

The Hairy Premium

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First Draft: March 14, 2023

This Draft: January 4, 2024

Abstract

This paper explores a puzzle originating from the market's persistent tendency to overestimate future spot rates, as evidenced by consistently overshooting forward rates. This results in unusually high positive long-term returns on net zero investments. We introduce the Hairy premium to quantify this puzzle. Since the 1990s, the 10-year US Hairy premium has averaged 3% p.a., ranging between 4.8% maximum and 1.1% minimum, consistently above 0, indicating asymmetric risk-reward. The Hairy premium spans over a century and it is a global phenomenon across G11 countries. About 45% is explained by a single global factor. While 14% of its variance is attributable to conventional term premiums, unlike them, it exhibits countercyclical dynamics, relating positively to recessions and inflation expectations, thus providing hedge during bad times. We show that a general equilibrium model with persistent degree of short-termism, motivated by interest rate swaps market structure and recent survey evidence, can explain the existence and dynamics of the Hairy premium.

JEL classification: E4, E52, G1, G2, G5, H63

Keywords: Interest rates, Expectations, Term premium, Inflation, Expectations, Uncertainty

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1 Introduction

Interest rates are a critical variable for both borrowers and investors, and the choice between fixed-rate and floating-rate debt or investment can significantly impact returns. Borrowers and investors, such as households, banks, corporations, and sovereigns, rely on forward curves to estimate the interest rates they will pay or receive for the duration of their projects. Figure 1, titled “Hairy Graph”, plots the forward curves that illustrate the degree of accuracy with which the market can anticipate short-term floating interest rates over the next five years at any given point in time.¹

[Insert Figure 1 about here]

The blue line represents the short-term floating spot rate, three-month LIBOR, while the grey lines represent the floating rate forward curves, depicting the market-predicted trajectory of the floating rate over the next five years. It is clear that the market consistently overestimates the trajectory of the floating rate. When the market underestimates the trajectory of the floating rate, it usually does so preceding and during periods of monetary policy tightening, when interest rates are raised sharply, which are historically disproportionately rare.

Motivated by the observations in this figure, we identify, measure, and analyze a new premium which we term the “Hairy Premium” (HP) capturing the systematic overshoots in the forward curves. The persistent overestimation of forward curves relative to eventually realized rates gives rise to a visual semblance of “hairs” across the time series, hence the term. We argue the magnitude, frequency, and serial correlation of these systematic overshooting patterns implies an additional premium, distinct from conventional term premiums, which has so far gone unrecognized by market participants and academic asset pricing theories.

¹Note that when we use the term “floating rates”, we are referring to any short-term interest rate, including Fed Funds, LIBOR, SOFR, short-term constant maturity rates on government bonds etc. In this study, we don’t favor any particular interest rate since we focus on comparing the difference between the fixed and floating rates of the same interest rate. The different interest rates are also largely interchangeable since they are all used when borrowing and investing and they are all primarily controlled by the FOMC monetary policy.

We show that the previously overlooked Hairy premium leads to significant impacts on returns of debt and derivative instruments tied to interest rate movements or employing discounting factors of the same. The persistent presence of this anomaly compels revising prevailing valuation methodologies to integrate the non-trivial effects of the Hairy premium across asset classes dependent on interest rates and their implying discount factors.

We measure the Hairy premium simply by the annual cost of being in a fixed rate minus the average “effective” cost of being in a floating rate over a specified holding period horizon. This corresponds to a strategy of entering a receive-fixed interest rate swap. Figure 2, for example, depicts the Hairy premium for the 10-year period horizon.²

[Insert Figure 2 about here]

Since the 1990s, borrowers and investors in the US have consistently earned an average 10-year Hairy premium profit of 3%, with a maximum of 4.8% and a minimum of 1.1% per year. Notably, there have been no instances of negative premiums since the 1990s. As a result, over a 10-year holding period horizon, it has always been more advantageous for a borrower (investor) to stick with floating rates (fixed rates) on their loan (bond investment) because they can expect to earn an annual profit equal to the Hairy premium for the duration of the borrowing (investment). Notice the asymmetric risk-reward profile in terms of magnitude because the Hairy premium rarely, if at all for longer maturities, goes negative. Thus, for borrowers, if being in floating debt is the right choice, then the benefit is significant and, if being in floating debt is the wrong choice, the loss is small.

We further demonstrate that the Hairy premium, while conceptually related, is distinct from the conventional term premium as outlined in [Campbell and Shiller \(1991\)](#) (CS) and supported by existing research as in [Fama and Bliss \(1987\)](#); [Cox, Ingersoll, and Ross \(1985\)](#); [Adrian, Crump, and Moench \(2013\)](#) (see discussion in [Dahlquist and Hasseltoft \(2015\)](#) and [Fontaine and Garcia \(2015\)](#)). We show

²The Hairy premium is equal to the difference between the 10-year fixed rate and the average effective floating rate over the next 10 years.

theoretically that the Hairy premium not only includes the conventional CS term premium but also encompasses all future expected term premiums as well as the serial and cross-covariances among them over the holding period horizon. Moreover, the Hairy premium increases as the maturity of the holding period extends. This increase is attributable to the persistent serial correlation of overshoots in upward-sloping forward curves throughout the holding period, implied by the persistence in investor expectations.

In the US, through variance decomposition, we demonstrate that only about 14% of the Hairy premium's variance is attributable to term premiums estimated from [Cox, Ingersoll, and Ross \(1985\)](#) (CIR), [Kim and Wright \(2005\)](#) (KW), [Cochrane and Piazzesi \(2008\)](#) (CPP), and [Adrian, Crump, and Moench \(2013\)](#) (ACM). We also observe that the Hairy premium's dynamics differ from those of other term premiums, with the exception of the CPP term premium. Specifically, the Hairy premium shows countercyclical behavior, increasing during recessions, thereby offering a hedging mechanism in adverse economic times. It is positively correlated with long-term inflation expectations and inversely related to periods of economic policy uncertainties and monetary policy tightening. However, there appears to be no significant link between the Hairy premium and uncertainties about monetary policy or equity markets.

To confirm that the theoretical distinction of the bond term premium and the Hairy premium has empirical support, we show that the pattern of persistent expectations is also apparent in survey data. First, based on the Fed's Survey of Professional Forecasters (SPF), the predictions consistently overshoot the trajectory of interest rates. In addition, recent survey and neuroscientific evidence shows that personal experience and economic conditions have a persistent impact on individual investors' degree of short-termism and present bias ([Sapolsky \(2017\)](#), [Kuralbayeva, Molnar, Rondinelli, and Wong \(2019\)](#), [Citanna and Siconolfi \(2022\)](#)). Finally, since the Hairy premium is given in nominal terms, we also show that the Hairy premium is positively correlated with long-term inflation expectations.

We also analyze the structure of the interest rate swaps market, documenting two end user groups. The first group, net payers of the Hairy premium, include insurers, pension funds and commercial banks ([Klingler and Sundaresan, 2019](#); [Begenau, Piazzesi, and Schneider, 2020](#)). The second group, net receivers of the Hairy premium, include mortgage servicers and relative-value mortgage investors like Fannie Mae and Freddie Mac ([Feldhutter and Lando, 2008](#); [Hanson, 2014](#)).

Motivated by the survey data and the structure of interest rate swaps market end users, we incorporate these features into a general equilibrium model to explain the Hairy premium's magnitude and dynamics. Consistent with survey and neuroscientific evidence, we allow a fraction of investors to be time inconsistent with a persistent level of short-termism (representing the first swaps market end user group - individual investors and mortgage agencies), while the rest are rational with stable time preferences (representing the second swaps market end user group - financial intermediaries), as in [Lundeby and Tancheva \(2023\)](#). We show that even if these retail investors with persistent degree of short-termism hold a small fraction of aggregate wealth, they generate endogenous discount rate risk. Intuitively, when retail investors realize they will be short-termistic in the near future, they tend to overconsume already today in order to equalize marginal rates of substitution. To induce the financial intermediaries to hold less capital, while keeping the same level of profit and costs, discount rates must increase and market value must fall. Thus, financial intermediaries realize that they may face capital outflows exactly when market value is low, so they require a compensation for this extra risk. We show that exposure to this risk increases with the maturity of financial securities, so long-term bonds yield a higher expected return than short-term bonds. This induces persistent overshoots in the trajectory of interest rates and as a result, financial intermediaries require receiving the Hairy premium for fixing interest rates in the future or being in the fixed rather than the floating leg of a swap. To match the nominal level of the Hairy premium we, in addition, incorporate inflation expectations to the model.

One possible alternative explanation for the Hairy premium is that it reflects frictions and con-

straints faced by financial intermediaries in the forwards and interest rate swaps market, such as regulatory, capital, margin, or collateral constraints. These constraints may lead to more expensive interest rate forward and swap contracts, thereby causing overshoot in the trajectory of short-term floating interest rates. However, unlike forward or interest rate swaps, the persistent expectations observed in the survey data are not tradable instruments by intermediaries. This suggests that the Hairy premium is reflective of the expectations and time preferences of professional and retail investors active in forward rate and swap markets, rather than representing intermediary frictions.

It is noteworthy that the Hairy premium, despite being among the highest in the US, is not limited to the US market or confined to recent periods. Instead, it is observed globally since at least the early 1900s, spanning over a century as far back as our data goes. Principal components analysis (PCA) reveals that approximately 45% of the premium can be attributed to a single factor and a cumulative 95% to the first five principal component factors present in all G11 economies.

The Hairy premium stands out from other anomalies because it is evident in tradable assets such as interest rate forwards and swaps. This creates actionable trading strategies for borrowers, investors, and financial intermediaries, enabling them to capitalize on the Hairy premium and generate significant profits by exploiting this anomaly. One such strategy involves borrowing a long-term loan at short-term floating interest rates, such as 3-month LIBOR, until maturity and then invest in long-term fixed-rate assets, like 10-year fixed-rate bonds, until the same matched maturity. Institutional investors can also employ more advanced strategies, such as engaging in a long-term interest rate swap (IRS) until maturity, where they pay the short-term floating interest rate, such as the 3-month LIBOR, while simultaneously receiving the long-term IRS rate, allowing them to secure the Hairy premium as a profit.

The fact that the Hairy premium has remained prevalent for more than a century, as far back as our data allows us to examine, raises an important question. We consider why investors do not trade to the point of eliminating this highly profitable opportunity. Such trading would adjust investor

expectations back to those predicted by rational expectations theory. The persistence of this deviation implies investors have not aligned their beliefs to the level rational expectations theory would forecast. We explore both the limits to arbitrage that prevent eliminating the deviation, as well as the expectation formation process that allows the profitable deviation from rational expectations to endure.

Turning to the practical implications of the Hairy premium for borrowers and investors, one of the fundamental decisions that borrowers and investors must make is whether to lock in a fixed interest rate or opt for a floating interest rate that may change over time. Borrowers benefit from rate certainty and lower risk with fixed rates, but face refinancing risks if rates fall. Investors gain predictable cashflows with fixed rates, but sacrifice flexibility if rates rise. The Hairy premium implies a tradeoff - borrowers pay extra premium for fixed rates that investors receive. Properly accounting for the Hairy premium is key for informed borrower and investor rate locking choices.

Overall, our study contributes new insights to the literature by introducing a new premium capturing the market's persistent tendency to overshoot forwards for over a century globally. We show how this premium impacts borrowers and investors in terms of cash flows, discount rates, and asymmetric risk-reward. These findings have significant implications for households, banks, corporations, and policymakers evaluating monetary policy's effect on long-term interest rates.

2 Related Literature

This study is broadly related to two streams of literature that explore deviations from rational expectations through overshoot of expectations and the effects of borrowing and investing via fixed versus floating interest rate financial instruments.

First, our research is linked to the existing body of literature that examines the time-varying term premium and testing the expectation hypothesis ([Fama and Bliss, 1987](#); [Campbell and Shiller, 1991](#); [Cochrane and Piazzesi, 2005](#); [Adrian, Crump, and Moench, 2013](#)), (see discussion in [Dahlquist and](#)

Hasseltoft (2015), Fontaine and Garcia (2015)). This empirically documented term premium represents the lower bound in our developed Hairy premium measure. Moreover, in particular we are related to the literature exploring departures from rational expectations through overshooting (Daniel, Hirshleifer, and Subrahmanyam, 1998), also focusing on surveys, such as Vissing-Jorgensen (2003); Carroll (2003); Souleles (2004); Mankiw, Reis, and Wolfers (2003); Giordani and Söderlind (2003); Manski (2017); Coibion, Gorodnichenko, and Kamdar (2018). To address this phenomenon and other evidence, earlier models maintained rational belief formation but introduced rational inattention and costs associated with obtaining or processing information (Sims, 2003; Woodford, 2003; Mankiw, Reis, and Wolfers, 2003; Gabaix, 2019). This approach effectively explained sluggish overshoots. However, recent findings, including our study, indicate more profound deviations from rationality, as the expectations of professionals in forecasting, corporate management, consumers, and investors consistently display biases towards overreacting to news (Souleles, 2004; Coibion and Gorodnichenko, 2012, 2015; Greenwood and Shleifer, 2014; Gennaioli, Shleifer, and Vishny, 2015; Gennaioli, Ma, and Shleifer, 2016; Gulen, Ion, and Rossi, 2019; d'Arienzo, 2020).

Additionally, macroeconomic models show that personal experience can shape individual beliefs over time regarding inflation, macroeconomic variables, and risk attitude (Malmendier and Nagel (2011), Malmendier and Nagel (2016), and Collin-Dufresne, Johannes, and Lochstoer (2017)). More recent survey data, demonstrate that personal experience and economic condition also shape attitude towards time (Sapolsky (2017), Kuralbayeva et al. (2019), and Citanna and Siconolfi (2022)). Embedding this evidence into a general equilibrium model, Lundebj and Tancheva (2023) show that persistence in the degree of investor short-termism gives rise to a positive unconditional bond term premium that stems primarily from the real interest rates, rather than inflation expectations, consistent with Albuquerque, Eichenbaum, Luo, and Rebelo (2016) and Gomez-Cram and Yaron (2020). We contribute to this string of literature by documenting how persistent expectations can give rise to a novel puzzle that goes beyond

the typical time-varying bond term premium, which we quantify by estimating the Hairy premium.

