# DATA AND MARKUPS: A MACRO-FINANCE PERSPECTIVE[*]

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**Abstract**

How can we measure the extent to which data-intensive firms exert market power? Economists typically look to markups as evidence of market power. Using a model with firms that price risk in their production decisions, we highlight the competing forces that make markups an unreliable measure of data-derived market power. Instead, we show how markups measured at different levels of aggregation reflect data and distinguish data from other intangible investments. These findings both reconcile seemingly contradictory empirical markup evidence and guide us to new ways of measuring data and its effects on markets.

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**JEL**. C6. D4. D5. L1.

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# I Introduction

Changes in firms' market power and the sources of those changes have become the focus of intense debate. Economists point to rising markups, economies of scale in information and the dominance of large, data-intensive firms as evidence that the unequal accumulation of data is responsible for a decline in competition (Jarsulic 2019). How should one measure the extent of this effect? To explore the link between data, competition, and measurable outcomes, we craft a model in which economies of scale in data induce a data-rich firm to invest in producing at a lower marginal cost and in capturing a larger market share. The model uncovers a rich set of interactions between data and market power measures, that make a markup measure a poor indicator of data-driven market competition. However, the model also reveals that the trade-off between data's competing effects on markups depends on the level of aggregation at which markups are measured. This makes the difference between product, firm and various industry-level markups a useful metric of data's effects. The model's predictions for markup measure divergence are consistently supported by existing empirical findings: Markups at the product, firm, and industry level diverge in trend and their cyclicality. Not only does the model reconcile seemingly-conflicting facts, it also teaches us that the difference in markups aggregated in different ways is exactly where we should be looking to see the effects of data.

To develop a tool to identify and understand the effects of data, Section II formulates a new framework, drawing on tools from multiple fields. As in macro theory, data is modeled as information; as in corporate finance, firms price risk; as in industrial organization theory, firms exploit market power. Data is digitized information. The essence of information is that it reduces uncertainty. Just like a larger data set reduces the econometrician's standard error, more data for firms reduces their forecast errors. Making future events more predictable allows the firm to make better decisions that raise expected profits, and also means the firm faces less uncertainty and less risk. While abstracting from risk is appropriate to study many questions, abstracting from risk and the price of risk when studying data removes its essential character. Since risk is central to the study of data, we adopt basic tools from corporate finance to model the ways in which firms use risk pricing to guide their production and investment decisions. Although the assumption that firms price risk is unusual in the firm competition literature, it is a bedrock principle of the field of corporate finance (Brealey, Myers, and Allen 2003; Eckbo 2008). That being said, our main results about markup divergence do not require priced risk. The realism of data-intensive firms that grow large and the ambiguity of data's effect on markups does.

When firms price risk in their decisions, markups conflate data and its competitive effects. A firm that prices risk requires a return to induce them to engage in risky production. This return is derived from their markup. In other words, markups compensate firms for the risk they bear. If firms use data to improve their forecasts and make their revenues more predictable and less risky, then these firms require less return to bear less risk. So, holding the size of the firm fixed, more data reduces markups. We call this the "risk channel."

However, data also makes it more profitable to grow large and develop a dominant market position. In the model, firms choose an up-front investment, which lowers their future marginal cost of production. Because the benefits of production are unknown, this up-front investment is risky. When data lowers that risk by predicting future demand, firms invest more. This is *investment-data complementarity*. More investment means the firm grows larger, produces at lower marginal cost, and earns higher markups. The notion that investment in cost reduction is a source of market power is in line with the view of Sutton (1991, 2001). In his language, our firms strategically use data to further differentiate themselves and thus create a dominant position. This force whereby data increases markups is what we call the "investment channel." The idea that there are socially good and bad aspects to markups, and that the balance between the two may change over time, is consistent with the evidence of Covarrubias, Gutiérrez, and Philippon (2020).

Section III shows that when a firm has more data, these two forces—the risk and investment channels—push markups in opposite directions. Therefore, markups are not a reliable indicator of data or its competitive effects. Furthermore, this trade-off between efficiency and risk compensation is similar to the trade-off between efficiency and compensation for fixed costs articulated in Weiss (2000). These two mechanisms would be difficult to distinguish based on product markups alone.

There are features of markups that allow us to identify the unique characteristics of data relative to other assets. Specifically, the growing volume of data can cause markups aggregated in different ways to diverge. The reason is that data facilitates prediction. Better predictions alter the composition of products and firms. Firms use data to adjust production. They produce more of goods that their data predicts are likely to be profitable. Of course, profitable goods are high-markup goods. Thus, even if two firms sell identical goods at identical prices with identical marginal costs, the firm with more data will be measured as a higher-markup firm because that firm uses data to skew the composition of its goods production toward higher-markup goods. Not only can data explain composition effects in markups, but some sort of information is also required to explain the change in composition. Every firm would like to produce more of the more profitable goods and less of the less profitable ones. This is only a feasible strategy if the firm can predict what will be profitable and what will not. Good prediction requires information.

Data also causes the markup of an average firm and its industry to diverge. Data-investment complementarity ensures that high-data firms invest more and sell more. But if these high-data firms also have high firm-level markups because they skew the composition of their goods toward high-markup goods, then high-markup firms are also larger firms. This creates another aggregation issue. The industry markup is likely to be higher than the average firm's markup because high-markup firms are bigger and more heavily weighted. Section III shows that the more data firms have, the more of a wedge will arise between product-level markups, firm-level markups, and industry markups measured as cost-weighted or sales-weighted aggregates. These wedges can then be used to back out the amount of data a firm has or the average amount of data held in an industry.

The fact that growing data creates wedges between various markup measures is not a mere curiosity. Such wedges exist in the data and are growing. Thus the model helps explain a curious feature of the data that has been at the heart of a debate about growing markups. From one perspective, markets are just as competitive today as in the past because good-level markups are stable (see for example Anderson, Rebelo, and Wong [2018]). Instead, growing firm-level and industry markups are evidence of declining competition (see Gutiérrez and Philippon [2016]; Furman and Orszag [2015]; Grullon, Larkin, and Michaely [2016]; De Loecker, Eeckhout, and Unger [2020]). Moreover, the distribution of markups and market shares has become more skewed, and as a result, the aggregation of markups gives rise to a different evolution of industry markups (see Hall [2018]). These facts validate our model. In turn, the model lends an economic interpretation to these facts beyond the explanation that composition effects must be at work.

Ultimately, most researchers are interested in markups because they are concerned about consumer welfare. Section IV discusses the relationship between markups, competitive outcomes, and welfare. Rising amounts of data can be good for consumers. After all, firms use data to produce more of goods that consumers want most. However, welfare may suffer when firms' data stocks become asymmetric. Our model can help to quantify that trade-off.

Another unexpected prediction of this model may help to reconcile an empirical debate about whether markups are pro- or countercyclical. This debate is central to the relevance of New Keynesian models. We find that data-intensive firms may have procyclical product markups, as in Nekarda and Ramey (2020), but countercyclical firm and industry markups (Bils 1985, 1987). In section VI, recessions are times when demand is lower on average, but also more volatile. The lower demand lowers markups. When demand is more volatile, firms that can use data to identify which product is currently in high demand can better adjust output to increase the firm's markup and profit by more than other firms. In short, higher volatility raises firm and industry markups because it creates a potential for larger composition effects. Understanding why markups measured differently have different properties allows researchers to determine which set of facts is most relevant for a given question.

The static model we explore is just one slice of a data economy. A central concern about market power for digital firms is that larger firms can generate more data, which reinforces their competitive position. To understand how this dynamic dovetails with our analysis, Section VII makes our static model dynamic. When data is a by-product of firms' economic activity, this changes markups. A new term arises that pushes prices and markups down. Firms that value data want to do more transactions to generate more data. To do more transactions, a firm must lower its price. The optimal production decision for a firm reveals that price and the marginal value of data enter as substitutes. Firms are effectively paid either with money or with their customers' data. This finding relates to the study of free digital goods. The idea that price does not fully capture the value of a transaction to a firm also provides one more reason that markups fail to capture market power in a data economy.

Finally, we turn to measurement. Since our contribution is to use theory to reinterpret and repurpose existing facts, we do not measure or contribute new facts. We show that if firms are increasingly using data to predict their profits, then subtle differences in how economists measure markups can deliver starkly different trends and cyclical patterns. So far, the debate surrounding these different patterns has been about the merits of measuring with micro or macro data. Our explanation—that the differences arise from firms' use of data—reframes this debate and lends new meaning to the facts. The differences no longer represent a mistake made by one group or the other, but rather an interesting and useful measure of the quantity of firms' data.

Section VIII offers guidance for how this framework might be used to measure data or the market power arising from that data. Our model teaches us that the difference between firm and product markups is a sufficient statistic for the amount of relevant data a profit-maximizing firm has about consumer demand. This would enable a sufficient statistics approach to measuring firms' data. The model could also be used for structural estimation. We discuss techniques to estimate firms' price of risk and approaches to measuring product characteristics, and we map the markup measures in our model to different empirical approaches in the markup literature.

RELATED LITERATURE Because we model data as digitized information, our tools are most similar to those in the information frictions literature in macroeconomics. Work by Lorenzoni (2009), Angeletos and La'O (2013), Asriyan, Laeven, and Martin (2022), David and Venkateswaran (2019), Nimark (2014) and Maćkowiak and Wiederholt (2009) feature similar information frictions, used to explain features of business cycles. Work by Rostek and Weretka (2012) uses similar tools to explore a reverse question: the effect of market size and market power on price informativeness. Similar tools are used in models of banking competition as well (Vives and Ye 2021), where banks use information for forecasting and pricing risk. However, banks differ from firms: while goods-producing firms choose freely how many units of a good to produce, lenders typically cannot lend twice the requested amount to a promising borrower. The ability to scale production in response to information is central to our study of market power.

Existing work on the digital economy explores whether data can be a source of market power. Kwon, Ma, and Zimmermann (2022) argue that the timing and degree of rising concentration in an industry correlate closely with the industry's investment in information technology. Jones and Tonetti (2020) explore what data ownership facilitates economic growth. In Kirpalani and Philippon (2020), data enables directed two-sided search. Acemoglu et al. (2022) and Bergemann and Bonatti (2019) model data as information and explore whether data markets are efficient. Ichihashi (2020) shows how firms can use consumer data to price discriminate, while Liang and Madsen (2021) explore the use of data in labor markets. De Ridder (2021) considers information technology to be something that raises fixed costs and reduces marginal costs. We do not dispute that data can be used for all of these purposes. However, at its essence, data is digitized information; information is used to reduce uncertainty or risk. The new elements we introduce are a product portfolio choice and non-indifference to risk. Both are well supported by evidence and central to our results that advance this literature by offering a new approach to data measurement and new considerations in welfare.

Our work obviously speaks to the large literature on markup measurement and complements it by providing new interpretations of results about trends and fluctuations in markups. Some new papers model the mechanisms that give rise to trending markups (see for example De Loecker, Eeckhout, and Mongey [2021]). Those models and Edmond, Midrigan, and Xu (2019) evaluate the welfare consequences of markups. Our approach differs because we explore the role of firms' data.

Empirical work on the data economy often necessarily focuses on specific markets.[1] Lambrecht and Tucker (2015) take a strategy perspective on whether data has the necessary features to confer market power. Similarly, Goldfarb and Tucker (2017) discuss the many ways in which this digital economy is transformative.

Recent work by Burstein, Carvalho, and Grassi (2020) analyzes the business cycle properties of markups. They show analytically how the sign of markup cyclicality varies with aggregation and

[1] Athey, Mobius, and Pal (2017) and Athey and Gans (2010) examine media competition; Brynjolfsson, Hu, and Smith (2003) study booksellers; and Rajgopal, Srivastava, and Zhao (2021) measure digital technology firms. de Cornière and Taylor (2020) categorize uses of data as pro- or anticompetitive.

they establish the economic importance of these markups. Our results complement these insights by proposing a specific mechanism that causes markups to fluctuate, one that is rooted in how firms use data to gain a competitive advantage.

# II Model

To explore the idea that data can create market power, we build a model with a few key features. First, firms face uncertainty about consumer demand. It is not essential that uncertainty is about demand, rather than advertising, hiring, product placement or costs. We simply need a variable that is profit-relevant and uncertain. Second, data is used to resolve this uncertainty. Data is used to predict the profitability of various actions. Third, firms face a cost of bearing risk. This price of risk is what governs the magnitude of the link between data, uncertainty, and investment. Fourth, to explore the relationship between data and the composition of the goods a firm produces, we model firms that choose quantities of multiple goods. Allowing those goods to have correlated attributes, as in Pellegrino (2020), makes data relevant to multiple goods. Finally, since the data competition hypothesis is about high-data firms growing large, we allow firms to choose an initial investment, which reduces their marginal cost of production. This allows us to explore if high-data firms invest to operate at a larger scale and thus grow to have more market power.

We first explore these features in a static model. Since our question is about what effects data has on competition measures, we take data to be exogenous and move it around in the model to observe its effect. Later, Section VII introduces dynamics and endogenizes data as a by-product of economic activity and something that can be purchased or sold. The forces we describe here will survive in that dynamic setting.

## II.A Setup

FIRMS There are $n_F$ firms, indexed by $i$: $i \in \{1, 2, \ldots, n_F\}$. Each firm chooses the number of units of each good they want to produce, an $N \times 1$ vector $\mathbf{q}_i$, to maximize risk-adjusted profit, where the price of risk is $\rho_i$.

$$U_i = \mathbf{E}\left[\pi_i | \mathcal{I}_i\right] - \frac{\rho_i}{2} \mathbf{Var}\left[\pi_i | \mathcal{I}_i\right] - g(\chi_c, \widetilde{\boldsymbol{c}}_i). \tag{1}$$

This mean-variance objective is consistent with empirical corporate finance evidence on firms' decisions (Eckbo 2008) with macro evidence on firms' investment response to uncertainty Kumar, Gorodnichenko, and Coibion (2023) and is a second-order approximation to a broader class of utility functions.

Firm production profit $\pi_i$ depends on quantities of each good, $\mathbf{q}_i$, the market price of each good, $\mathbf{p}$, and the marginal cost of production of that good, $\boldsymbol{c}_i$:

$$\pi_i = q_i' \left( \mathbf{p} - \boldsymbol{c}_i \right). \tag{2}$$

Prior to observing any of their data, each firm chooses an up-front investment. Let $\widetilde{\boldsymbol{c}}_i$ be the vector of marginal production costs for a unit of each attribute. The up-front investment choice is modeled as a choice of $\widetilde{\boldsymbol{c}}_i$ at an investment cost $g(\chi_c, \widetilde{\boldsymbol{c}}_i)$ to maximize $E[U_i]$. Assume that $g(\chi_c, \widetilde{\boldsymbol{c}}_i)$ is additively separable in attributes and strictly decreasing in $\widetilde{\boldsymbol{c}}_i$. Since lower choices of $\widetilde{\boldsymbol{c}}_i$ require a greater up-front investment, we interpret this as choosing a larger firm. Since we want to interpret $\chi_c$ as a parameter that governs the marginal cost of investment, we impose $\partial^2 g / \partial \chi_c \partial c_i < 0$. To guarantee non-negative interior marginal cost choices, one could impose $g\left(\chi_c, \widetilde{c}_i\right)$ is convex over $\widetilde{c}_i$, with $g\left(\chi_c, \bar{c}\right) = 0$ and $\lim_{\widetilde{c} \to 0} g\left(\chi_c, \widetilde{c}\right) = +\infty$. However, most of our results will not require this.

Products and Attributes   The product space has $N$ attributes, indexed by $j \in \{1, 2, \ldots, N\}$. Goods, indexed by $k$, are combinations of attributes.

Each good $k \in \{1, 2, \ldots, N\}$ can be represented as an $N \times 1$ vector $\boldsymbol{a}_k$ of weights that good places on each attribute. The $j$th entry of vector $\boldsymbol{a}_k$ describes how much of attribute $j$ the $k$th good requires. This collection of weights describes a good's location in the product space. Let the collection of $\boldsymbol{a}_k$'s for each good $k$ be an $N \times N$, full-rank matrix $A$, such that

$$\mathbf{q}_i = A \tilde{\mathbf{q}}_i. \tag{3}$$

Conversely, the quantity of attributes that a firm $i$ produces is a vector $\tilde{\mathbf{q}}_i$, with $j$th element $\tilde{q}_{ij}$. The attribute vector is the vector of firm $i$'s product quantities, $q_i$, times the inverse attribute matrix: $\tilde{\mathbf{q}}_i = A^{-1} \mathbf{q}_i$.

For now, the mapping between attributes and products is fixed. Initially, we can equate goods and attributes. Later, we allow firms to choose how to position their product in the product space by choosing $A$'s.

The marginal cost of producing a good depends on the up-front investment the firm makes and on that good's attributes. The firm's up-front investment of $g(\chi_c, \widetilde{\boldsymbol{c}}_i)$ allows it to produce each attribute $j$ at a unit cost of $\widetilde{c}_{ij}$. The vector $\widetilde{\boldsymbol{c}}_i$ is the $N$-by-1 vector of all marginal production costs of firm $i$ for each attribute. The vector $\boldsymbol{c}_i = A \widetilde{\boldsymbol{c}}_i$ is the vector of firm $i$'s marginal cost for each product. The cost of producing a unit of good $k$ for firm $i$ is therefore $\boldsymbol{c}_i = \boldsymbol{a}_k' \widetilde{\boldsymbol{c}}_i$. To keep the investment problem bounded, the investment cost function $g$ is convex in each element $\widetilde{c}_{ij}$.

PRICE Our demand system embodies the idea that goods with similar attributes are partial substitutes for each other. Therefore, the price of good $i$ can depend on the amount every firm produces of every good.

Each attribute $j$ has an average market price that depends on an attribute-specific constant and on the total quantity of that attribute that all firms produce:

$$\tilde{p}_j^M = \bar{p}_j - \frac{1}{\phi}\sum_{i=1}^{n_F} \tilde{q}_{ij}. \tag{4}$$

Each firm does not receive the market price, but rather faces an uncertain price that depends on a demand shock $\mathbf{b}_i$. The demand shock $\mathbf{b}_i$ is a vector with $j$th element $b_{ij}$. This vector is random and unknown to the firm: $\mathbf{b}_i \sim N(0, I)$.[2] Demand shocks can covary across firms: $\xi = \boldsymbol{Cov}(b_i, b_l)\ \forall i \neq l$. The price a firm receives for a unit of attribute $j$ is thus $\tilde{p}_j + b_{ij}$. We can express firm $i$'s price in vector form as

$$\tilde{\mathbf{p}}_i = \left[\tilde{p}_1^M, \tilde{p}_2^M, \ldots, \tilde{p}_N^M\right]' + b_i. \tag{5}$$

The firms's price of each good depends on its attributes. The price of good $k$ is the units of each attribute $a_{jk}$ times the price of each attribute $\tilde{p}_j$, summed over all the attributes:

$$p_k = \sum_{j=1}^{N} a_{jk}\tilde{p}_j. \tag{6}$$

INFORMATION Each firm generates $n_{di}$ data points. Each data point is a signal about the demands for each attribute: $\tilde{\mathbf{s}}_{i,z} = \boldsymbol{b}_i + \tilde{\boldsymbol{\varepsilon}}_{i,z}$, where $\tilde{\boldsymbol{\varepsilon}}_{i,z} \sim N(\mathbf{0}, I)$ is an $N \times 1$ vector. Signal noises are uncorrelated across attributes and across firms. [3]

Because we are interested in how data affects competition, we will take data ($n_{di}$ and $\tilde{\Sigma}_e$) as given and we exogenously change the amount of firms' data, $n_{di}$. Section VII explores what aspects of the results change when data is endogenously generated as a by-product of economic transactions.

We consider two possible information structures. In our baseline case, firms observe only their own data: $\mathcal{I}_i = \{\tilde{\mathbf{s}}_{i,z}\}_{z=1}^{ndi}$. However, we will also consider the case of public data. Public data means that all firms can observe every piece of data about every firm $\mathcal{I}_i = \{\{\tilde{\mathbf{s}}_{i',z}\}_{z=1}^{ndi}\}_{i'=1}^{nF}$. This structure simplifies results and clarifies that the mechanism does not rely on asymmetric information. When

[2]The fact that the variance matrix is diagonal is without loss. We simply define attributes to be an orthogonal decomposition of the demand variance-covariance space. We investigate the effect of changing $Var(b_i)$ in Section VI.

[3]For now, we assume each firm has $n_{di}$ data points about every attribute. Section VII relaxes this and allows data to differ by attribute. The assumption of signal variance $I$ is then without loss. By Bayes' law, two independent data points function in the same way as one data point with twice the precision. By assuming $V(\tilde{\boldsymbol{\varepsilon}}_{i,z}) = I$, we are simply letting $n_{di}$ govern data precision.

more relevant data exists about a firm $i$, then that firm will invest and produce more aggressively, regardless of who else observes that data.

EQUILIBRIUM

1. Each firm chooses a vector of marginal costs $\tilde{c}_i$, taking as given other firms' cost choices. Since the data realizations are unknown in this ex ante investment stage, the objective is the unconditional expectation of the utility in (1).

2. After observing the realized data, each firm updates beliefs with Bayes' law and then chooses the vector $\mathbf{q}_i$ of quantities to maximize conditional expected utility in (1), taking as given other firms' choices.

3. Prices clear the market for each good.

### II.B Discussion of Assumptions

Appendix C.1. solves the aggregate shock model and shows that all the main forces we identify here are present. The reason we relegate that model to the appendix is that the solution is complicated by firms' need to forecast what other firms know, as in Angeletos and Pavan (2007). In that setting, we can prove similar theoretical properties. But because the solutions are implicit functions, those results are less clear and thus less useful for expositing the main ideas.

FIRMS THAT PRICE RISK. Pricing risk ($\rho_i > 0$) is not essential. Our results cover the case where risk is not priced. However, when firms facing uncertain profit conditions scale back their investment and production plans, introducing data creates important economic trade-offs.

While risk pricing is novel in the markups literature, it is a bedrock principle of corporate finance and is well supported by numerous empirical studies in many domains.[4] Even if firms themselves are not risk averse, firms that take on risky projects will face a higher cost of capital.[5] So, the price-of-risk term could be interpreted as an adjustment to their expected profit. Specifically, Brealey, Myers, and Allen (2003) argues for a price of risk $\rho$ that matches the risk premium

[4]The Handbook of Empirical Corporate Finance fills chapter 18 with the evidence that firms price risk (Eckbo 2008). Most of the other chapters contain evidence in support of theories that are premised on risk pricing. For a textbook treatment of the topic, see Welch (2009), chapter 9. More recent work on this topic explores whether male and female CFOs are equally risk averse (Doan and Iskandar-Datta 2020). Management and psychology scholars (Lovallo et al. 2020) find that firms place too high a price on risk. In international economics, David, Ranciere, and Zeke (2022) document that multinational firms facing more risk hire less and compensate their capital owners with a greater share of income.

[5]While a risk price affects a firm's cost of borrowing, simply including a risky interest rate in marginal cost does not suffice. The risk of debt is far less than the risk to the enterprise value of the whole firm. Markups need to compensate the firm for risk to its debt and its equity.

on the S&P 500. If a firm gets less return per unit of risk than this, the firm would be better off not investing in production and instead returning the cash to investors to invest in a market portfolio of equity. Our $\rho_i$ term in (1) could capture both the price of risk and the covariance of the firm shock with market risk (the firm's beta).

In addition, our firms may price firm-specific risk because we are exploring a market with large players where firm-specific risk is not diversifiable. Furthermore, there is growing evidence that even idiosyncratic risk is priced, especially when firms face financial constraints (Hennesy and Whited 2007). Finally, it is also possible to interpret $\rho_i$ as the absolute risk aversion of a firm manager who is compensated with firm equity.

DATA ABOUT CONSUMER DEMAND. One might question whether data is used to forecast demand or marginal cost. Conceptually, it shouldn't matter. Firms that face risk from their cost structure should also face a higher cost of becoming more productive. If data helps firms reduce profit risk, whether from the cost or the revenue side, it should embolden them to invest more and produce more at a lower market price. The same forces operate. Why then choose to model demand uncertainty? Markups are price divided by marginal cost. Having the random variable in the denominator makes it nearly impossible to characterize the average value of markups. If empiricists typically studied inverse markups, then it would be more practical to study cost uncertainty.

LINEAR DEMAND. We assume a linear demand system, which is common in the information aggregation literature.[6] Not only does the linearity assumption permit transparent results, recent work also shows that the linear setup fits the data well. Our model builds on Pellegrino (2020)'s Generalized Hedonic-Linear demand system, used to study market power in a network economy (see also Galeotti et al. [2022]). A feature of this model is the declining demand elasticity in firm size. This generates realistic higher markups for larger firms. Using the nonparametric estimates of Baqaee, Farhi, and Sangani (2021) for the demand, Ederer and Pellegrino (2022) show that the linear demand system fits the data better than the iso-elastic demand system.

However, if one wanted to change the relationship between elasticity and firm size, introducing a function $\phi(c_i)$ would only change the magnitudes of our results. The same trade-offs arise.

COURNOT COMPETITION. We model conduct by firms in the market by means of Cournot competition. There are of course other sources of conduct and imperfect competition beyond Cournot

[6]See Chapters 7–8 in Veldkamp (2011) for a textbook treatment.

that lead to market power, including Bertrand competition, information frictions, transaction cost and barriers to entry. Appendix C.3. explores Bertrand competition.

GOODS AS BUNDLES OF ATTRIBUTES. We treat goods as collections of attributes, but this is not essential for our theoretical results. All results hold if $A = I$, in which case goods are attributes. However, the attribute structure creates correlation in demand across assets. That affects composition results and is important for measurement. To use this framework to measure a firm's data, it is crucial to recognize that information about one product can be informative about another. The correlated demand created by our attribute structure is what makes data relevant for multiple products.

Also, attribute-based demand is historically used in industrial organization (IO) economics because it allows researchers to predict what would happen if a new good was introduced.

NO ATTRIBUTE CHOICE. Appendix C.4. explores a model where a firm can choose the attributes of its good. The same forces are at work in that model. We choose to work with a simpler model without attribute choice to elucidate the main ideas more clearly.

NO VARIABLE CAPITAL COST. We made the investment in technology an up-front fixed cost. That means that the cost of capital is not part of the marginal cost that enters the markup calculation. One might object to that assumption on the grounds that the cost of capital is what captures the price of risk. Including a capital cost with a risk premium in marginal cost arguably absorbs the effect of risk on markups. This objection is tenuous. First, the capital cost is typically a borrowing cost. The risk premium on debt is not the same as the risk premium on equity. The firm cares about the variance of its cash flows, which is an equity claim. Second, the long-horizon risk that lenders care about is not the same as the short-term demand or cost fluctuations that data helps firms to forecast. These are substantially different risks. While including a variable capital cost with a risk premium in markup calculations probably improves their accuracy, this risk compensation has very little interaction with the way in which data helps to reduce operational uncertainties.

EXOGENOUS DATA. Section VII endogenizes data. The static forces are still present in that model and one new force emerges.

PUBLIC VS. PRIVATE DATA. The assumption that all data is public is obviously not realistic, but it is also not crucial for any of our main results. It clarifies what role private data plays. In a model with private data, firms face strategic uncertainty. They use data to forecast what other firms will do. Data thus reduces risk in two ways—by predicting demand for the firm's products and, if shocks are correlated ($\xi > 0$), by predicting the production decisions of other firms. Appendix C.2. shows that this strengthens the risk channel because data reduces both demand uncertainty and strategic uncertainty, prompting more production and more investment.

NO ENTRY OR EXIT. We take the number of firms as given. Adding entry would undoubtedly bring new insights. But that would also require a dynamic framework and a different paper. Since the static problem is not well understood, we start there. However, recent work by Baqaee and Farhi (2021) suggests that the aggregate distortions from market power are even larger once there is entry.

### II.C General Solution

We solve the model by backwards induction, starting with the quantity choices and then working backwards to determine optimal firm investments in lowering marginal costs $c_i$.

OPTIMAL PRODUCTION The first-order condition with respect to goods production $q_i$ is $\partial \mathcal{U}_i / \partial \boldsymbol{q}_i$ : $\mathbf{E}\left[\boldsymbol{p}_i|\mathcal{I}_i\right] - \boldsymbol{c}_i + \frac{\partial \mathbf{E}[\boldsymbol{p}_i|\mathcal{I}_i]}{\partial \boldsymbol{q}_i}\boldsymbol{q}_i - \rho_i \mathbf{Var}\left[\boldsymbol{p}_i|\mathcal{I}_i\right]\boldsymbol{q}_i \ = 0$. Rearranging delivers optimal production:

$$\boldsymbol{q}_i = \left(\rho_i \mathbf{Var}\left[\boldsymbol{p}_i|\mathcal{I}_i\right] - \frac{\partial \mathbf{E}\left[\boldsymbol{p}_i|\mathcal{I}_i\right]}{\partial \boldsymbol{q}_i}\right)^{-1} \left(\mathbf{E}\left[\boldsymbol{p}_i|\mathcal{I}_i\right] - \boldsymbol{c}_i\right). \tag{7}$$

From differentiating the attribute pricing function (4), we find that the price impact of one additional unit of attribute output is

$$\frac{\partial \mathbf{E}\left[\tilde{\boldsymbol{p}}_i|\mathcal{I}_i\right]}{\partial \tilde{\boldsymbol{q}}_i} = -\frac{1}{\phi} I_N. \tag{8}$$

Define the sensitivity of production to a change in expected profit as

$$\hat{\boldsymbol{H}}_i \equiv \left(\rho_i \mathbf{Var}\left[\boldsymbol{p}_i|\mathcal{I}_i\right] + \frac{\boldsymbol{I}_N}{\phi}\right)^{-1}. \tag{9}$$

To simplify the problem, we can change the choice variable and have firms choose the optimal vector of attribute production $\tilde{\boldsymbol{q}}_i$. If we rewrite (7), replacing $q$, $p$, and $c$ with attribute quantities, prices, and costs $\tilde{\boldsymbol{q}}_i$, $\tilde{\boldsymbol{p}}_i$, and $\tilde{\boldsymbol{c}}_i$, then we can substitute in the price impact (8) and conditional expectation (10) to get $\tilde{\boldsymbol{q}}_i = \hat{\boldsymbol{H}}_i\left(\overline{\boldsymbol{p}} + \boldsymbol{K}_i \boldsymbol{s}_i - \frac{1}{\phi}\sum_{i'} \tilde{\boldsymbol{q}}_{i'} - \tilde{\boldsymbol{c}}_i\right)$.

### II.D Solution with Public Data and Firm-Specific Shocks

While not as realistic, the special case of firm-specific shocks ($\xi = 0$) and public data allows us to more clearly illustrate the model's mechanics and the main economic forces of data.

BAYESIAN UPDATING According to Bayes' law for normal variables, observing $n_{di}$ signals, each with signal noise variance $\tilde{\Sigma}_e$, is the same as observing the average signal $\boldsymbol{s}_i = (1/n_{di}) \sum_{z=1}^{n_{di}} s_{iz} = \boldsymbol{b}_i + \boldsymbol{\varepsilon}_i$, where the variance of $\boldsymbol{\varepsilon}_i$ is $\Sigma_{\epsilon_i} = \tilde{\Sigma}_e / n_{di}$. Therefore, do a change of variable, replacing $\tilde{\Sigma}_e / n_{di}$ with $\Sigma_{\epsilon_i}$. In this representation, more data points (higher $n_{di}$) shows up as a lower composite signal noise $\Sigma_{\epsilon_i}$.

Define $\boldsymbol{K}_i$ to be the sensitivity of price beliefs to the signal $s_i$: $\boldsymbol{K}_i := (I_N + \Sigma_{\epsilon_i})^{-1}$.[7] Then, firm $i$'s expected value of the shock $b_i$ can be expressed simply as $E[b_i|\mathcal{I}_i] = K_i s_i$. The expectation and variance of the pricing function (4) are

$$\begin{aligned} \mathbf{E}\left[\tilde{\boldsymbol{p}}_i|\mathcal{I}_i\right] &= \bar{\boldsymbol{p}} + \mathbf{E}\left[\boldsymbol{b}_i|\mathcal{I}_i\right] - \frac{1}{\phi}\sum_{i'=1}^{n_F} q_{i'} \\ &= \bar{\boldsymbol{p}} + \boldsymbol{K}_i \boldsymbol{s}_i - \frac{1}{\phi}\sum_{i'=1}^{n_F} q_{i'}, \\ \mathbf{Var}\left[\tilde{\boldsymbol{p}}_i|\mathcal{I}_i\right] &= \mathbf{Var}\left[\boldsymbol{b}_i|\mathcal{I}_i\right] = \left(I_N + \Sigma_{\epsilon_i}\right)^{-1}\Sigma_{\epsilon_i}. \end{aligned} \tag{10}$$

OPTIMAL PRODUCTION Next, sum production $\tilde{\boldsymbol{q}}_i$ over all firms $i$ to get total production of each attribute $\sum_{i'} \tilde{\boldsymbol{q}}_{i'}$. This sum has a $\sum_{i'} \tilde{\boldsymbol{q}}_{i'}$ on both the left- and right-hand sides. Collect these terms and rearrange to get $\sum_{i'} \tilde{\boldsymbol{q}}_{i'} = \left(I + \frac{1}{\phi}\sum_i \hat{\boldsymbol{H}}_i\right)^{-1} \left[\sum_i \hat{\boldsymbol{H}}_i \left(\overline{\boldsymbol{p}} + \boldsymbol{K}_i \boldsymbol{s}_i - \widetilde{\boldsymbol{c}}_i\right)\right]$. Substituting this total production expression for $\sum_{i'=1}^{n_F} \tilde{\boldsymbol{q}}_{i'}$ in firm $i$'s optimal production ($\tilde{\boldsymbol{q}}_i{}^*$) yields the optimal production of each attribute by each firm $i$:[8]

$$\widetilde{\boldsymbol{q}}_i^* = \hat{\boldsymbol{H}}_i \left(\overline{\boldsymbol{p}} + \boldsymbol{K}_i \boldsymbol{s}_i - \widetilde{\boldsymbol{c}}_i - \left(I_N + \frac{1}{\phi}\sum_{i'} \hat{\boldsymbol{H}}_{i'}\right)^{-1} \left[\sum_{i'} \hat{\boldsymbol{H}}_{i'} \left(\overline{\boldsymbol{p}} + \boldsymbol{K}_{i'} \boldsymbol{s}_{i'} - \widetilde{\boldsymbol{c}}_{i'}\right)\right]\right).$$

Finally, the product-level optimal production function is the attribute weighting matrix $A$ times the optimal attribute production: $\boldsymbol{q}_i^* = A\tilde{\boldsymbol{q}}_i$.

[7] In a dynamic model, $K_i$ would be called the Kalman gain.

[8] Since all signals are normally distributed, this formula does tell us that production can potentially be negative. We could bound choices to be non-negative, but this would make analytical solutions for covariances impossible. If parameters are such that all firms want negative production of a good or attribute, then the solution is simply to redefine the product as its opposite. In the numerical results, we simply choose parameters that make negative production extremely unlikely.

