may default in bad states. We find that the price increases with borrower riskiness if the lender is optimistic about the asset value in the bad state of the world. Conversely, the price decreases with borrower riskiness when the borrower is pessimistic about the asset value in the bad state.

We consider three extensions regarding the method of financing the asset purchase. In each of these extensions, the equilibrium debt repayment is affected, but the price of the asset is not. In the first of these extensions, we let the borrower, the lender, and the seller bargain over the fraction financed ℓ rather than the repayment amount c. In the second, we consider the possibility that the fraction financed ℓ is the "Loan-to-Value" ratio (LTV), where "V" is the lesser of the price and the lender's value (rather than just the price, as modelled in the base model). In the third, we require the lender to rebate the difference between the collateral value and debt value back to the borrower if the collateral value exceeds the debt value in default. In each of these cases, the equilibrium asset price is as in Proposition 1. The only thing that changes is the equilibrium repayment.

Finally, we consider the possibility that the lender can sell the loan to a third party (as is the case in many applications, such as housing). We show that even if the borrower and the lender agree about the asset value, our main findings survive so long as the borrower and the ultimate buyer of the loan (e.g., a government-sponsored enterprise) disagree. Put simply, one can simply relabel the lender's belief as the investor's belief and obtain the same results.

4 Dynamic Model

To understand the implications of disagreement for returns, we extend the static model developed in Section 3 to continuous time. Importantly, we now model the source of disagreement. We assume that borrowers and lenders disagree because they place different weights on new information when updating their beliefs about collateral values. The dynamic model introduces other realistic features, such as estimates of value based on historical data, uncertainty about the arrival time of default, and recovery values based on market prices for the asset. The model combines the non-Bayesian information processing of Berrada (2009) with the linear price structure of Glaeser and Nathanson (2017).

4.1 Setup

Time is continuous and indexed by $t \in (-\infty, \infty)$. There are two types of agents: borrowers ($\mathcal{B}$) and lenders ($\mathcal{L}$). To focus on disagreement between borrowers and lenders, we focus on the case in which neither borrowers nor lenders earn surplus. We therefore omit the seller entirely. Agents are risk-neutral and discount utility flows at a common rate $r > 0$. There is an indivisible asset for which borrowers have unit demand. Suppose that the common stock value of the asset V_t evolves according to

$$dV_t = X_t dt + \sigma_V dB_t^V \tag{19}$$
$$dX_t = -\kappa X_t dt + \sqrt{2\kappa}\sigma_X dB_t^X, \tag{20}$$

where B_t^V and B_t^X are standard, uncorrelated one-dimensional Brownian motions, and κ, σ_V, and σ_X are positive, known constants (let $\tau_V = 1/\sigma_V^2$ and $\tau_X = 1/\sigma_X^2$ be the corresponding precisions). V_t is observed by all agents, and X_t is observed by none. Let τ_0 be the positive root of

$$0 = \tau_X \tau_V + 2\kappa(\tau_X - \tau_0)\tau_0. \tag{21}$$

To obtain a steady-state equilibrium, we assume that the unconditional prior belief is that X_t is normally distributed with mean zero and precision τ_0. Let $\widehat{X}_t^j$ denote agent j's subjective expectation of X_t at time t. For $j \in \{\mathcal{B}, \mathcal{L}\}$, we assume that $\widehat{X}_t^j$ evolves according to

$$d\widehat{X}_t^j = -\kappa \widehat{X}_t^j dt + w_j(dV_t - \widehat{X}_t^j dt), \tag{22}$$

where w_j is constant. equation (22) nests the Kalman-Bucy filter, which has coefficient $w_0 \equiv \tau_V/\tau_0$. Following the disagreement literature, we assume that $\mathcal{B}$ knows that $\mathcal{L}$'s growth rate estimate is $\widehat{X}_t^{\mathcal{L}}$, $\mathcal{L}$ knows that $\mathcal{B}$'s growth rate estimate is $\widehat{X}_t^{\mathcal{B}}$, and $\mathcal{B}$ and $\mathcal{L}$ agree to disagree (e.g., Banerjee and Kremer, 2010). The primary friction in the model is the possibility that $w_{\mathcal{B}}$ and $w_{\mathcal{L}}$ differ from each other and differ from the Bayesian weight w_0. This assumption is similar to that made by Berrada (2009).

Consider a borrower who buys the asset at time t. As in the static model, the lender finances a fraction $\ell \in (0, 1)$ of the loan. The time of default $T \geq t$ arrives exogenously with rate $\lambda > 0$. λ again measures borrower riskiness in the sense that when λ is large, borrowers default faster and are therefore "riskier." Let C_t be the perpetuity value of repayment (over a time interval of length ds, the borrower makes a repayment of $rC_t ds$ to the lender), P_t be the price of the collateral asset at time t, $U_s^{\mathcal{B}} = U^{\mathcal{B}}(s, C_t, P_t)$ be the borrower's value, and $U_s^{\mathcal{L}} = U^{\mathcal{L}}(s, C_t, P_t)$ be the lender's value. $U_s^{\mathcal{B}}$ and $U_s^{\mathcal{L}}$ evolve according to the following Bellman equations:

$$rU_s^{\mathcal{B}} ds = (rV_s - rC_t)ds - \lambda U_s^{\mathcal{B}} ds + \mathbb{E}_s^{\mathcal{B}}[dU_s^{\mathcal{B}}] \tag{23}$$
$$rU_s^{\mathcal{L}} ds = rC_t ds + \lambda(P_s - U_s^{\mathcal{L}})ds + \mathbb{E}_s^{\mathcal{L}}[dU_s^{\mathcal{L}}]. \tag{24}$$

The borrower's flow utility $rU_s^{\mathcal{B}} ds$ equals the flow utility she obtains from the asset $rV_s ds$

less the fixed repayment rC_tds she makes to the lender. With probability λds, she defaults and loses everything. Her continuation value is $\mathbb{E}_s^{\mathcal{B}}[dU_s^{\mathcal{B}}]$. The lender earns the fixed payment rC_tds, and with probability λds, the borrower defaults, the lender seizes the collateral, and the lender sells it for P_t. His continuation value is $\mathbb{E}_s^{\mathcal{L}}[dU_s^{\mathcal{L}}]$.

Integrating equations (23) and (24) yields

$$U^{\mathcal{B}}(t, C_t, P_t) = \mathbb{E}_t^{\mathcal{B}}\left[\int_t^T r(V_s - C_t)e^{-r(s-t)}ds\right]. \tag{25}$$

$$U^{\mathcal{L}}(t, C_t, P_t) = \mathbb{E}_t^{\mathcal{L}}\left[\int_t^T rC_t e^{-r(s-t)}ds + P_T e^{-r(T-t)}\right]. \tag{26}$$

We now use these utilities to determine the equilibrium debt repayment and asset price. Just as in our analysis of the static model, we focus on the case in which neither type of agent earns surplus. This assumption allows us to better study the role of disagreement between borrowers and lenders. Consider the indifference conditions for the borrower and the lender when neither agent obtains surplus:

$$0 = U^{\mathcal{B}}(t, C_t, P_t) - (1-\ell)P_t \tag{27}$$

$$0 = U^{\mathcal{L}}(t, C_t, P_t) - \ell P_t. \tag{28}$$

Equations (27) and (28) are the continuous-time analogues of equations (13) and (14). Adding equations (27) and (28), it follows that

$$P_t = \mathbb{E}_t\left[\int_t^T r\mathbb{E}_t^{\mathcal{B}}[V_s|T]e^{-r(s-t)}ds + \mathbb{E}_t^{\mathcal{L}}[P_T|T]e^{-r(T-t)}\right]. \tag{29}$$

equation (29) illustrates how the differing beliefs of borrowers and lenders are incorporated into prices. Up until default, the borrower expects to obtain a flow utility of $r\mathbb{E}_t^{\mathcal{B}}[V_s|T]$. In the event of default, the lender seizes the collateral and sells it for an expected market price of $\mathbb{E}_t^{\mathcal{L}}[P_T|T]$. In the next subsection, we show precisely how the agents' beliefs about the growth rate are reflected in the price of the collateral asset.

We have thus far been silent on why transactions occur, why buyers and sellers transact when they do, and how many transactions occur during a particular period of time. Suffice it to say, buyers and sellers are assumed to transact at a particular time for exogenous reasons (e.g., a firm acquires a vehicle or a piece of equipment to finish a project). Transactions are assumed to be sufficiently frequent so that agents can, in principle, extract the asset value from the history of asset prices. Although lack of liquidity is undoubtedly important in housing markets (Sagi, 2021), our goal is simply to illustrate how disagreement affects returns of assets that serve as collateral, absent search frictions.

4.2 Equilibrium

Following standard practice, we restrict attention to equilibria in which the price is linear in the state variables V_t, $\widehat{X}_t^{\mathcal{B}}$, and $\widehat{X}_t^{\mathcal{L}}$.

Proposition 2. *The unique linear equilibrium repayment C_t^* and price P_t^* are*

$$C_t^* = b_V(\lambda)V_t + b_{\mathcal{B}}(\lambda)\widehat{X}_t^{\mathcal{B}} + b_{\mathcal{L}}(\lambda)\widehat{X}_t^{\mathcal{L}} \text{ and} \tag{30}$$

$$P_t^* = V_t + c_{\mathcal{B}}(\lambda)\widehat{X}_t^{\mathcal{B}} + c_{\mathcal{L}}(\lambda)\widehat{X}_t^{\mathcal{L}}, \tag{31}$$

where

$$b_V(\lambda) = 1 - r^{-1}(r+\lambda)(1-\ell) \tag{32}$$

$$b_{\mathcal{B}}(\lambda) = (r+\kappa+\lambda)^{-1} - r^{-1}(r+\lambda)(1-\ell)c_{\mathcal{B}}(\lambda) \tag{33}$$

$$b_{\mathcal{L}}(\lambda) = -r^{-1}(r+\lambda)(1-\ell)c_{\mathcal{L}}(\lambda) \tag{34}$$

$$c_{\mathcal{B}}(\lambda) = r((w_{\mathcal{B}}+\kappa)+r)^{-1}(r+\lambda)^{-1}(r+\kappa+\lambda)^{-1}((w_{\mathcal{B}}+\kappa)+r+\lambda) \tag{35}$$

$$c_{\mathcal{L}}(\lambda) = \lambda(r+\kappa)^{-1}(r+\lambda)^{-1}\left(1 + w_{\mathcal{B}}r((w_{\mathcal{B}}+\kappa)+r)^{-1}(r+\kappa+\lambda)^{-1}\right). \tag{36}$$

Moreover, $c_{\mathcal{B}}(\lambda) + c_{\mathcal{L}}(\lambda) = 1/(r+\kappa)$, $c'_{\mathcal{B}}(\lambda) < 0$, and $c'_{\mathcal{L}}(\lambda) > 0$.

Proposition 2 generalizes the first part of Corollary 1 to the dynamic setting. In particular, it retains the feature that lenders' beliefs are more reflected in equilibrium prices when borrowers are riskier (i.e., $c'_{\mathcal{B}}(\lambda) < 0$ and $c'_{\mathcal{L}}(\lambda) > 0$).

4.2.1 Price and Lenders' Value

The lender's estimate of value is

$$A_t^{\mathcal{L}} = \mathbb{E}_t^{\mathcal{L}}\left[\int_t^\infty rV_s e^{-r(s-t)}ds\right] = V_t + (r+\kappa)^{-1}\widehat{X}_t^{\mathcal{L}}. \tag{37}$$

From Proposition 2, it follows that

$$P_t - A_t = c_{\mathcal{B}}(\lambda)(\widehat{X}_t^B - \widehat{X}_t^L). \tag{38}$$

equation (38) is the continuous-time analogue of the second part of Corollary 1. If lenders are more optimistic than borrowers ($\widehat{X}_t^L > \widehat{X}_t^B$), then the price is less than lenders' value ($P_t < A_t$). Since $c_{\mathcal{B}}$ is decreasing in λ, $P_t - A_t$ is increasing in λ (P_t increases towards A_t from below). If instead borrowers are more optimistic than lenders ($\widehat{X}_t^B > \widehat{X}_t^L$), then the price is greater than lenders' value ($P_t > A_t$). Again, since $c_{\mathcal{B}}$ is decreasing in λ, $P_t - A_t$ is decreasing in λ (P_t decreases towards A_t from above). Taken together, these results suggest that increasing borrower riskiness pulls the price towards lenders' value.

4.2.2 Predictability and Volatility of Returns

If agents follow Bayes' rule, they place a weight of w_0 on new information (specifically, $w_j = w_0$ in equation (22)). However, we consider the possibility that agents place different weights on new information than each other and than that prescribed by Bayes' rule. In this way, disagreement about growth rates stems from disagreement about how much weight to place on new information. In this subsection, we explain why this type of disagreement generates return predictability. This discussion will lay the groundwork for our discussion of return autocorrelation and return volatility.

For each agent $i \in \{\mathcal{B}, \mathcal{L}\}$, let

$$Z_t^i = X_t - \widehat{X}_t^i. \tag{39}$$

Z_t^i is the difference between the true growth rate X_t and agent i's estimate of the growth rate $\widehat{X}_t^i$. Z_t^i evolves according to

$$dZ_t^i = -(\kappa + w_i) Z_t dt - w_i \sigma_V dB_t^V + \sqrt{2\kappa}\sigma_X dB_t^X. \tag{40}$$

Let

$$\alpha_i(\lambda) = (r + \kappa + w_i) c_i(\lambda). \tag{41}$$

From equation (31) in Proposition 2 and the dynamics for the beliefs of the borrowers and lenders in equation (22), it follows that

$$\begin{aligned} dP_t + r(V_t - P_t)dt &= dV_t + c_{\mathcal{B}}(\lambda) d\widehat{X}_t^{\mathcal{B}} + c_{\mathcal{L}}(\lambda) d\widehat{X}_t^{\mathcal{L}} - r(c_{\mathcal{B}}(\lambda)\widehat{X}_t^{\mathcal{B}} + c_{\mathcal{L}}(\lambda)\widehat{X}_t^{\mathcal{L}})dt && (42) \\ &= \alpha_{\mathcal{B}}(\lambda)(dV_t - \widehat{X}_t^{\mathcal{B}} dt) + \alpha_{\mathcal{L}}(\lambda)(dV_t - \widehat{X}_t^{\mathcal{L}} dt) && (43) \\ &= \alpha_{\mathcal{B}}(\lambda)(Z_t^{\mathcal{B}} dt + \sigma_V dB_t^V) + \alpha_{\mathcal{L}}(\lambda)(Z_t^{\mathcal{L}} dt + \sigma_V dB_t^V). && (44) \end{aligned}$$

Note that $dP_t + r(V_t - P_t)dt$ represents the instantaneous total return. During a time interval of length dt, the owner of the asset enjoys a capital gain of dP_t plus utility from the asset $rV_t dt$ minus the financing/opportunity cost $rP_t dt$. Under the objective probability measure,

$$\mathbb{E}_t[dP_t + r(V_t - P_t)dt] = \alpha_{\mathcal{B}}(\lambda)(\mathbb{E}_t[X_t] - \widehat{X}_t^{\mathcal{B}})dt + \alpha_{\mathcal{L}}(\lambda)(\mathbb{E}_t[X_t] - \widehat{X}_t^{\mathcal{L}})dt. \tag{45}$$

Pessimism (in the sense that $\mathbb{E}[X_t] > \widehat{X}_t^j$) from either borrowers or lenders predicts positive future returns, and optimism ($\mathbb{E}[X_t] < \widehat{X}_t^j$) from either borrowers or sellers predicts negative future returns. Since $\alpha_{\mathcal{B}}$ is decreasing in λ and $\alpha_{\mathcal{L}}$ is increasing in λ, we additionally have that when borrowers are safer, borrowers' sentiment (i.e., optimism or pessimism) is more predictive of returns than lenders' sentiment. Conversely, when borrowers are riskier, borrowers' sentiment is less predictive of returns than lenders' sentiment.

We can similarly compute the variance of the instantaneous returns:

$$\mathbb{V}_t[dP_t + r(V_t - P_t)dt] = (\alpha_{\mathcal{B}}(\lambda) + \alpha_{\mathcal{L}}(\lambda))^2 \sigma_V^2 dt. \tag{46}$$

Now $\alpha'_{\mathcal{B}}(\lambda) + \alpha'_{\mathcal{L}}(\lambda) = (w_{\mathcal{B}} - w_{\mathcal{L}})c'_{\mathcal{B}}(\lambda)$, which is strictly negative if and only if $w_{\mathcal{B}} > w_{\mathcal{L}}$. If lenders update more slowly than borrowers ($w_{\mathcal{B}} > w_{\mathcal{L}}$), then returns are less volatile when borrowers are riskier. Intuitively, the asset price is more reflective of lenders' slowly updating beliefs.

4.2.3 Return Autocorrelation and Return Variance

We now explore how borrower riskiness affects return autocorrelation and return variance. To maintain tractability, we define the holding period return as in Kyle et al. (2023). The return to an investor who finances the purchase of the asset at the risk-free rate at time t, "reinvests" utility flows during the holding period at the risk-free rate, and sells the asset at time $t + \theta$ is

$$R_{t,t+\theta} \equiv P_{t+\theta} - P_t e^{r\theta} + e^{r\theta} \int_t^{t+\theta} rV_s e^{-r(s-t)} ds. \tag{47}$$

Note that $\lim_{\theta \to 0} R_{t,t+\theta} = dP_t + rV_t dt - rP_t dt$.

The unconditional return autocorrelation and return variance are

$$\rho_{t-\theta,t+\theta}(\lambda; w_{\mathcal{L}}, w_{\mathcal{B}}) \equiv \mathrm{Cov}(R_{t,t+\theta}, R_{t-\theta,t})/\mathrm{Var}(R_{t-\theta,t}) \tag{48}$$

$$\sigma_{t-\theta}^2(\lambda; w_{\mathcal{L}}, w_{\mathcal{B}}) \equiv \mathrm{Var}(R_{t-\theta,t}). \tag{49}$$

As we show in the proposition that follows, return autocorrelation and return variance are time-invariant. We therefore omit the time dependence in the discussion that follows. We are interested in the sign of the autocorrelation $\rho(\lambda; w_{\mathcal{B}}, w_{\mathcal{L}})$ and the sign of its derivative with respect to borrower riskiness $\partial_\lambda \rho(\lambda; w_{\mathcal{B}}, w_{\mathcal{L}})$. We are also interested in the sign of the derivative of the variance $\sigma^2(\lambda, w_{\mathcal{L}}, w_{\mathcal{B}})$ with respect to borrower riskiness $\partial_\lambda \sigma^2(\lambda, w_{\mathcal{L}}, w_{\mathcal{B}})$.

Before deriving any analytical results, we first consider a numerical simulation. Figure 1 shows the sign of the autocorrelation, the sign of its derivative with respect to λ, and the sign of the derivative of the variance with respect to λ in the $w_{\mathcal{B}}$-$w_{\mathcal{L}}$ plane for a particular set of parameters. The plane is demarcated into four regions, which correspond to the four combinations of signs of the autocorrelation and its derivative with respect to λ. When the sum (or average) of borrowers' weights and lenders' weights ($w_{\mathcal{B}} + w_{\mathcal{L}}$) is large (i.e., regions (2) and (3)), there is reversal (negative return autocorrelation). Here, agents place too much weight on new information. Conversely, when $w_{\mathcal{B}} + w_{\mathcal{L}}$ is small (i.e., regions (1) and (4)), there is momentum (positive return autocorrelation). Here, agents place too little weight on new information. When the difference between borrowers' weights and lenders' weights ($w_{\mathcal{B}} - w_{\mathcal{L}}$) is large (i.e., regions (1) and (2)), return autocorrelation increases and return variance decreases with the hazard rate of default, but when $w_{\mathcal{B}} - w_{\mathcal{L}}$ is small (i.e., regions (3) and (4)), return autocorrelation decreases and return variances increases with the hazard rate of default.

To develop intuition and prepare for the empirical work that follows, we focus on region (1). In this region, there is momentum because borrowers update like Bayesians, but lenders update more slowly. From an econometrician's point of view, prices adjust too slowly and returns exhibit momentum. When the hazard rate of default is small, prices do not reflect lenders' slow moving beliefs because they are unlikely to repossess the collateral anytime soon. When the hazard rate is high, prices are more reflective of lenders' beliefs relative to borrowers' beliefs, and returns exhibit stronger momentum and weaker volatility.

To conclude this section, we formally state our result regarding return autocorrelation and return variance.

