# Imperfect Flood Insurance Enforcement and Business Misallocation

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#### Abstract

Flood risk poses a widespread and growing threat to economic activities in the US. Under perfect market design, both residential households and commercial businesses would gradually move out of the flood zones, which agrees with socially optimal resource allocation. The current flood assistance system, however, fails to enforce the flood insurance take-up by the commercial property owners, while providing government aid to these uninsured properties ex post. In other words, the commercial property owners in the flood zones are free riding the federal flood assistance system and take an advantageous position in the rental market. The low commercial rent, counterproductively, attracts businesses back to the flood zone, which leads to resource misallocation. This project looks back at the past two decades (1999–2020) and empirically shows businesses grow 10% faster in the central business districts after they are designated as flood zones, which manifests the pulling force of businesses due to the free riding problem in the current flood assistance system. We build a spatial model to interpret the empirical finding as business misallocation, and quantify the welfare loss.

Keywords: Flood Insurance, Misallocation, Climate Change, Flood Risk, Business Growth

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## 1 Introduction

In 2020, 10.3% of US households were estimated to live in an area with substantial flood risk,<sup>1</sup> and this number will keep growing with global warming and rising sea levels. The anticipation of severe flood damage in the future has and will keep reshaping the landscape of economic activities in most urban areas. This project looks back at the past two decades (1999–2020), and studies how local business activities migrate in response to the rising flood risk, and whether the migration pattern is consistent with efficient resource allocation. This question is critical for understanding the economic implications of the increasing climate risks. And practically, the answer to this question will help guide the policy decisions under climate change.

In theory, flood risk is a negative productivity shock, which should lead to a gradual outflow of local businesses from the flood zones to non-flood zones. However, this efficient resource relocation process might be complicated and compromised by poor market design. This project shows that the current government-dominated flood risk management system in the US (federal flood insurance and government aid) is counterproductive, and leads to business misallocation.

In the US, the federal government dominates the flood risk management system, since flood risk is notoriously difficult for the private sector to insure. A flood event often results in a vast number of claims being filed simultaneously, which can easily go beyond the financial capacity of any private firm. The current government-dominated flood risk management system is built on three pillars. The first pillar is the flood risk information infrastructure, *i.e.* Federal Emergency Management Agency (FEMA) flood maps. Every year, FEMA spends more than \$200 million in assessing and updating flood risk for the nation to help the residents and businesses better understand their exposure to flood risk.<sup>2</sup> The second pillar is the provision of flood insurance through the National Flood Insurance Program. This is an *ex ante* protection measure. The third pillar is *ex post* government aid for areas that are hit by floods. This includes FEMA Individual Assistance Program and SBA Disaster Loan Assistance Program.

In practice, the coexistence of both *ex ante* (flood insurance) and *ex post* (government aid) risk protection measures creates a serious free riding problem. At risk property owners are discouraged

<sup>&</sup>lt;sup>1</sup>According to the national flood risk assessment report from the First Street Foundation, 14.6 million out of 142 million homes and properties across the US are located in the area with higher than 1% annual flood risk.

<sup>&</sup>lt;sup>2</sup>Recently, there are also private initiatives in the market, that try to use the most-up-to-date model and data to assess flood risk for the nation. They include First Street Foundation and Moody's Analytics.

to purchase flood insurance with the expectation of government aid for realized losses, even though either of the measures is well intended. To mitigate this free riding problem, the government enforces the purchase of flood insurance by bundling it with other government programs. For example, GSE (FHA, Fannie Mae, or Freddie Mac) mortgage borrowers have to show proof of flood insurance if the collateral property is in the flood zones. The significant presence of government intervention in the residential mortgage market makes flood insurance coverage enforceable for residential properties. But the story is sharply different for commercial properties. The lack of government presence in the commercial mortgage market makes it difficult to enforce the purchase of flood insurance for commercial properties. As a result, only 20% of the commercial properties (in dollar amount) in the flood zone is covered by NFIP flood insurance in 2020, while close to 70% of the residential properties is covered, as shown in Figure 1.

The imperfect flood insurance enforcement for commercial properties is distortionary and leads to business misallocation. Specifically, commercial property owners are able to gain competitive advantages in the rental market by avoiding flood insurance expenses while free riding government aids. In this way, they can lower their rent and attract more businesses flowing back to the flood zones, which is counterproductive.

To illustrate the misallocation effect of the imperfect flood insurance enforcement, this paper first builds a spatial model of the current flood risk management framework, and shows how it generates a force that drives more businesses into the flood zone. In this static model, flood risk is a negative productivity shock but can be perfectly insured by flood insurance. In addition to providing flood insurance, the government also gives aid to the property owners if they are hit by floods. A free riding problem arises in this setting—the property owners are not willing to cover their property with flood insurance, and will not purchase flood insurance unless they are forced to do so. We assume insurance coverage is only enforced for residential properties, but not for commercial properties. As a result, commercial rent drops with the increase of (perceived) flood risk, which motivates the businesses to move into the flood zone. This partial-equilibrium effect is the pull factor for the local businesses.

In general equilibrium, the movement of businesses is also affected by the other opposing force the rise of residential rent, and the linkage is through the complementarity between commercial and residential activities (*e.g.* labor source, customer base, *etc.*). The free riding of the commercial property owners on the flood risk management system would inflate the insurance cost of residential property owners, who are forced to pay. The inflated operation cost will be passed through to residential rent, which drives away residential tenants. This is a push factor for the local businesses, and the net business movement depends on the relative strength of the pull factor (free riding effect) and push factor (shrinking residential base). Our model also shows the relative strength of the two factors determined only by the commercial share of the location. Therefore, the model predicts businesses will move out of flood zones with low share of commercial properties (residential communities), and move into flood zones with high share of commercial properties (central business districts). Overall, the businesses are misallocated in either case, and welfare loss is incurred.