EQUILIBRIUM PRICE  Substituting this aggregate quantity in the pricing function (4) yields an equilibrium average price of each attribute:

$$\tilde{\mathbf{p}}^M = \bar{p} - \left(I_N + \frac{1}{\phi}\sum_i \hat{\boldsymbol{H}}_i\right)^{-1} \left[\sum_i \hat{\boldsymbol{H}}_i \left(\overline{\boldsymbol{p}} + \boldsymbol{K}_i \boldsymbol{s}_i - \widetilde{\boldsymbol{c}}_i\right)\right]. \tag{11}$$

The average price of a good $k$ with attribute vector $a_k$ is then simply $p_k^M = \mathbf{a}_k' \tilde{\mathbf{p}}$, and firm $i$ price of good $k$ is $\mathbf{a}_k'(\tilde{\mathbf{p}}^M + \mathbf{b}_i)$.

OPTIMAL INVESTMENT CHOICES  Firm i chooses cost $\boldsymbol{c}_i$ to maximize its unconditional expected utility $\mathbf{E}[U_i]$, taking all other firms' investment choices as given.

The optimal cost $\boldsymbol{c}_i$ for an interior solution satisfies (see Appendix A. for derivation):

$$\frac{\partial \mathbf{E}[U_i]}{\partial \tilde{\boldsymbol{c}}_i} = \frac{1}{2}\frac{\partial \mathbf{E}[\tilde{\boldsymbol{q}}_i]' \left(\frac{2}{\phi} I_N + \rho_i \mathbf{Var}\left[\boldsymbol{b}_i | \mathcal{I}_i\right]\right)^{-1} \mathbf{E}[\tilde{\boldsymbol{q}}]}{\partial \tilde{\boldsymbol{c}}_i} - \frac{\partial g(\chi_c, \tilde{\boldsymbol{c}}_i)}{\partial \tilde{\boldsymbol{c}}_i} = 0, \tag{12}$$

The first term is the marginal benefit. Lower production costs enable production at a greater scale and higher profit per unit. The second term is the marginal cost of the up-front investment.

## II.E Solution with Private Data and Common Shocks

The optimal production takes the same form as before. The difference is in the expectation $\mathbf{E}\left[\boldsymbol{p}_i | \mathcal{I}_i\right]$.

This is a challenging problem because Bayesian updating depends on the covariance between the observed data $s_i$ and the price the firm needs to forecast $p_i$. But, in this version of the model, that covariance is endogenous. The properties of the price depend on each firm's production choices. But each firms' production choices depend on their beliefs, which in turn, depend on the price-data covariance. To solve the fixed point of beliefs and output requires a state-space updating approach, as in Lambert, Ostrovsky, and Panov (2018). The state-space approach adopted to solve lemma 1 defines all the relevant model objects in terms of exogenous, orthogonal shocks and weights on those shocks, which may be endogenous. Then we can contruct conditional and unconditional variances and covariances, as functions of the weights. Finally, we solve the fixed point by choosing weights that maximize firms' objectives, given the beliefs that result.

**Lemma 1.** *With private data and common shocks, the equlibrium price takes the form*

$$
\begin{aligned}
\boldsymbol{p} &= \bar{\boldsymbol{p}}^M + \boldsymbol{F}\boldsymbol{b} - \frac{1}{\phi}\sum_{i=1}^{n_F}\hat{\boldsymbol{H}}_i\boldsymbol{K}_i\boldsymbol{\varepsilon}_i \qquad \textit{where} \\
\bar{\boldsymbol{p}}^M &= \left(\boldsymbol{I}_N + \frac{1}{\phi}\sum_{i=1}^{n_F}\hat{\boldsymbol{H}}_i\right)^{-1}\left(\bar{\boldsymbol{p}} + \frac{1}{\phi}\sum_{i=1}^{n_F}\hat{\boldsymbol{H}}_i\boldsymbol{c}_i\right) \\
\boldsymbol{F} &= \boldsymbol{I}_N - \frac{1}{\phi}\sum_{i=1}^{n_F}\hat{\boldsymbol{H}}_i\boldsymbol{K}_i \\
\boldsymbol{K}_i &= (\boldsymbol{F}n_{di} + \boldsymbol{h}_i)\,\frac{1}{n_{di}+1} \\
\hat{\boldsymbol{H}}_i &= \left[\rho_i\left(\boldsymbol{F}\boldsymbol{F}' + \frac{1}{n_{di}\phi^2}\sum_{i=1}^{n_F}(\hat{\boldsymbol{H}}_i\boldsymbol{K}_i)^2 - \frac{n_{di}}{n_{di}+1}\left(\boldsymbol{F} - \frac{1}{\phi n_{di}}\hat{\boldsymbol{H}}_i\boldsymbol{K}_i\right)\left(\boldsymbol{F} - \frac{1}{\phi n_{di}}\hat{\boldsymbol{H}}_i\boldsymbol{K}_i\right)'\right) + \frac{\boldsymbol{I}_N}{\phi}\right]^{-1}
\end{aligned}
\tag{13}
$$

Firms ' optimal production is

$$
\begin{aligned}
\tilde{\boldsymbol{q}}_i &= \hat{\boldsymbol{H}}_i(\bar{\boldsymbol{p}}^M - \boldsymbol{c}_i + \boldsymbol{K}_i\boldsymbol{s}_i) \\
\mathbf{E}[\tilde{\boldsymbol{q}}_i] &= \hat{\boldsymbol{H}}_i(\bar{\boldsymbol{p}}^M - \boldsymbol{c}_i)
\end{aligned}
\tag{14}
$$

# III Main Results: How Data Affects Markups

We begin by exploring how more data affects a firm's choices of how much to produce and how much to invest before production. By reducing the uncertainty a firm faces about consumer demand, data encourages the firm to produce more for a given level of investment. Reducing uncertainty also emboldens the firm to invest more in infrastructure that enables them to produce at a lower marginal cost. These two forces have opposite effects on markups. More production lowers prices, which in turn lowers markups. More initial investment lowers marginal cost, which raises markups. This section explores that tension.

We begin by defining a product markup.

**Definition 1** (Product markup)**.** *The product-level markup for product $k$ produced by firm $i$ is $M^p_{ik} :=$ $\mathbb{E}[\boldsymbol{p}_i(k)]/\boldsymbol{c}_i(k)$. The average product-level markup is*

$$
\overline{M}^p := \frac{1}{Nn_F}\sum_{i=1}^{n_F}\sum_{k=1}^{N} M^p_{ik}. \tag{15}
$$

To derive an expression for the product markup in the model, we simply divide each expected product price, using (11) and $E[\boldsymbol{b}_i] = 0$, by the marginal cost of that product, $\boldsymbol{c}_i = a_k' \widetilde{\boldsymbol{c}}_i$:

$$M_{ik}^p = \frac{1}{a_k' \widetilde{\boldsymbol{c}}_i} a_k' \left( \overline{\boldsymbol{p}} - \left( I_N + \frac{1}{\phi} \sum_{i=1}^{n_F} \hat{\boldsymbol{H}}_i \right)^{-1} \left( \sum_i \hat{\boldsymbol{H}}_i \boldsymbol{A} \left( \overline{\boldsymbol{p}} + \boldsymbol{K}_i \boldsymbol{s}_i - \widetilde{\boldsymbol{c}}_i \right) \right) \right). \tag{16}$$

Similarly, the average product markup for firm $i$ is $\overline{M}_i^p = (1/N) \sum_{k=1}^N M_{ik}^p$.

What makes a markup large? Some of these causes of high markups in equation (16) are not surprising. For example, having lots of valuable attributes raises a product's markup. In the model, valuable attributes are large $a_{ij}$'s, especially for attributes with high expected value $\bar{p}$, relative to their cost $c$. Also, having fewer firms raises markups: low $n_F$ lowers $\bar{H}$, which makes the negative term on the right smaller. This is the classic concern with concentrated markets.

Low price elasticity (low $\phi$) is also a cause of high markups in (16). However, unlike many models, infinite price elasticity ($\phi = \infty$) does not eliminate the markup. In this limiting case, where firms cannot exercise market power, the markup is not zero. The markup is still positive with infinite elasticity because even if firms have no power to affect prices, they still need to be compensated for risk. The markup that remains in the infinite elasticity case is a risk premium.

Other forces arise because firms price risk. When firms are more sensitive to risk, or the price of risk in capital markets is high (high $\rho$), this also raises markups. They need to charge a higher markup to compensate themselves for the higher financing costs that this risk will incur. This force shows up as high $\rho$ makes $\bar{H}$ low. When firms are very sensitive to risk, they are less sensitive to prices and cost. They won't produce more when there are small changes in profits, because they are too sensitive to the additional risk that might entail.

Finally, two forces show up in the markup formula that are affected by how much data a firm has. Those forces are risk and investment. They often compete and are at the heart of the results that follow. Therefore, we state and derive each formally.

Data, Investment, Output, and Markups The first two results encapsulate the standard logic about data and competition: Data enables firms to grow larger (invest more). These larger firms charge higher markups.

**Lemma 2.** ***Data-investment complementarity.*** *A firm with more data chooses a lower marginal cost $c_i$, which entails a higher cost investment and higher profitability $\Pi_i$.*

The proofs of this and all further results are in Appendix B.. The role of investment in data is to reduce the conditional variance of the firm's stochastic demand, which encourages the firm to produce more. Data increases the expected revenue of a firm by allowing it to produce more in states in which the price will be high. It also reduces the uncertainty around that investment and lowers the risk of the firm. Both of these effects increase the marginal benefit of production and the marginal benefit of investment. What this means is that high-data firms invest more and grow larger. As the next result shows, higher investment is also a channel through which data increases product markups.

**Lemma 3.** ***Higher investment raises product markups.*** *More investment (lower $c_i$ choice) in any attribute j of good k, s.t. $a_{jk} > 0$, increases the markup of attribute j. If the markup on attribute j is less than the markup on product k ($M_{i,k} \geq M^{\tilde{p}_{i,j}}$), then this also raises the markup on good k.*

A firm that invests in producing an attribute can produce that attribute at a lower cost. If a good $j$ does not load at all on that attribute ($a_{jk} = 0$), then the lower cost has no bearing on the cost or markup of that good. But if good $j$ contains some of that attribute ($a_{jk} > 0$), this investment lowers the cost of producing the good. Since markups are price divided by marginal cost, a lower cost raises the markup. Of course, a lower cost also lowers the equilibrium price of the good. However, the proof shows that price does not fall as much as cost. Therefore, the markup rises.

To see why not every attribute markup increase raises the product markup, consider a numerical example where a product uses 99% of an attribute with a price 101 and cost 100 and 1% of an attribute that costs 1 and has a price 5. The product markup is $\frac{99\% \cdot 101 + 1\% \cdot 5}{99\% \cdot 100 + 1\% \cdot 1} \approx 1.10403$. Now suppose we decrease the cost of the second attribute from 1 to 0.9 and reduce its price from 5 to 4.6. This implies that the second attribute's markup increases from 5 to 5.11. The new product markup is $\frac{99\% \cdot 101 + 1\% \cdot 4.6}{99\% \cdot 100 + 1\% \cdot 0.9} \approx 1.10373$, which is less than the original markup of 1.10403. Thus lowering the cost of a low-markup attribute can increase the product markup. This example is carefully contrived and not likely to be relevant. But it explains the reason for the parameter restriction in the Lemma.

However, investment is only one channel through which data affects markups. The model teaches us that there is a second channel: data reduces the risk of production, induces more production, and thereby lowers prices and markups. We isolate this channel by holding investment (firm size) fixed. Since $c_i$ is, of course, a choice variable which is not fixed, the correct formal statement is that this result holds when the marginal cost of adjusting investment $\chi$ is sufficiently high. Approximately, however, this parameter restriction simply serves to hold investment constant so that we can see the effect of data on output in isolation.

**Lemma 4.** ***Data reduces product markups (risk premium channel).*** *Holding all firms' investments fixed ($\chi_c$ sufficiently high), an increase in any firm's data about any attribute of good k reduces the markup of good k.*

Data reduces markups because it reduces the risk in production. This induces firms to produce more. This effect can be seen in the firm's first-order condition (7) where the conditional variance in the denominator represents risk. When this variance declines, optimal production rises. More production lowers price and lowers markups. When we reduce risk with data, firms do not need as much markup compensation to be willing to produce.

This effect is always present, regardless of the level of $\chi_c$. The restriction on $\chi_c$ is only there to isolate this channel from the investment channel, which is shut down when $\chi_c$ is sufficiently high. When $\chi_c$ is lower, this risk premium channel is still present. But it may be overpowered by the investment channel working in the opposite direction.

**Proposition 1.** ***Data in(de)creases product markups when risk price or marginal cost of investment is sufficiently low (high).*** *If the price of risk $\rho$ is sufficiently low or the investment cost $\chi_c$ is sufficiently low, then an increase in any firm's data about any attribute of good k increases the markup of good k, which loads positively on that attribute. Otherwise, an increase in any firm's data about any attribute of good k reduces the markup of good k.*

Equation (17) summarizes the effects of data on markups. The partial derivative of markups with respect to data is the difference between the risk premium effect and the investment effect.

$$\frac{\partial M_{i,j}^{\tilde{p}}}{\partial \Sigma_{\epsilon_{i,j}}^{-1}} = \underbrace{\frac{\tilde{\boldsymbol{c}}_{i,j}}{\tilde{\boldsymbol{c}}_{i,j}^2} \frac{\partial \bar{\boldsymbol{p}}_j^M}{\partial \Sigma_{\epsilon_{i,j}}^{-1}}}_{\text{Risk premium effect}} - \underbrace{\frac{\bar{\boldsymbol{p}}_j^M}{\tilde{\boldsymbol{c}}_{i,j}^2} \frac{\partial \tilde{\boldsymbol{c}}_{i,j}}{\partial \Sigma_{\epsilon_{i,j}}^{-1}}}_{\text{Investment effect}} . \tag{17}$$

where

$$\bar{\boldsymbol{p}}_j^M = \frac{\bar{\boldsymbol{p}}_j + \frac{1}{\phi}\sum_{i=1}^{n_F} \hat{\boldsymbol{H}}_{i,j}\boldsymbol{c}_{i,j}}{1 + \frac{1}{\phi}\sum_{i=1}^{n_F} \hat{\boldsymbol{H}}_{i,j}} \quad \text{and} \quad \hat{\boldsymbol{H}}_{i,j} = \left[\frac{1}{\phi} + \rho_i \left(1 + \Sigma_{\epsilon_{i,j}}^{-1}\right)^{-1}\right]^{-1} . \tag{18}$$

Proposition 1 simply identifies regions of the parameter space where the first or second term of (17) dominates. High risk aversion makes the risk premium effect large. In contrast, low marginal cost of investment makes investment very responsive to data and makes the investment channel the stronger effect.

Figures 1 and 2 illustrate how the risk reduction and investment forces compete. When firm investments greatly decrease marginal cost (low $\chi_c$), then the cost channel is dominant and more data primarily increases investment, lowers costs, and raises markups (Figure 1). When the cost-reduction investment is inefficient (high $\chi_c$), then data still prompts more investment, but this has little effect on marginal cost. Instead, the dominant force is risk reduction. Similarly, if the price of risk is high, risk reduction is also the dominant force. A data-rich firm faces less cost from taking on more risk with a large production plan. By producing more, data-rich firms drive prices down

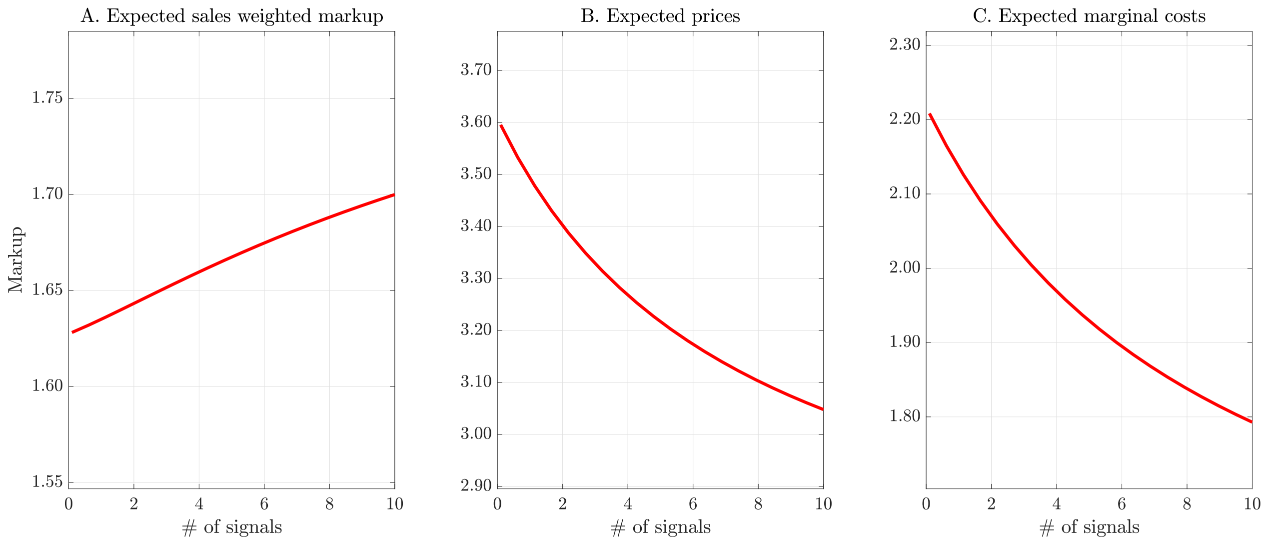



Figure 1: Data raises markups with low investment cost / price of risk

Notes: This comparative static exercise is constructed over a single-good duopoly example. The x-axis is the number of data points that both firms have. The investment cost function is assumed as $g(\chi_c, c_i) = \chi_c \left(\bar{c} - c_i\right)^2 / 2$ with $\chi_c = 1$ and $\bar{c} = 3$. Other parameters are: $\overline{p} = 5$, $\phi = 1$, $\sigma_b = 1$, $\mu_b = 0$, $\sigma_e = 2$, and $\rho_1 = \rho_2 = 1$.

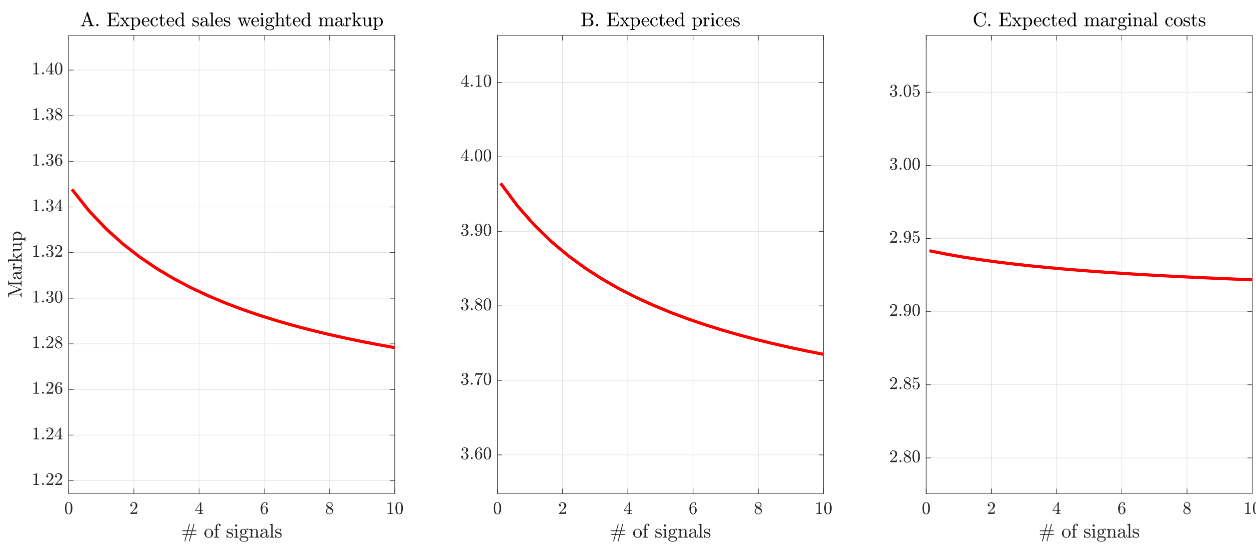



Figure 2: Data lowers markups with high investment cost / price of risk

Notes: This comparative static exercise is constructed over a single-good duopoly example. The x-axis is the number of data points that both firms have. The investment cost function is assumed as $g(\chi_c, c_i) = \chi_c \left(\bar{c} - c_i\right)^2 / 2$ with $\chi_c = 10$ and $\bar{c} = 3$. Other parameters are: $\overline{p} = 5$, $\phi = 1$, $\sigma_b = 1$, $\mu_b = 0$, $\sigma_e = 2$, and $\rho_1 = \rho_2 = 1$.

and lower markups (Figure 2). Which scenario prevails depends on the strength of each force in a particular industry.

Despite the fact that markups increase in one case and decrease in the other, both results paint a rosy picture of the role of data. Even when data increases markups, it decreases price. Markups only rise because the firm could produce at a lower cost. Both results point to the efficiency-enhancing and welfare-boosting effects of data. Unfortunately, these are not the only effects data can have. The following results point out the potential problems with this rosy scenario.

# IV Welfare

Typically, economists are interested in markups because they are assumed to be indicators of welfare loss or harmful market distortion. In this setting, markups perform a dual role—they are compensation for firm risk-taking and indicators of deadweight loss. This section characterizes efficient markups and welfare. We find that more data typically improves welfare, but it also makes distortions from market power more costly. When firms' stocks of data are asymmetric, exacerbating the data asymmetry can either improve welfare or harm it, depending on whether the risk or the investment effect dominates.

If firms are not compensated for the risk they bear, they will not produce. So a zero markup cannot be the efficient benchmark. Instead, we define prices to be efficient if they arise from production choices of firms that behave as if they were in a competitive market. This leads us to a new measure of market distortion, which we call the risk-adjusted markup.

Competitive firms are those who take market prices as given. In other words, they optimize as if price impact were zero: $\partial E_i[p]/\partial q_i = 0$. If we set price impact to zero in the firm's first-order condition, optimal production is

$$\boldsymbol{q}_i^{comp} = \frac{1}{\rho_i}\mathbf{Var}\left[\boldsymbol{p}_i|\mathcal{I}_i\right]^{-1}\left(\mathbf{E}\left[\boldsymbol{p}_i|\mathcal{I}_i\right] - \boldsymbol{c}_i\right). \tag{19}$$

In other words, production is the same, except that we redefine the sensitivity of production to changes in price or cost in (7) to be $H_i^{comp} = (1/\rho_i)\mathbf{Var}\left[\boldsymbol{p}_i|\mathcal{I}_i\right]^{-1}$.

The fact that market power enters only through the sensitivity term $H$ means that in firm production (7), more market power is mathematically equivalent to increasing the conditional variance $\mathbf{Var}\left[\boldsymbol{p}_i|\mathcal{I}_i\right]$. In other words, risk mimics market power. Both risk and market power restrain production. Both make firms less sensitive to expected changes in price or cost. In one case, it is because a risk-averse firm makes more conservative production decisions to manage its risk. In the other case, the firm makes more conservative decisions to minimize its price impact.

The fact that markups reflect risk, as well as market power, suggests that measuring market power should involve controlling for risk. One such measure of market power at the product level might be

$$H_{ik}^{p} - H_{ik}^{comp}.$$

The challenge this poses is that $H_{ik}^{comp}$ is not directly observed from firm behavior. Instead, it requires estimating a firm's data and price of risk. But using the markup wedges to measure data, as described in section VIII, makes this feasible.

Welfare Benefits of Data When all firms get more data, this can be a Pareto improvement. Firm owners benefit because more information improves forecasts, which reduce risk that they are averse to. Also, firms with more data invest to be more efficient. On top of that, consumer surplus increases because lower production cost and more information both tighten competition among firms. The next result formalizes this logic.

**Proposition 2.** ***Data improves welfare.*** *When firms are symmetric, then more data for every firm increases social welfare.*

Figure 3 illustrates this force. The upward slope of the lines tells us that welfare is increasing in the amount of data. This is true even when there is perfect competition. Even when there is no risk aversion, the ability to produce more goods to meet demand still enhances welfare.

Data Amplifies Market Power Costs Figure 3 decomposes the welfare loss into risk aversion and market power. The loss due to market power is much higher on the right, where data is abundant.

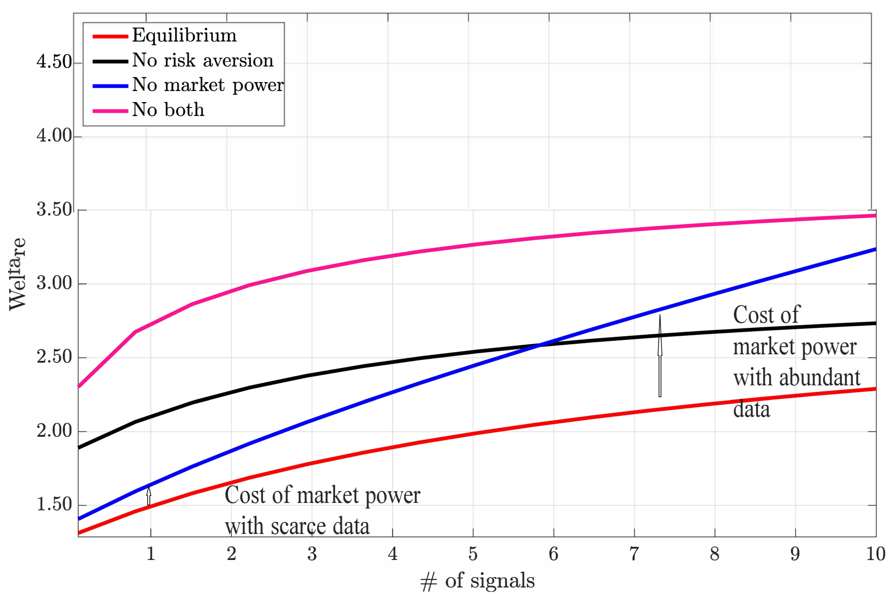



Figure 3: Welfare: Abundant data raises welfare, makes market power more costly

Notes: This counterfactual exercise is constructed over a single-good duopoly example. The x-axis is the number of data points that both firms have. The investment cost function is assumed as $g(\chi_c, c_i) = \chi_c \left(\bar{c} - c_i\right)^2 / 2$ with $\chi_c = 1$ and $\bar{c} = 3$. Other parameters are: $\overline{p} = 5$, $\phi = 1$, $\sigma_b = 1$, $\mu_b = 0$, $\sigma_e = 2$, and $\rho_1 = \rho_2 = 1$.

The reason that data makes market power more powerful can be seen in the first-order condition (7) of the firm's choice of production quantities $q$. The right term is expected profit per unit. That expected profit is divided by the term $\rho_i \mathbf{Var}[\boldsymbol{p}_i|\mathcal{I}_i] - \frac{\partial \mathbf{E}[\boldsymbol{p}_i|\mathcal{I}_i]}{\partial \boldsymbol{q}_i}$, which captures risk price $\rho_i$ times risk (the conditional variance), plus the expected price impact of a trade (market power). Imagine that the product of risk price and risk is large. Then, adding some market power to this large number does not change it by much. When we divide by a large number or a slightly larger number, the answer is almost the same. Thus, when data is scarce and variance is high, market power has little effect on production.

But when data is abundant, the conditional variance is low. Lots of data makes the firm less uncertain. If the first term is small, then adding market power to it makes a big difference. Dividing by a number close to zero or a number slightly less close to zero makes a big difference. Thus, when data is abundant and risk is low, market power has an outsized effect on production choices and thus on prices and markups.

DATA ASYMMETRY So far, we have explored what happens when all firms have more data. But a key concern for market competition is the possibility that firms have highly unequal stocks of data. In this case, an increase in data can reduce welfare.

To demonstrate this possibility, we consider an example with two firms. We fix the total number of data points and add data to one firm as we subtract it from second firm. Figure 4 highlights how the economy is affected by data dispersion.

Increasing data asymmetry has two opposite welfare effects: (1) increasing market power and hence deadweight loss, and (2) lower disutility from risks because the firm with more information will produce more. When the marginal cost $c_i$ is relatively small, a difference in data precision creates a large difference in investment and thus firm size. This force can easily enable one firm dominate the market.

While these results do not offer a simple answer or prediction about whether data is good or bad, they do provide an important input into the data policy debate. Data asymmetry and market dominance are not synonymous with welfare loss. Uncertainty is also a powerful drag on economic efficiency and on welfare. Sound data policy needs to trade off traditional market power harms against the benefits of resolving risk. This model produces a simple tool to evaluate that trade-off.

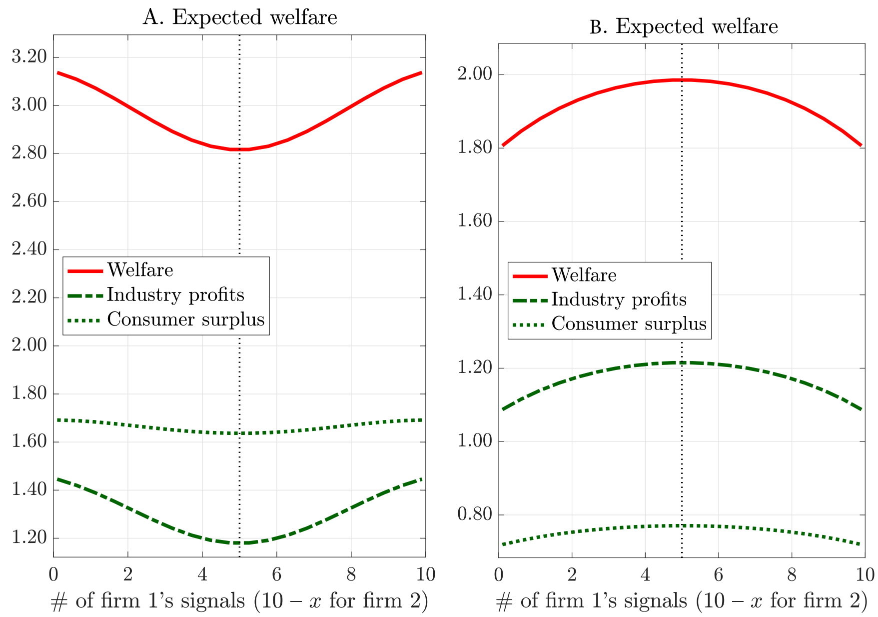



Figure 4: Data asymmetry and welfare with dominant risk channel (left) or investment channel (right).

Notes: This comparative static exercise is constructed over a single-good duopoly example. The investment cost function is assumed as $g(\chi_c, c_i) = \chi_c\,(\bar{c} - c_i)^2/2$. On the left, $\chi_c = 10$. On the right, $\chi_c = 1$. Other parameters are common to both plots: $\bar{c} = 3$, $\overline{p} = 5$, $\phi = 1$, $\sigma_b = 1$, $\mu_b = 0$, $\sigma_e = 2$, and $\rho_1 = \rho_2 = 1$. See Appendix B. for the computation of welfare.

# V Measuring Markups and Measuring Data

The previous analysis examined the forces that operate on product-level markups. But in empirical work, markups are often measured at the firm or industry level. Measuring markups at these more aggregated levels often yields different answers about how competition is evolving. The next set of results show why aggregate markups differ from product-level markups in ways that vary systematically with the amount of data firms have. In fact, the difference between a firm's product- and firm-level markups turns out to be a good proxy for the amount or quality of that firm's data.

These composition effects are not mere curiosities. They are also a feature of markup data. De Loecker, Eeckhout, and Unger (2020) find that two-thirds of the increase in measured industry markups comes from such composition effects. Crouzet and Eberly (2018) link the increase in markups to intangible assets, a broader category that includes data assets. They find that intangible-abundant firms have higher markups and that intangible-abundant industries have even higher markups. The results that follow contribute to this discussion by explaining why firms' use of predictive data can generate such statistical patterns.

## V.A Firm Markups

We begin by defining firm markups, exploring their relationship to product markups. In the following subsections, we will build up to the industry-level measures used by empirical researchers.

**Definition 2** (Firm Markup)**.** *The firm markup for firm i is the firm's revenue divided by the firm's total variable costs:*

$$M_i^f := \frac{\mathbf{E}[\boldsymbol{q}_i' \boldsymbol{p}_i]}{\mathbf{E}[\boldsymbol{q}_i' \boldsymbol{c}_i]}. \tag{20}$$

To understand the relationship between firm markups and product markups, we can rewrite the expectation of the product of price and quantity as the product of expectations, plus a covariance term:

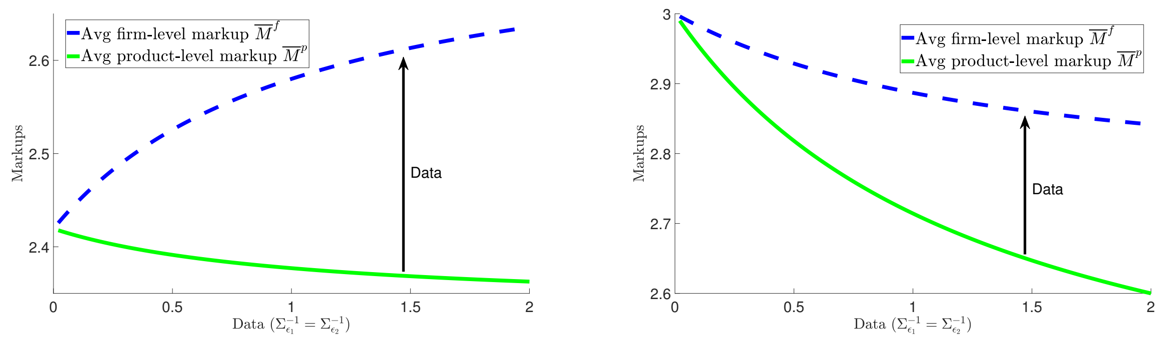
$$M_i^f = \frac{\mathbf{E}[\boldsymbol{q}_i]'\mathbf{E}[\boldsymbol{p}_i] + \mathbf{tr}\left[\mathbf{Cov}(\boldsymbol{p}_i, \boldsymbol{q}_i)\right]}{\mathbf{E}[\boldsymbol{q}_i' \boldsymbol{c}_i]} = \underbrace{\frac{\sum_{j=1}^N M_{ij}^p \boldsymbol{c}_i(j)\mathbf{E}[\boldsymbol{q}_i(j)]}{\sum_{j=1}^N \boldsymbol{c}_i(j)\mathbf{E}[\boldsymbol{q}_i(j)]}}_{\text{Cost-weighted product markups}} + \frac{\mathbf{tr}[\mathbf{Cov}\left(\boldsymbol{p}_i, \boldsymbol{q}_i\right)]}{\sum_{j=1}^N \boldsymbol{c}_i(j)\mathbf{E}[\boldsymbol{q}_i(j)]}. \tag{21}$$

The second equality just comes from using the definition of the product markup to substitute: $\mathbf{E}[\boldsymbol{p}_i] = M_i^{p\prime} \boldsymbol{c}_i$ and then rewriting the vector products as sums. We learn that the firm markup is a cost-weighted sum of product markups, plus a term that depends on the variance of prices and quantities. Firm data acts on this last term. It allows firms to produce more of goods that turn out to have high demand and thus high price.

