Sustainable finance under regulation*

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Abstract

We build a model analyzing optimal environmental regulation in the presence of socially responsible investors. Investors care about sustainability of their portfolios but cannot fully resolve the pollution externality. Regulations, such as pollution tax and subsidies to clean firms, reduce dirty firms' size but also reshape firms' shareholder compositions. Under the regulations, dirty firms' shareholders become on average less averse to holding polluting shares and hence these firms are less willing to adopt green technologies. We show that pollution can increase with regulation stringency. Optimal regulations do not always fully correct the externality and can deviate from the Pigouvian benchmark.

Keywords: Environmental regulation, socially responsible investment, pollution externality, shareholder composition

JEL: G11, G23, G32, H23, H41, Q58, M14

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1 Introduction

It is well acknowledged that human activities contribute to climate change and, hence, firms need to reduce their emissions to mitigate it (e.g., IPCC, 2018). In response to this challenge, there has been a dramatic increase in socially responsible investing in financial markets (e.g., GSIA, 2021). However, emissions are externalities, and so actions of investors alone are unlikely to fully resolve this issue. Therefore, government interventions are necessary. In fact, in recent years the number and the scope of environmental regulations have been growing worldwide (e.g., World Bank, 2022). The co-existence of the private and public approaches to address climate change raises fundamentally important questions: How do socially-concerned investors respond to environmental regulations? What is the optimal regulation in the presence of such investors?

To this end, we build a model of investors who differ in their attitudes toward investing in polluting firms, and firms that can adopt a green technology at a cost. Non-adopting firms pollute and thus contribute to the externality which negatively affects all investors. Investors' financing shape firms' sizes and shareholder compositions. Importantly, green technology adoption decisions are endogenously determined by shareholders' preferences. In the laissez-faire economy, investors' capital allocation and firms' adoption decisions are efficient if the pollution externality is absent. The presence of the externality calls for regulation. We show that the commonly used regulatory tools, such as pollution tax and subsidies to clean firms, have two countervailing effects. On the one hand, they make polluting firms less financially attractive which reduces the amount of capital they raise. On the other hand, they reshape firms' shareholder compositions such that fewer firms may end up adopting the green technology. Due to the latter effect, a more stringent regulation can in fact *increase* pollution. All in all, optimal regulation can deviate from a Pigouvian benchmark (where tax/subsidy equal to the marginal social cost of pollution).

More specifically, we consider an economy populated with atomistic heterogeneous investors and firms. There are two firm types; firms within each type are ex-ante identical but can differ ex-post. Firms of one type (c-firms) are less productive but $c$leaner than firms of the other type (d-firms). That is, c-firms never pollute, while d-firms are initially equipped with a polluting brown technology but can adopt a non-polluting green technology at an ex-ante unknown cost.[1] Throughout most of the paper, we consider a simple binary adoption cost structure. Namely, ex-post realizations of the adoption cost

[1]The model results are qualitatively unchanged if c-firms can pollute but are cleaner than d-firms, i.e. they face lower adoption costs and/or pollute less under the brown technology. Section 2.1 provides several real-life interpretations of c- and d-firms.

for some d-firms are relatively low; the adoption cost for other d-firms is prohibitively large such that they always operate the brown technology. Non-adopting d-firms—those facing the large adoption cost and those choosing not to adopt the green technology under the low adoption cost—contribute to the aggregate pollution externality. The externality negatively affects all investors. Since all agents in our model are atomistic, firms and investors take the externality as given.

On top of being adversely affected by the aggregate pollution externality, investors have heterogeneous preferences for sustainability of their investments. In particular, some investors care primarily about financial performance of their portfolios. Other investors suffer a high disutility from holding shares of polluting firms. Preferences of such investors can be driven by a non-pecuniary warm-glow disutility from being associated with polluting firms. These investors can be also managers of ESG funds, whose compensation and reputation hinge on identifying sustainable firms.[2] Using the terminology of Andreoni (1989, 1990), investors in our model are impure altruists as they suffer from the pollution externality and care about their individual investments beyond their contribution to the public bad. Notably, although investors do not internalize their impacts on the externality, their investment decisions still reduce aggregate pollution because they allocate less funds to polluting firms.[3]

We start by analyzing investors' capital allocation and firms' adoption decisions in the laissez-faire economy. The capital allocation is fully characterized by a threshold in the strength of investors' aversion to investing in polluting firms. Namely, investors whose aversions to holding polluting shares are above the threshold acquire shares of clean but less financially attractive c-firms, whereas remaining investors acquire shares of d-firms.[4] For a threshold investor, the d-firms' financial advantage and disutility caused by their higher expected pollution are exactly balanced. As a result, d-firms' capital is limited to wealth owned by investors with below-threshold aversions to holding polluting shares. Crucially, investors' shareholdings also determine firms' decisions to adopt the green technology. Specifically, a d-firm adopts the green technology if its average shareholder's

[2]See Crumpler and Grossman (2008), Bonnefon, Landier, Sastry, and Thesmar (2019), and Heeb, Kölbel, Paetzold, and Zeisberger (2022) for evidence on warm-glow preferences. Hartzmark and Sussman (2019) document that mutual funds ranked highly in sustainability by Morningstar attract additional flows.

[3]Our modeling of atomistic green investors' preferences is standard in the literature (e.g., Pástor, Stambaugh, and Taylor, 2021). If investors have size, they can internalize their impacts on the externality but tend to underinvest in the public good due to a free-rider problem.

[4]In reality, ESG funds tend to invest in clean companies. For example, according to the statement on the Vanguard's website, "most of our [ESG] funds are indexed and follow an exclusionary strategy that omits companies that don't meet certain ESG criteria," i.e. d-firms in our model. See https://investor.vanguard.com/investment-products/esg.

aversion to holding polluting shares exceeds the adoption cost. It means that there are no conflicts of interest between shareholders and firm managers in our model: Firm managers maximize average utility of their firms' shareholders when choosing whether to adopt the green technology (taking the aggregate pollution as given).

Two equilibria are possible in this environment, i.e. the *brown* equilibrium and the *green* equilibrium. In the brown equilibrium, a threshold investor is weakly averse to holding polluting shares, which implies that shareholder base of d-firms consists of mostly financially concerned investors. In this equilibrium, even d-firms facing low realizations of the adoption cost do not pay it, and so none of d-firms adopts the green technology. Expecting all d-firms to pollute, even investors with moderate aversions to holding polluting shares do not invest in them. In the green equilibrium, d-firms' shareholders are sufficiently strongly averse to holding polluting shares. Hence, d-firms facing low adoption costs switch to the green technology. Expecting d-firms to pollute less on average, investors with moderate aversions to holding polluting shares invest in them.

It follows that there are two main differences between the equilibria. In the brown equilibrium, d-firms pollute intensively but at the same time are relatively small because they raise capital only from investors who care mostly about financial performance of their portfolios. In the green equilibrium, in contrast, d-firms raise more capital but pollute less intensively. This can give rise to a counterintuitive outcome: Although firms pollute less intensively in the green equilibrium, the fact that they are also larger implies that the aggregate pollution can be higher in this equilibrium.[5] We assume that this is not the case. In particular, the aggregate pollution in the green equilibrium is lower if the fraction of d-firms with low realizations of the adoption cost is sufficiently high.

Characterizing conditions under which the two equilibria exist, we find that the green equilibrium exists only if the relative financial advantage of d-firms is sufficiently large. Intuitively, a high financial benefit is needed to incentivize investors who care strongly about sustainability of their portfolios to acquire shares of d-firms. Such investors in turn make d-firms' shareholder base sufficiently pro-green on average, and d-firms with low realizations of the adoption cost switch to the green technology. Analogously, the brown equilibrium exists only if the relative financial advantage of d-firms is not too large. Notably, we show that the two equilibria exist simultaneously if the difference in financial performances between c- and d-firms is moderate. In our analyses, we assume that the financial advantage of d-firms is sufficient for the green equilibrium to exist.

We then proceed to the welfare analysis. In our model, a natural measure of welfare

[5] Such a possibility is well-known in environmental economics (Jevons, 1866; Acemoglu and Rafey, 2019).

is the aggregate utility of all investors. It consists of three components: output produced by all firms net of any green technology adoption costs; aggregate disutility from holding polluting shares; and harm due to the pollution externality. We show that the welfare is higher in the green equilibrium, provided that the adoption cost paid by d-firms in this equilibrium is below a threshold. Furthermore, the threshold increases in the externality strength. This is intuitive: It is socially optimal to pay a higher cost for a pollution reduction as pollution becomes a more serious concern.

Importantly, we show that the laissez-faire economy is efficient if the externality is absent and investors coordinate on a socially preferable equilibrium when multiplicity is possible.[6] That is, the planner cannot achieve a higher welfare by choosing *any* different composition of firms' shareholder bases. This result is not immediate in our setting featuring incomplete markets. It is important because it implies that the only potential source of inefficiency is the pollution externality. In the presence of the externality, there is a room for government interventions.

We analyze two widely used policy tools. First, we consider subsidies to c-firms. A prominent example of such a policy is government support to renewable energy producers. In our model, the subsidy reduces the financial advantage of d-firms relative to c-firms. It has two effects on the economy. On the one hand, a higher subsidy implies that more investors become shareholders of c-firms as their financial performance improves. Consequently, d-firms receive less funding and, hence, pollute less in any given equilibrium. On the other hand, the policy reshapes firms' shareholder bases. As the financial advantage of d-firms diminishes because of the subsidy, most pro-green shareholders of these firms switch to become shareholders of c-firms. Consequently, an average d-firms' shareholder becomes less averse to holding polluting shares, and so these firms are less likely to adopt the green technology. We show that if the green equilibrium exists in the absence of the subsidy, it ceases to exist if the subsidy is sufficiently large. An increase in the subsidy can lead to a switch from the green to the brown equilibrium, which is associated with a sharp increase in pollution and an abrupt decline in welfare.

In light of the two effects the subsidy has on the economy, we characterize the optimal subsidy size as a function of the externality strength. If pollution is not a big social issue, i.e. the externality is weak, the optimal subsidy is small. Such a subsidy reduces the size of d-firms mildly and has a small effect on firms' shareholder compositions. As a result, firms' adoption decisions are unaffected, and the green equilibrium exists under the

[6]Naturally, if investors fail to coordinate on a socially preferable equilibrium, the planner can improve welfare by changing the type of equilibrium played.

optimal subsidy. In this case, the planner fully corrects the externality and thus reaches the Pigouvian benchmark: The optimal subsidy equals to the marginal social cost of pollution. As the externality strengthens, the planner optimally under-corrects it by setting the subsidy below the marginal cost of pollution. By doing so, the planner strikes a balance between reducing the size of d-firms and keeping their shareholders sufficiently pro-green for the green equilibrium to exist. Finally, if the externality is very strong, the planner subsidizes c-firms heavily, which leads to a large capital reallocation away from more productive d-firms. In this case, the brown equilibrium is the only possibility and thus all d-firms pollute. The overall pollution is low, however, because d-firms receive little capital. While such a large subsidy leads to a substantial output reduction, it is the only way to sufficiently ameliorate the pollution externality.

The second policy tool we consider is the pollution tax which is levied on those d-firms that do not adopt the green technology. Similar to the subsidy to c-firms, the tax has two effects on the economy. Because d-firms with high adoption cost realizations pollute in any equilibrium, the tax reduces expected financial performance of all d-firms. Therefore, a higher tax implies a reallocation of capital away from d-firms and, hence, a lower aggregate pollution in any equilibrium. The effect of the tax on green technology adoption decisions is more nuanced than that of the subsidy to c-firms. On the one hand, as the tax increases, d-firms' shareholders become on average less concerned about their portfolios' sustainability. On the other hand, it becomes more financially costly to operate the brown technology. While the latter effect always dominates if the tax is sufficiently high, the shareholder base effect can dominate if the tax is not too high. It is worth noting that in reality setting the tax to a high level is likely to be politically infeasible. Hence, we think that small and moderate tax levels are more empirically relevant.

We then characterize the optimal tax as a function of the externality strength. We find that the planner can achieve the Pigouvian benchmark if the externality is weak or strong. In the former case, the optimal tax is small and so is its effect on firms' shareholder compositions. In the latter case, the optimal tax is large and significantly reduces the size of d-firms. It also makes it very financially costly not to adopt the green technology. As a result, the green equilibrium exists in both cases. If the externality is moderately strong, fully correcting the externality may make the green equilibrium non-existent. In this case, the planner deviates from the Pigouvian benchmark.

Literature Our paper contributes to the theoretical literature on the impact of socially-concerned investors on firm production decisions. One strand of this literature studies how atomistic investors affect production decisions through divestment (Heinkel, Kraus, and Zechner, 2001; Pástor et al., 2021; De Angelis, Tankov, and Zerbib, 2022).[7] The threat of divestment by socially-concerned investors may incentivize firms to change their production decisions ex-ante.[8] Broccardo, Hart, and Zingales (2022) and Jagannathan, Kim, McDonald, and Xia (2022) compare the effectiveness of exit and voice strategies in inducing firms to reduce their negative externalities. In our model, green technology adoption decisions are made ex-post, reflecting preferences of firms' existing shareholders. Exit of pro-green investors from polluting firms leaves their shareholder bases less concerned about environmental harm, thus reducing the likelihood of the green technology adoption. Crucially, shareholder bases are formed endogenously as a result of investors' portfolio choices. This is different from Broccardo et al. (2022) and Jagannathan et al. (2022) who consider exit and voice strategies separately.

A complementary strand of the literature studies the effect of engagement by large impact investors on externalities generated by firms. Chowdhry, Davies, and Waters (2019) study impact investing when project owners cannot commit to social objectives. Oehmke and Opp (2020) characterize conditions under which large socially responsible investors can affect financially constrained entrepreneurs. Green and Roth (2021) analyze capital allocation in the presence of socially responsible investors who care about social value created only by firms they finance and those who care about the aggregate public good. Gollier and Pouget (2022) show that a large activist can generate positive financial returns by investing in non-responsible firms, turning them green, and selling back to socially responsible investors in the market. Edmans, Levit, and Schneemeier (2022) argue that a blockholder should not simply divest brown firms but instead should tilt toward those brown firms that have taken corrective actions. Landier and Lovo (2020) and Gupta, Kopytov, and Starmans (2022) explore the role of search frictions for efficacy of socially responsible investing. Morgan and Tumlinson (2019) emphasize a free-rider problem among socially responsible investors.

Relative to the above literature, an important distinguishing feature of our paper is the interaction between government policies and shareholder base compositions that ultimately determine firms' decisions to reduce negative externalities. To the best of our

[7]These papers also study asset pricing implications; other studies focusing on asset pricing in the presence of socially-concerned investors are Fama and French (2007); Luo and Balvers (2017); Pedersen, Fitzgibbons, and Pomorski (2021); Goldstein, Kopytov, Shen, and Xiang (2022); Zerbib (2022). Piatti, Shapiro, and Wang (2022) study asset pricing implications and public good provision in the portfolio-choice model featuring large socially-concerned and financial investors.

[8]Davies and Van Wesep (2018) show that coordinated divestment campaigns can be ineffective if managers care about long-run profitability.

knowledge, the shareholder base effect is novel to the literature on the environmental regulation. This literature is vast (see Nordhaus (2014) for a comprehensive discussion); most related to our paper are models studying environmental regulation in the presence of socially-concerned investors. Baker, Hollifield, and Osambela (2020) and Moisson (2020) consider firms with pre-determined technologies (green or brown) and study effects of Pigouvian taxation on socially-concerned investors' portfolio decisions. In contrast, in our model green technology adoption decisions are endogenous. Acemoglu and Rafey (2019) and Biais and Landier (2022) analyze green technology development and adoption under environmental regulations when the regulator lacks commitment power. We abstract from the regulator's commitment problem and instead focus on the effects of policies on shareholder base compositions. A recent paper by Inderst and Opp (2022) considers investors who care about investing in products labelled as sustainable and studies optimal labelling in this environment.[9]

Finally, our paper is related to papers studying firms' decisions under endogenous shareholder bases formed by investors with heterogeneous preferences. Baker and Wurgler (2004) show theoretically and empirically that firms cater to shareholders' demand for dividends. Levit, Malenko, and Maug (2022) analyze secondary market trading and voting in a one-firm setting. They emphasize inefficiencies arising when post-trade voting outcomes are determined by preferences of a median—not an average—shareholder. We abstract from this inefficiency in our model and instead focus on the pollution externality. Our paper features several other important differences. First, we consider a multiple-firm general equilibrium setting, which is crucial to analyze the aggregate pollution externality. Second, we consider the primary market for firms' shares. Thus, investors' portfolio choices affect not only firms' shareholder base compositions but also capital amounts they raise. Accordingly, we analyze environmental policies that simultaneously affect firm sizes and shareholder base compositions. These features also differentiate our paper from Gollier and Pouget (2022) who consider firms with pre-established assets and, similar to Levit et al. (2022), study secondary market trading. In a contemporaneous paper by Bisceglia, Piccolo, and Schneemeier (2022), secondary market trading may lead to concentration of responsible capital which crowds out socially responsible investment in excluded firms and increases markups for green products.

[9]In settings without socially-concerned investors, Hoffmann, Inderst, and Moslener (2017) and Heider and Inderst (2021) analyze environmental regulations when firms face financing frictions; Dávila and Walther (2021) and Oehmke and Opp (2022) study financial regulations such as green capital requirements aimed at mitigating financial risks and environmental externalities; Döttling and Rola-Janicka (2022) analyze optimal environmental and financial regulations jointly.

The remainder of the paper is organized as follows. We set up the model in Section 2 and analyze in without regulations in Section 3. Section 4 conducts policy analyses. Section 5 concludes. All derivations and proofs omitted from the main text are in Appendix.

2 Model setting

The economy is populated with a continuum of firms of $N = 2$ types, $i \in \{c, d\}$, and a continuum of heterogeneous atomistic investors. There are two periods, $t = 1, 2$, and no time discounting. All agents are risk-neutral. At $t = 1$, investors finance firms by buying a portfolio of firms' shares. At $t = 2$, firm managers choose whether to operate a brown technology or adopt a green technology at a cost. When choosing the technology, firm managers maximize an average utility of their shareholders. Firms then produce, investors consume firms' cash flows and potentially suffer from a negative externality.

2.1 Production technology

Throughout the paper, we focus on symmetric equilibria in which all firms of the same type receive identical funding and have identical shareholder bases. That is, at $t = 1$ firms of the same type are identical. As we specify below, they, however, may differ at $t = 2$ when production decisions are made.