We then test our theoretical prediction of business movement with business growth patterns in the past two decades. The empirical study focuses on the events that the flood risk surpasses 1% annual probability, making the local area designated as Special Flood Hazard Areas (SFHA, *aka*. flood zones), and analyzes its impact on business growth. Specifically, we compare the US flood maps published in 1996 and 2011 to trace the areas that change from non-flood zones to flood zones. These are our treated areas, and we will match them with comparable control areas in the same neighborhood which did not experience increased flood risk from 1999 to 2011.

Even though flood risk change is largely exogenous, different natural amenities between the treated areas and control areas might also confound the simple difference-in-differences analysis. In particular, the treated area tends to be the marginal place with underlying annual flood probability close to 1%, and its assessed flood risk happens to cross this threshold at some point in the 2000s. It might have drastically different amenities with areas that are always in the flood zone, *e.g.* beaches or islands. To solve this non-comparability problem, we take advantage of the continuous flood risk measures achieved from the First Street Foundation, and narrow our focus only on the marginal areas (annual flood risk between 0.5% and 2%) to accomplish a cleaner identification strategy.

With this identification strategy, we find businesses grow 10% faster in the central business districts in the 10-year period after they are designated as flood zones, which manifests the pull factor of businesses due to the free riding problem of the imperfect flood insurance enforcement. We then move to areas with less commercial properties and find the relative business growth decreases due to the increasing pushing force from the residential side. Ultimately, the push factor dominates in the residential communities, and businesses are flowing out after the designation of flood zone.

Literature review. Our project will contribute to the growing literature that studies the threat of floods and other climate hazards to the economy by focusing on the relocation of business activities. Most of the current studies focus on residential housing market (Baldauf et al., 2020; Bernstein et al., 2019; Murfin and Spiegel, 2020). One exception is Hino and Burke (2020), which finds that sophisticated commercial real estate buyers are more likely to price in the flood risk when making investment decisions. Our project will go beyond the simple price margin and study the relocation of businesses over a 25-year horizon. Another exception is Meltzer et al. (2021), which explores the business closures in New York City after Hurricane Sandy. Our project, however, aims to assess the impact on business activities due to the increased unrealized flood risk, which has implications for a broader set of areas.

Moreover, our project will be the first study on the distortionary effect of the federal flood assistance program. Several studies have attempted to capture the relationship between government aid and residents' decision to purchase flood insurance. Browne and Hoyt (2000) finds a positive correlation between flood insurance demand and FEMA disaster assistance using state-level data and fixed-effects models. Petrolia et al. (2013) surveyed roughly 1000 residents in Gulf Coast states showing a positive relationship between expectation of future aid and insurance. Kousky (2018) combine data on insurance purchases over the period 2000–2011 with two main U.S. post-disaster federal aid programs, estimating \$4000-\$5000 crowding-out effect of individual assistance grants on average quantity of insurance purchased the following year. This research seeks to fill gaps in the literature on commercial property and free-riding problems in the flood insurance market.

In this project, we propose investigating the spatial general equilibrium effect of climate risk, which has been understudied before. Desmet et al. (2018) use a dynamic spatial equilibrium model to estimate global population and economic activity shifts over the next 200 years under different greenhouse gas emission scenarios. Balboni (2021) employs a similar framework to predict future efficiency loss by ignoring climate risk for public infrastructure investments in Vietnam. To advance this line of literature, we apply a spatial equilibrium model to areas that experienced flood risk changes in the past 25 years and use the historical population and business migration patterns to discipline the model. In this way, we can better understand and estimate the behavior parameters (that summarize residents' or business owners' beliefs on the flood risk and moving frictions) in this model.

**Overview.** The paper proceeds as follows. Section 2 describes a model to illustrate how the current flood risk management system leads to business misallocation. Section 3 discusses the data and the setting for empirical tests. Section 4 outlines the empirical strategy and shows the empirical test results. Section 5 concludes.

# 2 Illustrative Model

#### 2.1 Baseline Setting without Flood Risk

There is a continuum of communities in the economy, and the measure of the communities is normalized to 1.

**Production Technology** Following the framework pioneered by Armington (1969), we assume each community i produces a variety (also indexed by i) with an endowed technology. Two factors are needed for production: residential properties  $l_i$  and commercial properties  $k_i$ . And the technology function takes the Cobb-Douglas form:

$$y_i = l_i^{1-\alpha_i} k_i^{\alpha_i}.$$

The technology parameter  $\alpha_i$  is exogenous, and community-specific. This is one of two characteristics that defines a community, and the other is its flood risk level, which will be introduced later.

Here we abstract away from the intermediate production inputs: labor and capital (or businesses). We assume one unit of residential property holds one unit of labor, and one unit of commercial property stores one unit of capital (or business). That implies  $l_i$  also measures the size of labor and  $k_i$  measures the size of capital (or businesses) in community *i*.

There is a competitive market for labors, who can freely move to different communities. The labor will accept an working offer in community i if and only her wage is enough to cover her residential rent  $\gamma_i$  plus the sustainable living cost, which is normalized to 0. Similarly, an unconstrained stock of capital can be rented by the producers. The capital rent is normalized to 0. But the producer in community i has to pay the storage cost, which is equal to the commercial property rent  $\kappa_i$ .

Therefore the producer's profit maximization problem can be written as

$$\max_{l_i,k_i} \Pi_i^P = p_i l_i^{1-\alpha_i} k_i^{\alpha_i} - \gamma_i l_i - \kappa_i k_i, \tag{1}$$

where  $p_i$  denotes the price of variety *i*. Solving the problem (1) with the first order condition, we can get

$$l_i = \frac{(1 - \alpha_i)p_i}{\gamma_i} y_i \tag{2}$$

$$k_i = \frac{\alpha_i p_i}{\kappa_i} y_i \tag{3}$$

Plugging (2) and (3) back to (1), we can write the producer's profit as

$$\Pi_i^P = \frac{\alpha_i^{\alpha_i} (1 - \alpha_i)^{1 - \alpha_i} p_i}{\kappa_i^{\alpha_i} \gamma_i^{1 - \alpha_i}} p_i y_i - p_i y_i.$$