**Proposition 3.** ***Data accumulation widens the wedge between product and firm markups.*** *Holding all firms' investments fixed ($(c_1, ..., c_{nF})$ given), an increase in firm i's data about any attribute increases $E[M_i^f - \overline{M}_i^p]$.*

Firm markups rise when data increases the covariance of firm's production decision $q_i$ with the price $p$ in (21). Without any data to predict demand, this covariance is low: without data, firms cannot know which markups would be high and which goods to produce more of. The positive effect of data on the price-quantity covariance shows up in the production first-order condition (7), where a reduction in the conditional variance of demand makes production decisions $q_i$ more sensitive to expected changes in price $p_i$. That higher sensitivity is a higher covariance.

Economists have long known that difference in markup measurement at different levels of aggregation represent composition effects. What is less well understood is why such composition effects might change. We show how firms' data accumulation naturally gives rise to changes in the composition of firms' products. Data is what makes it possible for the firm to skew the composition of their products in the direction of high-markup goods. So, data strengthens the composition effect and makes firm markups larger and larger relative to that firm's average product markup.




(a) Low price of risk. (b) High price of risk.

Figure 5: More data may raise or lower markups but always causes product and firm markups to diverge. Parameters used: $\bar{p} = 5$, $\phi = 0.1$, and $A = I$. Firm marginal costs are not chosen here. They are fixed as $c_1 = c_2 = 1$. On the left, $\rho_1 = \rho_2 = 1$. On the right, $\rho_1 = \rho_2 = 10$.

An Illustrative Example of the Product-Firm Markup Wedge To illustrate the mechanisms at work, Figure 5 plots the competing effects data has on product and firm/industry markups in a specific example. When the price of risk is high, the product-level markup falls as both firms' data rises. The reason the product markup is falling is that data is resolving risk. It is allowing the firms to be less uncertain because data allows them to forecast demand more precisely. Firms that are less uncertain require a lower markup to compensate them for the lower risk. When the price of risk is low, more data may result in higher firm markups, as high-data firms invest, grow, and lower their marginal costs.

Regardless of whether product markups rise or fall as data becomes more abundant, firm-level markups rise relative to those product markups. Data allows firms to forecast which products will have high markups and to produce more of those. In fact, as we explore later, this difference between product and firm markups can be used to measure a firm's stock of data.

What the model teaches us so far is that increases or decreases in markups, at either the product level or the firm level, are not indicative of a firm that has a larger stock of data. As a firm accumulates more data, both product and firm markups may increase, both may decrease, or they may move in opposite directions. Instead, data governs the difference in markups. Data changes the composition of products and firms and makes various measures of markups diverge. This is a theme that will recur as we proceed to explore markups at the industry level.

### V.B Measures of Markups in an Industry

Typically, researchers are interested in the markup for an industry because the regulatory question of interest is whether that industry is a competitive one or not. However, there are multiple ways to aggregate the markups for each firm into a single industry measure. We construct four of the most common measures here to understand how they differ. Then, we compare their theoretical predictions to empirical evidence. The model lends an interpretation to the different trends arising from the different ways empirical researchers measure industry markups.

**Definition 3.** *The unweighted average firm markup in an industry is*

$$\bar{M}^f := (1/N)\sum_{i=1}^{N} M_i^f. \tag{22}$$

**Definition 4.** *The cost-weighted markup for an industry is*

$$M^c := \sum_{i=1}^{N} w_i^c M_i^f \quad \textit{where cost weights are} \quad w_i^c = \frac{\mathbf{E}\left[\boldsymbol{q}_i' \boldsymbol{c}_i\right]}{\sum_{i=1}^{N} \mathbf{E}\left[\boldsymbol{q}_i' \boldsymbol{c}_i\right]}. \tag{23}$$

While the first definition is simply the markup of the average firm, this second definition weights larger firms more. With cost-weighted markups, larger firms are those that have larger variable costs compared with their industry competitors. The next definition also weights the markups of larger firms more. But in the sales-weighted markups, larger firms are those with larger gross revenues compared to the revenues of other firms in the same industry.

**Definition 5.** *The sales-weighted markup is*

$$M^s := \sum_{i=1}^{N} w_i^s M_i^f \quad \textit{where sales weights are} \quad w_i^s = \frac{\mathbf{E}\left[\boldsymbol{q}_i' \boldsymbol{p}_i\right]}{\sum_{i=1}^{N} \mathbf{E}\left[\boldsymbol{q}_i' \boldsymbol{p}_i\right]}. \tag{24}$$

**Definition 6.** *The industry- aggregates markup is*

$$M^{ind} := \frac{\mathbf{E}\left[\sum_{i=1}^{N} \boldsymbol{q}_i' \boldsymbol{p}_i\right]}{\mathbf{E}\left[\sum_{i=1}^{N} \boldsymbol{q}_i' \boldsymbol{c}_i\right]}. \tag{25}$$

Industry-aggregates markup is measured with data already aggregated at the industry level. It is the ratio of the total industry sales over the total industry variable cost. In theory, industry-aggregates markups are identical to cost-weighted markups:

$$M^c := \sum_{i=1}^{N} \frac{\mathbf{E}\left[\boldsymbol{q}_i' \boldsymbol{c}_i\right]}{\sum_{i=1}^{N} \mathbf{E}\left[\boldsymbol{q}_i' \boldsymbol{c}_i\right]} M_I^f = \sum_{i=1}^{N} \frac{\mathbf{E}\left[\boldsymbol{q}_i' \boldsymbol{c}_i\right]}{\sum_{i=1}^{N} \mathbf{E}\left[\boldsymbol{q}_i' \boldsymbol{c}_i\right]} \frac{\mathbf{E}\left[\boldsymbol{q}_i' \boldsymbol{p}_i\right]}{\mathbf{E}\left[\boldsymbol{q}_i' \boldsymbol{c}_i\right]} = \frac{\mathbf{E}\left[\sum_{i=1}^{N} \boldsymbol{q}_i' \boldsymbol{p}_i\right]}{\mathbf{E}\left[\sum_{i=1}^{N} \boldsymbol{q}_i' \boldsymbol{c}_i\right]} := M^{ind}. \tag{26}$$

However, in practice, with different sources of measurement error at the firm and aggregate level, each approach may deliver slightly different answers.

### V.C Data and Industry Markup Measures

Our theory of data provides an explanation for the widening gap between these various markup measures. Firms that have more data can reduce uncertainty. Lower uncertainty makes larger up-front investment optimal. So, high-data firms are large firms, which are weighted more by cost weights and sales weights, relative to the unweighted firm average. As explained in the firm markup section, firms use data to skew their production toward high-markup goods, making high-data firms likely to be higher-markup firms. Thus, the measures that weight large, high-data firms more will also weight high-markup firms more, generating a higher predicted industry markup.

**Proposition 4.** ***Growing data increases the wedges between industry markup measures.*** *Holding all firms' investments fixed ($(c_1, ..., c_{nF})$ given) and $c_i$ sufficiently small, an increase in firm i's data about any attribute*

*a. increases the difference between cost-weighted and unweighted firm markups $E[M^c - \overline{M}^f]$.*

*If, in addition, all firms are initially symmetric, then an increase in firm i's data about any attribute*

*b. increases the difference between sales-weighted and cost-weighted markups $E[M^s - M^c]$, and*

*c. increases the difference between the sales-weighted and industry-aggregates markup $E[M^s - M^{ind}]$.*

Mathematically, the key to each of these results is a covariance. In the first case, the covariance is between the firm markup and the total production of a firm. Cost-weighted markups are firm markups, weighted by the firm's share of variable cost of production. If $q_i$ is large for firms that have high markups, then the weighted average will have a higher markup than the unweighted average. This is related to data because, as discussed in the previous result, high-data firms skew their production to high-markup goods and thus have higher firm markups. High-data firms also produce more on average because data lowers their production risk. We can see this in the production first-order condition (7) where a reduction in the conditional variance reduces the denominator and makes production decisions $q_i$ larger on average.

Economically, this is another composition or aggregation effect. Data has economies of scale. Firms get the most value from their data if they grow large. The way they get value from data is to use the data to forecast which goods are high-margin and produce more of them. Thus, more data increases the covariance between size and markups and makes the aggregate markup larger than the average firm markup.

In case (b), the key covariance is between a firm's markup and the firm's revenue. Sales-weighted industry markups are firm markups weighted by the gross revenue of the firm. Cost-weighted industry markups are firm markups weighted by the variable cost of the firm. High-data firms are firms that are able to produce more of the products that have high price relative to their cost of production. Therefore, these high-data firms have higher sales-weighted markups relative to their cost-weighted markups.

The twist here is that "high-data firms" now means firms that have higher amounts of data than their competitors. Firms can obtain higher price relative to their cost because of information asymmetry. If all firms knew that demand would be high for an attribute and they all produced more of it, this would bring the price of that attribute back down. What we learn from this is that a divergence between sales-weighted and cost-weighted markups results from growth in cross-firm information asymmetry.

In part (c), firms' data stocks speak to the observed divergence in measures of markups using disaggregated firm data and to the measures that use total industry revenue and total industry cost. The third part of the proposition reveals that sales-weighted markups should also rise faster than markups measured on industry aggregates, if firms are accumulating more data over time. This third result follows from the second because of the theoretical equivalence between measurement using aggregates and cost-weighting the disaggregated firm markups, as shown in (26).

AN ILLUSTRATIVE EXAMPLE OF INDUSTRY MARKUP DIVERGENCE When firms choose their investment to lower their marginal cost of production, high-data firms choose to invest more. High-data firms, which we saw have higher firm-level markups, grow larger. As a result, their production accounts for a larger fraction of total production. Therefore, the higher markup of the high-data firms gets weighted more in the industry markup. Thus, investment choice amplifies the wedge between firm-level and industry markups.

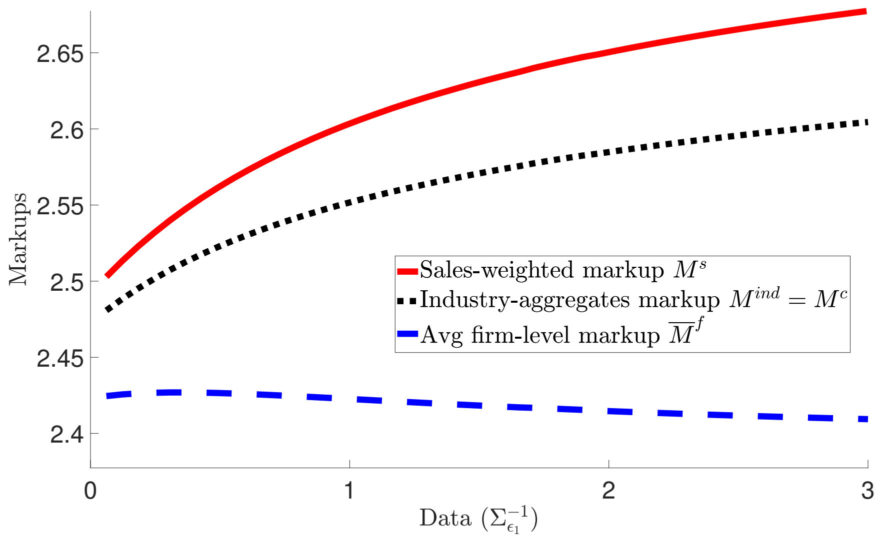


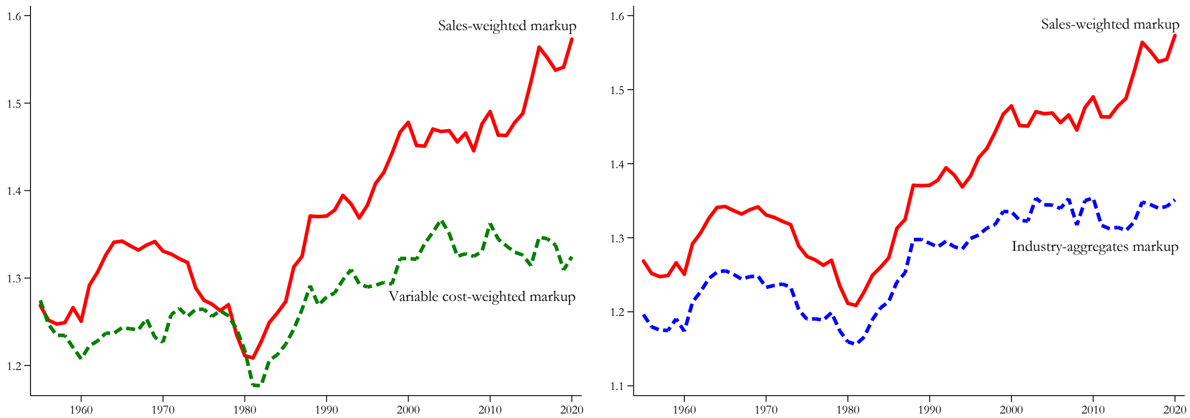

Figure 6: Data Accumulation Makes Industry Markup Measures Diverge. Investment cost function is $g(\chi_c, c_i) = \chi_c / c_i^2$, with $\chi_c = 1$. Parameters are $\bar{p} = 5$, $\rho_1 = 1$, $\rho_2 = 5$, $\phi = 0.8$, and $A = I$. Firm 1's data is measured on the x-axis. Firm 2's data is fixed at $\Sigma_{\epsilon_2}^{-1} = 1$.

In Figure 6, we see the gap between firm-level markup (blue dashed line at the bottom) and the industry markup (red solid line on top) widen relative to the previous results where that gap was much smaller. That market aggregation gap also grows as data becomes more abundant. That result suggests that as firms process more and more data, the differences between markups measured at various levels of aggregation will continue to grow.

With aggregate markups, there are now four ways in which data affects markups. Data increases markups because of investment, cross-product aggregation, and cross-firm aggregation. Data decreases markups because it induces firms to produce more (risk premium channel).

### V.D Empirical Evidence from Industry Markups

The empirical literature finds that there is a wedge between the sales-weighted markup and the cost-weighted markup, and that this wedge is growing over time since the early 1980s (see Figure 7a, from De Loecker, Eeckhout, and Unger (2020)). Firms that have market power sell at higher prices and therefore have higher revenue and relatively lower costs. This difference between sales and costs therefore drives a wedge between sales- and cost-weighted markup measures. This is consistent with what we find as firms that have market power boost their sales with fewer inputs since they have higher markups. In our model, firms who invest heavily in data do exactly that, and the more important the role of data, the bigger the wedge between the input- and output-weighted aggregate markup. Our contribution is to propose a theory based on the role of data in creating these wedges, and how they grow as the data becomes more important.




(a) Sales-weighted markups, $M^s$, (solid line) vs. cost-weighted markups, $M^c$, (dashed line)

(b) Sales-weighted markups, $M^s$, (solid line) vs. industry-aggregates markups, $M^{ind}$, (dashed line)

Figure 7: Markups Measured and Aggregated in Different Ways Diverged Over Time. Left panel is from De Loecker, Eeckhout, and Unger (2020), Figure XVI.A. Right panel is from De Loecker, Eeckhout, and Unger (2020), Figure V.

Our theory predicts differences between product, firm, and industry markups. To date, there is still limited evidence comparing product versus firm markups using the same data source. However, there is consistent evidence comparing firm markups to industry markups. In fact, the seminal paper on markup measurement by means of the production approach by Hall (1988) uses industry, not firm-level, data to construct aggregate markup measures (see also Hall [2018] for recent industry estimates using KLEMS data). With firm-level data and industry classification codes, we can mimic the industry aggregates using exactly the same set of firms underlying the industry aggregates. Based on De Loecker, Eeckhout, and Unger (2020) using data on publicly traded firms, Figure 7b shows that industry markups (blue dashed line) have increased by half as much as sales-weighted firm markups (red line). In other words, they find that there is a wedge between the industry markup and the sales-weighted firm markup, and that wedge is increasing as investment in data increases. Note that industry markups (in Figure 7b) look remarkably similar to cost-weighted firm markups (in Figure 7a). This is due to the systematic relation between input-weighting and industry aggregates in equation (26).

## VI Cyclicality of Markups

A key question for mainstream New Keynesian models of the type often used by central banks is whether markups are countercyclical. This question has created stark disagreement. Researchers who measure markups at the firm or industry level find clear evidence of countercyclical markups (Bils 1985, 1987). In contrast, researchers who measure markups at the product level do not find evidence of countercyclicality (Nekarda and Ramey 2020). Our model offers a way to reconcile these facts.

Our explanation builds on the progress in Burstein, Carvalho, and Grassi (2020). They show analytically how composition changes can turn procyclical markups into countercyclical ones, depending on how markups are aggregated. Our model provides a specific economic mechanism for these composition changes. The cyclical markup evidence, in turn, supports the realism of the model's assumptions.

To use the model to explore the cyclicality of markups, we first need to understand what is a boom or recession in the context of this model. Two relevant changes typically happen when an economy transitions from recession to boom. The first is that demand rises. The second is that the variances of demand and of output fall. Recessions are volatile, uncertain times. To formalize this new assumption, we introduce a variable *Boom* that is high in booms and low in recessions. Then, we make the level of demand procyclical and the demand variance countercyclical by assuming

$$\bar{p} = d_0 + d_1 * Boom \qquad \text{where } \; d_0, d_1 \geq 0, \tag{27}$$

$$\Sigma_b = d_2 - d_3 * Boom \qquad \text{where } \; d_2, d_3 \geq 0. \tag{28}$$

The change in the level of demand does not affect divergence, but it allows both lines to rotate. In other words, high demand in a boom regulates how countercyclical or acyclical product markups are. Falling variance in a boom is what makes the cyclical behavior of aggregate markups differ relative to product markups. The second statement is formalized in the next proposition.

**Proposition 5.** ***Product markups diverge from firm and industry markups when volatility rises.*** *Suppose the investment cost structure is such that firms choose identical investments ($c_i = c_j \;\; \forall\, i, j$).*

a. *The product-level markup is strictly increasing in demand variance, $\partial \mathbf{E}[M^p_{ij}]/\partial \Sigma_{b,j} > 0$, and converges to a constant as $\Sigma_{b,j} \to \infty$.*

b. *If demand variance is large enough, firm and industry markups are strictly increasing, $\partial \mathbf{E}[M^f_{ij}]/\partial \Sigma_{b,j} > 0$ and $\partial \mathbf{E}[M^m_{ij}]/\partial \Sigma_{b,j} > 0$, and asymptotic to a function increasing in variance, $\lim_{\Sigma_{b,k} \to \infty} \partial \mathbf{E}[M^f_{ij}]/\partial \Sigma_{b,j}, \partial \mathbf{E}[M^m_{ij}]/\partial \Sigma_{b,j} > 0$.*

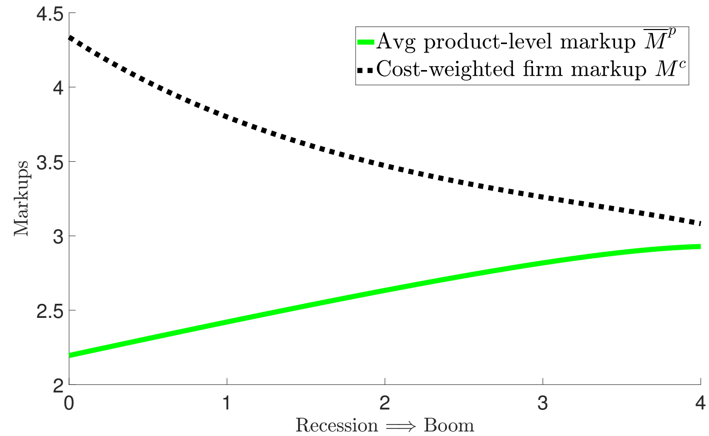



Figure 8: Procyclical product markups can coexist with countercyclical firm / industry markups. Left (right) on the x-axis represents recessions (booms), as described in (27) and (28), where $d_0 = 7/2$, $d_1 = 1/2$, $d_2 = 5$, and $d_3 = 1$. A decreasing line represents a countercyclical markup. Remaining parameters are $\rho_1 = \rho_2 = 1$, $c_1 = c_2 = 1$, $\phi = 1$, and $\Sigma_{\epsilon_1} = \Sigma_{\epsilon_2} = 1$.

The coexistence of a procyclical product markup and a countercyclical firm or industry markup is illustrated in Figure 8. The reason these two objects behave so differently is the covariance of demand and output. When the variance of demand rises, the covariance rises mechanically as well. The covariance of demand and output is what makes firm markups different from product markups. Firms have higher markups in more volatile environments because that volatility allows them to produce more of products that have extremely high markups. In other words, the volatility of recessions strengthens the composition effects that drive firm markups up but not product markups. This explains why Nekarda and Ramey (2020) found no change in markups but Bils (1985, 1987) did. Both may be right at the same time. Our model can then help researchers to think through which measure matters most for the economic question posed.

These results are for a high marginal cost of investment, which essentially holds firm size fixed. That may be a good assumption for a cyclical fluctuation. However, in the long run, investment may adjust. Appendix B.1. shows that when firms adjust investment flexibly in response to a change in demand and volatility, the effect is dampened.

## VII Where Does Data Come From? A Dynamic Model

So far, the paper has taken firms' data endowments as given, because the question at hand was what effect data has on markups. However, the broader context for this question is that economists are worried about market power, which might arise because of data production. To better understand the interaction of markups and data production, we set up a model that incorporates both. Considering data production and firm growth together introduces one new effect on markups: a role for data as a means of payment. A firm's marginal benefit of a transaction is the profit and the data it generates. Revenue and data enter in a substitutable way. The model teaches us that, when assessing competition, customers can pay with money or data. This is not just true for free digital apps. The price of every good should be affected. Despite this new role, the original insights derived from the static model survive.

**Dynamic model setup** Consider an economy where each firm $i$ chooses an $n \times 1$ vector $a_{it}$ that describes their location in the product space and a quantity $q_{it}$ to produce. As before, firms maximize expected profit, with a price of risk adjustment as in equation (1). There are $n$ attributes and demand shocks for each of those attributes.

What makes data an asset that retains value over multiple periods is that those demand shocks are persistent. If they were not persistent, if demand were independent each period, then data about yesterday's demand would have no value in predicting today's demand. Data would have one-period value. It would not be a long-lived asset. Therefore, we assume a persistent demand process that is an AR(1):

$$b_t = \rho b_{t-1} + \eta_{bt}, \qquad \eta_{bt} \sim iid\ N(0, \sigma_\eta I). \tag{29}$$

At the same time, there needs to be some transitory noise in prices. If there were not, the price of a good would be a sufficient statistic for all past data. If prices revealed all the information in past data, then data would confer only a one-period advantage. It would also not be a long-lived asset. Therefore the demand shock for each attribute is the persistent process (29), plus some transitory noise:

$$\tilde{b}_t = b_t + \epsilon_{bt} \qquad \epsilon_{bt} \sim iid\ N(0, \sigma_\epsilon I) \tag{30}$$

The fact that these demand shocks are common to all firms makes data from one firm relevant for another firm. This opens up the possibility of buying and selling data.

Data is produced as a by-product of economic activity. In other words, the more a firm produces and sells, the more it learns about its customers, its suppliers and its optimal choices. We can model this as a number of data points that depends on the amount produced $q$. Since data is about the demand for each attribute, the amount of data observed $d_{it}$ is also proportional to the good's loading on the attribute:

$$d_{it} = q_{i,t-1} a_{i,t-1}. \tag{31}$$

This captures the idea that firms learn more about attributes they produce. If they produce cell phones, they learn about the demand for (or cost of) electronics, not about demand for food. If they want to learn about the demand for food, they need to produce something edible, or buy such data. Notice that this makes production a form of active experimentation in the product space. Firms are like gamblers in a classic bandit problem, learning about the profitability of each action by observing its result.

The amount of data that a firm has to inform their decisions depends on data production as data purchases or sales: $D_{it} = d_{it} + m_{it}\mathcal{P}_t$ where $m_{it}$ is the amount of data purchased by firm $i$ at date $t$ and $\mathcal{P}_t$ is the time-$t$ market price per unit of data. Firms also choose an amount of data to sell $l_{it}$. Since data is non-rival, data that is sold is not lost. However, selling data may not be optimal if better-informed rivals reduce a firm's own production and profits.

Each data point is a signal about the demand shock vector $b_t$, with precision $\Sigma_e$ per signal. Firms update demand forecasts using Bayes law. Thus, when a firm obtains $D_{it}$ units of data about each attribute, Bayes law tells the firm to average the signals to arrive at a composite signal that has precision $D'_{i,t}\Sigma_e^{-1}D_{i,t}$. Notice that Bayes' law allows us to incorporate non-integer numbers of signals. So we can proceed considering $d_{it}$ and $D_{it}$ to be any real, non-negative numbers.

**Dynamic model solution** Let $\Omega_t$ be the set of all firm's data precisions $\{\omega_{it}\}_{i=1}^N$. The firms' optimal production choices $\{q_{i,t}, a_{i,t}\}$ and data purchases / sales $\{m_{i,t}, l_{i,t}\}$ solve the following recursive problem:

$$V(\Omega_t) = \max_{q_{i,t},a_{i,t},m_{i,t},l_{i,t}} (P_t - c)q_{i,t}a_{i,t} + \mathcal{P}_t(l_{i,t} - m_{i,t}) + \left(\frac{1}{1+r}\right) V(\Omega_{t+1}), \tag{32}$$

where the law of motion for each firm's $\omega_{i,t}$ is given by

$$\omega_{i,t+1} = \left[\rho^2\omega_{i,t}^{-1} + \sigma_\epsilon\right]^{-1} + (n_{i,t} + m_{i,t})\sigma_\epsilon^{-2} \tag{33}$$

and the number of data points produced by the firm is $n_{i,t} = q_{i,t}a_{i,t}$.

The first-order condition for the quantity of production looks similar to (7) in the static model. Optimal production depends on risk and price impact, in the denominator, and expected profit $(p - c)$, in the numerator.

$$\boldsymbol{q}_i a_i = \left(\rho_i \mathbf{Var}\left[\tilde{\boldsymbol{p}}|\mathcal{I}_i\right] + \frac{\partial \mathbf{E}\left[\tilde{\boldsymbol{p}}|\mathcal{I}_i\right]}{\partial \boldsymbol{q}_i}\right)^{-1} \left(\mathbf{E}\left[\tilde{\boldsymbol{p}}|\mathcal{I}_i\right] + \frac{\partial V(\Omega_{t+1})}{\partial q_i} - \boldsymbol{c}_i\right) \tag{34}$$

However, there is one new term in dynamic model: $\partial V/\partial q_i$ is the increase in the future value of the firm, from producing data.

Notice that the future value of data enters additively with the price. This means that monetary payments and data payments are substitutes for the firm. In other words, customers pay for goods, in part with data. This is a partial barter trade where goods are partly paid for with data, as when you receive a loyalty card discount at a supermarket or pharmacy. These discounts are similar to those in customer acquisition models (Nakamura and Steinsson 2011).

Data barter changes the interpretation of markups. The solution (34) reveals that the price of a good is not the complete payment for the good. The relevant measure of income from selling a unit of a good is $p + \partial V / \partial q_i$. So markups underestimate market power because they fail to account for the data payment that accompanies the monetary payment from customers. Firms in areas of the product space where data is valuable should keep their measured markups low, in order to generate more transactions, to generate more valuable data.

While this dynamic extension introduced new ideas about the interaction of data and markups, it did not change the main conclusions of the static model. Data still complicates the interpretation of markups as measures of market power. In this model, there are three main forces at work in dynamic product markups: (i) the classic effect of market power, (ii) a risk premium, and (iii) data barter. In a data-intensive sector, markups reflect the value of data and its effect on risk as well. Data still shows up as a force that changes how markups are aggregated. Firms use data to predict which goods will have high demand and produce more of those goods. Firms that do this prediction well will have higher firm markups and will grow bigger and get higher weights in their industry markup. But this model suggests that simply correcting markups for a risk premium will not be enough to solve the problem of measuring competition in data-intensive industries.

# VIII Mapping Theory to Data

One reason it is important to have models that describe the relationships between quantities like data and markups is that models inform measurement. In this case, the model teaches us how to measure the amount of data a firm has and how to determine what risks that data is about. While executing the measurement is a separate paper, this section is meant to aid others who might choose to use the model as a structure for empirical analysis.

The next result shows that we can measure the amount of data a firm has by looking at the gap between average product markups and firm markups. This is analogous to looking at the alpha of a fund manager to infer how much they know.

**Corollary 1.** ***Markup wedges are measures of data.*** *The production-aggregation wedge $E[M_i^f - \overline{M}_i^p]$ is a monotonic function of firm i's data.*

This result is a straightforward conclusion from Proposition 3. But it is key to measurement. For many measurement exercises, an econometrician may need to know how much data a firm or a collection of firms has. This suggests a measurement approach is to look at the markups at various degrees of aggregation and use the aggregation wedge to infer a corresponding level of data.

WHAT IS DATA ABOUT? MEASURING CHARACTERISTIC LOADINGS Measuring attributes is novel in finance, but more standard in IO. One way to gauge attribute loadings is by looking at demand variance-covariance across goods and extracting principal components. The eigenvectors are loadings. There are also other orthogonal decompositions one can use. But the eigen decomposition has a nice interpretation in terms of principal components.

Another way of measuring characteristic loadings is to use the Hoberg-Phillips measure of cosine similarity from textual analysis of firms' earnings reports. This measure determines how similarly different firms describe their products to their investors.

One might think of a characteristic of a good as being its location. Rossi-Hansberg, Sarte, and Trachter (2018) discovered a different divergence in measures of market power, one between local and national markets. That difference in market power is not expressed in markups but in concentration indices such as HHI (Herfindahl-Hirschman Index). Expressed in markups, there is no documented local-national divergence.[9]

Our predictions are consistent with the superstar firm economy of Autor et al. (2020) and the increasing span of control in Aghion et al. (2019) and Lashkari, Bauer, and Boussard (2018). The rise in firm concentration, the rise in average markups that comes from high-markup firms growing larger, and the correlation between productivity and concentration are all features of U.S. and international markets and are features of our model. Similarly, Crouzet and Eberly (2018) argue that large modern firms have high levels of intangible investment, which is correlated with having high markups. What our work adds is a mechanism—an explanation for why the accumulation of customer data can explain these trends.

MEASURING THE PRICE OF RISK Measuring risk price is novel in IO, but standard in finance. A key parameter that governs the sign of many of the predictions is $\rho$, the price of risk. Finance has developed a whole battery of tools to determine this risk price in various ways. A common approach is to use the market prices of equities to estimate the compensation investors demand for risk in that domain and then carry the same price over to determine the price of risk that a firm faces. The argument for doing that is that the manager should be maximizing equity holders'

[9] Benkard, Yurukoglu, and Zhang (2021) argue that HHI is defined over the market where consumers are located, whereas data used to measure HHI is based on the location of production, which leads to misleading and inconsistent findings when aggregating. Eeckhout (2020) argues that the discrepancy stems from a mechanical relation between population size and the market definition.

interests. The firm's equity holders are the same agents who hold other market equities, with the same risk preferences.[10]

Distinguishing Data from Competition Where data and market competition differ is in $cov(p, q_i)$. Data boosts the covariance between price and quantity by allowing firms to have better forecasts of demand and thereby price. Market competition also changes this covariance by making production decisions more sensitive to expected price changes. But data enhances that sensitivity and also makes expected price and actual price more highly correlated.

Data also enables more accurate forecasting, while market competition does not. Another approach to measuring and identifying firms' data would be to assess the accuracy of firms' forecasts.

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# IX Conclusion

The hypothesis that data encourages large firms to grow larger and gain market power is both plausible and incomplete. Because data improves both prediction and firms' profitability, we need to consider competive effects using a framework where firms compete and face uncertain outcomes that require prediction. In other words, wrestling with the competitive effects of data requires incorporating risk.

To guide out thinking and measurement, we constructed a new framework where firms use

[10] See Brealey, Myers, and Allen (2003) for a more complete explanation of the rationale and execution.

[11] Benkard, Yurukoglu, and Zhang (2021) argue that HHI is defined over the market where consumers are located, whereas data used to measure HHI is based on the location of production, which leads to misleading and inconsistent findings when aggregating. Eeckhout (2020) argues that the discrepancy stems from a mechanical relation between population size and the market definition.

data to reduce uncertainty about future demand for various products.We find that high-data firms do invest more, grow larger, and exert more impact on prices. However, if uncertain firms scale back production, then more data that resolves their uncertainty also pushes markups down. The effect of data may not be seen in markups.

Instead, the effects of data should show up in markup aggregation. Firms react to data about demand by shifting their production to high-demand goods. These are high-markup goods. So data changes the composition of production. This composition effect leads firms to shift production toward high-markup goods, which raises markups. The tug-of-war between risk reduction and the composition effects induced by data plays out differently for product, firm, and industry markups. A model designed to explore the logic of data and large firms turned out to explain why econometricians got different answers about what was happening to markups over time when they measured at different levels of aggregation. Our model suggests a new interpretation of existing facts. Constant product markups and rising firm and industry markups are not competing facts. They are consistent with an economy where firms are getting better and better at forecasting future demand. Both are helpful in the attempt to understand and measure firms' use of data.

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# Online Appendix

## A. Appendix: Solution Details

We start by solving the model with firm-specific shocks and public information for a general variance matrix of shocks for each firm, $\Sigma_{bi} = var(b_i)$. Then, to make the model more transparent, we further simplify to the case where all shocks have variance 1 and all signals have precision 1. Finally, we solve the model where shocks are aggregate, and show that the same properties hold.

### A.1. Solving the model with firm-specific shocks and public information

ATTRIBUTE SPACE The linear mapping $\boldsymbol{A}$ between good and attribute spaces allows us to transform the original model into attribute-competition model in which $n_F$ firms choose upfront investments and attributes to maximize their mean-variance utility.

INFORMATION Each firm indexed by $i$ has $n_{di}$ data points, each of which is a signal of the attribute demand shock $\boldsymbol{s}_{i,j} = \boldsymbol{b_i} + \boldsymbol{\varepsilon}_{i,j}$ where $j = 1, \ldots, n_{di}$. We assume signal noises are uncorrelated and normally distributed with zero mean and precision 1. The posterior variance conditional on $n_{di}$ signals is

$$\mathbf{Var}\left(\boldsymbol{b_i}|\{\boldsymbol{s}_{i,j}\}_{j=1}^{n_{di}}\right) = \frac{1}{1+n_{di}} I_N.$$

This is equivalent to a compound signal $\boldsymbol{s}_i$ with total data precision $n_{di} = \sum_{j=1}^{n_{di}} 1$, the number of data points about firm $i$'s demand.