As a result of investors' portfolio choice, a firm j of type $i \in \{c, d\}$ receives capital $k_{ij} = k_i$ and produces the final good according to the following technology,

$$y_{ij} = k_i \left[\alpha_i - f_{ij} \mathbb{I}\left\{\mathsf{a}_{ij} = 1\right\}\right],$$

where $\mathsf{a}_{ij} \in \{0, 1\}$ indicates if the firm adopts the green technology. If the firm operates the green technology, i.e. $\mathsf{a}_{ij} = 1$, it incurs a proportional cost $f_{ij} \geq 0$ that captures any expenses to install new equipment and change the production process. If the brown technology is operated, i.e. $\mathsf{a}_{ij} = 0$, the firm does not incur any additional costs. However, it generates h_i units of public bad per unit of capital. In what follows, we refer to it as pollution. That is, firm j of type i produces $h_i k_i \mathbb{I}\left\{\mathsf{a}_{ij} = 0\right\}$ units of pollution.[10]

Adoption costs f_{ij} are unknown at $t = 1$, and so investors are uncertain about exact values of adoption costs at the portfolio formation stage. A common prior about f_{ij} is described by a type-specific cumulative distribution function $\Phi_i(\cdot)$. Corresponding

[10]It is straightforward to extend the model such that the green technology adoption reduces—but not necessarily fully eliminates—pollution intensity.

probability density function is $\phi_i(\cdot)$. At $t = 2$, adoption costs are realized and become known to all agents. Because of the ex-post heterogeneity in adoption cost realizations, firms of the same type may make different adoption decisions. Cost realizations are independent across all firms. That is, the economy does not feature aggregate uncertainty.

Firms of the two types differ in terms of their financial performance and "cleanliness." Type-c firms are less productive but are ex-ante cleaner, in the sense that they pollute less under the brown technology and are expected to face lower adoption costs.

Assumption 1. *Type-c firms are less productive but cleaner. That is,*

(i) $\Delta \equiv \alpha_d - \alpha_c > 0$*;*

(ii) the green technology adoption cost for d-firms has first-order stochastic dominance over the green technology adoption cost for c-firms;

(iii) $h_c \leq h_d$*.*

There are few ways to think about firms of different types in our model. For example, c-firms can be thought as firms that are committed to invest significant funds in research and development of clean technologies. While their financial performances can be weakened because of these expenses, they are also more likely to succeed in developing an in-house clean technology that is cheap to adopt. In contrast, d-firms do not spend too much on R&D, which is attractive financially but makes the green technology adoption costlier. In particular, firms with unsuccessful R&D outcomes—predominantly d-firms—may rely more on buying expensive patents developed by other firms.

One may also think about c- and d-firms as using inherently different ways to produce. For example, c-firms can be thought as firms producing energy using renewable technologies, such as wind and solar. In contrast, d-firms operate well developed but more polluting fossil fuel-based technologies. Reducing their carbon footprints, e.g. through carbon capture, is naturally more expensive for d-firms. Under this interpretation, c-firms are also likely to pollute less intensively even if they do not adopt the green technology, i.e. $0 \leq h_c < h_d$.[11]

[11] In reality, firms producing renewable energy can still be environmentally harmful. For example, manufacturing and transporting photovoltaic elements or wind turbines is subject to emissions; renewable energy installations can also disrupt land use and wildlife habitat. Therefore, even for such firms there is room for becoming more sustainable. Correspondingly, in our framework c-firms can reduce their pollution intensity by adopting the green technology.

2.2 Investor preferences

There is a unit mass of heterogeneous atomistic investors. All investors are endowed with the same initial wealth w_0, which we normalize to one without loss of generality. Investors differ in terms of their attitudes toward holding shares of polluting firms in their portfolios. In particular, an investor γ holding s_{ij}^{γ} shares of firm j of type i incurs a nonpecuniary disutility $s_{ij}^{\gamma} \times \gamma h_i k_i \mathbb{I}\{\mathsf{a}_{ij} = 0\}$, where without loss of generality we normalize the total amount of shares issued by each firm to one. The cumulative distribution function and the probability density function of γ in the investor population are $G(\cdot)$ and $g(\cdot)$, respectively, and the support is $[0, \bar{\gamma}]$, where $\bar{\gamma} > 0$. We assume that $G(\cdot)$ is a strictly increasing continuous function on $[0, \bar{\gamma}]$ and that it is differentiable on $(0, \bar{\gamma})$.

In our model, investors with $\gamma > 0$ suffer a warm-glow disutility from holding shares of polluting firms. That is, high-γ investors care about greenness of their portfolios but, being atomistic, do not consider how their investments affect the aggregate pollution. Notably, high-γ investors can be also viewed as managers of ESG-dedicated funds whose fund flows depend on greenness of their portfolios. Under this interpretation, high-γ investors acquire shares of low-polluting firms because it helps attract fund inflows and, thus, increase their monetary payoffs.

Since firms of the same type are ex-ante identical and we focus on symmetric equilibria, an investor γ chooses the same holdings in all firms of the same type, $s_{ij}^{\gamma} = s_i^{\gamma}$. Furthermore, share prices, expected adoptions costs and expected adoption decisions are the same for firms of the same type. Therefore, the expected utility of investor γ holding a portfolio of stocks can be written as

$$\sum_{i=1}^{N=2} s_i^{\gamma} k_i \left[\alpha_i - \mathbb{E}\left(f_{ij} | \mathsf{a}_{ij} = 1\right) \mathbb{P}\left(\mathsf{a}_{ij} = 1\right) - \gamma h_i \mathbb{P}\left(\mathsf{a}_{ij} = 0\right)\right] + \left(1 - \sum_{i=1}^{N=2} s_i^{\gamma} p_i\right) R - \varepsilon \xi\left(P\right),$$

where p_i is the share price of type-i firms and R is the risk-free rate. $\xi(P)$ is disutility due to the aggregate pollution P,

$$P = \sum_{i=1}^{N=2} \left[h_i k_i \int_j \mathbb{I}\left\{\mathsf{a}_{ij} = 0\right\} dj\right],$$

where $\int_j \mathbb{I}\{\mathsf{a}_{ij} = 0\} dj$ is the mass of type-i firms operating the brown technology. Recall that there is no aggregate uncertainty in our model, and so the equilibrium aggregate pollution P is known to investors at $t = 1$. The parameter $\varepsilon \geq 0$ captures the strength of the pollution externality. We assume that $\xi(\cdot)$ is a strictly increasing and convex function, $\xi(0) = \xi'(0) = 0$. Our assumption on the convexity of the externality function $\xi(\cdot)$ is in line with Acemoglu and Rafey (2019).[12] Crucially, as we focus on atomistic investors and firms, all economic agents take the aggregate pollution as given.

2.3 Green technology adoption decision

At $t = 2$, after investors have chosen their portfolios and shareholder bases have been formed, firm-specific green technology adoption costs are realized and firm managers choose whether to adopt the green technology. We abstract away from any agency frictions and assume that managers pick the technology to maximize an average utility of their shareholders. Specifically, a manager of a given firm adopts the green technology if and only if the overall disutility from holding a brown firm among all shareholders of this firm exceeds the adoption cost. As discussed above, since all agents and firms are atomistic, managers do not consider the fact that operating the brown technology contributes to the aggregate pollution.

The following lemma establishes an intuitive result that the green technology is adopted if the adoption cost is sufficiently low.

Lemma 1. *There exists a threshold f_i^* such that the green technology is adopted by firm j of type i if and only if $f_{ij} \leq f_i^*$.*

Proof. See Appendix A.1. □

2.4 Portfolio choice

At $t = 1$, investors choose how to allocate their initial wealth $w_0 = 1$ expecting that firm j of type i is going to adopt the green technology if and only if the realized adoption cost f_{ij} will be below the threshold f_i^*. From the perspective of an investor γ, the value of investing one dollar in this firm is

$$v_i(\gamma, f_i^*) = \alpha_i - \mathbb{E}(f_{ij}|f_{ij} \leq f_i^*)\,\mathbb{P}(f_{ij} \leq f_i^*) - \gamma h_i \mathbb{P}(f_{ij} > f_i^*). \tag{1}$$

[12] Our main results hold if $\xi(\cdot)$ is weakly convex or linear (as in Golosov, Hassler, Krusell, and Tsyvinski, 2014). However, the analytic characterization becomes more tedious.

The portfolio choice problem is then

$$\max_{\{s_i^\gamma\}} \sum_{i=1}^{N=2} s_i^\gamma k_i v_i\left(\gamma, f_i^*\right) + \left(1 - \sum_{i=1}^{N=2} s_i^\gamma p_i\right) R - \varepsilon \xi(P), \tag{2}$$
$$\text{s.t. } \sum_{i=1}^{N=2} s_i^\gamma p_i \le 1, \quad s_i^\gamma \ge 0 \; \forall i.$$

The constraints restrict borrowing and short-selling.

Recall that each firm issues a unit mass of shares, so that the market clearing implies $\int s_i^\gamma dG(\gamma) = 1$. As a result, the amount of investment each type-i firm receives is $k_i = p_i \int s_i^\gamma dG(\gamma) = p_i$. Furthermore, we assume that bonds are in zero net supply. Therefore, the riskfree rate R is endogenously determined such that none of the investors is willing to save via bonds, and so the constraint $\sum_{i=1}^N s_i^\gamma p_i \le 1$ holds as equality for all investors.

Notice that utility per dollar invested in a type-i firm, $v_i\left(\gamma, f_i^*\right)$, is linear in γ. As a result, an investor γ invests all her wealth in shares of firms of type i with highest $v_i\left(\gamma, f_i^*\right)$. The following lemma formally establishes this result.

Lemma 2. *Mass of investors that hold shares of both firm types is zero. Furthermore, if investors γ_1 and $\gamma_2 > \gamma_1$ hold shares of type-i firms, all investors with $\gamma \in (\gamma_1, \gamma_2)$ hold shares of only type-i firms.*

Proof. See Appendix A.2. □

Lemma 2 implies that adoption decisions can be easily characterized. In particular, denote the set of investors who invest their entire wealths in shares of type-i firms by $\mathcal{S}_i$. In any symmetric equilibrium, all investors in $\mathcal{S}_i$ acquire the same number of all type-i firms' shares because they have identical initial wealths. As a result, the threshold adoption cost f_i^* is determined by the average γ of investors in $\mathcal{S}_i$, i.e.

$$\underbrace{f_i^* \int_{\gamma \in \mathcal{S}_i} dG(\gamma)}_{\text{Adoption cost}} = \underbrace{h_i \int_{\gamma \in \mathcal{S}_i} \gamma dG(\gamma)}_{\text{Disutility from holding a brown firm}} \Leftrightarrow f_i^* = \frac{h_i \int_{\gamma \in \mathcal{S}_i} \gamma dG(\gamma)}{\int_{\gamma \in \mathcal{S}_i} dG(\gamma)}. \tag{3}$$

As a result of investors' portfolio choices and green technology adoption decisions, type-i firms receive amount of capital K_i, produce output Y_i and pollution P_i as follows.

$$K_i = \int_{\gamma \in \mathcal{S}_i} dG(\gamma), \quad Y_i = K_i \left(\alpha_i - \int_0^{f_i^*} f d\Phi_i(f)\right), \quad P_i = h_i K_i \left(1 - \Phi_i\left(f_i^*\right)\right).$$

Finally, type-i firms generate the nonpecuniary disutility Υ_i for its shareholders,

$$\Upsilon_i = (1 - \Phi_i(f_i^*))\, h_i \int_{\gamma \in \mathcal{S}_i} \gamma dG(\gamma),$$

where $1 - \Phi_i(f_i^*)$ is the fraction of type-i firms not adopting the green technology.

3 Equilibrium characterization

In this section, we characterize the model in the absence of government interventions. We show that in the laissez-faire economy investors' capital allocation and firms' green technology adoption decisions are efficient if the pollution externality is absent. The presence of the externality calls for regulation which we analyze in Section 4.

We consider the model described in Section 2 under additional assumptions on the distribution of the adoption costs. These assumptions ease analytic characterization substantially but are general enough to deliver our main results. Specifically, we assume that c-firms are always green, i.e. their costs of adopting the green technology are zero or, equivalently, they do not pollute under the brown technology, $h_c = 0$. The adoption cost of d-firms follows a binary structure,

$$f_d = \begin{cases} f_h, & \text{with probability } \pi, \\ f_l, & \text{with probability } 1 - \pi, \end{cases}$$

where $f_h > f_l > 0$.

Because c-firms never pollute, investing one dollar in them generates $v_c = \alpha_c$ to all their shareholders. Investing one dollar in a d-firm, in contrast, generates heterogeneous utilities,

$$v_d(\gamma, f^*) = \alpha_d - \begin{cases} \gamma, & f^* < f_l, \\ (1-\pi) f_l + \pi \gamma, & f^* \in [f_l, f_h), \\ (1-\pi) f_l + \pi f_h, & f^* \geq f_h, \end{cases} \tag{4}$$

where we normalize $h_d = 1$. In the expression above, we omit the type-specific subscript in the adoption cost threshold f^*. This is because c-firms never pollute, and so the only meaningful threshold is the threshold for d-firms, which we denote by f^*.

Equation (4) implies that three cases are possible depending on the value of the adoption threshold f^*. In the first case, $f^* < f_l$ and even d-firms facing the low adoption cost do not adopt the green technology. In what follows, we refer to a corresponding equilibrium as *brown*. In the second case, $f^* \in [f_l, f_h)$ and d-firms adopt the green technology if the cost realization is f_l but not if it is f_h. We refer to equilibrium in this case as *green*. To keep the analyses compact, we assume that f_h is sufficiently high,

$$\alpha_d - \alpha_c = \Delta < (1-\pi) f_l + \pi f_h. \tag{5}$$

Suppose that d-firms are expected to adopt the green technology for any possible realization of the adoption cost, i.e. $f^* > f_h$. Under (5), d-firms are then less financially attractive than c-firms and, hence, raise zero capital.[13]

In equilibrium, f^* is endogenously determined by the preferences of d-firms' shareholders. In the next subsection, we characterize shareholder sorting and green technology adoption decisions in the brown and green equilibria.

3.1 Equilibrium characterization

Brown equilibrium: $f^* = f_b^* < f_l$. In this equilibrium, d-firms do not adopt the green technology even if the adoption cost turns out to be low. We have $v_d(\gamma) = \alpha_d - \gamma$, and so investors with $\gamma \in [0, \hat{\gamma}_b)$ invest in d-firms and investors with $\gamma \in [\hat{\gamma}_b, \bar{\gamma}]$ invest in c-firms, where $\hat{\gamma}_b$ is determined by

$$v_c = v_d(\hat{\gamma}_b) \Leftrightarrow \hat{\gamma}_b = \Delta. \tag{6}$$

Note that $\hat{\gamma}_b > 0$ under Assumption 1. Furthermore, if $\hat{\gamma}_b < \bar{\gamma}$, then both firm types receive nonzero financing in this equilibrium.

When making a portfolio choice, investors face a tradeoff. On the one hand, investing in d-firms is financially beneficial since they are more productive, i.e. $\Delta > 0$. On the other hand, d-firms pollute, and so investors with positive γ suffer from a nonpecuniary disutility when investing in such firms. In equilibrium, investors with sufficiently high $\gamma > \hat{\gamma}_b$ choose to hold shares of less productive but cleaner c-firms. Investors with $\gamma < \hat{\gamma}_b$ are less averse to holding polluting shares and thus invest in d-firms. A marginal investor with $\gamma = \hat{\gamma}_b$ is indifferent: her nonpecuniary disutility from holding d-firms' shares is

[13]Here we implicitly assume that if d-firms raise zero capital, they are not expected to adopt the green technology (for example, due to arbitrarily small fixed costs of adoption, which we do not explicitly model to keep the analyses transparent). That is, there is no equilibrium with $f^* \geq f_h$.

exactly offset by their higher financial performance, i.e. $\hat{\gamma}_b = \Delta$.

To verify that this is indeed an equilibrium, we need to check if the threshold adoption cost $f_b^* < f_l$. Using (3), we can write the threshold adoption cost as

$$f_b^* = f_b^*\left(\hat{\gamma}_b\right) = \frac{\int_0^{\hat{\gamma}_b} \gamma dG(\gamma)}{G\left(\hat{\gamma}_b\right)}. \tag{7}$$

Differentiating f_b^* with respect to $\hat{\gamma}_b$, we obtain

$$\frac{\partial f_b^*}{\partial \hat{\gamma}_b} = \frac{g\left(\hat{\gamma}_b\right)}{G\left(\hat{\gamma}_b\right)}\left(\hat{\gamma}_b - \frac{\int_0^{\hat{\gamma}_b} \gamma dG(\gamma)}{G\left(\hat{\gamma}_b\right)}\right) > 0. \tag{8}$$

As $\hat{\gamma}_b$ decreases, shareholders of d-firms become on average less concerned about sustainability of their portfolios. Consequently, they are less willing to sacrifice financial performance for the green technology adoption, and f_b^* declines. By definition of $\hat{\gamma}_b$ (6), this happens if d-firms become less financially attractive relative to c-firms, i.e. Δ shrinks. Intuitively, if the financial benefit from investing in d-firms is small, only investors who do not suffer too much from holding polluting shares, i.e. low-γ investors, are willing to invest in them. As a result, the brown equilibrium exists if d-firms' financial advantage is not too high, their shareholder bases consist of low-γ investors, and so their managers choose not to adopt the green technology. We formalize this result in Proposition 1 below.

To complete the description of the brown equilibrium, we compute output Y_b, pollution P_b and investors' welfare W_b in this equilibrium.

$$\begin{aligned}
Y_b &= \alpha_c\left(1 - G\left(\hat{\gamma}_b\right)\right) + \alpha_d G\left(\hat{\gamma}_b\right), \\
P_b &= G\left(\hat{\gamma}_b\right), \\
W_b &= Y_b - \int_0^{\hat{\gamma}_b} \gamma dG(\gamma) - \varepsilon\xi\left(P_b\right).
\end{aligned} \tag{9}$$

The welfare measure W_b aggregates utilities of individual investors and equals to the total output net of the aggregate nonpecuniary disutility of d-firms' shareholders and the pollution externality.

Green equilibrium: $f^* = f_g^* \in [f_l, f_h)$. In this equilibrium, d-firms with low adoption costs adopt the green technology. We have $v_d(\gamma) = \alpha_d - (1-\pi)f_l - \pi\gamma$, and so investors with $\gamma \in [0, \hat{\gamma}_g)$ invest in d-firms and investors with $\gamma \in [\hat{\gamma}_g, \bar{\gamma}]$ invest in c-firms, where $\hat{\gamma}_g$ is determined by

$$v_c = v_d\left(\hat{\gamma}_g\right) \Leftrightarrow \hat{\gamma}_g = \frac{1}{\pi}\Delta - \frac{1-\pi}{\pi} f_l. \tag{10}$$

Note that d-firms receive nonzero funding as long as $\hat{\gamma}_g > 0$, that is, if

$$\Delta > (1-\pi) f_l.$$

As in the brown equilibrium, if $\hat{\gamma}_g < \bar{\gamma}$, both firm types receive nonzero funding in the green equilibrium.