Second, our work is related to the body of literature examining the effects of choosing long- or short-term rates for asset financing or investing, and their impact on agents' outcomes, such as credit spreads etc., including [Bicksler and Chen \(1986\)](#); [Duffie and Liu \(2001\)](#); [Longstaff and Schwartz \(1995\)](#); [Titman \(1992\)](#), studies on demand for floating rate debt, timing, and maturity by [Faulkender \(2001\)](#); [Morellec, Valta, and Zhdanov \(2012\)](#); [Smithson and Wakeman \(1988\)](#); [Fenn, Post, and Sharpe \(1996\)](#), borrower characteristics of floating rate credit by [Goldberg and Heuson \(1992\)](#) for corporate borrowers and households ([Blacklow and Wells, 2010](#)), and [Dhillon, Shilling, and Sirmans \(1987\)](#) for mortgages. Studies investigating the outcomes of interest rate management strategies include [Oberoi \(2018\)](#); [Samant \(1996\)](#); [Scott and Peterson \(1986\)](#); [Mommel \(2011\)](#), which find strong correlations between changes in banks' market value and net interest income, regardless of portfolio composition. Elijah suggests that banks using derivatives to manage interest rate risk hold lower levels of capital. Larger banks are more likely to use derivatives, and derivative users tend to have lower systematic risk. Other studies explore the relationship between credit spreads and interest rates, effects of interest rate policy on equity and other debt ([Yang, Davis, and Leatham, 2001](#)), macroeconomic factors ([Carmichael and Handford, 2014](#)), firm characteristics such as credit ratings and leverage ratios, and the use of interest rate swaps to reduce default risk and finance capital investments ([Li and Mao, 2002](#)). [Ho and Saunders \(1983\)](#) demonstrates how banks manage fixed-rate loan commitment risk through a combination of financial futures contracts and a reservation fee.

3 What is the Hairy Premium?

3.1 Defining the Hairy Premium

The Hairy premium is a measure that we developed to quantify the magnitude, frequency, and serial correlation of the overestimations in forwards of the trajectory of the floating rate for the a specified holding period. We compute it by subtracting the average annual “effective” cost of being in a short-term floating rate, such as a 3-month rate, from the annual cost of being in a long-term fixed rate, such as a 10-year fixed rate, over the specified holding time, such as a 10-year period.

The Hairy premium is not just a formulated concept here; it can be effectively put into practice through a buy-and-hold till maturity trading strategy. To implement this strategy, one way is to engage in an interest rate swap (IRS) that pays a variable short-term interest rate, like the 3-month LIBOR and receives a fixed long-term rate, such as a 10-year fixed rate, holding the position until the IRS contract reaches maturity. It is important to note that this approach follows the conventional structure of a plain vanilla IRS, and the IRS market itself represents the largest and most liquid derivatives market globally. In Appendix A.1, the diagram depicts the Hairy premium cashflows derived from those of an IRS swap, as well as how the trading strategy is used to exploit the Hairy premium.

In addition to the aforementioned strategy, there is another approach to earning and benefiting from the Hairy premium. This involves borrowing a long-term loan at a short-term floating interest rate, such as the 3-month LIBOR, and investing it in a riskless long-term fixed-rate asset until the same matched maturity as the borrowed loan. By employing this strategy, one can naturally earn the Hairy premium (see Appendix A.2 for cashflow diagrams).

To formally define the Hairy premium, we first introduce some terminology. Let $r_t^{(N)}$ denote the annualized fixed interest rate one would receive for an N -term holding period. Further, let $r_t^{(1)}$ represent the annualized 1-period return on the short-term rate (specifically the 3-month rate). We also define the

“effective rate”, which is the average compounded rate an investor would pay annually if they stayed in the short-term 3-month rate and rolled it over risklessly for the entire N -term holding period. With these definitions in place, we can now define the N -term holding period annualized Hairy premium $HP_t^{(N)}$ using realized rates at time N as follows:

$$\begin{aligned} HP_t^{(N)} &= \exp \left\{ \frac{1}{N} \left(N \ln(1 + r_t^{(N)}) - \sum_{n=0}^{N-1} \ln(1 + r_{t+n}^{(1)}) \right) \right\} \\ &= \exp \left\{ \frac{1}{N} \left(N \tilde{r}_t^{(N)} - \sum_{n=0}^{N-1} \tilde{r}_{t+n}^{(1)} \right) \right\}, \end{aligned} \quad (1)$$

where the tilde indicates log returns. In essence, the Hairy premium represents the difference between the N -term annually compounded long-term fixed interest rate and the annualized effective rate resulting from rolling short-term rates over the same N -term holding period.

3.2 The Hairy Premium, the Expectations Hypothesis, and the Term Premium

The existence of the Hairy premium is a clear violation of the well known expectations hypothesis (“EH”), which states that the expected N -term return on an investment in a series of consecutive one-period bonds should be equal to the (certain) N -term return on a N -term bond. This implies that the annualized N -term long yield should be a geometric average of the expected short yields over the following N periods:

$$\left(1 + r_t^{(N)}\right)^N = \left(1 + r_t^{(1)}\right) \left(1 + E_t \left(r_{t+1}^{(1)}\right)\right) \dots \left(1 + E_t \left(r_{t+N-1}^{(1)}\right)\right) + c, \quad (2)$$

where E_t is the expectation conditional on the time t information set. While the EH offers a straightforward and intuitively appealing explanation of the yield curve, it overlooks the element of interest rate risk. Unless calculated until maturity, the actual return on a long-term bond is uncertain, prompting investors to seek compensation for this risk. The constant “term premium”, denoted as c , encompasses

this compensation and other factors that represent deviations from the strict EH.³

Extensive research has demonstrated the existence of a time-varying rather than a constant term premium, which has been investigated using numerous EH tests. This body of literature includes, for instance, [Fama \(1984\)](#); [Fama and Bliss \(1987\)](#); [Campbell and Shiller \(1991\)](#) (“CS”); [Cochrane and Piazzesi \(2005\)](#) (see discussion in [Dahlquist and Hasseltoft \(2015\)](#) and [Fontaine and Garcia \(2015\)](#)). Despite sharing a common underlying concept, the term “term premium” in this literature encompasses multiple distinct definitions. One commonly used definition is the expected holding period return on an N -period bond held for one period minus the holding period return on a one-period short-term bond (as in CS). We refer to this as the “term premium”.⁴

$$TP_t^{(N)} = E_t \left[R_{t,t+1}^{(N)} - R_{t,t+1}^{(1)} \right], \quad (3)$$

Alternative definitions yield different term premiums, but it is noteworthy that the term premiums in the empirical literature to our knowledge so far are represented, modeled, and tested as one-period returns since they presume that successive one-period returns are independently and identically distributed.

Importantly, in the following subsection, we show analytically how the term premium differs from our Hairy premium. While these premiums generally seem as similar concepts, their specific quantification and dynamics vary significantly. As a result, they carry distinct interpretations and implications for borrowers and lenders.

3.3 The Hairy Premium vs. The Term Premium

We now focus on demonstrating the distinctions between the term premium and our Hairy premium.

First, note that the term premium $TP_t^{(N)}$ from CS in equation (3) represents a single one-period return.

³The c reflects a constant term premium, that is a predictable excess return on the long-term T -period bond over the short term 1-period bond.

⁴Online Appendix A describes the relation to other definitions, such as the the “forward term premium” proposed by [Cochrane and Piazzesi \(2005\)](#) (“CP”), which is the forward rate minus the expected future spot rate.

In contrast, our Hairy premium $HP_t^{(N)}$ in equation (1) is an annualized return over an N -term holding period, employing a buy-and-hold strategy until reaching maturity.

Second, the term premium utilized by CS involves a strategy of purchasing and subsequently selling a long-term bond after a single period, periodically rebalancing until reaching maturity N . This differs from an annualized hold-to-maturity return over the entire holding period until maturity N in the Hairy premium. Importantly, when buying and selling the long-term bond, the one-period return is influenced not only by the interest rate risk but also by the leverage or magnification effect resulting from the duration of the long-term bond, which the term premium compensates for. This distinction does not apply to the Hairy premium since the strategy yielding it does not involve rebalancing the long-term bond, but rather holding it until maturity.

Third, the Hairy premium does not assume that expected returns within the buy-and-hold strategy exhibit are independently and identically distributed. To illustrate this and establish the relationship between our Hairy premium and the conventional term premium from CS, it is convenient to first express the gross holding period returns in terms of the prices at t and $t + 1$ and use the fact that the yield represents the interest rate that justifies the quoted price:

$$R_{t,t+1}^{(N)} = \frac{P_{t+1}^{(N-1)}}{P_t^{(N)}} = \frac{(1 + r_t^{(N)})^N}{(1 + r_{t+1}^{(N-1)})^{N-1}} \quad (4)$$

$$R_{t,t+1}^{(1)} = 1 + r_t^{(1)}. \quad (5)$$

To keep the expressions parsimonious, we can then alternatively express the term premium (eq. 3)

in terms of log returns by substituting equations (4) and (5) as follows:

$$\begin{aligned}
\widetilde{TP}_t^{(N)} &= E_t \left[\ln \left(R_{t,t+1}^{(N)} \right) - \ln \left(R_{t,t+1}^{(1)} \right) \right] = \\
&= N \ln \left(1 + r_t^{(N)} \right) - (N-1) \ln \left(1 + r_{t+1}^{(N-1)} \right) - \ln \left(1 + r_t^{(1)} \right) \\
&= N \tilde{r}_t^{(N)} - (N-1) \tilde{r}_{t+1}^{(N-1)} - \tilde{r}_t^{(1)}.
\end{aligned} \tag{6}$$

Generally, for an $N - n$ -period bond, the term premium would then be:

$$\widetilde{TP}_{t+n}^{(N-n)} = (N-n) \tilde{r}_{t+n}^{(N-n)} - (N-n-1) \tilde{r}_{t+n+1}^{(N-n-1)} - \tilde{r}_{t+n}^{(1)}. \tag{7}$$

Analogously, let us express our Hairy premium as a ratio and in terms of log returns as follows:

$$\begin{aligned}
HP_t^{(N)} &= \left(\frac{(1 + r_t^{(N)})^N}{\prod_{n=0}^{N-1} (1 + r_{t+n}^{(1)})} \right)^{\frac{1}{N}} \\
&= \exp \left\{ \frac{1}{N} \left(N \ln (1 + r_t^{(N)}) - \sum_{n=0}^{N-1} \ln (1 + r_{t+n}^{(1)}) \right) \right\} \\
&= \exp \left\{ \frac{1}{N} \left(N \tilde{r}_t^{(N)} - \sum_{n=0}^{N-1} \tilde{r}_{t+n}^{(1)} \right) \right\}.
\end{aligned} \tag{8}$$

Using equations (6) and (7) and taking the expectation, we can further express the expected Hairy premium in terms of the CS bond term premia at different horizons:

$$\begin{aligned}
E_t \left[HP_t^{(N)} \right] &= E_t \left[\exp \left\{ \frac{1}{N} \left(\widetilde{TP}_t^{(N)} + \sum_{n=1}^{(N-1)} \widetilde{TP}_{t+n}^{(N-n)} \right) \right\} \right] \\
&= \exp \left\{ \frac{1}{N} \left(E_t \left[\widetilde{TP}_t^{(N)} \right] + \sum_{n=1}^{(N-1)} E_t \left[\widetilde{TP}_{t+n}^{(N-n)} \right] + \frac{1}{2} \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} Cov_t \left(\widetilde{TP}_{t+n}^{(N-n)}, \widetilde{TP}_{t+m}^{(N-m)} \right) \right) \right\}.
\end{aligned} \tag{9}$$

In this step, as we take the expectation, our distinctive underlying assumption is that the term

premiums are normally distributed and that the short rate, $r_t^{(1)}$, and long-term rate, $r_t^{(N)}$, are serially correlated over time, implying that the term premiums $\widetilde{TP}_{t+n}^{(N-n)}$ and $\widetilde{TP}_{t+m}^{(N-m)}$ are also serially correlated over time. This correlation enables us to include the covariance term in the ultimate expression in equation (9).

Consequently, the Hairy premium is the conventional CS bond term premium, $TP_t^{(N)}$, for maturity N at time t plus the sum of all future expected term premiums and their cross-covariances at each date t until maturity N . As a result, if there is a positive serial correlation in future term premiums and if future short floating rates show persistence through positive serial correlation, the well-documented classic CS term premium serves as the lower bound for our Hairy premium measure.

It is important to emphasize the economic interpretation of this result. If the serial correlations between future term premiums are positive, the covariance terms in equation (9) will be positive. This, in turn, represents a persistent component in the expected future term premiums that reflects the persistent investor expectations of a positive term premium at different horizons. Hence, when these expected serial correlations are positive (negative), they drive the magnitude of the Hairy premium above (below) that of the conventional CS term premium.

In essence, since the Hairy premium is driven by the cross-covariances of expected future single period term premiums up to the N -term horizon, this implies serial correlation between the returns on an N -period strategy that is rolled over period-by-period. Since the single period returns are not independent, their sum over the N -periods does not equal the return on an N -period holding strategy. Similar patterns occur in equity markets, highlighting the difference between annualized long-term holding period returns and the compounded returns from a single-period rebalanced strategy.

Because both yields and term premiums exhibit serial and cross correlation over time, the following

inequality holds in expectations:

$$E_t[(1 + r_{t,t+N}^{(N)})] \neq E_t[(1 + r_t^{(1)})(1 + r_{t+1}^{(1)}) \dots (1 + r_{t+N-1}^{(1)})] \quad (10)$$

The N -period fixed rate long-term holding period return is not equal to the compounded return from rolling over 1-period investments for N periods.

Finally it is also important to keep in mind that the variance of the Hairy premium will differ from the variance of the conventional term premium. Because of the inclusion of the variance of the covariance terms, unless the covariance is stable over time, we anticipate that the variance of the Hairy premium will be higher than that of the conventional term premium, and the conventional term premium will act as a lower bound.

4 How Hairy is the Hairy Premium?

4.1 Data

We obtain monthly and daily data from Bloomberg on 3-month LIBOR and OIS floating rates and 2-, 5-, and 10-year interest rate swap rates (IRS) for the G11 countries. These include the United States (US), the United Kingdom (GBP), the Eurozone (EUR), Japan (JPY), Germany (DE), Switzerland (CHF), Canada (CAD), Australia (AUD), New Zealand (NZD), Sweden (SE), and Norway (NOK). Our sample period spans from January 1, 1990, to May 1, 2023. It covers significant periods such as the Russian crisis, the Global Financial Crisis, and the COVID-19 pandemic, as well as the intervals between these periods. Additionally, we collect monthly and daily data on 3-month and 10-year constant maturity rates (CMR) from Global Financial Data (GFD) for all the aforementioned G11 countries. Depending on the country and the frequency of the data, our sample periods range from January 1990 to May 2023.

[Insert Table 1 about here]

The data periods for each country in our sample are presented in Table 1. Additionally, we acquire economic indicator data for the US from FRED and Bloomberg. For a detailed summary of the data and more information about the data sources, please see Appendix B

4.2 Time-series Properties of the Hairy Premium for the US Economy

On a cashflow basis, the savings (costs) of being in a floating rather than a fixed interest paying debt (asset) are substantial for a borrower (investor) in the long run. The analysis below quantifies the historical difference between fixed and floating rates for a couple of maturity horizons for the US. For example, the following Figure 3 helps quantify the cost (profit) borrowers paid (investors received) over the 10-year term by fixing the rate. We use interest rate swap (IRS) rates to represent the fixed rate (red line) and LIBOR rates to represent the floating rate (blue). The crucial line on the graph is dark grey, representing the historical average of the floating rate over the next 10 years, called the (average) effective rate. This is the amount of annual interest the borrower (investor) would have paid (received) over the next 10 years if they had opted for a floating rate instead of a fixed rate. Thus, if the dark grey line is above the red line, the borrower (investor) would have been better (worse) off locking in a fixed rate on that day, as opposed to having a floating rate.