We assume the producers are perfectly competitive, thus make zero profit. The zero-profit condition  $(\Pi_i^P = 0)$  implies

$$p_i = \frac{\kappa_i^{\alpha_i} \gamma_i^{1-\alpha_i}}{\alpha_i^{\alpha_i} (1-\alpha_i)^{1-\alpha_i}}.$$
(4)

**Consumer's Problem** The consumer (developers and land owners) with income I is to maximize her utility from consuming each variety i, given the price of each variety  $p_i$ . The varieties are aggregated in the CES fashion.

$$\begin{aligned} \max_{y_i} & \left(\int_0^1 y_i^\sigma di\right)^{\frac{1}{\sigma}}, \ (\sigma < 1) \\ s.t. & \int_0^1 p_i y_i di = I. \end{aligned}$$

Solving the consumer's problem with the first order condition (technical appendix for details), we can get the downward sloping demand curve

$$y_i = p_i^{-\frac{1}{1-\sigma}} \bar{Y},\tag{5}$$

where  $\bar{Y} = \int_0^1 p_i y_i di / \int_0^1 p_i^{\frac{\sigma}{\sigma-1}} di$  is an aggregate outcome. We can note that the single parameter  $\sigma$  dictates the demand elasticity for all varieties.

**Housing Supply** In community *i*, a monopoly developer rents land from the local farmer at the cost of *r* for both residential and commercial development. Without loss of generality, the cost of development is normalized to 0. The developer sets residential rent  $\gamma_i$  and commercial rent  $\kappa_i$ , respectively, to maximize its profit, which is the surplus after paying for the rent of the land. Its profit maximization problem can be written as

$$\max_{\kappa_i,\gamma_i} \Pi^D(\kappa_i,\gamma_i) = \gamma_i l_i + \kappa_i k_i - r(l_i + k_i).$$
(6)

We can plug in the producer's profit maximization conditions (2), (3), (4) and the consumer's utility maximization condition (5) into the developer's profit function  $\Pi^D$ :

$$l_{i} + k_{i} = \left(\frac{l_{i}}{k_{i}} + 1\right) k_{i}$$

$$= \left(\frac{1 - \alpha_{i}}{\alpha_{i}} \frac{\kappa_{i}}{\gamma_{i}} + 1\right) \frac{\alpha_{i}}{\kappa_{i}} p_{i} y_{i}$$

$$= \frac{(1 - \alpha_{i}) \frac{\kappa_{i}}{\gamma_{i}} + \alpha_{i}}{\alpha_{i}^{\alpha_{i}} (1 - \alpha_{i})^{1 - \alpha_{i}} \left(\frac{\kappa_{i}}{\gamma_{i}}\right)^{1 - \alpha_{i}} p_{i} y_{i}}.$$
Plug in (4)
$$\Pi^{D} = p_{i} y_{i} - r \frac{(1 - \alpha_{i}) \frac{\kappa_{i}}{\gamma_{i}} + \alpha_{i}}{p_{i}} p_{i} y_{i}}.$$
Plug in  $\Pi^{P}_{i} = 0$ 

$$I^{D} = p_{i}y_{i} - r \frac{(1 - \alpha_{i})^{1 - \alpha_{i}}}{\alpha_{i}^{\alpha_{i}}(1 - \alpha_{i})^{1 - \alpha_{i}}} p_{i}y_{i} \qquad \text{Plug in } \Pi_{i}^{P} = 0$$

$$= p_{i}^{-\frac{\sigma}{1 - \sigma}}Y^{\frac{1}{\sigma}} \left[ 1 - \frac{r}{p_{i}} \frac{(1 - \alpha_{i})\frac{\kappa_{i}}{\gamma_{i}} + \alpha_{i}}{\alpha_{i}^{\alpha_{i}}(1 - \alpha_{i})^{1 - \alpha_{i}}} \left[ \frac{1 - \frac{r}{p_{i}}}{\alpha_{i}^{\alpha_{i}}(1 - \alpha_{i})^{1 - \alpha_{i}}} \left( \frac{\kappa_{i}}{\gamma_{i}} \right)^{1 - \alpha_{i}}} \right].$$
Plug in (5)

We can replace  $(\kappa_i, \gamma_i)$  with  $(p_i, \frac{\kappa_i}{\gamma_i})$  for the profit function  $\Pi^D$ , and transform the profit maximization problem into

$$\max_{\kappa_i,\gamma_i} \Pi(p_i, \frac{\kappa_i}{\gamma_i}) = p_i^{-\frac{\sigma}{1-\sigma}} Y^{\frac{1}{\sigma}} \left[ 1 - \frac{r}{p_i} \frac{(1-\alpha_i)\frac{\kappa_i}{\gamma_i} + \alpha_i}{\alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i} \left(\frac{\kappa_i}{\gamma_i}\right)^{1-\alpha_i}} \right]$$
(7)

Solving problem (7) with the first order condition, we can get

$$\kappa_i = \gamma_i = \frac{r}{\sigma};\tag{8}$$

$$p_i = \frac{r}{\sigma \alpha_i^{\alpha_i} (1 - \alpha_i)^{1 - \alpha_i}}.$$
(9)

The Rent Pricing Equation (8) shows that all properties (including residential and commercial in any community *i*) charge the same rent in this simple setting. And the rent pricing rule is that the developer simply charge an markup ( $\sigma^{-1}$ ) over its operation cost (land rent *r* in this setting). The developer's profit margin is  $\sigma^{-1}-1$ , which is coming from its monopoly power in community *i*'s housing supply. In other words, the developer's profit is derived from its ability to explore the consumer's non-perfectly-elastic demand of variety *i*, given the property rent will perfectly pass through to the variety price  $p_i$  shown in Equation (4). More generally, we can prove (proof in technical appendix)

**Theorem 1.** The monopoly developer charges an markup  $(\sigma^{-1})$  over its operation cost (including land rent and any additional cost) to price its property rent.

Here the specific market structure for the housing supply only matters for the specification of the developer's markup. The rent pricing rule still remains the same qualitatively, and this paper's main conclusion still goes through. In the technical appendix, we show this point with a duopoly market structure with one commercial developer and one residential developer.