Proposition 3 (Return Autocorrelation and Return Variance)**.** *The unconditional return autocorrelation and return variance are time-invariant. If lenders update more slowly than borrowers ($w_{\mathcal{L}} < w_{\mathcal{B}}$) and borrowers update like Bayesians ($w_{\mathcal{B}} = w_0$), returns are positively autocorrelated. Moreover, if θ is sufficiently small, return autocorrelation is strictly increasing in the hazard rate of default and return variance is strictly decreasing in the hazard rate of default.*

The condition that θ be sufficiently small does not appear to be necessary but greatly simplifies the proof. It transpires that when borrowers and lenders update at the Bayesian rate ($w_{\mathcal{B}} = w_{\mathcal{L}} = w_0$), there is no return autocorrelation. This is not to say that observed autocorrelation implies non-Bayesian updating or that non-Bayesian updating is the only explanation for momentum (e.g. Johnson, 2002). In contrast to the existing literature, however, our model uncovers a novel connection between borrower riskiness and momentum.

4.2.4 Capital Gains and Asset Heterogeneity

Before ending this section, we briefly discuss two practical issues: the autocorrelation of capital gains and the heterogeneity of assets. In many settings, utility flows or lease rates are difficult to observe, so returns are expressed exclusively in terms of capital gains. We therefore explore the applicability of the previous results to such settings. Let $\widetilde{R}_{t,t+\theta}$ be the change in the log-price:

$$\widetilde{R}_{t,t+\theta} \equiv \log(P_{t+\theta}) - \log(P_t). \tag{50}$$

The use of log-prices is common in the construction of price indices (e.g., Case and Shiller, 1989). Since existing models assume geometric growth and we consider arithmetic growth, the use of log-prices is a bit unnatural in our setting. Nevertheless, we compute the unconditional autocorrelation of the log-price changes, which is given by

$$\widetilde{\rho}_{t-\theta,t+\theta}(\lambda, w_{\mathcal{L}}, w_{\mathcal{B}}) = \frac{\mathrm{Cov}(\widetilde{R}_{t,t+\theta}, \widetilde{R}_{t-\theta,t})}{\mathrm{Var}(\widetilde{R}_{t-\theta,t})}. \tag{51}$$

In Figure 2, we plot the autocorrelation $\widetilde{\rho}$ for different values of λ under the assumptions that $w_{\mathcal{B}} = w_0$ and $w_{\mathcal{L}} < w_{\mathcal{B}}$. We draw two observations from Figure 2. First, return autocorrelation increases with borrower riskiness, just as in Proposition 3. Higher borrower riskiness increases the extent to which the price reflects lenders' beliefs, which update more slowly than prescribed by Bayes' rule. Second, return autocorrelation is non-zero when (i) borrowers do not default ($\lambda = 0$) and (ii) the price fully reflects borrowers' Bayesian beliefs. Return autocorrelation is non-zero in this case because even under no-arbitrage, capital gains are predictable (i.e., $\mathbb{E}_t[dP_t - rP_t dt] = -rV_t dt$).

One notable limitation of our analysis is our assumption that assets are homogeneous. In many of our applications, assets clearly offer heterogeneous amenities (e.g., houses, pieces of equipment, or vehicles). Such assets transact infrequently. Therefore, returns are computed at the asset-class level to make frequent observation possible. Ideally, our model would feature heterogeneous assets with stochastic sale times, and a model-implied price index could be constructed in the spirit of the repeat-sale index of Bailey et al. (1963). Such a model would need to take a stand on a variety of issues (Why does asset i trade at date t while asset j does not? Are other asset sales visible to market participants and to what extent can those sales serve as signals about market-wide growth rates?). We leave the development of such a model to future work. Alternatively, one might interpret our price P_t as a hedonic residual, where the value of observable amenities have already been projected out of the price.

5 Stylized Facts

In this section, we use U.S. residential real estate as a laboratory to explore the model's empirical relevance. We document a number of novel stylized facts in the process.

5.1 Institutional Details

The lender's belief is central to both the static and dynamic models. We use a home's appraised value as a proxy for the lender's belief about the value of collateral. A review of the institutional details surrounding appraisals is therefore in order.

In real estate, lenders use outside appraisers for at least two reasons. First, an outside appraiser helps the lender obtain an independent opinion of the market value of the property that a borrower pledges as collateral for a home loan (Eriksen et al., 2019, 2020). These valuations are important because they are used by lenders, investors, and other appraisers. Second, a home appraisal is often required by law. The Real Estate Appraisal Reform Act of 1988 requires that an appraisal be conducted by an independent and qualified appraiser for all federally-related mortgage loans. A federally-related mortgage loan includes any loan that is secured by a first lien or subordinate lien on residential real property and falls into one of the following categories: a loan made by a lender that is regulated by or whose deposits are insured by any agency of the federal government; a loan made by or insured by an agency of the federal government; a loan made in connection with a housing or urban development program administered by an agency of the federal government; a loan made and intended to be sold by the originating lender to FNMA, Government National Mortgage Association (GNMA), or FHLMC; or a loan that is the subject of a home equity conversion mortgage or reverse mortgage issued by a lender or creditor subject to the regulation.[4] As of early 2021,

[4]See RESPA and 12 USC §2602 for more details.

these mortgages accounted for at least 75% of all active single-family mortgages (Pendleton, 2021).

Although the requirements to become a licensed appraiser vary across states, most states require a combination of coursework and apprenticeships (Eriksen et al., 2020). Appraisers must follow the Universal Standards of Professional Appraisal Practice, which was adopted by Congress in 1989, in reaching an estimated value of the property. There are a number of valuation methods available to the appraiser, but the comparable sales method of valuation is the near universally-adopted approach used by appraisers to value residential property (Eriksen et al., 2020). This method can be described in three steps (Eriksen et al., 2019): First, find transactions of comparable properties that best match the subject property in physical attributes, geographic proximity, and temporal proximity; second, adjust for differences in attributes between each comparable transaction and the subject property to estimate an adjusted value for each transaction; third, apply weights to each comparable transaction to arrive at an appraised value of the subject property. Upon completion of these steps, the appraiser provides the lender an appraised value of the subject property.

5.2 Micro Evidence: Sale Prices and Lenders' Values

In this subsection, we use mortgage data to empirically examine Corollary 1, which states that greater borrower riskiness is associated with a smaller distance between the lender's estimate of value and the asset price. Unlike data on secured lending in most settings, data on mortgages for residential real estate are unique in that they allow us to observe both the equilibrium price (i.e., sale price) and the lender's estimate of value (i.e., appraised value). As discussed in Section B of the Appendix, one may alternatively think of the appraised value as the value of some ultimate investor, such as a GSE. Although we are unable to observe an objective estimate of borrower riskiness, we are able to observe variables that are highly correlated with borrowers' default risk. A meta-analysis of the determinants of residential mortgage default shows that home equity and FICO score are consistently negatively associated with default risk, and LTV ratio is consistently positively associated with default risk (Jones and Sirmans, 2015). We use these three variables as proxies for borrower riskiness.

5.2.1 Empirical Specification

We begin by mapping the model's prediction to our baseline specification. In the static model, we take $a_{\mathcal{B}}$, $a_{\mathcal{L}}$, and λ to be constants that are known (at least to the parties of a particular transaction). We now take the perspective of an econometrician and treat these quantities as random variables.

To see the relation between borrower riskiness and the distance between the lender's value and the price of the asset, we consider two cases. Suppose we condition on the appraised value being greater than the sale price: $a_{\mathcal{L}} > p^*(\lambda)$. According to Corollary 1, the lender is optimistic relative to the borrower ($a_{\mathcal{L}} > a_{\mathcal{B}}$). Rearranging and taking the natural logarithm of equation (18), we obtain

$$\log(a_{\mathcal{L}} - p^*(\lambda)) = \log(1 - \lambda) + \log(a_{\mathcal{L}} - a_{\mathcal{B}}). \tag{52}$$

Therefore, this equation predicts a negative relation between borrower riskiness and the difference between the appraised value and the sale price.

Conversely, if we condition on $p^*(\lambda) > a_{\mathcal{L}}$, then according to Corollary 1, the borrower is optimistic relative to the lender ($a_{\mathcal{B}} > a_{\mathcal{L}}$). Rearranging and taking the natural logarithm of equation (18), we obtain

$$\log(p^*(\lambda) - a_{\mathcal{L}}) = \log(1 - \lambda) + \log(a_{\mathcal{B}} - a_{\mathcal{L}}). \tag{53}$$

Therefore, this equation predicts a negative relation between borrower riskiness and the difference between the sale price and the appraised value. Taken together, the two equations predict that increasing borrower riskiness pulls the price towards the lender's value.

5.2.2 Data: Corelogic

For each of the tests in this subsection, we use data from CoreLogic Loan-Level Market Analytics. CoreLogic collects detailed data on both conforming and non-conforming mortgages (appraised value, sale price, LTV, FICO score, etc.) at origination from the 25 largest mortgages servicers in the U.S. (Lewis, 2023). These data span 1985 through 2021. Since the home price indexes we use in later tests are for single-family homes whose mortgages were originated for purchase, we focus our loan-level analysis on the same types of mortgages. We exclude observations in which any of the following are missing: appraised value, sale price, original loan balance, original LTV, zip code. Since LTV ratios use the lesser of appraised value and sale price, we use original loan balances, appraised values, and sale prices to check the integrity of the original LTV ratios provided by CoreLogic. If the LTV ratio we estimate using the variables individually are not within one percent of the original LTV ratio provided by CoreLogic, we drop the observation. We assume that loans whose LTV ratios are above 100% are collateralized in part with assets we cannot observe, so we drop those observations. We also drop observations that, in our estimation, likely have wrong values for appraised value or sale price. These include observations in which appraised value is less than or equal to 20% (1/5) of the sale price or greater than or equal to 500% (5×) of the sale price. Lastly, we drop singleton observations, which can overstate statistical significance and lead to incorrect inferences (Correia, 2015).

We present summary statistics of our variables in Table 1. Panel A focuses on observations in which the appraised value is larger than the sale price, Panel B focuses on observations in which the appraised value is smaller than the sale price, and Panel C focuses on observations in which the appraised value equals the sale price. *Appraised Value* is the reported fair market value of the property (presented in thousands). *Sale Price* is the sale price of the property (presented in thousands). *log(A-P)* is the natural logarithm of the *Appraised Value* less *Sale Price*. *log*$(P - A)$ is the natural logarithm of the opposite. *log(HE)* is the natural logarithm of home equity, which is the sale price minus loan amount at the time of loan origination (presented in thousands). *LTV* is the original mortgage amount divided by the lesser of *Appraised Value* or *Sale Price*. *FICO* is the borrower's FICO score at the time of loan origination. All variables but *Appraised Value* and *Sale Price* are winsorized at the 0.5% and 99.5% levels.

Two things deserve mention. First, there are far more observations in Panel A than there are in Panel B. We discuss this discrepancy below. Second, both appraised values and sale prices are larger in Panel B than they are in Panel A. Intuitively, instances in which the sales price is larger than the appraised value are more likely to occur with higher-priced homes because the buyers of those homes are more likely to be wealthy, and thus, be less constrained by low appraisals (via the LTV channel). We see further evidence of this lack of financial constraints in Panel B with relatively low LTV ratios.

To better understand the distribution of the difference between sale prices and appraised values, we present four histograms of the difference in Figure 3. Panel A presents the raw difference between sale prices and appraised values (bin width of $1,000). Nearly 40% of all raw differences are between -$1,000 and $0. This concentration is driven by observations in which the sale price and appraised value are identical, which occurs in 36% of all observations.

Just over 60% of all sale prices are less than appraised values, and 4% of the sale prices are above appraised values. These percentages are similar to those found in previous work (e.g., Chinloy et al., 1997, Eriksen et al., 2020, Calem et al., 2021). We also present a trimmed version of the distribution of the raw differences in Panel B. The data are trimmed at raw differences of -$25,000 and $0, relative differences of -15% and 0%, and comprise 89% of the overall sample. Of these differences, over 50% are between -$1,000 and $0.

Panel C of Figure 3 presents the relative difference between sale prices and appraised values (bin width of 0.5%). Nearly 50% of all differences are between -0.5% and 0%, and of course, the overwhelming majority of these differences are 0%. Panel D shows that in the trimmed data, there is a similarly steep decline in the distribution of differences from the concentration around zero.

There are several reasons why there are many more observations with sale price less than (or equal to) appraised value. One reason for this asymmetry is that when the sale price is greater than the appraised value, the buyer can use the low appraisal to successfully renegotiate a lower sale price, which will lead to more observations in which the appraised value equals the sale price (Calem et al., 2021). Another reason for this distribution asymmetry is when the sale price is greater than the appraised value, renegotiation fails, and no mortgage is originated (Nakamura, 2010; Fout and Yao, 2016). These observations do not show up in our data. The last reason for this distribution asymmetry is the upward bias in appraised values.[5] This bias has been estimated to be about 5% to 6% (Agarwal et al., 2015; Eriksen et al., 2020). The results in Figure 2 of Eriksen et al. (2020) suggest that the bias predominantly affects observations in which the appraised value is at most 5% higher than the sale price. Agarwal et al. (2015) finds greater biases for more highly levered loans, which biases us against finding results.[6]

The reasons for this distribution asymmetry can therefore affect our results if the reasons are in some way correlated with our proxies for borrower riskiness. We discuss and address this potential issue below.

[5]See Cho and Megbolugbe (1996), Chinloy et al. (1997), Nakamura (2010), Agarwal et al. (2015), Ding and Nakamura (2016), Fout and Yao (2016), Eriksen et al. (2019, 2020), Calem et al. (2021), and Mayer and Nothaft (2022) for examples.

[6]For example, consider a loan for which the "true" appraisal, A, is less than the price P ($A < P$). However, the appraiser biases the appraisal upward by an amount B, so that the observed appraisal, $A+B$, is greater than the price ($A+B > P$) and the loan is approved. This loan is used to estimate the coefficients of the regression of $\log(A+B-P)$ on a variety of risk measures, including LTV. Agarwal et al. (2015) claims that B is increasing in leverage (i.e., LTV), which biases us against finding the predicted, negative coefficient.

5.2.3 Appraisal, Price, and Borrower Riskiness

Corollary 1 states that increasing borrower riskiness pulls the price towards the lender's value. In Table 2, we present results consistent with this corollary. Specifically, Table 2 presents results from regressions of the appraised value minus the sale price (or the the sale price minus the appraised value) on *log(HE)*, *LTV*, and *FICO* and their first principal component. We take the negative of *log(HE)* and *FICO*, so that the predicted sign of the coefficient is negative for all proxies for borrower riskiness. Since the relation between borrower riskiness and the distance between sale price and appraised value likely varies by geographic area and time, we include zip code by time period (i.e., year by month) fixed effects in all our specifications. Standard errors are likely to be correlated in both the cross section and the time series, so we cluster standard errors by zip code and time period.

Panel A of Table 2 focuses on observations in which the appraised value is greater than or equal to the the price, so the dependent variable is either $log(A-P)$ (for OLS) or $A-P$ (for Poisson). The first four columns present results from our OLS regressions. As predicted by Corollary 1, the first three columns show that the coefficients on all three proxies for borrower riskiness are negative and highly significant. For example, the coefficient on *LTV* is -0.134, which implies that a one standard deviation increase in the LTV ratio is associated with a 13.4% decline in the difference between appraised value and sale price.

Our three proxies for borrower riskiness are highly correlated with each other, so including them all in the same regression leads to multicollinearity issues. To include information from all three proxies in the same specification and avoid these issues, we use the first principal component (i.e., *PC1*) from a principal component analysis of the three proxies. The first principal component contains nearly 70% of the explained variance. The fourth column in Panel A of Table 2 shows that the coefficient on *PC1* is -0.169, which implies that a one standard deviation increase in the first principal component is associated with a 16.9% decrease in the difference between appraised value and sale price. Since *PC1* is 91%, 89%, and 64% correlated with *log(HE)*, *LTV*, and *FICO*, respectively, the results are consistent with Corollary 1.

Using OLS allows us to better map the dependent variable in Corollary 1 to our empirical test, but since the sale price often equals the appraised value, we must drop a significant number of observations when taking the natural log of the difference between the two. To avoid dropping these observations, we also estimate our specification using Poisson regressions, which are well suited to accommodate outcomes with a value of zero (Cohn et al., 2022).

In the last four columns in Panel A of Table 2, we present results from our Poisson regressions. In these specifications, we include all observations from the first four columns and all observations in which the sale price equals the appraised value. Importantly, the last four columns of Panel A show that coefficients obtained from estimating our Poisson regressions are also negative and highly significant, which is consistent with our results obtained from our OLS regressions and Corollary 1.

Panel B of Table 2 focuses on observations in which the sale price is greater than or equal to the the appraised value. Since sale prices are seldomly higher than appraised values, the number of observations in these regressions is much smaller. Nonetheless, the results are largely consistent with those in Panel A. Specifically, all coefficients from both OLS regressions and Poisson regressions are negative, and all but one are highly significant.

Overall, the results in Table 2 provide evidence consistent with Corollary 1, which states that increasing borrower riskiness pulls the price toward the lender's value.

5.2.4 Robustness

As discussed above, we work through several extensions of the static model in Section B of the Appendix. In this subsection, we discuss the results from empirical tests that map to two of those extensions and discuss the results from an empirical test that incorporates appraisal bias. These results can be found in Section C of the Appendix.

In our first empirical exploration of our theoretical extensions, we consider the possibility that the lender is unable to recover the full value of the asset in the event of default (a possibility explored in Corollary 2 in Section B.2 in the Appendix). In this case, borrower riskiness is negatively associated with the distance between the equilibrium price and the lender's effective value, which we define as the lender's value net of repossession costs. After adjusting appraised values for repossession costs of 5% and 10%, we find that our results are consistent with Corollary 2 for all proxies for borrower riskiness and their first principal component.

In Corollary 3 (Section B.6 in the Appendix), we focus on the situation in which the lender can sell the loan. In this setting, the lender agrees with both the borrower and the seller about the value of the asset, but an outside investor of the loan holds a different view on the value of the asset. The investor is able to impose her beliefs on the value of the asset, because if those beliefs are not reflected, the investor will not purchase the collateralized loan. Corollary 3 therefore implies that borrower riskiness is negatively associated with the distance between sale price and appraised value among loans that are sold to outside investors. Using conforming mortgages as loans that are likely to be sold to outside investors, we find that our results are consistent with Corollary 3 for all proxies for borrower riskiness and their first principal component.

In our last robustness test of the micro evidence, we ensure our results are robust to adjusting appraised values for appraisal bias. As discussed above, a significant portion of our observations might be affected by appraisal bias or renegotiation, and these affected observations might lead to a mechanical relation between our dependent variable and independent variables. However, when we split our samples into observations that are less likely to be affected by appraisal bias and observations that are more likely to be affected by appraisal bias, we find that our results are consistent with Corollary 1 in each group.

5.3 Macro Evidence: Return Momentum and Return Volatility

We now turn our attention to the predictions of the dynamic model. In particular, Proposition 3 predicts that return autocorrelation should be strongest and return volatility should be weakest during episodes of high borrower riskiness.

5.3.1 Data: Freddie Mac and FHFA HPI

For the tests in this section, we obtain data on our proxies for borrower riskiness (i.e., initial home equity, LTV, and FICO) from Freddie Mac. These data are available at quarterly intervals, cover loans for purchase of single-family homes, and span 1999 through 2021.[7] Freddie Mac is one of two sources of mortgages for the Federal Housing Finance Agency House Price Index (FHFA HPI), which we use to estimate housing returns and their associated volatility. We use the FHFA quarterly purchase-only indexes of the 100 largest metropolitan statistical areas.

We present summary statistics of the data in Table 3. *Log Return* is the annual change in the log value using quarterly data of the non-seasonally adjusted FHFA HPI at the CBSA (Core-Based Statistical Area) level. *Volatility* is the time-varying standard deviation of the error term from a first-order GARCH model of *Log Return* on a one-year lag of itself within each CBSA. We use the full sample of FHFA HPI data going back to 1991 to estimate *Volatility* before matching it with our loan data from Freddie Mac. Our proxies for borrower riskiness are at the loan level, so to obtain estimates at the CBSA level, we estimate the weighted average of each proxy within each CBSA×year×quarter combination and use loan

[7]See https://www.freddiemac.com/research/datasets/sf-loanlevel-dataset for more details.

value as the weight. The distributions of our proxies are similar to those in Table 1, but since we average loan-level observations at the CBSA×year×quarter level before estimating the distribution, the ranges and standard deviations are smaller.