According to Bayes's law, we have

$$\begin{aligned} \mathbf{E}\left[\tilde{\boldsymbol{p}}_i|\mathcal{I}_i\right] &= \bar{\boldsymbol{p}} + \mathbf{E}\left[\boldsymbol{b}_i|\mathcal{I}_i\right] - \frac{1}{\phi}\sum_{j=1}^{n_F} \tilde{\boldsymbol{q}}_j = \bar{\boldsymbol{p}} + \boldsymbol{K}_i\boldsymbol{s}_i - \frac{1}{\phi}\sum_{j=1}^{n_F}\tilde{\boldsymbol{q}}_j \\ \mathbf{Var}\left[\tilde{\boldsymbol{p}}_i|\mathcal{I}_i\right] &= \mathbf{Var}\left[\boldsymbol{b}_i|\mathcal{I}_i\right] = \frac{1}{1+n_{di}} I_N \end{aligned} \tag{35}$$

For results that refer to more data, we mean a derivative with respect to the number of data points $n_{di}$.

MAXIMIZING RISK-ADJUSTED PROFIT Take first-order condition of firm's utility function, we get an expression for optimal attribute choices.

$$\tilde{\boldsymbol{q}}_i = \left(\rho_i \mathbf{Var}\left[\tilde{\boldsymbol{p}}_i|\mathcal{I}_i\right] - \frac{\partial \mathbf{E}\left[\tilde{\boldsymbol{p}}_i|\mathcal{I}_i\right]}{\partial \tilde{\boldsymbol{q}}_i}\right)^{-1}\left(\mathbf{E}\left[\tilde{\boldsymbol{p}}_i|\mathcal{I}_i\right] - \tilde{\boldsymbol{c}}_i\right)$$

Differentiating the inverse demand curve $\tilde{\boldsymbol{p}}_i = \bar{\boldsymbol{p}} + \boldsymbol{b}_i - \frac{1}{\phi}\sum_{j=1}^{n_F}\tilde{\boldsymbol{q}}_j$ reveals that market power is constant:

$$\frac{\partial \mathbf{E}\left[\tilde{\boldsymbol{p}}_i|\mathcal{I}_i\right]}{\partial \tilde{\boldsymbol{q}}_i} = \frac{\partial \mathbf{E}\left[\boldsymbol{p}_i|\mathcal{I}_i\right]}{\partial \boldsymbol{q}_i} = -\frac{1}{\phi}\boldsymbol{I}_N \tag{36}$$

Substituting this constant market power into the first order condition for optimal output yields the next expression for optimal attribute production. But this expression has the attribute choice $\tilde{q}_i$ on both the left and the right sides of the equality. It arises on the right side because firm $i$'s production choice $\tilde{q}_i$ affects the expected price $\mathbf{E}\left[\tilde{\boldsymbol{p}}_i|\mathcal{I}_i\right]$. Therefore, we substitute in the price and re-arrange to collect all $\tilde{q}_i$ terms and reveal the optimal production choice:

$$\tilde{\boldsymbol{q}}_i = \left(\rho_i \mathbf{Var}\left[\tilde{\boldsymbol{p}}_i|\mathcal{I}_i\right] - \frac{\partial \mathbf{E}\left[\tilde{\boldsymbol{p}}_i|\mathcal{I}_i\right]}{\partial \tilde{\boldsymbol{q}}_i}\right)^{-1} \left(\mathbf{E}\left[\tilde{\boldsymbol{p}}_i|\mathcal{I}_i\right] - \tilde{\boldsymbol{c}}_i\right) \tag{37}$$

Define the sensitivity of supply to a change in the expected profit as:

$$\hat{\boldsymbol{H}}_i := \left(\frac{\boldsymbol{I}_N}{\phi} + \rho_i \mathbf{Var}\left[\boldsymbol{p}_i|\mathcal{I}_i\right]\right)^{-1}. \tag{38}$$

Use (36) to substitute out $\partial \mathbf{E}\left[\tilde{\boldsymbol{p}}_i|\mathcal{I}_i\right] / \partial \tilde{\boldsymbol{q}}_i$. Then use Bayes law to replace the expectation $\mathbf{E}\left[\boldsymbol{b}_i|\mathcal{I}_i\right]$ with the weighted sum of signals $\boldsymbol{K}_i \boldsymbol{s}_i$, with $\boldsymbol{K}_i = \frac{n_{di}}{n_{di}+1} I_N$. Using these three substitutions, we can rewrite (37) as:

$$\tilde{\boldsymbol{q}}_i = \hat{\boldsymbol{H}}_i \left(\bar{\boldsymbol{p}} + \boldsymbol{K}_i \boldsymbol{s}_i - \frac{1}{\phi} \sum_{i'=1}^{n_F} \tilde{\boldsymbol{q}}_{i'} - \tilde{\boldsymbol{c}}_i\right) \tag{39}$$

The solution above generates the best-response function, given the aggregate output. When all data is public, this aggregate output is known. But firm $i$ output choice still shows up on the left and right sides. Before continuing on to solve for aggregate output and prices, we first stop to correctly express firm $i$'s best response, as a function of all other firms' output choices. Define $\boldsymbol{H}_i = \left(\rho_i \mathbf{Var}\left[\boldsymbol{b}_i|\mathcal{I}_i\right] + \frac{2}{\phi} \boldsymbol{I}_N\right)^{-1}$. Then, we can collect the terms in $q_i$ to get the best response:

$$\tilde{\boldsymbol{q}}_i = \boldsymbol{H}_i \left(\bar{\boldsymbol{p}} + \boldsymbol{K}_i \boldsymbol{s}_i - \frac{1}{\phi} \sum_{j \neq i} \tilde{\boldsymbol{q}}_j - \tilde{\boldsymbol{c}}_i\right). \tag{40}$$

SUB-GAME EQUILIBRIUM We solve the sub-game Nash equilibrium by summing both sides of (39) over all firms to express the aggregate output:

$$\begin{aligned}
\sum_i \tilde{\boldsymbol{q}}_i &= \sum_i \hat{\boldsymbol{H}}_i \left(\bar{\boldsymbol{p}} + \boldsymbol{K}_i \boldsymbol{s}_i - \frac{1}{\phi} \sum_{i=1}^{n_F} \tilde{\boldsymbol{q}}_i - \tilde{\boldsymbol{c}}_i\right) \\
\left(I_N + \frac{1}{\phi} \sum_i \hat{\boldsymbol{H}}_i^{-1}\right) \sum_i \tilde{\boldsymbol{q}}_i &= \sum_i \hat{\boldsymbol{H}}_i \left(\bar{\boldsymbol{p}} + \boldsymbol{K}_i \boldsymbol{s}_i - \tilde{\boldsymbol{c}}_i\right) \\
\sum_i \tilde{\boldsymbol{q}}_i &= \left(I_N + \frac{1}{\phi} \sum_i \hat{\boldsymbol{H}}_i^{-1}\right)^{-1} \sum_i \hat{\boldsymbol{H}}_i \left(\bar{\boldsymbol{p}} + \boldsymbol{K}_i \boldsymbol{s}_i - \tilde{\boldsymbol{c}}_i\right)
\end{aligned} \tag{41}$$

From aggregate output, we can express the market price of each attribute as $\tilde{p}^M = \bar{p} - \frac{1}{\phi} \sum_i \tilde{\boldsymbol{q}}_i$. Define $\bar{\boldsymbol{p}}^M$ to be the average market price $\boldsymbol{p}^M$. Given that the signals $\boldsymbol{s}_i$ are, on average, equal to zero, the average market price is

$$\bar{\boldsymbol{p}}^M := \left(\boldsymbol{I}_N + \frac{1}{\phi} \sum_{i=1}^{n_F} \hat{\boldsymbol{H}}_i\right)^{-1} \left(\bar{\boldsymbol{p}} + \frac{1}{\phi} \sum_{i=1}^{n_F} \hat{\boldsymbol{H}}_i \tilde{\boldsymbol{c}}_i\right) \tag{42}$$

Finally, we can express the equilibrium output and price as functions of marginals costs, parameters and firms' data $s_i$:

$$\tilde{\boldsymbol{q}}_i = \hat{\boldsymbol{H}}_i(\bar{\boldsymbol{p}}^M - \tilde{\boldsymbol{c}}_i) + \hat{\boldsymbol{H}}_i\boldsymbol{K}_i\boldsymbol{s}_i - \frac{\hat{\boldsymbol{H}}_i}{\phi}\left(\boldsymbol{I}_N + \frac{1}{\phi}\sum_{i=1}^{n_F}\hat{\boldsymbol{H}}_i\right)^{-1}\sum_{j=1}^{n_F}\hat{\boldsymbol{H}}_j\boldsymbol{K}_j\boldsymbol{s}_j \tag{43}$$

$$\tilde{\boldsymbol{p}}_i \equiv \boldsymbol{p}^M + \boldsymbol{b}_i = \bar{\boldsymbol{p}}^M + \boldsymbol{b}_i - \frac{1}{\phi}\left(\boldsymbol{I}_N + \frac{1}{\phi}\sum_{i=j}^{n_F}\hat{\boldsymbol{H}}_j\right)^{-1}\sum_{j=1}^{n_F}\hat{\boldsymbol{H}}_j\boldsymbol{K}_j\boldsymbol{s}_j, \tag{44}$$

where the last term describes how the realized market price $\boldsymbol{p}^M$ differs from its average $\bar{\boldsymbol{p}}^M$ because of a weighted sum of the firms' random data realizations $\boldsymbol{s}_j$.

PRODUCT-LEVEL MARKUP The product-level markup produced by firm $i$ is $M^{\tilde{p}}_{i,j} := \mathbf{E}[\tilde{\boldsymbol{p}}_{i,j}]/\tilde{\boldsymbol{c}}_{i,j}$. The average product-level markup on the attributes is

$$\overline{M}^{\tilde{p}} = \frac{1}{N}\frac{1}{n_F}\sum_{i=1}^{n_F}\sum_{j=1}^{N} M^{\tilde{p}}_{i,j} = \frac{1}{n_F N}\sum_{i=1}^{n_F}\sum_{j=1}^{N}\frac{\mathbf{E}[\tilde{\boldsymbol{p}}_{i,j}]}{\tilde{\boldsymbol{c}}_{i,j}} = \frac{1}{n_F N}\sum_{i=1}^{n_F}\sum_{j=1}^{N}\frac{\bar{\boldsymbol{p}}^M_j}{\tilde{\boldsymbol{c}}_{i,j}} \tag{45}$$

Since the posterior variance is $\mathbf{Var}\left[\boldsymbol{b}_i|\mathcal{I}_i\right] = 1/(1+n_{di})I_N$, thus

$$\frac{\partial\bar{\boldsymbol{p}}^M_j}{\partial n_{di}} = \frac{1}{\phi}(1 + \frac{1}{\phi}\sum_{s=1}^{n_F}\hat{\boldsymbol{H}}_{s,j})^{-1}\rho_i\hat{\boldsymbol{H}}^2_{i,j}1/(1+n_{di})\left(\tilde{\boldsymbol{c}}_{i,j} - \bar{\boldsymbol{p}}^M_j\right) < 0 \tag{46}$$

which means that $\partial\overline{M}^{\tilde{p}}/\partial\Sigma^{-1}_{\epsilon_i,k} < 0$

FIRM-LEVEL MARKUP The firm-level markup for firm $i$ is the quantity-weighted prices divided by quantity-weighted costs:

$$M^f_i = \frac{\mathbf{E}[\tilde{\boldsymbol{q}}'_i\tilde{\boldsymbol{p}}_i]}{\mathbf{E}[\tilde{\boldsymbol{q}}'_i\tilde{\boldsymbol{c}}_i]} = \frac{\mathbf{E}[\tilde{\boldsymbol{q}}_i]'\mathbf{E}[\tilde{\boldsymbol{c}}] + \mathbf{Trace}[\mathbf{Cov}\left(\tilde{\boldsymbol{p}}_i, \tilde{\boldsymbol{q}}_i\right)]}{\mathbf{E}[\tilde{\boldsymbol{q}}'_i\tilde{\boldsymbol{c}}_i]} \tag{47}$$

Thus, the average firm-level markup is $\overline{M}^f = (1/n_F)\sum_{i=1}^{n_F} M^f_i$. As for the denominator, the equilibrium output increases with more data since

$$\frac{\partial\mathbf{E}\tilde{\boldsymbol{q}}_{i,j}}{\partial n_{di}} = \rho_i\left(\frac{1}{1+n_{di}}\right)^2\hat{\boldsymbol{H}}^2_{i,j}\left(\bar{\boldsymbol{p}}^M_j - \tilde{\boldsymbol{c}}_{i,j}\right)\left[1 - (\phi + \sum_{k=1}^{n_F}\hat{\boldsymbol{H}}_{k,j})^{-1}\hat{\boldsymbol{H}}_{i,j}\right] > 0 \tag{48}$$

Although price decreases with more data, the revenue rises.

$$\begin{aligned}\frac{\partial\mathbf{E}\tilde{\boldsymbol{q}}_{i,j}\mathbf{E}\tilde{\boldsymbol{p}}_{i,j}}{\partial n_{di}} &= \rho_i\left(\frac{1}{1+n_{di}}\right)^2\hat{\boldsymbol{H}}^2_{i,j}\left(\bar{\boldsymbol{p}}^M_j - \tilde{\boldsymbol{c}}_{i,j}\right)\left[\bar{\boldsymbol{p}}^M_j\left(1 - (\phi + \sum_{k=1}^{n_F}\hat{\boldsymbol{H}}_{k,j})^{-1}2\hat{\boldsymbol{H}}_{i,j}\right) + \tilde{\boldsymbol{c}}'_{i,j}(\phi + \sum_{k=1}^{n_F}\hat{\boldsymbol{H}}_{k,j})^{-1}\hat{\boldsymbol{H}}_{i,j}\right]\\ &= \rho_i\left(\frac{1}{1+n_{di}}\right)^2\hat{\boldsymbol{H}}^2_{i,j}\left(\bar{\boldsymbol{p}}^M_j - \tilde{\boldsymbol{c}}_{i,j}\right)\left[\bar{\boldsymbol{p}}^M_j\left(\phi - \hat{\boldsymbol{H}}_{i,j} + \frac{1}{\phi}\sum_{k=1,k\neq i}^{n_F}\hat{\boldsymbol{H}}_{k,j}\right) + \tilde{\boldsymbol{c}}'_{i,j}\hat{\boldsymbol{H}}_{i,j}\right](\phi + \sum_{k=1}^{n_F}\hat{\boldsymbol{H}}_{k,j})^{-1} > 0\end{aligned} \tag{49}$$

where we can swap the order of the $H$ matrices because they are diagonal.

COST-WEIGHTED INDUSTRY MARKUP The industry markup weighted by cost is

$$M^{m,cost} := \frac{\mathbf{E}\left[\sum_{i=1}^{n_F} \tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{p}}_i\right]}{\mathbf{E}\left[\sum_{i=1}^{n_F} \tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{c}}_i\right]} = \frac{\sum_{i=1}^{n_F} \mathbf{E}\left[\tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{p}}_i\right]}{\sum_{i=1}^{n_F} \mathbf{E}\left[\tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{c}}_i\right]} = \sum_{i=1}^{n_F} w_i^{cost} M_i^f \quad \text{where} \quad w_i^{cost} = \frac{\mathbf{E}\left[\tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{c}}_i\right]}{\sum_{i=1}^{n_F} \mathbf{E}\left[\tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{c}}_i\right]}. \tag{50}$$

The weight $w_{i,j}^{cost}$ increases with more data $n_{di}$ as

$$\frac{\partial w_{i,j}^{cost}}{\partial n_{di}} = \frac{\tilde{\boldsymbol{c}}_{i,j}}{\left(\sum_{i=1}^{n_F} \mathbf{E}\left[\tilde{\boldsymbol{q}}_{i,j}' \tilde{\boldsymbol{c}}_{i,j}\right]\right)^2} \left[\frac{\partial \mathbf{E} \tilde{\boldsymbol{q}}_{i,j}}{\partial n_{di}} \left(\sum_{k=1,k\neq i}^{n_F} \mathbf{E}\left[\tilde{\boldsymbol{q}}_{k,j}' \tilde{\boldsymbol{c}}_{k,j}\right]\right) - \mathbf{E}\tilde{\boldsymbol{q}}_{i,j} \left(\sum_{k=1,k\neq i}^{n_F} \frac{\partial \mathbf{E}(\tilde{\boldsymbol{q}}_{k,j})}{\partial n_{di}} \tilde{\boldsymbol{c}}_{k,j}\right)\right] > 0 \tag{51}$$

The last inequality is due to the existing results $\frac{\partial \mathbf{E}\tilde{\boldsymbol{q}}_{i,j}}{\partial n_{di}} > 0$ and $\frac{\partial \mathbf{E}(\tilde{\boldsymbol{q}}_{i,j})}{\partial n_{di}} = \hat{\boldsymbol{H}}_{i,j} \frac{\partial \bar{\boldsymbol{p}}_j^M}{\partial n_{di}} < 0$.

SALES-WEIGHTED INDUSTRY MARKUP The industry markup weighted by sales is

$$M^{m,sale} := \sum_{i=1}^{n_F} w_i^{sale} M_i^f = \frac{\sum_{i=1}^{n_F} \frac{\mathbf{E}^2\left[\tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{p}}_i\right]}{\mathbf{E}\left[\tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{c}}_i\right]}}{\sum_{i=1}^{n_F} \mathbf{E}\left[\tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{p}}_i\right]} \quad \text{where} \quad w_i^{sale} = \frac{\mathbf{E}\left[\tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{p}}_i\right]}{\sum_{i=1}^{n_F} \mathbf{E}\left[\tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{p}}_i\right]}. \tag{52}$$

EXPECTED RISK-ADJUSTED FIRM PROFIT To solve for the firms' cost choices, we need to solve for expected utility of each firm. Substituting in the definition of firm profit into the objective function (1)

$$U_i = \mathbf{E}\left[\tilde{\boldsymbol{q}}_i' (\tilde{\boldsymbol{p}}_i - \tilde{\boldsymbol{c}}_i)\right] - \frac{\rho_i}{2} \mathbf{E}\left[\tilde{\boldsymbol{q}}_i' \mathbf{Var}\left[\tilde{\boldsymbol{p}}_i | \mathcal{I}_i\right] \tilde{\boldsymbol{q}}_i\right] - g(\chi_c, \tilde{\boldsymbol{c}}_i) \tag{53}$$

Note that the first expected profit term is $\mathbf{E}\left[\tilde{\boldsymbol{q}}_i' (\mathbf{E}\left[\tilde{\boldsymbol{p}}_i | \mathcal{I}_i\right] - \tilde{\boldsymbol{c}}_i)\right]$. Using the first order condition (7), we can substitute $(\mathbf{E}\left[\tilde{\boldsymbol{p}}_i | \mathcal{I}_i\right] - \tilde{\boldsymbol{c}}_i)$ out with $\hat{\boldsymbol{H}}_i^{-1} \tilde{\boldsymbol{q}}_i$. That substitution allows us to write the firm objective as

$$\begin{aligned} U_i &= \tilde{\boldsymbol{q}}_i' \left(\hat{\boldsymbol{H}}_i^{-1} - \frac{\rho_i}{2} \mathbf{Var}\left[\tilde{\boldsymbol{p}}_i | \mathcal{I}_i\right]\right) \tilde{\boldsymbol{q}}_i - g(\chi_c, \tilde{\boldsymbol{c}}_i) \quad = \tilde{\boldsymbol{q}}_i' \left(\frac{I_N}{\phi} + \frac{\rho_i}{2} \mathbf{Var}\left[\tilde{\boldsymbol{p}}_i | \mathcal{I}_i\right]\right) \tilde{\boldsymbol{q}}_i - g(\chi_c, \tilde{\boldsymbol{c}}_i) \\ &= \frac{1}{2} \tilde{\boldsymbol{q}}_i' \boldsymbol{H}_i^{-1} \tilde{\boldsymbol{q}}_i - g(\chi_c, \tilde{\boldsymbol{c}}_i) \end{aligned} \tag{54}$$

The expression above is utility conditional on a firm's data. To choose marginal cost, we need to compute expected utility that is not conditional on the firm's signals because firms choose cost before signals are observed. This utility could be expressed as expected profit minus the price of risk. Substituting in the expected profit expression above, we get

$$\begin{aligned} \mathbf{E}[U_i] &= \frac{1}{2} \mathbf{E}\left[\tilde{\boldsymbol{q}}_i' \boldsymbol{H}_i^{-1} \tilde{\boldsymbol{q}}_i\right] - g(\chi_c, \tilde{\boldsymbol{c}}_i) \\ &= \frac{1}{2} \left(\mathbf{E}[\tilde{\boldsymbol{q}}_i]' \boldsymbol{H}_i^{-1} \mathbf{E}[\tilde{\boldsymbol{q}}_i] + Trace\left(\boldsymbol{H}_i^{-1} \mathbf{Var}[\tilde{\boldsymbol{q}}_i]\right)\right) - g(\chi_c, \tilde{\boldsymbol{c}}_i) \end{aligned} \tag{55}$$

We can compute the mean of the firm's quantity choice by taking an expectation of (43), using the fact that the prior means of all data points $s_i$ and $s_j$ are zero:

$$E[\tilde{\boldsymbol{q}}_i] = \hat{\boldsymbol{H}}_i (\bar{\boldsymbol{p}}^M - \tilde{\boldsymbol{c}}_i). \tag{56}$$

To work out the variance term, use the first order condition (7) to rewrite $\mathbf{Var}[\tilde{\boldsymbol{q}}_i] = \hat{\boldsymbol{H}}_i' \mathbf{Var}\left(\tilde{\boldsymbol{p}}_i\right) \hat{\boldsymbol{H}}_i$, since the only ex-ante unknown variable in the first order condition is the price. Next, recognize that the price is a sum of two independent terms, the market price and the demand shock: $\tilde{\boldsymbol{p}}_i = p^M + b_i$, where $p^M$ is given by (44). Thus,

$$\mathbf{Var}\left(\tilde{\boldsymbol{p}}_i\right) = I_N + \frac{1}{\phi^2}\left(\sum_{i=j}^{n_F}\hat{\boldsymbol{H}}_j\right)^{-2}\sum_{j=1}^{n_F}(\hat{\boldsymbol{H}}_j\boldsymbol{K}_j)'\mathbf{Var}(\boldsymbol{s}_j)(\hat{\boldsymbol{H}}_j\boldsymbol{K}_j)' \tag{57}$$

If there are $n_{di}$ data points about each attribute, each with precision 1, then the variance of a firm's average data point $s_j$ is $(1 + 1/n_{di})I_N$. In this case, the formula for the variance of output is $\mathbf{Var}[\tilde{\boldsymbol{q}}_i] = \hat{\boldsymbol{H}}_i'\mathbf{Var}\left(\tilde{\boldsymbol{p}}_i\right)\hat{\boldsymbol{H}}_i$, with the variance of the price given by (57). This yields

$$\mathbf{Var}[\tilde{\boldsymbol{q}}_i] = \hat{\boldsymbol{H}}_i'\left[I_N + \frac{1}{\phi^2}\left(\frac{n_{di}}{n_{di}+1}\right)\left(\sum_{i=j}^{n_F}\hat{\boldsymbol{H}}_j'\right)^{-1}\left(\sum_{i=j}^{n_F}\hat{\boldsymbol{H}}_j\right)^{-1}\sum_{j=1}^{n_F}\hat{\boldsymbol{H}}_j\hat{\boldsymbol{H}}_j'\right]\hat{\boldsymbol{H}}_i. \tag{58}$$

Notice that this part of expected utility is independent of the firm's cost choices.

OPTIMAL CHOICES OF MARGINAL COST The first and second order condition for the optimal marginal cost choice $\boldsymbol{c}_i$ is

$$\begin{aligned}
\frac{\partial \mathbf{E}[U_i]}{\partial \tilde{\boldsymbol{c}}_i} &= \frac{1}{2}\frac{\partial \mathbf{E}[\tilde{\boldsymbol{q}}_i]'\boldsymbol{H}_i^{-1}\mathbf{E}[\tilde{\boldsymbol{q}}_i]}{\partial \tilde{\boldsymbol{c}}_i} - \frac{\partial g(\chi_c, \tilde{\boldsymbol{c}}_i)}{\partial \tilde{\boldsymbol{c}}_i} = 0 \\
\frac{\partial^2 \mathbf{E}[U_i]}{\partial \tilde{\boldsymbol{c}}_i \partial \tilde{\boldsymbol{c}}_i'} &= \frac{1}{2}\frac{\partial^2 \mathbf{E}[\tilde{\boldsymbol{q}}_i]'\boldsymbol{H}_i^{-1}\mathbf{E}[\tilde{\boldsymbol{q}}_i]}{\partial \tilde{\boldsymbol{c}}_i \partial \tilde{\boldsymbol{c}}_i'} - \frac{\partial^2 g(\chi_c, \tilde{\boldsymbol{c}}_i)}{\partial \tilde{\boldsymbol{c}}_i \partial \tilde{\boldsymbol{c}}_i'} \text{ is negative semi-definite}
\end{aligned} \tag{59}$$

Since signal noise is diagonal, we have $\boldsymbol{H}_{i,j}^{-1} = \frac{2}{\phi} + \rho_i \mathbb{V}\left[b_i|\mathcal{I}_i\right]$ and $\mathbb{V}\left[b_i|\mathcal{I}_i\right] = (1 + n_{di})^{-1}$. Thus the FOC and SOC could be written as

$$\begin{aligned}
\frac{\partial \mathbf{E}[U_i]}{\partial \tilde{\boldsymbol{c}}_i} &= \frac{1}{2}\frac{\partial}{\partial \tilde{\boldsymbol{c}}_i}\left(\sum_{s=1}^{N}\left(\bar{\boldsymbol{p}}_s^M - \tilde{\boldsymbol{c}}_{i,s}\right)^2\hat{\boldsymbol{H}}_{i,s}^2\boldsymbol{H}_{i,s}^{-1}\right) - \frac{\partial g(\chi_c, \tilde{\boldsymbol{c}}_i)}{\partial \tilde{\boldsymbol{c}}_i} = 0 \\
\frac{\partial \mathbf{E}[U_i]}{\partial \tilde{\boldsymbol{c}}_{i,j}} &= \left(\bar{\boldsymbol{p}}_j^M - \tilde{\boldsymbol{c}}_{i,j}\right)\hat{\boldsymbol{H}}_{i,j}^2\boldsymbol{H}_{i,j}^{-1}\left[\frac{\hat{\boldsymbol{H}}_{i,j}\phi^{-1}}{1 + \frac{1}{\phi}\sum_{s=1}^{n_F}\hat{\boldsymbol{H}}_{s,j}} - 1\right] - \frac{\partial g(\chi_c, \tilde{\boldsymbol{c}}_i)}{\partial \tilde{\boldsymbol{c}}_{i,j}} = 0 \\
\frac{\partial^2 \mathbf{E}[U_i]}{\partial \tilde{\boldsymbol{c}}_{i,j}\partial \tilde{\boldsymbol{c}}_{i,k}} &= \hat{\boldsymbol{H}}_{i,j}^2\boldsymbol{H}_{i,j}^{-1}\left[\frac{\hat{\boldsymbol{H}}_{i,j}\phi^{-1}}{1 + \frac{1}{\phi}\sum_{s=1}^{n_F}\hat{\boldsymbol{H}}_{s,j}} - 1\right]\frac{\hat{\boldsymbol{H}}_{i,j}\phi^{-1}}{1 + \frac{1}{\phi}\sum_{s=1}^{n_F}\hat{\boldsymbol{H}}_{s,j}} - \frac{\partial g(\chi_c, \tilde{\boldsymbol{c}}_i)}{\partial \tilde{\boldsymbol{c}}_{i,j}\partial \tilde{\boldsymbol{c}}_{i,k}}
\end{aligned} \tag{60}$$

since the average market price $D_j$ for attribute $j$ is

$$\bar{\boldsymbol{p}}_j^M = \frac{\bar{\boldsymbol{p}}_j + \frac{1}{\phi}\sum_{s=1}^{n_F}\hat{\boldsymbol{H}}_{s,j}\tilde{\boldsymbol{c}}_{s,j}}{1 + \frac{1}{\phi}\sum_{s=1}^{n_F}\hat{\boldsymbol{H}}_{s,j}} \quad \text{and} \quad \frac{\partial \bar{\boldsymbol{p}}_j^M}{\partial \tilde{\boldsymbol{c}}_{i,k}} = \frac{\hat{\boldsymbol{H}}_{i,j}\phi^{-1}}{1 + \frac{1}{\phi}\sum_{s=1}^{n_F}\hat{\boldsymbol{H}}_{s,j}} \tag{61}$$

### A.2. Simplest Model with Symmetric Firms

To make the results more transparent, consider the case where all firms are symmetric. They have an equal number of data points $n_{d,i} = n_d\ \forall i$ and equal prices of risk (which could be zero) $\rho_i = \rho\ \forall i$. Recall that we can consider WLOG the case where each signal has precision 1: $\widetilde{\Sigma} = 1$. If this is not the case, we can always scale the number of signals $n_d$ to create an equivalent representation of the model, where each signal has precision 1. As stated in the model setup, the variance of $b_i$, the demand shocks, is $I_N$.

In this case, the total data precision is simply the number of signals: $n_d$. Since the prior precision of the demand shock $b$ is also 1, by Bayes' law, the posterior precision is $\mathbb{V}\left[b_i|\mathcal{I}_i\right] = (1+n_d)^{-1}I_N$ and the Bayesian updating weight on the average data point is $\boldsymbol{K}_i = n_d/(n_d+1)I_N$. Substituting in these simplified terms, we can rewrite the sensitivities of firm production to changes in the price $\widehat{\boldsymbol{H}}_i$ and sensitivities to changes in their cost $\boldsymbol{H}_i$ as:

$$\widehat{\boldsymbol{H}}_i := \left(\frac{1}{\phi} + \rho_i \mathbb{V}\left[b_i|\mathcal{I}_i\right]\right)^{-1} = \frac{\phi(1+n_d)}{1+n_d+\rho\phi} I_N \tag{62}$$

$$= \phi\zeta I_N \qquad \text{where} \qquad \zeta = \frac{(1+n_d)}{1+n_d+\rho\phi} \tag{63}$$

$$\sum_{i=1}^{nF} \widehat{\boldsymbol{H}}_i = n_F \phi\zeta I_N \tag{64}$$

$$\boldsymbol{H}_i := \left(\frac{2}{\phi} + \rho_i \mathbb{V}\left[b_i|\mathcal{I}_i\right]\right)^{-1} = \frac{\phi(1+n_d)}{2(1+n_d)+\rho\phi} I_N \tag{65}$$

$$\text{where} \qquad \boldsymbol{H}_i^{-1} = \widehat{\boldsymbol{H}}_i^{-1} + \frac{1}{\phi} I_N = \frac{1}{\phi}\left(\frac{1}{\zeta}+1\right) I_N \tag{66}$$

$$\sum_{i=1}^{nF} \boldsymbol{H}_i = \frac{n_F(1+n_d)}{2(1+n_d)+\rho\phi} I_N \tag{67}$$

$$\tag{68}$$

We can then substitute these firm-level and aggregate supply sensitivities into (42), (56) and (58) to get simplified expressions for the expected market price and the mean and variance of the firm-level supply of each good:

$$\bar{\boldsymbol{p}}^M = \left(1 + \frac{1}{\phi}\sum_{i=1}^{n_F} \widehat{\boldsymbol{H}}_i\right)^{-1} \left(\overline{\boldsymbol{p}} + \frac{1}{\phi}\sum_{i=1}^{n_F} \widehat{H}_i \overline{\boldsymbol{c}}\right) = \frac{1}{1+n_F\zeta}(\overline{\boldsymbol{p}} + n_F\zeta\overline{\boldsymbol{c}}) \tag{69}$$

$$\mathbf{E}[\tilde{\boldsymbol{q}}_i] = \boldsymbol{H}_i \left(\bar{\boldsymbol{p}}^M - \overline{\boldsymbol{c}}\right) = \frac{\phi\zeta}{1+n_F\zeta}\left(\bar{\boldsymbol{p}} - \overline{\boldsymbol{c}}\right) \tag{70}$$

$$\mathbf{Var}[\tilde{\boldsymbol{q}}_i] = \phi^2\zeta^2 \frac{n_d}{n_d+1}\left[1 - n_F\left(\frac{\zeta}{1+n_F\zeta}\right)^2\right] I_N \tag{71}$$

# B. Proofs and Auxiliary Results

**Proof of Lemma 2: Data-Investment Complementarity**

*Proof.* To show this complementarity between information and costs, we first differentiate $\mathbb{E}\left[U_i\right]$ in (85) with respect to marginal cost. Here, $\tilde{c}_{ij}$ denotes firm $i$'s marginal cost of producing attribute $j$. $\widehat{\boldsymbol{H}}_{ij}$ denotes the $jj$-th entry of the diagonal matrix $\widehat{\boldsymbol{H}}_i$, which captures the sensitivity of $i$'s production of attribute $j$ to a marginal change in the expected profit of producing attribute $j$. Then,

$$\begin{aligned}\frac{\partial \mathbb{E}\left[U_i\right]}{\partial \widetilde{c}_{ij}} &= \frac{\partial}{\partial \widetilde{c}_{ij}}\left\{\frac{1}{2}\left[\mathbb{E}\left[\widetilde{\boldsymbol{q}}_i\right]' \boldsymbol{H}_i^{-1}\mathbb{E}\left[\widetilde{\boldsymbol{q}}_i\right] + \operatorname{tr}\left(\boldsymbol{H}_i^{-1}\mathbb{V}\left[\widetilde{\boldsymbol{q}}_i\right]\right)\right] - g\left(\chi_c, \widetilde{\boldsymbol{c}}_i\right)\right\} \\ &= -\widehat{\boldsymbol{H}}_{ij}^2 \boldsymbol{H}_{ij}^{-1}\left(\phi + \sum_{i=1}^{n_F}\widehat{\boldsymbol{H}}_{ij}\right)^{-2}\left[\phi\left(\overline{\boldsymbol{p}} - \widetilde{c}_{ij}\right) + \sum_{s\neq i}^{n_F}\widehat{H}_{sj}\left(\widetilde{c}_{sj} - \widetilde{c}_{ij}\right)\right]\left(\phi + \sum_{s\neq i}\widehat{H}_{sj}\right) \\ &\quad - \frac{\partial g\left(\chi_c, \widetilde{\boldsymbol{c}}_i\right)}{\partial \widetilde{c}_i}\end{aligned} \tag{72}$$

Denote $\mathbb{V} = \mathbb{V}\left[\boldsymbol{b}_{ij}|\mathcal{I}\right]$. As a short hand, we use matrix exponents as $X^2 := X'X$. We do not need to respect matrix orderings because all matrices regarding attributes are square and diagonal. Then, the second order cross derivative is:

$$\begin{aligned}\frac{\partial^2 \mathbb{E}\left[U_i\right]}{\partial \widetilde{c}_{ij}\partial \mathbb{V}\left[\boldsymbol{b}_{ij}|\mathcal{I}\right]} &= \frac{\partial}{\partial \mathbb{V}}\left\{-\widehat{\boldsymbol{H}}_{ij}^2 \boldsymbol{H}_{ij}^{-1}\left(\phi + \sum_{i=1}^{n_F}\widehat{\boldsymbol{H}}_{ij}\right)^{-2}\left[\phi\left(\overline{\boldsymbol{p}} - \widetilde{c}_{ij}\right) + \sum_{l\neq i}^{n_F}\widehat{H}_{lj}\left(\widetilde{c}_{lj} - \widetilde{c}_{ij}\right)\right]\left(\phi + \sum_{l\neq i}\widehat{H}_{lj}\right) - \frac{\partial g\left(\chi_c, \widetilde{\boldsymbol{c}}_i\right)}{\partial \widetilde{c}_i}\right\} \\ &= \underbrace{-\left\{\left[\phi\left(\overline{\boldsymbol{p}} - \widetilde{c}_{ij}\right) + \sum_{l\neq i}^{n_F}\widehat{H}_{lj}\left(\widetilde{c}_{lj} - \widetilde{c}_{ij}\right)\right]\left(\phi + \sum_{l\neq i}\widehat{H}_{lj}\right)\right\}}_{\text{A negative term, denote } L^-} \\ &\qquad \cdot \frac{\partial}{\partial \mathbb{V}_i}\left(\phi + \sum_{s=1}^{n_F}\left(\frac{1}{\phi} + \rho_s \mathbb{V}_s\right)^{-1}\right)^{-2}\left(\frac{1}{\phi} + \rho_i \mathbb{V}_i\right)^{-2}\left(\frac{2}{\phi} + \rho_i \mathbb{V}_i\right) \\ &= \left(L^-\right)\cdot \underbrace{\frac{\partial}{\partial \mathbb{V}_i}\left\{\left(2 + \phi\rho_i \mathbb{V}_i + \sum_{s\neq i}^{n_F}\frac{\frac{1}{\phi} + \rho_i \mathbb{V}_i}{\frac{1}{\phi} + \rho_s \mathbb{V}_s}\right)\left(\phi + \sum_{s\neq i}^{n_F}\left[\frac{1}{\frac{1}{\phi} + \rho_s \mathbb{V}_s}\left(1 - \frac{1}{2 + \phi\rho_i \mathbb{V}_i}\right)\right]\right)\right\}^{-1}}_{\text{Decreasing n } \mathbb{V}_i\text{, i.e., } <0} > 0\end{aligned} \tag{73}$$

Hence, we get $\frac{\partial^2 \mathbb{E}[U_i]}{\partial \widetilde{c}_{ij}\partial \mathbb{V}} > 0$, which means the marginal benefit from reducing costs is higher (more negative) when firms have better information (lower variance).