With price solutions, we can then solve all the quantity variables:

$$y_i = \left(\frac{\sigma \alpha_i^{\alpha_i} (1 - \alpha_i)^{1 - \alpha_i}}{r}\right)^{\frac{1}{1 - \sigma}} \bar{Y}$$
(10)

$$k_i = \left(\frac{\alpha_i}{1 - \alpha_i}\right)^{1 - \alpha_i} \left(\frac{\sigma \alpha_i^{\alpha_i} (1 - \alpha_i)^{1 - \alpha_i}}{r}\right)^{\frac{1}{1 - \sigma}} \bar{Y}$$
(11)

$$l_i = \left(\frac{1-\alpha_i}{\alpha_i}\right)^{\alpha_i} \left(\frac{\sigma \alpha_i^{\alpha_i} (1-\alpha_i)^{1-\alpha_i}}{r}\right)^{\frac{1}{1-\sigma}} \bar{Y}$$
(12)

Community Mix of Residential and Commercial Activities In this static model, all properties can only generate one-period rent income, so the property value is equal to its total rent income. In the similar fashion with the empirical analysis, we define the commercial-ness of community ias its commercial property value share  $\kappa_i k_i / (\gamma_i l_i + \kappa_i k_i)$ . From the producer's profit maximization conditions (2) and (3), we know **Theorem 2.** The commercial-ness of the community is determined by and equal to the single technology parameter  $\alpha_i$ .

#### 2.2 A Benchmark Model with Flood Risk and Perfectly Enforced Flood Insurance

**Flood Risk** After FEMA evaluation, f measure of communities are designated as the flood zones. Without loss of generality, we assign indices in [0, f] to communities in the flood zone, and communities in (f, 1] are non-flood zones. With probability  $P_F$ , a flood will hit the economy, and all communities in the flood zone will suffer the loss. The realization of flood event takes place after the development is completed, and the flood will submerge both residential and commercial properties. So the employed labors and capital have to be relocated for production.

**Flood Insurance** The government provides flood insurance *ex ante* to the flood zone. The premium of the federal flood insurance is  $\pi^*$ , and will provide non-flood farmland (and development) for the flooded producers to restore production.

The flood insurance is mandatory, so all developers in the flood zone have to buy the flood insurance for their residential and commercial properties. The government has a balanced fiscal budget and uses the premium income to pay for relocation cost when the flood realizes. The fiscal balance condition is

$$\pi^* \int_0^f (l_i + k_i) di = P_F \int_0^f r(l_i + k_i) di$$

which implies

$$\pi^* = r P_F \tag{13}$$

Land Rent With heterogeneous flood conditions across different communities, the land rents will be heterogeneous in the economy. Specifically, land rent remains the same at r for non-flood-zone communities. In the flood zones, however, the rent of land will drop, since the farmers' opportunity cost depreciates due to the flood risk. When the flood comes, the farmers will get 0 for agricultural use. This implies

$$r_{i \in [0,f]} = r_F = r(1 - P_F) \tag{14}$$

$$r_{i\in(f,1]} = r_{NF} = r \tag{15}$$

**Resolving Developer's Problem** In the flood zone, the developers' operation cost is the sum of land rent and flood insurance:

$$r_F + \pi^* = r \tag{16}$$

For the communities in the non-flood zone, the developers' operation cost still comprises of only the land rent:  $r_{NF} = r$ . Therefore, the effective operation cost for the developers is the same for all communities as in the setting without flood risk. With Theorem 1, the residential and commercial rents are still

$$\kappa_{i \in [0,f] \cup (f,1]} = \gamma_{i \in [0,f] \cup (f,1]} = \frac{r}{\sigma}$$
(17)

as in the baseline setting without flood risk.

**Optimal Resource Allocation** With the perfect insurance and the same factor prices in (17), the producers and consumers are solving the same problem as if there were no flood risk. Therefore, we can conclude

**Theorem 3.** In an economy with perfectly insurable flood risk, the optimal resource allocation can be restored with mandatory flood insurance.

Quantitatively, the labor and capital (or business) allocation is at the same level as in the baseline

setting

$$y_{i\in[0,f]\cup(f,1]} = \left(\frac{\sigma\alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i}}{r}\right)^{\frac{1}{1-\sigma}}\bar{Y}$$

$$\tag{18}$$

$$k_{i\in[0,f]\cup(f,1]} = \left(\frac{\alpha_i}{1-\alpha_i}\right)^{1-\alpha_i} \left(\frac{\sigma\alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i}}{r}\right)^{\frac{1}{1-\sigma}} \bar{Y}$$
(19)

$$l_{i\in[0,f]\cup(f,1]} = \left(\frac{1-\alpha_i}{\alpha_i}\right)^{\alpha_i} \left(\frac{\sigma\alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i}}{r}\right)^{\frac{1}{1-\sigma}} \bar{Y}$$
(20)

### 2.3 A Model with Government Aid and Imperfect Flood Insurance Enforcement

This parts deviates from the setting with perfectly enforcement of the flood insurance, and builds in the flood risk management framework employed by the US government.

Flood insurance and Government Aid The government provides flood insurance *ex ante*, and government aid *ex post* to the flood zone. The flood insurance works in the same way as in the previous setting. It costs an *ex ante* premium  $\pi$ , and will provide non-flood farmland (and development) for the flooded producers to restore production.

The flood insurance is only mandatory for residential properties. For the uninsured commercial properties, the government will provide full-compensation government aid if the flood hits. The existence of government aid disincentives the developer to buy any flood insurance for their commercial properties (free riding problem).