5.3.2 Slow Appraisals

As discussed above, the near universally-adopted approach used by appraisers to value residential property is the comparable sales valuation method. However, this method is likely a major contributor for why appraisers engage in appraisal smoothing, or appraisal lag (Clayton et al., 2001).[8] Appraisal lag leads to appraised values that tend to lag the real estate price cycle (Matysiak and Wang, 1995). Appraisers tend to underestimate value in rising markets and overestimate value in falling markets (Diaz III and Wolverton, 1998). In other words, appraisers, and by extension lenders, update their estimates of asset values more slowly than buyers and sellers.[9]

This phenomenon can be seen in Figure 4, which plots two national FHFA HPIs. The FHFA HPIs are weighted, repeat-sales indexes estimated from repeat mortgage transactions on single-family properties whose mortgages have been purchased or securitized by Fannie Mae or Freddie Mac. The all-transactions index is estimated using sale prices and appraised values, and the purchase-only index is estimated using only sale prices.[10] Since the all-transactions index includes estimates from both sales and appraisals, and not just appraisals, we can interpret any differences between the two indexes as a lower bound on the actual distance between a hypothetical appraisal-only index and the actual purchase-only index.

With that interpretation in mind, Figure 4 shows that after home prices peaked in early 2007, the all-transactions index stayed above the purchase-only index for several years (i.e., overestimated value in falling markets). Once the purchase-only index started to rise again at the beginning of 2012, it quickly surpassed the all-transactions index and stayed above it through 2021 (i.e., underestimated value in rising markets).

[8]See Geltner et al. (2003) and the references therein for a review of the literature on appraisal smoothing.

[9]It is important to note that appraisal lag and appraisal bias are not incompatible with each other. Appraised values may be systematically higher than sale prices but changes in appraised values may still lag changes in sale prices. Since indexes, such as those in Figure 4, are estimated as a function of the starting value of the same series, only comparisons of changes, and not levels, can be made between indexes.

[10]Both indexes, which are at the national level, have been indexed to 100 beginning the first quarter of 1991, for that is the first available data for the purchase-only index. See https://www.fhfa.gov/DataTools/Downloads/Pages/House-Price-Index-Datasets.aspx#mpo for more details.

5.3.3 Return Momentum and Borrower Riskiness

Table 4 presents results that map to the first part of Proposition 3, which states that return autocorrelation increases with the hazard rate of default. Rather than simply interact lagged return with our proxies for borrower riskiness, as stated in Proposition 3, we interact lagged return with dummy variables that turn on when our proxies are in a given part of the distribution. Specifically, we interact lagged return with *Low*, *Mid*, and *High*, which are dummy variables indicating whether the proxy for borrower riskiness of interest is in the lowest, middle, or highest time-series tercile within a given CBSA. Using dummies allows us to account for any non-linearities in the relation between returns and the interaction of our proxies and lagged return. Since standard errors are likely to be correlated in both the cross section and the time series, we cluster standard errors by CBSA and time period (year by quarter).

The first column of Table 4 shows that annual residential real estate returns are autocorrelated, which is consistent with previous work. In the next three columns, we interact lagged return with our dummy variables. Recall that borrower riskiness is higher when home equity is lower, LTV is higher, and FICO is lower. Therefore, the results in these three columns show that return autocorrelation increases nearly monotonically as borrower riskiness increases.

In the last column, we sort by *PC1*, which is the first principal component of the three variables. We find that return autocorrelation increases as *PC1* decreases. Since *PC1* is positively correlated with both *log(HE)* and *FICO* and negatively correlated with *LTV*, the results in the last column also show that return autocorrelation increases with borrower riskiness.[11]

In summary, the results in Table 4 provide evidence consistent with Proposition 3, which states that return autocorrelation is higher when default is more likely.

5.3.4 Return Volatility and Borrower Riskiness

The second part of Proposition 3 states that return variance (and therefore volatility) is decreasing with the hazard rate of default. We investigate the empirical relevance of this prediction by analyzing simple means of volatilities after sorting firms by our different prox-

[11] Table C5 in the Appendix shows that our momentum results are nearly identical when using simple, rather than weighted, averages to calculate our quarter-level proxies for borrower riskiness. Table C6 in the Appendix shows that our results are robust to interacting lagged returns with our proxies for borrower riskiness directly rather than interacting lagged returns for dummies.

ies for borrower riskiness. As discussed above, we estimate volatility as the time-varying standard deviation of the error term from a first-order GARCH model of *Log Return* on a one-year lag of itself within each CBSA. We present the results in Table 5.

Panel A of Table 5 shows that average volatilities decline monotonically with borrower riskiness for each one of our proxies and their first principal component. For example, the first column shows that average volatility is 3.54% per quarter for observations in the lowest *log(HE)* tercile and 6.22% per quarter for observations in the highest *log(HE)* tercile. Since borrower riskiness decreases with *log(HE)*, the results in the first column show that volatility decreases with borrower riskiness.

Importantly, Panel B of Table 5 shows that many of these differences are both economically and statistically significant. For example, the last column shows that volatilities for observations in the highest *PC1* tercile are 2.79 percentage points higher than those in the lowest *PC1*. The associated clustered standard error, which is obtained from a regression of volatility on the *Low* and *Mid* tercile dummy, is less than a quarter of the coefficient.[12,13]

In summary, the results in Table 5 provide evidence consistent with the second part Proposition 3, which states that return variance, and therefore volatility, decreases with borrower riskiness.

6 Conclusion

In this paper, we present a model of secured lending in which borrowers and lenders disagree about the value of collateral. The prices of assets that serve as collateral reflect the beliefs of both borrowers and lenders. Specifically, prices are more reflective of lenders' beliefs when borrowers are riskier and more reflective of borrowers' beliefs when borrowers are safer. In a dynamic version of the model, we consider the possibility that borrowers and lenders place different (and potentially non-Bayesian) weights on new information. We establish conditions under which returns exhibit momentum or reversal. Most relevant for collateralized lending, we explore the role of borrower riskiness as a mediating factor. Specifically, we investigate conditions under which borrower riskiness exacerbates or attenuates momentum/reversal and how borrower riskiness affects return volatility.

[12] Each standard error is obtained from a regression of volatility on a combination (depending on the volatility difference of interest) of tercile dummies for *Low*, *Middle*, and *High*.

[13] Table C7 in the Appendix shows that our volatility results are nearly identical when using simple, rather than weighted, averages to calculate our quarter-level proxies for borrower riskiness. Table C8 in the Appendix shows that our results hold when we run simple univariate regressions of return volatility on our proxies for borrower riskiness.

To demonstrate the model's empirical relevance, we investigate its implications for the U.S. residential real estate market. Using a large sample of U.S. home loans, we show that sale prices are closer to appraised values when borrower are riskier. Using housing returns in the 100 largest U.S. metro areas, we show that momentum is strongest and variance weakest when borrowers are riskier. Taken together, our evidence is consistent with a specification of the model in which borrowers update like Bayesians, and lenders update less aggressively than prescribed by Bayes' rule. Although we focus on residential real estate, the model applies to any setting in which an asset collateralizes its own financing, such as accounts receivable, buildings, equipment, inventory, land, and vehicles.

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Figure 1: Return Autocorrelation and Return Variance

This figure shows how return autocorrelation ($\rho(\lambda, w_{\mathcal{B}}, w_{\mathcal{L}})$) and return volatility ($\sigma^2(\lambda; w_{\mathcal{L}}, w_{\mathcal{B}})$) in our model changes as borrowers' weight on new information ($w_{\mathcal{B}}$) and lenders' weight on new information ($w_{\mathcal{L}}$) change. The parameters used in the simulation to generate this figure are $r = \kappa = 0.1$, $\tau_X = \tau_V = 1$, $\lambda = 0.05$, and $\theta = 0.2$. The hatched line in region (1) is the beliefs space we consider in Proposition 3.

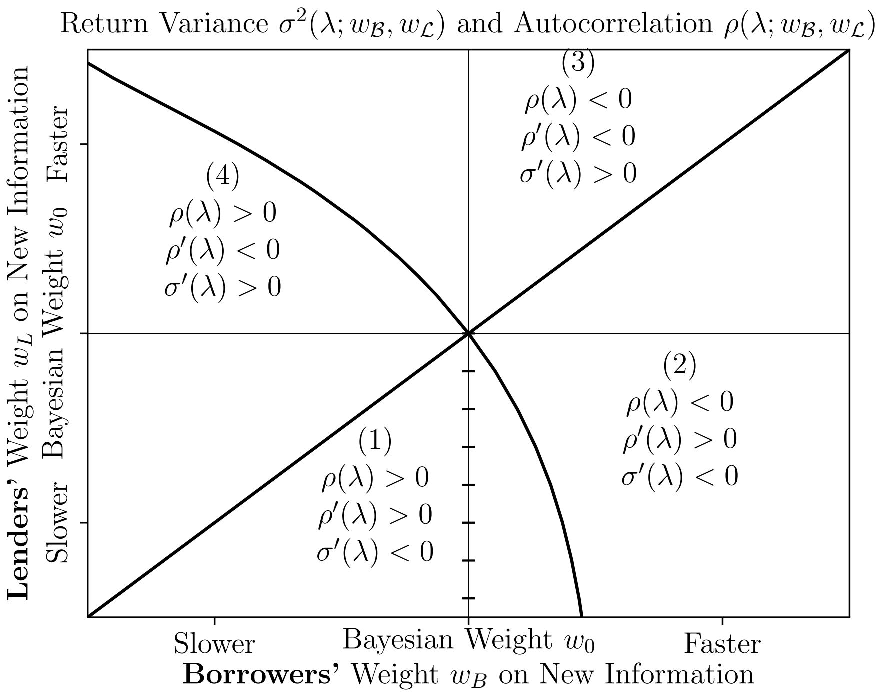

Figure 2: Capital Gain Rate Autocorrelation

This figure presents the log capital gain rate autocorrelation for different values of the hazard rate of default λ. The parameters used in the simulation to generate this figure are $r = \kappa = 0.1$, $\tau_X = \tau_V = 1$, $\theta = 0.2$, $w_{\mathcal{B}} = w_0$, and $w_{\mathcal{L}} = w_0/2$. For each λ, we simulate 10 million return series over the time interval $[0, 20]$ to compute the autocorrelation.

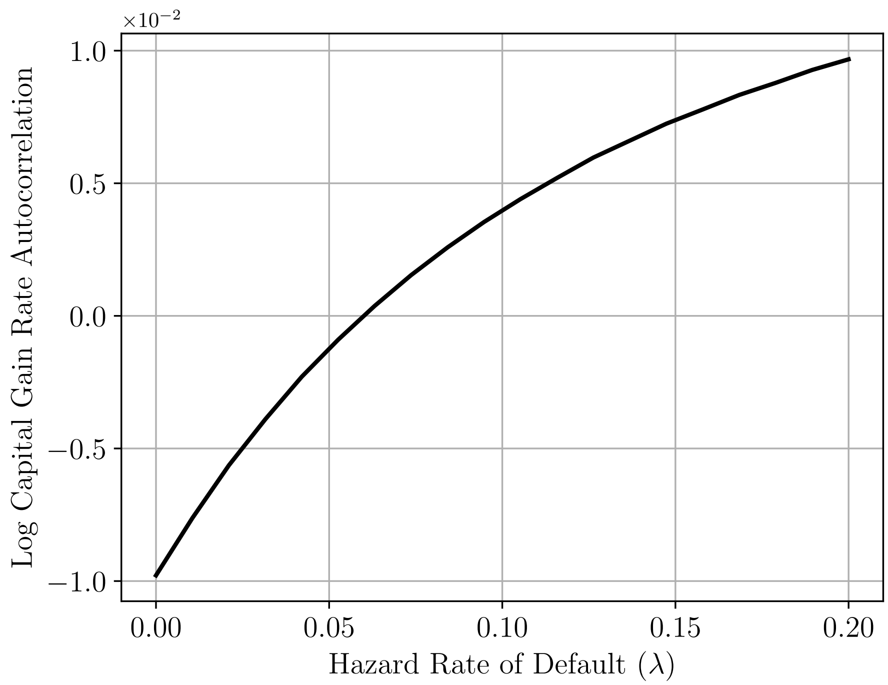

Figure 3: Sale Price and Appraised Value

This figure presents differences between sale prices and appraised values in our loan-level data from CoreLogic. Panel A presents the raw difference (in thousands of dollars). Panel B presents the raw difference (in thousands of dollars) after trimming the data. Panel C presents the relative difference. Panel D presents the relative difference after trimming the data. In Panels A and C, both *Sale Price* and *Appraised Value* are winsorized at the 0.5% and 99.5% levels. In Panels B and D, data are trimmed at raw differences of -$25,000 and $0 and relative differences of -15% and 0%.

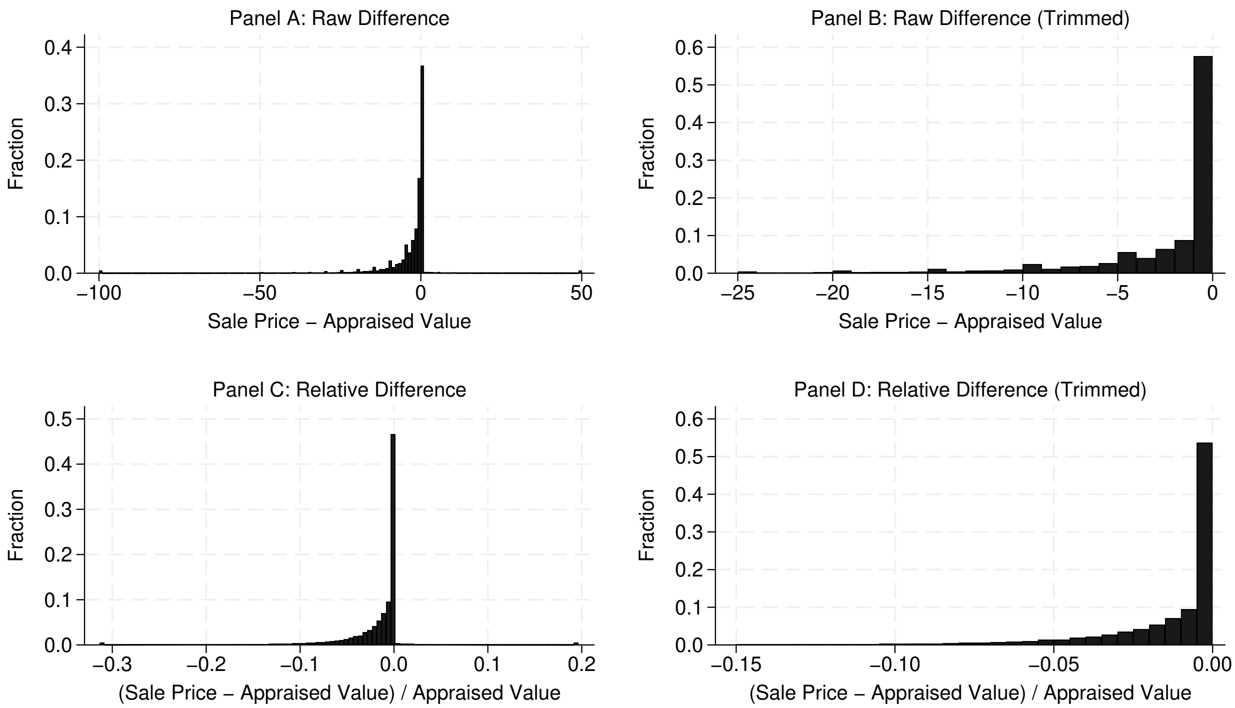

Figure 4: Slow Appraisals

This figure presents two price indexes from the Federal Housing Finance Agency. The All-Transactions Index is estimated using appraised values and sale prices. The Purchase-Only Index is estimated using sale prices. Both indexes are constructed using national observations at the quarterly level and are indexed to 100 beginning 1991.

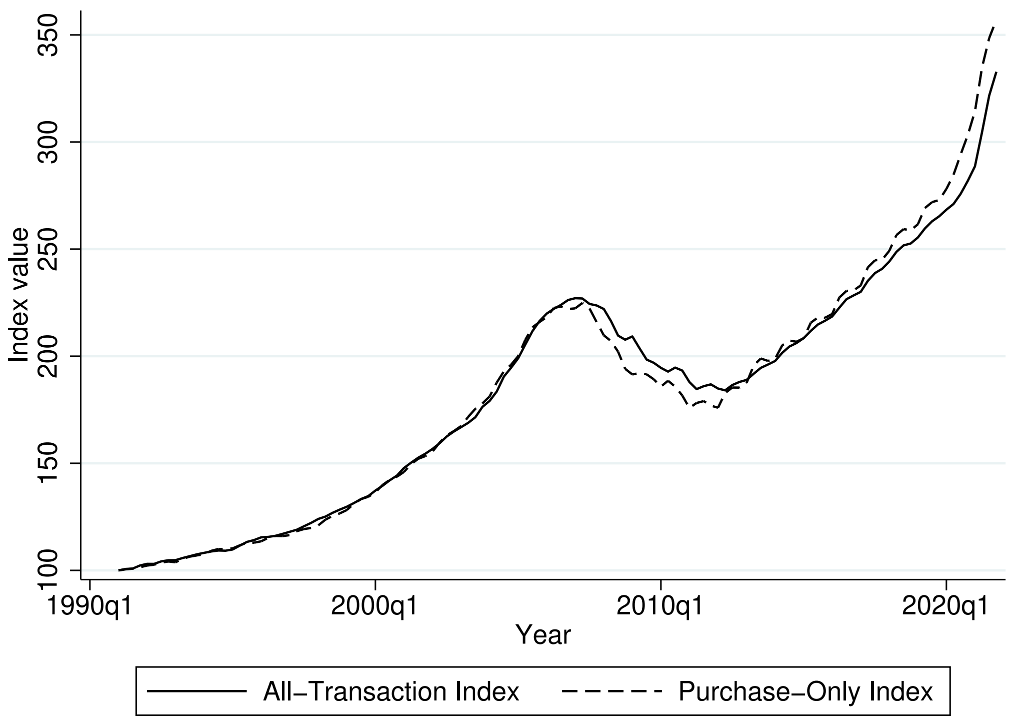

Table 1: Summary Statistics: CoreLogic

This table presents summary statistics of our loan-level data from CoreLogic. Panel A focuses on observations in which the appraised value is larger than the sale price, Panel B focuses on observations in which the appraised value is smaller than the sale price, and Panel C focuses on observations in which the appraised value equals the sale price. *Appraised Value* is the reported fair market value of the property (presented in thousands). *Sale Price* is the sale price of the property (presented in thousands). *log(A-P)* is the natural logarithm of the *Appraised Value* less *Sale Price*. $log(P-A)$ is the natural logarithm of the opposite. *log(HE)* is the natural logarithm of home equity, which is the sale price minus loan amount at the time of loan origination. *LTV* is the original mortgage amount divided by the lesser of *Appraised Value* or *Sale Price*. *FICO* is the borrower's FICO score at the time of loan origination. All variables but *Appraised Value* and *Sale Price* are winsorized at the 0.5% and 99.5% levels.