[Insert Figure 3 about here]

Fixing the rate has thus never been less expensive for a borrower, as the dark grey line is never above the red in the decades examined in this Figure 3. On the other hand, fixing the rate has always been profitable for investors. This is further illustrated in Figure 2, which depicts the difference between the 10-year fixed rate and the 10-year effective rate (the difference between the red and dark grey lines), which is our defined Hairy premium metric.

It is also particularly notable that, based on the data in Table 2 Panel A, since the 1990s, the average Hairy premium in the United States has been 3%, with a minimum non-negative 0.7% annually and a maximum non-negative 4.7% annually over the next 10 years, with no single instance being negative, which would be a loss (gain) for a borrower (investor) paying (receiving) a fixed (floating) interest rate. Note that this is the so-called “LIBOR-based Hairy premium”.

[Insert Table 2 about here]

Adjacent to the “LIBOR-based Hairy premium” data is shown another premium denoted as the “CMR-based Hairy premium”, which stands for the government constant maturity rates-based Hairy premium. This alternate computation employs the government 3-month constant maturity rate (CMR) as the short rate and a government 10-year CMR for the long fixed rate. Notably, regardless of whether the Hairy premium is derived from LIBOR or CMR, its characteristics are nearly identical in levels and exhibit a correlation of over 97%. Consequently, we use these interchangeably.

The reason for introducing the CMR-based Hairy premium is to enable the expansion of our sample data to time periods preceding the 1990s. This is due to the unavailability of reliable data on 3-month LIBOR and IRS prices for these earlier periods. Thus, in our analysis of the complete sample, which encompasses timeframes preceding the 1990s, government CMRs will be utilized in place of the aforementioned data sources.

Upon examination of the Hairy premium further back in time and referring to Table 2, it is notable that the Hairy premium has been there since the 1900s and remained consistent throughout different time periods. Notice the Hairy premium’s statistical significance is robust at 1% even after adjusting for 119 overlapping monthly observations over the 10 years using the Hansen-Hodrick method, as shown in Panel D. Moreover, Panel B encompasses the period from 1961 to 1990, while Panel C extends even further back and runs from 1900 until 1960. What varies across these periods is primarily the magnitude of the Hairy premium, rather than its existence. Notably, during the 1960 to 1990 period, the average

annual 10-year Hairy premium, for instance, appears to have bottomed, but still at a reasonable bounded lower level of -1%. This period encompasses the Volcker era, characterized by a deliberate disinflation strategy implemented in the late 1980s. This strategy involved aggressive tightening monetary policy, leading to a significant increase in the federal funds rate, which reached as high as 19%.

4.2.1 Holding Period Maturity Horizon

Rerunning the above analysis from 1990s onward for a 5-year maturity horizon yields the same result, with the exception that the average Hairy premium is now a bit lower, but still positive, than in the 10-year case, and there is now a small chance of 10% rather than 0% as in the 10-year case that it is better for the borrower (investor) to fix (not fix) the rate of the debt (asset), as shown in Figure 4.

[Insert Figure 4 about here]

The average Hairy premium is still a healthy 1.5% per annum, but the minimum is now a negative -1%, while the maximum is 4.9% annually, as shown in Figure 5 and Table 2 Panel A. This means that, for instance, if a borrower had chosen not to fix the rate on their five-year debt, they would have had a 10% chance of overpaying by 1% annually, and a 90% chance of saving an impressive 4.9% profit annually over the next five years.

[Insert Figure 5 about here]

The evidence presented above suggests that the risk-reward of floating versus fixed interest rates is asymmetric. For borrowers (investors), when it is right to be in a floating (fixed) rate, the savings are an impressive 4.9% per year, and when it is wrong to be in a floating (fixed) rate, the costs are only -1% per year.

Looking at the 2-year Hairy premium case, the annual average premium is lower than the 5-year average, at 0.6% per year. Therefore, the longer the term, the more the forward curves overshoot

the spot interest rates and the greater the Hairy premium. Thus, the longer the term, the higher the likelihood that the borrower (investor) will save (lose) money by choosing floating interest rates over the fixed-rate alternative.

4.2.2 Recessions and Monetary Policy Tightening Cycles

When analyzing the Hairy graph in Figure 1, the market tends to systematically overestimate the trajectory of floating rates, except for periods immediately preceding a monetary tightening cycle. The shaded green boxes in Figure 6 indicate these periods.

[Insert Figure 6 about here]

The figure highlights two notable takeaways. First, monetary policy tightening cycles are relatively short-lived. Historically, the duration between the first rate hike and the first rate cut has not generally exceeded 3.2 years, as illustrated by the period from June 2004 to August 2007. Second, the market often fails to factor in the inevitable economic downturn. Within two years of the start of a monetary tightening cycle, there is a high likelihood that floating rates are already beginning to trend lower. While rates may initially rise, they eventually fall, following the adage that “what goes up must come down”.

The Hairy premium exhibits its highest performance at the onset and during monetary easing cycles. Interestingly, this finding goes against intuition, as we would expect a preference for investing in floating rate assets during such periods given that the short-term interest rate surpasses or is close to the long-term fixed rate during the period just before the monetary easing.

Nevertheless, it is important to note that the Hairy premium does not solely capture the immediate disparity between the long-term fixed and short-term floating rates. Rather, it accounts for the average difference over the holding period time as well as their future (serial) covariances over the holding period. Consequently, periods characterized by flat yield curves, which are relatively short-lived and infrequent,

do not have a substantial adverse effect on the overall behavior of the Hairy premium. This is primarily due to the yield curve being predominantly upward-sloping and steep persistently.

The aforementioned behavior of the Hairy premium can be observed in Figure 7, which displays the 10-year CMR-based Hairy premium for the US since the early 1900s. Within the figure, the periods characterized by economic recession, which coincide with yield curve inversion, are visually highlighted in pink.

[Insert Figure 7 about here]

It is evident that the Hairy premium increases during recessions, hence it is countercyclical and acts as a hedge during bad times.

4.2.3 Level and Shape of Yield Curve

We also conduct a formal test to determine whether the Hairy premium is associated with variations in the level of short-term rates, which are more directly influenced by monetary policy, or to shifts in long-term rates that may be unaffected by monetary policy if monetary policy is ineffective. The results from the regression analysis are presented in Table 4.

[Insert Table 4 about here]

The relationship between the US 10-year Hairy premium and the levels of short-term and long-term constant maturity rates (CMR) is statistically significant. When either the short- or long-term rates are higher (column (1) and (2)), the Hairy premium tends to be higher as well. However, this interpretation can be slightly misleading because increasing short-term rates are typically accompanied by decreasing long-term rates. In a combined analysis in column (3), for instance, when short-term rates increase by 1% while long-term rates remain constant, the Hairy premium actually decreases by 0.398% annually. On the other hand, if long-term rates increase while short-term rates are held constant, the

Hairy premium surges by 0.745% annually, almost in a one-to-one relationship. This indicates that the Hairy premium is influenced by the shape and steepness of the yield curve, which is further confirmed by the additional analysis in column (4). When the yield curve steepens, as measured by the increased difference between 10-year rates and 3-month rates, the Hairy premium also increases. This makes sense because a steep yield curve indicates that long-term interest rates overshoot short-term interest rates. Similarly, when the yield curve is inverted, meaning the 10-year rate is lower than the 3-month rate, the Hairy premium falls.

4.2.4 Inflation Expectations

If there is a connection between the Hairy premium and recessions, as well as the shape of the yield curve, it is logical to expect a relationship with inflation. However, considering the time delay in reporting inflation data, it becomes crucial for us to analyze inflation expectations that provide advance information compared to the actual reported figures.

Examining inflationary expectations becomes even more interesting because, surprisingly, we find that the Hairy Graph phenomenon, where forward rates overshoot spot short floating rates (as shown in Figure 1), also exists when using survey data on expected future long-term rates. Thus, this phenomenon extends beyond forwards and interest rate swaps and includes surveys overshooting expectations of future interest rates.

In Figure 8, we present the same Hairy Graph as in Figure 1, but instead of using the 10-year forward interest rates to illustrate the “hairs”, we use the Mean Fed’s Survey of Professional Forecasters (SPF) to represent expected future 10-year interest rates on government bonds.

[Insert Figure 8 about here]

The pink line represents the 10-year realized rate, while the “hairs” represent the mean of the

Federal Reserve Bank of Philadelphia’s SPF for the expected 10-year government rate.⁵ It is clear that the survey also consistently overestimates the trajectory of the realized interest rates. In both graphs, it is noticeable that, when the market or survey underestimates the trajectory of the interest rates, they usually do so preceding and during periods of monetary policy tightening, when interest rates are sharply raised, which are disproportionately rare in the data. The above finding suggests that the Hairy premium may be a result of overshooting expectations regarding future interest rates, rather than financial frictions within the forward and IRS instruments’ financial markets. This is supported by the presence of the premium, not only in the financial instruments but also in the projected interest rates derived from professionals’ survey data.

One possible explanation for these overshoots in interest-rate expectations is an overshoot in inflation expectations. To test this hypothesis, we conduct a formal regression analysis, regressing changes in the US 10-year Hairy premium on changes in the Federal Reserve Bank of Cleveland’s Expected inflation⁶ as well as the expected inflation from Michigan University.⁷ The former is intended to represent inflation expectations among professionals, while the latter is intended to represent “subjective” inflation expectations among US households. Table 5 summarizes the findings.

[Insert Table 5 about here]

Columns (1) and (2) focus on long-term inflation expectations, such as 5-10 years ahead. Both the professionals’ inflation expectation for the 10-year-ahead inflation, the Federal Reserve Bank of

⁵The Survey of Professional Forecasters (SPF) is a quarterly economic forecast survey conducted by the Federal Reserve Bank of Philadelphia, providing macroeconomic predictions for the United States. It holds the distinction of being the longest-running survey of its kind in the country.

⁶The Federal Reserve Bank of Cleveland provides estimates of the expected inflation rate for the next 30 years across different time periods. These estimates are generated using a model that accounts for treasury yields, inflation data, inflation swaps and professional survey-based measures of inflation expectations. The reported 10-year expected inflation estimate represents the projected average rate of inflation over the upcoming 10 years.

⁷The Michigan University Consumer Expectations survey concentrates on three key aspects: how consumers perceive their own financial outlook, their short-term outlook for the general economy, and their long-term outlook for the economy. Each monthly survey comprises around 50 core questions, each assessing various facets of consumer attitudes and expectations. The survey samples are statistically designed to represent all households in the United States, excluding Alaska and Hawaii. A minimum of 500 telephone interviews are conducted each month for the Surveys of Consumers.

Cleveland's Expected inflation in column (1), and the consumers' inflation expectation for the 5-10 year-ahead inflation, the mean of the Michigan University Survey in column (2), are statistically significant. However, the R^2 in column (1) is significantly higher, at 41%, than in column (2), where it is only 2.5%; note that these specifications are in changes rather than levels. Thus, the Hairy premium is better explained by professionals' long-term inflation expectations than by consumers' long-term inflation expectations. This is also confirmed in column (3), where when the two measures of long-term inflation expectations are added together, only the professionals' expectation from the Federal Reserve Bank of Cleveland remains significant.

Column (4) also shows that consumer inflation expectation over the next 12 months, as measured by the Mean of the Michigan University Survey, is marginally significant but with a very low coefficient and R^2 . However, realized contemporaneous inflation in column (5), as measured by CPI, is significant for the Hairy premium, but with much smaller R^2 when compared to long-term 5-10 years-ahead professional inflation expectations.

In column (2), we also test for the uncertainty around the mean of the Michigan University Survey forecasts for the 5-10 year inflation measured by the standard deviation (SD) and we find that it is not significant for the Hairy premium. Nonetheless, when it comes to expectations, the use of different reporting methods can have significant implications. Even if forecasters have the same probabilistic beliefs, they might offer different point predictions. On the other hand, forecasters with different beliefs may provide identical point predictions. Comparing point predictions among forecasters can therefore be challenging. Divergent predictions do not necessarily indicate disagreement among forecasters, and uniform predictions do not necessarily indicate agreement.

4.2.5 Economic Uncertainty

In Table 6, we further investigate the time-series relationship of the US Hairy premium with other economic variables linked to uncertainty.

[Insert Table 6 about here]

The results in columns (1) and (2) confirm that the Hairy premium rises during recessions, as shown in Figure 7, and when there is uncertainty about US economic policy, as measured by the news-based FRED Economic Policy Index⁸ developed by Baker, Bloom, and Davis (2016). When paired, they both remain significant in column (3) and explain about 33% of the Hairy premium variation. Although the coefficient is small, what is interesting is that increased economic uncertainty is associated with a decrease in the Hairy premium, contrary to what might be expected if the Hairy premium was a remuneration for bearing economic uncertainty risk. Indeed, economic uncertainty coincides with yield curve flattening or inversion when the forwards curves are downward sloping and undershoot the realized short interest rates as monetary policy tightening is underway making the Hairy premium compress.

To further investigate the sources of uncertainty, we report results from regressing the Hairy premium in monetary policy uncertainty and equity market-related uncertainty in columns (5) and (6), but they are insignificant. As a result, the Hairy premium is found to be unrelated to equity market premium uncertainties or uncertainties about the direction of monetary policy according to these measures. Finally, as shown in column (4), the Hairy premium is unrelated to overall economic activity as measured by the Chicago Fed National Activity Index⁹ (CFNAI). The index may be overly broad.

⁸This index is a news-based economic policy uncertainty index. It is built on newspaper articles about policy uncertainty from major US newspapers. It counts the number of newspaper articles that include the words uncertain and economy, and one or more policy-relevant terms. For additional details, see Baker, Bloom, and Davis (2016)

⁹The CFNAI is a weighted average of 85 monthly national economic activity indicators. The CFNAI is a single-measure summary of a common factor in these national economic data. As a result, historical movements in this Chicago Fed index are intended to closely track periods of economic expansion and contraction, as well as rising and falling inflationary pressures.

4.3 Cross-sectional Properties of the Hairy Premium for G11 Economies

Up to this point our analysis has centered on the US economy, yet a question naturally arises regarding the presence of the Hairy premium as a phenomenon beyond the borders of the US. Remarkably, the Hairy premium exists everywhere, making it a global phenomenon rather than one confined to the US. For instance, Figure 9 plots the 10-year CMR-based Hairy premium for G11 countries as far back as the early 1900s. Within the figure, the periods characterized by economic recession in the US economy only, which coincide with higher Hairy premium in the US, are highlighted in pink.

[Insert Figure 9 about here]

The figure shows that the Hairy premium is a global phenomenon and that premiums are related between the different countries over time, particularly prior to the late 1970s and after the late 1980s. The period in between is known as the Volcker period, and some G11 countries, namely DE, GBP, and JPY, appear to have had diverging premiums during that time.