The government funds both flood insurance payout and government aid with their premium income:

$$\pi \int_0^f l_i di = P_F \int_0^f r(l_i + k_i) di$$

which implies

$$\pi > \pi^* = r P_F \tag{21}$$

**Resolving Developer's Problem** In the flood zone, the developers' operation cost for residential property is  $(1 - P_F)r + \pi$  and the operation cost for commercial property is  $(1 - P_F)r$ . According to

Theorem 1, the developers will apply the same markup to their operation cost, which implies:

$$\kappa_{i(F)} = \frac{(1 - P_F)r}{\sigma} < \kappa_{i(NF)} \tag{22}$$

$$\gamma_{i(F)} = \frac{(1 - P_F)r + \pi}{\sigma} > \gamma_{i(NF)}$$
(23)

which implies

$$k_{i(F)} = \frac{k_{i(NF)}}{\left[\underbrace{\left(1 - P_F + \frac{\pi}{r}\right)^{1 - \alpha_i} (1 - P_F)^{\alpha_i}}_{\text{composite developing cost}}\right]^{\frac{\sigma}{1 - \sigma}} (1 - P_F)}$$
(24)

 $\operatorname{So}$ 

$$lnk_{i(F)} - lnk_{i(NF)} = \underbrace{-\frac{\sigma}{1-\sigma}ln\left[\left(1-P_F+\frac{\pi}{r}\right)^{1-\alpha_i}(1-P_F)^{\alpha_i}\right]}_{(\downarrow) \text{ flood effect}} + \underbrace{ln\frac{1}{1-P_F}}_{(\uparrow) \text{ free riding effect}}$$
(25)
$$= \frac{\sigma}{1-\sigma}ln\frac{1-P_F+\frac{\pi}{r}}{1-P_F}\alpha_i - ln(1-P_F) - \frac{\sigma}{1-\sigma}ln(1-P_F+\frac{\pi}{r})$$
(26)

**Parametrization** If the developer's profit margin is 10%, flood risk is 1%, and flood insurance premium is 2%,

$$lnk_{i(F)} - lnk_{i(NF)} \approx 0.2\alpha_i - 0.089$$

For commercial-ness below 0.445, businesses move out of the community; for commercial-ness above 0.445, businesses move into the community. As shown in Figure 2, the movement of the businesses is monotonically determined by the commercial-ness of the location.

**Theorem 4.** After a community is designated as a flood zone from non-flood zone, the business movement is an increasing function with the commercial-ness of the community.

# 3 Data and Empirical Setting

This section describes the data and empirical strategy for this study.

#### 3.1 Data

The flood maps come from FEMA's Flood Map Service Center. We download the map in 2020 that is publicly available. And we have acquired the historical flood map in 1999 (Q3) and 2011 archived in the Univesity of Wisconsin-Milwaukee Library. These maps assign flood zone designations at the polygon level and are used to determine national flood insurance premiums. The Special Flood Hazard Zones identified from these maps represent areas that will be inundated by a flood event with a 1% chance in any given year. In our analysis, we consider areas in Special Flood Hazard Zones as flood zones (areas with flood risk). The flood maps does not cover ever corner of the nation, and the covered areas in general expands over years. In our analysis, we only focus on the areas that are covered by flood maps in all three years (1999, 2011, 2020), as shown in Figure 3.

We obtain information on establishments from the Infogroup historical business database, a longitudinal panel of establishments from 1997 to 2020 constructed by Infogroup. Infogroup identifies establishments using yellow pages, phone books and newspapers, and incorporates phone verification for the entire database. The dataset reports the number of employees, industry classification, sales, and precise location of each establishment. Most importantly for this analysis, the data track the precise latitude and longitude of the businesses, so we can assign them to the small polygons in the flood map. The previous studies have documented Infogroup dataset is more comprehensive than than public records, like County Business Pattern, since it is more likely to capture non-employer firms and small chain establishments (Meltzer et al., 2021).

Our commercial real estate valuation comes from CoreLogic's property sales and characteristics data, which compiles deed transaction records and property tax roll information from US County assessors and recorder offices.

#### 3.2 Discretization of the Maps

The flood zone polygons do not coincide with any administrative geographies, even for small units, like ZIP code or census tract. And we need to aggregate the businesses from its precise locations to a certain area to summarize business growth. We discretize the US map into very small grids. We start with grid unit 0.06 latitude degree long (approximately 0.5 mile) and 0.06 longitude degree wide. We then compute the population living in this grid (from 2010 census), and cut the grid equally into 4 smaller grids if the containing population is above 100. We iterate this process till all grid contains population smaller than 100. Figure 4(b) illustrates the discretized map in Miami.

# 4 Empirical Strategy and Results

### 4.1 Empirical Strategy

This study will focus on the events that the flood risk surpasses 1% annually, making the local area designated as Special Flood Hazard Areas (SFHA, *aka.* flood zones), and analyze its impact on business relocations. Specifically, we will compare the US flood maps published in 1996 and 2011 to trace the areas that changed from non-flood zones to flood zones.

Figure 5 illustrates this idea with Miami as an example. Panel (a) plots the flood map of Miami in 1999 with the discretized grids, and panel (b) plots the flood map of Miami in 2011. In both maps, the purple area denotes the designated flood zone, and pink area is the non-flood zone. Our strategy is to overlay these two flood map snapshots and generate the figure in panel (c). There are four different groups of grid areas: (1) the purple area is in flood zone in both 1999 and 2011; (2) the pink area is in non-flood zone in both 1999 and 2011; (3) the red area changes from flood zone in 1999 to non-flood zone in 2011; and (4) the dark blue area changes from non-flood zone in 1999 to flood zone in 2011. The dark blue areas are our treated areas, and we will match them with comparable control areas in the same neighborhood.

**Identification Strategy** A naïve strategy is just to compare the treated areas (dark blue) with the rest of the places (purple, pink and red). Panel (a) of Figure 7 follows this naïve strategy, and shows the normalized business counts (in *log* scale) in the four different areas. A problem immediate jumps out for this naïve strategy: the pretrends for different groups are not paralleled.

Even though flood risk change is largely exogenous, different natural amenities between the treated areas and control areas might also confound the comparability of the treated and control groups. Specifically, the treated area tends to be the marginal place with true annual flood probability close to 1%, and its assessed flood risk happened to cross this threshold between in 2000s. It might have drastically different amenities with areas that are always in the flood zone, e.g. beaches or islands. To solve this problem, we take advantage of the continuous flood risk measure assessed

by First Street Foundation, and narrow our focus only on the marginal areas to achieve a cleaner identification strategy.