To verify that this is indeed an equilibrium, we need to check if the threshold adoption cost $f_g^* \in [f_l, f_h)$. Using (3), we can write the threshold adoption cost as

$$f_g^* = f_g^*\left(\hat{\gamma}_g\right) = \frac{\int_0^{\hat{\gamma}_g} \gamma dG(\gamma)}{G\left(\hat{\gamma}_g\right)}. \tag{11}$$

First, note that $f_g^* < f_h$ because

$$f_g^* < \hat{\gamma}_g = \frac{1}{\pi}\Delta - \frac{1-\pi}{\pi} f_l < f_h,$$

where the last inequality holds due to (5). Hence, we only need to check if $f_g^* \geq f_l$. Differentiating f_g^* with respect to $\hat{\gamma}_g$ we find that, as in (8), $\frac{\partial f_g^*}{\partial \hat{\gamma}_g} > 0$. Thus, the green equilibrium exists if $\hat{\gamma}_g$ is sufficiently high, which by definition of $\hat{\gamma}_g$ (10) is the case if Δ is large. Intuition behind this result mirrors that in the brown equilibrium case. As the financial advantage of d-firms increases, these firms are able to attract more high-γ investors. As a result, d-firms' managers become more willing to pay the adoption cost f_l and switch to the green technology. We formalize this result in Proposition 1 below.

By the law of large numbers, in the green equilibrium fraction $1-\pi$ of d-firms have the adoption cost f_l and hence adopt the green technology. The remaining d-firms have the adoption cost f_h and operate the brown technology. Therefore, output Y_g, pollution P_g and investors' welfare W_g are as follows.

$$\begin{aligned}
Y_g &= \alpha_c\left(1 - G\left(\hat{\gamma}_g\right)\right) + \left(\alpha_d - (1-\pi) f_l\right) G\left(\hat{\gamma}_g\right), \\
P_g &= \pi G\left(\hat{\gamma}_g\right), \\
W_g &= Y_g - \pi \int_0^{\hat{\gamma}_g} \gamma dG(\gamma) - \varepsilon \xi\left(P_g\right).
\end{aligned} \tag{12}$$

Proposition 1 summarizes conditions for the existence of the two equilibria.

Proposition 1. *If $f_l > \int_0^{\bar{\gamma}} \gamma dG(\gamma)$, only the brown equilibrium exists. If $f_l \leq \int_0^{\bar{\gamma}} \gamma dG(\gamma)$, there exist $\bar{\gamma} \geq \bar{\Delta} > \underline{\Delta} > f_l$ such that*

(i) if $\Delta < \underline{\Delta}$, only the brown equilibrium exists;

(ii) if $\Delta \geq \bar{\Delta}$, only the green equilibrium exists;

(iii) if $\Delta \in [\underline{\Delta}, \bar{\Delta})$, the green and brown equilibria coexist.

Furthermore, if the green and brown equilibria coexist, d-firms receive more funding in the green equilibrium, i.e. $\hat{\gamma}_g > \hat{\gamma}_b$, where $\hat{\gamma}_b$ and $\hat{\gamma}_g$ are given by (6) and (10), respectively.

Proof. See Appendix A.3. □

Proposition 1 delivers several results. First, it states that if investors' sustainability preferences are weak on average, i.e. $\int_0^{\bar{\gamma}} \gamma dG(\gamma) < f_l$, only the brown equilibrium exists. This is intuitive. It follows from our analyses that shares of d-firms are held by low-γ investors. Therefore, if the adoption cost f_l is so large that even an average investor in population prefers not to adopt the green technology, then only the brown equilibrium is possible.

If the adoption cost f_l is not that large, both equilibria are possible. As discussed above, whether the brown or green equilibrium exists depends on the difference in productivities between the two firm types Δ. In particular, the brown equilibrium exists if the difference is not too large, $\Delta < \bar{\Delta}$, and the green equilibrium exists if the difference is not too small, $\Delta \geq \underline{\Delta}$.

Proposition 1 also shows that the two equilibria can coexist, i.e. $\underline{\Delta} < \bar{\Delta}$. In this parameter region, the shareholder base of d-firms is larger in the green equilibrium, i.e. $\hat{\gamma}_g > \hat{\gamma}_b$. This is because in the green equilibrium d-firms are expected to adopt the green technology with a positive probability and so are expected to be less polluting than in the brown equilibrium. This in turn attracts investors with stronger aversions to holding polluting shares $\gamma \in [\hat{\gamma}_b, \hat{\gamma}_g)$ to become d-firms' shareholders.

Figure 1 illustrates investor sorting in the green and brown equilibria in case the two equilibria coexist. Specifically, panel (A) shows that the marginal investor is more averse to holding polluting shares in the green equilibrium, $\hat{\gamma}_g > \hat{\gamma}_b$, which implies that d-firms receive more funding in the green equilibrium. Furthermore, in the green equilibrium shareholders of d-firms have a higher average γ. Therefore, they are more willing to adopt the green technology, meaning that the adoption cost threshold $f_g^* > f_b^*$, where the adoption thresholds are given by (7) and (11). For the brown and green equilibria to coexist, it must be that $f_b^* < f_l < f_b^*$ (panel B).

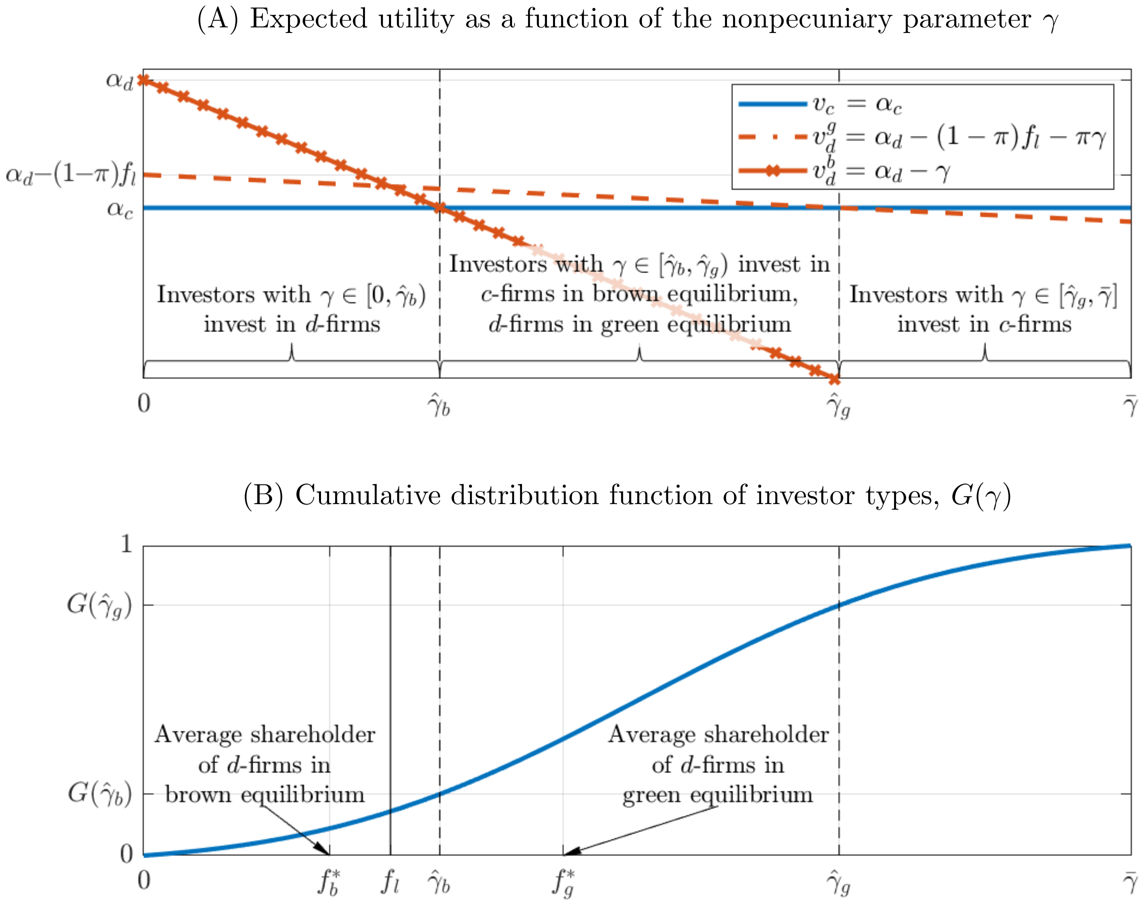

Figure 1: Panel (A): Expected utilities of investing one dollar in shares of c-firms (blue solid line, v_c), d-firms in green (red dashed line, v_d^g) and brown equilibria (red line with circle markers, v_d^b) as functions of the nonpecuniary disutility parameter γ. Panel (B): Cumulative distribution function of investor types, $G(\gamma)$ (blue solid line); marginal investors, $\hat{\gamma}_b$ and $\hat{\gamma}_g$; adoption thresholds, f_b^* and f_g^* in the brown and green equilibria, respectively. Since $f_b^* < f_l < f_g^*$, the two equilibria coexist.

3.2 Brown and green equilibria: Comparison

As discussed in Section 3.1, the brown and green equilibria differ in two main aspects. On the one hand, in the brown equilibrium d-firms never adopt the green technology. On the other hand, they only attract investors with low disutilities from holding polluting shares, and so the amount of capital d-firms raise in the brown equilibrium is smaller than in the green equilibrium, $G(\hat{\gamma}_b) < G(\hat{\gamma}_g)$. Therefore, although in the green equilibrium d-firms pollute less intensively on average, they are also larger, suggesting that the overall pollution can in principle be higher in the green equilibrium.[14] In line with Acemoglu and Rafey (2019), we assume that this is not the case.

[14]This possibility is known as the Jevons paradox (Jevons, 1866) in environmental economics.

Assumption 2. *The aggregate pollution is smaller in the green equilibrium, i.e.*

$$P_b > P_g \Leftrightarrow G\left(\hat{\gamma}_b\right) > \pi G\left(\hat{\gamma}_g\right).$$

Economically, Assumption 2 implies that adopting the green technology leads to a reduction in the aggregate pollution. Using (6) and (10) that define $\hat{\gamma}_b$ and $\hat{\gamma}_g$, we can see that Assumption 2 is satisfied if $G(\cdot)$ is weakly concave. Another sufficient condition under which this assumption holds is that the probability for d-firms to adopt the green technology in the green equilibrium $1-\pi$ is sufficiently large.

Even though pollution in the green equilibrium is lower than in the brown equilibrium under Assumption 2, adopting the green technology might be socially undesirable if the adoption cost is too large. The following lemma proves an intuitive result that the green equilibrium is socially preferable if the adoption cost f_l is sufficiently low.

Lemma 3. *There exists a threshold $\bar{f}_l = \bar{f}_l(\varepsilon) > 0$ such that the green equilibrium exists and is socially preferable, $W_g > W_b$, if $f < \bar{f}_l$. Moreover, $\bar{f}_l(\varepsilon)$ is a (weakly) increasing function.*

Proof. See Appendix A.4. □

Lemma 3 also establishes that the threshold $\bar{f}_l\left(\varepsilon\right)$ is an increasing function of the externality strength parameter ε. If the externality is strong, reducing pollution is socially valuable even if it is subject to a substantial financial cost.

In what follows, we make an assumption on the equilibrium selection.

Assumption 3. *If the two equilibria coexist, a socially preferable one is played.*

This assumption is conservative. It ensures that our results are driven by fundamental economic forces in the model rather than by coordination failures.

3.3 Discussion

The key premise of our model is that investors dislike investing in polluting firms. Consequently, high-γ investors acquire shares of c-firms that are less attractive financially but always operate the green technology. Investors with lower but positive γ become shareholders of d-firms and can induce their managers to adopt the green technology. Therefore, even though investors are atomistic and do not internalize their impacts on the aggregate pollution, in equilibrium their investments can still lead to a pollution reduction and hence alleviate negative effects of the externality. Notably, if investors did not incur any disutility from holding polluting shares, i.e. $\gamma = 0$ for all investors, only more financially attractive d-firms would receive funding and these firms would never adopt the green technology.

Importantly, in the absence of the pollution externality, i.e. if $\varepsilon = 0$, investors' capital allocation and firms' adoption decisions are efficient. That is, relative to the laissez-faire allocation described above, a social planner cannot achieve a higher welfare by choosing a different shareholder composition. In particular, if the green equilibrium does not exist, the planner implements the brown equilibrium by selecting investors with $\gamma \in [0, \hat{\gamma}_b)$ as shareholders of d-firms, where $\hat{\gamma}_b$ is given by (6). Similarly, if the brown equilibrium does not exist, the planner implements the green equilibrium by selecting investors with $\gamma \in [0, \hat{\gamma}_g)$ as shareholders of d-firms, where $\hat{\gamma}_g$ is given by (10). Finally, if the two equilibria coexist, the planner chooses either $\hat{\gamma}_g$ or $\hat{\gamma}_b$ as the threshold investor type, depending on whether the green or brown equilibrium is socially preferable.[15] For example, under conditions of Lemma 3, the planner sets $\hat{\gamma} = \hat{\gamma}_g$. Notably, such allocations are socially optimal even for the planner who can choose *any* shareholder composition, not necessarily satisfying the threshold property.

Proposition 2. *Suppose that the aggregate pollution externality is absent, $\varepsilon = 0$. Relative to the laissez-faire allocation, a social planner cannot achieve a higher aggregate welfare by choosing a different shareholder base composition.*

Proof. See Appendix A.5. □

Proposition 2 implies that if $\varepsilon = 0$, the planner wants to change neither the firms' shareholder bases in a given equilibrium, nor the type of equilibrium played. In particular, if the green equilibrium does not exist in the laissez-faire economy, it is not optimal for the planner to choose such shareholder bases that d-firms adopt the green technology when facing the f_l adoption cost.[16]

The constrained efficiency result of Proposition 2 is not obvious in our incomplete market economy. There are two main features that lie behind this results. First, managers make adoption decisions based on the average preferences of their shareholders, i.e. there are no conflicts of interests between managers and shareholders. Second, there is

[15] See Proposition 1 for the conditions under which the equilibria exist.

[16] If in the laissez-faire economy investors do not coordinate on a socially preferable equilibrium, i.e. Assumption 3 is violated, the planner can improve welfare by changing the type of equilibrium played.

no aggregate pollution externality. If such an externality is present, i.e. $\varepsilon > 0$, atomistic investors cannot fully resolve the externality issue, even though they dislike holding polluting shares. Consequently, there is room for policy interventions.

The next section conducts policy analyses when the pollution externality is present. An important novel point of our paper is that we analyze how planner's interventions affect firms' shareholder bases and, through that channel, change firms' sizes and their green technology adoption decisions. More specifically, we show that commonly discussed policy interventions aimed at curbing pollution, such as carbon taxation or subsidies to clean firms, can at times cause opposite effects and be socially undesirable.

4 Policy analyses

In this section, we analyze two commonly used policy tools. Section 4.1 considers subsidies to c-firms. Such subsidies are frequently used to support renewable energy producers; for example, the recent US Inflation Reduction Act of 2022 includes production and investment tax credits toward clean energy manufacturing, as well as consumer tax credits for using energy-efficient goods (e.g., rooftop solar panels).[17] Section 4.2 considers taxes levied on polluting d-firms, akin to carbon pricing instruments such as carbon taxes and carbon crediting mechanisms.[18] Section 4.3 discusses the results and their robustness.

4.1 Subsidy to clean firms

4.1.1 Setting

At the beginning of $t = 1$, before investors make their portfolio choices, the planner imposes a subsidy $\varsigma \geq 0$. Specifically, the planner commits to transfer ςk_c to each c-firm at $t = 2$, where k_c is the amount of capital a c-firm raises from investors. The subsidy is financed by lump sum taxation of all investors. Effectively, the subsidy makes c-firms more financially attractive by reducing the difference between productivities of firms of different types: without the subsidy, this difference is Δ; under the subsidy, it is $\Delta - \varsigma$.

The equilibrium characterization is analogous to that of the baseline model in Section 3.1. The main difference is that a marginal investor, who is indifferent between holding

[17] See the report by the International Renewable Energy Agency (Taylor, 2020) for a detailed discussion of energy sector subsidies worldwide.

[18] According to the World Bank's report (World Bank, 2022), there are 68 carbon pricing instruments implemented worldwide as of 2022, covering around 23% of global greenhouse gas emissions.

shares of c- and d-firms, is less concerned about sustainability of her portfolio in any equilibrium in the presence of the subsidy, i.e.

$$\hat{\gamma}_b(\varsigma) = \Delta - \varsigma, \tag{13}$$

$$\hat{\gamma}_g(\varsigma) = \frac{1}{\pi}(\Delta - \varsigma) - \frac{1-\pi}{\pi} f_l. \tag{14}$$

Expressions for the threshold cost f^*, output Y, pollution P and welfare W remain unchanged for both equilibria, where in this section we define output as the overall firms' output net of taxes imposed on investors.

We start by characterizing how the subsidy size ς reshapes the regions in which the two equilibria exist and equilibrium quantities. We focus on the most interesting case in which the average disutility from holding polluting shares is not too small, $f_l < \int_0^{\bar{\gamma}} \gamma dG(\gamma)$, and the difference between productivities Δ is sufficiently large so that only the green equilibrium exists in the absence of the subsidy, i.e. $\Delta \geq \bar{\Delta}$.

Proposition 3. *Suppose that $f_l \leq \int_0^{\bar{\gamma}} \gamma dG(\gamma)$ and $\Delta \geq \bar{\Delta}$, where $\bar{\Delta}$ is defined in Proposition 1. Then there exist $0 \leq \underline{\varsigma} < \bar{\varsigma} < \Delta - (1-\pi) f_l$ such that*

(i) if $\varsigma \leq \underline{\varsigma}$, only the green equilibrium exists;

(ii) if $\varsigma > \bar{\varsigma}$, only the brown equilibrium exists;

(iii) if $\varsigma \in (\underline{\varsigma}, \bar{\varsigma}]$, the green and brown equilibria coexist.

Furthermore, in any equilibrium an increase in ς leads to a reduction in the amount of capital raised by d-firms, aggregate output and pollution.

Proof. See Appendix A.6. □

An increase in the subsidy size ς makes d-firms relatively less attractive financially, which implies that these firms receive less funding and pollute less in any given equilibrium. At the same time, the aggregate output (net of taxes) also declines as a result of capital reallocation to less productive c-firms.

Importantly, the subsidy also changes the compositions of firms' shareholder bases, as can be seen from the expressions for the threshold types (13)–(14). As d-firms become relatively less financially attractive, investors who are moderately averse to holding polluting shares switch to holding shares of c-firms. Remaining d-firms' shareholders care less about sustainability of their portfolios, and so managers of d-firms are less willing to pay the cost to adopt the green technology. Even if the green equilibrium exists under the zero subsidy, it ceases to exist if the subsidy size is sufficiently large, $\varsigma > \bar{\varsigma}$. This result echoes Proposition 1 which shows that the green equilibrium does not exist if the productivities of the two firm types are sufficiently close.