Panel A: Appraised value larger than price								
	Mean	SD	10th	25th	Median	75th	90th	Obs.
Appraised Value (A)	230.8	211.7	82.0	117.0	171.0	275.0	430.0	11,030,416
Sale Price (P)	222.8	203.9	78.5	112.9	165.6	267.0	419.0	11,030,416
log(A-P)	7.79	1.65	5.52	6.91	8.01	8.85	9.77	11,030,416
log(HE)	9.67	1.73	7.27	8.60	10.04	10.90	11.57	11,030,416
LTV	83.8%	14.3%	67.2%	80.0%	80.0%	95.0%	98.6%	11,030,416
FICO	711	66.6	619	666	720	766	790	7,274,646
Panel B: Price larger than appraised value								
	Mean	SD	10th	25th	Median	75th	90th	Obs.
Appraised Value (A)	283.4	278.1	92.5	137.5	215.0	345.0	522.4	309,186
Sale Price (P)	318.8	309.4	106.5	155.0	243.2	389.0	590.2	309,186
log(P-A)	9.04	2.00	6.40	7.82	9.21	10.55	11.52	309,186
log(HE)	10.99	1.25	9.29	10.38	11.84	12.39	12.71	309,186
LTV	77.5%	16.4%	54.8%	71.2%	80.0%	90.0%	96.5%	309,186
FICO	721	63.8	631	680	732	774	795	126,969
Panel C: Price equals appraised value								
	Mean	SD	10th	25th	Median	75th	90th	Obs.
Appraised Value (A)	257.0	280.4	78.4	115.0	178.0	305.0	500.0	8,550,607
Sale Price (P)	257.0	280.4	78.4	115.0	178.0	305.0	500.0	8,550,607
log(HE)	10.01	10.33	7.64	9.06	10.33	11.18	11.88	8,550,607
LTV	80.6%	80.0%	59.6%	75.0%	80.0%	94.5%	98.4%	8,550,607
FICO	710	67.7	617	665	720	766	790	4,397,448

Table 2: Appraisal, Price, and Borrower Riskiness

This table presents results from regressions of the distance between appraised value and sale price on borrower riskiness. In our ordinary least squares (OLS) regressions, the dependent variable is $log(A - P)$ in Panel A and $log(P - A)$ in Panel B. In our Poisson regressions, the dependent variable is *A-P* in Panel A and *P-A* in Panel B. *A* is *Appraised Value*, and *P* is *Sale Price*. Our proxies for borrower riskiness are *log(HE)*, *LTV*, and *FICO*. We take the negative of *log(HE)* and *FICO* so that the sign of the predicted coefficient is the same for all variables. *PC1* is the first principal component of the three proxies. See Table 1 for variable definitions. All regression variables are winsorized at the 0.5% and 99.5% levels. *LTV*, *FICO*, and *PC1* are standardized to zero mean and unit standard deviation. Standard errors below coefficients are adjusted for clustering at the zip code and year by month levels. Statistical significance at the 1%, 5%, and 10% levels is denoted by ***, **, and *.

Panel A: Appraised value greater than or equal to the price

	OLS				Poisson			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log(HE)	-0.122***				-0.079***			
	(0.002)				(0.002)			
LTV		-0.134***				-0.058***		
		(0.002)				(0.003)		
FICO			-0.076***				-0.060***	
			(0.002)				(0.003)	
PC1				-0.169***				-0.120***
				(0.003)				(0.003)
Zip×Year×Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	17.8%	16.5%	15.0%	15.9%				
Pseudo R^2					37.2%	34.9%	35.7%	36.3%
Observations	11,030,416	11,030,416	7,274,646	7,274,646	20,095,354	20,095,354	12,318,206	12,318,206

Panel B: Price greater than or equal to the appraised value

	OLS				Poisson			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log(HE)	-0.988***				-1.967***			
	(0.021)				(0.031)			
LTV		-0.069***				-0.055***		
		(0.017)				(0.017)		
FICO			-0.083***				-0.016	
			(0.016)				(0.029)	
PC1				-0.521***				-0.771***
				(0.011)				(0.019)
Zip×Year×Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	56.8%	39.4%	34.3%	39.0%				
Pseudo R^2					72.2%	45.1%	45.0%	50.3%
Observations	309,186	309,186	126,969	126,969	3,524,647	3,524,647	1,532,085	1,532,085

Table 3: Summary Statistics: Freddie Mac and FHFA HPI

This table presents summary statistics of returns derived from FHFA HPI and our loan data from Freddie Mac. Data from both sources focus on single-family homes that were purchased, not refinanced, and are of the 100 largest CBSAs. *Log Return* is the annual change in the log value using quarterly data of the non-seasonally adjusted FHFA HPI at the CBSA (Core-Based Statistical Area) level. *Volatility* is the time-varying standard deviation of the error term from a first-order GARCH model of *Log Return* on a one-year lag of itself within each CBSA. *log(HE)* is the weighted average of quarterly home equity values (i.e., natural logarithm of home equity estimated from LTV and the loan amount at the time of loan origination) within a CBSA. *LTV* is the weighted average of quarterly loan-to-value ratios within a CBSA. *FICO* is the weighted average of quarterly FICO scores within a CBSA. All averages are calculated with value used as the weight.

	Mean	SD	10th	25th	Median	75th	90th	Obs.
Log Return	4.40%	8.02%	-0.05%	1.33%	4.67%	8.34%	13.32%	8,772
Volatility	4.72%	4.71%	1.39%	1.96%	3.08%	5.70%	9.95%	8,772
log(HE)	11.32	0.53	10.75	10.95	11.24	11.61	12.06	8,772
LTV	76.8%	6.1%	70.3%	74.3%	77.6%	81.0%	83.2%	8,772
FICO	746	16.2	722	734	751	757	763	8,772

Table 4: Return Autocorrelation and Borrower Riskiness

This table presents results from regressions of *Log Return* on a one-year lag of itself interacted with dummies for our different proxies for borrower riskiness. *Log Return* is the annual change in log value using quarterly data of the non-seasonally adjusted FHFA HPI. *Low*, *Mid*, and *High* are dummy variables indicating whether the proxy in that column is in the lowest, middle, or highest tercile within a given CBSA. Our proxies for borrower riskiness are *log(HE)*, *LTV*, and *FICO*. *PC1* is the first principal component of the three proxies. See Table 3 for variable definitions. Panel A presents estimates of return autocorrelation for each tercile. Panel B presents return autocorrelation differences between terciles. Standard errors below coefficients are adjusted for clustering at the CBSA and year by quarter levels. Statistical significance at the 1%, 5%, and 10% levels is denoted by ***, **, and *.

Panel A: Return autocorrelation

	No Sort (1)	log(HE) (2)	LTV (3)	FICO (4)	PC1 (5)
Log $Return_{t-4}$	0.68***				
	(0.07)				
Log $Return_{t-4}$ × Low		0.79***	0.56***	0.78***	0.90***
		(0.10)	(0.07)	(0.08)	(0.10)
Log $Return_{t-4}$ × Mid		0.73***	0.72***	0.65***	0.77***
		(0.08)	(0.09)	(0.09)	(0.09)
Log $Return_{t-4}$ × High		0.58***	0.93***	0.58***	0.53***
		(0.08)	(0.11)	(0.11)	(0.07)
Adjusted R^2	41.4%	42.3%	43.6%	42.1%	43.9%
Observations	8,772	8,772	8,772	8,772	8,772

Panel B: Differences in return autocorrelation

	No Sort (1)	log(HE) (2)	LTV (3)	FICO (4)	PC1 (5)
Log $Return_{t-4}$ ×	0.68***	0.79***	0.56***	0.78***	0.90***
	(0.07)	(0.10)	(0.07)	(0.08)	(0.10)
Log $Return_{t-4}$ × Mid		-0.05	0.16**	-0.13*	-0.12***
		(0.05)	(0.08)	(0.07)	(0.05)
Log $Return_{t-4}$ × High		-0.21**	0.37***	-0.20	-0.37***
		(0.10)	(0.11)	(0.13)	(0.10)
Adjusted R^2	41.4%	42.3%	43.6%	42.1%	44.1%
Observations	8,772	8,772	8,772	8,772	8,772

Table 5: Return Volatility and Borrower Riskiness

This table presents average volatilities within different subsamples. *Volatility* is the time-varying standard deviation of the error term from a first-order GARCH model of *Log Return* on a one-year lag of itself within each CBSA. *Log Return* is the annual change in log value using quarterly data of the non-seasonally adjusted FHFA HPI. *Low*, *Mid* , and *High* indicate whether an observation is in the lowest, middle, or highest tercile of a given proxy for borrower riskiness within a given CBSA. Our proxies for borrower riskiness are *log(HE)*, *LTV*, and *FICO*. *PC1* is the first principal component of the three proxies. See Table 3 for variable definitions. Panel A presents average volatilities. Panel B presents differences in volatilities (i.e., percentage points) between terciles that are identified within each proxy for borrower riskiness. Standard errors below differences are obtained from regressions of volatility on tercile dummies for the proxies for borrower riskiness. Standard errors are adjusted for clustering at the CBSA and year by quarter levels. Statistical significance at the 1%, 5%, and 10% levels is denoted by ***, **, and *.

Panel A: Return volatility

	log(HE) (1)	LTV (2)	FICO (3)	PC1 (4)
Low	3.54%	5.66%	3.79%	3.41%
Mid	4.45%	4.64%	4.71%	4.41%
High	6.22%	3.85%	5.70%	6.39%

Panel B: Differences in volatility

	log(HE) (1)	LTV (2)	FICO (3)	PC1 (4)
High-Low	2.68***	1.81***	1.91***	2.98***
	(0.39)	(0.56)	(0.44)	(0.43)
High-Mid	1.77***	0.79*	0.99*	1.98***
	(0.31)	(0.40)	(0.52)	(0.36)
Mid-Low	0.91***	1.02***	0.93*	1.00***
	(0.18)	(0.33)	(0.50)	(0.21)

A Proofs

A.1 Proof of Proposition 1

Let ψ be the log of the product in equation (10):

$$\psi(c,p) = \sum_j \eta_j \log\left(\mathbb{E}^j[S_j(c,p)]\right). \tag{54}$$

It can be shown that $\psi_c(c^*(\lambda), p^*(\lambda)) = \psi_p(c^*(\lambda), p^*(\lambda)) = 0$ (first-order conditions), and $\psi_{cc}(c^*(\lambda), p^*(\lambda)) < 0$ and $\psi_{cc}(c^*(\lambda), p^*(\lambda))\psi_{pp}(c^*(\lambda), p^*(\lambda)) > \psi_{cp}(c^*(\lambda), p^*(\lambda))^2$ (second-order conditions for local maximum).

A.2 Proof of Proposition 2

Conjecture a linear equilibrium of the form

$$P_t = c_V V_t + c_{\mathcal{B}} \widehat{X}_t^{\mathcal{B}} + c_{\mathcal{L}} \widehat{X}_t^{\mathcal{L}}, \tag{55}$$

where c_V, $c_{\mathcal{B}}$, and $c_{\mathcal{L}}$ are known constants. Integrating equation (20) from t to $s > t$, we obtain

$$X_s = X_t e^{-\kappa(s-t)} + \sqrt{2\kappa}\sigma_X \int_t^s e^{-\kappa(s-\tau)} dB_\tau^X. \tag{56}$$

Therefore,

$$\mathbb{E}_t^j[X_s] = \widehat{X}_t^j e^{-\kappa(s-t)}. \tag{57}$$

Integrating equation (22) from t to T, we obtain

$$\widehat{X}_T^j = e^{-(\kappa+w_j)(T-t)} \widehat{X}_t^j + w_j \int_t^T e^{-(\kappa+w_j)(T-s)} dV_s. \tag{58}$$

Therefore, i's expectation of j's belief is

$$\begin{aligned} \mathbb{E}_t^i[\widehat{X}_T^j|T] &= e^{-(\kappa+w_j)(T-t)} \widehat{X}_t^j + w_j \int_t^T e^{-(\kappa+w_j)(T-s)} \widehat{X}_t^i e^{-\kappa(s-t)} ds && (59) \\ &= e^{-(\kappa+w_j)(T-t)} \widehat{X}_t^j + \left(e^{-\kappa(T-t)} - e^{-(\kappa+w_j)(T-t)}\right) \widehat{X}_t^i, && (60) \end{aligned}$$

having used the fact that

$$\mathbb{E}_t^i[dV_s] = \mathbb{E}_t^i[X_s ds + \sigma_V dB_s^V] = \widehat{X}_t^i e^{-\kappa(s-t)} ds. \tag{61}$$

The value at time T is given by

$$V_T = V_t + \int_t^T X_s ds + \sigma_V \left(B_T^V - B_t^V\right). \tag{62}$$

Hence,

$$\mathbb{E}_t^j[V_T] = V_t + \int_t^T \mathbb{E}_t^j[X_s] ds = V_t + \int_t^T \widehat{X}_t^j e^{-\kappa(s-t)} ds = V_t + \kappa^{-1}\left(1 - e^{-\kappa(T-t)}\right)\widehat{X}_t^j. \tag{63}$$

For any constant $a > 0$,

$$\mathbb{E}_t\left[e^{-a(T-t)}\right] = \int_t^\infty \lambda e^{-(a+\lambda)(T-t)} dT = \frac{\lambda}{a+\lambda}. \tag{64}$$

First, the buyer's expected value from owning the asset is

$$\mathcal{V}_t = \mathbb{E}_t\left[\int_t^T r\mathbb{E}_t^{\mathcal{B}}[V_s|T] e^{-r(s-t)} ds\right] \tag{65}$$

$$= \mathbb{E}_t\left[\int_t^T r\left(V_t + \kappa^{-1}\left(1 - e^{-\kappa(s-t)}\right)\widehat{X}_t^{\mathcal{B}}\right) e^{-r(s-t)} ds\right] \tag{66}$$

$$= r(r+\lambda)^{-1} V_t + r(r+\lambda)^{-1}(r+\kappa+\lambda)^{-1}\widehat{X}_t^{\mathcal{B}}. \tag{67}$$

Next, we compute the lender's expected recovery value. From equation (60),

$$\mathbb{E}_t^{\mathcal{L}}[V_T|T] = V_t + \kappa^{-1}\left(1 - e^{-\kappa(T-t)}\right)\widehat{X}_t^{\mathcal{L}} \tag{68}$$

$$\mathbb{E}_t^{\mathcal{L}}[\widehat{X}_T^{\mathcal{B}}|T] = e^{-(w_{\mathcal{B}}+\kappa)(T-t)}\widehat{X}_t^{\mathcal{B}} + \left(e^{-\kappa(T-t)} - e^{-(w_{\mathcal{B}}+\kappa)(T-t)}\right)\widehat{X}_t^{\mathcal{L}} \tag{69}$$

$$\mathbb{E}_t^{\mathcal{L}}[\widehat{X}_T^{\mathcal{L}}|T] = e^{-\kappa(T-t)}\widehat{X}_t^{\mathcal{L}}. \tag{70}$$

Therefore,

$$\mathbb{E}_t\left[\mathbb{E}_t^{\mathcal{L}}[V_T|T]e^{-r(T-t)}\right] = \lambda(r+\lambda)^{-1}V_t + \lambda(r+\lambda)^{-1}(r+\kappa+\lambda)^{-1}\widehat{X}_t^{\mathcal{L}} \tag{71}$$

$$\mathbb{E}_t\left[\mathbb{E}_t^{\mathcal{L}}[\widehat{X}_T^{\mathcal{B}}|T]e^{-r(T-t)}\right] = \lambda(w_{\mathcal{B}}+\kappa+r+\lambda)^{-1}\widehat{X}_t^{\mathcal{B}} + \lambda w_{\mathcal{B}}(r+\kappa+\lambda)^{-1}(r+w_{\mathcal{B}}+\kappa+\lambda)^{-1}\widehat{X}_t^{\mathcal{L}} \tag{72}$$

$$\mathbb{E}_t\left[\mathbb{E}_t^{\mathcal{L}}[\widehat{X}_T^{\mathcal{L}}|T]e^{-r(T-t)}\right] = \lambda(r+\kappa+\lambda)^{-1}\widehat{X}_t^{\mathcal{L}} \tag{73}$$

and therefore, the expected recovery value is

$$\mathcal{R}_t = \mathbb{E}_t\left[\mathbb{E}_t^{\mathcal{L}}[P_T|T]e^{-r(T-t)}\right] \tag{74}$$

$$= \mathbb{E}_t\left[\left(c_V\mathbb{E}_t^{\mathcal{L}}[V_T|T] + c_{\mathcal{B}}\mathbb{E}_t^{\mathcal{L}}[\widehat{X}_T^{\mathcal{B}}|T] + c_{\mathcal{L}}\mathbb{E}_t^{\mathcal{L}}[\widehat{X}_T^{\mathcal{L}}|T]\right)e^{-r(T-t)}\right], \tag{75}$$

which can be computed using equations (71) through (73). Rewriting equation (29),

$$P_t = \mathcal{V}_t + \mathcal{R}_t \tag{76}$$

$$\begin{aligned} &= \left(r(r+\lambda)^{-1} + c_V\lambda(r+\lambda)^{-1}\right)V_t \\ &\quad + \left(r(r+\lambda)^{-1}(r+\kappa+\lambda)^{-1} + c_{\mathcal{B}}\lambda((w_{\mathcal{B}}+\kappa)+r+\lambda)^{-1}\right)\widehat{X}_t^{\mathcal{B}} \\ &\quad + \lambda(r+\kappa+\lambda)^{-1}\left(c_V(r+\lambda)^{-1} + c_{\mathcal{B}}w_{\mathcal{B}}(r+(w_{\mathcal{B}}+\kappa)+\lambda)^{-1} + c_{\mathcal{L}}\right)\widehat{X}_t^{\mathcal{L}}. \end{aligned} \tag{77}$$

Comparing terms, we have that $c_V(\lambda) = 1$, and $c_{\mathcal{B}}$ and $c_{\mathcal{L}}$ are given by the expressions in equations (35) and (36) respectively. It follows that

$$c'_{\mathcal{L}}(\lambda) = \frac{r\left(w_{\mathcal{B}}(\kappa+2(\lambda+r)) + (\kappa+\lambda+r)^2\right)}{(\lambda+r)^2(\kappa+\lambda+r)^2(\kappa+r+w_{\mathcal{B}})} > 0. \tag{78}$$

and

$$(r+\kappa)(c_{\mathcal{B}}(\lambda) + c_{\mathcal{L}}(\lambda)) = 1. \tag{79}$$

It remains to compute the equilibrium repayment. Rearranging the no-surplus condition for the borrower, we obtain

$$(1-\ell)P_t = \mathbb{E}_t\left[\int_t^T r\mathbb{E}_t^{\mathcal{B}}[V_s|T]e^{-r(s-t)}ds\right] + \mathbb{E}_t\left[\int_t^T rC_te^{-r(s-t)}\right] \tag{80}$$

$$= r(r+\lambda)^{-1}V_t + r(r+\lambda)^{-1}(r+\kappa+\lambda)^{-1}\widehat{X}_t^{\mathcal{B}} - r(r+\lambda)^{-1}C_t, \tag{81}$$

and hence

$$C_t = V_t + (r+\kappa+\lambda)^{-1}\widehat{X}_t^{\mathcal{B}} - r^{-1}(r+\lambda)(1-\ell)P_t \tag{82}$$

$$= b_V(\lambda)V_t + b_{\mathcal{B}}(\lambda)\widehat{X}_t^{\mathcal{B}} + b_{\mathcal{L}}(\lambda)\widehat{X}_t^{\mathcal{L}}, \tag{83}$$

where b_V, $b_{\mathcal{B}}$, and $b_{\mathcal{L}}$ are given by equations (32), (33), and (34) respectively. This concludes the proof. ■

A.3 Proof of Proposition 3

We begin by establishing some facts about several unconditional expectations that will appear later in the proof. For each $i \in \{\mathcal{B}, \mathcal{L}\}$, let $\eta_i = r + w_i$, $\zeta_{i,j} = (r+\eta_i)(r+\eta_j)$,

$$h_i(s,t,\tau) = e^{r(\tau-t)+(\kappa+w_i)(s-t)}, \text{ and} \tag{84}$$

$$H_{i,\tau_0,\tau_1}(s,\tau) = \int_{\tau_0}^{\tau_1} h_i(s,t,\tau)dt. \tag{85}$$

In particular,

$$H_{i,s,\tau}(s,\tau) = (r+\kappa+w_i)^{-1}(e^{r(\tau-s)} - e^{-(\kappa+w_i)(\tau-s)}) \tag{86}$$

$$H_{i,\tau-\theta,\tau}(s,\tau) = (r+\kappa+w_i)^{-1}(e^{(r+\kappa+w_i)\theta} - 1)e^{-(\kappa+w_i)(\tau-s)} \tag{87}$$

$$H_{i,\tau,\tau+\theta}(s,\tau) = (r+\kappa+w_i)^{-1}(1 - e^{-(r+\kappa+w_i)\theta})e^{-(\kappa+w_i)(\tau-s)}. \tag{88}$$

In what follows, we omit arguments for parsimony. For a standard 1-dimensional Brownian motion B_t, one can write

$$\int_{\tau-\theta}^{\tau}\int_{-\infty}^{t} h_i dB_s dt = \int_{-\infty}^{\tau-\theta} H_{i,\tau-\theta,\tau}dB_s + \int_{\tau-\theta}^{\tau} H_{i,s,\tau}dB_s \tag{89}$$

$$\int_{\tau}^{\tau+\theta}\int_{-\infty}^{t} h_i dB_s dt = \int_{-\infty}^{\tau-\theta} H_{i,\tau,\tau+\theta}dB_s + \int_{\tau-\theta}^{\tau} H_{i,\tau,\tau+\theta}dB_s + \int_{\tau}^{\tau+\theta} H_{i,s,\tau+\theta}dB_s. \tag{90}$$