The drawback of the FEMA flood map is that it does not provides the level of flood risk for each location. Thus, we supplement the flood map with flood risk score data provided by the First Street Foundation (FSF) in the year of 2020. The benefit of the FSF risk score is that it gives us a continuous measure of flood risk with 10 discrete scores. Even though the flood risk score is achieve in the year of 2020, we believe it still captures the true underlying flood risk since the natural flood risk is stable in a 20-year window. Panel (a) of Figure 6 plots the FSF flood risk scores for the city of Miami, which illustrate the problem of the comparison naïve strategy. The coastal areas and islands have flood risk score close 10 (with "Extreme" flood risk), but are used as control areas for the treated areas with flood risk close to the threshold probability 1%.

Our identification strategy is to exclude areas with underlying flood risk far from this threshold probability. Panel (b) of Figure 6 plots the legend for the FSF flood risk score. The FSF scores a location based on two factors: probability of a flood and the depth of flooding conditional on flood. FEMA flood zone designation, however, only takes the probability of flood into consideration. To make the two flood risk assessment comparable, we only look at the last row (flood depth 0" to 3") in FSF flood risk matrix. Another difference between these two flood risk assessment system is that FEMA 1% threshold is quotes as the annual flood probability, while FSF quotes 30-year cumulative flood probability. We assume the flood events are independent across years so we can convert the 30-year cumulative flood probability back to annual flood probability. For example, the threshold 30year probability (27%) between FSF score 4 and 5 can be converted to 1% annual flood probability, which coincides with the FEMA flood zone threshold annual flood probability. Similarly, 12% 30-year flood probability corresponds to 0.5% annual flood probability and 47% 30-year flood probability corresponds to 2% annual flood probability.

We restrict our sample to a small window around the FSF risk score 5 to focus on the marginal flood risk areas. Panel (b) of Figure 7 narrows the focus to the areas with flood risk score between 3 and 7, and we can see the pretrends are more paralleled, compared with the non-restricted sample in panel (a). Panel (c) further restricts the sample to flood risk score between 4 and 6, and the parallel trends are even better established. We then employ the classic difference-in-differences regression framework to formalize our analysis, with following specification:

$$y_{qt} = \beta Post_t * Treat_q + \mu_q + \lambda_t + \epsilon_{qt}$$
<sup>(27)</sup>

where the sample is restricted to areas with FSF flood score between 4 and 6, and no flood zone designation change between 2011 and 2020. Grid g is defined as treated if it was not in flood zone in 1999, but is designated as a flood zone in 2011.  $\mu_g$  is the grid fix effect and  $\lambda_t$  is the year fixed effect. For regressions, we cluster the standard errors at the county level. Clustering at the county level allows for arbitrary within-county correlation and assumes that there is no cross-county correlation.

#### 4.2 Heterogeneous Effect on Business Growth

Figure 8 shows businesses grow 10% faster in the central business districts in the 10-year period after they are designated as flood zones, which manifests the pulling force of businesses due to the free riding problem of the flood insurance. We then move to areas with less commercial properties and find the relative business growth decreases due to the increasing pushing force from the residential side. Ultimately, the push factor dominates in the residential communities, and businesses are flowing out after the designation of flood zone.

This is consistent with the theoretical prediction in Theorem 4, which is visualized in Figure 2.

#### 4.3 Average Effect on Business Growth

Table 2 shows that overall, businesses grow 1.9% slower in areas designated as flood zones. In general equilibrium, the movement of businesses is also affected by the other opposing force—the rise of residential rent, and the linkage is through the complementarity between commercial and residential activities (*e.g.* labor source, customer base, *ect.*). The free riding of the commercial property owners on the flood risk management system would inflate the insurance cost of residential property owners, who are enforced to pay. The inflated operation cost will be passed through to residential rent, which drives away residential tenants. This is a push factor for the local businesses, and the net business movement depends on the relative strength of the pull factor (free riding effect) and push factor (shrinking residential base). Our model also show the relative strength of the two factors determined

only by the commercial share of the location. Therefore, the model predicts businesses will move out of flood zones with low share of commercial properties (residential communities), and move into flood zones with high share of commercial properties (central business districts). Overall, the businesses are misallocated in either case, and welfare loss is incurred.

# 5 Conclusion

Flood risk poses a widespread and growing threat to economic activities in the US. Under perfect market design, both residential households and commercial businesses would gradually move out of the flood zones, which agrees with socially optimal resource allocation. The current flood assistance system, however, fails to enforce the flood insurance take-up by the commercial property owners, while providing government aid to these uninsured properties ex post. In other words, the commercial property owners in the flood zones are free riding the federal flood assistance system and takes an advantageous position in the rental market. The low commercial rent, counterproductively, attracts businesses back to the flood zone, which leads to resource misallocation. This project looks back at the past two decades (1999-2020) and empirically shows businesses grow 10% faster in the central business districts after they are designated as flood zones, which manifests the pulling force of businesses due to the free riding problem in the current flood assistance system.

To illustrate misallocation effect of the imperfect flood insurance enforcement, this paper first builds a spatial model of the current flood risk management framework, and shows it generates a force that drives more businesses into the flood zone. In this static model, flood risk is a negative productivity shock but can be perfectly insured by flood insurance. In addition to providing the flood insurance, the government also gives aid to the property owners if they are hit by floods. A free riding problem arises in this setting—the property owners are not willing to cover their property with flood insurance, and will not purchase flood insurance unless they are forced to do so. We assume insurance coverage is only enforced for residential properties, but not for commercial properties. As a result, commercial rent drops with the increase of (perceived) flood risk, which motivates the businesses to move into the flood zone. This partial-equilibrium effect is the pull factor for the local businesses.