It is worth noting that Proposition 3 can be straightforwardly extended to handle other values of the productivity difference Δ. For example, if $\Delta \in [\underline{\Delta}, \bar{\Delta})$, where both thresholds are defined in Proposition 1, then there are no nonnegative subsidy levels for which only the green equilibrium exists, i.e. $\underline{\varsigma} \geq 0$ does not exist. If $\Delta < \underline{\Delta}$, the green equilibrium does not exist for any $\varsigma \geq 0$.

4.1.2 Optimal policy

We proceed by characterizing the welfare-maximizing subsidy. In Appendix A.7, we show that the derivative of the welfare measure W with respect to the subsidy size ς in any equilibrium can be written as

$$\frac{dW}{d\varsigma} = \frac{dK_d}{d\varsigma}\left(\varsigma - \varepsilon\frac{d\xi\left(P(K_d)\right)}{dK_d}\right), \tag{15}$$

where K_d is the amount of capital raised by d-firms and $P(K_d)$ is the amount of pollution produced by d-firms.

The effect of the subsidy on welfare depends on two components. The first component is the sensitivity of the amount of capital invested in d-firms to the subsidy size, $\frac{dK_d}{d\varsigma}$. By Proposition 3, $\frac{dK_d}{d\varsigma} < 0$. The second component is *Pigouvian wedge*, i.e. the size of uncorrected externality. If this wedge is negative, then the subsidy size is smaller than the marginal externality. The externality is under-corrected and the regulation is too lenient. If the wedge is positive, the externality is over-corrected and the regulation is too stringent.

Our goal in this section is to characterize the optimal subsidy as a function of the externality strength ε. To do so, it is instructive to first consider a special case in which the type of equilibrium played is fixed.

Exogenous adoption decision Specifically, suppose that in the brown equilibrium managers of d-firms commit to never adopt the green technology, and in the green equilibrium they commit to adopt the green technology if the adoption cost is f_l, *irrespective* of their firms' shareholder bases. The following lemma characterizes the optimal subsidy under exogenous adoption decisions.

Lemma 4. *There exist unique increasing functions $\varsigma_g^*(\varepsilon) < \varsigma_b^*(\varepsilon)$ that maximize investors'* *welfare in the green and brown equilibria, respectively. Furthermore, there exists an* $\bar{f}_l^{\varsigma} > 0$ *such that if* $f_l \leq \bar{f}_l^{\varsigma}$, $W_g\left(\varsigma_g^*(\varepsilon)\right) > W_b\left(\varsigma_b^*(\varepsilon)\right)$.

Proof. See Appendix A.7. □

In a given equilibrium, the optimal subsidy level sets the derivative $\frac{dW}{d\varsigma}$, given by (15), to zero. Since the externality function $\xi(\cdot)$ is convex, the optimal subsidy levels, $\varsigma_g^*(\varepsilon)$ and $\varsigma_b^*(\varepsilon)$, are unique. Furthermore, because the green equilibrium features less pollution than the brown equilibrium by Assumption 2, $\varsigma_g^*(\varepsilon) < \varsigma_b^*(\varepsilon)$. Furthermore, Lemma 4 establishes sufficient conditions under which the maximum welfare that can be achieved in the green equilibrium is higher than that in the brown equilibrium. Specifically, if the adoption cost f_l is sufficiently low, it is socially optimal for d-firms to adopt the green technology when facing the low adoption cost. This is intuitive: if the adoption cost is prohibitively large, the financial loss due to adoption outweighs a social benefit due to a reduction in pollution.[19]

As follows from Lemma 4, it is socially optimal for d-firms to adopt the green technology and for the planner to set $\varsigma = \varsigma_g^*(\varepsilon)$ if f_l is not too large. Therefore, if managers of d-firms can commit to adopt the green technology when facing the low adoption cost irrespective of shareholders' preferences, the planner sets the Pigouvian wedge in the green equilibrium (dubbed as the green Pigouvian wedge) to zero.

Crucially, government interventions reshape firms' shareholder bases and so affect managers' decisions to adopt the green technology if they cannot commit to ignore shareholders' preferences. We discuss this case below.

Endogenous adoption decision In the baseline model, the planner takes into account how its policy affects firms' shareholder bases and, hence, managers' decisions to adopt the green technology. The following proposition characterizes the optimal subsidy as a function of the externality strength when managers' decisions are endogenous.

Proposition 4. *Suppose that* $f_l \leq \min\left[\bar{f}_l^{\varsigma}, \int_0^{\bar{\gamma}} \gamma dG(\gamma)\right]$ *and* $\Delta \geq \underline{\Delta}$, *where* $\bar{f}_l^{\varsigma}$ *is defined in Lemma 4 and* $\underline{\Delta}$ *is defined in Proposition 1. Then there exist* $0 \leq \underline{\varepsilon}^{\varsigma} < \bar{\varepsilon}^{\varsigma}$ *such that*

(i) if $\varepsilon \leq \underline{\varepsilon}^{\varsigma}$, *optimal subsidy is* $\varsigma^*(\varepsilon) = \varsigma_g^*(\varepsilon)$, *and green Pigouvian wedge is zero.*

(ii) if $\varepsilon > \bar{\varepsilon}^{\varsigma}$, *optimal subsidy is* $\varsigma^*(\varepsilon) = \varsigma_b^*(\varepsilon)$, *and green Pigouvian wedge is positive.*

(iii) if $\varepsilon \in (\underline{\varepsilon}^{\varsigma}, \bar{\varepsilon}^{\varsigma}]$, *optimal subsidy is* $\varsigma^*(\varepsilon) = \bar{\varsigma}$, *where* $\bar{\varsigma}$ *is defined in Proposition 3, and green Pigouvian wedge is negative.*

[19]This result is similar to the one implied by Lemma 3. Note, however, that Lemma 3 compares welfare in the two equilibria for a given set of underlying parameters, while Lemma 4 compares them under the optimal subsidies that differ across the two equilibria.

Proof. See Appendix A.8. □

Proposition 4 describes how the optimal subsidy and the green Pigouvian wedge depend on the externality strength. Under the conditions stated in this proposition, d-firms' managers adopt the green technology when facing the f_l adoption cost in the absence of government interventions. However, the amount of capital d-firms raise—and, hence, the amount of pollution they produce—is too large from the social perspective. By providing a subsidy to c-firms, the planner reduces the amounts of capital d-firms receive and pollution they produce. If the externality is weak, $\varepsilon \leq \underline{\varepsilon}^{\varsigma}$, the optimal government intervention is of a small scale, $\varsigma^*(\varepsilon) = \varsigma_g^*(\varepsilon) < \bar{\varsigma}$. Such an intervention sets the green Pigouvian wedge to zero and, at the same time, does not change the shareholder base composition of d-firms too much. As a result, the green equilibrium exists by Proposition 3. This case is illustrated by panel (A) of Figure 2.

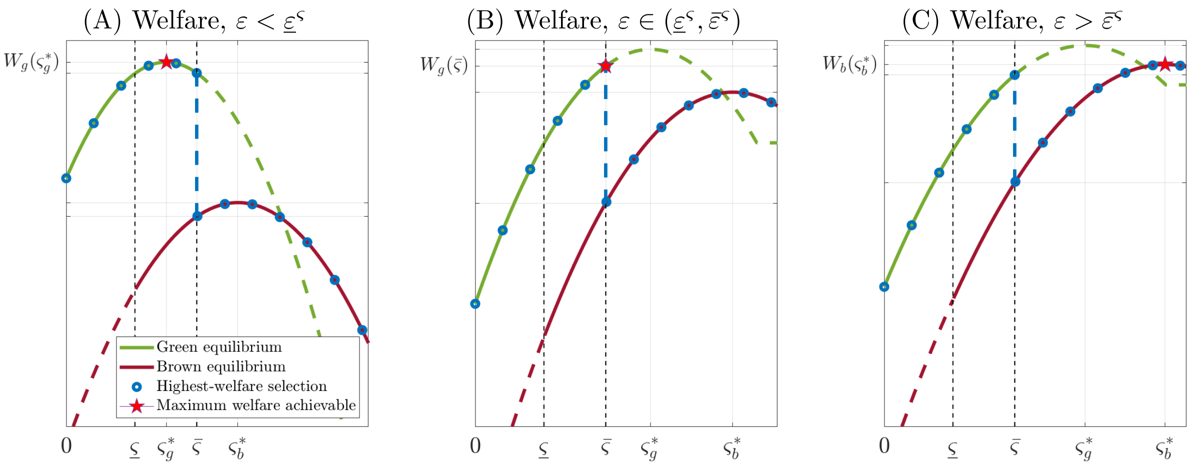

Figure 2: Welfare in the green equilibrium, the brown equilibrium and under the highest-welfare selection criterion as functions of the subsidy size ς. Panel (A): Low externality strength, $\varepsilon < \underline{\varepsilon}^{\varsigma}$. Panel (B): Intermediate externality strength, $\varepsilon \in (\underline{\varepsilon}^{\varsigma}, \bar{\varepsilon}^{\varsigma})$. Panel (C): High externality strength, $\varepsilon > \bar{\varepsilon}^{\varsigma}$. For green and brown equilibria in all panels, solid/dashed lines show welfare if a corresponding equilibrium exists/does not exist when adoption decisions are based on shareholders' preferences. In all panels, star marks welfare under the optimal subsidy size. Appendix B.1 provides numerical values used for these graphs.

An important result of Proposition 4 is that the green Pigouvian wedge deviates from zero if the externality is sufficiently strong. As ε increases, so does the subsidy needed to set the green Pigouvian wedge to zero (Lemma 4). If $\varepsilon = \underline{\varepsilon}^{\varsigma}$ and $\varsigma = \varsigma_g^*(\underline{\varepsilon}^{\varsigma}) = \bar{\varsigma}$, the shareholder base of d-firms shrinks to the extent that their average shareholder is indifferent between spending f_l to adopt the green technology and operating the brown technology. A further increase in the subsidy size makes the green equilibrium non-existent and leads to an equilibrium switch, which is associated with a sharp decline in welfare and *increase* in pollution (Figure 3). Consequently, if $\varepsilon > \underline{\varepsilon}^{\varsigma}$, the highest welfare in the green equilibrium is achieved at $\varsigma = \bar{\varsigma}$, where the green Pigouvian wedge is negative, i.e. the externality is under-corrected. However, as long as ε is not too high, i.e. $\varepsilon < \bar{\varepsilon}^{\varsigma}$, the size of this under-correction at $\bar{\varsigma}$ is relatively small, and the planner cannot achieve a higher welfare in the brown equilibrium even by setting $\varsigma = \varsigma_b^*(\varepsilon)$. All in all, if the externality is of intermediate strength, i.e. $\varepsilon \in (\underline{\varepsilon}^{\varsigma}, \bar{\varepsilon}^{\varsigma}]$, the planner keeps d-firms relatively large so that their shareholder bases are sufficiently green-friendly. This is beneficial financially, since d-firms are more productive, and at the same time not too costly environmentally because pollution intensity of d-firms in the green equilibrium is low. This case is illustrated by panel (B) of Figure 2.

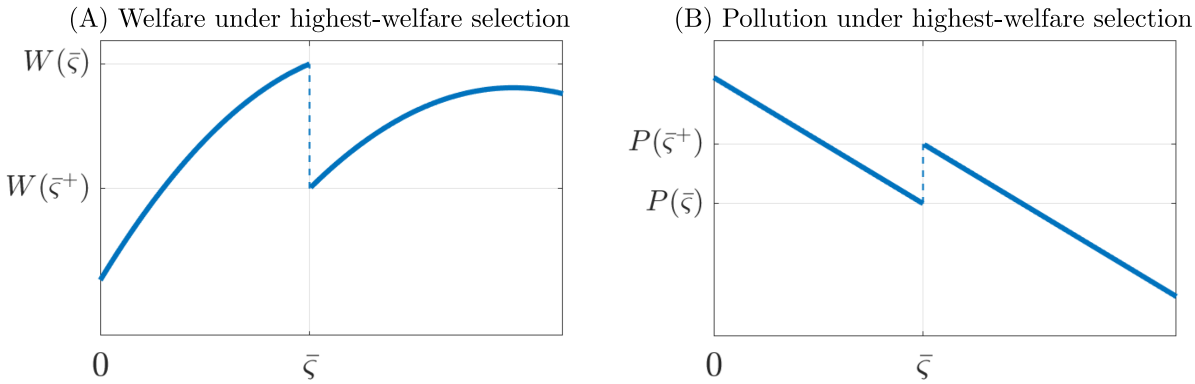

Figure 3: Panel (A): Welfare under the highest-welfare selection criterion as a function of the subsidy size ς. Panel (B): Pollution under the highest-welfare selection criterion as a function of the subsidy size ς. The externality strength is $\varepsilon \in (\underline{\varepsilon}^{\varsigma}, \bar{\varepsilon}^{\varsigma})$. Appendix B.1 provides numerical values used for these graphs.

If the externality is very strong, $\varepsilon > \bar{\varepsilon}^{\varsigma}$, the planner reduces the size of d-firms drastically by incentivizing investors to acquire shares of heavily subsidized c-firms. The planner sets $\varsigma = \varsigma_b^*(\varepsilon)$, under which the green equilibrium does not exist. Therefore, all d-firms pollute, but the overall pollution amount is low because these firms receive little capital. While this is costly financially, it is the only way for the planner to substantially ameliorate the pollution externality. This case is illustrated by panel (C) of Figure 2.

4.2 Pollution tax

4.2.1 Setting

At the beginning of $t = 1$, before investors make their portfolio choices, the planner imposes a tax $\tau \geq 0$ on polluting firms. Specifically, the planner commits to collect an amount τk_j from each polluting firm j at $t = 2$, where k_j is the amount of capital firm j raises. Tax proceeds are rebated to investors. Since c-firms never pollute, they are not taxed. Importantly, d-firms that choose to adopt the green technology are also not taxed. Therefore, unlike the subsidy to c-firms, the tax directly affects incentives of d-firms' managers to adopt the green technology.

The equilibrium characterization is analogous to that of the baseline model in Section 3.1. Specifically, expressions for output Y, pollution P and welfare W remain unchanged for both equilibria, where in this section we define output as the aggregate output plus tax proceeds rebated to investors. There are, however, two important differences due to taxation.

First, in the presence of the tax, the value of investing a dollar in a d-firm becomes

$$v_d(\gamma, \tau) = \alpha_d - \begin{cases} \gamma + \tau, & \text{in the brown equilibrium,} \\ (1-\pi) f_l + \pi(\gamma + \tau), & \text{in the green equilibrium.} \end{cases}$$

The value of investing a dollar in a c-firm remains $v_c = \alpha_c$. Therefore, compared to the no-tax scenario, a marginal investor is less concerned about sustainability of her portfolio in any equilibrium, i.e.

$$\hat{\gamma}_b(\tau) = \Delta - \tau, \tag{16}$$

$$\hat{\gamma}_g(\tau) = \frac{1}{\pi}\Delta - \frac{1-\pi}{\pi} f_l - \tau. \tag{17}$$

A higher tax makes a marginal investor less averse to holding polluting shares, i.e. $\frac{d\hat{\gamma}}{d\tau} = -1$ in both equilibria. Since expected pollution of d-firms is positive in any equilibrium, a higher tax means that they become less financially attractive, and so only low-γ investors are willing to invest in them. This effect is similar to that of the subsidy to c-firms.

Second, managers of d-firms take into account that their outputs are going to be taxed unless they adopt the green technology. Therefore, the threshold adoption cost becomes

$$f^*(\hat{\gamma}, \tau) = \tau + \frac{\int_0^{\hat{\gamma}} \gamma dG(\gamma)}{G(\hat{\gamma})}, \tag{18}$$

where $\hat{\gamma} = \hat{\gamma}_g(\tau)$ in the green equilibrium and $\hat{\gamma} = \hat{\gamma}_b(\tau)$ in the brown equilibrium. The effect of τ on the threshold cost can be decomposed in two channels,

$$\frac{df^*}{d\tau} = \underbrace{\frac{\partial f^*}{\partial \tau}}_{\text{Direct effect}} + \underbrace{\frac{d\hat{\gamma}}{d\tau}\frac{\partial f^*}{\partial \hat{\gamma}}}_{\text{Shareholder base effect}} = 1 - \frac{g(\hat{\gamma})}{G(\hat{\gamma})}\left(\hat{\gamma} - \frac{\int_0^{\hat{\gamma}} \gamma dG(\gamma)}{G(\hat{\gamma})}\right).$$

Holding shareholder composition fixed, an increase in τ makes it more expensive to operate the polluting technology. As a result, managers of d-firms are willing to adopt the green technology under a higher adoption cost. That is, the direct effect of τ on f^* is positive, $\frac{\partial f^*}{\partial \tau} > 0$. On the other hand, an increase in τ makes d-firms' shareholders on average less concerned about their portfolios' sustainability, which means that managers of d-firms are less willing to pay the adoption cost. Therefore, the shareholder base effect of τ on f^* is negative, $\frac{d\hat{\gamma}}{d\tau}\frac{\partial f^*}{\partial \hat{\gamma}} < 0$. Importantly, the shareholder base effect can be stronger than the direct effect. If this is the case, an increase in tax on polluting firms can make d-firms' managers less willing to adopt the green technology, that is, the green equilibrium might cease to exist. Below, we first describe conditions under which the shareholder base effect dominates, and then discuss optimal taxation.

Intuitively, the shareholder base effect is strong if a small change in the preference of a marginal investor $\hat{\gamma}$ leads to a large change in the average preference of d-firms' shareholders. To illustrate that, we consider the following example. Suppose that the distribution $g(\gamma)$ of investor types is two-peaked, as shown in panel (A) of Figure 4. Such a distribution implies that there are two major groups of investors. The first group consists of investors who are highly concerned about greenness of their portfolios, e.g. dedicated ESG funds (right peak). The second group consists of investors who mostly care about financial performance of their portfolios (left peak). In the absence of a pollution tax, $\tau = 0$, a marginal investor $\hat{\gamma}_g(0)$ belongs to the first group. As a result, the shareholder base of d-firms is on average sufficiently pro-green, and managers adopt the green technology (in panel (D), $f_g^*(0) > f_l$).

An increase in the pollution tax makes a marginal investor less concerned about pollution, i.e. $\hat{\gamma}_g(\tau)$ declines (panel (B) in the same figure). Such a change has a strong impact on the average preference of the shareholder base (panel C): a small reduction in $\hat{\gamma}_g$ induces a large mass of high-γ investors to become shareholders of c-firms. As a result, d-firms' shareholders become substantially less pro-green on average. The shareholder base effect here is particularly strong and dominates the direct effect, and so the adoption threshold $f_g^*(\tau)$ declines and crosses f_l at $\tau = \tau_1$ (panel D).