Consider the following unconditional expectations:

$$\mathcal{I}^{(1)} \equiv \mathbb{E}\left[\left(\int_{\tau-\theta}^{\tau} e^{r(\tau-t)} dB_t\right)^2\right] \tag{91}$$

$$\mathcal{I}_i^{(2)} \equiv \mathbb{E}\left[\int_{\tau-\theta}^{\tau} e^{r(\tau-t)} dB_t \cdot \int_{\tau-\theta}^{\tau}\int_{-\infty}^{t} h_i dB_s dt\right] \tag{92}$$

$$\mathcal{I}_{i,j}^{(3)} \equiv \mathbb{E}\left[\int_{\tau-\theta}^{\tau}\int_{-\infty}^{t} h_i dB_s dt \cdot \int_{\tau-\theta}^{\tau}\int_{-\infty}^{t} h_j dB_s dt\right] \tag{93}$$

$$\mathcal{I}_i^{(4)} \equiv \mathbb{E}\left[\int_{\tau-\theta}^{\tau} e^{r(\tau-t)} dB_t \cdot \int_{\tau}^{\tau+\theta}\int_{-\infty}^{t} h_i dB_s dt\right] \tag{94}$$

$$\mathcal{I}_{i,j}^{(5)} \equiv \mathbb{E}\left[\int_{\tau-\theta}^{\tau}\int_{-\infty}^{t} h_i dB_s dt \cdot \int_{\tau}^{\tau+\theta}\int_{-\infty}^{t} h_j dB_s dt\right]. \tag{95}$$

By the Itô isometry (and equations (89) and (90)), we have

$$\mathcal{I}^{(1)} = \int_{\tau-\theta}^{\tau} e^{2r(\tau-s)} ds = \frac{e^{2r\theta}-1}{2r} \tag{96}$$

$$\mathcal{I}_i^{(2)} = \int_{\tau-\theta}^{\tau} e^{r(\tau-s)} H_{i,s,\tau} ds = \frac{1}{r+\eta_i}\left(\mathcal{I}^{(1)} + \frac{1-e^{(r-\eta_i)\theta}}{r-\eta_i}\right) \tag{97}$$

$$\mathcal{I}_{i,j}^{(3,a)} \equiv \int_{-\infty}^{\tau-\theta} H_{i,\tau-\theta,\tau} H_{j,\tau-\theta,\tau} ds = \frac{(e^{(r+\eta_i)\theta}-1)(e^{(r+\eta_j)\theta}-1)e^{-(\eta_i+\eta_j)\theta}}{(\eta_i+\eta_j)\zeta_{i,j}} \tag{98}$$

$$\mathcal{I}_{i,j}^{(3,b)} \equiv \int_{\tau-\theta}^{\tau} H_{i,s,\tau} H_{j,s,\tau} ds = \frac{e^{2r\theta}-1}{2r\zeta_{i,j}} - \frac{e^{(r-\eta_i)\theta}-1}{(r-\eta_i)\zeta_{i,j}} - \frac{e^{(r-\eta_j)\theta}-1}{(r-\eta_j)\zeta_{i,j}} + \frac{1-e^{-(\eta_i+\eta_j)\theta}}{(\eta_i+\eta_j)\zeta_{i,j}} \tag{99}$$

$$\mathcal{I}_{i,j}^{(3)} = \mathcal{I}_{i,j}^{(3,a)} + \mathcal{I}_{i,j}^{(3,b)} \tag{100}$$

and

$$\mathcal{I}_i^{(4)} = \int_{\tau-\theta}^{\tau} e^{r(\tau-s)} H_{i,\tau,\tau+\theta} ds = \frac{2(\cosh(r\theta)-\cosh(\eta_i\theta))e^{-\eta_i\theta}}{(r+\eta_i)(r-\eta_i)} \tag{101}$$

$$\mathcal{I}_{i,j}^{(5,a)} = \int_{-\infty}^{\tau-\theta} H_{i,\tau-\theta,\tau} H_{j,\tau,\tau+\theta} ds \tag{102}$$

$$\mathcal{I}_{i,j}^{(5,b)} = \int_{\tau-\theta}^{\tau} H_{i,s,\tau} H_{j,\tau,\tau+\theta} ds \tag{103}$$

$$\mathcal{I}_{i,j}^{(5)} = \mathcal{I}_{i,j}^{(5,a)} + \mathcal{I}_{i,j}^{(5,b)} = \frac{2(\cosh(r\theta)-\cosh(\eta_j\theta))e^{-\eta_j\theta}}{(\eta_i+\eta_j)(r+\eta_j)(r-\eta_j)}. \tag{104}$$

Note that

$$\frac{\mathcal{I}_i^{(2)} + \mathcal{I}_j^{(2)}}{\mathcal{I}_{i,j}^{(3)}} = \frac{\mathcal{I}_j^{(4)}}{\mathcal{I}_{i,j}^{(5)}} = \eta_i + \eta_j \tag{105}$$

and

$$\mathcal{I}^{(1)\prime}(\theta) = e^{2r\theta} \tag{106}$$

$$\mathcal{I}_i^{(2)\prime}(\theta) = \frac{1}{r+\eta_i}\left(e^{2r\theta} - e^{(r-\eta_i)\theta}\right) \tag{107}$$

$$\mathcal{I}_{i,j}^{(3)\prime}(\theta) = \frac{1}{\eta_i+\eta_j}\left(\mathcal{I}_i^{(2)\prime}(\theta) + \mathcal{I}_j^{(2)\prime}(\theta)\right). \tag{108}$$

In particular, $\mathcal{I}^{(1)\prime}(\theta) = 1$ and $\mathcal{I}_i^{(2)\prime}(\theta) = \mathcal{I}_{i,j}^{(3)\prime}(\theta) = 0$. Having established these facts, we now turn our attention to the error in borrowers' and lenders' estimates of the growth rate. Integrating equation (40) from $-\infty$ to t, we obtain

$$Z_t^i = \sqrt{2\kappa}\sigma_X M_t^{X,i} - w_i \sigma_V M_t^{V,i}, \tag{109}$$

where

$$M_t^{X,j} \equiv \int_{-\infty}^{t} e^{(\kappa+w_j)(s-t)} dB_s^X \tag{110}$$

$$M_t^{V,j} \equiv \int_{-\infty}^{t} e^{(\kappa+w_j)(s-t)} dB_s^V. \tag{111}$$

Importantly, $\mathbb{E}[M_{\tau_0}^{X,j} M_{\tau_1}^{V,j}] = 0$ for any times τ_0 and τ_1. Define the integrands

$$N_{\tau-\theta,\tau}^{Z^j} \equiv \alpha_j(\lambda) \int_{\tau-\theta}^{\tau} e^{r(\tau-t)} Z_t^j dt \tag{112}$$

$$N_{\tau-\theta,\tau}^{V} \equiv \overline{\alpha} \int_{\tau-\theta}^{\tau} e^{r(\tau-t)} dB_t^V, \tag{113}$$

where $\overline{\alpha} = (\eta_{\mathcal{B}} + \eta_{\mathcal{L}})\sigma_V$. It follows that

$$N_{\tau-\theta,\tau}^{Z^j} = \alpha_j(\lambda) \int_{\tau-\theta}^{\tau} e^{r(\tau-t)} \left(\sqrt{2\kappa}\sigma_X M_t^{X,j} - w_j \sigma_V M_t^{V,j}\right) dt. \tag{114}$$

Now consider the return from time τ to time $\tau+\theta$ as defined by equation (47):

$$R_{\tau,\tau+\theta} = \int_{\tau}^{\tau+\theta} e^{r((\tau+\theta)-t)}(dP_t + r(V_t - P_t)dt) \tag{115}$$

$$\begin{aligned} &= \alpha_{\mathcal{B}}(\lambda)\int_{\tau}^{\tau+\theta} e^{r((\tau+\theta)-t)}(Z_t^{\mathcal{B}}dt + \sigma_V dB_t^V) \\ &\quad + \alpha_{\mathcal{L}}(\lambda)\int_{\tau}^{\tau+\theta} e^{r((\tau+\theta)-t)}(Z_t^{\mathcal{L}}dt + \sigma_V dB_t^V) \end{aligned} \tag{116}$$

$$= N_{\tau,\tau+\theta}^{Z^{\mathcal{B}}} + N_{\tau,\tau+\theta}^{Z^{\mathcal{L}}} + N_{\tau,\tau+\theta}^{V}. \tag{117}$$

It follows immediately that $R_{\tau-\theta,\tau} = N_{\tau-\theta,\tau}^{Z^{\mathcal{B}}} + N_{\tau-\theta,\tau}^{Z^{\mathcal{L}}} + N_{\tau-\theta,\tau}^{V}$. Let

$$\varphi_{i,j} = 2\kappa\tau_X^{-1} + w_i w_j \tau_V^{-1}. \tag{118}$$

Consider the following variances and covariances:

$$\mathbb{E}[(N_{\tau-\theta,\tau}^{V})^2] = (\alpha_{\mathcal{B}}(\lambda) + \alpha_{\mathcal{L}}(\lambda))^2\tau_V^{-1}\mathcal{I}^{(1)} \tag{119}$$

$$\mathbb{E}[N_{\tau-\theta,\tau}^{Z^i} N_{\tau-\theta,\tau}^{V}] = -w_i\alpha_i(\lambda)(\alpha_{\mathcal{B}}(\lambda) + \alpha_{\mathcal{L}}(\lambda))\tau_V^{-1}\mathcal{I}_i^{(2)} \tag{120}$$

$$\mathbb{E}[N_{\tau-\theta,\tau}^{Z^i} N_{\tau-\theta,\tau}^{Z^j}] = \varphi_{i,j}\alpha_i(\lambda)\alpha_j(\lambda)\mathcal{I}_{i,j}^{(3)} \tag{121}$$

$$\mathbb{E}[N_{\tau-\theta,\tau}^{V} N_{\tau,\tau+\theta}^{Z^i}] = -w_i\alpha_i(\lambda)(\alpha_{\mathcal{B}}(\lambda) + \alpha_{\mathcal{L}}(\lambda))\tau_V^{-1}e^{r\theta}\mathcal{I}_i^{(4)} \tag{122}$$

$$\mathbb{E}[N_{\tau-\theta,\tau}^{Z^i} N_{\tau,\tau+\theta}^{Z^j}] = \varphi_{i,j}\alpha_i(\lambda)\alpha_j(\lambda)e^{r\theta}\mathcal{I}_{i,j}^{(5)}. \tag{123}$$

The unconditional variance of $R_{\tau-\theta,\tau}$ is

$$\sigma_1^2(\lambda,\theta) \equiv \mathbb{E}[(R_{\tau-\theta,\tau})^2] \tag{124}$$

$$\begin{aligned} &= \mathbb{E}[(N_{\tau-\theta,\tau}^{Z^{\mathcal{B}}})^2] + \mathbb{E}[(N_{\tau-\theta,\tau}^{Z^{\mathcal{L}}})^2] + \mathbb{E}[(N_{\tau-\theta,\tau}^{V})^2] \\ &\quad + 2\mathbb{E}[N_{\tau-\theta,\tau}^{Z^{\mathcal{B}}} N_{\tau-\theta,\tau}^{Z^{\mathcal{L}}}] + 2\mathbb{E}[N_{\tau-\theta,\tau}^{V} N_{\tau-\theta,\tau}^{Z^{\mathcal{B}}}] + 2\mathbb{E}[N_{\tau-\theta,\tau}^{V} N_{\tau-\theta,\tau}^{Z^{\mathcal{L}}}] \end{aligned} \tag{125}$$

$$\begin{aligned} &= \varphi_{\mathcal{B},\mathcal{B}}\alpha_{\mathcal{B}}(\lambda)^2\mathcal{I}_{\mathcal{B},\mathcal{B}}^{(3)}(\theta) + \varphi_{\mathcal{L},\mathcal{L}}\alpha_{\mathcal{L}}(\lambda)^2\mathcal{I}_{\mathcal{L},\mathcal{L}}^{(3)}(\theta) + (\alpha_{\mathcal{B}}(\lambda) + \alpha_{\mathcal{L}}(\lambda))^2\tau_V^{-1}\mathcal{I}^{(1)}(\theta) \\ &\quad + 2\varphi_{\mathcal{B},\mathcal{L}}\alpha_{\mathcal{B}}(\lambda)\alpha_{\mathcal{L}}(\lambda)\mathcal{I}_{\mathcal{B},\mathcal{L}}^{(3)}(\theta) - 2w_{\mathcal{B}}\alpha_{\mathcal{B}}(\lambda)(\alpha_{\mathcal{B}}(\lambda) + \alpha_{\mathcal{L}}(\lambda))\tau_V^{-1}\mathcal{I}_{\mathcal{B}}^{(2)}(\theta) \\ &\quad - 2w_{\mathcal{L}}\alpha_{\mathcal{L}}(\lambda)(\alpha_{\mathcal{B}}(\lambda) + \alpha_{\mathcal{L}}(\lambda))\tau_V^{-1}\mathcal{I}_{\mathcal{L}}^{(2)}(\theta) \end{aligned} \tag{126}$$

$$\begin{aligned} &= \psi(w_{\mathcal{B}})\mathcal{I}_{\mathcal{B},\mathcal{B}}^{(3)}\alpha_{\mathcal{B}}(\lambda)^2 + \psi(w_{\mathcal{L}})\mathcal{I}_{\mathcal{L},\mathcal{L}}^{(3)}\alpha_{\mathcal{L}}(\lambda)^2 + \tau_V^{-1}\mathcal{I}^{(1)}(\alpha_{\mathcal{B}}(\lambda) + \alpha_{\mathcal{L}}(\lambda))^2 \\ &\quad + 2(\psi(w_{\mathcal{B}})\mathcal{I}_{\mathcal{B},\mathcal{I}}^{(3)} + \tau_V^{-1}(w_{\mathcal{B}} - w_{\mathcal{L}})\mathcal{I}_{\mathcal{L}}^{(2)})\alpha_{\mathcal{B}}(\lambda)\alpha_{\mathcal{L}}(\lambda) \end{aligned} \tag{127}$$

which does not depend on τ. Let

$$\beta_{i,j}(\lambda) = \alpha_i'(\lambda)\alpha_j(\lambda) + \alpha_i(\lambda)\alpha_j'(\lambda). \tag{128}$$

Differentiating with respect to λ:

$$\begin{aligned}\partial_\lambda \sigma_1^2(\lambda) = \psi(w_\mathcal{B})\mathcal{I}^{(3)}_{\mathcal{B},\mathcal{B}}\beta_{\mathcal{B},\mathcal{B}}(\lambda) + \psi(w_\mathcal{L})\mathcal{I}^{(3)}_{\mathcal{L},\mathcal{L}}\beta_{\mathcal{L},\mathcal{L}}(\lambda) \\ + \tau_V^{-1}\mathcal{I}^{(1)}(\beta_{\mathcal{B},\mathcal{B}}(\lambda) + 2\beta_{\mathcal{B},\mathcal{L}}(\lambda) + \beta_{\mathcal{L},\mathcal{L}}(\lambda)) \\ + 2(\psi(w_\mathcal{B})\mathcal{I}^{(3)}_{\mathcal{B},\mathcal{L}} + \tau_V^{-1}(w_\mathcal{B} - w_\mathcal{L})\mathcal{I}^{(2)}_\mathcal{L})\beta_{\mathcal{B},\mathcal{L}}(\lambda).\end{aligned} \tag{129}$$

Define

$$\psi(w) = 2\kappa\tau_X^{-1} - w(w+2\kappa)\tau_V^{-1}. \tag{130}$$

Recall that w_0 is the positive root of ψ. Let w_{-1} denote the negative root so that

$$\psi(w) = (w_0 - w)(w - w_{-1}). \tag{131}$$

Note that $\psi(w) > 0$ for $w \in [0, w_0)$, $\psi(w_0) = 0$, and $\psi(w) < 0$ for $w > w_0$. Let

$$D_i(\lambda) = \frac{\alpha_i(\lambda)(w_i\alpha_{-i}(\lambda) + w_{-i}\alpha_i(\lambda) + (2\kappa + w_i)(\alpha_i(\lambda) + \alpha_{-i}(\lambda)))\psi(w_i)}{\eta_i + \eta_{-i}}. \tag{132}$$

We have written D_i as a function of λ as we will need the derivative of D_i with respect to λ:

$$D_i'(\lambda) = \frac{(w_i\beta_{i,-i}(\lambda) + w_{-i}\beta_{i,i}(\lambda) + (2\kappa + w_i)(\beta_{i,i}(\lambda) + \beta_{i,-i}(\lambda)))\psi(w_i)}{\eta_i + \eta_{-i}}. \tag{133}$$

The unconditional auto-covariance is

$$\sigma_{1,2}(\lambda,\theta) \equiv \mathbb{E}[R_{\tau-\theta,\tau}R_{\tau,\tau+\theta}] \tag{134}$$

$$= \sum_{j\in\{\mathcal{B},\mathcal{L}\}} \mathbb{E}[N^{Z^\mathcal{B}}_{\tau-\theta,\tau}N^{Z^j}_{\tau,\tau+\theta}] + \mathbb{E}[N^{Z^\mathcal{L}}_{\tau-\theta,\tau}N^{Z^j}_{\tau,\tau+\theta}] + \mathbb{E}[N^{V}_{\tau-\theta,\tau}N^{Z^j}_{\tau,\tau+\theta}] \tag{135}$$

$$\begin{aligned}= \varphi_{\mathcal{B},\mathcal{B}}\alpha_\mathcal{B}(\lambda)^2 e^{r\theta}\mathcal{I}^{(5)}_{\mathcal{B},\mathcal{B}} + \varphi_{\mathcal{B},\mathcal{L}}\alpha_\mathcal{B}(\lambda)\alpha_\mathcal{L}(\lambda)e^{r\theta}(\mathcal{I}^{(5)}_{\mathcal{B},\mathcal{L}} + \mathcal{I}^{(5)}_{\mathcal{L},\mathcal{B}}) \\ + \varphi_{\mathcal{L},\mathcal{L}}\alpha_\mathcal{L}(\lambda)^2 e^{r\theta}\mathcal{I}^{(5)}_{\mathcal{L},\mathcal{L}} - w_\mathcal{B}\alpha_\mathcal{B}(\lambda)(\alpha_\mathcal{B}(\lambda) + \alpha_\mathcal{L}(\lambda))\tau_V^{-1}e^{r\theta}\mathcal{I}^{(4)}_\mathcal{B} \\ - w_\mathcal{L}\alpha_\mathcal{L}(\lambda)(\alpha_\mathcal{B}(\lambda) + \alpha_\mathcal{L}(\lambda))\tau_V^{-1}e^{r\theta}\mathcal{I}^{(4)}_\mathcal{L}\end{aligned} \tag{136}$$

$$= (D_\mathcal{B}(\lambda)\mathcal{I}^{(5)}_{\mathcal{B},\mathcal{B}}(\theta) + D_\mathcal{L}(\lambda)\mathcal{I}^{(5)}_{\mathcal{L},\mathcal{L}}(\theta))e^{r\theta}, \tag{137}$$

having used the fact that $\mathbb{E}[N^{Z^{\mathcal{B}}}_{\tau-\theta,\tau} N^V_{\tau,\tau+\theta}] = \mathbb{E}[N^{Z^{\mathcal{L}}}_{\tau-\theta,\tau} N^V_{\tau,\tau+\theta}] = \mathbb{E}[N^V_{\tau-\theta,\tau} N^V_{\tau,\tau+\theta}] = 0$. Therefore,

$$\partial_\lambda \sigma_{1,2}(\lambda) = (D'_{\mathcal{B}}(\lambda)\mathcal{I}^{(5)}_{\mathcal{B},\mathcal{B}} + D'_{\mathcal{L}}(\lambda)\mathcal{I}^{(5)}_{\mathcal{L},\mathcal{L}})e^{r\theta}. \tag{138}$$

Finally,

$$\rho(\lambda,\theta;w_{\mathcal{B}},w_{\mathcal{L}}) = \frac{\sigma_{1,2}(\lambda,\theta;w_{\mathcal{B}},w_{\mathcal{L}})}{\sigma_1(\lambda,\theta;w_{\mathcal{B}},w_{\mathcal{L}})\sigma_2(\lambda,w_{\mathcal{B}},w_{\mathcal{L}})} = \frac{\sigma_{1,2}(\lambda,\theta;w_{\mathcal{B}},w_{\mathcal{L}})}{\sigma_1^2(\lambda,\theta;w_{\mathcal{B}},w_{\mathcal{L}})}. \tag{139}$$