In general equilibrium, the movement of businesses is also affected by the other opposing force the rise of residential rent, and the linkage is through the complementarity between commercial and residential activities (*e.g.* labor source, customer base, *ect.*). The free riding of the commercial property owners on the flood risk management system would inflate the insurance cost of residential property owners, who are enforced to pay. The inflated operation cost will be passed through to residential rent, which drives away residential tenants. This is a push factor for the local businesses, and the net business movement depends on the relative strength of the pull factor (free riding effect) and push factor (shrinking residential base). Our model also show the relative strength of the two factors determined only by the commercial share of the location. Therefore, the model predicts businesses will move out of flood zones with low share of commercial properties (residential communities), and move into flood zones with high share of commercial properties (central business districts). Overall, the businesses are misallocated in either case, and welfare loss is incurred.

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# Tables

Dependent Variable	Log(Business Count)				
Marginal Area (Risk Score)	[2, 8]	[3, 7]	[4, 6]	[4.5, 5.5]	
$Treated*(Year \ge 2011)$	-0.0049	-0.0085	-0.0209**	-0.0237**	
	(-0.88)	(-1.37)	(-2.50)	(-2.43)	
Year FE	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	
Grid FE	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	
$R^2$	0.83	0.83	0.83	0.83	
Obs.	$12,\!933,\!792$	$7,\!609,\!824$	$3,\!929,\!616$	$1,\!520,\!304$	
Pop.	$21,\!366,\!778$	$12,\!419,\!132$	$6,\!389,\!913$	$2,\!415,\!526$	
Grids	$538,\!908$	$317,\!076$	163,734	$63,\!346$	

Table 1 Average Effect on Business Growth After Flood Zone Designation

*Note*: This is DID regression table with specification 27. The sample is restricted to areas with FSF flood risk score between 4 and 6, and no flood zone designation change between 2011 and 2020. An area is defined as treated if it was not in flood zone in 1999, but is designated as a flood zone in 2011. Grid fixed effects and year fixed effects are controlled. Standard Errors cluster at the county level.

Dependent Variable	Log(Business Count)			
Marginal Area (Risk Score)	[2, 8]	[3, 7]	[4,  6]	[4.5, 5.5]
$\boxed{\text{Treated}^*(\text{Year} \ge 2011)}$	-0.0036	-0.0070	-0.0194**	-0.0222**
	(-0.65)	(-1.12)	(-2.26)	(-2.24)
Year FE	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
Grid FE	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
$R^2$	0.82	0.82	0.82	0.83
Obs.	$12,\!261,\!120$	$7,\!152,\!528$	$3,\!679,\!944$	$1,\!427,\!304$
Pop.	$20,\!189,\!203$	$11,\!621,\!262$	$5,\!953,\!731$	$2,\!255,\!174$
Grids	510,880	298,022	$153,\!331$	$59,\!471$

Table 2Average Effect on Business Growth After Flood Zone Designation (Excluding Improving<br/>Areas)

*Note*: This is DID regression table with specification 27. The sample is restricted to areas with FSF flood risk score between 4 and 6, and no flood zone designation change between 2011 and 2020. An area is defined as treated if it was not in flood zone in 1999, but is designated as a flood zone in 2011. Grid fixed effects and year fixed effects are controlled. Standard Errors cluster at the county level.

	log(Business Count)
Commercial Share in [0, 25%]	-0.0257***
	(-2.91)
Commercial Share in $[25\%, 50\%]$	0.0054
	(0.23)
Commercial Share in [50%, 75%]	0.0269
	(0.83)
Commercial Share in $[75\%, 100\%]$	$0.0971^{***}$
	(3.53)
Year FE	$\checkmark$
Grid FE	$\checkmark$
$R^2$	0.84
Obs.	3,062,208

Table 3 Heterogeneous Effect: Businesses Movement Across Ares with Different Commercial Property Share

*Note*: The sample is restricted to areas with FSF flood risk score between 4 and 6, and no flood zone designation change between 2011 and 2020. An area is defined as treated if it was not in flood zone in 1999, but is designated as a flood zone in 2011. Grid fixed effects and year fixed effects are controlled. Standard Errors cluster at the county level.

# Figures



Figure 1 Flood Insurance Coverage in 2020



Figure 2 Model Prediction of Business Movement After Flood Zone Designation



Figure 3 Focused Areas (Covered in both 1999, 2011 and 2020) Note: This figure plots the areas that are covered by flood maps in all three years (1999, 2011, 2020).



Figure 4 Illustration of Discretizing Maps Into Grids (Miami as an Example)







(c) Miami, 1999 vs 2011



Figure 5 Change of Flood Zone Designation (Miami as an Example)

Note: Panel (a) plots the flood map of Miami in 1999 with the discretized grids, and panel (b) plots the flood map of Miami in 2011. In both maps, the purple area denotes the designated flood zone, and pink area is the non-flood zone. Panel (c) overlays these two flood map snapshots, in which there are four different groups of grid areas: (1) the purple area is in flood zone in both 1999 and 2011; (2) the pink area is in non-flood zone in both 1999 and 2011; (3) the red area changes from non-flood zone in 1999 to flood zone in 2011; and (4) the dark blue area changes from flood zone in 1999 to non-flood zone in 2011.

(a) Flood Risk Score Map of Miami



(b) Flood Risk Score Legend Flood Factor Matrix



Figure 6 First Street Foundation Flood Risk Score (Miami as an Example)



(a) Normalized Business Counts for Different Groups without Restriction

(b) Normalized Business Counts for Different Groups with Flood Risk Score in [3,7]



(c) Normalized Business Counts for Different Groups with Flood Risk Score in [4,6]



Figure 7 Illustration of Identification Strategy



Figure 8  $\,$  Heterogeneous Effect: Businesses Movement Across Ares with Different Commercial Property Share

# Appendix A Technical Appendix

### A.1 A Model with Duopoly Developers

In community *i*, one commercial developer rents land from the land owner to develop commercial properties, and one residential developer rents land to develop residential properties. The commercial developer sets commercial rent at  $\kappa_i$ , and the residential developer sets residential rent at  $\gamma_i$ .

**Developer's problem** The developer collects rent from the local producer, and its profit is the surplus after paying for the rent of the land.