To formally explore the relationship between the Hairy premiums in the G11 economies over time, we further conduct a principal components analysis (PCA). The data period utilised in the PCA analysis is from 1980 onwards because this is the time period for which we have data for all countries at the same time. Figure 10 depicts the contribution of each principal component to the variation in the G11 10-year CMR Hairy premiums.

[Insert Figure 10 about here]

The first principal component accounts for 44.5% of the variation in the premiums among the G11 countries, while the combination of the first, second and third components accounts for 84.7% of the variation in premiums among the G11 countries. The remarkable aspect is that the opportunity to profit from the Hairy premium has not been limited to the US since the 1900s; instead, it has been a

global phenomenon. This suggests the presence of common global systemic factors, as evidenced by the PCA analysis indicating such factors from the 1980s onwards.

Additionally, we examine the countries' loading to these global systemic factors and the direction of their loading, as depicted in Figure 11.

[Insert Figure 11 about here]

All of the countries exhibit positive loadings on the first factor, ranging from 6.6% and 7.7% for NZD and USD to 43.9% for GBP. In contrast, the loading directions for the second and third factors vary by country. On the second factor, CAD, CHF, DE/EUR, GBP, JPY, and the US have negative loadings, while AUD, NOK, NZD, and SE have positive loadings. Notably, the US exhibits a large positive loading on the third factor, as do AUD, CAD, NOK, and JPY, albeit to a lesser extent, whereas the remaining countries all exhibit a negative loading.

5 Empirical Comparison between Hairy and Term Premiums

After conceptually distinguishing the Hairy premium from the conventional term premium in Section 3, we now conduct an empirical comparison between our model-free Hairy premium and estimated term premiums from the empirical literature.

Most existing empirical research on term premiums employs affine term structure models often estimated through maximum likelihood techniques. Without taking a stand on any model, such models include [Cox, Ingersoll, and Ross \(1985\)](#) (CIR) and [Kim and Wright \(2005\)](#) (KW) (for other examples also see [Longstaff \(1990\)](#); [Chen and Scott \(1993\)](#); [Duffie and Kan \(1996\)](#); [Dai and Singleton \(2000\)](#); [Duffee \(2002\)](#); [Collin-Dufresne and Goldstein \(2002\)](#); [Piazzesi \(2003\)](#); [Singleton \(2006\)](#)). These models rely on distributional assumptions and no-arbitrage constraints for computational tractability. More importantly, they assume bond yields are an affine function of state variables (pricing factors) and that

yield pricing innovations are serially uncorrelated across time and maturities.

The benefit of our model-free estimated Hairy premiums is that it explicitly allows and accounts for serially and cross-sectionally correlated yields. As conceptually shown in Section 3, the Hairy premium is, in major part, fundamentally borne and influenced by the serial correlation in yields, as reflected in the serial correlation in term premiums across the holding period maturity horizon of the Hairy premium.

Therefore, a useful empirical comparison can be made between the Hairy premium and term premium estimates generated by maximum likelihood estimated affine term structure models. This contrast helps isolate the contribution of two key components to the Hairy premium. First, the constraints imposed in affine models omit the persistent serial covariances in yield term premiums that exist across maturities and time. Second, the cross-sectionally and serially uncorrelated yield pricing innovations assumptions in affine models fail to capture also pricing errors that, while independently and identically distributed (iid) over time, are not spanned by the affine models' prescribed pricing factors. As a result, the difference between the model-free Hairy premium and affine term premium estimates quantifies the joint contribution of these two factors - serial yield innovation correlations and unmodeled iid yield pricing innovations - which are fundamental determinants of the Hairy premium but are muted in standard maximum-likelihood affine specifications.

Furthermore, some recent studies have recognized the limitations of constrained standard maximum likelihood affine models. They propose less restrictive regression-based estimations that the serial correlation in yield pricing innovations. Without taking a stand on a specific model, two such examples are [Cochrane and Piazzesi \(2008\)](#) (CPP) and [Adrian, Crump, and Moench \(2013\)](#) (ACM).

Empirically contrasting term premium estimates from the ACM and CPP versus standard maximum likelihood affine models and the model-free Hairy premium helps us further decompose and quantify the influence of the serial correlation structure excluded from these standard affine models as well as

the unmodeled iid yield pricing innovations contributions to the Hairy premium. Of course, we must acknowledge there will be residual mismeasurement in the form of statistical measurement error in all of these estimates, as none are calculated free of statistical measurement error. Nevertheless, this empirical comparison offers an insightful initial quantitative evaluation of the relative magnitudes of the serial correlation of the term premiums and unmodeled iid yield pricing innovations which fundamentally drive the model-free Hairy premium.

In a CIR affine model term premiums are estimated from interest rates that are a standard Gaussian affine function of a single mean-reverting stochastic factor following a square-root diffusion. Given its simplicity, it imposes an overly restrictive assumption of perfect correlation of rate volatilities across maturities. In practice, this model is often calibrated using short-term interest rate data and we calibrate it here, following standard literature, with data from [Gürkaynak, Sack, and Wright \(2006\)](#). The KW model also employs a standard affine Gaussian methodology as implemented in [Kim2012](#) to estimate term premiums. However, it utilizes a three-factor arbitrage-free term structure model with latent statistical factors. Here we use the authors' published estimates of the model, but their estimation utilizes 3-month T-bill survey forecasts along with yields data. Both frameworks have limitations capturing yield dynamics during effective lower bound (ELB) periods, a shortcoming common across standard Gaussian affine models. The persistent serial correlation of short rates stuck near the effective lower bound in ELB periods induces correlated yield pricing innovations over time, violating the iid assumption. Thus, the standard affine Gaussian structure struggles to model yields when short rate persistence is high, as the constraining iid yield pricing error assumption is inconsistent with the serial correlation arising from lower bound effects.

CPP estimates term premiums using predictive regressions of excess Treasury bond returns on yield curve factors - the first three principal components, namely level, slope and curvature, and a return forecasting factor (CPP factor). The CPP factor is comprised of a linear combination of forward

rates. While statistically derived, CPP argues this CPP factor represents time-varying compensation for interest rate risk that rises in recessions with heightened uncertainty. CPP implicitly accommodate serially correlated yield innovations. Here we follow the estimation procedure of the authors but, following standard literature, we employ data on yields and forwards from [Gürkaynak et al. \(2006\)](#).

ACM estimates term premiums by explicitly allowing for serially correlated yield pricing innovations using a multi-stage regression approach with five yield factors, overcoming the constraints in standard maximum likelihood affine Gaussian models. However, some limitations include assuming efficient markets and a VAR(1) factors' process, which may not fully capture the serial correlation dynamics or nonlinear affine pricing of interest rates. Here we use the authors' published estimates of their model. Overall, both ACM and CPP employ flexible regression approaches to estimate term premiums without tightly constraining yield serial correlation dynamics like standard affine Gaussian models. Comparing the two model approaches, ACM finds better out-of-sample fit using five factors instead of CPP's four factors.

Figure 12 plots the estimated term premiums over time from the models - the CIR and KW Gaussian affine models, the ACM regression method, and the CPP predictive regression approach - comparing them to the model-free Hairy premium. Furthermore, Table 7 displays the summary statistics of the Hairy premium and the CIR, KW, ACM, and CPP term premiums. Appendix C illustrates in depth additional details on the how each of the term premium models are estimated.

[Insert Figure 12 about here]

[Insert Table 7 about here]

It is evident the model-free HP exhibits different dynamics over time compared to the CIR, KW and ACM premium estimates, often trending in opposing directions. The CIR, KW and ACM premiums appear procyclical, while the HP is largely countercyclical. The ACM premium has a closer mean to

the HP at 1.12% from 1991 to 2023 and 1.52% from 1961 to 2023, versus 3% and 2% for the HP over the same periods.

Notably, the CPP premium matches the HP's time series dynamics and is countercyclical as the HP, but persists at a much lower level, with a mean of -0.49% from 1991 to 2023 versus 3% for the HP, and 0% from 1961 to 2023 versus 2% for the HP. The CPP premium dynamics match the HP because the fourth factor in the CPP model encompasses a linear combination of forwards, essentially capturing the overshoot in forwards underlying the Hairy premium anomaly.

Furthermore, we decompose the contribution of the KW, ACM, and CPP term premium estimates to the model-free Hairy premium using a vector error correction model (VECM). The VECM framework is warranted given the persistence exhibited in the time series dynamics of all the term premiums, including the Hairy premium. Modeling their relationships through a VECM allows us to understand the links between the KW, ACM, and CPP premiums and the Hairy premium, as well as disentangle the proportional variance contribution of each estimated premium to the Hairy premium.

The VECM specification is:

$$\Delta \begin{bmatrix} HP_t \\ HW_t \\ ACM_t \\ CPP_t \end{bmatrix} = \mu + \begin{bmatrix} \alpha_{HP} \\ \alpha_{HW} \\ \alpha_{ACM} \\ \alpha_{CPP} \end{bmatrix} \left(\begin{bmatrix} HP_{t-1} \\ HW_{t-1} \\ ACM_{t-1} \\ CPP_{t-1} \end{bmatrix} - \beta' \begin{bmatrix} 1 \\ HW_{t-1} \\ ACM_{t-1} \\ CPP_{t-1} \end{bmatrix} \right) + \Gamma_1 \Delta \begin{bmatrix} HP_{t-1} \\ HW_{t-1} \\ ACM_{t-1} \\ CPP_{t-1} \end{bmatrix} + \dots + \Gamma_p \Delta \begin{bmatrix} HP_{t-p} \\ HW_{t-p} \\ ACM_{t-p} \\ CPP_{t-p} \end{bmatrix} + \begin{bmatrix} \epsilon_{HP,t} \\ \epsilon_{HW,t} \\ \epsilon_{ACM,t} \\ \epsilon_{CP,t} \end{bmatrix}$$

The VECM equation elucidates both the short-term and long-term adjustments among the Hairy premium (HP), KW, ACM, and CP term premiums in response to deviations from their equilibrium relationship. In this equation, Δ denotes the first difference operator applied to each variable. The vector μ represents the constant term in each equation of the system. The vector α embodies the loading coefficients, while the term in parentheses is the error correction term, with β' being the transposed cointegration vector (excluding the constant). The Γ_i matrices capture the short-term dynamics via the

lagged differences of the variables. $\epsilon_{it} \sim N(0, \Sigma)$ denotes the error terms associated with each variable at time t assumed to be iid. The α coefficients indicate the speed and direction of adjustment towards long-term equilibrium when deviations occur, the Γ_i coefficients reflect the short-term relationships, and the β coefficients represent the long-run equilibrium relationships among the premiums. Overall, the VECM characterizes both the short and long-run relationships among the various term premium estimates in relation to the Hairy premium. Table 8 displays the results from the VECM regression.

[Insert Table 8 about here]

We then compute impulse response functions (IRFs) obtaining the isolated response of each term premium to a one-unit shock in the other premiums, holding other shocks at zero and then calculate the forecast error variance decomposition quantifying the proportion of the forecast error variance for the Hairy premium stemming from shocks to each of the other premiums. Figure 14 displays the variance decomposition of each premium's contribution to the Hairy premium at 12 monthly periods (1 year) ahead (see Table 9 for up to 36 monthly periods ahead). The lion's share, 86%, is contributed by the Hairy premium itself, followed by 6.9% from HW, 3.7% from CPP, and 3.3% from ACM. Therefore, the Hairy premium's forecast error variance primarily comes from its own lags, with some contributions also from ACM and KW, and CPP.

[Insert Figure 14 about here]

In summary, the Hairy premium exhibits different cyclical dynamics over time compared to other premium estimates. The CIR, KW and ACM premiums largely trend procyclically. Conversely, the Hairy premium is countercyclical. While no model premium matches the model-free Hairy premium, the CPP premium better reflects its time series dynamics and is countercyclical like the Hairy premium, because the fourth CPP factor captures the forward rates' overshoots underlying the Hairy premium. However, the CPP premium mean is consistently much lower in magnitude. The ACM premium mean

is closer to the Hairy premium's mean, yet its dynamics diverge from the Hairy premium. Forecast error variance decomposition shows the Hairy premium variance mainly derives from its own lags, with secondary contributions from the ACM, CPP and KW premiums.

6 Hairy Premium-Based Affine Regime-Switching Model

In this section, we incorporate the Hairy premium dynamics into a standard affine model to showcase the possibility of generating and modeling the Hairy premium in this framework. However, we simply overlay the standard affine model with the Hairy premium dynamics, so while we can model the premium, this does not inform the underlying microfoundations driving the Hairy premium. For those microfoundations we turn to a more comprehensive general equilibrium model in the next section.

We start by proposing a basic affine term structure model where the short rate follows a CIR process, with the Hairy premium influencing the CIR parameters over time across separate regimes.

The dynamics of the short rate, denoted as r_t , are captured by the CIR model, which is expressed as:

$$dr_t = \kappa_{hp_t}(\theta_{hp_t} - r_t)dt + \sigma_{hp_t}\sqrt{r_t}dW_{r_t} \quad (11)$$

where κ_{hp_t} represents the mean-reversion speed of adjustment in regime hp_t , θ_{hp_t} is the long-term mean level in regime hp_t , σ the volatility in regime hp_t , and dW_{r_t} is a standard Wiener process.

The model incorporates a regime-switching mechanism based on the Hairy premium at time t , denoted hp_t , which is determined based on the level of the HP, with cut-offs denoted as α and β :

$$hp_t = \begin{cases} Low, & \text{if } HP_t \leq \alpha \\ Medium, & \text{if } HP_t \text{ is between } \alpha \text{ and } \beta \\ High, & \text{if } HP_t > \beta \end{cases} \quad (12)$$

The yield of a zero-coupon bond is influenced by the short-term interest rate and the prevailing Hairy premium market regime:

$$Y(t, T) = A_{hp_t}(t, T) + B_{hp_t}(t, T) \cdot r_t \quad (13)$$

where $A_{hp_t}(t, T)$ and $B_{hp_t}(t, T)$ are functions determined from the model's parameters and the current regime hp_t .

For model estimation and calibration, we use yields data from [Gürkaynak, Sack, and Wright \(2006\)](#) and employ Maximum Likelihood Estimation, enabling us to estimate the parameters κ_{hp_t} , θ_{hp_t} , and σ_{hp_t} for each regime, alongside determining the regime cutoffs α and β . We ensure model robustness through extensive historical validation. [Figure 13](#) plots the estimated term premiums over time from the Hairy premium CIR regime-switching model (CIRHP) versus the model-free Hairy premium. [Appendix D](#) illustrates in depth additional details on the how the model is designed and estimated.

[Insert [Figure 13](#) about here]

The estimated term premium closely matches the model-free Hairy premium. However, this merely reproduces the observed Hairy premium properties rather than explaining the premium's origins. In essence, while we can replicate Hairy premium attributes via time-varying parameters, this does not inform the microfoundations underlying the temporal parameter variations giving rise to the Hairy premium. Thus, we next turn to a more comprehensive, microfounded general equilibrium model to explain the origins and dynamics of the Hairy premium.