Now consider the case in which $w_{\mathcal{B}} = w_0$. Then $D_{\mathcal{B}}(\lambda;w_0,w_{\mathcal{L}}) = D'_{\mathcal{B}}(\lambda;w_0,w_{\mathcal{L}}) = 0$ and hence

$$\sigma_{1,2}(\lambda,\theta;w_0,w_{\mathcal{L}}) = D_{\mathcal{L}}(\lambda;w_0,w_{\mathcal{L}})\mathcal{I}^{(5)}_{\mathcal{L},\mathcal{L}}(\theta;w_0,w_{\mathcal{L}})e^{r\theta} \tag{140}$$

$$\partial_\lambda\sigma_{1,2}(\lambda,\theta;w_0,w_{\mathcal{L}}) = D'_{\mathcal{L}}(\lambda;w_0,w_{\mathcal{L}})\mathcal{I}^{(5)}_{\mathcal{L},\mathcal{L}}(\theta;w_0,w_{\mathcal{L}})e^{r\theta}. \tag{141}$$

We now wish to show that if $w_{\mathcal{L}} < w_0$ as assumed, then $\sigma_{1,2}(\lambda;w_0,w_{\mathcal{L}}) > 0$. Note first that since $w_{\mathcal{L}} < w_0$, $D_{\mathcal{L}}(\lambda;w_0,w_{\mathcal{L}}) > 0$. Next,

$$\mathcal{I}^{(5)}_{\mathcal{L},\mathcal{L}} = \frac{2(\cosh(r\theta) - \cosh(\eta_{\mathcal{L}}\theta))e^{-\eta_{\mathcal{L}}\theta}}{(\eta_{\mathcal{L}}+\eta_{\mathcal{L}})(r+\eta_{\mathcal{L}})(r-\eta_{\mathcal{L}})} > 0, \tag{142}$$

which follows from the fact that $(\cosh(r\theta) - \cosh(\eta_{\mathcal{L}}\theta))/(r\theta - \eta_{\mathcal{L}}\theta) > 0$. We conclude that $\sigma_{1,2}(\lambda;w_0,w_{\mathcal{L}}) > 0$. We have that $D_{\mathcal{L}}(\lambda) = \alpha_{\mathcal{L}}(\lambda)\widetilde{D}_{\mathcal{L}}(\lambda)\psi(w_{\mathcal{L}})/(2\kappa + w_{\mathcal{B}} + w_{\mathcal{L}})$, where

$$\widetilde{D}_{\mathcal{L}}(\lambda) = w_{\mathcal{L}}\alpha_{\mathcal{B}}(\lambda) + w_{\mathcal{B}}\alpha_{\mathcal{L}}(\lambda) + (2\kappa + w_{\mathcal{L}})(\alpha_{\mathcal{L}}(\lambda) + \alpha_{\mathcal{B}}(\lambda)) \tag{143}$$

$$= (2\kappa + w_0 + w_{\mathcal{L}})\alpha_{\mathcal{B}}(\lambda) + 2(\kappa + w_{\mathcal{L}})\alpha_{\mathcal{L}}(\lambda). \tag{144}$$

Let

$$m_0 = 2(2\kappa + w_0 + w_{\mathcal{L}})(r + \kappa + w_{\mathcal{L}}) \tag{145}$$

$$m_1 = 2(\kappa + w_{\mathcal{L}})(r + \kappa + w_0)(r + \kappa + w_{\mathcal{L}}) \tag{146}$$

so that

$$(\eta_{\mathcal{L}}\widetilde{D}_{\mathcal{L}})'(\lambda) = m_0 c_{\mathcal{L}}(\lambda)c'_{\mathcal{L}}(\lambda) + m_1(c_{\mathcal{B}}(\lambda)c'_{\mathcal{L}}(\lambda) + c'_{\mathcal{B}}(\lambda)c_{\mathcal{L}}(\lambda)) \tag{147}$$

$$= (m_0 c_{\mathcal{L}}(\lambda) + m_1(c_{\mathcal{B}}(\lambda) - c_{\mathcal{L}}(\lambda)))c'_{\mathcal{L}}(\lambda) \tag{148}$$

$$= (m_1 c_{\mathcal{B}}(\lambda) + (m_0 - m_1)c_{\mathcal{L}}(\lambda))c'_{\mathcal{L}}(\lambda) \tag{149}$$

$$> 0, \tag{150}$$

which follows from the fact that $m_0 - m_1 = 2(r(\kappa + w_0) + (\kappa + w_{\mathcal{L}})^2)$. We conclude that $\partial_\lambda \sigma_{1,2}(\lambda; w_0, w_{\mathcal{L}}) > 0$. Finally, we consider

$$\begin{aligned}\partial_\lambda \sigma_1^2(\lambda, \theta; w_0, w_{\mathcal{L}}) &= \psi(w_{\mathcal{L}})\beta_{\mathcal{L},\mathcal{L}}(\lambda)\mathcal{I}^{(3)}_{\mathcal{L},\mathcal{L}}(\theta) + \tau_V^{-1}(\beta_{\mathcal{B},\mathcal{B}}(\lambda) + 2\beta_{\mathcal{B},\mathcal{L}}(\lambda) + \beta_{\mathcal{L},\mathcal{L}}(\lambda))\mathcal{I}^{(1)}(\theta) \\ &\quad + 2\tau_V^{-1}(w_0 - w_{\mathcal{L}})\beta_{\mathcal{B},\mathcal{L}}(\lambda)\mathcal{I}^{(2)}_{\mathcal{L}}(\theta) \qquad (151) \\ &= (\psi(w_{\mathcal{L}})\beta_{\mathcal{L},\mathcal{L}}(\lambda) + 2(\kappa + w_{\mathcal{L}})\tau_V^{-1}(w_0 - w_{\mathcal{L}})\beta_{\mathcal{B},\mathcal{L}}(\lambda))\mathcal{I}^{(3)}_{\mathcal{L},\mathcal{L}}(\theta) \\ &\quad + \tau_V^{-1}(\beta_{\mathcal{B},\mathcal{B}}(\lambda) + 2\beta_{\mathcal{B},\mathcal{L}}(\lambda) + \beta_{\mathcal{L},\mathcal{L}}(\lambda))\mathcal{I}^{(1)}(\theta). \qquad (152)\end{aligned}$$

Note that

$$\begin{aligned}&\beta_{\mathcal{L},\mathcal{L}}(\lambda) + 2\beta_{\mathcal{B},\mathcal{L}}(\lambda) + \beta_{\mathcal{B},\mathcal{B}}(\lambda) \\ &\quad = 2(\alpha_{\mathcal{L}}(\lambda) + \alpha_{\mathcal{B}}(\lambda))(\eta'_{\mathcal{L}}(\lambda) + \eta'_{\mathcal{B}}(\lambda)) \qquad (153) \\ &\quad = 2(\alpha_{\mathcal{L}}(\lambda) + \alpha_{\mathcal{B}}(\lambda))((r + \kappa + w_{\mathcal{L}})c'_{\mathcal{L}}(\lambda) + (r + \kappa + w_0)c'_{\mathcal{B}}(\lambda)) \qquad (154) \\ &\quad = 2(\alpha_{\mathcal{L}}(\lambda) + \alpha_{\mathcal{B}}(\lambda))((r + \kappa + w_{\mathcal{L}}) - (r + \kappa + w_0))c'_{\mathcal{L}}(\lambda) \qquad (155) \\ &\quad = 2(\alpha_{\mathcal{L}}(\lambda) + \alpha_{\mathcal{B}}(\lambda))(w_{\mathcal{L}} - w_0)c'_{\mathcal{L}}(\lambda) \qquad (156) \\ &\quad < 0. \qquad (157)\end{aligned}$$

Since $\mathcal{I}^{(1)}(0) = \mathcal{I}^{(3)}_{\mathcal{L},\mathcal{L}}(0) = \mathcal{I}^{(3)\prime}_{\mathcal{L},\mathcal{L}}(0) = 0$ and $\mathcal{I}^{(1)\prime}(0) = 1$, we have $\partial_\lambda \sigma_1^2(\lambda, 0; w_0, w_{\mathcal{L}}) = 0$ and

$$\partial_{\theta\lambda} \sigma_1^2(\lambda, 0; w_0, w_{\mathcal{L}}) = \beta_{\mathcal{L},\mathcal{L}}(\lambda) + 2\beta_{\mathcal{B},\mathcal{L}}(\lambda) + \beta_{\mathcal{B},\mathcal{B}}(\lambda) < 0. \qquad (158)$$

Therefore, $\partial_\lambda \sigma_1^2$ is strictly less than zero in a neighborhood of $\theta = 0$. ■

B Model Extensions

The base model is admittedly stylized. We therefore present a number of extensions to demonstrate that the the basic intuition of Proposition 1 is robust to alternative specifications of the model.

B.1 Sequential Nash Bargaining

In practice, the repayment and the price are determined through multiple rounds of bilateral bargaining. For example, a small business owner may first determine the loan terms he will be able to obtain before approaching the seller for a piece of equipment or vehicle. Alternatively, a home buyer may first negotiate the price of a house with a seller before approaching a lender for financing. With these possibilities in mind, we now consider two extensions of the model. We first consider a version of the model in which the borrower and lender bargain over a repayment, and the borrower can commit to the repayment in a subsequent round of bargaining with the seller over the price. We then consider a version of the model in which the borrower (i.e., buyer) and seller bargain over a price, and the borrower can commit to the price in a subsequent round of bargaining with the lender over the repayment. The goal of these extensions is to illustrate that the multilateral nature of price formation in the previous section does not materially affect the equilibrium outcome.

Consider the case in which the borrower and lender bargain first. There are now three dates, $t = 0, 1, 2$. On date $t = 0$, the borrower and the lender bargain over a repayment, c. On date $t = 1$, the borrower and the seller bargain over the price p. On date $t = 2$, payoffs are realized. Since each stage of bargaining is bilateral, we appeal to Nash (1950) directly for the bargaining solution.

Proposition B.1.1 (Borrower and Lender Bargain First)**.** *If the borrower and lender bargain first, the equilibrium repayment and price are*

$$\begin{aligned} c^*(\lambda) = {} & (1-\lambda)^{-1}(\eta_{\mathcal{B}}+\eta_{\mathcal{L}})^{-1}((1-\ell)\eta_{\mathcal{B}}+\eta_{\mathcal{S}})^{-1} \\ & \times [(1-\lambda)((1-\ell)\eta_{\mathcal{B}}\eta_{\mathcal{L}}+(\ell\eta_{\mathcal{B}}+\eta_{\mathcal{L}})\eta_{\mathcal{S}})a_{\mathcal{B}} - \lambda(1-\ell)\eta_{\mathcal{B}}(\eta_{\mathcal{B}}+\eta_{\mathcal{S}})a_{\mathcal{L}} \\ & - (1-\ell)((1-\eta_{\mathcal{L}})\eta_{\mathcal{L}} - \ell\eta_{\mathcal{B}}(\eta_{\mathcal{B}}+\eta_{\mathcal{L}}))a_{\mathcal{S}}] \end{aligned} \tag{159}$$

$$\begin{aligned} p^*(\lambda) = {} & (\eta_{\mathcal{B}}+\eta_{\mathcal{L}})^{-1}((1-\ell)\eta_{\mathcal{B}}+\eta_{\mathcal{S}})^{-1} \\ & \times [\eta_{\mathcal{B}}\eta_{\mathcal{S}}((1-\lambda)a_{\mathcal{B}}+\lambda a_{\mathcal{L}}) + ((1-\ell)\eta_{\mathcal{B}}(1-\eta_{\mathcal{S}})+\eta_{\mathcal{L}}\eta_{\mathcal{S}})a_{\mathcal{S}}]\,. \end{aligned} \tag{160}$$

Proof. We proceed by backwards induction, starting on date $t = 1$. Suppose that the borrower had committed to a repayment c on date $t = 0$. At $t = 1$, the borrower's and seller's, subjective expected surpluses are

$$\mathbb{E}^{\mathcal{B}}[S_{\mathcal{B}}(c,p)] = (1-\lambda)(a_{\mathcal{B}} - c) - (1-\ell)p \tag{161}$$
$$\mathbb{E}^{\mathcal{S}}[S_{\mathcal{S}}(c,p)] = p - a_{\mathcal{S}}. \tag{162}$$

Given c, the $p(c)$ that maximizes the Nash product $\mathbb{E}^{\mathcal{B}}[S_{\mathcal{B}}(c,p)]^{\eta_{\mathcal{B}}}\mathbb{E}^{\mathcal{S}}[S_{\mathcal{S}}(c,p)]^{\eta_{\mathcal{S}}}$ is

$$p(c) = \frac{\eta_{\mathcal{B}}}{\eta_{\mathcal{B}} + \eta_{\mathcal{S}}} \cdot a_{\mathcal{S}} + \frac{\eta_{\mathcal{S}}}{\eta_{\mathcal{B}} + \eta_{\mathcal{S}}} \cdot \frac{1-\lambda}{1-\ell} \cdot (a_{\mathcal{B}} - c). \tag{163}$$

Therefore, at $t = 0$, the borrower's and lender's subjective, expected surpluses are

$$\mathbb{E}^{\mathcal{B}}[S_{\mathcal{B}}(c,p(c))] = (1-\lambda)(a_{\mathcal{B}} - c) - (1-\ell)p(c) \tag{164}$$
$$\mathbb{E}^{\mathcal{L}}[S_{\mathcal{L}}(c,p(c))] = \lambda a_{\mathcal{L}} + (1-\lambda)c - \ell p(c). \tag{165}$$

The equilibrium c maximizes the Nash product $\mathbb{E}^{\mathcal{B}}[S_{\mathcal{B}}(c,p(c))]^{\eta_{\mathcal{B}}}\mathbb{E}^{\mathcal{L}}[S_{\mathcal{L}}(c,p(c))]^{\eta_{\mathcal{L}}}$ is given by equation (160). ■

Consider the limit as $\eta_{\mathcal{S}} \to 0$. To simplify the analysis, suppose that the ratio $\eta_0 \equiv \eta_{\mathcal{B}}/(\eta_{\mathcal{B}} + \eta_{\mathcal{L}})$ remains constant as $\eta_{\mathcal{B}} \to 0$ and $\eta_{\mathcal{L}} \to 0$. Then

$$\lim_{\eta_{\mathcal{S}} \to 1} p^*(\lambda) = \eta_0((1-\lambda)a_{\mathcal{B}} + \lambda a_{\mathcal{L}}) + (1-\eta_0)a_{\mathcal{S}}. \tag{166}$$

Moreover, $p^{*\prime}(\lambda) > 0$ if $a_{\mathcal{L}} > a_{\mathcal{B}}$ and $p^{*\prime}(\lambda) < 0$ if $a_{\mathcal{L}} < a_{\mathcal{B}}$, just as in Proposition 1.

Now consider the case in which the borrower (i.e., buyer) and seller bargain first. There are again three dates, $t = 0, 1, 2$. On date $t = 0$, the borrower and the seller bargain over a price, p. On date $t = 1$, the borrower and the lender bargain over the repayment c. On date $t = 2$, payoffs are realized.

Proposition B.1.2 (Borrower and Seller Bargain First)**.** *If the borrower and seller bargain first, the equilibrium repayment and price are*

$$\begin{aligned} c^*(\lambda) = {} & (1-\lambda)^{-1}(\eta_{\mathcal{B}} + \eta_{\mathcal{L}})^{-1}(\eta_{\mathcal{B}} + \eta_{\mathcal{S}})^{-1} \\ & \times \big[(1-\lambda)(\eta_{\mathcal{B}}\eta_{\mathcal{L}} + \ell(\eta_{\mathcal{B}} + \eta_{\mathcal{L}})\eta_{\mathcal{S}})a_{\mathcal{B}} - \lambda(\eta_{\mathcal{B}}^2 + (1-\ell)(\eta_{\mathcal{B}} + \eta_{\mathcal{L}})\eta_{\mathcal{S}})a_{\mathcal{L}} \\ & \quad - (\eta_{\mathcal{B}}\eta_{\mathcal{L}} - \ell(\eta_{\mathcal{B}} + \eta_{\mathcal{L}})\eta_{\mathcal{B}})a_{\mathcal{S}}\ \big] \end{aligned} \tag{167}$$
$$p^*(\lambda) = (\eta_{\mathcal{B}} + \eta_{\mathcal{S}})^{-1}(\eta_{\mathcal{S}}((1-\lambda)a_{\mathcal{B}} + \lambda a_{\mathcal{L}}) + \eta_{\mathcal{B}}a_{\mathcal{S}}). \tag{168}$$

Proof. We proceed by backwards induction, starting on date $t = 1$. Suppose that the borrower had committed to a price p on date $t = 0$. At $t = 1$, the borrower's and lender's subjective, expected surpluses are

$$\mathbb{E}^{\mathcal{B}}[S_{\mathcal{B}}(c,p)] = (1-\lambda)(a_{\mathcal{B}} - c) - (1-\ell)p \tag{169}$$

$$\mathbb{E}^{\mathcal{L}}[S_{\mathcal{L}}(c,p)] = \lambda a_{\mathcal{L}} + (1-\lambda)c - \ell. \tag{170}$$

Given p, $c(p)$ that maximizes the Nash product $\mathbb{E}^{\mathcal{B}}[S_{\mathcal{B}}(c,p)]^{\eta_{\mathcal{B}}}\mathbb{E}^{\mathcal{L}}[S_{\mathcal{L}}(c,p)]^{\eta_{\mathcal{L}}}$ is

$$c(p) = \frac{\eta_{\mathcal{L}}}{(\eta_{\mathcal{B}} + \eta_{\mathcal{L}})(1-\lambda)} \cdot ((1-\lambda)a_{\mathcal{B}} - (1-\ell)p) - \frac{\eta_{\mathcal{B}}}{(\eta_{\mathcal{B}} + \eta_{\mathcal{L}})(1-\lambda)} \cdot (\lambda a_{\mathcal{L}} - \ell p). \tag{171}$$

The seller's and borrower's expected surpluses

$$\mathbb{E}^{\mathcal{S}}[S_{\mathcal{S}}(c(p),p)] = p - a_{\mathcal{S}} \tag{172}$$

$$\mathbb{E}^{\mathcal{B}}[S_{\mathcal{B}}(c(p),p)] = (1-\lambda)(a_{\mathcal{B}} - c(p)) - (1-\ell)p. \tag{173}$$

The equilibrium p maximizes the Nash product $\mathbb{E}^{\mathcal{B}}[S_{\mathcal{B}}(c(p),p)]^{\eta_{\mathcal{B}}}\mathbb{E}^{\mathcal{S}}[S_{\mathcal{S}}(c(p),p)]^{\eta_{\mathcal{S}}}$ is given by equation (168). ■

Note that as the surpluses of the borrower and the lender go to zero, the prices under sequential bargaining and simultaneous bargaining converge to $p(\lambda) = (1-\lambda)a_{\mathcal{B}} + \lambda a_{\mathcal{L}}$. Moreover, $p^{*\prime}(\lambda) > 0$ if $a_{\mathcal{L}} > a_{\mathcal{B}}$ and $p^{*\prime}(\lambda) < 0$ if $a_{\mathcal{L}} < a_{\mathcal{B}}$, just as in Proposition 1.

B.2 Repossession Costs

In many applications, collateral repossession is costly; lenders may incur substantial holding costs (in the case of equipment repossession) or have to sell at a foreclosure sale discount (Conklin et al., 2023). In this subsection, we extend the model to include repossession costs. We show that increasing borrower riskiness pulls the price of the asset towards the lender's effective value (his value net of repossession costs).

Suppose that when the borrower defaults, the lender is able to recover only a fraction $\xi \in (0,1)$ of the collateral value. The subjective, expected surpluses of the borrower and the seller are as before (see equations (6) and (8)), but the lender's expected surplus is now

$$\mathbb{E}^{\mathcal{L}}[S_{\mathcal{L}}(c,p)] = \lambda\xi a_{\mathcal{L}} + (1-\lambda)c - \ell p. \tag{174}$$

Solving for the equilibrium repayment and price amounts to substituting $\xi a_{\mathcal{L}}$ for $a_{\mathcal{L}}$ in Proposition 1.