If the tax is sufficiently high, e.g. $\tau = \tau_2$, almost all investors within the first pro-green group switch to become shareholders of c-firms. A marginal increase in τ has a small effect on the average preference of d-firms' shareholders. At this point, the shareholder base effect is weaker than the direct effect, and $f_g^*(\tau)$ becomes an increasing function.

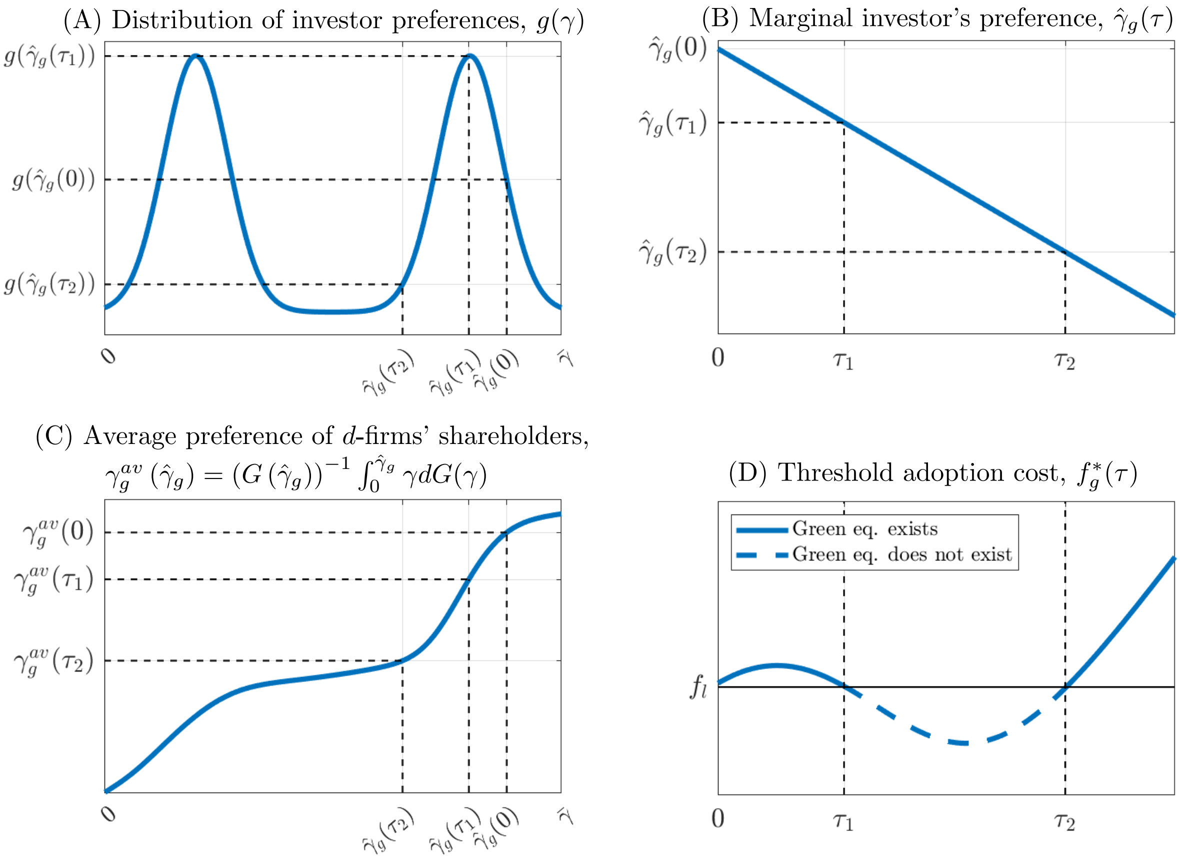

Figure 4: Panel (A): Probability density function of the distribution of investor preferences, $g(\gamma)$. Panel (B): Preference of a marginal investor as a function of pollution tax, $\hat{\gamma}_g(\tau)$. Panel (C): Average preference of d-firms' shareholders, $\gamma_g^{av}(\hat{\gamma}_g) = (G(\hat{\gamma}_g))^{-1} \int_0^{\hat{\gamma}_g} \gamma dG(\gamma)$, as a function of the preference of a marginal investor $\hat{\gamma}_g$. Panel (D): Threshold adoption cost as a function of pollution tax, $f_g^*(\tau)$; green equilibrium exists if and only if $f_g^*(\tau) \geq f_l$. Appendix B.2 provides numerical values used for these graphs.

Overall, if the tax is sufficiently low, $\tau < \tau_1$, d-firms' shareholders are pro-green and managers adopt the green technology. If the tax is high, $\tau > \tau_2$, the financial cost of operating the brown technology is so high that d-firms adopt the green technology even though their shareholder base consists of low-γ investors. If τ is in the intermediate range, $\tau \in (\tau_1, \tau_2)$, the financial cost of operating the brown technology is not too high but preferences of d-firms' shareholders are not sufficiently pro-green. As a result, d-firms' managers choose not to adopt the green technology.

Proposition 5 below confirms that the results of the example above hold in a more general setting. We again focus on the most interesting case in which the green equilibrium exists in the absence of the tax. Proposition 5 shows that the green equilibrium exists if the tax is sufficiently low or sufficiently high. However, the green equilibrium might cease to exist for the intermediate tax levels.

Proposition 5. *Suppose that $f_l < \int_0^{\bar{\gamma}} \gamma dG(\gamma)$ and $\Delta \geq \underline{\Delta}$, where $\underline{\Delta} > f_l$ is defined in Proposition 1. Then there exist $0 \leq \underline{\tau} < \bar{\tau} < f_l$ such that*

(i) the green equilibrium exists if $\tau \in [0, \underline{\tau}]$ and $\tau \in \left[\bar{\tau}, \frac{1}{\pi}(\Delta - (1-\pi) f_l)\right]$;[20]

(ii) the green equilibrium might not exist for intermediate levels of τ; specifically, if Δ is sufficiently close to $\underline{\Delta}$ and $\frac{df_g^*}{d\tau}\big|_{\tau=0} < 0$, it does not exist for $\tau \in (\tau_1, \tau_2)$ for some $\underline{\tau} < \tau_1 < \tau_2 < \bar{\tau}$;

(iii) if the green equilibrium does not exist, the brown equilibrium exists.

Furthermore, in any equilibrium an increase in τ leads to a reduction in the amount of capital raised by d-firms, aggregate output and pollution.

Proof. See Appendix A.9. □

As in the case of the subsidy to c-firms considered in Section 4.1, an increase in the tax makes d-firms relatively less financially attractive. They receive less funding and pollute less in any given equilibrium. The aggregate output (adjusted for tax proceeds rebated to investors) also declines as a result of capital reallocation to less productive c-firms.

The effect of the tax on d-firms' green technology adoption decisions is more nuanced than that of the subsidy to c-firms. On the one hand, as τ increases, shareholder base of d-firms become less pro-green on average. On the other hand, it becomes more financially costly to operate the brown technology. If the shareholder base effect dominates, the green equilibrium might cease to exist for intermediate levels of τ.

4.2.2 Optimal policy

We proceed by characterizing the welfare-maximizing tax. In Appendix A.10, we show that the derivative of the welfare measure W with respect to the tax τ in any equilibrium

[20] If $\tau > \frac{1}{\pi}(\Delta - (1-\pi) f_l)$, d-firms receive no funding in the green equilibrium (from (17), $\hat{\gamma}_g(\tau) < 0$).

can be written as

$$\frac{dW}{d\tau} = \frac{dP}{d\tau}\left(\tau - \varepsilon \frac{d\xi\left(P\right)}{dP}\right), \tag{19}$$

where P is the amount of pollution produced by d-firms. This expression is analogous to (15) in Section 4.1. The effect of the tax on welfare can be decomposed in two components: the sensitivity of the pollution amount to τ, $\frac{dP}{d\tau} < 0$, and the Pigouvian wedge.

The difference between (15) and (19) is due to the fact that the subsidy to c-firms and the pollution tax affect firms differently. The subsidy makes all d-firms less financially attractive, irrespective of the technology they operate. Effectively, it reduces return on capital K_d invested in d-firms. The tax penalizes only d-firms operating the brown technology, i.e. it directly targets pollution.

Next, we characterize the optimal tax as a function of the externality strength ε. As in Section 4.1, we start by considering a special case in which adoption decisions are exogenous.

Exogenous adoption decision Suppose that in the brown equilibrium managers of d-firms commit not to adopt the green technology, and in the green equilibrium they commit to adopt the green technology if the adoption cost is f_l, *irrespective* of their firms' shareholder bases. The following lemma characterizes the optimal tax under exogenous adoption decisions.

Lemma 5. *There exist unique increasing functions $\tau_g^*(\varepsilon)$ and $\tau_b^*(\varepsilon)$ that maximize investors' welfare in the green and brown equilibria, respectively. Furthermore, there exists an $\bar{f}_l^\tau > 0$ such that if $f_l \leq \bar{f}_l^\tau$, $W_g\left(\tau_g^*(\varepsilon)\right) > W_b\left(\tau_b^*(\varepsilon)\right)$.*

Proof. See Appendix A.10. □

Lemma 5 is an analogue of Lemma 4. If the adoption decisions are exogenous, there exist unique optimal taxes $\tau_g^*(\varepsilon)$ and $\tau_b^*(\varepsilon)$ that fully correct the pollution externality in the green and brown equilibria, respectively.[21] Furthermore, unless the adoption cost f_l is too large, the green equilibrium is socially preferable. That is, if managers of d-firms can commit to adopt the green technology when facing the low adoption cost irrespective of shareholders' preferences, the planner fully corrects the externality in the green equilibrium by setting $\tau^*(\varepsilon) = \tau_g^*(\varepsilon)$.

[21] Unlike Lemma 4, Lemma 5 does not rank $\tau_g^*(\varepsilon)$ and $\tau_b^*(\varepsilon)$. While $\tau_g^*(\varepsilon) < \tau_b^*(\varepsilon)$ if ε is small, this is no longer the case if ε is large. Intuitively, if the externality is very large, it is optimal to keep d-firms small. In the brown equilibrium, this is achieved if $\hat{\gamma}_b(\tau_b^*)$ is close to zero, i.e. τ_b^* is close to Δ. In the green equilibrium, this is achieved if $\hat{\gamma}_g(\tau_g^*)$ is close to zero, i.e. τ_g^* is close to $\frac{1}{\pi}(\Delta - (1-\pi)f_l) > \Delta$.

Endogenous adoption decision If firm managers' make adoption decisions based on their shareholders' preferences, the planner should take into account how taxation reshapes the shareholder bases. The following proposition characterizes the optimal tax as a function of the externality strength in this case.

Proposition 6. *Suppose that* $f_l \leq \min\left[\bar{f}_l^\tau, \int_0^{\bar{\gamma}} \gamma dG(\gamma)\right]$ *and* $\Delta \geq \underline{\Delta}$*, where* $\bar{f}_l^\tau$ *is defined in Lemma 5 and* $\underline{\Delta}$ *is defined in Proposition 1. Then there exist* $0 \leq \underline{\varepsilon}^\tau < \bar{\varepsilon}^\tau$ *such that*

(i) if $\varepsilon \leq \underline{\varepsilon}^\tau$ *or* $\varepsilon \geq \bar{\varepsilon}^\tau$*, optimal tax is* $\tau^*(\varepsilon) = \tau_g^*(\varepsilon)$*, and green Pigouvian wedge is zero;*

(ii) if externality is of intermediate strength, the green equilibrium does not always exist for $\tau = \tau_g^*(\varepsilon)$*. If it exists, optimal tax is* $\tau^*(\varepsilon) = \tau_g^*(\varepsilon)$*, and green Pigouvian wedge is zero; otherwise,* $\tau^*(\varepsilon) \neq \tau_g^*(\varepsilon)$*, and green Pigouvian wedge is nonzero.*

Proof. See Appendix A.11. □

This proposition is again analogous to its counterpart from Section 4.1, i.e. Proposition 4. Specifically, the planner sets a nonzero tax to reduce the size of d-firms and, hence, the amount of pollution they produce. If the externality is weak, i.e. $\varepsilon < \underline{\varepsilon}^\tau$, only a small intervention is needed to fully correct the externality and set the green Pigouvian wedge to zero. Such an intervention does not affect the shareholder base composition of d-firms too much, and d-firms' managers choose to adopt the green technology when facing the f_l adoption cost (that is, the green equilibrium exists).

If the externality is strong, $\varepsilon > \bar{\varepsilon}^\tau$, the optimal tax is high and also fully corrects the externality in the green equilibrium. Recall that the pollution tax provides a financial incentive to d-firms' managers to adopt the green technology. If the tax is sufficiently high, d-firms' managers adopt the green technology even if their shareholders are low-γ investors, i.e. the green equilibrium exists. This is different from the subsidy to c-firms that reduces financial attractiveness of all d-firms irrespective of technology they operate.

Importantly, if the externality is of intermediate strength, the tax that sets the green Pigouvian wedge to zero, $\tau_g^*(\varepsilon)$, might make the green equilibrium non-existent. As follows from our discussion in Section 4.2.1, this is the case if the shareholder base effect dominates the direct effect of taxation. The planner then deviates from $\tau_g^*(\varepsilon)$. Panel (A) of Figure 5 illustrates this scenario for the same parametrization as for Figure 4. Specifically, there exists a range in taxes, (τ_1, τ_2), for which the green equilibrium does not exist. In this example, $\tau_g^*(\varepsilon)$ falls into this range, and so the planner sets $\tau^*(\varepsilon) = \tau_2 > \tau_g^*(\varepsilon)$. Although it implies an over-correction of the externality, it also provides managers of d-firms' a sufficiently strong financial incentive to adopt the green technology.

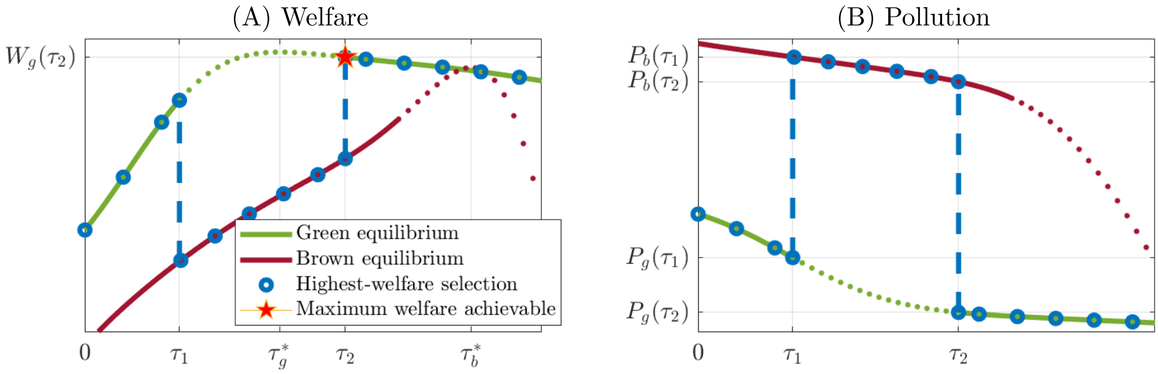

Figure 5: Welfare (panel A) and pollution (panel B) in the green equilibrium, the brown equilibrium and under the highest-welfare selection criterion as functions of pollution tax τ. For green and brown equilibria in both panels, solid/dotted lines show variables if a corresponding equilibrium exists/does not exist when adoption decisions are made based on shareholders' preferences. In panel (A), star marks welfare under the optimal tax. The externality strength is $\varepsilon \in (\underline{\varepsilon}^\tau, \bar{\varepsilon}^\tau)$. Appendix B.2 provides numerical values used for these graphs.

4.3 Discussion

Our policy analyses underscore that the conventional environmental policies—pollution tax and subsidy to clean firms—have two effects on the economy. First, both policies make polluting firms less financially attractive, reducing the amount of capital these firms raise from investors. This effect is intuitive and is commonly used to justify these policies. The second effect arises from the endogenous response of shareholder bases to the policies and is novel to the literature. As c-firms become more financially attractive, investors who care about their portfolios' sustainability exit d-firms and become shareholders of c-firms. The remaining d-firms' shareholders are less averse to holding polluting shares and thus are less willing to sacrifice financial returns for pollution reduction. This in turn makes it less likely for these firms to adopt the green technology. Overall, the regulator should consider a trade-off between reducing the size of d-firms and keeping their shareholder bases sufficiently pro-green. In particular, the shareholder base effect might induce the planner to under- or over-correct the pollution externality relative to the exogenous shareholder base benchmark.

Notably, there are differences in the way the subsidy to c-firms and the pollution tax affect the green technology adoption. Unlike the subsidy, the tax incentivizes the green technology adoption by imposing a financial burden only on polluting d-firms (see Equation (18)). The differences between the policies become evident if the pollution externality is strong. In this case, the regulator wants to significantly reduce pollution by either heavily subsidizing c-firms or imposing a high tax on polluting d-firms. A large subsidy to c-firms keeps d-firms small but, at the same time, they never adopt the green technology. In contrast, if the tax is large, it is financially costly to pollute, and d-firms adopt the green technology even though their shareholders are not particularly pro-green. Therefore, unlike a high subsidy, a high tax can both keep d-firms small and incentivize their managers to adopt the green technology.

In reality, the differences between the two policies are likely to be less pronounced than the model suggests. Hefty government interventions might be infeasible in the first place, for example, due to political economy considerations. If the regulator can impose only a moderate level of taxes or subsidies due to such considerations, the effects of the two policies are alike (see Propositions 3 and 5).[22] Furthermore, the green technology adoption may be challenging for small firms. In our model, the adoption cost is proportional to the amount of capital a firm raises. In practice, this cost might include a substantial fixed component, for example, due to acquisition of green patents. In the presence of fixed adoption costs, it is necessary for d-firms to have sufficient capital to be able to cover them. Thus, neither a high subsidy nor a high tax can simultaneously reduce the size of d-firms and incentivize their managers to adopt the green technology.

5 Conclusion

Climate change is one of the biggest challenges of our times. In response to this challenge, there has been a substantial increase in socially responsible investing in financial markets. At the same time, regulators all over the world have been imposing more environmental regulations. In this paper, we analyze the public and private approaches to tackle climate change jointly. We investigate how regulations affect allocation of socially responsible capital and firms' incentives to reduce pollution. We highlight that such commonly used policy tools as subsidies to clean firms and pollution tax reshape firms' shareholder compositions so that fewer firms may end up adopting the green technology. As a result, pollution can increase with regulation stringency, and optimal regulations do not always fully correct the externality.