□

**Proof of Lemma 3: Greater investment raises a firm's product markup.**

*Proof.* More investment would lower marginal cost $\tilde{\boldsymbol{c}}_{i,j}$ and its derivative is

$$\frac{\partial M_{i,j}^{\tilde{p}}}{\partial \tilde{\boldsymbol{c}}_{i,j}} = \frac{\frac{\partial \bar{\boldsymbol{p}}_j^M}{\partial \tilde{\boldsymbol{c}}_{i,j}}\tilde{\boldsymbol{c}}_{i,j} - \bar{\boldsymbol{p}}_j^M}{\tilde{\boldsymbol{c}}_{i,j}^2} = \frac{\frac{1}{\phi}\hat{\boldsymbol{H}}_{i,j}\tilde{\boldsymbol{c}}_{i,j} - \bar{\boldsymbol{p}}_j - \frac{1}{\phi}\sum_{s=1}^{n_F}\hat{\boldsymbol{H}}_{s,j}\tilde{\boldsymbol{c}}_{s,j}}{\tilde{\boldsymbol{c}}_{i,j}^2\left(1 + \frac{1}{\phi}\sum_{s=1}^{n_F}\hat{\boldsymbol{H}}_{s,j}\right)} = -\frac{\bar{\boldsymbol{p}}_j + \frac{1}{\phi}\sum_{s=1,s\neq i}^{n_F}\hat{\boldsymbol{H}}_{s,j}\tilde{\boldsymbol{c}}_{s,j}}{\tilde{\boldsymbol{c}}_{i,j}^2\left(1 + \frac{1}{\phi}\sum_{s=1}^{n_F}\hat{\boldsymbol{H}}_{s,j}\right)} \leq 0 \tag{74}$$

The negative derivative confirms that more investment leads to higher attribute-level markup. Similarly, for the other attributes $j'$ we have $\frac{\partial M_{i,j'}^{\tilde{p}}}{\partial \tilde{\boldsymbol{c}}_{i,j}} = 0$.

Next, differentiate this product markup with respect to the marginal cost of attribute $j$. Consider the markup on product $k$ that used attribute $j$ ($A_{kj} > 0$). This markup is $\sum_j A_{k,j}\mathbb{E}[\tilde{p}_{i,j}]/(\sum_j A_{kj}\tilde{\boldsymbol{c}}_{i,j})$.

Its derivative is

$$\frac{dM_{i,k}}{d\tilde{\boldsymbol{c}}_{i,j}} = \frac{[\sum_j A_{kj}\tilde{\boldsymbol{c}}_{i,j}]A_{kj}\frac{\partial}{\partial\tilde{\boldsymbol{c}}_{i,j}}\mathbb{E}[\tilde{p}_{i,j}] - [\sum_j A_{k,j}\mathbb{E}[\tilde{p}_{i,j}]]A_{kj}}{[\sum_j A_{kj}\tilde{\boldsymbol{c}}_{i,j}]^2} \tag{75}$$

$$= \frac{A_{kj}}{\sum_j A_{kj}\tilde{\boldsymbol{c}}_{i,j}}\left[\frac{\partial}{\partial\tilde{\boldsymbol{c}}_{i,j}}\mathbb{E}[\tilde{p}_{i,j}] - M_{i,k}\right]. \tag{76}$$

We know that $\frac{\partial}{\partial\tilde{c}_{i,j}}\mathbb{E}[p_{i,j}] < M^{\tilde{p}_{i,j}}$ because earlier in the proof, we established that

$$\frac{dM^{\tilde{p}}_{i,j}}{d\tilde{\boldsymbol{c}}_{i,j}} = \frac{1}{\tilde{\boldsymbol{c}}_{i,j}}\left[\frac{\partial}{\partial\tilde{\boldsymbol{c}}_{i,j}}\mathbb{E}[\tilde{p}_{i,j}] - M_{i,k}\right] \leq 0. \tag{77}$$

Therefore, (76) is negative if the markup on product $k$ is greater than the markup on attribute $j$: $M_{i,k} \geq M^{\tilde{p}_{i,j}}$. □

**Proof of Lemma 4: (Risk premium channel) Product-level markup decreases in data.** When investment is sufficiently inflexible (high $\chi_c$), and product $i$ loads positively on all attributes ($a_{ij} \geq 0$), then the product markup $\mathbf{E}\left(p_i/c_i\right) = \mathbf{E}\left(p_i\right)/c_i$ is decreasing in data.

*Proof.* Assume each firm is endowed with a fixed investment ($\boldsymbol{c}_i$). By continuity, the result will extend to cases where the investment is close to fixed, which is when $\chi_c$ is sufficiently high. The markup on the attribute $j$, produced by firm $i$ is $M^{\tilde{p}}_{i,j} := \mathbf{E}[\tilde{\boldsymbol{p}}_{i,j}]/\boldsymbol{c}_{i,j}$. The average markup on the attributes is

$$\overline{M}^{\tilde{p}} = \frac{1}{N}\frac{1}{n_F}\sum_{i=1}^{n_F}\sum_{j=1}^{N} M^{\tilde{p}}_{i,j} = \frac{1}{n_F N}\sum_{i=1}^{n_F}\sum_{j=1}^{N}\frac{\mathbf{E}[\tilde{\boldsymbol{p}}_{i,j}]}{\boldsymbol{c}_{i,j}} = \frac{1}{n_F N}\sum_{i=1}^{n_F}\sum_{j=1}^{N}\frac{\bar{\boldsymbol{p}}^M_j}{\boldsymbol{c}_{i,j}} \tag{78}$$

We denote the posterior variance $\Sigma_{b_i} = \mathbf{Var}\left[\boldsymbol{b}_i|\mathcal{I}_i\right] = \left(I_N + \Sigma^{-1}_{\epsilon_i}\right)^{-1}$. The $j^{th}$ term of equilibrium price $\bar{\boldsymbol{p}}^M_j$ is

$$\bar{\boldsymbol{p}}^M_j = \frac{\bar{\boldsymbol{p}}_j + \frac{1}{\phi}\sum_{i=1}^{n_F}\hat{\boldsymbol{H}}_{i,j}\boldsymbol{c}_{i,j}}{1 + \frac{1}{\phi}\sum_{i=1}^{n_F}\hat{\boldsymbol{H}}_{i,j}} \quad \text{where} \quad \hat{\boldsymbol{H}}_{i,j} = \left[\frac{1}{\phi} + \rho_i\left(1 + \Sigma^{-1}_{\epsilon_{i,j}}\right)^{-1}\right]^{-1} \tag{79}$$

The positive output means $\bar{\boldsymbol{p}}^M \geq \boldsymbol{c}_i$, thus

$$\frac{\partial\bar{\boldsymbol{p}}^M_j}{\partial n_{di}} = \delta_{jk}\frac{1}{\phi}\frac{\rho_i\hat{\boldsymbol{H}}^2_{i,j}\Sigma^2_{b_{i,j}}}{1 + \frac{1}{\phi}\sum_{s=1}^{n_F}\hat{\boldsymbol{H}}_{s,j}}\left(\boldsymbol{c}_{i,j} - \bar{\boldsymbol{p}}^M_j\right) < 0 \Rightarrow \frac{\partial\overline{M}^{\tilde{p}}}{\partial n_{di}} < 0 \tag{80}$$

Since the price of a good is $a_i$ times the vector of attribute prices, and all the attribute prices are decreasing in data, the good price and thus the product-level markup is decreasing in data as well.

We prove the negative first order derivative for fixed choices of cost $\tilde{\boldsymbol{c}}_i$, which corresponds to infinitely high marginal cost $\chi_c \to \infty$. This result is strictly negative and continuous in $\tilde{\boldsymbol{c}}_i$. If we assume $\chi_c$ is sufficiently high, this is arbitrarily close to fixed $c$. By continuity, the inequality will still hold. □

□

**Proof of Proposition 1: Product markups increase or decrease in data (net change).**

*Proof.* The product-level markup is $M^{\tilde{p}}_{i,j} = \mathbf{E}[\tilde{\boldsymbol{p}}_{i,j}]/\tilde{\boldsymbol{c}}_{i,j} = \bar{\boldsymbol{p}}^M_j/\tilde{\boldsymbol{c}}_{i,j}$. Its partial derivative to data is

$$\frac{\partial M^{\tilde{p}}_{i,j}}{\partial n_{di}} = \frac{1}{\tilde{\boldsymbol{c}}^2_{i,j}}\left(\frac{\partial \bar{\boldsymbol{p}}^M_j}{\partial n_{di}}\tilde{\boldsymbol{c}}_{i,j} - \bar{\boldsymbol{p}}^M_j\frac{\partial \tilde{\boldsymbol{c}}_{i,j}}{\partial n_{di}}\right) \tag{81}$$

According to (46) and Lemma 2, we have

$$\frac{\partial \bar{\boldsymbol{p}}^M_j}{\partial n_{di}} = \frac{1}{\phi}\frac{\rho_i \hat{\boldsymbol{H}}^2_{i,j}\Sigma^2_{b_i,j}}{1+\frac{1}{\phi}\sum_{s=1}^{n_F}\hat{\boldsymbol{H}}_{s,j}}\left(\tilde{\boldsymbol{c}}_{i,j} - \bar{\boldsymbol{p}}^M_j\right) < 0 \quad \text{and} \quad \frac{\partial \tilde{\boldsymbol{c}}_i}{\partial n_{di}} = \left(-\frac{\partial^2 \mathbf{E}[U_i]}{\partial \tilde{\boldsymbol{c}}_i \partial \tilde{\boldsymbol{c}}_i'}\right)^{-1}\Lambda \le 0 \tag{82}$$

If marginal cost $\tilde{\boldsymbol{c}}_{i,j}$ or price of risk $\rho_i$ is sufficiently low, the second term in the numerator $-\bar{\boldsymbol{p}}^M_j\frac{\partial \tilde{c}_{i,j}}{\partial n_{di}} > 0$ dominates the marginal effect, thus increasing product markups.

□

**Proof of Proposition 2: Welfare** In this section, we work out the components of welfare, as preliminaries to the two welfare results that follow. Firm profits are given by (54), where the production decisions are given by (7).

Consumer surplus is the area under the demand curve that lies above the equilibrium price. Since there are $N$ attributes, we sum up the surplus from each attribute. In (4), we that the demand curve is a linear function, $\bar{p} - 1/\phi\sum_i \tilde{q}_i + b_i$. Consumer surplus is the triangle under this demand curve, that lies above the equilibrium price. The height of this triangle is $\bar{p} - (\bar{p} - 1/\phi\sum_i \tilde{q}_i + b_i)$ $= \sum_i \tilde{q}_i + b_i$. The width of this triangle is $\sum_i \tilde{q}_i$. The area of this triangle is $1/2$ its base times the height: $1/2(\sum_i \tilde{q}_i + b_i)\sum_i \tilde{q}_i$. We take the expectation of this product as the measure of expected consumer surplus.

$$\begin{aligned}\mathbb{E}CS &= \frac{1}{2\phi}\mathbb{E}\left[(\sum_i \tilde{q}_i)(\sum_i \tilde{q}_i + b_i)\right] &(83)\\ &= \frac{1}{2\phi}\left(\mathbb{E}\left[\sum_i \tilde{q}_i\right]^2 + \mathbb{V}\left[\sum_i \tilde{q}_i\right] + \mathbb{C}\left[\sum_i \tilde{q}_i, \sum_i b_i\right]\right) &(84)\end{aligned}$$

Welfare is consumer surplus (84) plus the sum of all firms' profits (54).

Since the result considers a case with symmertic firms, we can use the expressions for the symmetric model in (62) - (71) to write firms' profit (54) as

$$\begin{aligned}\mathbf{E}[U_i] &= \frac{1}{2}\left(\mathbf{E}[\tilde{\boldsymbol{q}}_i]'\boldsymbol{H}_i^{-1}\mathbf{E}[\tilde{\boldsymbol{q}}_i] + \mathbf{tr}\left(\boldsymbol{H}_i^{-1}\mathbf{Var}[\tilde{\boldsymbol{q}}_i]\right)\right) - g(\chi_c, \tilde{\boldsymbol{c}}_i)\\ &= \frac{\phi\zeta}{2(1+n_F\zeta)^2}\left(\bar{\boldsymbol{p}} - \bar{\boldsymbol{c}}\right)'\left(\bar{\boldsymbol{p}} - \bar{\boldsymbol{c}}\right) + \frac{N}{2}\phi\zeta(1+\zeta)\frac{n_d}{n_d+1}\left[1 - n_F\left(\frac{\zeta}{1+n_F\zeta}\right)^2\right] - g(\chi_c, \tilde{\boldsymbol{c}}_i)\end{aligned} \tag{85}$$

where $tr$ is a matrix trace and the first term of the last equality comes from rewriting $H_i^{-1} = \hat{\boldsymbol{H}}_i^{-1} + 1/\phi = n_F + \zeta/(\phi\zeta)$.

Aggregate output $\sum_i \tilde{q}_i$ is given by (41). In the special case where firms' demand shocks are independent, firms are symmetric and shocks and signals have variance 1, we can express aggregate output as

$$\sum_i \widetilde{q}_i = \frac{n_F \zeta^2}{n_F + \zeta} \left( \bar{\boldsymbol{p}} - \bar{\boldsymbol{c}} + \frac{n_d}{n_d + 1} \sum_i s_i \right) \tag{86}$$

where $s_i$ is the average data point of firm $i$. The covariance of output with the demand shocks works through the data $s_i = b_i + \bar{\epsilon}_i$, where $\bar{\epsilon}_i$ is the average signal noise, independent across firms and attributes, with mean zero and variance $1/n_{di}$.

Since data realizations $s_i$ have mean zero, the expected aggregate output is

$$E[\sum_i \widetilde{q}_i] = \frac{n_F \zeta^2}{n_F + \zeta} \left( \bar{\boldsymbol{p}} - \bar{\boldsymbol{c}} \right) \tag{87}$$

Since the variance of the average data point is the variance of $b_i$ plus $1/n_d$ times the variance of the noise of each data point, it is $var(s_i) = 1 + 1/n_d \cdot 1$. Thus,

$$V[\sum_i \widetilde{q}_i] = \left( \frac{n_F \zeta^2}{n_F + \zeta} \right)^2 \frac{n_d n_F}{n_d + 1}. \tag{88}$$

Finally, the covariance of aggregate output and demand shocks arises because the data points are $s_i = b_i + \bar{\epsilon}_i$, where $\bar{\epsilon}_i$ is the average signal noise, independent of $b_i$. Thus, the covariance is $Cov(\sum_i \widetilde{q}_i, \sum_i b_i) = n_F \zeta^2 / (n_F + \zeta) \cdot n_d / (n_d + 1)$, times the variance of $b_i$, which is one.

Thus consumer surplus (84) can be simplified to

$$\mathbb{E}CS = \frac{1}{2\phi} \left( \frac{n_F \zeta^2}{n_F + \zeta} \right) \left( \left( \frac{n_F \zeta^2}{n_F + \zeta} \right) (\bar{\boldsymbol{p}} - \bar{\boldsymbol{c}})' (\bar{\boldsymbol{p}} - \bar{\boldsymbol{c}}) + \left( \frac{n_F^2 \zeta^2}{n_F + \zeta} + 1 \right) \frac{n_d}{n_d + 1} \right) \tag{89}$$

The next step is to differentiate each of these terms with respect to $n_d$, to show that welfare increases in the number of data points. We differentiate each term in the firms' profit function, holding the marginal cost of the firm $\bar{c}$ fixed. Then, we come back at the end to include the marginal effect of data on firms' marginal cost choices as well. First, define

$$\zeta_d := \partial \zeta / \partial d_n = \frac{n_F \phi \rho}{(1 + n_d + \phi \rho)^2} > 0$$

Then, consider the first term of consumer surplus and the first term of firms' profits jointly. These terms include the surplus that is shifted from firms to consumers when equilibrium prices change.

$$ECS1 + E\Pi 1 = \frac{\phi}{2} \left( \frac{n_F^2 \zeta^4 + n_F \zeta}{(1 + n_F \zeta)^2} \right) (\bar{\boldsymbol{p}} - \bar{\boldsymbol{c}})' (\bar{\boldsymbol{p}} - \bar{\boldsymbol{c}}) \tag{90}$$

The squared difference between $\bar{\boldsymbol{p}}$ and $\bar{\boldsymbol{c}}$ is always positive, as is $\phi/2$. Therefore, $ECS1 + E\Pi 1$ is weakly increasing in data if

$$\frac{\partial}{\partial \zeta} \left( \frac{n_F^2 \zeta^4 + n_F \zeta}{(1 + n_F \zeta)^2} \right) \geq 0$$

Simple differentiation reveals that this term is always positive. Thus, this part of welfare is increasing in $\zeta$. Since $\zeta_d > 0$, they are increasing in $n_d$ as well, holding the cost, $\bar{c}$, fixed, for the moment. However, this increase masks the fact that it is the consumers who are gaining from lower prices and firms that lose profits. The firms lose because more data makes firms less uncertain. Less uncertain firms produce more. More output lowers firms' markups. However, firms will still gain from the reduction in risk, if they factor risk pricing into their objective, i.e. if $\rho > 0$.

Next, consider the second term in the firm's profit function, $\frac{N}{2}\phi\zeta(1+\zeta)\frac{n_d}{n_d+1}\left[1 - n_F\left(\frac{\zeta}{1+n_F\zeta}\right)^2\right]$. Since $N\phi/2 > 0$, and $\zeta(1+\zeta)$ is clearly increasing in $\zeta$, it suffices to sign the derivative:

$$\frac{\partial}{\partial n_d}\left[1 - n_F\left(\frac{\zeta}{1+n_F\zeta}\right)^2\right] = \frac{\partial}{\partial n_d}\left(\frac{1+2n_F\zeta + n_F(n_F-1)\zeta^2}{1+n_F\zeta}\right)^2$$

This is non-negative, as long as there is at least one firm ($n_F \geq 1$).

Finally, consider the last term of consumer surplus, $ECS2 = \frac{1}{2\phi}\left(\frac{n_F\zeta^2}{n_F+\zeta}\right)\left(\frac{n_F^2\zeta^2}{n_F+\zeta}+1\right)\cdot\frac{n_d}{n_d+1}$. Since the coefficients are all positive, $\zeta^2$ is increasing in data and $n_d/(n_d+1)$ is increasing in data $n_d$, this last term of welfare is increasing as well.

The last step of the proof relaxes the assumption that marginal costs $\bar{c}$ stay fixed. $\bar{c}$ enters through the first term in both firm profits and consumer surplus that we combined as $ECS1 + E\Pi 1$ in (90). Note that firms will not produce anything on average if $\bar{\boldsymbol{p}} - \bar{\boldsymbol{c}} < 0$. So we can restrict attention to cases where that difference is non-negative. Thus, the squared difference is decreasing in $\bar{c}$. Furthermore, all the coefficients multiplying the term are positive. Thus, welfare is decreasing in the marginal cost of production $\bar{c}$. Lemma 1 proves data - investment complementarity: $\partial\bar{c}/\partial n_d < 0$.

Thus, more symmetric data for firms increases welfare, both by raising welfare for a given amount of investment, and by inducing firms to choose lower marginal costs. □

**Welfare with Asymmetric Firms** Consider the case where firms have different numbers of data points. All firms but one, $n_F - 1$ firms in total, have $n_{d2}$ data points, while one firm has asymmetric data, with $n_{d1}$ data points.

In this asymmetric case, the supply sensitivity terms $H_i$ and $\hat{H}_i$ take the same form as in (65) and (62). But they differ across firms. Define the firm-specific and the aggreagate sensitivity of supply to changes in expected profit as:

$$\zeta_i = \frac{1+n_{di}}{1+n_{di}+\rho_i\phi} \qquad \zeta_a = \frac{1}{n_F}\left((n_F-1)\zeta_1 + \zeta_2\right) \tag{91}$$

Then, $\sum_i \hat{\boldsymbol{H}}_i = n_F\zeta_a\phi$. That allows us to express the expected market price as

$$\bar{p}^M = \frac{1}{1+n_F\zeta_a}\left(\bar{\boldsymbol{p}} + (n_F-1)\zeta_1\bar{\boldsymbol{c}}_1 + \zeta_2\bar{\boldsymbol{c}}_2\right) \tag{92}$$

Then using (41), we can expected express aggregate output as

$$\mathbf{E}[\sum_i \tilde{q}_i] = \frac{1}{1+n_F\zeta_a}\phi\zeta_a\left(\bar{\boldsymbol{p}} - \bar{\boldsymbol{c}}\right) \tag{93}$$

Using the same substitutions, we can use (41) to express the variance of output and the covariance of output with the aggregate demand shocks $\sum_i b_i$ as

$$\mathbf{V}[\sum_i \tilde{q}_i] = \left(\frac{\phi}{1+n_F\zeta_a}\right)^2 \sum_i \frac{n_{di}}{n_{di}+1}\zeta_i^2. \tag{94}$$

$$Cov[\sum_i \tilde{q}_i, \sum_i b_i] = \left(\frac{\phi}{1+n_F\zeta_a}\right)\sum_i \frac{n_{di}\zeta_i}{n_{di}+1}. \tag{95}$$

Then, consumer surplus is

$$\mathbb{E}CS = \left(\frac{\phi}{1+n_F\zeta_a}\right)^2 \left(\phi^2\zeta_a^2\left(\overline{\boldsymbol{p}}-\overline{\boldsymbol{c}}\right)'\left(\overline{\boldsymbol{p}}-\overline{\boldsymbol{c}}\right) + \sum_i \frac{n_{di}}{n_{di}+1}\zeta_i^2\right) + \left(\frac{2\phi}{1+n_F\zeta_a}\right)\sum_i \frac{n_{di}\zeta_i}{n_{di}+1} \tag{96}$$

Each firm's profit takes the same form as before (85).

**Proof of Proposition 3: The firm-level markup wedge increases in data.**

*Proof.* Firm-level markup for firm $i$ is $M_i^f$ is defined as

$$M_i^f = \frac{\mathbf{E}[\tilde{\boldsymbol{q}}_i'\tilde{\boldsymbol{p}}_i]}{\mathbf{E}[\tilde{\boldsymbol{q}}_i'\tilde{\boldsymbol{c}}_i]} = \frac{\mathbf{E}[\tilde{\boldsymbol{q}}_i]'\mathbf{E}[\tilde{\boldsymbol{p}}_i] + \mathbf{trCov}\left(\tilde{\boldsymbol{p}}_i, \tilde{\boldsymbol{q}}_i\right)}{\mathbf{E}[\tilde{\boldsymbol{q}}_i'\tilde{\boldsymbol{c}}_i]} = \frac{\sum_{l=1}^N \mathbf{E}[\tilde{\boldsymbol{q}}_{i,l}]\mathbf{E}[\tilde{\boldsymbol{p}}_{i,l}] + \sum_{l=1}^N \mathbf{Cov}_{i,l}}{\sum_{l=1}^N \mathbf{E}[\tilde{\boldsymbol{q}}_{i,l}]\tilde{\boldsymbol{c}}_{i,l}} \tag{97}$$

where $\mathbf{Cov}_{i,l}$ is the $l^{th}$ diagonal value of the price-quantity covariance matrix $cov(\tilde{p}_i, \tilde{q}_i)$. From (95), this is

$$\mathbf{Cov}_{i,l} = \frac{\hat{\boldsymbol{H}}_{i,l}}{\left(1+\sum_{s=1}^{n_F}\frac{1}{\phi}\hat{\boldsymbol{H}}_{s,l}\right)^2}\left[\sum_{s=1,s\neq i}^{n_F}\left(\frac{\hat{\boldsymbol{H}}_{s,l}}{\phi}\right)^2 \boldsymbol{K}_{s,l} + \boldsymbol{K}_{i,l}\left(1+\sum_{s=1,s\neq i}^{n_F}\frac{\hat{\boldsymbol{H}}_{s,l}}{\phi}\right)^2\right] \tag{98}$$

where $K_{il}$ is firm $i$'s Bayesian updating weight on the signal about attribute $l$.

Taking partial derivative of the Bayesian updating weight on the data (Kalman gain) $\boldsymbol{K}_i = \left(1+n_{di}^{-1}\right)^{-1}$, with respect to $n_{di}$ yields

$$\frac{\partial \boldsymbol{K}_{i,j}}{\partial n_{di}} = -n_{di}^2 \tag{99}$$

Recall that $\hat{\boldsymbol{H}}_{k,l} = (\phi^{-1}+\rho_k(1+n_{di})^{-1}))^{-1}$. This implies

$$\frac{\partial \hat{\boldsymbol{H}}_{k,l}}{\partial n_{di}} = \rho_k \hat{\boldsymbol{H}}_{k,l}^2(1+n_{di})^{-2} \tag{100}$$

Similarly, using $\mathbb{E}\left[\tilde{\boldsymbol{p}}_{k,l}\right] = \bar{\boldsymbol{p}}_l^M = \frac{\bar{p}_l + \frac{1}{\phi}\sum_{i'=1}^{n_F}\hat{\boldsymbol{H}}_{i',l}\tilde{\boldsymbol{c}}_{i'l}}{1+\frac{1}{\phi}\sum_{i'=1}^{n_F}\hat{\boldsymbol{H}}_{i',l}}$, we obtain

$$\frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{p}}_{k,l}\right]}{\partial n_{di}} = \frac{(1+\frac{1}{\phi}\sum_{i'=1}^{n_F}\hat{\boldsymbol{H}}_{i',l})\frac{1}{\phi}\sum_{i'=1}^{n_F}\frac{\partial \hat{\boldsymbol{H}}_{i',l}}{\partial n_{di}}\tilde{\boldsymbol{c}}_{i'l} - (\bar{p}_l + \frac{1}{\phi}\sum_{i'=1}^{n_F}\hat{\boldsymbol{H}}_{i',l}\tilde{\boldsymbol{c}}_{il})\frac{1}{\phi}\sum_{i'=1}^{n_F}\frac{\partial \hat{\boldsymbol{H}}_{i',l}}{\partial n_{di}}}{(1+\frac{1}{\phi}\sum_{i'=1}^{n_F}\hat{\boldsymbol{H}}_{i',l})^2} \tag{101}$$

$$= \frac{(1+\frac{1}{\phi}\sum_{i'=1}^{n_F}\hat{\boldsymbol{H}}_{i',l})\frac{1}{\phi}\hat{\boldsymbol{H}}_{i,l}^2\rho_i(1+n_{di})^{-2}\tilde{\boldsymbol{c}}_{il} - (\bar{p}_l + \frac{1}{\phi}\sum_{i'=1}^{n_F}\hat{\boldsymbol{H}}_{i',l}\tilde{\boldsymbol{c}}_{i'l})\frac{1}{\phi}\hat{\boldsymbol{H}}_{i,l}^2\rho_i(1+n_{di})^{-2}}{(1+\frac{1}{\phi}\sum_{i'=1}^{n_F}\hat{\boldsymbol{H}}_{i',l})^2} \tag{102}$$

$$= \frac{1}{\phi}\hat{\boldsymbol{H}}_{i,l}^2\rho_i(1+n_{di})^{-2}\frac{(1+\frac{1}{\phi}\sum_{i'=1}^{n_F}\hat{\boldsymbol{H}}_{i',l})\tilde{\boldsymbol{c}}_{il} - (\bar{p}_l + \frac{1}{\phi}\sum_{i'=1}^{n_F}\hat{\boldsymbol{H}}_{i',l}\tilde{\boldsymbol{c}}_{i'l})}{(1+\frac{1}{\phi}\sum_{i'=1}^{n_F}\hat{\boldsymbol{H}}_{i',l})^2} \tag{103}$$

$$= \frac{1}{\phi}\hat{\boldsymbol{H}}_{i,l}^2\rho_i(1+n_{di})^{-2}\frac{(1+\frac{1}{\phi}\sum_{i'=1}^{n_F}\hat{\boldsymbol{H}}_{i',l})\tilde{\boldsymbol{c}}_{il} - (\bar{p}_l + \frac{1}{\phi}\sum_{i'=1}^{n_F}\hat{\boldsymbol{H}}_{i',l}\tilde{\boldsymbol{c}}_{i'l})}{(1+\frac{1}{\phi}\sum_{i'=1}^{n_F}\hat{\boldsymbol{H}}_{i',l})^2} \tag{104}$$

$$= \frac{\hat{\boldsymbol{H}}_{i,l}^2\rho_i(1+n_{di})^{-2}}{\phi+\sum_{i'=1}^{n_F}\hat{\boldsymbol{H}}_{i',l}}(\tilde{\boldsymbol{c}}_{i,l} - \bar{\boldsymbol{p}}_l^M) \tag{105}$$

Similarly, for the expected quantity produced $\mathbb{E}\left[\tilde{\boldsymbol{q}}_{k,l}\right] = \hat{\boldsymbol{H}}_{k,l}(\bar{\boldsymbol{p}}_l^M - \tilde{\boldsymbol{c}}_{k,l})$:

$$\frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_{k,l}\right]}{\partial n_{di}} = \frac{\partial \hat{\boldsymbol{H}}_{k,l}}{\partial n_{di}}(\bar{\boldsymbol{p}}_l^M - \tilde{\boldsymbol{c}}_{k,l}) + \hat{\boldsymbol{H}}_{k,l}\frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{p}}_{k,l}\right]}{\partial n_{di}} \tag{106}$$

$$= \hat{\boldsymbol{H}}_{k,l}^2\rho_k(\Sigma_{b_k,l}^{-1}+n_{di})^{-2}(\bar{\boldsymbol{p}}_l^M - \tilde{\boldsymbol{c}}_{k,l}) + \hat{\boldsymbol{H}}_{k,l}\frac{\hat{\boldsymbol{H}}_{i,l}^2\rho_i(\Sigma_{b_i,l}^{-1}+n_{di})^{-2}}{\phi+\sum_{i'=1}^{n_F}\hat{\boldsymbol{H}}_{i',l}}(\tilde{\boldsymbol{c}}_{i,l} - \bar{\boldsymbol{p}}_l^M) \tag{107}$$

$$= \hat{\boldsymbol{H}}_{i,l}^2\rho_i(\Sigma_{b_i,l}^{-1}+n_{di})^{-2}(\bar{\boldsymbol{p}}_l^M - \tilde{\boldsymbol{c}}_{i,l})\left(I_N - \frac{\hat{\boldsymbol{H}}_{k,l}}{\phi+\sum_{i'=1}^{n_F}\hat{\boldsymbol{H}}_{i',l}}\right) \tag{108}$$

Thus the derivative of numerator is

$$\frac{\partial \mathbf{E}\tilde{\boldsymbol{q}}_{i,l}\mathbf{E}\tilde{\boldsymbol{p}}_{i,l}}{\partial n_{di}} = \rho_i\hat{\boldsymbol{H}}_{i,j}^2\Sigma_{b_i,j}^2\left(\bar{\boldsymbol{p}}_j^M - \tilde{\boldsymbol{c}}_{i,j}\right)\left(\frac{1-\frac{1}{\phi}\hat{\boldsymbol{H}}_{i,j}+\frac{1}{\phi}\sum_{s=1,s\neq i}^{n_F}\hat{\boldsymbol{H}}_{s,j}}{1+\frac{1}{\phi}\sum_{s=1}^{n_F}\hat{\boldsymbol{H}}_{s,j}}\bar{\boldsymbol{p}}_j^M + \frac{\frac{1}{\phi}\hat{\boldsymbol{H}}_{i,j}\tilde{\boldsymbol{c}}_{i,j}}{1+\frac{1}{\phi}\sum_{s=1}^{n_F}\hat{\boldsymbol{H}}_{s,j}}\right) \geq 0$$

$$\frac{\partial \mathbf{Cov}_{i,l}}{\partial n_{di}} = \hat{\boldsymbol{H}}_{i,j}\Sigma_{b_i,j}^2\left(\frac{\rho_i\left(1-\frac{1}{\phi}\hat{\boldsymbol{H}}_{i,j}+\frac{1}{\phi}\sum_{s=1,s\neq i}^{n_F}\hat{\boldsymbol{H}}_{s,j}\right)}{\left(1+\sum_{s=1}^{n_F}\frac{1}{\phi}\hat{\boldsymbol{H}}_{s,j}\right)}\mathbf{Cov}_{i,j} + \frac{\left(1+\sum_{s=1,s\neq i}^{n_F}\frac{1}{\phi}\hat{\boldsymbol{H}}_{s,j}\right)^2}{\left(1+\sum_{s=1}^{n_F}\frac{1}{\phi}\hat{\boldsymbol{H}}_{s,j}\right)^2}\right) \geq 0 \tag{109}$$

Since the covariance term is the difference between the firm markup and the average product markup, this proves that that difference, the firm-level markup wedge in increasing in the firm's data. Moreover, firm-level markup increases with more data with small price of risk $\rho_i$ since

$$\frac{\partial M_i^f}{\partial n_{di}} = \frac{\left(\frac{\partial \mathbf{E}\tilde{\boldsymbol{q}}_{i,j}\mathbf{E}\tilde{\boldsymbol{p}}_{i,j}}{\partial n_{di}} + \frac{\partial \mathbf{Cov}_{i,j}}{\partial n_{di}}\right)}{\sum_{l=1}^N \mathbf{E}\tilde{\boldsymbol{q}}_{i,l}\tilde{\boldsymbol{c}}_{i,l}} - \frac{\left(\sum_{l=1}^N \mathbf{E}[\tilde{\boldsymbol{q}}_{i,l}\tilde{\boldsymbol{p}}_{i,l}]\right)\frac{\partial \mathbf{E}\tilde{\boldsymbol{q}}_{i,j}}{\partial n_{di}}\tilde{\boldsymbol{c}}_{i,j}}{\left(\sum_{l=1}^N \mathbf{E}\tilde{\boldsymbol{q}}_{i,l}\tilde{\boldsymbol{c}}_{i,l}\right)^2} \quad \text{and} \lim_{\rho_i\to 0}\frac{\partial M_i^f}{\partial n_{di}} = \frac{\hat{\boldsymbol{H}}_{i,j}\Sigma_{b_i,j}^2\frac{\left(1+\sum_{s=1,s\neq i}^{n_F}\frac{1}{\phi}\hat{\boldsymbol{H}}_{s,j}\right)^2}{\left(1+\sum_{s=1}^{n_F}\frac{1}{\phi}\hat{\boldsymbol{H}}_{s,j}\right)^2}}{\sum_{l=1}^N \mathbf{E}\tilde{\boldsymbol{q}}_{i,l}\tilde{\boldsymbol{c}}_{i,l}} > 0 \tag{110}$$

We prove the negative first order derivative for fixed choices of cost $\tilde{\boldsymbol{c}}_i$, which corresponds to infinite high marginal cost $\chi_c \to \infty$. This result is strictly negative and continuous in $\tilde{\boldsymbol{c}}_i$. If we assume $\chi_c$ is sufficiently high, by continuity, the inequality will still hold. $\square$

**Proof of Proposition 4a: Wedge between cost-weighted firm markup and average firm markup.** This proof shows that high-data firms produce more on average. Therefore, they have larger impacts on cost-weighted industry markup, increasing the industry-level markup wedge.