7 Why the Hairy premium? A Theory of Persistent Short-termism

According to the rational expectations theory, the term structure of interest rates reflects the market's current expectations of future short-term rates. The Hairy premium documented so far represents a

clear and systematic deviation from the rational expectations theory. We consider a possible reason for the deviations from rational expectations that could give rise to the Hairy premium. In particular, we propose an explanation that captures the participants on the interest rate swap market and is motivated by the survey evidence on investors' time preferences and beliefs regarding their degree of short-termism.

7.1 Participants in the Interest Rates Swaps Market

The interest rate swaps market is one of the most liquid market globally, with \$466 trillion in outstanding notional value in 2022. In the US alone there is \$168 trillion outstanding notional (BIS, 2022). The market exhibits extreme liquidity with \$500 billion in average daily trading volume and a 0.5 basis points bid-ask yield spread for 10-year swaps.

We follow TBAC (2021) and Hanson, Malkhozov, and Venter (2022) and group swap market participants into “end users” and “market-makers”. There are two main groups of end users. The first group are typically net payers of the fixed swap rate and thus net payers of the Hairy premium. This includes mortgage servicers and relative-value mortgage investors like Fannie Mae and Freddie Mac, who pay the fixed swap rate on net to take advantage when mortgage-backed securities (MBS) trade cheap compared to interest rate swaps (Feldhutter and Lando, 2008; Hanson, 2014). Swaps have historically been a more effective hedge for MBS than Treasuries. However, when long-term rates fall, expected mortgage prepayments increase, reducing MBS duration due to negative convexity (Perli and Sack, 2003). This leads mortgage investors to start entering receive-fixed swaps, thus receiving the Hairy premium, to offset their existing long-term net pay-fixed positions. In addition, fixed-income money managers are included in the second group. These managers use swaps to adjust the duration of their portfolios and tend to be net payers of the fixed swap rate.

The second group are typically net receivers of the fixed swap rate and thus net receivers of the Hairy premium. This includes insurers and pension funds, who receive the fixed rate on net to add duration

to their portfolios since the duration of their liabilities exceeds their assets (Klingler and Sundaresan, 2019). Their demand to receive the fixed rate rises when rates fall due to convexity hedging needs (Domanski, Shin, and Sushko, 2017). Commercial banks also typically receive the fixed rate on net (Begenau, Piazzesi, and Schneider, 2020). They are generally hurt by declining rates as loans reprice faster than deposits (Driscoll and Judson, 2013; Drechsler, Savov, and Schnabl, 2021), thus receiving the fixed rate helps offset this exposure. Non-financial corporations receive the fixed rate on net as well in order to convert fixed-rate debt issues into synthetic floating-rate funding.

Market-makers simply accommodate the demands from both end user groups. They usually hedge their interest rate risk in the Treasuries market and as a result are predominantly concerned with the relative valuation of swaps versus Treasuries - i.e., with the level of swap spreads (the difference between swap rates and like-maturity government bond yields) and any differential between the short-term rate referenced by swaps and the short-term Treasury financing rate (TBAC, 2021), but do not pay or receive the Hairy premium on net. Overall, the first end user group tends to be net receivers of the fixed swap rate and thus the Hairy premium, while the second group tends to be the net payers of both the fixed swap rate and thus the Hairy premium.

7.2 Investor preferences

Based on the structure of the interest rate swaps market having two groups of end user types, we consider a general equilibrium model with a continuum of two types of agents as in Lundeby and Tancheva (2023): retail investors, corresponding to the first end user group, who are time-inconsistent (TI) and susceptible to changes in their degree of short-termism and financial intermediaries, corresponding to the second end user group, who are time-consistent (TC) and rational. Both types have Epstein-Zin preferences and identical risk preferences γ , elasticity of intertemporal substitution ψ , and time discount factors β . The TC investors are assumed to have stable time preferences and exponential discounting, while the

TI investors are present biased and use quasi-hyperbolic discounting as in [Laibson \(1997\)](#). Hence, the TI agents are more impatient in the near future than in the distant future, discounting it with discount factor $\delta\beta$. We denote the wealth share of the TI agents as the ratio of their wealth to the aggregate wealth $s_t = \frac{W_{TI,t}}{W_t}$.

Furthermore, embedding recent survey evidence that personal experience can have a lasting impact on individual's level of impatience ([Sapolsky, 2017](#); [Kuralbayeva, Molnar, Rondinelli, and Wong, 2019](#); [Citanna and Siconolfi, 2022](#)), we allow the TI agents to have time-varying degree of short-termism. We model this feature by assuming that with probability θ the TI agents realize they will remain short-termistic in the future, and with probability $1 - \theta$ they believe they will become as long-termistic as the TC agents. This specification is equivalent to partial awareness of the present bias as in [O'Donoghue and Rabin \(1999\)](#), but we allow it to vary over time as in [Citanna and Siconolfi \(2022\)](#) and [Lundeby and Tancheva \(2023\)](#).

To ensure that the model is stationary and both types of agents survive in the long run, we consider a discrete time version of the [Blanchard \(1985\)](#) and [Garleanu and Panageas \(2015\)](#) overlapping-generations (OLG) model. At each time period a fraction $1 - \lambda$ of agents is born while a randomly chosen mass $1 - \lambda$ die and their wealth is distributed proportionally back in the economy. The newborn agents of each type are born with an equal share of wealth and as a result no type can be completely crowded out in the long run.

7.3 The Economy

We assume a simple economy with a risk-free and a risky asset that pays dividend D . The sum of the consumption of the TC and TI agents yields the aggregate consumption:

$$C_t \equiv C_{TI,t} + C_{TC,t}. \tag{14}$$

Considering the real part of the Hairy premium, we model consumption and dividend growth as i.i.d., where dividend growth is more volatile:

$$\Delta c_{t+1} = \mu + \varepsilon_{t+1} \quad (15)$$

$$\Delta d_{t+1} = \varrho \Delta c_{t+1}. \quad (16)$$

In addition, we model the subjective beliefs of the TI agent about her degree of short-termism as a Markov process that can take K different values $\hat{\theta}$ with transition probabilities of moving from state $\hat{\theta}_k$ to $\hat{\theta}_l$ given by $\Pi_{\theta,k,l} \equiv \mathbb{P}(\theta_{t+1} = \hat{\theta}_l | \theta_t = \hat{\theta}_k)$. Consistent with survey data, we model the beliefs about the degree of short-termism as highly persistent over time with a low likelihood of large jumps between states.

To match the nominal level of the Hairy premium we follow [Bansal and Shaliastovich \(2013\)](#) and model the logarithm of the inflation rate π_{t+1} as follows:

$$\pi_{t+1} = \mu_\pi + x_{\pi,t} + \sigma_{\pi,t} \eta_{\pi,t+1}, \quad (17)$$

where $\eta_{\pi,t+1}$ is a transitory component and $x_{\pi,t+1}$ is a predictable component in inflation:

$$x_{\pi,t+1} = \rho_\pi x_{\pi,t} + \sigma_{x_\pi,t} \eta_{x_\pi,t+1} \quad (18)$$

and $\sigma_{x_\pi,t}$ is the time-varying volatility of inflation. This feeds back in consumption growth as follows:

$$\Delta c_{t+1} = \mu + \rho_{c,\pi} x_{\pi,t} + \varepsilon_{t+1}. \quad (19)$$

7.4 Equilibrium

The time-consistent financial intermediaries (TC) and time-inconsistent (TI) retail investors maximize their utility, subject to their budget constraints:

$$V_{TI,t} = \max_{C_{TI,t}, W_{TI,t+1}} \left[C_{TI,t}^\rho + \beta \delta \mathbb{E}_t \left[\theta_t V_{TI,t+1}^\alpha + (1 - \theta_t) V_{TC|TI,t+1}^\alpha \right] \frac{\rho}{\alpha} \right]^{\frac{1}{\rho}} \quad (20)$$

$$V_{TC,t} = \max_{C_{TC,t}, W_{TC,t+1}} \left[C_{TC,t}^\rho + \beta \mathbb{E}_t \left[V_{TC,t+1}^\alpha \right] \frac{\rho}{\alpha} \right]^{\frac{1}{\rho}} \quad (21)$$

$$\text{s.t. } C_{TI,t} + \mathbb{E}_t[M_{t,t+1} W_{TI,t+1}] = W_{TI,t} \quad (22)$$

$$C_{TC,t} + \mathbb{E}_t[M_{t,t+1} W_{TC,t+1}] = W_{TC,t}, \quad (23)$$

where $M_{t,t+1}$ denotes the stochastic discount factor. $V_{TC,t}$ and $V_{TI,t}$ represent the utility of the TC and TI agents at time t . We denote risk aversion parameter as $\alpha = 1 - \gamma$ and EIS parameter as $\rho = 1 - \frac{1}{\psi}$. The planned continuation value function of the TI agent $V_{TI,t+1}$ is a weighted sum of her utility in case she remains short-termistic with probability θ or becomes as long-termistic as the TC agents with probability $1 - \theta$. Even though her beliefs about short-termism change over time, in reality the TI agent always remains time inconsistent. However, we assume that each individual TI agent is fully aware of the bias of all other agents of her type, consistent with experimental evidence (Fedyk, 2022). Since we consider a continuum of agents who are atomistic price-takers and their decisions do not affect the overall wealth of their type, each TI agent knows the correct level of wealth of the entire TI group, $W_{TI,t+1}$, even if she wrongly believes she can become long-termistic with utility $V_{TC|TI,t+1}$.

Based on this we derive the equilibrium condition that determines the wealth share of each type,

which equalizes the stochastic discount factors of the surviving agents of each type:

$$\begin{aligned} & \delta\beta \left(\frac{C_{TI,t+1}}{C_{TI,t}} \right)^{\rho-1} \frac{\theta_t V_{TI,t+1}^{\alpha-\rho} + (1-\theta_t) \left(\frac{W_{TI,t+1}}{W_{TC,t+1}} V_{TC,t+1} \right)^{\alpha-\rho}}{\mathbb{E}_t \left[\theta_t V_{TI,t+1}^\alpha + (1-\theta_t) \left(\frac{W_{TI,t+1}}{W_{TC,t+1}} V_{TC,t+1} \right)^\alpha \right]^{\frac{\alpha-\rho}{\alpha}}} \left[\frac{1}{1-\lambda} \left(1 - \frac{1}{2}\lambda \left(1 + \frac{W_{TC,t+1}}{W_{TI,t+1}} \right) \right) \right]^{\rho-1} = \\ & \beta \left(\frac{C_{TC,t+1}}{C_{TC,t}} \right)^{\rho-1} \frac{V_{TC,t+1}^{\alpha-\rho}}{\mathbb{E}_t \left[V_{TC,t+1}^\alpha \right]^{\frac{\alpha-\rho}{\alpha}}} \left[\frac{1}{1-\lambda} \left(1 - \frac{1}{2}\lambda \left(1 + \frac{W_{TI,t+1}}{W_{TC,t+1}} \right) \right) \right]^{\rho-1}. \end{aligned} \quad (24)$$

7.5 Model Implications

We price the time t real short-term and long-term risk-free rates as:

$$R_{f,t} = \mathbb{E}_t \left[\frac{1}{M_{t,t+1}} \right] \quad (25)$$

$$R_{f,t,t+n} = \mathbb{E}_t \left[\frac{1}{M_{t,t+n}} \right] = \mathbb{E}_t \left[\frac{1}{M_{t,t+1} \times M_{t+1,t+2} \times \dots \times M_{t+n-1,t+n}} \right]. \quad (26)$$

To price the nominal interest rates, we specify the nominal discount factor by dividing the real one by the change in price levels: $M_{t+1}^{\$} = M_{t+1} e^{-\pi_{t+1}}$.

Even if the retail TI investors with persistent degree of short-termism hold a small share of aggregate wealth, they generate endogenous and time-varying discount rate risk. Intuitively, when they know they will be short-termistic in the near future, the TI agents tend to overconsume already today in order to equalize marginal rates of substitution. To prompt the financial intermediaries to hold less capital, while keeping the same level of profit and costs, discount rates must increase and market value must fall.

Since financial intermediaries realize they may face capital outflows exactly when market value is low and discount rates are high, they are more sensitive to the discount rate risk. Hence, they are more averse to holding financial securities with longer maturity, that are more exposed to this risk. As a result, they require a premium in order to hold long-term bonds or enter the fixed rather than the floating leg of an interest rate swap. This induces persistent overshoots of the trajectory of interest

rates and gives rise to the Hairy premium.

To explain the nominal level of the Hairy premium we, in addition, incorporate inflation expectations to the model. In our setting the variation in real rates rather than inflation expectations is the key driver of the positive yield curve slope and the higher expected returns on long-term bonds, consistent with [Albuquerque, Eichenbaum, Luo, and Rebelo \(2016\)](#) and [Gomez-Cram and Yaron \(2020\)](#).

8 Hairy Premium: Historical Advantages for Borrowers & Investors

8.1 A Brief Illustration

To illustrate what benefit the Hairy premium suggests, we consider two example scenarios: one in which a borrower is breaking even, and one in which a borrower saves money by choosing a floating rate. Let's examine [Figure 4](#). For example, in 1996, all three lines converge at 5.90% and the blue shade indicates that 3-months LIBOR was also 5.90%. The red line represents the fixed rate that a borrower could have switched to – a fixed rate for five years at 5.90% – while the dark gray line indicates that if a borrower opted to stay with the floating rate, the interest rate would have averaged 5.90% over the next five years (ending in 2001). Therefore, in 1996, no matter if the borrower chose to remain floating or switch to a fixed rate, they would have paid the same 5.90% interest rate for the next five years. Consequently, whether the borrower remained floating or switched to fixed in 2006, they would still have paid 5.90% for the next five years.

As a contrast, let's consider another period, such as mid-2007. The blue shade in [Figure 4](#) shows that 3-months LIBOR was around 5.70%. The red line shows that a borrower could have switched to a fixed rate for five years at 5.00%, while the dark gray line shows that if the borrower remained floating, the interest rate would have averaged 1.50% annually over the next five years (ending in mid-2012). In this case, the borrower must have faced a difficult decision because the fixed rate was lower than the

floating rate. If they had opted to stay floating, they would have paid an average effective interest rate of only 1.50% annually over the next five years because the Fed dramatically reduced rates during the Great Financial Crisis (GFC).

Figures 2 and 5 display the distribution of savings or costs resulting from floating debt. These figures summarize the analysis presented above and highlight three critical observations when considering cashflows until maturity. Firstly, there are significant savings associated with floating debt for borrowers and fixed-rate assets for investors. Secondly, there is an asymmetry in the risk-reward ratio between fixed and floating alternatives. Lastly, fixing interest rates immediately before a monetary policy tightening cycle, which occurs less frequently, is the optimal approach for borrowers, but not for investors. Appendix E illustrates in depth additional details on the benefits and costs of the Hairy premium for borrowers and investors at different economic time periods.