To keep our analyses transparent, we have not included several important features in our model. In our framework, one can investigate the role of large investors who internalize their impacts on the aggregate externality. It would be also interesting to

[22] A simple way to extend our model to include such considerations is to add a cost that is convex in the intervention size to the planner's objective.

analyze a dynamic version of our model, in which investors not only fund firms in the primary market but can also trade shares in the secondary market. Another important extension is to study interactions between investors' funding decisions and government interventions when the regulator cannot credibly commit to policies ex-ante. We leave formal analyses of these extensions to future work.

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Appendix

A Proofs

A.1 Lemma 1.

Proof. Consider a firm j of type i. Denote the total wealth invested in it by its shareholders by m_{ij} and denote the total disutility of its shareholders (weighed by wealth contributed by each shareholder to the firm) from holding polluting shares by γ_{ij}^{tot}. In any symmetric equilibrium m_{ij} and γ_{ij}^{tot} depend only on firm type i. Thus, $m_{ij} = m_i$ and $\gamma_{ij}^{tot} = \gamma_i^{tot}$. Firm j's manager adopts the green technology if and only if the total cost of adoption $f_{ij} m_i$ does not exceed γ_i^{tot}, that is, $f_{ij} \leq f_i^* = \frac{\gamma_i^{tot}}{m_i}$. □

A.2 Lemma 2.

Proof. First, we prove that in any symmetric equilibrium $v_c(\gamma, f_c^*)$ and $v_d(\gamma, f_d^*)$, given by (1), are distinct linear functions of γ, in the sense that they have different slope coefficients and/or different intercepts. We prove this result by contradiction. Suppose that $v_c(\gamma, f_c^*)$ and $v_d(\gamma, f_d^*)$ are the same linear functions of γ. Then all investors are indifferent between holding shares of c- and d-firms, which implies that c- and d-firms have identical shareholder bases. Therefore, the threshold adoption costs are identical, $f_c^* = f_d^* = f^*$. But then the slope coefficients of $v_c(\gamma, f^*)$ and $v_d(\gamma, f^*)$ are different because $h_c \mathbb{P}(f_{cj} > f^*) < h_d \mathbb{P}(f_{dj} > f^*)$ by Assumption 1. Thus, $v_c(\gamma, f_c^*)$ and $v_d(\gamma, f_d^*)$ are distinct linear functions of γ.

In the second step, we prove that at most mass zero of investors hold shares of both firm types. Suppose not. Consider an investor γ' holding nonzero number of shares of both firm types. For this investor, the first-order condition of (2) is

$$k_i v_i(\gamma', f_i^*) = p_i \left(\lambda^{\gamma'} + R\right), \tag{A.1}$$

where $\lambda^{\gamma'} > 0$ is the Lagrange multiplier of the budget constraint $\sum_{i=1}^{N=2} s_i^{\gamma'} p_i \leq 1$. Since the investor holds nonzero number of shares of both firm types, (A.1) holds for $i = c, d$, which implies

$$\frac{v_c(\gamma', f_c^*)}{v_d(\gamma', f_d^*)} = \frac{p_c}{p_d}\frac{k_d}{k_c} = 1, \tag{A.2}$$

where the last equality holds because for any firm type i, $k_i = p_i \int s_i^{\gamma} dG(\gamma) = p_i$. Because $v_c(\gamma, f_c^*)$ and $v_d(\gamma, f_d^*)$ are different linear functions of γ, there exists at most one $\gamma' \in [0, \bar{\gamma}]$ solving (A.2). Since the cumulative distribution function of investor types $G(\cdot)$ is continuous, the mass of such investors is zero.

We proceed to prove the second part of Lemma 2. Suppose that investors γ_1 and $\gamma_2 > \gamma_1$ hold shares of c-firms. Then $v_c(\gamma_1, f_c^*) \geq v_d(\gamma_1, f_d^*)$ and $v_c(\gamma_2, f_c^*) \geq v_d(\gamma_2, f_d^*)$, with at least one inequality being strict. Since $v_c(\gamma, f_c^*)$ and $v_d(\gamma, f_d^*)$ are linear functions of γ, $v_c(\gamma_3, f_c^*) > v_d(\gamma_3, f_d^*)$ for any $\gamma_3 \in (\gamma_1, \gamma_2)$, which implies that all investors with $\gamma \in (\gamma_1, \gamma_2)$ hold shares of c-firms. We can use similar steps to prove that if investors γ_1 and $\gamma_2 > \gamma_1$ hold shares of d-firms, all investors with $\gamma \in (\gamma_1, \gamma_2)$ hold shares of only d-firms. □

A.3 Proposition 1.

Proof. To simplify notation, we introduce a function $\psi(\hat{\gamma}) = \frac{\int_0^{\hat{\gamma}} \gamma dG(\gamma)}{G(\hat{\gamma})}$. It captures the average disutility of d-firms' shareholders from holding polluting shares. This function satisfies three properties: (1) $\psi(\hat{\gamma}) < \hat{\gamma}$; (2) $\lim_{\hat{\gamma} \to 0} \psi(\hat{\gamma}) = 0$ by L'Hospital's rule; (3) $\psi(\hat{\gamma})$ is an increasing function because

$$\frac{\partial \psi}{\partial \hat{\gamma}} = \frac{g(\hat{\gamma})}{G(\hat{\gamma})} \left(\hat{\gamma} - \frac{\int_0^{\hat{\gamma}} \gamma dG(\gamma)}{G(\hat{\gamma})} \right) > 0.$$

The green equilibrium exists if $\psi(\hat{\gamma}_g) \geq f_l$, where $\hat{\gamma}_g$ is given by Equation (10). The brown equilibrium exists if $\psi(\hat{\gamma}_b) < f_l$, where $\hat{\gamma}_b$ is given by Equation (6). The equilibrium characterization can be split in two cases.

Case 1: $f_l > \int_0^{\bar{\gamma}} \gamma dG(\gamma)$. In this case,

$$\psi(\hat{\gamma}) \leq \int_0^{\bar{\gamma}} \gamma dG(\gamma) < f_l$$

for any $\hat{\gamma} \in [0, \bar{\gamma}]$. Therefore, the adoption cost f_l is always higher than the average disutility of d-firms' shareholders from holding polluting shares. Hence, the green equilibrium does not exist and the brown equilibrium exists.

Case 2: $f_l \leq \int_0^{\bar{\gamma}} \gamma dG(\gamma)$. Recall that $\psi(\hat{\gamma})$ is a continuous increasing function of $\hat{\gamma}$. Furthermore, $\lim_{\hat{\gamma} \to 0} \psi(\hat{\gamma}) = 0$ and $\psi(\bar{\gamma}) \geq f_l$. Therefore, there exists a unique $\underline{\Delta} > 0$ such that $\psi(\hat{\gamma}_g(\underline{\Delta})) = f_l$, where $\hat{\gamma}_g(\Delta)$ is given by (10). Similarly, there exists a unique

$\bar{\Delta} \leq \bar{\gamma}$ such that $\psi\left(\hat{\gamma}_b\left(\bar{\Delta}\right)\right) = f_l$, where $\hat{\gamma}_b\left(\Delta\right)$ is given by (6).

Next, we prove that $\underline{\Delta} < \bar{\Delta}$. Suppose that $\Delta = \bar{\Delta}$ so that $\psi\left(\hat{\gamma}_b\left(\bar{\Delta}\right)\right) = f_l$, which implies $\hat{\gamma}_b\left(\bar{\Delta}\right) = \bar{\Delta} > f_l$, where we use the definition of $\hat{\gamma}_b$ (6) and the fact that $\psi\left(\hat{\gamma}\right) < \hat{\gamma}$. Since $\bar{\Delta} > f_l$, we have

$$\hat{\gamma}_g\left(\bar{\Delta}\right) = \frac{1}{\pi}\bar{\Delta} - \frac{1-\pi}{\pi}f_l > \bar{\Delta} = \hat{\gamma}_b\left(\bar{\Delta}\right).$$

Because $\psi\left(\hat{\gamma}\right)$ increases in $\hat{\gamma}$, we have $\psi\left(\hat{\gamma}_g\left(\bar{\Delta}\right)\right) > \psi\left(\hat{\gamma}_b\left(\bar{\Delta}\right)\right) = f_l$, which means that the green equilibrium exists if $\Delta = \bar{\Delta}$. Moreover, $\psi\left(\hat{\gamma}_g(\bar{\Delta})\right) > \psi\left(\hat{\gamma}_g(\underline{\Delta})\right) = f_l$, which implies that $\bar{\Delta} > \underline{\Delta}$ because $\psi(\hat{\gamma})$ increases in $\hat{\gamma}$ and $\hat{\gamma}_g(\Delta)$ increases in Δ.

Finally, we prove that $\hat{\gamma}_g > \hat{\gamma}_b$ if the green and brown equilibria coexist. Using (6) and (10), we can write

$$\hat{\gamma}_g - \hat{\gamma}_b = \frac{1-\pi}{\pi}\left(\Delta - f_l\right). \tag{A.3}$$

Note that $\hat{\gamma}_g\left(f_l\right) = f_l$ and $\psi\left(\hat{\gamma}_g\left(f_l\right)\right) = \psi\left(f_l\right) < f_l$, which implies that the green equilibrium does not exist if $\Delta = f_l$. This in turn implies that $\underline{\Delta} > f_l$. Therefore, if the green equilibrium exists, then $\Delta \geq \underline{\Delta} > f_l$ and (A.3) is positive. □

A.4 Lemma 3.

Proof. Define f_l^1 implicitly as a solution to

$$f_l - \frac{\int_0^{\hat{\gamma}_g(f_l)} \gamma dG(\gamma)}{G\left(\hat{\gamma}_g\left(f_l\right)\right)} = 0, \tag{A.4}$$

where we write $\hat{\gamma}_g$, defined by (10), as a function of f_l, i.e. $\hat{\gamma}_g = \hat{\gamma}_g(f_l)$. Clearly, there exists a unique $f_l^1 > 0$ solving (A.4) because the left-hand side is an increasing function that is negative at $f_l = 0$ and positive as f_l approaches $\frac{\Delta}{1-\pi}$ (and $\hat{\gamma}_g(f_l)$ approaches zero). The green equilibrium exists if $f_l \in [0, f_l^1]$.

Next, we compare welfare in the green and brown equilibria. They are given by (9) and (12), respectively, and can be written as

$$W_b = W_b^1 - \varepsilon\xi\left(G\left(\hat{\gamma}_b\right)\right) = \alpha_c + G\left(\hat{\gamma}_b\right)\Delta - \int_0^{\hat{\gamma}_b} \gamma dG(\gamma) - \varepsilon\xi\left(G\left(\hat{\gamma}_b\right)\right),$$

$$W_g = W_g^1 - \varepsilon\xi\left(\pi G\left(\hat{\gamma}_g\right)\right) = \alpha_c + G\left(\hat{\gamma}_g\right)\left(\Delta - (1-\pi)f_l\right) - \pi\int_0^{\hat{\gamma}_g} \gamma dG(\gamma) - \varepsilon\xi\left(\pi G\left(\hat{\gamma}_g\right)\right),$$

where $\hat{\gamma}_b$ and $\hat{\gamma}_g$ are given by (6) and (10), respectively. Write $\hat{\gamma}_g = \hat{\gamma}_g\left(f_l, \pi\right)$, $W_g^1 = W_g^1\left(f_l, \pi\right)$ and $W_g = W_g\left(f_l, \pi\right)$ as functions of f_l and π. Note that $\hat{\gamma}_b$, W_b^1 and W_b do not depend on f_l and π.

Suppose that $f_l = 0$. Differentiating W_g^1 with respect to π, we find

$$\frac{dW_g^1(0,\pi)}{d\pi} = -\int_0^{\hat{\gamma}_g(0,\pi)} \gamma dG(\gamma) < 0.$$

Therefore,

$$\begin{aligned} W_g(0,\pi) =& W_g^1(0,\pi) - \varepsilon\xi\left(\pi G\left(\hat{\gamma}_g(0,\pi)\right)\right) \overset{\pi<1}{>} W_g^1(0,1) - \varepsilon\xi\left(G\left(\pi\hat{\gamma}_g(0,\pi)\right)\right) = \\ & W_1^b - \varepsilon\xi\left(G\left(\pi\hat{\gamma}_g(0,\pi)\right)\right) \overset{\text{Asm } 2}{>} W_1^b - \varepsilon\xi\left(G\left(\hat{\gamma}_b\right)\right) = W^b. \end{aligned} \tag{A.5}$$

Consider the range of f_l for which the green equilibrium exists, i.e. $f_l \in [0, f_l^1]$. Define $\bar{f}_l = f_l^1$ if $W_g(f_l) > W_b \; \forall f_l \in [0, f_l^1]$. In this case, $\bar{f}_l$ does not depend on ε.

If $\exists \check{f}_l \in [0, f_l^1]$ such that $W_g(\check{f}_l) \leq W_b$, then define $\bar{f}_l = \inf\{f_l \in [0, f_l^1] : W_g(f_l) \leq W_b\}$. In this case, $\bar{f}_l > 0$ because $W_g(f_l)$ is a continuous function and $W_g(0) > W_b$ by (A.5). Furthermore,

$$W_g\left(\bar{f}_l, \varepsilon\right) - W_b(\varepsilon) = 0 \Rightarrow \left.\frac{dW_g}{df_l}\right|_{f_l=\bar{f}_l} \times \frac{d\bar{f}_l}{d\varepsilon} + \frac{dW_g}{d\varepsilon} - \frac{dW_b}{d\varepsilon} = 0. \tag{A.6}$$

Note that $\frac{dW_g}{d\varepsilon} - \frac{dW_b}{d\varepsilon} = \xi\left(G\left(\hat{\gamma}_b\right)\right) - \xi\left(\pi G\left(\hat{\gamma}_g\right)\right) > 0$ by Assumption 2. Furthermore, since $\bar{f}_l$ is an infimum, $\left.\frac{dW_g}{df_l}\right|_{f_l=\bar{f}_l} < 0$. From (A.6) it then follows that $\frac{d\bar{f}_l}{d\varepsilon} > 0$. □

A.5 Proposition 2.

Proof. We prove this proposition in two steps. In the first step, we show that the planner follows a threshold rule when allocating investors across *c*- and *d*-firms. That is, the planner never picks a shareholder base composition in which there exist such groups of investors $\mathcal{S}_1$ and $\mathcal{S}_2$ that investors in $\mathcal{S}_1$ and $\mathcal{S}_2$ invest nonzero wealths in c-firms and d-firms, respectively, and all investors in $\mathcal{S}_1$ have lower disutilities from holding polluting shares (i.e., lower γ) than any investor in $\mathcal{S}_2$.[23] In the second step, we show that the thresholds the planner picks are the same as in the decentralized case.

Step 1. Suppose that our premise is wrong and such $\mathcal{S}_1$ and $\mathcal{S}_2$ exist. In what follows, we are going to show that there exists an alternative shareholder base composition that achieves a higher aggregate welfare. Three cases are possible.

Case 1: Initial equilibrium is brown. In this case, none of d-firms adopts the green technology under the initial shareholder base composition.

Define two sets $\mathcal{S}_1' \subseteq \mathcal{S}_1$ and $\mathcal{S}_2' \subseteq \mathcal{S}_2$ such that the total wealth invested in c-firms by investors in $\mathcal{S}_1'$ is nonzero and is the same as the total wealth invested in d-firms by investors in $\mathcal{S}_2'$. Such sets $\mathcal{S}_1'$ and $\mathcal{S}_2'$ exist because investors in $\mathcal{S}_1$ and $\mathcal{S}_2$ invest nonzero wealths in c-firms and d-firms, respectively.

Consider an alternative shareholder composition in which, first, the total wealth of investors in $\mathcal{S}_1'$ invested in c-firms in the initial allocation is invested in d-firms instead; second, the total wealth of investors in $\mathcal{S}_2'$ invested in d-firms in the initial allocation is invested in c-firms. Effectively, the alternative allocation implies a "swap" in shareholdings by investors in $\mathcal{S}_1'$ and $\mathcal{S}_2'$. Such a swap reduces the aggregate disutility of d-firms' shareholders from holding polluting shares. This is because all investors in $\mathcal{S}_1'$ have lower disutilities from holding polluting shares than any investor in $\mathcal{S}_2'$. At the same time, it does not affect the aggregate output because total wealths invested in c- and d-firms remain the same. Therefore, the alternative allocation is socially preferable.

Case 2: Initial equilibrium is green. In this case, d-firms facing the f_l adoption cost adopt the green technology under the initial shareholder base composition. Denote the total wealths invested in each d-firm and c-firm under the initial shareholder base composition by $m_{d,0}$ and $m_{c,0}$, respectively. Similarly, denote the total disutilities of c- and d-firms' shareholders from holding polluting shares by $\gamma_{c,0}^{tot}$ and $\gamma_{d,0}^{tot}$, respectively. Two subcases are possible.

Subcase 2.1: $\gamma_{d,0}^{tot} > f_l m_{d,0}$. Define $\mathcal{S}_1'$ and $\mathcal{S}_2'$ in the same way as in *Case 1* and consider an alternative allocation that implies a "swap" in shareholdings by investors in $\mathcal{S}_1'$ and $\mathcal{S}_2'$. In the alternative allocation, the total wealths invested in each d- and c-firm remain the same, $m_{d,1} = m_{d,0}$ and $m_{c,1} = m_{c,0}$. Moreover, the total disutility of d-firms' shareholders from holding polluting shares declines, $\gamma_{d,1}^{tot} < \gamma_{d,0}^{tot}$. However, because

[23] Note that the planner is indifferent between shareholder compositions that differ in shareholdings of zero mass.

$\gamma_{d,0}^{tot}$ is strictly greater than $f_l m_{d,0}$, we can always find such subsets $\mathcal{S}_1'$ and $\mathcal{S}_2'$ that the green equilibrium is sustainable under the alternative shareholder base composition, i.e. $\gamma_{d,0}^{tot} > \gamma_{d,1}^{tot} > f_l m_{d,1} = f_l m_{d,0}$.

As in *Case 1*, the aggregate output under the alternative shareholder base composition is the same as under the initial shareholder base composition. Furthermore, the aggregate non-pecuniary disutility is lower under the alternative shareholder base composition, $\Upsilon_1 < \Upsilon_0 \Leftrightarrow \pi\gamma_{d,0}^{tot} < \pi\gamma_{d,1}^{tot}$. Therefore, the alternative allocation is socially preferable.