*Proof.* The cost weight for firm $i$ is

$$w_i^{cost} = \frac{\mathbf{E}\left[\tilde{\boldsymbol{q}}_i'\tilde{\boldsymbol{c}}_i\right]}{\sum_{k=1}^{n_F}\mathbf{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{c}}_k\right]} = \frac{\sum_{l=1}^{N}\mathbf{E}[\tilde{\boldsymbol{q}}_{i,l}]\tilde{\boldsymbol{c}}_{i,l}}{\sum_{k=1}^{n_F}\sum_{l=1}^{N}\mathbf{E}[\tilde{\boldsymbol{q}}_{k,l}]\tilde{\boldsymbol{c}}_{k,l}} \tag{111}$$

This weight is increasing in data for the firm $i$ since

$$\begin{aligned}\frac{\partial w_i^{cost}}{\partial n_{di}} &= \frac{\frac{\partial \mathbf{E}[\tilde{\boldsymbol{q}}_{i,j}]}{\partial n_{di}}\tilde{\boldsymbol{c}}_{i,j}\left(\sum_{k=1,k\neq i}^{n_F}\mathbf{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{c}}_k\right]\right) - \mathbf{E}\left[\tilde{\boldsymbol{q}}_i'\tilde{\boldsymbol{c}}_i\right]\sum_{k=1,k\neq i}^{n_F}\tilde{\boldsymbol{c}}_{k,j}\frac{\partial \mathbf{E}[\tilde{\boldsymbol{q}}_{k,j}]}{\partial n_{di}}}{\left(\sum_{k=1}^{n_F}\mathbf{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{c}}_k\right]\right)^2} \\ &= \rho_i\hat{\boldsymbol{H}}_{i,j}^2\Sigma_{b_i,j}^2\left(\bar{\boldsymbol{p}}_j^M - \tilde{\boldsymbol{c}}_{i,j}\right)\left[\frac{\tilde{\boldsymbol{c}}_{i,j}\left(\sum_{k=1,k\neq i}^{n_F}\mathbf{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{c}}_k\right]\right)}{\left(\sum_{k=1}^{n_F}\mathbf{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{c}}_k\right]\right)^2}\frac{1+\sum_{s\neq i,s=1}^{n_F}\frac{\hat{\boldsymbol{H}}_{s,j}}{\phi}}{1+\sum_{s=1}^{n_F}\frac{\hat{\boldsymbol{H}}_{s,j}}{\phi}} + \frac{\mathbf{E}\left[\tilde{\boldsymbol{q}}_i'\tilde{\boldsymbol{c}}_i\right]\sum_{k=1,k\neq i}^{n_F}\frac{\frac{1}{\phi}\hat{\boldsymbol{H}}_{k,j}\tilde{\boldsymbol{c}}_{k,j}}{1+\frac{1}{\phi}\sum_{s=1}^{n_F}\hat{\boldsymbol{H}}_{s,j}}}{\left(\sum_{k=1}^{n_F}\mathbf{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{c}}_k\right]\right)^2}\right] \geq 0\end{aligned} \tag{112}$$

This inequality indicates that high-data firms produce more on average and have larger impacts on cost-weighted industry markup. Furthermore, firm-level markup increases in data if cost is small enough and $N > 1$ since

$$\frac{\partial M_i^f}{\partial n_{di}} = \frac{\left(\frac{\partial \mathbb{E}\tilde{\boldsymbol{q}}_{i,j}\mathbb{E}\tilde{\boldsymbol{p}}_{i,j}}{\partial n_{di}} + \frac{\partial \mathbf{Cov}_{i,j}}{\partial n_{di}}\right)}{\sum_{l=1}^{N}\mathbb{E}\tilde{\boldsymbol{q}}_{i,l}\tilde{\boldsymbol{c}}_{i,l}} - \frac{\left(\sum_{l=1}^{N}\mathbb{E}[\tilde{\boldsymbol{q}}_{i,l}\tilde{\boldsymbol{p}}_{i,l}]\right)\frac{\partial \mathbb{E}\tilde{\boldsymbol{q}}_{i,j}}{\partial n_{di}}\tilde{\boldsymbol{c}}_{i,j}}{\left(\sum_{l=1}^{N}\mathbb{E}\tilde{\boldsymbol{q}}_{i,l}\tilde{\boldsymbol{c}}_{i,l}\right)^2} \Rightarrow \lim_{\tilde{\boldsymbol{c}}_{i,j}\to 0}\frac{\partial M_i^f}{\partial n_{di}} = \frac{\left(\frac{\partial \mathbb{E}\tilde{\boldsymbol{q}}_{i,j}\mathbb{E}\tilde{\boldsymbol{p}}_{i,j}}{\partial n_{di}} + \frac{\partial \mathbf{Cov}_{i,j}}{\partial n_{di}}\right)}{\sum_{l\neq j}^{N}\mathbb{E}\tilde{\boldsymbol{q}}_{i,l}\tilde{\boldsymbol{c}}_{i,l}} > 0 \tag{113}$$

These two forces intensify each other and drive up the industry-level markups compared to unweighted firm-level markups, leading to increasing wedge between these two markups. This proof holds for fixed choices of cost $\tilde{\boldsymbol{c}}_i$, which corresponds to infinite high marginal cost $\chi_c \to \infty$. The inequality still holds for large enough $\chi_c$ by using continuity. $\square$

**Proof of Proposition 4b: Sales weighted vs cost-weighted markup** Notice that the wedge between the sales- and cost-weighted markups is

$$M^{m,sales} - M^m = \sum_{i=1}^{N}\left(\underbrace{\frac{\mathbb{E}\left[\tilde{\boldsymbol{q}}_i'\tilde{\boldsymbol{p}}_i\right]}{\sum_{i=1}^{N}\mathbb{E}\left[\tilde{\boldsymbol{q}}_i'\tilde{\boldsymbol{p}}_i\right]}}_{w_i^{sales}} - \underbrace{\frac{\mathbb{E}\left[\tilde{\boldsymbol{q}}_i'\tilde{\boldsymbol{c}}_i\right]}{\sum_{i=1}^{N}\mathbb{E}\left[\tilde{\boldsymbol{q}}_i'\tilde{\boldsymbol{c}}_i\right]}}_{w_i^{m}}\right)\frac{\mathbb{E}\left[\tilde{\boldsymbol{q}}_i'\tilde{\boldsymbol{p}}_i\right]}{\mathbb{E}\left[\tilde{\boldsymbol{q}}_i'\tilde{\boldsymbol{c}}_i\right]} \tag{114}$$

When firms are ex ante identical, this wedge is zero $M^{m,sales} - M^m$.

To see how data $n_{di}$ affects the wedge, let's first take a loot at how it affects the difference between the sales weight and the cost weight of and firm $k$:

$$\frac{\partial}{\partial n_{di}}(w_k^{sales} - w_k^m) = w_k^{sales}\left(\frac{1}{\mathbb{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{p}}_k\right]}\frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{p}}_k\right]}{\partial n_{di}} - \frac{1}{\sum_{k'}\mathbb{E}\left[\tilde{\boldsymbol{q}}_{k'}'\tilde{\boldsymbol{p}}_{k'}\right]}\sum_{k'}\frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_{k'}'\tilde{\boldsymbol{p}}_{k'}\right]}{\partial n_{di}}\right) \tag{115}$$

$$- w_k^m\left(\frac{1}{\mathbb{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{c}}_k\right]}\frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{c}}_k\right]}{\partial n_{di}} - \frac{1}{\sum_{k'}\mathbb{E}\left[\tilde{\boldsymbol{q}}_{k'}'\tilde{\boldsymbol{c}}_{k'}\right]}\sum_{k'}\frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_{k'}'\tilde{\boldsymbol{c}}_{k'}\right]}{\partial n_{di}}\right) \tag{116}$$

$$= w_k^{sales}\left(\frac{1 - w_k^{sales}}{\mathbb{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{p}}_k\right]}\frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{p}}_k\right]}{\partial n_{di}} - \frac{1}{\sum_{k'}\mathbb{E}\left[\tilde{\boldsymbol{q}}_{k'}'\tilde{\boldsymbol{p}}_{k'}\right]}\sum_{k'\neq k}\frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_{k'}'\tilde{\boldsymbol{p}}_{k'}\right]}{\partial n_{di}}\right) \tag{117}$$

$$- w_k^m\left(\frac{1 - w_k^m}{\mathbb{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{c}}_k\right]}\frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{c}}_k\right]}{\partial n_{di}} - \frac{1}{\sum_{k'}\mathbb{E}\left[\tilde{\boldsymbol{q}}_{k'}'\tilde{\boldsymbol{c}}_{k'}\right]}\sum_{k'\neq k}\frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_{k'}'\tilde{\boldsymbol{c}}_{k'}\right]}{\partial n_{di}}\right) \tag{118}$$

$$\tag{119}$$

Using the assumptions that firms are ex ante identical, we have $w_k^{sales} = w_k^m = \frac{1}{n_F}$ and $\mathbb{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{p}}_k\right] = \mathbb{E}\left[\tilde{\boldsymbol{q}}_i'\tilde{\boldsymbol{p}}_i\right], \forall k, i$ and the effect of information on the weights can be simplified to

$$\frac{\partial}{\partial n_{di}}(w_k^{sales} - w_k^m) = \frac{1}{n_F}\left(\frac{1}{\mathbb{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{p}}_k\right]}\frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{p}}_k\right]}{\partial n_{di}} - \frac{n_F - 1}{n_F\mathbb{E}\left[\tilde{\boldsymbol{q}}_{k'}'\tilde{\boldsymbol{p}}_{k'}\right]}\frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_{k'}'\tilde{\boldsymbol{p}}_{k'}\right]}{\partial n_{di}} - \frac{1}{n_F\mathbb{E}\left[\tilde{\boldsymbol{q}}_{k'}'\tilde{\boldsymbol{p}}_{k'}\right]}\frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_i'\tilde{\boldsymbol{p}}_i\right]}{\partial n_{di}}\right) \tag{120}$$

$$- \frac{1}{n_F}\left(\frac{1}{\mathbb{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{c}}_k\right]}\frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{c}}_k\right]}{\partial n_{di}} - \frac{n_F - 1}{n_F\mathbb{E}\left[\tilde{\boldsymbol{q}}_{k'}'\tilde{\boldsymbol{c}}_{k'}\right]}\frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_{k'}'\tilde{\boldsymbol{c}}_{k'}\right]}{\partial n_{di}} - \frac{1}{n_F\mathbb{E}\left[\tilde{\boldsymbol{q}}_{k'}'\tilde{\boldsymbol{c}}_{k'}\right]}\frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_i'\tilde{\boldsymbol{c}}_i\right]}{\partial n_{di}}\right) \tag{121}$$

$$= \frac{1}{n_F^2\mathbb{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{p}}_k\right]}\left(\frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{p}}_k\right]}{\partial n_{di}} - \frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_i'\tilde{\boldsymbol{p}}_i\right]}{\partial n_{di}}\right) \tag{122}$$

$$- \frac{1}{n_F^2\mathbb{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{c}}_k\right]}\left(\frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{c}}_k\right]}{\partial n_{di}} - \frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_i'\tilde{\boldsymbol{c}}_i\right]}{\partial n_{di}}\right), \forall k \neq i \tag{123}$$

$$\tag{124}$$

and similarly for firm $i$ itself

$$\frac{\partial}{\partial n_{di}}(w_i^{sales} - w_i^m) = \frac{n_F - 1}{n_F^2\mathbb{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{p}}_k\right]}\left(\frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_i'\tilde{\boldsymbol{p}}_i\right]}{\partial n_{di}} - \frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{p}}_k\right]}{\partial n_{di}}\right) \tag{125}$$

$$- \frac{n_F - 1}{n_F^2\mathbb{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{c}}_k\right]}\left(\frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_i'\tilde{\boldsymbol{c}}_i\right]}{\partial n_{di}} - \frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{c}}_k\right]}{\partial n_{di}}\right) \tag{126}$$

where $k$ is any firm different from $i$.

Notice that the condition that the wedge in the weights widens amount to showing that the elasticity in the different of sales in higher than that of the cost

$$\frac{1}{\mathbb{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{p}}_k\right]}\left(\frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_i'\tilde{\boldsymbol{p}}_i\right]}{\partial n_{di}^{-1}} - \frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{p}}_k\right]}{\partial n_{di}}\right) \geq \frac{1}{\mathbb{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{c}}_k\right]}\left(\frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_i'\tilde{\boldsymbol{c}}_i\right]}{\partial n_{di}} - \frac{\partial \mathbb{E}\left[\tilde{\boldsymbol{q}}_k'\tilde{\boldsymbol{c}}_k\right]}{\partial n_{di}}\right) \tag{127}$$

which is equivalent to the information

$$\frac{\partial}{\partial n_{di}} \frac{\mathbb{E}\left[\tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{p}}_i\right]}{\mathbb{E}\left[\tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{c}}_i\right]} \geq \frac{\partial}{\partial n_{di}} \frac{\mathbb{E}\left[\tilde{\boldsymbol{q}}_k' \tilde{\boldsymbol{p}}_k\right]}{\mathbb{E}\left[\tilde{\boldsymbol{q}}_k' \tilde{\boldsymbol{c}}_k\right]} \tag{128}$$

This means if difference in weights $w_i^{m,sales} - w_i^m$ turns positive whenever it increases the markup of firm $i$ relatively more than other firms. Therefore, the wedge between the sales weighted markup and the cost-weighted markup is always weakly increasing in $n_{di}$. And it is strict if the information of firm $i$ affects the markup of firm $i$ differently from that of firm $k$, which is generically true as the information firm $i$ to concentrate more on high-markup products while the opposite for the other firms.

Indeed, this result reflect the fact that the wedge between the sales-weighted markup and the cost-weighted markup is always non-negative and it is zero if and only if all firms are symmetric. To see this point, notice we can write the wedge as

$$M^{m,sales} - M^m = \frac{\sum_{i=1}^N \mathbb{E}\left[\tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{c}}_i\right] \sum_{i=1}^N \frac{\mathbb{E}\left[\tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{p}}_i\right]^2}{\mathbb{E}\left[\tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{c}}_i\right]} - \left(\sum_{i=1}^N \mathbb{E}\left[\tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{p}}_i\right]\right)^2}{\sum_{i=1}^N \mathbb{E}\left[\tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{p}}_i\right] \sum_{i=1}^N \mathbb{E}\left[\tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{c}}_i\right]} \tag{129}$$

Recall Cauchy-Schwarz inequality $\left(\sum_{i=1}^N u_i v_i\right)^2 \leq \left(\sum_{i=1}^N u_i^2\right)\left(\sum_{i=1}^N v_i^2\right)$. Let $u_i = \sqrt{\mathbb{E}\left[\tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{c}}_i\right]}$ $v_i = \sqrt{\frac{\mathbb{E}\left[\tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{p}}_i\right]^2}{\mathbb{E}\left[\tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{c}}_i\right]}}$. The Cauchy-Schwarz inequality says [12]

$$\sum_{i=1}^N \mathbb{E}\left[\tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{c}}_i\right] \sum_{i=1}^N \frac{\mathbb{E}\left[\tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{p}}_i\right]^2}{\mathbb{E}\left[\tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{c}}_i\right]} - \left(\sum_{i=1}^N \mathbb{E}\left[\tilde{\boldsymbol{q}}_i' \tilde{\boldsymbol{p}}_i\right]\right)^2 \geq 0 \tag{130}$$

and the equality holds if and only if all firms have the same markup.

Intuitively, a high-markup firm has higher sales relative to it's costs so, it has a higher sales-weight than its cost weights. Similarly, low-markup firm tends to have lower sales-weight than cost-weight but is out-weighted by the high-markup firms. The wedge achieves the minimum 0 when all firms are symmetric and it gets larger as the information brings more asymmetry in the production.

**Proof of Proposition 4c: Sales-weighted vs. industry aggregates markup** The reason this corollary follows directly from Proposition 4b, that the cost-weighted industry markup and the aggregate markup are the same, in our setting. This is a version of the aggregation results of Edmond, Midrigan and Xu (2019), extended to our linear demand system. The proof is just algebraic manipulation:

$$M^{ag} := \frac{\mathbf{E}\left[\sum_{i=1}^N \boldsymbol{q}_i' \boldsymbol{p}_i\right]}{\mathbf{E}\left[\sum_{i=1}^N \boldsymbol{q}_i' \boldsymbol{c}_i\right]} = \frac{\sum_{i=1}^N \mathbf{E}\left[\boldsymbol{q}_i' \boldsymbol{p}_i\right]}{\sum_{i=1}^N \mathbf{E}\left[\boldsymbol{q}_i' \boldsymbol{c}_i\right]} = \sum_{i=1}^N w_i^m M_i^f = M^m \quad \text{where} \quad w_i^c = \frac{\mathbf{E}\left[\boldsymbol{q}_i' \boldsymbol{c}_i\right]}{\sum_{i=1}^N \mathbf{E}\left[\boldsymbol{q}_i' \boldsymbol{c}_i\right]}. \tag{131}$$

[12] This special case is also referred to as Sedrakyan's inequality, Bergström's inequality, Engel's form, the T2 lemma, or Titu's lemma.

### B.1. Cyclical Markups

**Proof of proposition 5** Part a: product markups are increasing in demand variance and converge to a constant.

*Proof.* Let $\sigma_b I_N$ denote the variance of demand shocks $b$. According to the definition of $\hat{\boldsymbol{H}}_i$, we have

$$\begin{aligned} \hat{\boldsymbol{H}}_i &= \left(\frac{\boldsymbol{I}_N}{\phi} + \rho_i \mathbf{Var}(\tilde{\boldsymbol{p}}_i|\mathcal{I}_i)\right)^{-1} \quad \text{and} \quad \mathbf{Var}(\tilde{\boldsymbol{p}}_i|\mathcal{I}_i) = \left(\sigma_b^{-1} + n_{di}\right)^{-1} \\ \Rightarrow \lim_{\sigma_b\to\infty} \mathbf{Var}(\tilde{\boldsymbol{p}}_i|\mathcal{I}_i) &= 1/n_{di}, \quad \tilde{\hat{\boldsymbol{H}}}_i := \lim_{\sigma_b\to\infty} \hat{\boldsymbol{H}}_i = \left(\frac{\boldsymbol{I}_N}{\phi} + \rho_i/n_{di}\right)^{-1} \end{aligned} \tag{132}$$

The equilibrium price is given by

$$\mathbf{E}\left[\tilde{\boldsymbol{p}}_i\right] = \bar{\boldsymbol{p}}^M = \left(\boldsymbol{I}_N + \frac{1}{\phi}\sum_{i=1}^{n_F} \hat{\boldsymbol{H}}_i\right)^{-1} \left(\bar{\boldsymbol{p}} + \frac{1}{\phi}\sum_{i=1}^{n_F} \hat{\boldsymbol{H}}_i \boldsymbol{c}_i\right) \tag{133}$$

It clearly converges due to convergent $\hat{\boldsymbol{H}}_i$, so we have

$$\begin{aligned} \tilde{\boldsymbol{p}} := \lim_{\sigma_b\to\infty} \mathbf{E}\left[\tilde{\boldsymbol{p}}_i\right] &= \left(\boldsymbol{I}_N + \frac{1}{\phi}\sum_{i=1}^{n_F} \lim_{\sigma_b\to\infty} \hat{\boldsymbol{H}}_i\right)^{-1} \left(\bar{\boldsymbol{p}} + \frac{1}{\phi}\sum_{i=1}^{n_F} \lim_{\sigma_b\to\infty} \hat{\boldsymbol{H}}_i \boldsymbol{c}_i\right) \\ &= \left[\boldsymbol{I}_N + \sum_{i=1}^{n_F} \left(\boldsymbol{I}_N + \phi\rho_i/n_{di}\right)^{-1}\right]^{-1} \left[\bar{\boldsymbol{p}} + \sum_{i=1}^{n_F} \boldsymbol{c}_i \left(\boldsymbol{I}_N + \phi\rho_i/n_{di}\right)^{-1}\right] \end{aligned} \tag{134}$$

This result implies convergent product-level markup on the attributes as $\lim_{\sigma_b\to\infty} \overline{M}^p$ exists. Since equilibrium price on the goods is a linear combination of weight matrix $\boldsymbol{A}$ and $\tilde{\boldsymbol{p}}_i$, the product-level markup on the goods converges.

$$\boldsymbol{q}_i = \boldsymbol{A}\tilde{\boldsymbol{q}}_i \quad \text{and} \quad \boldsymbol{p}_i = \boldsymbol{A}\tilde{\boldsymbol{p}}_i \Rightarrow \overline{M}^p = \frac{1}{N}\frac{1}{n_F}\sum_{i=1}^{n_F}\sum_{j=1}^{N} \frac{\left(\boldsymbol{A}\mathbf{E}\left[\tilde{\boldsymbol{p}}_i\right]\right)_j}{\left(\boldsymbol{A}\boldsymbol{c}_i\right)_j} \quad \text{converges.} \tag{135}$$

If all the firms have identical sizes ($\boldsymbol{c}_i = \bar{\boldsymbol{c}}$), the derivative of equilibrium price for specific attribute $j$ is

$$\frac{\partial \mathbf{E}[\tilde{\boldsymbol{p}}_{i,j}]}{\partial \Sigma_{b,j}} = \frac{(\bar{\boldsymbol{c}}_j - \bar{\boldsymbol{p}}_j)\frac{1}{\phi}\sum_{i=1}^{n_F} \frac{\partial \hat{\boldsymbol{H}}_{i,j}}{\partial \Sigma_{b,j}}}{\left(1 + \frac{1}{\phi}\sum_{i=1}^{n_F} \hat{\boldsymbol{H}}_{i,j}\right)^2} \quad \text{and} \quad \frac{\partial \hat{\boldsymbol{H}}_{i,j}}{\partial \Sigma_{b,j}} = -\frac{\hat{\boldsymbol{H}}_{i,j}^2 \rho_i/n_{di}^2}{\left(\Sigma_{b.j} + 1/n_{di}\right)^2} \le 0 \tag{136}$$

Since positive production implies lower marginal cost ($\bar{\boldsymbol{c}}_j \le \bar{\boldsymbol{p}}_j$), the numerator of the derivative is positive. □

Part b: Firm and industry level markups are increasing in demand variance. They asymptote to a linearly increasing function of demand variance.

*Proof.* First, We will show that the trace of the covariance $\mathbf{tr}[\mathbf{Cov}(\tilde{\boldsymbol{p}}_i, \tilde{\boldsymbol{q}}_i)]$ is always positive.

$$\begin{aligned}\mathbf{Cov}(\tilde{\boldsymbol{p}}_i, \tilde{\boldsymbol{q}}_i) = &\left(\boldsymbol{I}_N + \sum_{j=1}^{n_F} \frac{\hat{\boldsymbol{H}}_j}{\phi}\right)^{-1} \sum_{j=1}^{n_F} \hat{\boldsymbol{H}}_j \mathbf{Var}(\boldsymbol{K}_j \boldsymbol{s}_j) \hat{\boldsymbol{H}}_j \left(\boldsymbol{I}_N + \sum_{j=1}^{n_F} \frac{\hat{\boldsymbol{H}}_j}{\phi}\right)^{-1} \frac{\hat{\boldsymbol{H}}_i}{\phi^2} + \mathbf{Var}(\boldsymbol{K}_i \boldsymbol{s}_i) \hat{\boldsymbol{H}}_i \\ &- \left(\boldsymbol{I}_N + \sum_{j=1}^{n_F} \frac{\hat{\boldsymbol{H}}_j}{\phi}\right)^{-1} \hat{\boldsymbol{H}}_i \mathbf{Var}(\boldsymbol{K}_i \boldsymbol{s}_i) \frac{\hat{\boldsymbol{H}}_i}{\phi} - \mathbf{Var}(\boldsymbol{K}_i \boldsymbol{s}_i) \hat{\boldsymbol{H}}_i \left(\boldsymbol{I}_N + \sum_{j=1}^{n_F} \frac{\hat{\boldsymbol{H}}_j}{\phi}\right)^{-1} \frac{\hat{\boldsymbol{H}}_i}{\phi}\end{aligned} \tag{137}$$

Denote $\boldsymbol{Y}$ the sum of price impacts $\boldsymbol{Y} = \boldsymbol{I}_N + \sum_{j=1}^{n_F} \frac{\hat{\boldsymbol{H}}_j}{\phi}$. The trace could be written as

$$\begin{aligned}&\mathbf{tr}[\mathbf{Cov}(\tilde{\boldsymbol{p}}_i, \tilde{\boldsymbol{q}}_i)] \\ =&\mathbf{tr}\left[\boldsymbol{Y}^{-1} \sum_{j=1}^{n_F} \hat{\boldsymbol{H}}_j \mathbf{Var}(\boldsymbol{K}_j \boldsymbol{s}_j) \hat{\boldsymbol{H}}_j \boldsymbol{Y}^{-1} \frac{\hat{\boldsymbol{H}}_i}{\phi^2}\right] + \mathbf{tr}\left[\mathbf{Var}(\boldsymbol{K}_i \boldsymbol{s}_i) \hat{\boldsymbol{H}}_i\right] - \mathbf{tr}\left[\boldsymbol{Y}^{-1} \hat{\boldsymbol{H}}_i \mathbf{Var}(\boldsymbol{K}_i \boldsymbol{s}_i) \frac{\hat{\boldsymbol{H}}_i}{\phi}\right] - \mathbf{tr}\left[\mathbf{Var}(\boldsymbol{K}_i \boldsymbol{s}_i) \hat{\boldsymbol{H}}_i \boldsymbol{Y}^{-1} \frac{\hat{\boldsymbol{H}}_i}{\phi}\right] \\ \geq&\mathbf{tr}\left[\boldsymbol{Y}^{-1} \hat{\boldsymbol{H}}_i \mathbf{Var}(\boldsymbol{K}_i \boldsymbol{s}_i) \hat{\boldsymbol{H}}_i \boldsymbol{Y}^{-1} \frac{\hat{\boldsymbol{H}}_i}{\phi^2}\right] + \mathbf{tr}\left[\mathbf{Var}(\boldsymbol{K}_i \boldsymbol{s}_i) \hat{\boldsymbol{H}}_i\right] - \mathbf{tr}\left[\boldsymbol{Y}^{-1} \hat{\boldsymbol{H}}_i \mathbf{Var}(\boldsymbol{K}_i \boldsymbol{s}_i) \frac{\hat{\boldsymbol{H}}_i}{\phi}\right] - \mathbf{tr}\left[\mathbf{Var}(\boldsymbol{K}_i \boldsymbol{s}_i) \hat{\boldsymbol{H}}_i \boldsymbol{Y}^{-1} \frac{\hat{\boldsymbol{H}}_i}{\phi}\right] \\ =&\mathbf{tr}\left[\mathbf{Var}(\boldsymbol{K}_i \boldsymbol{s}_i) \hat{\boldsymbol{H}}_i \boldsymbol{Y}^{-1} \frac{\hat{\boldsymbol{H}}_i}{\phi^2} \boldsymbol{Y}^{-1} \hat{\boldsymbol{H}}_i\right] + \mathbf{tr}\left[\mathbf{Var}(\boldsymbol{K}_i \boldsymbol{s}_i) \hat{\boldsymbol{H}}_i\right] - \mathbf{tr}\left[\mathbf{Var}(\boldsymbol{K}_i \boldsymbol{s}_i) \frac{\hat{\boldsymbol{H}}_i}{\phi} \boldsymbol{Y}^{-1} \hat{\boldsymbol{H}}_i\right] - \mathbf{tr}\left[\mathbf{Var}(\boldsymbol{K}_i \boldsymbol{s}_i) \hat{\boldsymbol{H}}_i \boldsymbol{Y}^{-1} \frac{\hat{\boldsymbol{H}}_i}{\phi}\right] \\ =&\phi \mathbf{tr}\left[\mathbf{Var}(\boldsymbol{K}_i \boldsymbol{s}_i) \left(\frac{\hat{\boldsymbol{H}}_i}{\phi} \boldsymbol{Y}^{-1} \frac{\hat{\boldsymbol{H}}_i}{\phi} \boldsymbol{Y}^{-1} \frac{\hat{\boldsymbol{H}}_i}{\phi} + \frac{\hat{\boldsymbol{H}}_i}{\phi} - \frac{\hat{\boldsymbol{H}}_i}{\phi} \boldsymbol{Y}^{-1} \frac{\hat{\boldsymbol{H}}_i}{\phi} - \frac{\hat{\boldsymbol{H}}_i}{\phi} \boldsymbol{Y}^{-1} \frac{\hat{\boldsymbol{H}}_i}{\phi}\right)\right] \\ =&\phi \mathbf{tr}\left[\mathbf{Var}(\boldsymbol{K}_i \boldsymbol{s}_i) \frac{\hat{\boldsymbol{H}}_i}{\phi} \left(\boldsymbol{Y}^{-1} \frac{\hat{\boldsymbol{H}}_i}{\phi} - \boldsymbol{I}_N\right)^2\right] \geq 0\end{aligned} \tag{138}$$

We denote $\boldsymbol{x}_i = \frac{\hat{\boldsymbol{H}}_i}{\phi}$ and $Z_i = \mathbf{Var}(\boldsymbol{K}_i \boldsymbol{s}_i) = \frac{\Sigma_b^2}{\sigma_b + \Sigma_i}$ and consider diagonal shock and signal variance. $\boldsymbol{x}_i$, $\boldsymbol{Y}$ and $Z_i$ are diagonal under our assumption. The covariance matrix is simplified as

$$\mathbf{Cov}(\tilde{\boldsymbol{p}}_i, \tilde{\boldsymbol{q}}_i) = \phi \left[\boldsymbol{Y}^{-1} \sum_{j=1}^{n_F} \boldsymbol{x}_i Z_j \boldsymbol{x}_i \boldsymbol{Y}^{-1} \boldsymbol{x}_i + Z_i \boldsymbol{x}_i - \boldsymbol{Y}^{-1} \boldsymbol{x}_i Z_i \boldsymbol{x}_i - Z_i \boldsymbol{x}_i \boldsymbol{Y}^{-1} \boldsymbol{x}_i\right] \tag{139}$$

The covariance matrix is also diagonal and denote the $k^{th}$ diagonal $\mathbf{Cov}_{i,k}$. Subscript $k$ refers to the $k^{th}$ diagonal value.

$$\begin{aligned}\mathbf{Cov}_{i,k} &:= \mathbf{Cov}\left(\boldsymbol{p}_{i,k}, \tilde{\boldsymbol{q}}_{i,k}\right) \\ &= \phi \left[\boldsymbol{Y}_k^{-1} \sum_{j=1}^{n_F} \boldsymbol{x}_{j,k} Z_{j,k} \boldsymbol{x}_{j,k} \boldsymbol{Y}_k^{-1} \boldsymbol{x}_{i,k} + Z_i \boldsymbol{x}_{i,k} - \boldsymbol{Y}_k^{-1} \boldsymbol{x}_{i,k} Z_{i,k} \boldsymbol{x}_{i,k} - Z_{i,k} \boldsymbol{x}_{i,k} \boldsymbol{Y}_k^{-1} \boldsymbol{x}_{i,k}\right] \\ &= \phi \frac{\boldsymbol{x}_{i,k}}{\boldsymbol{Y}_k^2} \left[\sum_{j \neq i, j=1}^{n_F} \boldsymbol{x}_{j,k}^2 Z_{j,k} + Z_{i,k} \left(\boldsymbol{x}_{i,k} - \boldsymbol{Y}_k\right)^2\right]\end{aligned} \tag{140}$$

The limiting behavioral for all variables are

$$\begin{aligned}
&\lim_{\sigma_b\to\infty} \boldsymbol{x}_{i,k} = (1+\phi\rho_i/n_{di})^{-1} \\
&\lim_{\sigma_b\to\infty} \boldsymbol{Y}_k = 1+\sum_{j=1}^{n_F}\lim_{\sigma_b\to\infty}\boldsymbol{x}_{j,k} = 1+\sum_{j=1}^{n_F}\left(1+\phi\rho_j/n_{di}\right)^{-1} \\
&\lim_{\sigma_b\to\infty}\frac{Z_{i,k}}{\sigma_b} = \lim_{\sigma_b\to\infty}\frac{\frac{\Sigma_{b,k}^2}{\sigma_b+\Sigma_{i,k}}}{\sigma_b} = 1
\end{aligned} \tag{141}$$