The commercial developer's profit maximization problem is

$$\max_{\kappa_i} \Pi_c(\kappa_i) = (\kappa_i - r)k_i, \tag{A1}$$

And the residential developer's profit maximization problem is

$$\max_{\gamma_i} \ \Pi_r(\gamma_i) = (\gamma_i - r)l_i.$$
(A2)

From the local producer's profit maximization we know

$$k_{i} \stackrel{(3)}{=} \frac{\alpha_{i}}{\kappa_{i}} p_{i} y_{i}$$

$$\stackrel{(5)}{=} \frac{\alpha_{i}}{\kappa_{i}} p_{i}^{-\frac{\sigma}{1-\sigma}} \bar{Y}$$

$$\stackrel{(4)}{=} \left(\frac{\gamma_{i}}{1-\alpha_{i}}\right)^{-\frac{(1-\alpha_{i})\sigma}{1-\sigma}} \left(\frac{\kappa_{i}}{\alpha_{i}}\right)^{-\frac{1-(1-\alpha_{i})\sigma}{1-\sigma}} \bar{Y}$$

$$l_{i} \stackrel{(2)}{=} \frac{1-\alpha_{i}}{\alpha_{i}} \frac{\kappa_{i}}{\gamma_{i}} k_{i}$$

$$= \left(\frac{\gamma_{i}}{1-\alpha_{i}}\right)^{-\frac{(1-\alpha_{i})\sigma}{1-\sigma}-1} \left(\frac{\kappa_{i}}{\alpha_{i}}\right)^{-\frac{\alpha_{i}\sigma}{1-\sigma}} \bar{Y}$$

We can solve the derivatives:

$$\begin{split} \frac{\partial k_i}{\partial \kappa_i} &= \left(-1 - \frac{\alpha_i \sigma}{1 - \sigma}\right) \frac{k_i}{\kappa_i} \\ \frac{\partial l_i}{\partial \gamma_i} &= -\frac{1 - \alpha_i \sigma}{1 - \sigma} \frac{l_i}{\gamma_i} \end{split}$$

Solving the problem (A1) and (A2) with the first order conditions:

$$0 = \frac{\partial \Pi_c}{\partial \kappa_i} = k_i + (\kappa_i - r) \frac{\partial k_i}{\partial \kappa_i} = \frac{k_i}{\kappa_i} \left[ \kappa_i - (1 + \frac{\alpha_i \sigma}{1 - \sigma})(\kappa_i - r) \right];$$
  
$$0 = \frac{\partial \Pi_r}{\partial \gamma_i} = l_i + (\gamma_i - r) \frac{\partial l_i}{\partial \gamma_i} = \frac{l_i}{\gamma_i} \left[ \gamma_i - \frac{1 - \alpha_i \sigma}{1 - \sigma}(\gamma_i - r) \right];$$

we can get

$$\kappa_i = \left[1 + \frac{(1-\sigma)(1-\alpha_i)}{\alpha_i}\right] \frac{r}{\sigma}$$
(A3)

$$\gamma_i = \left[1 + \frac{\alpha_i(1-\sigma)}{1-\alpha_i}\right] \frac{r}{\sigma} \tag{A4}$$

**Discussion** A few observations:

- 1. In this economy, the rent of all properties are priced with markup on top of the developing cost (land rent).
- 2. The profit margin of both commercial and residential properties is above the profit margin in an economy with one monopolistic mixed-developer in each community  $\sigma^{-1} 1$ .
- 3. The commercial-ness of community *i* is only determined by technology parameter  $\alpha_i$ .

$$\frac{k_i}{l_i} = \frac{\alpha_i}{1-\alpha_i} \gamma_i \frac{1}{\kappa_i}$$

(all three factors are increasing with  $\alpha_i$ ).

4. As the commercial-ness of a community increases, the residential rent is increasing while the commercial rent is decreasing.

**Resolving Developer's Problem with Flood Risk and Imperfect Enforcement of Flood Insurance** In the flood zone, the developers' operation cost for residential property is  $(1 - P_F)r + \pi$ and the operation cost for commercial property is  $(1 - P_F)r$ . According to Theorem 1, the developers will apply the same markup to their operation cost, which implies:

$$\kappa_{i(F)} = \left[1 + \frac{(1-\sigma)(1-\alpha_i)}{\alpha_i}\right] \frac{(1-P_F)r}{\sigma} < \kappa_{i(NF)}$$
(A5)

$$\gamma_{i(F)} = \left[1 + \frac{\alpha_i(1-\sigma)}{1-\alpha_i}\right] \frac{(1-P_F)r + \pi}{\sigma} > \gamma_{i(NF)}$$
(A6)

which implies

$$k_{i(F)} = \frac{k_{i(NF)}}{\left[\underbrace{\left(1 - P_F + \frac{\pi}{r}\right)^{1-\alpha_i} (1 - P_F)^{\alpha_i}}_{\text{composite developing cost}}\right]^{\frac{\sigma}{1-\sigma}} (1 - P_F)}$$
(A7)

 $\mathbf{So}$ 

$$lnk_{i(F)} - lnk_{i(NF)} = \underbrace{-\frac{\sigma}{1-\sigma}ln\left[\left(1-P_F + \frac{\pi}{r}\right)^{1-\alpha_i}(1-P_F)^{\alpha_i}\right]}_{(\downarrow) \text{ flood effect}} + \underbrace{ln\frac{1}{1-P_F}}_{(\uparrow) \text{ free riding effect}}$$
(A8)
$$= \frac{\sigma}{-ln\frac{1-P_F + \frac{\pi}{r}}{\sigma}\alpha_i - ln(1-P_F) - \frac{\sigma}{-ln(1-P_F + \frac{\pi}{r})}}$$
(A9)

$$= \frac{\sigma}{1-\sigma} ln \frac{1-P_F + \frac{\pi}{r}}{1-P_F} \alpha_i - ln(1-P_F) - \frac{\sigma}{1-\sigma} ln(1-P_F + \frac{\pi}{r})$$
(A9)