8.2 What about Intermediate Mark-to-Market Considerations?

The conclusions drawn above show that for borrowers floating-rate long-term debt is dominating strategy except during infrequent monetary policy-induced yield curve inversions when, at worst, they would be indifferent. This holds even considering mark-to-market effects as any floating-rate debt sells at par if sold before maturity, with no price fluctuation risk, as long as there is no correlation between credit risk and interest rates.

One may logically assume if borrowers consistently gain from floating rates, lenders should profit by fixing rates. However, the existence of the Hairy premium challenges this view when considering mark-to-market impacts on investors seeking to sell assets before maturity. Unlike floating-rate assets which always trade at par when sold prior to their maturity, fixed-rate asset prices depend on prevailing yield curves. When policy tightening is expected, fixed asset prices mark-to-market down in proportion to their duration.

For instance, we consider the period just before the 1994 Fed rate hike. An investor could have gained a profit of about 1.70% in the first year before the 1994 yield inversion (as shown in Figure 5) if they fixed their rate for 5 years on their asset. However, the 5-year Hairy premium at that time was in fact negative at approximately -0.50%, as indicated by the purple bars in Figure 4, meaning the investor would have incurred losses over the next five years with the five-year fixed rate. Furthermore, in 1994, right after fixing their rate, the Fed raised rates by 3%. Assuming a flat yield curve shifted parallelly up at the rate rise, the fixed rate asset price would have mark-to-market down by the duration times the hike, about 15% ($3\% \times 5$ years) for that investor.

In essence, although investors usually benefit from long-term fixed rate assets earning the Hairy premium, which is often positive, in times of infrequent monetary policy tightening they could incur losses from the negative Hairy premium prevailing when buying the long term fixed rate asset, reflected in significant mark-to-market declines as illustrated after the 1994 Fed 3% rate hike example. Thereby, considering the Hairy premium and mark-to-market impacts, the Hairy premium serves as a warning indication to investors for the potential mark-to-market losses on fixed rate assets.

9 Conclusions

In the study, we identify a global anomaly spanning over a century arising from the market's persistent tendency to overestimate the future path of spot interest rates which we measure with a new premium called the Hairy premium. We analyze the impact of this Hairy premium on borrowers and investors as well as on asset pricing. We show that a general equilibrium model with rational financial intermediaries and time-inconsistent retail investors with persistent degree of short-termism can explain the existence of the Hairy premium.

We further document the properties of this Hairy premium. The Hairy premium is countercyclical, positively correlated with recessions and long-term inflation expectations and negatively correlated

with economic policy uncertainties and monetary policy tightening cycles. It is also evident in surveys of professional interest rate forecasters, suggesting that the premium reflects professional investors' expectations, preferences, and risk aversion rather than regulatory or financial intermediaries' frictions associated with interest rate forward and swap instrument markets.

The Hairy premium exhibits an asymmetric risk-reward profile in terms of magnitudes, and the longer the term of the debt (asset), the more likely it is that floating interest rates will save money (cost money) for the borrowers (investors) relative to the fixed rate alternative. The findings have significant implications for households, banks, and corporations, as well as for policymakers evaluating the impact of monetary policy on long-term interest rates.

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Table 1: Country-Specific Sample Date Ranges

Country	From	To
Australia (AUD)	July, 1928	May, 2023
Canada (CAD)	March, 1934	May, 2023
Switzerland (CHF)	January, 1980	May, 2023
Germany (DE)	January, 1953	May, 2023
Eurozone (EUR)	January, 1984	December, 2023
United Kingdom (GBP)	January, 1900	May, 2023
Japan (JPY)	January, 1960	May, 2023
Norway (NOK)	December, 1941	May, 2023
New Zealand (NZD)	March, 1978	May, 2023
Sweden (SE)	January, 1955	May, 2023
United States (US)	January, 1920	May, 2023

Table 2: Summary Statistics for 10-year Constant Maturity Rate (CMR) Based Hairy Premium (HP) (monthly frequency and in percentage points)

Panel A: January 1, 1991 - May 1, 2023

Country	N	Mean	SD	Min	Max
US	270	3.0	0.8	1.1	4.8
GBP	270	2.0	0.9	-0.4	4.0
DE	248	3.0	0.7	1.0	4.6
EUR	121	3.1	0.9	0.9	4.6
JPY	270	2.1	1.3	0.4	5.8
CHF	267	2.6	0.7	1.1	4.6
CAD	270	3.0	0.9	0.9	5.4
AUD	270	2.1	1.3	-0.6	5.9
NZD	165	1.3	0.9	-0.7	5.1
SE	270	3.4	1.3	0.9	7.5
NOK	270	2.4	0.9	0.2	4.7

Panel B: January 1, 1961 - December 31, 1990

Country	N	Mean	SD	Min	Max
US	372	1.3	2.5	-1.9	7.6
GBP	372	1.1	1.7	-1.8	6.3
DE	372	2.4	1.1	-0.1	5.0
EUR	96	2.1	1.7	-1.1	4.8
JPY	372	2.9	1.4	0.7	9.0
CHF	144	0.7	1.3	-0.5	3.9
CAD	372	1.0	2.2	-2.3	6.4
AUD	372	0.8	2.7	-3.3	7.3
NZD	166	2.1	3.3	-3.8	7.5
SE	372	0.8	1.8	-1.8	6.6
NOK	372	1.6	1.9	-2.5	5.9

Panel C: January 1, 1900 - December 31, 1960

Country	N	Mean	SD	Min	Max
US	492	1.5	1.1	-0.3	4.0
GBP	732	1.1	1.2	-1.0	3.9
DE	96	2.9	0.9	1.6	4.7
JPY	12	4.2	0.3	3.9	4.7
CAD	322	1.6	1.0	-0.4	3.3
AUD	390	2.3	0.6	0.8	5.8
SE	72	-0.1	0.3	-0.8	0.5
NOK	229	2.0	0.6	1.1	3.0

Panel D: Whole Sample January 1, 1900 - May 1, 2023

Country	N	Mean	SD	Min	Max	AR	SE (HH)	T-stat (HH)
US	1122	1.8***	1.8	-1.9	7.6	0.99	0.48	3.65
GBP	1362	1.2***	1.3	-1.8	6.3	0.98	0.31	4.06
DE	704	2.7***	1.0	-0.1	5.0	0.97	0.12	22.31
EUR	205	2.6***	1.4	-1.1	4.8	0.98	0.30	8.49
JPY	642	2.5***	1.4	0.4	9.0	0.98	0.37	6.90
CHF	399	1.9***	1.3	-0.5	4.6	0.99	0.47	3.97
CAD	952	1.7***	1.7	-2.3	6.4	0.99	0.50	3.45
AUD	1020	1.7***	1.9	-3.3	7.3	0.99	0.52	3.20
NZD	319	1.6**	2.5	-3.8	7.5	0.97	0.70	2.35
SE	702	1.6**	2.0	-1.8	7.5	0.99	0.69	2.36
NOK	859	1.9***	1.4	-2.5	5.9	0.99	0.38	4.99

Note: *** p<0.01, ** p<0.05, * p<0.1 based on Hansen-Hodrick (HH) adjustment for q = 119 overlapping observations for t-tests. "AR" stands for autocorrelation coefficient. N represents the number of monthly time series observations. Please note that the value of N varies across countries, depending on the availability of coverage data for each country during that period, as indicated in Table 1.

Table 3: Summary Statistics for Libor Based Hairy Premium (HP)
(monthly frequency and in percentage points)

Panel A: January 1, 1991 - May 1, 2023

Country	HP	N	Mean	SD	Min	Max
EUR	10-year Libor-based	172	2.8	0.7	0.8	4.6
	5-year Libor-based	232	1.4	0.9	-0.3	3.8
	2-year Libor-based	268	0.5	0.7	-1.1	3.3
GBP	10-year Libor-based	60	2.5	0.9	0.8	4.4
	5-year Libor-based	120	1.1	1.0	-0.3	4.3
	2-year Libor-based	245	0.3	0.7	-1.5	3.4
JPY	10-year Libor-based	44	1.3	0.2	0.7	1.4
	5-year Libor-based	104	0.4	0.2	-0.1	0.7
	2-year Libor-based	140	0.2	0.1	-0.1	0.4
USD	10-year Libor-based	268	3.0	0.9	0.7	4.7
	5-year Libor-based	328	1.6	1.3	-1.0	4.9
	2-year Libor-based	364	0.7	1.1	-1.6	3.5

Note: N represents the number of monthly time series observations. Please note that the value of N varies across countries, depending on the availability of coverage data for each country during that period.

Table 4: Relationship between the US 10-year Hairy Premium (HP) and US Interest Rate Term Structure and Yield Curve Steepness Variables (in levels at monthly frequency)

	<i>Dependent variable:</i>			
	10-year HP			
	(1)	(2)	(3)	(4)
3-month CMR	0.204** (0.071)		-0.398** (0.152)	
10-year CMR		0.331*** (0.029)	0.745*** (0.179)	
(10-year - 3-month CMR)				0.329*** (0.069)
Const.	0.647 (0.383)	-0.471 (0.498)	-1.146* (0.560)	1.212*** (0.350)
Observations	641	641	641	641
R ²	0.081	0.166	0.215	0.034

Note: *** p<0.01, ** p<0.05, * p<0.1.

HAC-adjusted SE at conservative 12-monthly lags are shown in parenthesis for all specifications except the one with the difference in rates (10-year - 3-month CMR), for which Robust SE is reported. This is because this variable is not persistent, making HAC-adjusted SE inadequate.

Table 5: Relationship between the US 10-year Hairy Premium (HP) and Realized and Expected Inflation (in changes at monthly frequency)

	<i>Dependent variable:</i>				
	$\Delta 10\text{-year HP}$				
	(1)	(2)	(3)	(4)	(5)
<u><i>Long-term Expectations:</i></u>					
Δ 10 year Fed of Cleveland Mean	1.316*** (0.082)		1.315*** (0.083)		
Δ 5-10 year UofM Mean		0.210*** (0.066)	0.006 (0.062)		
SD of 5-10 year UofM Mean		0.0006 (0.256)			
<u><i>Short-term Expectations:</i></u>					
Δ 12 month UofM Mean				0.064* (0.035)	
<u><i>Realized:</i></u>					
Δ CPI					0.115*** (0.029)
Const.	0.001 (0.011)	0.005 (0.016)	0.001 (0.011)	0.006 (0.016)	0.0003 (0.011)
Observations	376	411	376	424	640
R ²	0.410	0.025	0.410	0.008	0.024

Note: *p<0.1; **p<0.05; ***p<0.01. Robust SE are shown in parenthesis.

Table 6: Relationship between the US 10-year Hairy Premium (HP) and other Economic Indicators (in levels at monthly frequency)

	<i>Dependent variable:</i>					
	10-year HP					
	(1)	(2)	(3)	(4)	(5)	(6)
Recession Dummy	0.838*** (0.245)		1.079*** (0.218)			
FRED Economic Policy Uncertainty Index		-0.009*** (0.001)	-0.010*** (0.002)			
CFNAI				-0.159 (0.100)		
FRED Economic Policy Uncertainty Index: Monetary					-0.001 (0.001)	
FRED Equity Market-related Economic Uncertainty Index						-0.001 (0.001)
Constant	2.721*** (0.099)	3.790*** (0.123)	3.707*** (0.197)	2.849*** (0.055)	2.930*** (0.106)	2.920*** (0.092)
Observations	98	294	98	294	294	294
R ²	0.109	0.188	0.328	0.009	0.002	0.002

Note: *** p<0.01, ** p<0.05, * p<0.1.

HAC-adjusted SE at 12-monthly lags are shown in parenthesis for all specifications.

Table 7: Summary Statistics for the US 10-year Hairy Premium (HP) and US Term Premiums Estimated by HW, CPP, and ACM at Different Periods (in levels at monthly frequency)

Period: 1991-2023

	N	Mean	SD	Min	Max
HP	270	3.03	0.84	1.07	4.77
HW	308	0.65	0.96	-1.66	4.22
CPP	383	-0.49	1.19	-3.23	2.53
ACM	392	1.12	1.25	-1.38	4.09

Period: 1961-2023

	N	Mean	SD	Min	Max
HP	630	2.00	2.18	-1.86	7.57
HW	308	0.65	0.96	-1.66	4.22
CPP	616	0.00	2.16	-6.69	6.37
ACM	747	1.52	1.37	-1.38	5.29

Table 8: VECM Relationship between the US 10-year Hairy Premium (HP) and US Term Premiums Estimated by HW, CPP, and ACM (in levels at monthly frequency)

Cointegrating Equation			
Coefficient	Estimate		
$\beta(\text{HP})$	1.00	<i>(Normalized)</i>	
$\beta(\text{CPP})$	-0.63		
$\beta(\text{ACM})$	0.84		
$\beta(\text{HW})$	-0.80		
Error Correction			
Coefficient	Estimate	Std. Error	t-value
$\alpha(\text{HP})$	-0.02	0.03	-0.59
$\alpha(\text{CPP})$	0.16***	0.04	3.69
$\alpha(\text{ACM})$	0.04	0.04	1.01
$\alpha(\text{HW})$	0.12**	0.04	2.67
Observations	186		

Note: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

The optimal lag order (1) and order of variables selected are based on the Akaike Information Criterion (AIC).