Subcase 2.2: $\gamma_{d,0}^{tot} = f_l m_{d,0}$. Define $\mathcal{S}_1'$ and $\mathcal{S}_2'$ in the same way as in *Case 1* and consider an alternative allocation that implies a "swap" in shareholdings by investors in $\mathcal{S}_1'$ and $\mathcal{S}_2'$. As in *Subcase 2.1*, $\gamma_{d,0}^{tot} > \gamma_{d,1}^{tot}$, $m_{d,0} = m_{d,1}$ and $m_{c,0} = m_{c,1}$. In contrast to *Subcase 2.1*, $\gamma_{d,1}^{tot} < f_l m_{d,1}$, and thus the brown equilibrium is played under the alternative shareholder base composition. Since the aggregate output is the same under the initial and alternative allocations, the difference in welfare measures is

$$W_1 - W_0 = -\gamma_{d,1}^{tot} + \pi\gamma_{d,0}^{tot} + (1-\pi) f_l m_{d,0} > -(1-\pi)\gamma_{d,0}^{tot} + (1-\pi) f_l m_{d,0} = 0.$$

Therefore, the alternative allocation is socially preferable.

Step 2. In the first step, we have established that the planner follows a threshold rule when allocating investors across c- and d-firms. In this step, we show that the thresholds the planner picks are the same as in the decentralized case. For a given threshold $\hat{\gamma}$, welfare in the green and brown equilibria can be written as follows (recall that $\varepsilon = 0$).

$$W_g(\hat{\gamma}) = \alpha_c + G(\hat{\gamma})(\Delta - (1-\pi) f_l) - \pi \int_0^{\hat{\gamma}} \gamma dG(\gamma),$$
$$W_b(\hat{\gamma}) = \alpha_c + G(\hat{\gamma})\Delta - \int_0^{\hat{\gamma}} \gamma dG(\gamma).$$

Differentiating these expressions with respect to $\hat{\gamma}$, we find

$$\left.\frac{dW_g}{d\hat{\gamma}}\right|_{\hat{\gamma}=\hat{\gamma}^*} = 0 \Leftrightarrow g(\hat{\gamma}^*)(\Delta - (1-\pi) f_l - \pi\hat{\gamma}^*) = 0 \Rightarrow \hat{\gamma}^* = \frac{1}{\pi}\Delta - \frac{1-\pi}{\pi} f_l = \hat{\gamma}_g, \quad \text{(A.7)}$$
$$\left.\frac{dW_b}{d\hat{\gamma}}\right|_{\hat{\gamma}=\hat{\gamma}^*} = 0 \Leftrightarrow g(\hat{\gamma}^*)(\Delta - \hat{\gamma}^*) \Rightarrow \hat{\gamma}^* = \Delta = \hat{\gamma}_b, \quad \text{(A.8)}$$

where $\hat{\gamma}_b$ and $\hat{\gamma}_g$ are given by (6) and (10), respectively. It is also easy to see that W_g and W_b have negative second derivatives at $\hat{\gamma}^* = \hat{\gamma}_g$ and $\hat{\gamma}^* = \hat{\gamma}_b$, respectively. We split our analyses in four cases.

Case 1: $f_l > \int_0^{\bar{\gamma}} \gamma dG(\gamma)$. By Proposition 1, in this case only the brown equilibrium exists in the decentralized economy. Moreover, the planner cannot implement the green equilibrium by picking any $\hat{\gamma} \in [0, \bar{\gamma}]$. Therefore, the planner sets $\gamma = \hat{\gamma}_b$ that maximizes $W_b(\hat{\gamma})$ by (A.8), and the brown equilibrium is played. This outcome is the same as in the decentralized economy.

Case 2: $f_l \leq \int_0^{\bar{\gamma}} \gamma dG(\gamma)$ *and* $\Delta < \underline{\Delta}$. By Proposition 1, in this case only the brown equilibrium exists in the decentralized economy. The fact that the green equilibrium does not exist implies

$$f_l G\left(\hat{\gamma}_g\right) > \int_0^{\hat{\gamma}_g} \gamma dG(\gamma). \tag{A.9}$$

Different from *Case 1*, the planner can implement the green equilibrium by picking such $\check{\gamma}_g \in (\hat{\gamma}_g, \bar{\gamma}]$ that $f_l G\left(\check{\gamma}_g\right) \leq \int_0^{\check{\gamma}_g} \gamma dG(\gamma)$. However, the planner never finds it optimal to do so because, as we show below, $W_g\left(\hat{\gamma}_g\right) < W_b\left(\hat{\gamma}_b\right)$. Since $\hat{\gamma}_g$ maximizes $W_g\left(\hat{\gamma}\right)$ by (A.7), it then means that the planner prefers to set $\hat{\gamma} = \hat{\gamma}_b$, such that the brown equilibrium is played as in the decentralized case.

Comparing $W_g\left(\hat{\gamma}_g\right)$ and $W_b\left(\hat{\gamma}_b\right)$, we find

$$\begin{aligned}
&W_b\left(\hat{\gamma}_b\right) - W_g\left(\hat{\gamma}_g\right) = \\
&\qquad \Delta\left(G\left(\hat{\gamma}_b\right) - G\left(\hat{\gamma}_g\right)\right) + \pi \int_0^{\hat{\gamma}_g} \gamma dG(\gamma) + G\left(\hat{\gamma}_g\right)(1-\pi) f_l - \int_0^{\hat{\gamma}_b} \gamma dG(\gamma) \overset{(A.9)}{>} \\
&\qquad \Delta\left(G\left(\hat{\gamma}_b\right) - G\left(\hat{\gamma}_g\right)\right) + \int_0^{\hat{\gamma}_g} \gamma dG(\gamma) - \int_0^{\hat{\gamma}_b} \gamma dG(\gamma) \geq \\
&\qquad \operatorname{sgn}\left(\hat{\gamma}_g - \hat{\gamma}_b\right)\left(\hat{\gamma}_b - \Delta\right)\left(G\left(\hat{\gamma}_g\right) - G\left(\hat{\gamma}_b\right)\right) \overset{(6)}{=} 0.
\end{aligned}$$

Case 3: $f_l \leq \int_0^{\bar{\gamma}} \gamma dG(\gamma)$ *and* $\Delta \geq \bar{\Delta}$. By Proposition 1, in this case only the green equilibrium exists in the decentralized economy. The fact that the brown equilibrium does not exist implies

$$f_l G\left(\hat{\gamma}_b\right) \leq \int_0^{\hat{\gamma}_b} \gamma dG(\gamma), \tag{A.10}$$

Note that $\hat{\gamma}_g > \hat{\gamma}_b$. Indeed, using (6) and (10), we can write

$$\hat{\gamma}_g - \hat{\gamma}_b = \frac{1-\pi}{\pi}\left(\Delta - f_l\right) > 0, \tag{A.11}$$

where the latter inequality holds because $\Delta \geq \bar{\Delta} > f_l$ by Proposition 1.

The planner can implement the brown equilibrium by picking such $\check{\gamma}_b \in (0, \hat{\gamma}_b)$ that $f_l G\left(\check{\gamma}_b\right) > \int_0^{\check{\gamma}_b} \gamma dG(\gamma)$. However, the planner never finds it optimal to do so because, as we show below, $W_g\left(\hat{\gamma}_g\right) > W_b\left(\hat{\gamma}_b\right)$. Since $\hat{\gamma}_b$ maximizes $W_b\left(\hat{\gamma}\right)$ by (A.8), it then means that the planner prefers to set $\hat{\gamma} = \hat{\gamma}_g$, such that the green equilibrium is played as in the decentralized case.

Comparing $W_g\left(\hat{\gamma}_g\right)$ and $W_b\left(\hat{\gamma}_b\right)$, we find

$$\begin{aligned}
W_g\left(\hat{\gamma}_g\right) - W_b\left(\hat{\gamma}_b\right) = \\
&\Delta\left(G\left(\hat{\gamma}_g\right) - G\left(\hat{\gamma}_b\right)\right) - \pi \int_0^{\hat{\gamma}_g} \gamma dG(\gamma) - G\left(\hat{\gamma}_g\right)(1-\pi) f_l + \int_0^{\hat{\gamma}_b} \gamma dG(\gamma) \overset{(A.10)}{\geq} \\
&\left(\Delta - (1-\pi) f_l\right)\left(G\left(\hat{\gamma}_g\right) - G\left(\hat{\gamma}_b\right)\right) - \pi\left(\int_0^{\hat{\gamma}_g} \gamma dG(\gamma) - \int_0^{\hat{\gamma}_b} \gamma dG(\gamma)\right) \overset{(A.11)}{>} \\
&\left(\Delta - (1-\pi) f_l - \pi \hat{\gamma}_g\right)\left(G\left(\hat{\gamma}_g\right) - G\left(\hat{\gamma}_b\right)\right) \overset{(10)}{=} 0.
\end{aligned}$$

Case 4: $f_l \leq \int_0^{\bar{\gamma}} \gamma dG(\gamma)$ and $\Delta \in \left[\underline{\Delta}, \bar{\Delta}\right)$. By Proposition 1, in this case both the green and brown equilibria exist in the decentralized economy. From (A.7)–(A.8) it follows that the planner sets $\hat{\gamma} = \hat{\gamma}_g$ if $W_g\left(\hat{\gamma}_g\right) \geq W_b\left(\hat{\gamma}_b\right)$ and sets $\hat{\gamma} = \hat{\gamma}_b$ otherwise. If in the decentralized economy a socially preferable equilibrium is played in case of multiplicity (Assumption 3), then the planner cannot improve welfare relative to the decentralized economy. □

A.6 Proposition 3.

Proof. As discussed in Section 4.1, introducing subsidy to c-firms affects only the preference of a marginal investor, $\hat{\gamma}_b(\varsigma)$ and $\hat{\gamma}_g(\varsigma)$ in the brown and green equilibria, respectively. They are given by (13) and (14). From (13) and (14), it follows that increasing ς is equivalent to reducing Δ. Therefore, the existence of $\underline{\varsigma}$ and $\bar{\varsigma}$, defined in Proposition 3, directly follows from Proposition 1.

Note that at $\varsigma = \varsigma_1 \equiv \Delta - (1-\pi) f_l$ the green equilibrium does not exist because $\hat{\gamma}_g(\varsigma_1) = 0$. Furthermore, for any $f_l > 0$, it also does not exist if ς is slightly above ς_1, which implies that $\bar{\varsigma} < \varsigma_1 = \Delta - (1-\pi) f_l$.

We proceed by conducting the comparative statics of capital raised by d-firms, output, and pollution with respect to ς. Capital raised by d-firms in the green and brown equilibria can be written as

$$K_{d,g} = G\left(\hat{\gamma}_g\left(\varsigma\right)\right) \Rightarrow \frac{dK_{d,g}}{d\varsigma} = -g\left(\hat{\gamma}_g\left(\varsigma\right)\right)\frac{1}{\pi} < 0,$$
$$K_{d,b} = G\left(\hat{\gamma}_b\left(\varsigma\right)\right) \Rightarrow \frac{dK_{d,b}}{d\varsigma} = -g\left(\hat{\gamma}_b\left(\varsigma\right)\right) < 0.$$

The negative impact of ς on capital raised by d-firms implies that pollution in the green and brown equilibria declines in ς because $P_g = \pi K_{d,g}$ and $P_b = K_{d,b}$, respectively. Output in the green and brown equilibria also declines in ς because $Y_g = \alpha_c + K_{d,g}\left(\Delta - (1-\pi) f_l\right)$ and $Y_b = \alpha_c + K_{d,b}\Delta$, respectively. Note that if the green equilibrium exists, it must be that $\hat{\gamma}_g > 0 \Rightarrow \Delta > (1-\pi) f_l$. □

A.7 Lemma 4.

Proof. Welfare in the green and brown equilibria can be written as

$$W_g\left(\varsigma\right) = \alpha_c + G\left(\hat{\gamma}_g\left(\varsigma\right)\right)\left(\Delta - (1-\pi) f_l\right) - \pi \int_0^{\hat{\gamma}_g(\varsigma)} \gamma dG(\gamma) - \varepsilon\xi\left(\pi G\left(\hat{\gamma}_g\left(\varsigma\right)\right)\right),$$
$$W_b\left(\varsigma\right) = \alpha_c + G\left(\hat{\gamma}_b\left(\varsigma\right)\right)\Delta - \int_0^{\hat{\gamma}_b(\varsigma)} \gamma dG(\gamma) - \varepsilon\xi\left(G\left(\hat{\gamma}_b\left(\varsigma\right)\right)\right), \tag{A.12}$$

where $\hat{\gamma}_b\left(\varsigma\right)$ and $\hat{\gamma}_g\left(\varsigma\right)$ are given by (13) and (14), respectively. Differentiating $W_g\left(\varsigma\right)$ and $W_b\left(\varsigma\right)$ with respect to ς, we find

$$\frac{dW_g}{d\varsigma} = -\frac{1}{\pi} g\left(\hat{\gamma}_g\left(\varsigma\right)\right)\left(\varsigma - \pi\varepsilon\xi'\left(\pi G\left(\hat{\gamma}_g\left(\varsigma\right)\right)\right)\right), \tag{A.13}$$
$$\frac{dW_b}{d\varsigma} = -g\left(\hat{\gamma}_b\left(\varsigma\right)\right)\left(\varsigma - \varepsilon\xi'\left(G\left(\hat{\gamma}_b\left(\varsigma\right)\right)\right)\right). \tag{A.14}$$

Note that (A.13)–(A.14) can be written as (15), where capital invested in d-firms in the green and brown equilibria is, respectively, $K_{d,g} = G\left(\hat{\gamma}_g\left(\varsigma\right)\right)$, $K_{d,b} = G\left(\hat{\gamma}_b\left(\varsigma\right)\right)$, and pollution in the green and brown equilibria is, respectively, $P_g\left(K_{d,g}\right) = \pi K_{d,g}$ and $P_b\left(K_{b,g}\right) = K_{b,g}$.

Setting the derivatives (A.13)–(A.14) to zero, we find

$$\varsigma_g^* - \pi\varepsilon\xi'\left(\pi G\left(\hat{\gamma}_g\left(\varsigma_g^*\right)\right)\right) = 0, \tag{A.15}$$

$$\varsigma_b^* - \varepsilon\xi'\left(G\left(\hat{\gamma}_b\left(\varsigma_b^*\right)\right)\right) = 0. \tag{A.16}$$

Recall that the externality function $\xi(\cdot)$ is an increasing convex function such that $\xi(0) = \xi'(0) = 0$. Then there exists a unique solution to (A.15), $\varsigma_g^* \in (0, \Delta - (1-\pi)f_l)$ because the left-hand side of this equation is an increasing function of ς_g^* that switches sign on $(0, \Delta - (1-\pi)f_l)$. Similarly, there exists a unique $\varsigma_b^* \in (0, \Delta)$ solving (A.16). Furthermore, $\varsigma_g^* = \varsigma_g^*(\varepsilon)$ and $\varsigma_b^* = \varsigma_b^*(\varepsilon)$ are increasing functions of ε because the left-hand sides of (A.15)–(A.16) decrease in ε. Finally, $\varsigma_g^*(\varepsilon) < \varsigma_b^*(\varepsilon)$ because, by Assumption 2, pollution in the green equilibrium is lower than in the brown equilibrium for a given level of subsidy, i.e. $\pi G\left(\hat{\gamma}_g\left(\varsigma\right)\right) < G\left(\hat{\gamma}_b\left(\varsigma\right)\right)$.

Next, we prove the existence of the threshold $\bar{f}_l^{\varsigma}$. Write $W_g = W_g\left(\varsigma, f_l, \pi\right)$, $\hat{\gamma}_g = \hat{\gamma}_g\left(\varsigma, f_l, \pi\right)$ and $\varsigma_g^* = \varsigma_g^*(f_l, \pi)$. Note that $W_b\left(\varsigma\right)$, $\hat{\gamma}_b\left(\varsigma\right)$ and ς_b^* do not depend on f_l and π. Furthermore, $W_g(\varsigma, f_l, 1) = W_b(\varsigma)$, $\hat{\gamma}_g(\varsigma, f_l, 1) = \hat{\gamma}_b(\varsigma)$ and $\varsigma_g^*(f_l, 1) = \varsigma_b^*$.

Suppose that $f_l = 0$. By the envelope theorem,

$$\frac{dW_g(\varsigma_g^*(0,\pi), 0, \pi)}{d\pi} = \frac{\partial W_g(\varsigma_g^*, 0, \pi)}{\partial \pi} = - \int_0^{\hat{\gamma}_g(\varsigma_g^*, 0, \pi)} \gamma dG(\gamma) - \varepsilon G\left(\hat{\gamma}_g(\varsigma_g^*, 0, \pi)\right)\xi'\left(\pi G\left(\hat{\gamma}_g(\varsigma_g^*, 0, \pi)\right)\right) < 0.$$

Therefore,

$$W_g\left(\varsigma_g^*(0,\pi), 0, \pi\right) \overset{\pi<1}{>} W_g\left(\varsigma_g^*(0,1), 0, 1\right) = W^b(\varsigma_b^*).$$

By continuity of $W_g\left(\varsigma_g^*\left(f_l, \pi\right), f_l, \pi\right)$ in f_l, there exists an $\bar{f}_l^{\varsigma} > 0$ such that if $f_l \leq \bar{f}_l^{\varsigma}$, $W_g\left(\varsigma_g^*(f_l, \pi), f_l, \pi\right) > W_b\left(\varsigma_b^*\right)$. □

A.8 Proposition 4.

Proof. Recall that $\varsigma_g^*(\varepsilon)$ and $\varsigma_b^*(\varepsilon)$ solve (A.15) and (A.16), respectively. Using the definitions of $\hat{\gamma}_b(\varsigma)$ and $\hat{\gamma}_g(\varsigma)$, (13) and (14), respectively, we find

$$\begin{aligned} \varsigma_g^*(0) &= 0, \quad \lim_{\varepsilon\to\infty} \varsigma_g^*(\varepsilon) = \Delta - (1-\pi) f_l, \\ \varsigma_b^*(0) &= 0, \quad \lim_{\varepsilon\to\infty} \varsigma_b^*(\varepsilon) = \Delta. \end{aligned} \tag{A.17}$$

Furthermore, by Lemma 4, $\varsigma_g^*(\varepsilon)$ and $\varsigma_b^*(\varepsilon)$ are increasing functions. Therefore, there exists a unique $\underline{\varepsilon}^{\varsigma}$ such that

$$\varsigma_g^*\left(\underline{\varepsilon}^{\varsigma}\right) = \bar{\varsigma},$$

where $\bar{\varsigma}$ is defined in Proposition 3.

If $\varepsilon \leq \underline{\varepsilon}^{\varsigma}$, the green equilibrium exists under the subsidy level $\varsigma_g^*(\varepsilon)$ because $\varsigma_g^*(\varepsilon) < \bar{\varsigma}$. From Lemma 4 it then follows that the optimal subsidy is $\varsigma_g^*(\varepsilon)$. In this case, the green equilibrium is played under the optimal subsidy, and the green Pigouvian wedge is zero.