The ratio of covariance to shock variance converges as

$$\lim_{\sigma_b\to\infty}\frac{\mathbf{Cov}_{i,k}}{\sigma_b} = \frac{\phi\left(1+\phi\rho_i/n_{di}\right)^{-1}\left[\sum_{j\neq i,j=1}^{n_F}\left(1+\phi\rho_j/n_{di}\right)^{-2}+\left(1+\sum_{j=1,j\neq i}^{n_F}\left(1+\phi\rho_j/n_{di}\right)^{-1}\right)^2\right]}{\left(1+\sum_{j=1}^{n_F}\left(1+\phi\rho_j/n_{di}\right)^{-1}\right)^2} \tag{142}$$

we have

$$\begin{aligned}
M_i^f &= \frac{\mathbf{E}[\tilde{\boldsymbol{q}}_i'\tilde{\boldsymbol{p}}_i]}{\mathbf{E}[\tilde{\boldsymbol{q}}_i'\boldsymbol{c}_i]} = \frac{\mathbf{E}[\tilde{\boldsymbol{q}}_i]'\mathbf{E}[\boldsymbol{p}]+\mathbf{tr}\left[\mathbf{Cov}(\tilde{\boldsymbol{p}}_i,\tilde{\boldsymbol{q}}_i)\right]}{\mathbf{E}[\tilde{\boldsymbol{q}}_i'\boldsymbol{c}_i]} \\
&= \frac{\sum_{j=1}^N(\mathbf{E}(\tilde{\boldsymbol{p}}_{i,j})-\boldsymbol{c}_{i,j})\mathbf{E}(\tilde{\boldsymbol{p}}_{i,j})\hat{\boldsymbol{H}}_{i,j}+\sum_{j=1}^N\mathbf{Cov}_{i,j}}{\sum_{j=1}^N(\mathbf{E}(\tilde{\boldsymbol{p}}_{i,j})-\boldsymbol{c}_{i,j})\boldsymbol{c}_{i,j}\hat{\boldsymbol{H}}_{i,j}}
\end{aligned} \tag{143}$$

We assume the diagonal values of shock variance are the same, so the asymptote of $M_i^f$ is

$$\begin{aligned}
\alpha_i &:= \lim_{\sigma_b\to\infty}\frac{M_i^f}{\sigma_b} = \frac{\sum_{j=1}^N\lim_{\Sigma_b\to\infty}\frac{\mathbf{Cov}_{i,j}}{\sigma_b}}{\sum_{j=1}^N(\tilde{\boldsymbol{p}}_j-\boldsymbol{c}_{i,j})\boldsymbol{c}_{i,j}\tilde{\hat{\boldsymbol{H}}}_{i,j}} > 0 \\
\gamma_i &:= \lim_{\sigma_b\to\infty}\left(M_i^f-\alpha_i\sigma_b\right) = \frac{\sum_{j=1}^N(\tilde{\boldsymbol{p}}_j-\boldsymbol{c}_{i,j})\tilde{\boldsymbol{p}}_j\tilde{\hat{\boldsymbol{H}}}_{i,j}+\widetilde{\mathbf{Cov}}_{i,j}}{\sum_{j=1}^N(\tilde{\boldsymbol{p}}_j-\boldsymbol{c}_{i,j})\boldsymbol{c}_{i,j}\tilde{\hat{\boldsymbol{H}}}_{i,j}}
\end{aligned} \tag{144}$$

where the difference $\widetilde{\mathbf{Cov}}_i$ is defined as

$$\begin{aligned}
\widetilde{\mathbf{Cov}}_i &:= \lim_{\sigma_b\to\infty}\left(\mathbf{Cov}_{i,k}-\left(\lim_{\sigma_b\to\infty}\frac{\mathbf{Cov}_{i,k}}{\sigma_b}\right)\sigma_b\right) \\
&= -\frac{\phi\left(1+\phi\rho_i/n_{di}\right)^{-1}\left[\sum_{j\neq i,j=1}^{n_F}\left(1+\phi\rho_j/n_{di}\right)^{-2}n_{di}^{-1}+\left(1+\sum_{j=1,j\neq i}^{n_F}\left(1+\phi\rho_j/n_{di}\right)^{-1}\right)^2/n_{di}\right]}{\left(1+\sum_{j=1}^{n_F}\left(1+\phi\rho_j/n_{di}\right)^{-1}\right)^2}
\end{aligned} \tag{145}$$

The average firm-level markup $\overline{M}^f = (1/n_F)\sum_{i=1}^{n_F}M_i^f$ approaches $\sum_{i=1}^{n_F}\frac{\alpha_i}{n_F}\sigma_b+\sum_{i=1}^{n_F}\frac{\gamma_i}{n_F}$ in the long run. The economy-level markup is $M^m = \sum_{i=1}^{n_F}w^{\boldsymbol{H}_i}M_i^f$ with $w^{\boldsymbol{H}_i} = \frac{\mathbf{E}\left[\tilde{q}_i'c_i\right]}{\sum_{i=1}^{n_F}\mathbf{E}\left[\tilde{q}_i'c_i\right]}$. The weight $w^{\boldsymbol{H}_i}$ converges to $w_i$ as shock variance goes to infinity, implying an asymptote of economy-level markup.

$$w_i := \lim_{\sigma_b\to\infty}w^{\boldsymbol{H}_i} = \frac{\sum_{j=1}^N(\tilde{\boldsymbol{p}}_j-\boldsymbol{c}_{i,j})\boldsymbol{c}_{i,j}\tilde{\hat{\boldsymbol{H}}}_{i,j}}{\sum_{i=1}^{n_F}\sum_{j=1}^N(\tilde{\boldsymbol{p}}_j-\boldsymbol{c}_{i,j})\boldsymbol{c}_{i,j}\tilde{\hat{\boldsymbol{H}}}_{i,j}} \Rightarrow M^m \text{ approaches } \sum_{i=1}^{n_F}w_i\alpha_i\sigma_b+\sum_{i=1}^{n_F}w_i\gamma_i \tag{146}$$

Finally, the derivative of each component of covariance is

$$
\begin{aligned}
\frac{\partial \boldsymbol{x}_{i,k}}{\partial \sigma_b} &= -\phi\rho_i \boldsymbol{x}_{i,k}^2 \left(n_{di}\sigma_b + I_N\right)^{-2} = -\frac{\boldsymbol{x}_{i,k}(1-\boldsymbol{x}_{i,k})}{\sigma_b(\sigma_b n_{di} + I_N)} \\
\frac{\partial \boldsymbol{Y}_k}{\partial \sigma_b} &= \sum_{j=1}^{n_F} \frac{\partial \boldsymbol{x}_{j,k}}{\partial \sigma_b} = -\sum_{j=1}^{n_F} \frac{\boldsymbol{x}_{j,k}(1-\boldsymbol{x}_{j,k})}{\sigma_b(n_{di}\sigma_b + I_N)} \\
\frac{\partial Z_{i,k}}{\partial \sigma_b} &= \frac{\sigma_b(n_{di}\sigma_b + 2I_N)}{(n_{di}\sigma_b + I_N)^2} = \frac{Z_{i,k}(n_{di}\sigma_b + 2I_N)}{\Sigma_{b,k}\left(n_{di}\sigma_b + I_N\right)} \\
\frac{\partial \frac{\boldsymbol{x}_{i,k}}{\boldsymbol{Y}_k^2}}{\partial \sigma_b} &= \frac{\boldsymbol{x}_{i,k}}{\sigma_b \boldsymbol{Y}_k^2}\left[\frac{2}{\boldsymbol{Y}_k}\sum_{j=1}^{n_F}\frac{\boldsymbol{x}_{j,k}(1-\boldsymbol{x}_{j,k})}{n_{di}\sigma_b + I_N} - \frac{(1-\boldsymbol{x}_{i,k})}{n_{di}\sigma_b + I_N}\right] \\
\frac{\partial \sum_{j\neq i,j=1}^{n_F} \boldsymbol{x}_{j,k}^2 Z_{j,k}}{\partial \sigma_b} &= \sum_{j\neq i,j=1}^{n_F} \frac{\boldsymbol{x}_{j,k}^2 Z_{j,k}}{(n_{di}\sigma_b + I_N)\sigma_b}\left[n_{di}\sigma_b + 2I_N - 2(1-\boldsymbol{x}_{j,k})\right] \\
\frac{\partial Z_{i,k}\left(\boldsymbol{x}_{i,k} - \boldsymbol{Y}_k\right)^2}{\partial \sigma_b} &= \frac{Z_{i,k}\left(\boldsymbol{x}_{i,k} - \boldsymbol{Y}_k\right)^2\left(n_{di}\sigma_b + 2I_N\right)}{(n_{di}\sigma_b + 2I_N)\sigma_b} - 2\left(\boldsymbol{Y}_k - \boldsymbol{x}_{i,k}\right)\frac{Z_{i,k}}{\sigma_b}\sum_{j\neq i,j=1}^{n_F} \boldsymbol{x}_{j,k}(1-\boldsymbol{x}_{j,k})(n_{di}\sigma_b + I_N)^{-1}
\end{aligned}
\tag{147}
$$

So the derivative of covariance $\mathbf{Cov}_{i,k}$ could be decomposed into two parts

$$
\frac{\partial \mathbf{Cov}_{i,k}}{\partial \sigma_b} = \phi \frac{\boldsymbol{x}_{i,k}}{\boldsymbol{Y}_k^2}\left[\boldsymbol{G}_1 + \boldsymbol{G}_2\right] \tag{148}
$$

where

$$
\begin{aligned}
\boldsymbol{G}_1 &\coloneqq Z_{i,k}\left(\boldsymbol{x}_{i,k} - \boldsymbol{Y}_k\right)^2 \frac{n_{di}\sigma_b + (1+\boldsymbol{x}_{i,k})}{n_{di}\sigma_b + 1} - 2Z_{i,k}\left(\boldsymbol{Y}_k - \boldsymbol{x}_{i,k}\right)\frac{\boldsymbol{x}_{i,k}}{\boldsymbol{Y}_k}\sum_{j\neq i,j=1}^{n_F}\frac{\boldsymbol{x}_{j,k}(1-\boldsymbol{x}_{j,k})}{n_{di}\sigma_b + 1} \\
\boldsymbol{G}_2 &\coloneqq \frac{n_{di}\sigma_b + \boldsymbol{x}_{i,k}}{n_{di}\sigma_b + 1}\sum_{j\neq i,j=1}^{n_F}\boldsymbol{x}_{j,k}^2 Z_{j,k} + \frac{2}{\boldsymbol{Y}_k}\sum_{j=1}^{n_F}\frac{\boldsymbol{x}_{j,k}(1-\boldsymbol{x}_{j,k})}{\sigma_b + 1}\sum_{j\neq i,j=1}^{n_F}\boldsymbol{x}_{j,k}^2 Z_{j,k} \\
&\qquad + \sum_{j\neq i,j=1}^{n_F}\boldsymbol{x}_{j,k}^2 Z_{j,k}\frac{1}{n_{di}\sigma_b + 1}\left[1 - 2(1-\boldsymbol{x}_{j,k})\right]
\end{aligned}
\tag{149}
$$

We can prove that $\boldsymbol{G}_1$ is always positive

$$
\boldsymbol{G}_1 \geq 0 \Leftrightarrow Z_{i,k}\left(\boldsymbol{x}_{i,k} - \boldsymbol{Y}_k\right)^2 \frac{\sigma_b + (1+\boldsymbol{x}_{i,k})}{n_{di}\sigma_b + 1} \geq 2Z_{i,k}\left(\boldsymbol{Y}_k - \boldsymbol{x}_{i,k}\right)\frac{\boldsymbol{x}_{i,k}}{\boldsymbol{Y}_k}\sum_{j\neq i,j=1}^{n_F}\frac{\boldsymbol{x}_{j,k}(1-\boldsymbol{x}_{j,k})}{n_{di}\sigma_b + 1} \tag{150}
$$

Since $\boldsymbol{Y}_k \geq 1 + \boldsymbol{x}_{i,k}$ and $0 \leq \boldsymbol{x}_{i,k} \leq 1$, we have

$$
\begin{aligned}
Z_{i,k}\left(\boldsymbol{x}_{i,k} - \boldsymbol{Y}_k\right)^2 \frac{n_{di}\sigma_b + (1+\boldsymbol{x}_{i,k})}{n_{di}\sigma_b + 1} &\geq Z_{i,k}\left(\boldsymbol{Y}_k - \boldsymbol{x}_{i,k}\right)^2 \\
&\geq Z_{i,k}\left(\boldsymbol{Y}_k - \boldsymbol{x}_{i,k}\right)\frac{2\boldsymbol{x}_{i,k}}{\boldsymbol{Y}_k}\left(1 + \sum_{j\neq i,j=1}^{n_F}\boldsymbol{x}_{j,k}\right) \\
&\geq Z_{i,k}\left(\boldsymbol{Y}_k - \boldsymbol{x}_{i,k}\right)\frac{2\boldsymbol{x}_{i,k}}{\boldsymbol{Y}_k}\sum_{j\neq i,j=1}^{n_F}\frac{\boldsymbol{x}_{j,k}(1-\boldsymbol{x}_{j,k})}{n_{di}\sigma_b + 1}
\end{aligned}
\tag{151}
$$

As for the $\boldsymbol{G}_2$, large shock variance ($\sigma_b \geq 1/n_{di}, \forall j$) guarantees its positivity since

$$\begin{aligned}
&\sigma_b \geq 1/n_{di} \Rightarrow \frac{n_{di}\sigma_b}{n_{di}\sigma_b+1} \geq \frac{1}{n_{di}\sigma_b+n_{di}} \\
&\Rightarrow \frac{n_{di}\sigma_b+\boldsymbol{x}_{i,k}}{n_{di}\sigma_b+1}\sum_{j\neq i,j=1}^{n_F}\boldsymbol{x}_{j,k}^2 Z_{j,k} \geq \sum_{j\neq i,j=1}^{n_F}\boldsymbol{x}_{j,k}^2 Z_{j,k}\frac{1}{n_{di}\sigma_b+1} \\
&\Rightarrow \boldsymbol{G}_2 \geq \sum_{j\neq i,j=1}^{n_F}\boldsymbol{x}_{j,k}^2 Z_{j,k}\frac{1}{n_{di}\sigma_b+1}\left[2-2(1-\boldsymbol{x}_{j,k})\right] \geq 0
\end{aligned} \tag{152}$$

So the derivative of covariance $\mathbf{Cov}_{i,k}$ is positive when shock variance is large enough.

This proof held marginal costs $\tilde{c}$ fixed. If we assume the marginal cost of adjusting $c$ is sufficiently high, by continuity, the inequality will still hold. □

Cyclical Markups with Efficient Investment The trade-off between the risk premium and motivation effect still exists here. When the variances of shocks increase, the firm has a tendency to charge a higher price in order to compensate for the increasing risk. On the other hand, they will become less willing to invest, which leads to higher production costs and thus drive markups down. As we show here, depending on the parameters, the aggregated markups can both increase or decrease with the economic cycle.

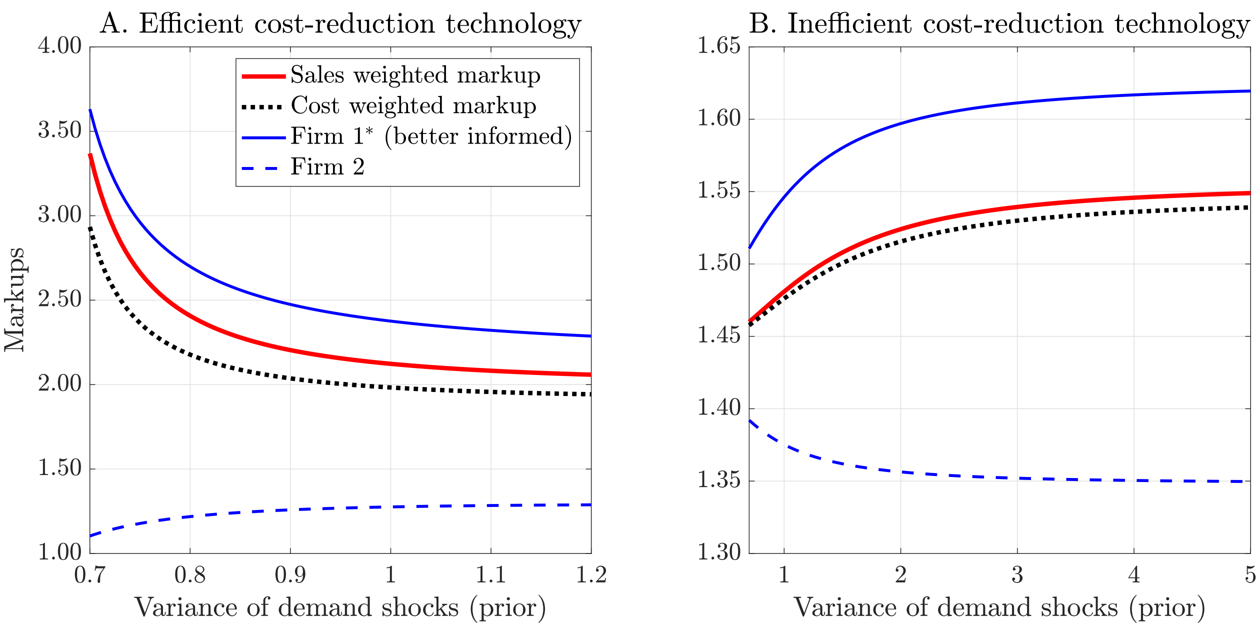


Figure 9: Comparative static: markups and economic cycles, $\rho = 1$

Notes: These two panels depict how markups change with the variance of demand shock. For tractability, the weighted markups are weighted by expected sales (costs) over expected markups. At firm level, firm 1 has eight data points while firm 2 only observes two. The parameter for investment, $\chi_c$, is 1 and 1.5 for the two panels, respectively. Moreover, $\rho_1 = \rho_2 = 1$.

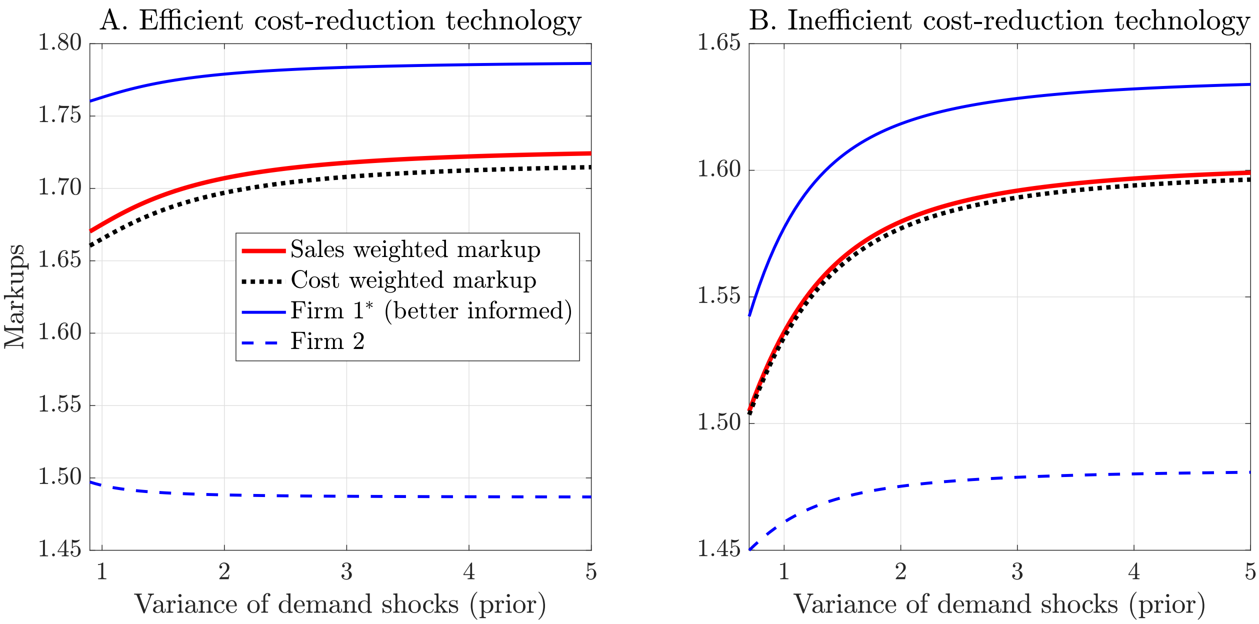


Figure 10: Comparative static: markups and economic cycles, $\rho = 4$

Notes: These two panels depict how markups change with the variance of demand shock. For tractability, the weighted markups are weighted by expected sales (costs) over expected markups. At firm level, firm 1 has eight data points while firm 2 only observes two. The parameter for investment, $\chi_c$, is 1 and 1.5 for the two panels, respectively. Moreover, $\rho_1 = \rho_2 = 4$.

# C. Solutions to Alternative Models

## C.1. A Model with Aggregate Demand Shocks

The new complication in this model is that the solution is not explicit. The solution is characterized by a set of $n_F + 3$ equations in $n_F + 3$ unknowns.

Since firm $i$ could only observe $\boldsymbol{s}_i$, its expectation of the price is $\mathbf{E}[\boldsymbol{p}|\boldsymbol{s}_i] = \bar{\boldsymbol{p}}^M + \boldsymbol{K}_i \boldsymbol{s}_i$, where

$$\boldsymbol{K}_i = \mathbf{Cov}(\boldsymbol{p}, \boldsymbol{s}_i)\mathbf{Var}(\boldsymbol{s}_i)^{-1} \tag{153}$$

The variance of price forecast error is

$$\mathbf{Var}[\boldsymbol{p}|\boldsymbol{s}_i] = \mathbf{Var}(\boldsymbol{p}) - \mathbf{Cov}(\boldsymbol{p}, \boldsymbol{s}_i)\mathbf{Var}(\boldsymbol{s}_i)^{-1}\mathbf{Cov}(\boldsymbol{p}, \boldsymbol{s}_i)' \tag{154}$$

SOLUTION We can rearrange the pricing equation (4) as $\frac{1}{\phi}\sum_{i=1}^{n_F} \tilde{\boldsymbol{q}}_i = \bar{\boldsymbol{p}} + \boldsymbol{b} - \tilde{\boldsymbol{p}}$ and then substitute in the first order condition (7) and the linear pricing rule guess from lemma 1:

$$\begin{aligned}
&\Rightarrow \frac{1}{\phi}\sum_{i=1}^{n_F} \hat{\boldsymbol{H}}_i(\bar{\boldsymbol{p}}^M - \boldsymbol{c}_i + \boldsymbol{K}_i \boldsymbol{s}_i) = \bar{\boldsymbol{p}} + \boldsymbol{b} - \left(\bar{\boldsymbol{p}}^M + \boldsymbol{F}\boldsymbol{b} + \sum_{i=1}^{n_F} \boldsymbol{h}_i \boldsymbol{\varepsilon}_i\right) \\
&\Rightarrow \left(\boldsymbol{F} - \boldsymbol{I}_N + \frac{1}{\phi}\sum_{i=1}^{n_F} \hat{\boldsymbol{H}}_i \boldsymbol{K}_i\right)\boldsymbol{b} + \sum_{i=1}^{n_F}\left(\boldsymbol{h}_i + \frac{1}{\phi}\hat{\boldsymbol{H}}_i \boldsymbol{K}_i\right)\boldsymbol{\varepsilon}_i + \left(\boldsymbol{I}_N + \frac{1}{\phi}\sum_{i=1}^{n_F}\hat{\boldsymbol{H}}_i\right)\bar{\boldsymbol{p}}^M - \bar{\boldsymbol{p}} - \frac{1}{\phi}\sum_{i=1}^{n_F}\hat{\boldsymbol{H}}_i \boldsymbol{c}_i = 0
\end{aligned} \tag{155}$$

Matching coefficients yields the solutions in lemma 1.

FIRM-LEVEL MARKUP The firm-level markup for firm $i$ is the quantity-weighted prices divided by quantity-weighted costs:

$$\begin{aligned} M_i^f &= \frac{\mathbf{E}[\tilde{\boldsymbol{q}}_i'\boldsymbol{p}]}{\mathbf{E}[\tilde{\boldsymbol{q}}_i'\boldsymbol{c}_i]} = \frac{\mathbf{E}[\tilde{\boldsymbol{q}}_i]'\mathbf{E}[\boldsymbol{p}] + \mathbf{tr}\left[\mathbf{Cov}(\boldsymbol{p}_i, \tilde{\boldsymbol{q}}_i)\right]}{\mathbf{E}[\tilde{\boldsymbol{q}}_i'\boldsymbol{c}_i]} \\ &= \frac{(\bar{\boldsymbol{p}}^M - \boldsymbol{c}_i)'\hat{\boldsymbol{H}}_i\bar{\boldsymbol{p}}^M + \mathbf{tr}\left(\hat{\boldsymbol{H}}_i\boldsymbol{K}_i\mathbf{Var}(\boldsymbol{s}_i)\boldsymbol{K}_i'\right)}{(\bar{\boldsymbol{p}}^M - \boldsymbol{c}_i)'\hat{\boldsymbol{H}}_i\boldsymbol{c}_i} > \frac{(\bar{\boldsymbol{p}}^M - \boldsymbol{c}_i)'\hat{\boldsymbol{H}}_i\bar{\boldsymbol{p}}^M}{(\bar{\boldsymbol{p}}^M - \boldsymbol{c}_i)'\hat{\boldsymbol{H}}_i\boldsymbol{c}_i} \end{aligned} \tag{156}$$

Without data, the updating weight $K_i$ is zero because there is no information in the data to weight. The weight $K_i$ is strictly increasing in the number of data points $n_{di}$. $\hat{\boldsymbol{H}}_i$ is also strictly increasing in data $n_{di}$. Thus, if one firm acquires more data and we hold the equilibrium price fixed, the firm markup increases, relative to the average product markup.

ECONOMY-LEVEL MARKUP The industry markup is

$$M^m := \frac{\mathbf{E}\left[\sum_{i=1}^{n_F}\tilde{\boldsymbol{q}}_i'\boldsymbol{p}_i\right]}{\mathbf{E}\left[\sum_{i=1}^{n_F}\tilde{\boldsymbol{q}}_i'\boldsymbol{c}_i\right]} = \frac{\sum_{i=1}^{n_F}\mathbf{E}\left[\tilde{\boldsymbol{q}}_i'\boldsymbol{p}_i\right]}{\sum_{i=1}^{n_F}\mathbf{E}\left[\tilde{\boldsymbol{q}}_i'\boldsymbol{c}_i\right]} = \sum_{i=1}^{n_F} w^{\boldsymbol{H}_i} M_i^f \quad \text{where} \quad w^{\boldsymbol{H}_i} = \frac{\mathbf{E}\left[\tilde{\boldsymbol{q}}_i'\boldsymbol{c}_i\right]}{\sum_{i=1}^{n_F}\mathbf{E}\left[\tilde{\boldsymbol{q}}_i'\boldsymbol{c}_i\right]}. \tag{157}$$

*Aggregate Demand Model: Cyclical Markup Fluctuations*

**Proposition 6.** *The product-level markup converges as shock variance tends to infinity given identical risk aversion and signal precision across all firms.*

*Proof.* At this point, we need to relax the assumption that $var(b) = I$ and allow the variance of demand shocks to be $\sigma_b I_N$. It is helpful to grow the variance of all attributed demand shocks together with a scalar $\sigma_b$ so that we can take a comparative static with respect to one scalar parameter.

Define $M_i = \left(\sigma_b + \frac{1}{n_{di}}\left(I_N + \frac{\hat{\boldsymbol{H}}_i}{\phi}\right)\right)^{-1}$, we have $\lim_{\sigma_b\to\infty} M_i\sigma_b = I_N$. The unknown coefficients could be expressed in $\hat{\boldsymbol{H}}_i$ and $M_i$.

$$\begin{aligned} \boldsymbol{K}_i &= [I_N + \sum_{j=1}^{n_F}\sigma_b M_j\frac{\hat{\boldsymbol{H}}_j}{\phi}]^{-1}\sigma_b M_i \\ \boldsymbol{h}_i &= -\boldsymbol{K}_i\frac{\hat{\boldsymbol{H}}_i}{\phi} = -\frac{\sigma_b M_i\frac{\hat{\boldsymbol{H}}_i}{\phi}}{I_N + \sum_{j=1}^{n_F}\sigma_b M_j\frac{\hat{\boldsymbol{H}}_j}{\phi}} \\ \boldsymbol{F} &= I_N + \sum_{j=1}^{n_F}\boldsymbol{h}_i = \frac{1}{1 + \sum_{j=1}^{n_F}\sigma_b M_j\frac{\hat{\boldsymbol{H}}_j}{\phi}} \end{aligned} \tag{158}$$

The price impact $\hat{\boldsymbol{H}}_i$ is

$$\phi\hat{\boldsymbol{H}}_i^{-1} = I_N + \rho_i\phi\left(\sigma_b\boldsymbol{F}'\boldsymbol{F} + \sum_{i=1}^{n_F}\boldsymbol{h}_i^2\frac{1}{n_{di}} - \boldsymbol{K}_i^2(\sigma_b + \frac{1}{n_{di}})\right) \tag{159}$$

By symmetry, all firm choose the same impact function $\hat{\boldsymbol{H}}_i$, thus

$$\begin{aligned}
\phi \hat{\boldsymbol{H}}_{i,k}^{-1} &= 1+\rho_i\phi \frac{\left(\frac{1}{n_{di}}+\sigma_b+\frac{\hat{\boldsymbol{H}}_{i,k}}{\phi n_{di}}\right)^2\sigma_b + n_F\sigma_b^2\frac{\hat{\boldsymbol{H}}_{i,k}^2}{\phi^2 n_{di}} - \sigma_b^2(\sigma_b+\frac{1}{n_{di}}I_N)}{\left(\sigma_b+\Sigma_{\epsilon_{i,k}}+(1/n_{di}+n_F\sigma_b)\frac{\hat{\boldsymbol{H}}_{i,k}}{\phi}\right)^2} \\
&= 1+\rho_i\phi\frac{\left(1+\frac{\hat{\boldsymbol{H}}_{i,k}}{\phi}\right)\left(2+\left(1+\frac{\hat{\boldsymbol{H}}_{i,k}}{\phi}\right)\frac{1}{n_{di}\sigma_b}\right)+n_F\frac{\hat{\boldsymbol{H}}_{i,k}^2}{\phi^2 n_{di}}-\frac{1}{n_{di}}}{\left(1+\frac{1}{n_{di}\sigma_b}+(\frac{1}{n_{di}\sigma_b}+n_F)\frac{\hat{\boldsymbol{H}}_{i,k}}{\phi}\right)^2}
\end{aligned} \tag{160}$$

This is a cubic equation for $\hat{\boldsymbol{H}}_{i,k}$ and has explicit solution. Moreover, the solution is convergent since all coefficients converge as shock variance $\sigma_b$ goes to infinity. Another observation is that $\boldsymbol{F}^2\sigma_b - \boldsymbol{K}_i^2\sigma_b$ is bounded since

$$(\boldsymbol{F}^2-\boldsymbol{K}_i^2)\sigma_b = \left(1+\sum_{j=1}^{n_F}\sigma_b M_j\frac{\hat{\boldsymbol{H}}_j}{\phi}\right)^{-2}\frac{2\Sigma_{\epsilon_i}\left(1+\frac{\hat{\boldsymbol{H}}_i}{\phi}\right)+\sigma_b^{-1}\left(\Sigma_{\epsilon_i}\left(1+\frac{\hat{\boldsymbol{H}}_i}{\phi}\right)\right)^2}{\left(1+\frac{I}{n_{di}\sigma_b}\left(1+\frac{\hat{\boldsymbol{H}}_i}{\phi}\right)\right)^2} \tag{161}$$

So the RHS of equation (159) is bounded and $\hat{\boldsymbol{H}}_i$ is positive in the limit. The product-level markup is clearly convergent because $\lim_{\sigma_b\to\infty}\mathbf{E}\left[\boldsymbol{p}_i\right] = \lim_{\sigma_b\to\infty}\left(\boldsymbol{I}_N+\frac{1}{\phi}\sum_{i=1}^{n_F}\hat{\boldsymbol{H}}_i\right)^{-1}\left(\bar{\boldsymbol{p}}+\frac{1}{\phi}\sum_{i=1}^{n_F}\hat{\boldsymbol{H}}_i\boldsymbol{c}_i\right)$ exists.

This proof held marginal costs $\tilde{c}$ fixed. If we assume the marginal cost of adjusting $c$ is sufficiently high, by continuity, the inequality will still hold. □

**Proposition 7.** *The firm-level and economy-level markups are strictly increasing in demand shock variance, if the shock variance is large enough. In the high-variance limit, markups approach linear asymptotes.*

*Proof.* The covariance term is $\boldsymbol{K}_i\mathbf{Var}(\boldsymbol{s}_i)\boldsymbol{K}_i'\hat{\boldsymbol{H}}_i$ and we have

$$\begin{aligned}
M_i^f &= \frac{\mathbf{E}[\tilde{\boldsymbol{q}}_i'\boldsymbol{p}_i]}{\mathbf{E}[\tilde{\boldsymbol{q}}_i'\boldsymbol{c}_i]} = \frac{\mathbf{E}[\tilde{\boldsymbol{q}}_i]'\mathbf{E}[\boldsymbol{p}]+\mathbf{tr}\left[\mathbf{Cov}(\boldsymbol{p}_i,\tilde{\boldsymbol{q}}_i)\right]}{\mathbf{E}[\tilde{\boldsymbol{q}}_i'\boldsymbol{c}_i]} \\
&= \frac{\sum_{j=1}^N(\mathbf{E}(\boldsymbol{p}_{i,j})-\boldsymbol{c}_{i,j})\mathbf{E}(\boldsymbol{p}_{i,j})\hat{\boldsymbol{H}}_{i,j}+\sum_{j=1}^N\hat{\boldsymbol{H}}_{i,j}\boldsymbol{K}_{i,j}^2\frac{n_{di}\sigma_b^2}{n_{di}\sigma_b+1}}{\sum_{j=1}^N(\mathbf{E}(\boldsymbol{p}_{i,j})-\boldsymbol{c}_{i,j})\boldsymbol{c}_{i,j}\hat{\boldsymbol{H}}_{i,j}}
\end{aligned} \tag{162}$$

The $K_i$ converges as $\lim_{\sigma_b\to\infty}M_i\sigma_b = 1$. The asymptote for $M_i^f$ is

$$\alpha_i := \lim_{\sigma_b\to\infty}\frac{M_i^f}{\sigma_b} = \frac{\sum_{j=1}^N\tilde{\hat{\boldsymbol{H}}}_{i,j}\tilde{\boldsymbol{K}}_j^2}{\sum_{j=1}^N(\tilde{\boldsymbol{p}}_j-\boldsymbol{c}_{i,j})\boldsymbol{c}_{i,j}\tilde{\hat{\boldsymbol{H}}}_{i,j}}, \quad \gamma_i := \lim_{\sigma_b\to\infty}\left(M_i^f-\alpha_i\sigma_b\right) = \frac{\sum_{j=1}^N\left((\tilde{\boldsymbol{p}}_j-\boldsymbol{c}_{i,j})\tilde{\boldsymbol{p}}_j-\tilde{\boldsymbol{K}}_j^2/n_{di}\right)\tilde{\hat{\boldsymbol{H}}}_{i,j}}{\sum_{j=1}^N(\tilde{\boldsymbol{p}}_j-\boldsymbol{c}_{i,j})\boldsymbol{c}_{i,j}\tilde{\hat{\boldsymbol{H}}}_{i,j}} \tag{163}$$

Where $\lim_{\sigma_b\to\infty}\hat{\boldsymbol{H}}_i = \tilde{\hat{\boldsymbol{H}}}_i$ and $\lim_{\sigma_b\to\infty}\boldsymbol{K}_i = \tilde{\boldsymbol{K}}_i = \left(I_N+\sum_{i=1}^{n_F}\tilde{\hat{\boldsymbol{H}}}_i\right)^{-1}$. The average firm-level markup $\overline{M}^f = (1/n_F)\sum_{i=1}^{n_F}M_i^f$ approaches $\sum_{i=1}^{n_F}\frac{\alpha_i}{n_F}\sigma_b+\sum_{i=1}^{n_F}\frac{\gamma_i}{n_F}$ in the long run. The economy-level markup is $M^m = \sum_{i=1}^{n_F}w^{\boldsymbol{H}_i}M_i^f$ with $w^{\boldsymbol{H}_i} = \frac{\mathbf{E}\left[\tilde{\boldsymbol{q}}_i'\boldsymbol{c}_i\right]}{\sum_{i=1}^{n_F}\mathbf{E}\left[\tilde{\boldsymbol{q}}_i'\boldsymbol{c}_i\right]}$. The weight $w^{\boldsymbol{H}_i}$ converges to $w_i$ as shock variance goes to infinity.

$$w_i := \lim_{\sigma_b \to \infty} w^{\boldsymbol{H}_i} = \frac{\sum_{j=1}^{N} (\tilde{\boldsymbol{p}}_j - \boldsymbol{c}_{i,j}) \boldsymbol{c}_{i,j} \tilde{\tilde{\boldsymbol{H}}}_{i,j}}{\sum_{i=1}^{n_F} \sum_{j=1}^{N} (\tilde{\boldsymbol{p}}_j - \boldsymbol{c}_{i,j}) \boldsymbol{c}_{i,j} \tilde{\tilde{\boldsymbol{H}}}_{i,j}} \Rightarrow M^m \text{ approaches } \sum_{i=1}^{n_F} w_i \alpha_i \sigma_b + \sum_{i=1}^{n_F} w_i \gamma_i \tag{164}$$

This proof held marginal costs $\tilde{c}$ fixed. If we assume the marginal cost of adjusting $c$ is sufficiently high, by continuity, the inequality will still hold. □

## C.2. A Model with Firm-Specific Data that is Private Information

For simplicity, we assumed that all firms see the signals of all other firms in the economy. In this appendix we solve a model with signals that are privately observed by one firm only. We compare the solution in the private and public signal models and find modest differences.