Table 9: % Forecast Error Variance Decomposition (FEVD) of *HP* for Months 1 to 36 (3 years)

Months Ahead	HP	CPP	ACM	HW
1	100.00	0.00	0.00	0.00
2	88.22	0.32	7.29	4.17
3	85.61	0.60	6.60	7.20
4	86.28	0.99	5.25	7.47
5	86.58	1.48	4.63	7.31
6	86.42	1.93	4.35	7.30
7	86.33	2.32	4.08	7.28
8	86.29	2.67	3.85	7.19
9	86.24	2.98	3.68	7.10
10	86.17	3.25	3.55	7.03
11	86.11	3.49	3.44	6.97
12	86.06	3.70	3.34	6.90
13	86.02	3.88	3.25	6.85
14	85.97	4.05	3.18	6.80
15	85.93	4.20	3.11	6.75
16	85.90	4.33	3.06	6.71
17	85.87	4.45	3.01	6.67
18	85.84	4.56	2.96	6.64
19	85.81	4.66	2.92	6.61
20	85.79	4.75	2.88	6.58
21	85.77	4.83	2.85	6.56
22	85.75	4.91	2.81	6.53
23	85.73	4.97	2.78	6.51
24	85.71	5.04	2.76	6.49
25	85.70	5.10	2.73	6.47
26	85.68	5.15	2.71	6.46
27	85.67	5.20	2.69	6.44
28	85.66	5.25	2.67	6.42
29	85.64	5.29	2.65	6.41
30	85.63	5.34	2.63	6.40
31	85.62	5.38	2.62	6.39
32	85.61	5.41	2.60	6.37
33	85.60	5.45	2.59	6.36
34	85.59	5.48	2.57	6.35
35	85.59	5.51	2.56	6.34
36	85.58	5.54	2.55	6.33
37	85.57	5.57	2.54	6.32

Figure 1: The Hairy Graph

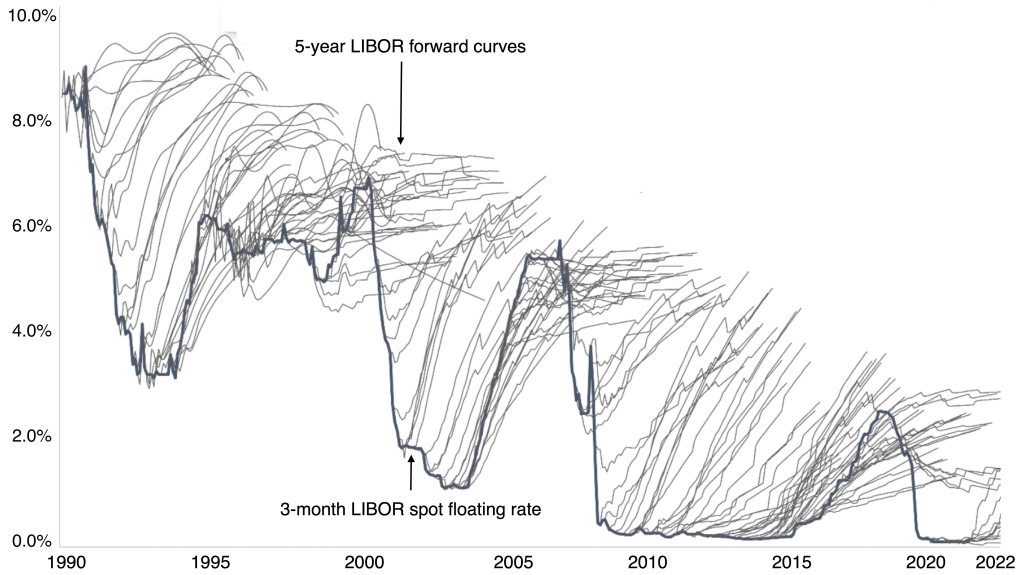


Figure 2: The Hairy Premium to Fix for 10-Year Tenor

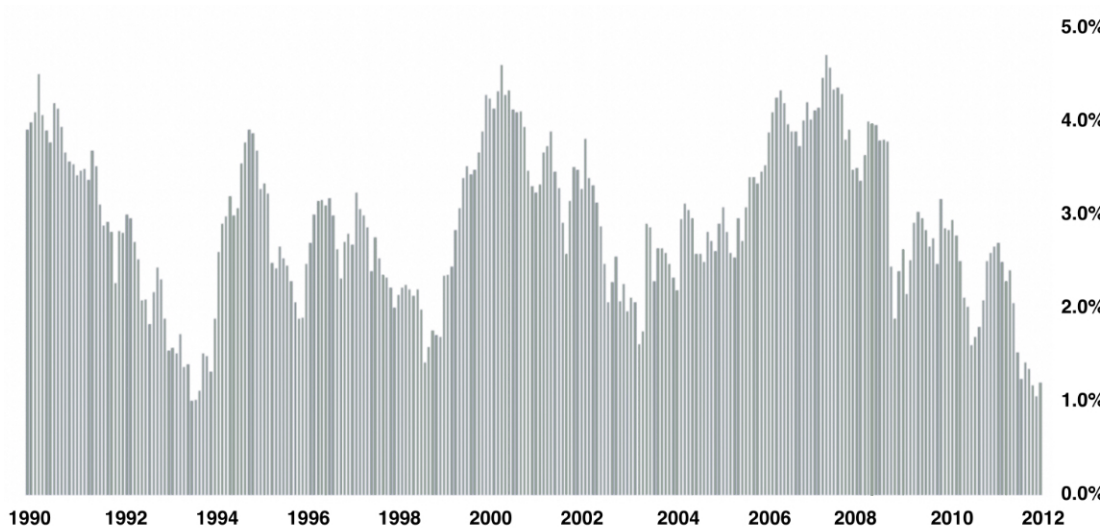


Figure 3: 10-Year Tenor - Fixed vs. Floating

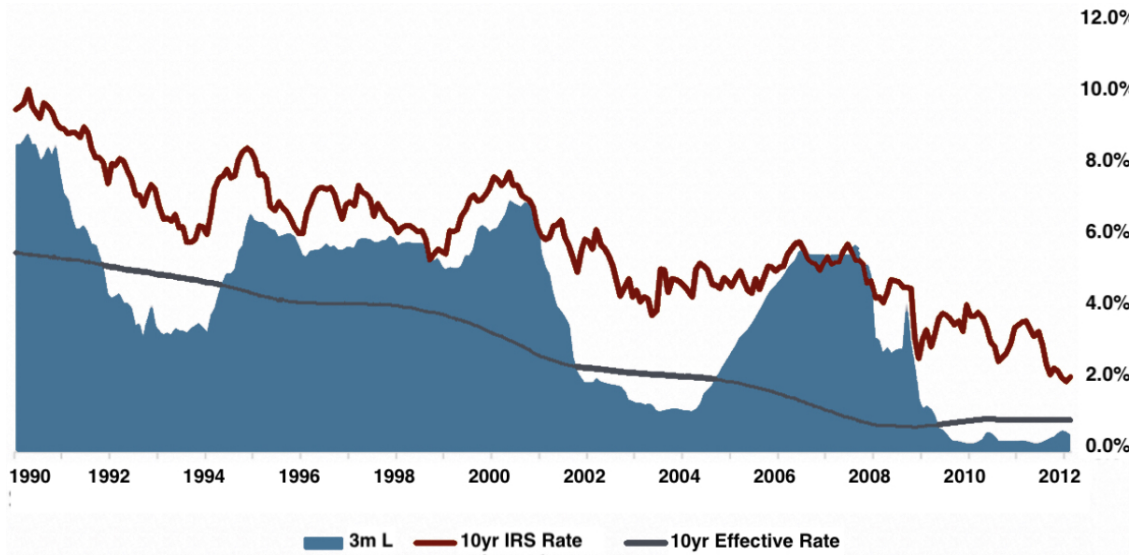


Figure 4: Five-Year Tenor - Fixed vs. Floating

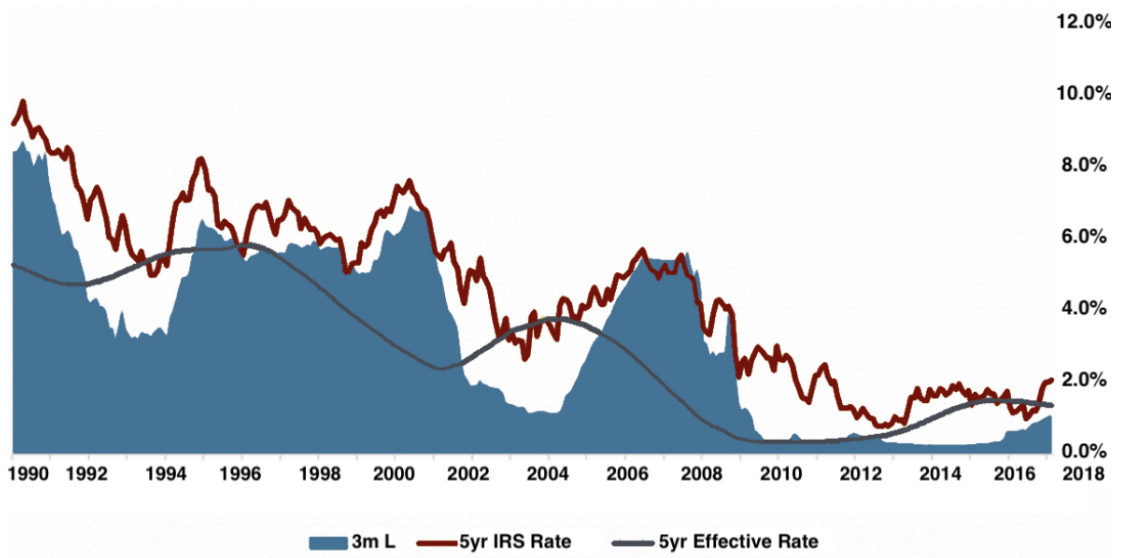


Figure 5: The Hairy Premium to Fix for Five-Year Tenor

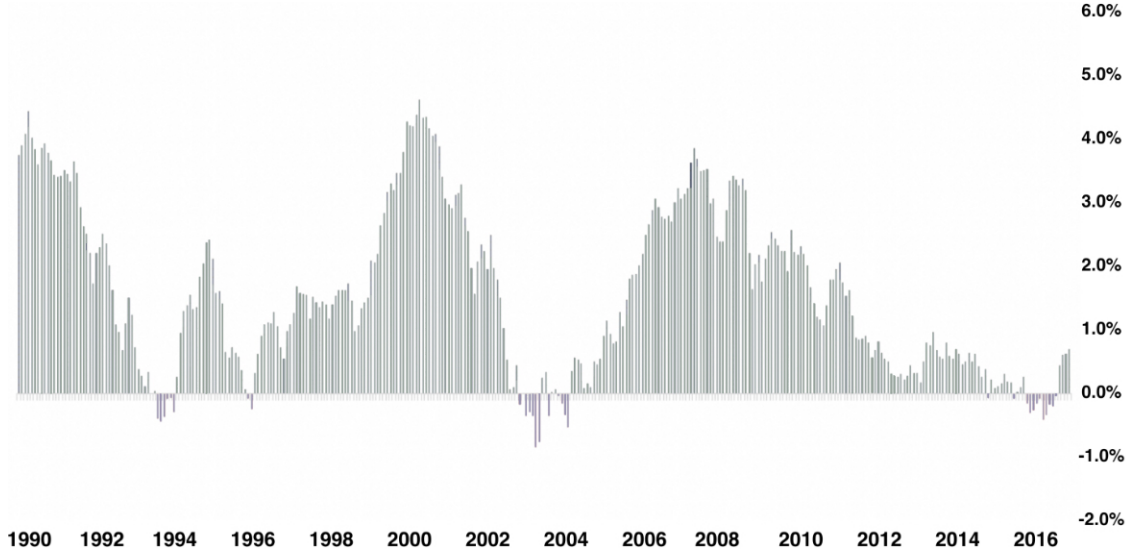


Figure 6: Periods of Monetary Tightening

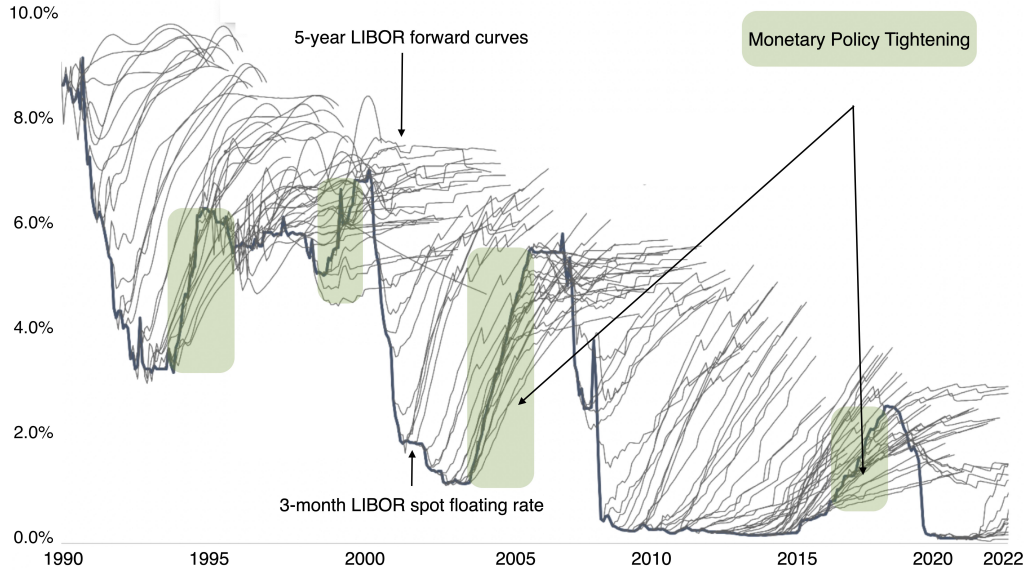


Figure 7: US CMR-based 10-year Hairy Premium and US Economic Recessions

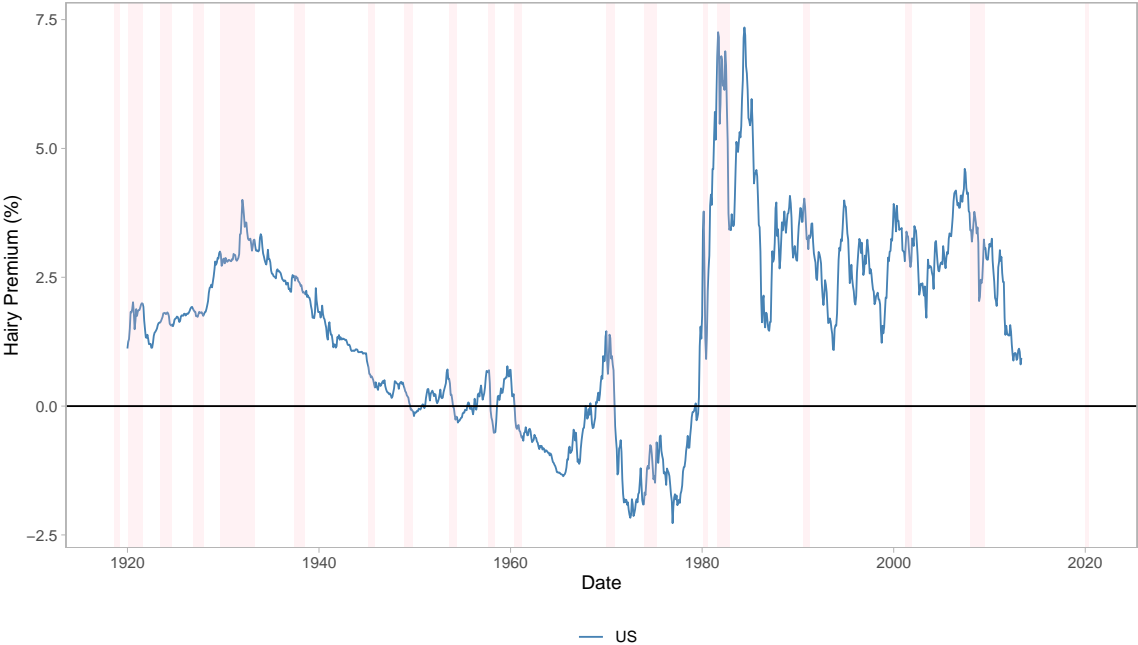


Figure 8

10-year Realized Rate vs. 10-year Forecasted Rate from Fed's Survey of Professional Forecasters