Next, we characterize the planner's optimal subsidy when $\varepsilon > \underline{\varepsilon}^{\varsigma}$. In this case, $\varsigma_g^*(\varepsilon) > \bar{\varsigma}$, and the highest welfare in the green equilibrium is reached at $\bar{\varsigma}$ because $W_g(\varsigma)$ increases on $\varsigma \in [0, \bar{\varsigma}]$. Thus, the planner either chooses $\bar{\varsigma}$ such that the green equilibrium is played, or $\varsigma_b^*(\varepsilon)$ to achieve the highest welfare in the brown equilibrium, $W_b\left(\varsigma_b^*(\varepsilon)\right)$. At $\varsigma = \bar{\varsigma}$, welfare in the green equilibrium is

$$W_g(\bar{\varsigma}) = \alpha_c + G\left(\hat{\gamma}_g(\bar{\varsigma})\right)\left(\Delta - (1-\pi) f_l\right) - \pi \int_0^{\hat{\gamma}_g(\bar{\varsigma})} \gamma dG(\gamma) - \varepsilon \xi\left(\pi G\left(\hat{\gamma}_g(\bar{\varsigma})\right)\right). \tag{A.18}$$

Define

$$\bar{\varepsilon}^{\varsigma} = \inf\{\varepsilon \geq \underline{\varepsilon}^{\varsigma} : W_b\left(\varsigma_b^*(\varepsilon)\right) \geq W_g(\bar{\varsigma})\}, \tag{A.19}$$

where $W_b(\varsigma)$ is given by (A.12). Such $\bar{\varepsilon}^{\varsigma}$ exists and is above $\underline{\varepsilon}^{\varsigma}$. Indeed, $W_g\left(\varsigma_g^*\left(\underline{\varepsilon}^{\varsigma}\right)\right) = W_g(\bar{\varsigma}) > W_b\left(\varsigma_b^*\left(\underline{\varepsilon}^{\varsigma}\right)\right)$ by Lemma 4. Furthermore, $\lim_{\varepsilon\to\infty} W_g(\bar{\varsigma}) = -\infty$, and from (A.17) it follows that $\lim_{\varepsilon\to\infty} \hat{\gamma}_b\left(\varsigma_b^*(\varepsilon)\right) = 0$, which implies $\lim_{\varepsilon\to\infty} W_b\left(\varsigma_b^*(\varepsilon)\right) = \alpha_c > 0$.

By definition of $\bar{\varepsilon}^{\varsigma}$ (A.19), $W_g(\bar{\varsigma}) \geq W_b\left(\varsigma_b^*(\varepsilon)\right)$ for $\varepsilon \in (\underline{\varepsilon}^{\varsigma}, \bar{\varepsilon}^{\varsigma}]$. Therefore, for such values of ε the planner sets $\varsigma = \bar{\varsigma}$, and the green equilibrium is played. However, the green Pigouvian wedge is negative because $\bar{\varsigma} < \varsigma_g^*(\varepsilon)$.

Finally, we show that the planner sets $\varsigma = \varsigma_b^*(\varepsilon)$ if $\varepsilon > \bar{\varepsilon}^{\varsigma}$. Note that the brown equilibrium exists under $\varsigma_b^*(\varepsilon)$ if $\varepsilon > \bar{\varepsilon}^{\varsigma}$ because $\varsigma_b^*(\varepsilon) > \varsigma_g^*(\varepsilon) > \bar{\varsigma}$. At the same time, the highest welfare achievable in the green equilibrium is $W_g(\bar{\varsigma})$. Therefore, it is sufficient to show that $W_b(\varsigma_b^*(\varepsilon)) > W_g(\bar{\varsigma})$ for $\varepsilon > \bar{\varepsilon}^{\varsigma}$. At $\varepsilon = \bar{\varepsilon}^{\varsigma}$

$$\left.\frac{d\left(W_g(\bar{\varsigma}) - W_b\left(\varsigma_b^*(\varepsilon)\right)\right)}{d\varepsilon}\right|_{\varepsilon=\bar{\varepsilon}^{\varsigma}} \leq 0$$

because otherwise $\bar{\varepsilon}^{\varsigma}$ is not an infimum. Using (A.12) and (A.18), we find that the derivative above is strictly negative for $\varepsilon > \bar{\varepsilon}^{\varsigma}$ because

$$\frac{d^2 W_g(\bar{\varsigma})}{d\varepsilon^2} = 0 \quad \text{and} \quad \frac{d^2 W_b\left(\varsigma_b^*(\varepsilon)\right)}{d\varepsilon^2} = \frac{d\varsigma_b^*(\varepsilon)}{d\varepsilon} g\left(\hat{\gamma}_b\left(\varsigma_b^*(\varepsilon)\right)\right) \xi'\left(G\left(\hat{\gamma}_b\left(\varsigma_b^*(\varepsilon)\right)\right)\right) > 0,$$

where the latter inequality holds because $\varsigma_b^*(\varepsilon)$ is an increasing function (Lemma 4). Therefore, for $\varepsilon > \bar{\varepsilon}^{\varsigma}$, $W_g(\bar{\varsigma}) < W_b(\varsigma_b^*(\varepsilon))$, and the planner sets $\varsigma = \varsigma_b^*(\varepsilon) > \varsigma_g^*(\varepsilon)$. Because $\varsigma_b^*(\varepsilon) > \varsigma_g^*(\varepsilon)$, the green Pigouvian wedge is positive. □

A.9 Proposition 5.

Proof. We start by proving part (i) of Proposition 5. By Proposition 1, the green equilibrium exists if $\Delta \geq \underline{\Delta} > f_l$, $f_l < \int_0^{\bar{\gamma}} \gamma dG(\gamma)$, and $\tau = 0$, i.e.

$$\psi\left(\hat{\gamma}_g(0)\right) \equiv \frac{\int_0^{\hat{\gamma}_g(0)} \gamma dG(\gamma)}{G\left(\hat{\gamma}_g(0)\right)} \geq f_l,$$

where $\hat{\gamma}_g(\tau)$ is given by (17). Because $\psi\left(\hat{\gamma}_g(\tau)\right)$ is a continuous function of τ, it then follows that there exists $\underline{\tau} \geq 0$ such that $\psi\left(\hat{\gamma}_g(\tau)\right) + \tau \geq f_l$ on $\tau \in [0, \underline{\tau}]$.

Next, note that if $\tau \in \left[f_l, \frac{1}{\pi}\left(\Delta - (1-\pi) f_l\right)\right]$, the green equilibrium exists. This is because d-firms have nonempty shareholder base, $\hat{\gamma}_g(\tau) \geq 0$, and d-firms' managers find it optimal to pay the adoption cost, i.e. $\psi\left(\hat{\gamma}_g(\tau)\right) + \tau \geq f_l$. Furthermore, at $\tau = f_l$, $\hat{\gamma}_g(f_l) = \frac{1}{\pi}\left(\Delta - f_l\right) > 0$ and $\psi\left(\hat{\gamma}_g(f_l)\right) > 0$, and hence there exists a $\bar{\tau} < f_l$ such that the green equilibrium exists on $\tau \in \left[\bar{\tau}, \frac{1}{\pi}(\Delta - (1-\pi) f_l)\right]$.

We proceed to show that the green equilibrium might not exist for some intermediate range of τ (part (ii) of Proposition 5). Suppose that $\Delta = \underline{\Delta}$. Then, by definition of $\underline{\Delta}$,

$$f^*\left(\hat{\gamma}_g(0, \underline{\Delta}), 0\right) = \psi\left(\hat{\gamma}_g(0, \underline{\Delta})\right) = f_l,$$

where $f^*(\hat{\gamma}_g(\tau, \Delta), \tau)$ is given by (18), and where we write $\hat{\gamma}_g$, given by (17), as a function of τ and Δ, i.e. $\hat{\gamma}_g = \hat{\gamma}_g(\tau, \Delta)$. If $\frac{df_g^*}{d\tau}\Big|_{\tau=0} < 0$, then a marginal increase in τ from 0 implies that the green equilibrium ceases to exist. Then part (ii) of Proposition 5 follows because $f_g^*\left(\hat{\gamma}_g(\tau, \Delta), \tau\right)$ is a smooth function of τ and Δ.

We now prove part (iii) of Proposition 5. Note that

$$\hat{\gamma}_g(\tau) > \hat{\gamma}_b(\tau) \Leftrightarrow \Delta > f_l,$$

which is true because $\Delta \geq \underline{\Delta} > f_l$. Here $\hat{\gamma}_b(\tau)$ and $\hat{\gamma}_g(\tau)$ are given by (16) and (17), respectively. Therefore, if the green equilibrium does not exist, i.e. $f^*(\hat{\gamma}_g(\tau), \tau) < f_l$, then the brown equilibrium exists because $f^*(\hat{\gamma}, \tau)$ is an increasing function of $\hat{\gamma}$ and so $f^*(\hat{\gamma}_b(\tau), \tau) < f^*(\hat{\gamma}_g(\tau), \tau) < f_l$.

We are left to conduct the comparative statics of capital raised by d-firms, output, and pollution with respect to τ. Capital raised by d-firms in the green and brown equilibria can be written as

$$K_{d,g} = G\left(\hat{\gamma}_g\left(\tau\right)\right) \Rightarrow \frac{dK_{d,g}}{d\tau} = -g\left(\hat{\gamma}_g\left(\tau\right)\right) < 0,$$
$$K_{d,b} = G\left(\hat{\gamma}_b\left(\tau\right)\right) \Rightarrow \frac{dK_{d,b}}{d\tau} = -g\left(\hat{\gamma}_b\left(\tau\right)\right) < 0.$$

The negative impact of τ on capital raised by d-firms implies that pollution in the green and brown equilibria declines in ς because $P_g = \pi K_{d,g}$ and $P_b = K_{d,b}$, respectively. Output in the green and brown equilibria also declines in τ because $Y_g = \alpha_c + K_{d,g}\left(\Delta - (1-\pi)f_l\right)$ and $Y_b = \alpha_c + K_{d,b}\Delta$, respectively. Note that if the green equilibrium exists, it must be that $\hat{\gamma}_g > 0 \Rightarrow \Delta > (1-\pi)f_l$. □

A.10 Lemma 5.

Proof. Welfare in the green and brown equilibria can be written as

$$W_g\left(\tau\right) = \alpha_c + G\left(\hat{\gamma}_g\left(\tau\right)\right)\left(\Delta - (1-\pi)f_l\right) - \pi \int_0^{\hat{\gamma}_g(\tau)} \gamma dG(\gamma) - \varepsilon\xi\left(\pi G\left(\hat{\gamma}_g\left(\tau\right)\right)\right), \quad \text{(A.20)}$$

$$W_b\left(\tau\right) = \alpha_c + G\left(\hat{\gamma}_b\left(\tau\right)\right)\Delta - \int_0^{\hat{\gamma}_b(\tau)} \gamma dG(\gamma) - \varepsilon\xi\left(G\left(\hat{\gamma}_b\left(\tau\right)\right)\right), \quad \text{(A.21)}$$

where $\hat{\gamma}_b(\tau)$ and $\hat{\gamma}_g(\tau)$ are given by (16) and (17), respectively. Differentiating $W_g(\tau)$ and $W_b(\tau)$ with respect to τ, we find

$$\frac{dW_g}{d\tau} = -\pi g(\hat{\gamma}_g(\tau))(\tau - \varepsilon\xi'(\pi G(\hat{\gamma}_g(\tau)))), \tag{A.22}$$

$$\frac{dW_b}{d\tau} = -g(\hat{\gamma}_b(\tau))(\tau - \varepsilon\xi'(G(\hat{\gamma}_b(\tau)))). \tag{A.23}$$

Note that (A.22)–(A.23) can be written as (19), where pollution in the green and brown equilibria is, respectively, $P_g = \pi G(\hat{\gamma}_g(\tau))$ and $P_b = G(\hat{\gamma}_b(\tau))$.

Setting the derivatives (A.22)–(A.23) to zero, we find

$$\tau_g^* - \varepsilon\xi'\left(\pi G\left(\hat{\gamma}_g\left(\tau_g^*\right)\right)\right) = 0, \tag{A.24}$$

$$\tau_b^* - \varepsilon\xi'(G(\hat{\gamma}_b(\tau_b^*))) = 0. \tag{A.25}$$

Recall that the externality function $\xi(\cdot)$ is an increasing convex function such that $\xi(0) = \xi'(0) = 0$. Then there exists a unique solution to (A.24), $\tau_g^* \in \left(0, \frac{1}{\pi}(\Delta - (1-\pi)f_l)\right)$, because the left-hand side of this equation is an increasing function of τ_g^* that switches sign on $\left(0, \frac{1}{\pi}(\Delta - (1-\pi)f_l)\right)$. Similarly, there exists a unique $\tau_b^* \in (0, \Delta)$ solving (A.16). Furthermore, $\tau_g^* = \tau_g^*(\varepsilon)$ and $\tau_b^* = \tau_b^*(\varepsilon)$ are increasing functions of ε because the left-hand sides of (A.24)–(A.25) decrease in ε.

Next, we prove the existence of the threshold $\bar{f}_l^\tau$. Write $W_g = W_g(\tau, f_l, \pi)$, $\hat{\gamma}_g = \hat{\gamma}_g(\tau, f_l, \pi)$ and $\tau_g^* = \tau_g^*(f_l, \pi)$. Note that $W_b(\tau)$, $\hat{\gamma}_b(\tau)$ and τ_b^* do not depend on f_l and π. Furthermore, $W_g(\tau, f_l, 1) = W_b(\tau)$, $\hat{\gamma}_g(\tau, f_l, 1) = \hat{\gamma}_b(\tau)$ and $\tau_g^*(f_l, 1) = \tau_b^*$.

Suppose that $f_l = 0$. By the envelope theorem,

$$\frac{dW_g(\tau_g^*(0,\pi), 0, \pi)}{d\pi} = \frac{\partial W_g(\tau_g^*, 0, \pi)}{\partial \pi} = -\int_0^{\hat{\gamma}_g(\tau_g^*, 0, \pi)} \gamma dG(\gamma) - \varepsilon G\left(\hat{\gamma}_g(\tau_g^*, 0, \pi)\right)\xi'\left(\pi G\left(\hat{\gamma}_g(\tau_g^*, 0, \pi)\right)\right) < 0.$$

Therefore,

$$W_g\left(\tau_g^*(0,\pi), 0, \pi\right) \overset{\pi<1}{>} W_g\left(\tau_g^*(0,1), 0, 1\right) = W^b(\tau_b^*).$$

By continuity of $W_g\left(\tau_g^*(f_l, \pi), f_l, \pi\right)$ in f_l, there exists an $\bar{f}_l^\tau > 0$ such that if $f_l \leq \bar{f}_l^\tau$, $W_g\left(\tau_g^*(f_l, \pi), f_l, \pi\right) > W_b(\tau_b^*)$. □

A.11 Proposition 6.

Proof. Recall that $\tau_g^*(\varepsilon)$ and $\tau_b^*(\varepsilon)$ solve (A.24) and (A.25), respectively. Using the definitions of $\hat{\gamma}_b(\tau)$ and $\hat{\gamma}_g(\tau)$, (16) and (17), respectively, we find

$$\tau_g^*(0) = 0, \quad \lim_{\varepsilon\to\infty} \tau_g^*(\varepsilon) = \frac{1}{\pi}\left(\Delta - (1-\pi)f_l\right),$$
$$\tau_b^*(0) = 0, \quad \lim_{\varepsilon\to\infty} \tau_b^*(\varepsilon) = \Delta.$$

Furthermore, by Lemma 5, $\tau_g^*(\varepsilon)$ and $\tau_b^*(\varepsilon)$ are increasing functions. Therefore, there exist $0 \leq \underline{\varepsilon}^\tau < \bar{\varepsilon}^\tau$ such that

$$\tau_g^*(\underline{\varepsilon}^\tau) = \underline{\tau} < \tau_g^*(\bar{\varepsilon}^\tau) = \bar{\tau},$$

where $\underline{\tau}$ and $\bar{\tau}$ are defined in Proposition 5.

If $\varepsilon \leq \underline{\varepsilon}^\tau$ or $\varepsilon \geq \bar{\varepsilon}^\tau$, the green equilibrium exists under $\tau_g^*(\varepsilon)$. From Lemma 5 it then follows that the optimal tax is $\tau_g^*(\varepsilon)$. In this case, the green equilibrium is played under the optimal tax, and the green Pigouvian wedge is zero.

Suppose there exists a set $\mathcal{S}_\tau^g \subseteq (\underline{\tau}, \bar{\tau})$ such that if $\tau \in S_\tau$, then the green equilibrium does not exist for such a τ. Part (ii) of Proposition 5 identifies sufficient conditions under which $\mathcal{S}_\tau^g$ is nonempty. By continuity of $\tau_g^*(\varepsilon)$, there exists $\varepsilon' \in (\underline{\varepsilon}^\tau, \bar{\varepsilon}^\tau)$ such that $\tau_g^*(\varepsilon') \in \mathcal{S}_\tau^g$. In this case, the planner cannot achieve a zero green Pigovian wedge and sets optimal tax as follows,

$$\tau^*(\varepsilon') \in \arg\max\left\{\arg\max_{\tau\notin\mathcal{S}_\tau^g} W_g(\tau,\varepsilon'), \arg\max_{\tau\notin\mathcal{S}_\tau^b} W_b(\tau,\varepsilon')\right\}.$$

Here $\mathcal{S}_\tau^b$ is a subset of $[0,\Delta)$ for which the brown equilibrium does not exist, and $W_g(\tau,\varepsilon)$ and $W_b(\tau,\varepsilon)$ are given by (A.20) and (A.21), respectively. □

B Parametrizations used for figures

B.1 Section 4.1

Figure 2: $\alpha_d = 1.5$, $\alpha_c = 1$, $\pi = 0.55$, $f_l = 0.2$, externality function is $\xi(x) = \frac{1}{2}x^2$, distribution of γ is uniform on $[0,1]$, $\varepsilon = 1$ (panel A), $\varepsilon = 3$ (panel B), $\varepsilon = 4.5$ (panel C).

Figure 3: same as for panel (B) of Figure 2.

B.2 Section 4.2

Figure 4: $\alpha_d = 1.6$, $\alpha_c = 1$, $\pi = 0.325$, $f_l = 0.465$. Distribution of γ: $\bar{\gamma} = 1$, $g(\gamma) = 0.2 + 0.8\,(0.5g_1(\gamma) + 0.5g_2(\gamma))$, where $g_1(\cdot)$ is a probability density function of a truncated normal distribution with $\mu_1 = 0.2$, $\sigma_1 = 0.07$ and support $[0, 0.5)$; $g_2(\cdot)$ is a probability density function of a truncated normal distribution with $\mu_2 = 0.8$, $\sigma_2 = 0.07$ and support $[0.5, 1]$.

Figure 5: same as for Figure 4; externality function is $\xi(x) = \frac{1}{2}x^2$, $\varepsilon = 0.9$.