The only change to the setup of the main model is the information set. Firm $i$ observes only the $n_{di}$ data points generated by firm $i$, not the data produced by other firms. This is equivalent to conditioning expectations on the composite signal $\tilde{\mathbf{s}}_i$.

The first-order condition for firms still holds given their beliefs and strategies adopted by other firms. We denote the conditional expectation $\mathbf{E}_i(\cdot) = \mathbf{E}(\cdot|\mathcal{I}_i)$ for firm $i$. The inverse demand function is given by

$$\begin{aligned}
\boldsymbol{p}_i &= \bar{\boldsymbol{p}} + \boldsymbol{b}_i - \frac{1}{\phi} \sum_{j=1}^{n_F} \tilde{\boldsymbol{q}}_j \\
\Rightarrow \mathbf{E}\left[\boldsymbol{p}_i|\mathcal{I}_i\right] &= \bar{\boldsymbol{p}} + \mathbf{E}\left[\boldsymbol{b}_i|\mathcal{I}_i\right] - \frac{1}{\phi} \sum_{j=1}^{n_F} \mathbf{E}\left[\tilde{\boldsymbol{q}}_j|\mathcal{I}_i\right] \\
\Rightarrow \mathbf{E}_i \boldsymbol{p}_i &= \bar{\boldsymbol{p}} + \mathbf{E}_i \boldsymbol{b}_i - \frac{1}{\phi} \sum_{j=1}^{n_F} \mathbf{E}_i \tilde{\boldsymbol{q}}_j
\end{aligned} \tag{165}$$

So the optimal output in the incomplete information setup is

$$\begin{aligned}
\tilde{\boldsymbol{q}}_i &= \left(\rho_i \mathbf{Var}\left[\boldsymbol{p}_i|\mathcal{I}_i\right] - \frac{\partial \mathbf{E}\left[\boldsymbol{p}_i|\mathcal{I}_i\right]}{\partial \tilde{\boldsymbol{q}}_i}\right)^{-1} \left(\mathbf{E}\left[\boldsymbol{p}_i|\mathcal{I}_i\right] - \boldsymbol{c}_i\right) \\
\Rightarrow &\left(\rho_i \mathbf{Var}\left[\boldsymbol{p}_i|\mathcal{I}_i\right] - \frac{\partial \mathbf{E}\left[\boldsymbol{p}_i|\mathcal{I}_i\right]}{\partial \tilde{\boldsymbol{q}}_i}\right) \tilde{\boldsymbol{q}}_i = \mathbf{E}\left[\boldsymbol{p}_i|\mathcal{I}_i\right] - \boldsymbol{c}_i \\
\Rightarrow &\left(\rho_i \mathbf{Var}\left[\boldsymbol{p}_i|\mathcal{I}_i\right] + \frac{1}{\phi} \boldsymbol{I}_N\right) \tilde{\boldsymbol{q}}_i = \bar{\boldsymbol{p}} + \mathbf{E}_i \boldsymbol{b}_i - \frac{1}{\phi} \sum_{j=1}^{n_F} \mathbf{E}_i \tilde{\boldsymbol{q}}_j - \boldsymbol{c}_i \\
\Rightarrow &\left(\rho_i \mathbf{Var}\left[\boldsymbol{p}_i|\mathcal{I}_i\right] + \frac{2}{\phi} \boldsymbol{I}_N\right) \tilde{\boldsymbol{q}}_i = \bar{\boldsymbol{p}} + \mathbf{E}_i \boldsymbol{b}_i - \frac{1}{\phi} \sum_{j=1, j \neq i}^{n_F} \mathbf{E}_i \tilde{\boldsymbol{q}}_j - \boldsymbol{c}_i \\
\Rightarrow &\tilde{\boldsymbol{q}}_i = \boldsymbol{H}_i \left(\bar{\boldsymbol{p}} + \mathbf{E}_i \boldsymbol{b}_i - \boldsymbol{c}_i - \frac{1}{\phi} \sum_{j=1, j \neq i}^{n_F} \mathbf{E}_i \tilde{\boldsymbol{q}}_j\right), \ \forall\, i = 1, \ldots, n_F
\end{aligned} \tag{166}$$

**Linear Equilibrium** We first solve for a linear equilibrium in which optimal output is a linear function of signal. Suppose that the each firm follows a linear strategy of the form

$$\tilde{\boldsymbol{q}}_i = \boldsymbol{\alpha}_i + \boldsymbol{\gamma}_i \mathbf{E}_i \boldsymbol{b}_i = \boldsymbol{\alpha}_i + \boldsymbol{\gamma}_i \boldsymbol{K}_i \boldsymbol{s}_i \tag{167}$$

Then the optimal action function (166) across all firms is

$$\begin{aligned}
&\tilde{\boldsymbol{q}}_i = \boldsymbol{H}_i \left( \bar{\boldsymbol{p}} + \mathbf{E}_i \boldsymbol{b}_i - \boldsymbol{c}_i - \frac{1}{\phi} \sum_{j=1, j\neq i}^{n_F} \mathbf{E}_i \tilde{\boldsymbol{q}}_j \right) \\
\Rightarrow &\boldsymbol{\alpha}_i + \gamma_i \mathbf{E}_i \boldsymbol{b}_i = \boldsymbol{H}_i \left( \bar{\boldsymbol{p}} + \mathbf{E}_i \boldsymbol{b}_i - \boldsymbol{c}_i - \frac{1}{\phi} \sum_{j=1, j\neq i}^{n_F} \mathbf{E}_i \left( \boldsymbol{\alpha}_j + \gamma_j \mathbf{E}_j \boldsymbol{b}_j \right) \right) \\
\Rightarrow &\boldsymbol{\alpha}_i + \gamma_i \mathbf{E}_i \boldsymbol{b}_i = \boldsymbol{H}_i \left( \bar{\boldsymbol{p}} + \mathbf{E}_i \boldsymbol{b}_i - \boldsymbol{c}_i - \frac{1}{\phi} \sum_{j=1, j\neq i}^{n_F} \boldsymbol{\alpha}_j \right) \\
\Rightarrow &\boldsymbol{\alpha}_i - \boldsymbol{H}_i \left( \bar{\boldsymbol{p}} - \boldsymbol{c}_i - \frac{1}{\phi} \sum_{j=1, j\neq i}^{n_F} \boldsymbol{\alpha}_j \right) + (\gamma_i - \boldsymbol{H}_i) \mathbf{E}_i \boldsymbol{b}_i = 0, \ \forall\, i = 1, \ldots, n_F
\end{aligned} \tag{168}$$

Since the last equation holds for arbitrary $\mathbf{E}_i \boldsymbol{b}_i$, we must have

$$\begin{aligned}
\boldsymbol{\alpha}_i &= \boldsymbol{H}_i \left( \bar{\boldsymbol{p}} - \boldsymbol{c}_i - \frac{1}{\phi} \sum_{j=1, j\neq i}^{n_F} \boldsymbol{\alpha}_j \right) \\
\gamma_i &= \boldsymbol{H}_i = \hat{\boldsymbol{H}}_i \left( \boldsymbol{I}_N + \frac{1}{\phi} \hat{\boldsymbol{H}}_i \right)^{-1}
\end{aligned} \tag{169}$$

From the first equation we can solve for $\boldsymbol{\alpha}_i$ (similar to section 8)

$$\begin{aligned}
\boldsymbol{\alpha}_i &= \left( \boldsymbol{H}_i^{-1} - \frac{\boldsymbol{I}_N}{\phi} \right)^{-1} \left[ \left( \boldsymbol{I}_N + \frac{1}{\phi} \sum_{j=1}^{n_F} \left( \boldsymbol{H}_j^{-1} - \frac{\boldsymbol{I}_N}{\phi} \right)^{-1} \right)^{-1} \left( \bar{\boldsymbol{p}} + \frac{1}{\phi} \sum_{j=1}^{n_F} \left( \boldsymbol{H}_j^{-1} - \frac{\boldsymbol{I}_N}{\phi} \right)^{-1} \boldsymbol{c}_j \right) - \boldsymbol{c}_i \right] \\
&= \hat{\boldsymbol{H}}_i \left[ \left( \boldsymbol{I}_N + \frac{1}{\phi} \sum_{j=1}^{n_F} \hat{\boldsymbol{H}}_j \right)^{-1} \left( \bar{\boldsymbol{p}} + \frac{1}{\phi} \sum_{j=1}^{n_F} \hat{\boldsymbol{H}}_j \boldsymbol{c}_j \right) - \boldsymbol{c}_i \right]
\end{aligned} \tag{170}$$

Finally the equilibrium output $\tilde{\boldsymbol{q}}_i$ is given by

$$\begin{aligned}
\tilde{\boldsymbol{q}}_i &= \hat{\boldsymbol{H}}_i \left[ \left( \boldsymbol{I}_N + \frac{1}{\phi} \sum_{j=1}^{n_F} \hat{\boldsymbol{H}}_j \right)^{-1} \left( \bar{\boldsymbol{p}} + \frac{1}{\phi} \sum_{j=1}^{n_F} \hat{\boldsymbol{H}}_j \boldsymbol{c}_j \right) - \boldsymbol{c}_i \right] + \boldsymbol{H}_i \mathbf{E}_i \boldsymbol{b}_i \\
&= \hat{\boldsymbol{H}}_i \left( \bar{\boldsymbol{p}}^M - \boldsymbol{c}_i \right) + \hat{\boldsymbol{H}}_i \left( \boldsymbol{I}_N + \frac{1}{\phi} \hat{\boldsymbol{H}}_i \right)^{-1} \boldsymbol{K}_i \boldsymbol{s}_i
\end{aligned} \tag{171}$$

The equilibrium price and output are

$$\begin{aligned}
\mathbf{E}(\tilde{\boldsymbol{q}}_i) &= \hat{\boldsymbol{H}}_i \left( \bar{\boldsymbol{p}}^M - \boldsymbol{c}_i \right) \\
\boldsymbol{p}_i &= \bar{\boldsymbol{p}}^M + \boldsymbol{b}_i - \frac{1}{\phi} \sum_{j=1}^{n_F} \hat{\boldsymbol{H}}_j \left( \boldsymbol{I}_N + \frac{1}{\phi} \hat{\boldsymbol{H}}_j \right)^{-1} \boldsymbol{K}_j \boldsymbol{s}_j \Rightarrow \mathbf{E}(\boldsymbol{p}_i) = \bar{\boldsymbol{p}}^M
\end{aligned} \tag{172}$$

In the case where there are two firms, we can prove that this equilibrium exists and is unique. Proof available on request. We omit it here for now because it is lengthy.

PRODUCT-LEVEL MARKUP The product-level markup for product $k$ produced by firm $i$ is $M_{ik}^p := \mathbf{E}[\boldsymbol{p}_i(j)]/\boldsymbol{c}_i(j)$. The average product-level markup is

$$\overline{M}^p = \frac{1}{N}\frac{1}{n_F}\sum_{i=1}^{n_F}\sum_{j=1}^{N} M_{ij}^p = \frac{1}{N}\frac{1}{n_F}\sum_{i=1}^{n_F}\sum_{j=1}^{N}\frac{\bar{\boldsymbol{p}}^M(j)}{\boldsymbol{c}_i(j)} \tag{173}$$

FIRM-LEVEL MARKUP The firm-level markup for firm $i$ is the quantity-weighted prices divided by quantity-weighted costs:

$$\begin{aligned} M_i^f &= \frac{\mathbf{E}[\tilde{\boldsymbol{q}}_i'\boldsymbol{p}]}{\mathbf{E}[\tilde{\boldsymbol{q}}_i'\boldsymbol{c}_i]} = \frac{\mathbf{E}[\tilde{\boldsymbol{q}}_i]'\mathbf{E}[\boldsymbol{p}] + \mathbf{tr}\left[\mathbf{Cov}(\boldsymbol{p}_i, \tilde{\boldsymbol{q}}_i)\right]}{\mathbf{E}[\tilde{\boldsymbol{q}}_i'\boldsymbol{c}_i]} \\ &= \frac{(\bar{\boldsymbol{p}}^M - \boldsymbol{c}_i)'\hat{\boldsymbol{H}}_i\bar{\boldsymbol{p}}^M + \mathbf{tr}\left(\left(\boldsymbol{I}_N - \frac{\boldsymbol{H}_i}{\phi}\right)\boldsymbol{K}_i\mathbf{Var}(\boldsymbol{s}_i)\boldsymbol{K}_i'\boldsymbol{H}_i\right)}{(\bar{\boldsymbol{p}}^M - \boldsymbol{c}_i)'\hat{\boldsymbol{H}}_i\boldsymbol{c}_i} > \frac{(\bar{\boldsymbol{p}}^M - \boldsymbol{c}_i)'\hat{\boldsymbol{H}}_i\bar{\boldsymbol{p}}^M}{(\bar{\boldsymbol{p}}^M - \boldsymbol{c}_i)'\hat{\boldsymbol{H}}_i\boldsymbol{c}_i} \end{aligned} \tag{174}$$

Thus, the average firm-level markup is $\overline{M}^f = (1/n_F)\sum_{i=1}^{n_F} M_i^f$.

INDUSTRY MARKUP The industry markup is

$$M^m := \frac{\mathbf{E}\left[\sum_{i=1}^{n_F}\tilde{\boldsymbol{q}}_i'\boldsymbol{p}_i\right]}{\mathbf{E}\left[\sum_{i=1}^{n_F}\tilde{\boldsymbol{q}}_i'\boldsymbol{c}_i\right]} = \frac{\sum_{i=1}^{n_F}\mathbf{E}\left[\tilde{\boldsymbol{q}}_i'\boldsymbol{p}_i\right]}{\sum_{i=1}^{n_F}\mathbf{E}\left[\tilde{\boldsymbol{q}}_i'\boldsymbol{c}_i\right]} = \sum_{i=1}^{n_F} w^{\boldsymbol{H}_i} M_i^f \quad \text{where} \quad w^{\boldsymbol{H}_i} = \frac{\mathbf{E}\left[\tilde{\boldsymbol{q}}_i'\boldsymbol{c}_i\right]}{\sum_{i=1}^{n_F}\mathbf{E}\left[\tilde{\boldsymbol{q}}_i'\boldsymbol{c}_i\right]}. \tag{175}$$

*Private Information Model: Cyclical Markup Behavior*

**Proposition 8.** *The product-level markup converges as shock variance goes to infinity given identical risk aversion and signal precision across all firms.*

*Proof.* We analyze an economy consisted of identical firms with diagonal firm and shock variance matrices. The price impact $\hat{\boldsymbol{H}}_i$ satisfies the following equation

$$\begin{aligned} &\hat{\boldsymbol{H}}_i = \left[\rho_i\mathbf{Var}(\boldsymbol{b}_i|\mathcal{I}_i) + \frac{\boldsymbol{I}_N}{\phi} + \rho_i(n_F-1)\frac{\hat{\boldsymbol{H}}_i}{\phi}\left(\boldsymbol{I}_N + \frac{1}{\phi}\hat{\boldsymbol{H}}_i\right)^{-1}\mathbf{Var}(\boldsymbol{K}_i\boldsymbol{s}_i)\left(\boldsymbol{I}_N + \frac{1}{\phi}\hat{\boldsymbol{H}}_i\right)^{-1}\frac{\hat{\boldsymbol{H}}_i}{\phi}\right]^{-1} \\ &\Rightarrow \hat{\boldsymbol{H}}_{i,k}^{-1} = \rho_i\frac{\sigma_b\Sigma_{\epsilon_i,k}}{\sigma_b + \Sigma_{\epsilon_i,k}} + \frac{1}{\phi} + \rho_i(n_F-1)\left(\frac{\hat{\boldsymbol{H}}_{i,k}}{\phi + \hat{\boldsymbol{H}}_{i,k}}\right)^2\frac{\Sigma_{b,k}^2}{\sigma_b + \Sigma_{\epsilon_i,k}},\ \forall k = 1,\ldots,N \end{aligned} \tag{176}$$

Taking derivative with respect to $\sigma_b$ for both sides, we have

$$\begin{aligned} &-\hat{\boldsymbol{H}}_{i,k}^{-2}\frac{\partial\hat{\boldsymbol{H}}_{i,k}}{\partial\sigma_b} = \frac{\rho_i\Sigma_{\epsilon_i,k}^2}{\sigma_b + \Sigma_{\epsilon_i,k}} + \rho_i(n_F-1)\left[\frac{\Sigma_{b,k}^2}{\sigma_b + \Sigma_{\epsilon_i,k}}\frac{2\phi\hat{\boldsymbol{H}}_{i,k}}{(\hat{\boldsymbol{H}}_{i,k} + \phi)^3}\frac{\partial\hat{\boldsymbol{H}}_{i,k}}{\partial\sigma_b} + \left(\frac{\hat{\boldsymbol{H}}_{i,k}}{\phi + \hat{\boldsymbol{H}}_{i,k}}\right)^2\frac{\sigma_b(\sigma_b + 2\Sigma_{\epsilon_i,k})}{\sigma_b + \Sigma_{\epsilon_i,k}}\right] \\ &-\left(\hat{\boldsymbol{H}}_{i,k}^{-2} + \rho_i(n_F-1)\frac{\Sigma_{b,k}^2}{\sigma_b + \Sigma_{\epsilon_i,k}}\frac{2\phi\hat{\boldsymbol{H}}_{i,k}}{(\hat{\boldsymbol{H}}_{i,k} + \phi)^3}\right)\frac{\partial\hat{\boldsymbol{H}}_{i,k}}{\partial\sigma_b} = \frac{\rho_i\Sigma_{\epsilon_i,k}^2}{\sigma_b + \Sigma_{\epsilon_i,k}} + \rho_i(n_F-1)\left(\frac{\hat{\boldsymbol{H}}_{i,k}}{\phi + \hat{\boldsymbol{H}}_{i,k}}\right)^2\frac{\sigma_b(\sigma_b + 2\Sigma_{\epsilon_i,k})}{\sigma_b + \Sigma_{\epsilon_i,k}} \end{aligned} \tag{177}$$

The derivative $\frac{\partial\hat{\boldsymbol{H}}_{i,k}}{\partial\sigma_b}$ is clearly negative, implying convergent $\hat{\boldsymbol{H}}_{i,k}$ (decreasing and non-negative) as shock variance goes to infinity. Furthermore, $\hat{\boldsymbol{H}}_{i,k}$ must converges to zero, otherwise the RHS of equation (176) is unbounded while the LHS is bounded. The product-level markup $\overline{M}^p$ is convergent:

$$\overline{M}^p = \frac{1}{N}\frac{1}{n_F}\sum_{i=1}^{n_F}\sum_{j=1}^{N} M_{ij}^p = \frac{1}{N}\frac{1}{n_F}\sum_{i=1}^{n_F}\sum_{j=1}^{N}\frac{\mathbf{E}(\boldsymbol{p}_{i,j})}{\boldsymbol{c}_{i,j}} \quad \text{and} \quad \lim_{\sigma_b\to\infty}\overline{M}^p = \frac{1}{N}\frac{1}{n_F}\sum_{i=1}^{n_F}\sum_{j=1}^{N}\frac{\bar{\boldsymbol{p}}_j}{\boldsymbol{c}_{i,j}} \tag{178}$$

since $\mathbf{E}\left[\boldsymbol{p}_i\right] = \bar{\boldsymbol{p}}^M = \left(\boldsymbol{I}_N + \frac{1}{\phi}\sum_{i=1}^{n_F}\hat{\boldsymbol{H}}_i\right)^{-1}\left(\bar{\boldsymbol{p}} + \frac{1}{\phi}\sum_{i=1}^{n_F}\hat{\boldsymbol{H}}_i\boldsymbol{c}_i\right)$ and $\lim_{\sigma_b\to\infty}\mathbf{E}\left[\boldsymbol{p}_i\right] = \bar{\boldsymbol{p}}$

□

**Proposition 9.** *The firm-level and economy-level markups are strictly increasing if the shock variance is large enough, and approach their linear asymptotes.*

*Proof.* The firm-level markup for firm $i$ is the quantity-weighted prices divided by quantity-weighted costs:

$$\begin{aligned} M_i^f &= \frac{\mathbf{E}[\tilde{\boldsymbol{q}}_i'\boldsymbol{p}_i]}{\mathbf{E}[\tilde{\boldsymbol{q}}_i'\boldsymbol{c}_i]} = \frac{\mathbf{E}[\tilde{\boldsymbol{q}}_i]'\mathbf{E}[\boldsymbol{p}] + \mathbf{tr}\left[\mathbf{Cov}(\boldsymbol{p}_i, \tilde{\boldsymbol{q}}_i)\right]}{\mathbf{E}[\tilde{\boldsymbol{q}}_i'\boldsymbol{c}_i]} \\ &= \frac{(\bar{\boldsymbol{p}}^M - \boldsymbol{c}_i)'\hat{\boldsymbol{H}}_i\bar{\boldsymbol{p}}^M + \mathbf{tr}\left(\left(\boldsymbol{I}_N + \frac{1}{\phi}\hat{\boldsymbol{H}}_i\right)^{-1}\mathbf{Var}(\boldsymbol{K}_i\boldsymbol{s}_i)\left(\boldsymbol{I}_N + \frac{1}{\phi}\hat{\boldsymbol{H}}_i\right)^{-1}\hat{\boldsymbol{H}}_i\right)}{(\bar{\boldsymbol{p}}^M - \boldsymbol{c}_i)'\hat{\boldsymbol{H}}_i\boldsymbol{c}_i} \end{aligned} \tag{179}$$

We further assume all the products have same shock $\sigma_b$ and signal variance, thus ensuring the same diagonal values of $\hat{\boldsymbol{H}}_i$. The $M_i^f$ can be simplified as

$$M_i^f = \frac{\sum_{j=1}^N(\bar{\boldsymbol{p}}_j^M - \boldsymbol{c}_{i,j})\bar{\boldsymbol{p}}_j^M + \sum_{j=1}^N\left(1 + \frac{1}{\phi}\hat{\boldsymbol{H}}_{i,j}\right)^{-2}\frac{\Sigma_{b,j}^2}{\Sigma_{b,j}+\Sigma_{\epsilon_{i,j}}}}{\sum_{j=1}^N(\bar{\boldsymbol{p}}_j^M - \boldsymbol{c}_{i,j})\boldsymbol{c}_{i,j}} \tag{180}$$

$M_i^f$ also admits an asymptote as

$$\alpha_i := \lim_{\sigma_b\to\infty}\frac{M_i^f}{\sigma_b} = \frac{N}{\sum_{j=1}^N(\bar{\boldsymbol{p}}_j - \boldsymbol{c}_{i,j})\boldsymbol{c}_{i,j}} \quad \text{and} \quad \gamma_i := \lim_{\sigma_b\to\infty}\left(M_i^f - \alpha_i\sigma_b\right) = \frac{\sum_{j=1}^N\left((\bar{\boldsymbol{p}}_j - \boldsymbol{c}_{i,j})\bar{\boldsymbol{p}}_j - \Sigma_{\epsilon_{i,j}}\right)}{\sum_{j=1}^N(\bar{\boldsymbol{p}}_j - \boldsymbol{c}_{i,j})\boldsymbol{c}_{i,j}} \tag{181}$$

The average firm-level markup $\overline{M}^f = (1/n_F)\sum_{i=1}^{n_F}M_i^f$ approaches $\sum_{i=1}^{n_F}\frac{\alpha_i}{n_F}\sigma_b + \sum_{i=1}^{n_F}\frac{\gamma_i}{n_F}$ in the long run. The economy-level markup is $M^m = \sum_{i=1}^{n_F}w^{\boldsymbol{H}_i}M_i^f$ with $w^{\boldsymbol{H}_i} = \frac{\mathbf{E}\left[\tilde{\boldsymbol{q}}_i'\boldsymbol{c}_i\right]}{\sum_{i=1}^{n_F}\mathbf{E}\left[\tilde{\boldsymbol{q}}_i'\boldsymbol{c}_i\right]}$. The weight $w^{\boldsymbol{H}_i}$ converges to $w_i$ as shock variance goes to infinity.

$$w_i := \lim_{\sigma_b\to\infty}w^{\boldsymbol{H}_i} = \frac{\sum_{j=1}^N(\bar{\boldsymbol{p}}_j - \boldsymbol{c}_{i,j})\boldsymbol{c}_{i,j}}{\sum_{i=1}^{n_F}\sum_{j=1}^N(\bar{\boldsymbol{p}}_j - \boldsymbol{c}_{i,j})\boldsymbol{c}_{i,j}} \Rightarrow M^m \text{ approaches } \sum_{i=1}^{n_F}w_i\alpha_i\sigma_b + \sum_{i=1}^{n_F}w_i\gamma_i \tag{182}$$

□

### C.3. Bertrand Competition

In our baseline model with Cournot competition, there are $n_F$ firms producing $n$ attributes and aggregating these attributes to $n$ goods. Each firm produce all attributes and all goods, were all goods are perfect substitutes.

Solving the model, we transform the profit maximization problem for goods into a profit maximization problem for attributes. The market-level inverse demand function for attributes is

$$\mathbf{p^M} = \mathbf{\bar{p}} - \frac{1}{\phi}\sum_{i=1}^{n_F}\mathbf{q_i} \tag{183}$$

where $\mathbf{p^M}$ is the market level price of attributes (an n-dimensional vector). For individual firm $i$, the price for attributes is $\mathbf{\tilde{p}_i} = \mathbf{p^M} + \mathbf{b_i}$ where $\mathbf{b_i}$ is the demand shock for each of the $n$ attributes, faced by firm $i$.

All attribute $j$ are produced by all firms, and given final goods are perfect substitutes, so are the attributes. Under Bertrand competition, when the firm chooses the price, the quantity demanded for the attribute $j$ is

$$\widetilde{q}_{ij} = \begin{cases} \phi(\bar{p}_j - \tilde{p}_{ij} + b_{ij}) & \text{if} \quad p^M_{ij} = \tilde{p}_{ij} - b_{ij} \quad \text{is the smallest} \\ 0 & \text{if} \quad p^M_{ij} = \tilde{p}_{ij} - b_{ij} \quad \text{isn't the smallest} \end{cases} \tag{184}$$

Formally, the profit maximization can be written as before

$$U_i = \mathbf{E}\left[\pi_i \mid \mathcal{I}_i\right] - \frac{\rho_i}{2}\mathbf{Var}\left[\pi_i \mid \mathcal{I}_i\right] - g\left(\chi_c, \tilde{c}_i\right) \tag{185}$$

$$= (p_i - c_i)\mathbf{E}\left[q_i \mid \mathcal{I}_i\right] - \frac{\rho_i}{2}\mathbf{Var}\left[q_i \mid \mathcal{I}_i\right](p_i - c_i)^2 - g\left(\chi_c, \tilde{c}_i\right). \tag{186}$$

Now, given the demand (184), the solution will involve a corner solution where, under Bertrand competition, the firm chooses the price for each attribute to undercut any competitor, resulting in the lowest cost firm taking the entire market at a price that is just below the cost of the second lowest cost firm.

## C.4. Choosing A Location in Product Space

In the previous problem, we introduced the idea of product attributes so that a piece of data might be informative about the demand of multiple products. But we held the attributes of each product fixed. In reality, firms can choose the type of product to produce. They choose attributes. We show that the insights of the previous analysis carry over, with one small change. Data will allow a firm to choose a product that has higher-markup attributes. This makes product markups more like firm markups in the original model.

Each firm produces a single product, or bundle of products, with attributes chosen by the firm. Then the firm chooses how many units of the product or product bundle to produce. Formally, firm $i \in \{1, 2, \ldots, n_F\}$ chooses an $n \times 1$ vector $a_i$ that describes their location in the product space, such that $\sum_j a_{ij} = 1$. As before, The $j$th entry of vector $a_i$ describes how much of attribte $i$ firm $i$'s good contains.

The rest of the model assumptions, including consumer demand and the nature of data are the same as before. Thus, the firm's production problem is

$$max_{a_i,q_i}\mathbf{E}\left[q_i\boldsymbol{a}_i'\left(\mathbf{\tilde{p}} - \boldsymbol{c}_i\right)|\mathcal{I}_i\right] - \frac{\rho_i}{2}\mathbf{Var}\left[q_i\boldsymbol{a}_i'\left(\mathbf{\tilde{p}} - \boldsymbol{c}_i\right)|\mathcal{I}_i\right] - g(\chi_c, \boldsymbol{c}_i), \tag{187}$$

s.t. $\sum_j a_{ij} = 1$.

Just like the previous problem, prior to observing any of their data, each firm also chooses their cost vector $\boldsymbol{c}_i$. Since the data realizations are unknown in this ex-ante invetment stage, the objective is the unconditional expectation of the utility in 1

$$max_{c_i}\mathbf{E}\left[\;\mathbf{E}\left[q_i\boldsymbol{a}_i'\left(\tilde{\mathbf{p}}-\boldsymbol{c}_i\right)|\mathcal{I}_i\right]-\frac{\rho_i}{2}\mathbf{Var}\left[q_i\boldsymbol{a}_i'\left(\tilde{\mathbf{p}}-\boldsymbol{c}_i\right)|\mathcal{I}_i\right]\right]-g(\chi_c,\boldsymbol{c}_i). \tag{188}$$

SOLUTION Firm $i$'s optimal production from the first order condition looks identical to the one before, except that now it is the the product of quantity and attributes that achieves this solution.

$$q_i\boldsymbol{a}_i=\left(\rho_i\mathbf{Var}\left[\boldsymbol{p}_i|\mathcal{I}_i\right]+\frac{\partial\mathbf{E}\left[\boldsymbol{p}_j|\mathcal{I}_j\right]}{\partial\boldsymbol{q}_i}\right)^{-1}\left(\mathbf{E}\left[\boldsymbol{p}_i|\mathcal{I}_i\right]-\boldsymbol{c}_i\right) \tag{189}$$

This tells us that the solution to the problem is exactly the same. In the previous problem, a firm choice produce any quantity of attributes it wanted with the right mix of products. In this problem, the firm can also choose any quantity of attributes it likes with the right quantity and product location.

The only thing that changes in this formulation of the problem is the interpretation of what constitutes a product. In the previous problem, a product had a fixed set of attributes. In this problem, a product is a fraction of the total output of the firm. Therefore the product markup here is more like what the firm markup was before. In other words, data affects the composition of a product now. Firms with data choose to produce products with higher-value attributes. This is a force that can make markups flat or increasing in data.

**Proposition 10.** *When firms choose attributes, product markups will increase in data, for a low enough risk aversion $\rho_i$.*

*Proof.* Comparing first-order condition (189) with original optimal choice (37), we could solve this extension model by substituting $\tilde{\boldsymbol{q}}_i$ in (37) with $q_i\boldsymbol{a}_i$ and further extend existing propositions for $q_i$ and $\boldsymbol{a}_i$ by one-to-one mapping

$$q_i=\sum_{j=1}^{N}\tilde{\boldsymbol{q}}_{i,j}\quad\text{and}\quad\boldsymbol{a}_i=\frac{\tilde{\boldsymbol{q}}_i}{\sum_{j=1}^{N}\tilde{\boldsymbol{q}}_{i,j}} \tag{190}$$

Since firms optimize their choices in product space, the product markup is then the weighted average of attributes markups

$$M_i^p:=\frac{\mathbb{E}[\boldsymbol{a}_i'\tilde{\boldsymbol{p}}_i]}{\mathbb{E}[\boldsymbol{a}_i'\tilde{\boldsymbol{c}}_i]}=\frac{\mathbb{E}[q_i\boldsymbol{a}_i'\tilde{\boldsymbol{p}}_i]}{\mathbb{E}[q_i\boldsymbol{a}_i'\tilde{\boldsymbol{c}}_i]}=\frac{\mathbb{E}[\tilde{\boldsymbol{q}}_i'\tilde{\boldsymbol{p}}_i]}{\mathbb{E}[\tilde{\boldsymbol{q}}_i'\tilde{\boldsymbol{c}}_i]}=M_i^f \tag{191}$$

This tells us that the product markups is equivalent to the firm-level markup of the original model. We already know that data boost firm-level markup with small risk aversion $\rho_i$ (Proposition 3), thus the product markup will increase in data for a low enough risk aversion $\rho_i$.

This proof held marginal costs $\tilde{c}$ fixed, which corresponds to infinitely high marginal cost of adjusting $c$: $\chi_c\to\infty$. If we assume $\chi_c$ is sufficiently high, by continuity, the inequality will still hold.

□

This result shows why this extension is helpful for the model to match data showing flat or increasing product markups. The fact that markups had to be declining in the previous model was an artifact of the assumption that product characteristics are fixed. While that simplified the model and allowed us to focus on explaining the many other forces at play, the richer model paints a more realistic and data-consistent picture of how data, competition and markups interact.