Proposition B.2.1 (Repossession Costs)**.** *The equilibrium repayment and price are*

$$\begin{aligned} c^*(\lambda) = (1-\lambda)^{-1} & [(1-\lambda)(\eta_{\mathcal{L}} + \ell\eta_{\mathcal{S}})a_{\mathcal{B}} \\ & + \lambda(\eta_{\mathcal{L}} - (1-\ell\eta_{\mathcal{S}}))\xi a_{\mathcal{L}} - (\eta_{\mathcal{L}} - \ell(1-\eta_{\mathcal{S}}))a_{\mathcal{S}}] \end{aligned} \tag{175}$$

$$p^*(\lambda) = \eta_{\mathcal{S}}((1-\lambda)a_{\mathcal{B}} + \lambda\xi a_{\mathcal{L}}) + (1-\eta_{\mathcal{S}})a_{\mathcal{S}}. \tag{176}$$

Now $p^{*\prime}(\lambda) > 0$ if $\xi a_{\mathcal{L}} > a_{\mathcal{B}}$, and $p^{*\prime}(\lambda) < 0$ if $\xi a_{\mathcal{L}} < a_{\mathcal{B}}$. The analogue of Corollary 1 follows:

Corollary 2. *If neither the borrower nor the lender earn surplus* ($\eta_{\mathcal{B}} = \eta_{\mathcal{L}} = 0$)*, then*

$$p^*(\lambda) = (1-\lambda)a_{\mathcal{B}} + \lambda\xi a_{\mathcal{L}} \tag{177}$$

and hence

$$p^{*\prime}(\lambda) = \xi a_{\mathcal{L}} - a_{\mathcal{B}} = (1-\lambda)^{-1}(\xi a_{\mathcal{L}} - p^*(\lambda)). \tag{178}$$

As in Corollary 1, the price is decreasing in borrower riskiness when the price is substantially greater than the lender's value ($p^*(\lambda) >> a_{\mathcal{L}}$) and increasing in borrower riskiness when the price is substantially less than the lender's value ($p^*(\lambda) << a_{\mathcal{L}}$). Mathematically, the point at which the sign of $p^{*\prime}(\lambda)$ flips now depends on the magnitude of the repossession costs; when repossession costs are higher, the point at which the sign of $p^{*\prime}(\lambda)$ flips is lower. These results suggest that increasing borrower riskiness pulls the price towards the lender's value less repossession costs. In our empirical section, we provide evidence supporting Corollary 2 assuming different estimates of repossession costs.

B.3 Endogenous Default Probabilities

We now consider an extension of the base model in which the probability of default is correlated with the value of collateral. In practice, borrowers are most likely to default when the asset value is low, either for non-strategic reasons (asset values are low precisely when borrowers are illiquid) or strategic reasons (the borrower is underwater and chooses to walk away from both the loan and the asset).[14] In this extension, the borrower and the lender agree about borrower riskiness and the probabilities of good and bad states of the world

[14]In housing, a relatively small fraction of homeowners default explicitly because of a decline in home values (i.e., strategic default). Estimates range from 6.4% (Foote et al., 2008) to 35.1% (Guiso et al., 2013).

(e.g., expansions and recessions), but they disagree about the value of collateral in these different states of the world.[15]

To keep the extension simple, we modify our original distributional assumptions. Suppose there are two states indexed by $s \in \{H, L\}$. $s = H$ with probability $\pi \in (0,1)$ and $s = L$ with probability $1 - \pi$. If $s = H$, the borrower does not default. If $s = L$, the borrower defaults with probability $\lambda \in (0,1)$ and remains solvent with probability $1 - \lambda$. Agent j believes that the asset is worth a_j^s in state s. In general, agents believe that the asset is less valuable in state L than in state H: $a_j^L < a_j^H$. This simple specification captures the idea that the collateral value should be low precisely when default is most likely.

We can now write the subjective, expected surpluses of the borrower and the lender:

$$\mathbb{E}^{\mathcal{B}}[S_{\mathcal{B}}(c,p)] = \pi(a_{\mathcal{B}}^H - c) + (1-\pi)(1-\lambda)(a_{\mathcal{B}}^L - c) - (1-\ell)p \tag{179}$$

$$\mathbb{E}^{\mathcal{L}}[S_{\mathcal{L}}(c,p)] = \pi c + (1-\pi)(\lambda a_{\mathcal{L}}^L + (1-\lambda)c) - \ell p. \tag{180}$$

The following proposition gives the equilibrium repayment and price with endogenous default probabilities.

Proposition B.3.1 (Endogenous Default)**.** *If neither the borrower nor the lender earn surplus ($\eta_{\mathcal{B}} = \eta_{\mathcal{L}} = 0$), the equilibrium repayment and price are*

$$c^*(\lambda) = (1-(1-\pi)\lambda)^{-1}(\ell(\pi a_{\mathcal{B}}^H + (1-\pi)(1-\lambda)a_{\mathcal{B}}^L) - (1-\ell)(1-\pi)\lambda a_{\mathcal{L}}^L) \tag{181}$$

$$p^*(\lambda) = \pi a_{\mathcal{B}}^H + (1-\pi)((1-\lambda)a_{\mathcal{B}}^L + \lambda a_{\mathcal{L}}^L). \tag{182}$$

Moreover, $p^{*\prime}(\lambda) > 0$ if the lender is optimistic, in the sense that $a_{\mathcal{L}}^L > a_{\mathcal{B}}^L$, and $p^{*\prime}(\lambda) < 0$ if the borrower is pessimistic, in the sense that $a_{\mathcal{L}}^L < a_{\mathcal{B}}^L$.

Proof. Set equations (179) and (180) equal to zero and solve for c and p. ∎

Although the exact notions of "optimistic" and "pessimistic" are different from those of Proposition 1, the basic finding that prices increase with borrower riskiness when the lender is optimistic and decrease when the borrower is optimistic survives.

Note that if default always occurs in the L-state ($\lambda = 1$), the price is given by

$$p(\pi) = \pi a_{\mathcal{B}}^H + (1-\pi)a_{\mathcal{L}}^L. \tag{183}$$

[15] See Simsek (2013) for a thorough analysis of disagreement about the distribution of collateral values. Our primary departure is the consideration of borrower riskiness (λ), which determines how the views of the borrower and the lender are incorporated into the asset price.

In this case, π plays the same role as λ in mediating beliefs, except that now, π represents the risk of the asset rather than the risk of the borrower. The price is a convex combination of the borrowers' belief about the asset in the H-state (when she consumes the asset) and the lenders' belief about the asset in the L-state (when he consumes the asset).

From equation (183), it follows that

$$p'(\pi) = \pi^{-1}(a^L_{\mathcal{L}} - p(\pi)), \tag{184}$$

which just says that the point at which the sign of $p'(\pi)$ flips depends on the lender's value for the asset specifically in the event of default.

B.4 Endogenous Leverage

In the base model, the fraction of the asset price financed by the lender is taken to be exogenous. In practice, borrowers choose how much they borrow (perhaps subject to a constraint on loan-to-value). In this subsection, we take the repayment c as fixed and let agents bargain over the fraction of the purchase price to be financed ℓ as well as the asset price p.

Proposition B.4.1 (Endogenous Leverage)**.** *The equilibrium leverage and price are*

$$p^*(\lambda) = \eta_{\mathcal{S}}((1-\lambda)a_{\mathcal{B}} + \lambda a_{\mathcal{L}}) + (1-\eta_{\mathcal{S}})a_{\mathcal{S}} \tag{185}$$

$$\ell^*(\lambda) = p^*(\lambda)^{-1}((1-\lambda)c + \lambda a_{\mathcal{L}} + \eta_{\mathcal{L}}(a_{\mathcal{S}} - ((1-\lambda)a_{\mathcal{B}} + \lambda a_{\mathcal{L}}))). \tag{186}$$

Proof. Just as in the proof of Proposition 1, but now solve for ℓ and p (vs. c and p). ■

The asset price in equation (185) of Proposition B.4.1 is identical to the asset price in equation (12) of Proposition 1. The reason is simple: total surplus (equation (9) in both cases is the same. Neither the total surplus nor the seller's surplus depend explicitly on how the borrower finances her purchase (i.e., the fraction of the price to be financed ℓ or the repayment c). We conclude that $p^{*\prime}(\lambda) > 0$ if $a_{\mathcal{L}} > a_{\mathcal{B}}$ and $p^{*\prime}(\lambda) < 0$ if $a_{\mathcal{L}} < a_{\mathcal{B}}$, just as in Proposition 1.

B.5 LTV Constraints and Surplus Funds

In this section, we explore two related extensions to the baseline model: LTV constraints and surplus fund rebates. In housing applications, lenders use the so-called loan-to-value

(LTV) ratio to determine how much of the purchase price to finance. In the LTV ratio, the "L" denotes the loan amount, while the "V" denotes the minimum of the price and the appraise value. We therefore rewrite the bargaining problem to include the LTV constraint. Let $\underline{\ell} \in (0,1)$ denote the LTV constraint (i.e., 80% in U.S. housing markets) and m denote the borrower's down payment. Then

$$\underline{\ell} = \frac{p - m}{\min\{p, a_{\mathcal{L}}\}}. \tag{187}$$

Rearranging, we obtain

$$\underline{m}(p) \equiv p - \underline{\ell} \min\{p, a_{\mathcal{L}}\}. \tag{188}$$

We now consider bargaining over the price p and repayment c.

$$\mathbb{E}^{\mathcal{B}}[S_{\mathcal{B}}(c,p)] = (1-\lambda)(a_{\mathcal{B}} - c) - \underline{m}(p) \tag{189}$$

$$\mathbb{E}^{\mathcal{L}}[S_{\mathcal{L}}(c,p)] = \lambda a_{\mathcal{L}} + (1-\lambda)c - (p - \underline{m}(p)) \tag{190}$$

To ease the exposition, we focus on the case in which neither borrower nor lender earn surplus ($\eta_{\mathcal{B}} = \eta_{\mathcal{L}} = 0$ and $\eta_{\mathcal{S}} = 1$). Note that $\bar{c}$ is the repayment in the base model. We find a result similar to that of Proposition B.4.1: The existence of the appraisal constraint distorts the financing, but not the price of the asset. The total expected surplus is exactly as in equation (9).

Proposition B.5.1 (LTV Constraint)**.** *The equilibrium price and repayment are*

$$c^*(\lambda) = (1-\lambda)^{-1}\left[\underline{\ell} \min\{(1-\lambda)a_{\mathcal{B}} + \lambda a_{\mathcal{L}}, a_{\mathcal{L}}\} - \lambda a_{\mathcal{L}}\right] \tag{191}$$

$$p^*(\lambda) = (1-\lambda)a_{\mathcal{B}} + \lambda a_{\mathcal{L}}. \tag{192}$$

Proof. Set (189) and (190) equal zero. Solve for c and p. ∎

In practice, lenders may rebate any surplus to the borrower in the event of default.

$$\mathbb{E}^{\mathcal{B}}[S_{\mathcal{B}}(c,p)] = \lambda \max\{a_{\mathcal{L}} - c, 0\} + (1-\lambda)(a_{\mathcal{B}} - c) - (1-\ell)p \tag{193}$$

$$\mathbb{E}^{\mathcal{L}}[S_{\mathcal{L}}(c,p)] = \lambda \min\{a_{\mathcal{L}}, c\} + (1-\lambda)c - \ell p \tag{194}$$

We find a result similar to that of Proposition B.4.1: The existence of the appraisal constraint distorts the financing, but not the price of the asset. To ease the exposition, we focus on the case in which neither borrower nor lender earn surplus ($\eta_{\mathcal{B}} = \eta_{\mathcal{L}} = 0$ and $\eta_{\mathcal{S}} = 1$).

Proposition B.5.2 (Surplus Funds)**.** *The equilibrium price and repayment are*

$$c^*(\lambda) = \max\{\ell((1-\lambda)a_{\mathcal{B}} + \lambda a_{\mathcal{L}}), (1-\lambda)^{-1}[\ell((1-\lambda)a_{\mathcal{B}} + \lambda a_{\mathcal{L}}) - \lambda a_{\mathcal{L}}]\} \tag{195}$$

$$p^*(\lambda) = (1-\lambda)a_{\mathcal{B}} + \lambda a_{\mathcal{L}}. \tag{196}$$

Proof. Set (193) and (194) equal zero. Solve for c and p. ∎

One of the shortcomings of the baseline model is that it sometimes predicts negative interest rates in the sense that $c^* < \ell p^*(\lambda)$. As an example, suppose that $\eta_{\mathcal{S}} = 1$, $\eta_{\mathcal{B}} = \eta_{\mathcal{L}} = 0$, and $a_{\mathcal{B}} = a_{\mathcal{L}} = a_{\mathcal{S}} = a > 0$. In both the baseline model and in the extension considered in this section, $p^*(\lambda) = a$. In the baseline model,

$$c^*(\lambda) = (1-\lambda)^{-1}(\ell - \lambda)a = (1-\lambda)^{-1}(\ell-\lambda)p^*(\lambda) < \ell p^*(\lambda). \tag{197}$$

In other words, the present value of debt repayments is less than the amount financed by the lender. When surplus funds are rebated back to the borrower, this is not the case. Specifically, $\overline{c}(\lambda) = (1-\lambda)^{-1}(\ell-\lambda)a$, $\underline{c}(\lambda) = a$, and therefore.

$$c^*(\lambda) = \max\{(1-\lambda)^{-1}(\ell-\lambda), 1\}a = \max\{(1-\lambda)^{-1}(\ell-\lambda), 1\}p^*(\lambda) = p^*(\lambda) > \ell p^*(\lambda). \tag{198}$$

B.6 Secondary Market

We now consider the possibility that the lender can sell the loan. We therefore add an investor $\mathcal{I}$ to our model (e.g., a government-sponsored enterprise or an investor in an asset-backed security). In this case, the lender may agree with the borrower and the seller about the asset value. Alternatively, the lender may use his informational advantage to sell riskier loans (Agarwal et al., 2012) or engage in lax borrower screening because the lender is going to sell the loan to an outside investor regardless of the asset's value (Keys et al., 2010). In either situation, the ultimate investor holds a differing view from the borrower and the seller.

Consider a lender who originates home loans. He will not make a particular loan unless the loan can be sold to an outside investor. However, the investor will not buy the loan unless the originator uses a particular appraised value of the home, even if that appraised value differs markedly from the originator's value. In this way, an investor's beliefs can distort the asset price just as much as a lender's beliefs can.

We reiterate that although we consider a bargaining game between four agents, we could reinterpret the bargaining weights as agents' respective market power in a competitive market. Let p_A be the price of the asset, p_L be the price of the loan, and $\varphi \in (0,1)$ be the fraction of the loan retained by the lender. Let $\eta_{\mathcal{I}}$ be the investor's bargaining weight, and suppose that all agents' weights sum to unity. The surpluses are

$$S_{\mathcal{B}}(c, p_A, p_L) = (1-d)(v_1 - c) - (1-\ell)p_A \tag{199}$$

$$S_{\mathcal{L}}(c, p_A, p_L) = p_L + \varphi(dv_1 + (1-d)c) - \ell p_A \tag{200}$$

$$S_{\mathcal{S}}(c, p_A, p_L) = p_A - v_1 \tag{201}$$

$$S_{\mathcal{I}}(c, p_A, p_L) = (1-\varphi)(dv_1 + (1-d)c) - p_L. \tag{202}$$

Therefore, the subjective, expected surpluses are

$$\mathbb{E}^{\mathcal{B}}[S_{\mathcal{B}}(c, p_A, p_L)] = (1-\lambda)(a_{\mathcal{B}} - c) - (1-\ell)p_A \tag{203}$$

$$\mathbb{E}^{\mathcal{L}}[S_{\mathcal{L}}(c, p_A, p_L)] = p_L + \varphi(\lambda a_{\mathcal{L}} + (1-\lambda)c) - \ell p_A \tag{204}$$

$$\mathbb{E}^{\mathcal{S}}[S_{\mathcal{S}}(c, p_A, p_L)] = p_A - a_{\mathcal{S}} \tag{205}$$

$$\mathbb{E}^{\mathcal{I}}[S_{\mathcal{I}}(c, p_A, p_L)] = (1-\varphi)(\lambda a_{\mathcal{I}} + (1-\lambda)c) - p_L. \tag{206}$$

The total expected surplus is therefore

$$\sum_j \mathbb{E}^j[S_j(c,p)] = (1-\lambda)a_{\mathcal{B}} + \lambda(\varphi a_{\mathcal{L}} + (1-\varphi)a_{\mathcal{I}}) - a_{\mathcal{S}}. \tag{207}$$

We characterize the equilibrium in the following proposition.

Proposition B.6.1 (Secondary Market)**.** *If neither the borrower, the lender, nor the investor earn surplus and the borrower and the lender agree on the estimated value of the asset, then the equilibrium repayment, price of the asset, and price of the loan are*

$$c^*(\lambda,\varphi) = (1-\lambda)^{-1}(((1-\lambda) - \ell(1-(1-\varphi)\lambda))a_{\mathcal{B}} - \ell(1-\varphi)\lambda a_{\mathcal{I}}) \tag{208}$$

$$p_A^*(\lambda,\varphi) = (1-\varphi)\lambda a_{\mathcal{I}} + (1-(1-\varphi)\lambda)a_{\mathcal{B}} \tag{209}$$

$$p_L^*(\lambda,\varphi) = (1-\varphi)(((1-\lambda) - \ell(1-(1-\varphi)\lambda))a_{\mathcal{B}} + (\lambda - \ell(1-\varphi)\lambda)a_{\mathcal{I}}). \tag{210}$$

Proof. Set equations (206), (204), and (203) equal to zero and solve for c, p_L, and p_A. ∎

Corollary 3. *If neither the borrower, the lender, nor the investor earn surplus and the* *borrower and the lender agree on the estimated value of the asset, then*

$$\frac{\partial p_A^*}{\partial \lambda} = (1-\varphi)(a_{\mathcal{I}} - a_{\mathcal{B}}) = (1-(1-\varphi)\lambda)^{-1}(1-\varphi)(a_{\mathcal{I}} - p_A^*(\lambda,\varphi)). \tag{211}$$

Corollary 3 states that even if the lender has the option to sell the loan on a secondary market, there is still an effect of borrower riskiness on the asset price as mediated by disagreement between the borrower and the ultimate investor. That is, the price is decreasing in borrower riskiness when the price is greater than the investor's value ($p_A^*(\lambda,\varphi) > a_{\mathcal{I}}$) and increasing in borrower riskiness when the price is less than the investor's value ($p_A^*(\lambda,\varphi) < a_{\mathcal{I}}$).

C Supplementary Empirical Results

In this section, we provide and discuss supplementary empirical results.

C.1 Repossession Costs

In Corollary 2, we consider the possibility that the lender is unable to recover the full value of the asset in the event of default. Specifically, equation (178) in Corollary 2 states that borrower riskiness is negatively associated with the distance between the equilibrium price and the lender's effective value, which we define as the lender's value net of repossession costs. In housing, these costs are largely driven by a foreclosure sale discount. There are four broad reasons for this discount (Conklin et al., 2023). First, there are observable and unobservable prior differences between distressed and non-distressed properties (Frame, 2010). Second, there are differences in the condition of the house that is caused by distressed homeowners' reduction in maintenance (Lambie-Hanson, 2015). Third, distressed sellers, which are often financial institutions (i.e., lenders) have greater urgency, and the briefer time the house is on the market is correlated with lower price (Clauretie and Daneshvary, 2009). Fourth, there is a stigma associated with distressed sales, which is often driven by the asymmetric information endemic to real estate transactions (Stroebel, 2016; Lopez, 2021).

Although there is wide agreement that there is a material foreclosure sale discount, there is no consensus on the size of the discount (Conklin et al., 2023). Some estimates been shown to be as high as 25% or 30% (e.g., Campbell et al., 2011), but others have found discounts to be as low as 5% or 10% (e.g., Conklin et al., 2023, Clauretie and Daneshvary, 2009). We use both 5% and 10% as our estimates of repossession costs. Note that regardless of the exact value of repossession costs, Corollaries 1 and 2 make the same prediction when the appraised value and sale price are sufficiently far from each other.

Table C1 presents results using an estimated repossession costs of 5%. Panel A focuses on observations in which sale price is less than or equal to 95% of appraised value. The dependent variable is $log(0.95\times A\text{-}P)$ for our OLS regressions and $0.95\times A\text{-}P$ for our Poisson regressions. All coefficients across all proxies for borrower riskiness and both types of specifications are negative and highly significant.

Panel B of Table C1 considers observations in which sale price is greater than or equal to 95% of appraised value and includes the overwhelming number of observations in the sample. The dependent variable here is $log(P\text{-}0.95\times A)$ for our OLS regressions and $P\text{-}0.95\times A$ for our Poisson regressions. Consistent with both Panel A of Table C1 and Corollary 2, coefficients on each proxy for borrower riskiness and on their first principal component are negative and highly significant.