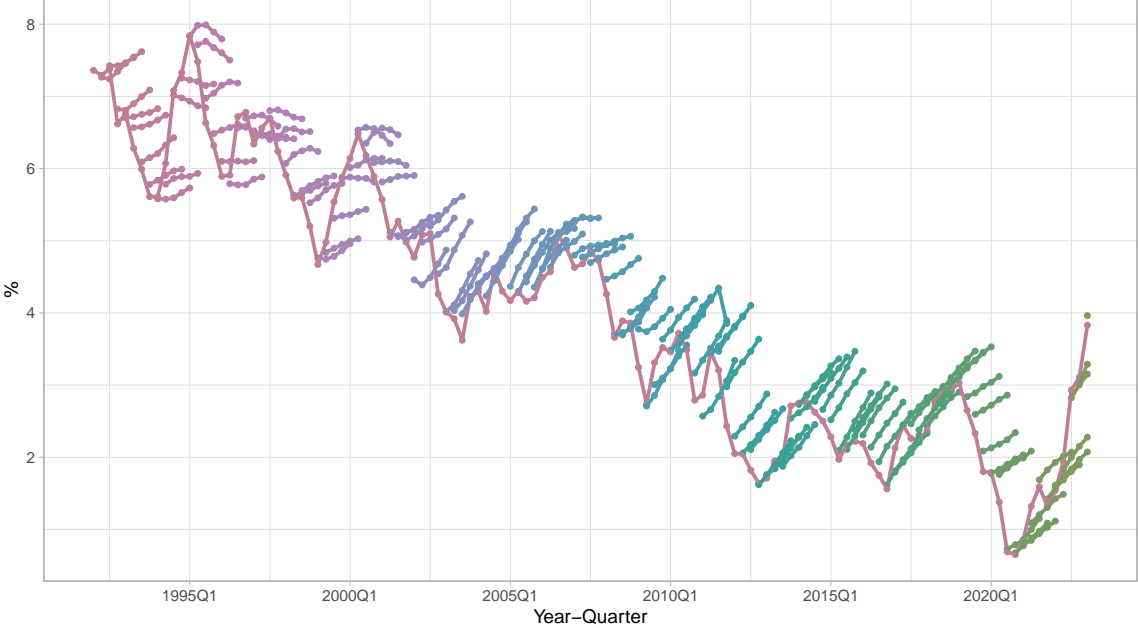


Figure 9: CMR-based 10-year Hairy Premium in G11 Countries and US Economic Recessions

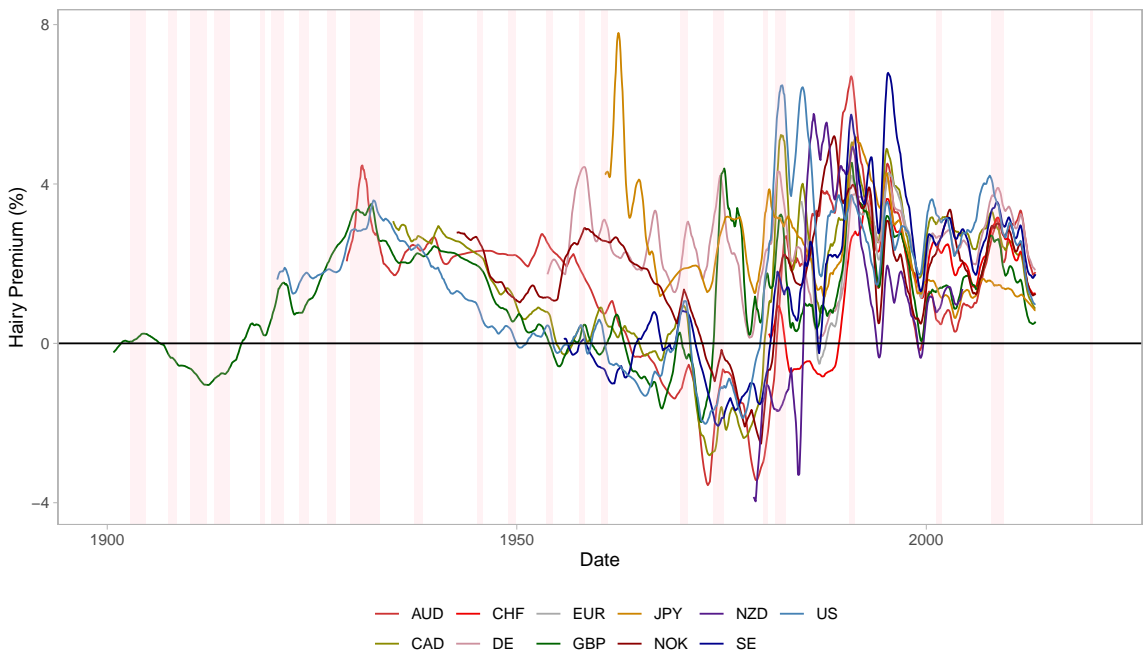


Figure 10: Each Principal Component % Contribution to the Variation of the 10-year Hairy Premiums of G11 Economies

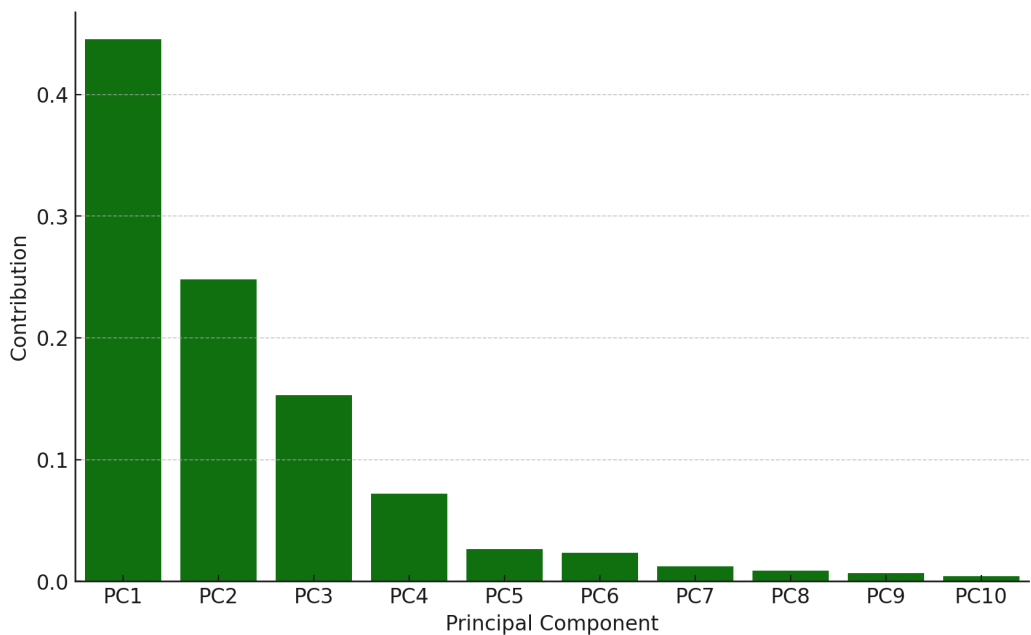


Figure 11: Each Country Sign and % Loading on the 1st , 2nd, and 3rd Principal Component

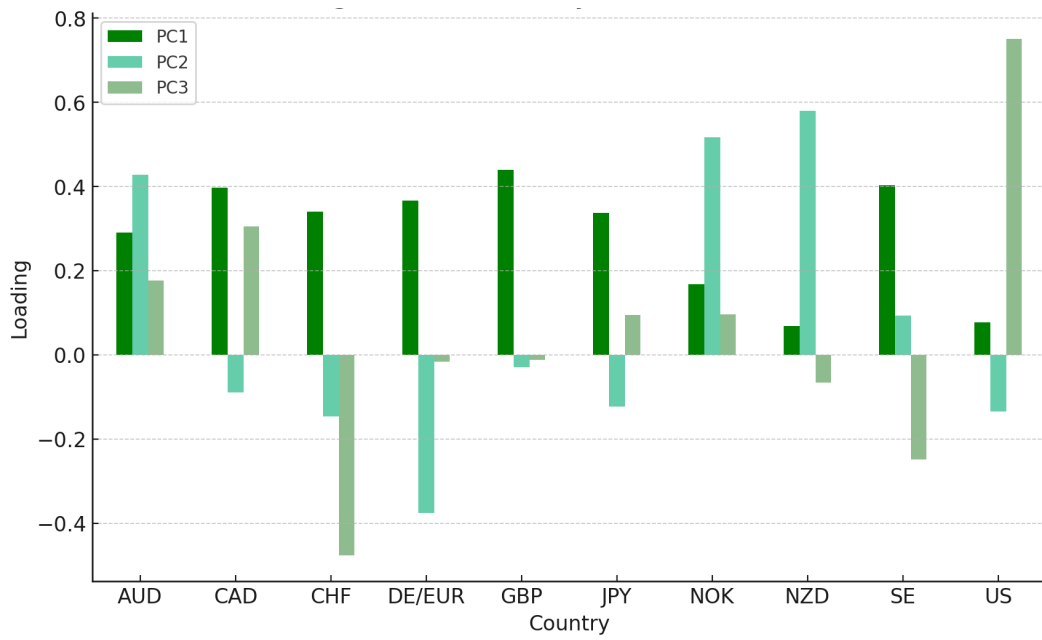


Figure 12: Comparison between 10-year model-free Hairy premium (HP) and model-based [Cox, Ingersoll, and Ross \(1985\)](#) term premium (CIR), [Kim and Wright \(2005\)](#) term premium (HW), [Cochrane and Piazzesi \(2008\)](#) term premium (CPP), and [Adrian, Crump, and Moench \(2013\)](#) term premium (ACM) over Time

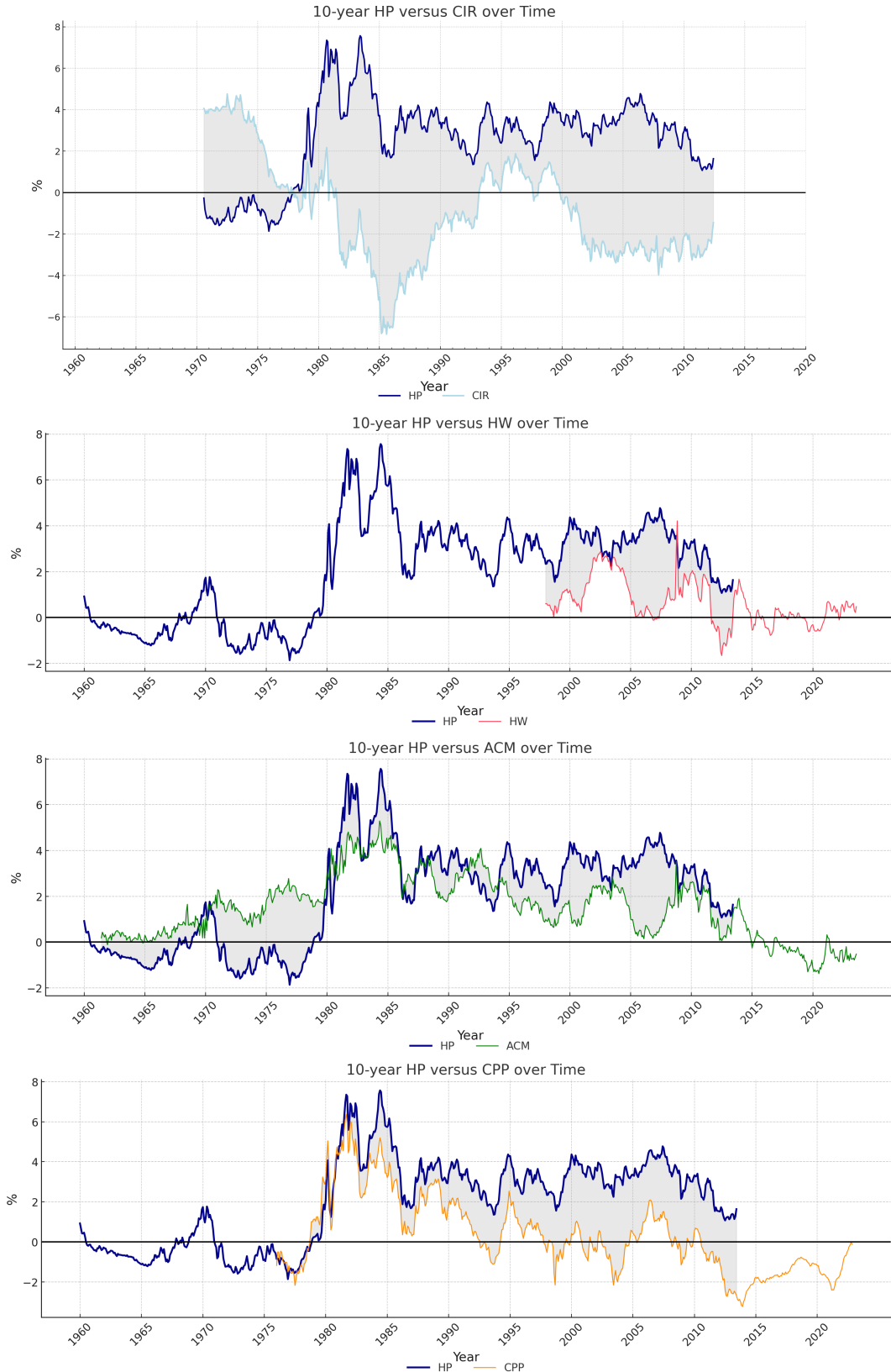


Figure 13: Comparison between 10-year model-free Hairy premium (HP) and Hairy premium-based Regime-Switching CIR model term premium (CIRHP) over Time

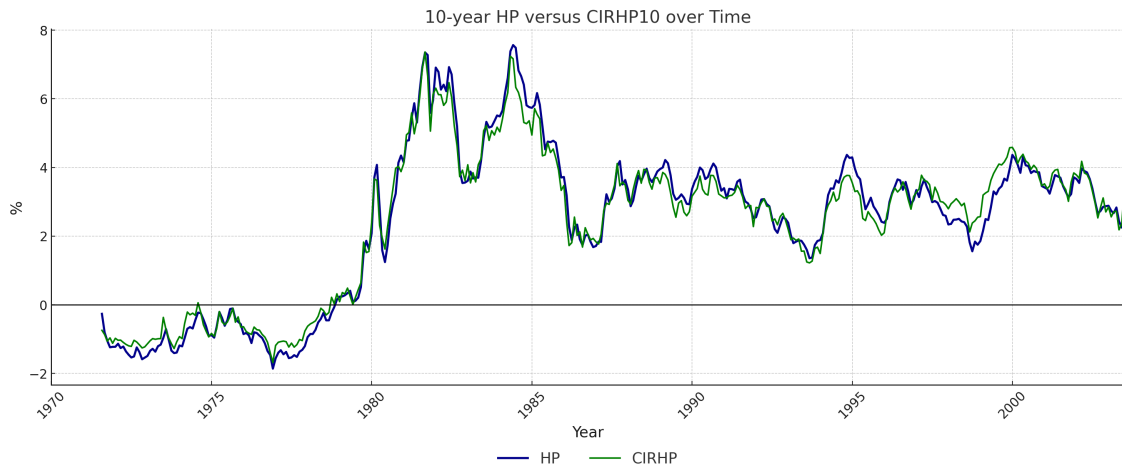


Figure 14: Variance Decomposition of 10-year Hairy premium (HP)