Table C2 shows that our results are similar when we consider repossession costs of 10%. Overall, the results in Tables C1 and C2 are consistent with Corollary 2.

C.2 Secondary Market

In Corollary 3, we focus on the situation in which the lender can sell the loan. In this setting, the lender agrees with the borrower and the seller about the value of the asset, but an outside investor of the loan holds a different view on the value of the asset. The investor is able to impose her beliefs on the value of the asset because if those beliefs are not reflected, the investor will not purchase the loan. Corollary 3 therefore implies that borrower riskiness is negatively associated with the distance between appraised value and sale price among loans that are sold to outside investors.

To understand the empirical relevance of this corollary, we would ideally have information on which loans are sold to outside investors and which loans are kept on lenders' balance sheets. Although we unfortunately do not have that information, we can proxy for this delineation by using the GSE Eligible Flag provided by CoreLogic. This flag turns on when the loan conforms to the GSE standard eligible requirements criteria. The GSEs do not buy every conforming loan, but according to the Federal Housing Finance Agency (FHFA), GSE share of all conforming mortgages has been between 50% and 65% over the past fifteen years.[C1] Therefore, we estimate our baseline specification on the subsample of conforming loans. We present the results in Table C3.

Since almost 90% of all mortgages in our main sample are conforming mortgages, the results in Table C3 are very similar to those in Table 2. Specifically, Panel A, which focuses on observations in which appraised value is greater than or equal to the sale price, shows that the coefficients on all proxies for borrower riskiness and their first principal component are negative and highly significant. The coefficients in Panel B, which focuses on all other observations, are not all statistically significant, but they are almost all the correct sign.

Overall, Table C3 shows that our baseline prediction holds for mortgages that are likely to be sold to outside investors.

[C1] See https://www.fhfa.gov/Media/Blog/Pages/What-Types-of-Mortgages-Do-Fannie-Mae-and-Freddie-Mac-Acquire.aspx for more details.

C.3 Appraisal Bias

As discussed in Section 5.2.2, a significant portion of our observations might be affected by appraisal bias or renegotiation, and these affected observations might lead to a mechanical relation between our dependent variable and independent variables. To alleviate this concern, we separately estimate our specification for subsamples that include less affected observations and more affected observations. Less affected observations are defined to be those in which the appraised value is at least 5% higher than the sale price, and more affected observations are those in which the appraised value is at most 5% higher than the sale price. We present the results in Table C4.

Panel A presents results for less affected observations. Since these observations are less likely to be affected by appraisal bias, the dependent variable is unchanged relative to that in Table 2 (i.e., $log(A-P)$ for OLS, $A-P$ for Poisson). Consistent with Corollary 1, coefficients on all proxies for borrower riskiness and their first principal component are negative and highly significant.

In Panel B of Table C4, we focus on observations that are likely more affected by appraisal bias. As discussed in discussed in Section 5.2.2, this bias has been estimated to be about 5% to 6%. Therefore, the dependent variable in these regressions is $log(P - A/1.05)$ in our OLS regressions and $P - A/1.05$ in our Poisson regressions.[C2] Importantly, Panel B shows that after accounting for appraisal bias in the difference between sale price and appraised value, the estimates on all proxies for borrower riskiness and their first principal component are negative and highly significant.

In summary, Table C4 shows that our results are robust to adjusting appraised values for the well-documented appraisal bias.

[C2]We recognize that appraisal bias is likely more than 5% for observations whose sale price is further from the unobservable, unbiased appraised value and likely less than 5% for observations whose sale price is closer to the unobservable, unbiased appraised value. However, without more details on the amount of bias of different types of observations, we use the average bias of 5% as a rough estimate.

Table C1: Appraisal, Price, and Borrower Riskiness: 5% Repossession Costs

This table presents results from regressions of the distance between the effective estimate of collateral value (i.e., estimate of collateral value less repossession costs) and sale price on borrower riskiness. We assume repossession costs are 5% of the appraised value. In our ordinary least squares (OLS) regressions, the dependent variable is *log(0.95×A-P)* in Panel A and *log(P-0.95×A)* in Panel B. In our Poisson regressions, the dependent variable is *0.95×A-P* in Panel A and *P-0.95×A* in Panel B. *A* is *Appraised Value*, and *P* is *Sale Price*. Our proxies for borrower riskiness are *log(HE)*, *LTV*, and *FICO*. We take the negative of *log(HE)* and *FICO* so that the sign of the predicted coefficient is the same for all variables. *PC1* is the first principal component of the three proxies. See Table 1 for variable definitions. All regression variables are winsorized at the 0.5% and 99.5% levels. *LTV*, *FICO*, and *PC1* are standardized to zero mean and unit standard deviation. Standard errors below coefficients are adjusted for clustering at the zip code and year by month levels. Statistical significance at the 1%, 5%, and 10% levels is denoted by ***, **, and *.

Panel A: Sale price less than or equal to 95% of appraised value

	OLS				Poisson			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log(HE)	-0.131***				-0.141***			
	(0.002)				(0.003)			
LTV		-0.146***				-0.144***		
		(0.003)				(0.003)		
FICO			-0.075***				-0.072***	
			(0.004)				(0.004)	
PC1				-0.164***				-0.166***
				(0.003)				(0.004)
Zip×Year×Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	25.2%	24.8%	22.5%	23.1%				
Pseudo R^2					62.0%	61.8%	60.4%	61.0%
Observations	1,460,622	1,460,622	940,219	940,219	1,489,309	1,489,309	958,997	958,997

Panel B: Sale price greater than or equal to 95% of appraised value

	OLS				Poisson			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log(HE)	-0.191***				-0.313***			
	(0.004)				(0.008)			
LTV		-0.137***				-0.128***		
		(0.001)				(0.002)		
FICO			-0.063***				-0.064***	
			(0.001)				(0.002)	
PC1				-0.208***				-0.244***
				(0.002)				(0.002)
Zip×Year×Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	54.8%	49.0%	48.1%	51.2%				
Pseudo R^2					72.4%	62.9%	65.5%	69.0%
Observations	19,255,219	19,255,219	11,459,270	11,459,270	19,292,239	19,292,239	11,485,196	11,485,196

Table C2: Appraisal, Price, and Borrower Riskiness: 10% Repossession Costs

This table presents results from regressions of the distance between the effective estimate of collateral value (i.e., estimate of collateral value less repossession costs) and sale price on borrower riskiness. We assume repossession costs are 10% of the appraised value. In our ordinary least squares (OLS) regressions, the dependent variable is *log(0.90×A-P)* in Panel A and *log(P-0.90×A)* in Panel B. In our Poisson regressions, the dependent variable is *0.90×A-P* in Panel A and *P-0.90×A* in Panel B. *A* is *Appraised Value*, and *P* is *Sale Price*. Our proxies for borrower riskiness are *log(HE)*, *LTV*, and *FICO*. We take the negative of *log(HE)* and *FICO* so that the sign of the predicted coefficient is the same for all variables. *PC1* is the first principal component of the three proxies. See Table 1 for variable definitions. All regression variables are winsorized at the 0.5% and 99.5% levels. *LTV*, *FICO*, and *PC1* are standardized to zero mean and unit standard deviation. Standard errors below coefficients are adjusted for clustering at the zip code and year by month levels. Statistical significance at the 1%, 5%, and 10% levels is denoted by ***, **, and *.

Panel A: Sale price less than or equal to 90% of appraised value

	OLS				Poisson			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log(HE)	-0.125***				-0.129***			
	(0.004)				(0.005)			
LTV		-0.143***				-0.135***		
		(0.004)				(0.004)		
FICO			-0.070***				-0.061***	
			(0.005)				(0.006)	
PC1				-0.147***				-0.147***
				(0.006)				(0.006)
Zip×Year×Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	31.9%	31.7%	27.5%	28.0%				
Pseudo R^2					67.5%	67.4%	65.1%	65.6%
Observations	418,699	418,699	266,132	266,132	431,213	431,213	274,489	274,489

Panel B: Sale price greater than or equal to 90% of appraised value

	OLS				Poisson			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log(HE)	-0.180***				-0.268***			
	(0.004)				(0.006)			
LTV		-0.128***				-0.122***		
		(0.001)				(0.002)		
FICO			-0.062***				-0.064***	
			(0.001)				(0.002)	
PC1				-0.201***				-0.227***
				(0.002)				(0.002)
Zip×Year×Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	63.3%	57.0%	55.5%	59.0%				
Pseudo R^2					77.5%	69.0%	70.2%	73.7%
Observations	20,702,445	20,702,445	12,477,035	12,477,035	20,726,204	20,726,204	12,493,944	12,493,944

Table C3: Appraisal, Price, and Borrower Riskiness: Secondary Market

This table presents results from regressions of the distance between appraised value and sale price on borrower riskiness among mortgages that are likely to be sold to an outside investor. In our ordinary least squares (OLS) regressions, the dependent variable is $log(A-P)$ in Panel A and $log(P-A)$ in Panel B. In our Poisson regressions, the dependent variable is *A-P* in Panel A and *P-A* in Panel B. *A* is *Appraised Value*, and *P* is *Sale Price*. Our proxies for borrower riskiness are *log(HE)*, *LTV*, and *FICO*. We take the negative of *log(HE)* and *FICO* so that the sign of the predicted coefficient is the same for all variables. *PC1* is the first principal component of the three proxies. See Table 1 for variable definitions. All regression variables are winsorized at the 0.5% and 99.5% levels. *LTV*, *FICO*, and *PC1* are standardized to zero mean and unit standard deviation. Standard errors below coefficients are adjusted for clustering at the zip code and year by month levels. Statistical significance at the 1%, 5%, and 10% levels is denoted by ***, **, and *.

Panel A: Appraised value greater than or equal to the price

	OLS				Poisson			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log(HE)	-0.108***				-0.050***			
	(0.002)				(0.002)			
LTV		-0.131***				-0.048***		
		(0.002)				(0.003)		
FICO			-0.077***				-0.062***	
			(0.003)				(0.003)	
PC1				-0.156***				-0.097***
				(0.003)				(0.003)
Zip×Year×Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	15.3%	15.1%	14.1%	14.6%				
Pseudo R^2					33.8%	33.7%	35.9%	36.0%
Observations	10,206,248	10,206,248	6,747,257	6,747,257	18,232,404	18,232,404	11,217,204	11,217,204

Panel B: Price greater than or equal to the appraised value

	OLS				Poisson			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log(HE)	-0.978***				-2.057***			
	(0.022)				(0.038)			
LTV		-0.050***				-0.015		
		(0.019)				(0.019)		
FICO			-0.074***				0.002	
			(0.017)				(0.031)	
PC1				-0.503***				-0.770***
				(0.011)				(0.022)
Zip×Year×Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	58.5%	41.1%	36.3%	40.8%				
Pseudo R^2					75.3%	46.6%	46.3%	51.8%
Observations	267,148	267,148	108,721	108,721	2,972,029	2,972,029	1,279,052	1,279,052

Table C4: Appraisal, Price, and Borrower Riskiness: Appraisal bias

This table presents results from regressions of the distance between the appraisal bias-adjusted appraised value and sale price on borrower riskiness. Panel A includes observations that are less affected by appraisal bias, which we define as observations in which the appraised value is greater than or equal to 5% more than the sale price. Panel B includes observations that are more affected by appraisal bias, which we define as observations in which the appraised value is less than or equal to 5% greater than the sale price. In our ordinary least squares (OLS) regressions, the dependent variable is *log(A-P)* in Panel A and *log(P-A/1.05)* in Panel B. In our Poisson regressions, the dependent variable is *A-P* in Panel A and *P-A/1.05* in Panel B. *A* is *Appraised Value* (i.e., lender's value), and *P* is *Sale Price* (i.e., price). Our proxies for borrower riskiness are *log(HE)*, *LTV*, and *FICO*. We take the negative of *log(HE)* and *FICO* so that the sign of the predicted coefficient is the same across all variables. *PC1* is the first principal component of the three proxies. See Table 1 for variable definitions. All regression variables are winsorized at the 0.5% and 99.5% levels. *LTV*, *FICO*, and *PC1* are standardized to zero mean and unit standard deviation. Standard errors below coefficients are adjusted for clustering at the zip code and year by month levels. Statistical significance at the 1%, 5%, and 10% levels is denoted by ***, **, and *.

Panel A: Less affected by appraisal bias

	OLS				Poisson			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log(HE)	-0.147***				-0.174***			
	(0.002)				(0.003)			
LTV		-0.128***				-0.135***		
		(0.002)				(0.002)		
FICO			-0.060***				-0.067***	
			(0.002)				(0.003)	
PC1				-0.170***				-0.186***
				(0.002)				(0.003)
Zip×Year×Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	51.1%	48.5%	45.6%	47.8%				
Pseudo R^2					70.2%	68.8%	67.3%	68.7%
Observations	1,603,591	1,603,591	1,032,505	1,032,505	1,603,591	1,603,591	1,032,505	1,032,505

Panel B: More affected by appraisal bias

	OLS				Poisson			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log(HE)	-0.192***				-0.317***			
	(0.004)				(0.008)			
LTV		-0.137***				-0.129***		
		(0.001)				(0.002)		
FICO			-0.063***				-0.064***	
			(0.001)				(0.002)	
PC1				-0.209***				-0.246***
				(0.002)				(0.002)
Zip×Year×Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	54.3%	48.6%	47.7%	50.7%				
Pseudo R^2					72.0%	62.5%	65.1%	65.1%
Observations	19,118,683	19,118,683	11,364,711	11,364,711	19,153,826	19,153,826	11,389,122	11,389,122

Table C5: Return Autocorrelation and Borrower Riskiness: Simple Averages

This table presents results from regressions of *Log Return* on a one-year lag of itself interacted with dummies for our different proxies for borrower riskiness. *Log Return* is the annual change in log value using quarterly data of the non-seasonally adjusted FHFA HPI. *Low*, *Mid*, and *High* are dummy variables indicating whether the proxy in that column is in the lowest, middle, or highest tercile within a given CBSA. Our proxies for borrower riskiness are *log(HE)*, *LTV*, and *FICO*, which are defined as in Table 3 except they are simple averages rather than weighted averages. *PC1* is the first principal component of the three proxies. Panel A presents estimates of return autocorrelation for each tercile. Panel B presents return autocorrelation differences between terciles. Standard errors below coefficients are adjusted for clustering at the CBSA and year by quarter levels. Statistical significance at the 1%, 5%, and 10% levels is denoted by ***, **, and *.

Panel A: Return autocorrelation

	No Sort (1)	log(HE) (2)	LTV (3)	FICO (4)	PC1 (5)
Log $Return_{t-4}$	0.68***				
	(0.07)				
Log $Return_{t-4}$ × Low		0.73***	0.54***	0.78***	0.85***
		(0.10)	(0.07)	(0.08)	(0.11)
Log $Return_{t-4}$ × Mid		0.73***	0.73***	0.65***	0.79***
		(0.08)	(0.09)	(0.08)	(0.09)
Log $Return_{t-4}$ × High		0.60***	0.98***	0.58***	0.52***
		(0.09)	(0.12)	(0.11)	(0.07)
Adjusted R^2	41.4%	42.3%	43.6%	42.1%	44.1%
Observations	8,772	8,772	8,772	8,772	8,772

Panel B: Differences in return autocorrelation

	No Sort (1)	log(HE) (2)	LTV (3)	FICO (4)	PC1 (5)
Log $Return_{t-4}$	0.68***	0.73***	0.54***	0.78***	0.85***
	(0.07)	(0.10)	(0.07)	(0.08)	(0.11)
Log $Return_{t-4}$ × Mid		0.00	0.19**	-0.14**	-0.06
		(0.06)	(0.08)	(0.07)	(0.06)
Log $Return_{t-4}$ × High		-0.13	0.43***	-0.20	-0.33***
		(0.11)	(0.12)	(0.12)	(0.11)
Adjusted R^2	41.4%	42.3%	43.6%	42.1%	44.1%
Observations	8,772	8,772	8,772	8,772	8,772

Table C6: Return Autocorrelation and Borrower Riskiness: Interactions

This table presents results from regressions of *Log Return* on a one-year lag of itself interacted with different proxies for borrower riskiness. *Log Return* is the annual change in log value using quarterly data of the non-seasonally adjusted FHFA HPI. Our proxies for borrower riskiness are *log(HE)*, *LTV*, and *FICO*. *PC1* is the first principal component of the three proxies. See Table 3 for variable definitions. Standard errors below coefficients are adjusted for clustering at the CBSA and year by quarter levels. Statistical significance at the 1%, 5%, and 10% levels is denoted by ***, **, and *.

	(1)	(2)	(3)	(4)
Log Return$_{t-4}$	2.01***	0.36	9.91***	0.66***
	(0.50)	(0.45)	(2.54)	(0.08)
log(HE)	-0.05***			
	(0.01)			
Log Return$_{t-4}$ × log(HE)	-0.12***			
	(0.04)			
LTV		0.004***		
		(0.0009)		
Log Return$_{t-4}$ × LTV		0.004		
		(0.006)		
FICO			0.09***	
			(0.03)	
Log Return$_{t-4}$ × FICO			-1.24***	
			(0.34)	
PC1				-0.02***
				(0.00)
Log Return$_{t-4}$ × PC1				-0.06**
				(0.02)
Intercept	0.62***	-0.27***	-0.68***	0.02***
	(0.13)	(0.07)	(0.25)	(0.01)
CBSA FE	Yes	Yes	Yes	Yes
Adjusted R^2	42.8%	44.2%	44.6%	43.7%
Observations	8,772	8,772	8,772	8,772

Table C7: Return Volatility and Borrower Riskiness: Simple Averages

This table presents average volatilities within different subsamples. *Volatility* is the time-varying standard deviation of the error term from a first-order GARCH model of *Log Return* on a one-year lag of itself within each CBSA. *Log Return* is the annual change in log value using quarterly data of the non-seasonally adjusted FHFA HPI. *Low*, *Mid*, and *High* indicate whether an observation is in the lowest, middle, or highest tercile of a given proxy for borrower riskiness within a given CBSA. Our proxies for borrower riskiness are *log(HE)*, *LTV*, and *FICO*, which are defined as in Table 3 except they are simple averages rather than weighted averages. *PC1* is the first principal component of the three proxies. Panel A presents average volatilities. Panel B presents differences in volatilities (i.e., percentage points) between terciles that are identified within each proxy for borrower riskiness. Standard errors below differences are obtained from regressions of volatility on tercile dummies for the proxies for borrower riskiness. Standard errors are adjusted for clustering at the CBSA and year by quarter levels. Statistical significance at the 1%, 5%, and 10% levels is denoted by ***, **, and *.

Panel A: Average Volatility

	log(HE) (1)	LTV (2)	FICO (3)	PC1 (4)
Low	3.74%	5.76%	3.82%	3.53%
Mid	4.45%	4.55%	4.70%	4.31%
High	6.01%	3.83%	5.68%	6.37%

Panel B: Differences in Volatility

	log(HE) (1)	LTV (2)	FICO (3)	PC1 (4)
High-Low	2.27***	1.93***	1.86***	2.84***
	(0.39)	(0.57)	(0.45)	(0.42)
High-Mid	1.56***	0.72*	0.98*	2.06***
	(0.32)	(0.40)	(0.53)	(0.37)
Mid-Low	0.71***	1.21***	0.88*	0.78***
	(0.19)	(0.34)	(0.50)	(0.20)

Table C8: Return Volatility and Borrower Riskiness: Univariate Regression

This table presents results from regressions of *Volatility* on different proxies for borrower riskiness. *Volatility* is the time-varying standard deviation of the error term from a first-order GARCH model of *Log Return* on a one-year lag of itself within each CBSA. *Log Return* is the annual change in log value of the non-seasonally adjusted FHFA HPI. Our proxies for borrower riskiness are *log(HE)*, *LTV*, and *FICO*. *PC1* is the first principal component of the three proxies. See Table 3 for variable definitions. Standard errors below coefficients are adjusted for clustering at the CBSA and year by quarter levels. Statistical significance at the 1%, 5%, and 10% levels is denoted by ***, **, and *.

	(1)	(2)	(3)	(4)
log(HE)	2.19***			
	(0.35)			
LTV		-0.78***		
		(0.27)		
FICO			0.92***	
			(0.17)	
PC1				1.82***
				(0.35)
Intercept	4.72***	4.72***	4.72***	4.72***
	(0.31)	(0.33)	(0.34)	(0.31)
CBSA FE	Yes	Yes	Yes	Yes
Adjusted R^2	25.4%	21.0%	23.0%	24.9%
Observations	8,772	8,772	8,772	8,772