Export-Platform FDI: Cannibalization or Complementarity?

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The dominant branch of the economics literature on multinational firms (MNEs) treats their final-good production location choices as substitutes. In these models, global firms face a 'proximity-concentration tradeoff' in which production location decisions depend on the cost of production in each country, the size of trade costs between production and consumption locations, and the desire to concentrate production for many markets in a single location to conserve on fixed costs of production (Helpman et al., 2004; Tintelnot, 2017). In these settings, improvements in locationspecific productivity generate cannibalization effects that reduce the profitability of operating affiliates in other countries.

Recent empirical work, however, suggests that MNEs' plant locations may not always be substitutes. Garetto et al. (2019) find that US MNEs' affiliate sales in certain countries are unaffected by their affiliate activities in other countries. Using newly merged data on US firms' trade and multinational activity by country, Antràs et al. (2023) show that US MNEs are not only more likely to export to countries in which they have affiliates, but also to other countries that are proximate to those affiliates, a fact that is hard to square with canonical 'export-platform' FDI models.

This paper explores conditions under which a model of export-platform FDI generates cannibalization effects. We first develop a baseline model similar to Tintelnot (2017) in which final goods are produced only with labor and there are no fixed costs of exporting. Perhaps surprisingly, this model does *not* necessarily generate cannibalization effects. The key condition that

determines whether an MNE's plant sales across locations are substitutes or complements depends on whether the goods produced by the MNE's plants are more or less substitutable than the goods produced by other firms. While past work assumes, with few exceptions, a common elasticity of substitution within and across firms' products, such a knife-edge case need not hold empirically. For example, Whirlpool's washing machines are close substitutes to Mavtags washing machines but are likely complements to its own dryers, whereas Ford's Explorer seems to be a closer substitute for its Escape than any model produced by a rival company.¹

Having developed this baseline model, we introduce destination-specific fixed costs of exporting that are incurred at the *firm level*, and show that this extension expands the range of parameter values for which the model delivers complementarity across MNEs' production locations. Finally, we introduce tradable intermediate inputs and show that whenever global sourcing entails firm-by-country-specific fixed costs of sourcing shared across all the MNE's plants, the range of parameter values for which assembly location decisions are complements is again expanded.

I. A Model Export-Platform FDI

We begin by developing a simple multicountry model of export-platform FDI similar to Tintelnot (2017).

A. Environment and Preferences

We consider a world in which individuals in J countries consume differentiated manufactured goods produced by heterogeneous

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¹For more details, see www.hawkfordstcharles.com/research-escape-vs-explorer. The Explorer is produced at a plant in Illinois, while the Escape is made in Kentucky.

firms. Although each firm produces a single good, we assume that this firm's good is differentiated based on its production country and that the same firm may produce in multiple countries.

We index firms by φ and varieties within firms by k. Given our Armington assumption, k also corresponds to an index for production locations. We assume a nested CES structure in which the degree of substitutability $\sigma > 1$ across varieties produced by different firms may differ from the degree of substitutability $\varepsilon > 1$ across varieties produced by the same firm.² More formally, preferences are represented by

(1)
$$U_{Mi} = \left(\int_{\varphi \in \Omega_i} \mathbf{q}_i(\varphi)^{\frac{\sigma-1}{\sigma}} d\varphi \right)^{\frac{\sigma}{\sigma-1}},$$

where Ω_i is the endogenous measure of firms selling differentiated goods in country i, and where the firm-specific composite $\mathbf{q}_i(\varphi)$ is

$$\boldsymbol{q}_{i}\left(\boldsymbol{\varphi}\right) = \left(\sum_{k \in \mathcal{K}\left(\boldsymbol{\varphi}\right)} q_{i}\left(\boldsymbol{\varphi}, k\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

The set $\mathcal{K}(\varphi) \subseteq J$ includes the assembly locations from which firm φ sells varieties in country i.

It is straightforward to show that these preferences imply that consumers in country i spend an amount

(2)
$$S_{ki}(\varphi) = \left(\frac{p_i(\varphi, k)}{\boldsymbol{p}_i(\varphi)}\right)^{1-\varepsilon} \times \left(\frac{\boldsymbol{p}_i(\varphi)}{P_i}\right)^{1-\sigma} E_i$$

of their income on variety k produced by firm φ . In this expression, $p_i(\varphi, k)$ is the price charged for that variety k, $\boldsymbol{p}_i(\varphi)$ is the overall price index for varieties sold by

firm φ , and P_i is the economy-wide ideal price index in country i (see Appendix A for formal definitions). E_i is total spending on manufactured goods in country $i \in J$.

The sales equation (2) illustrates that whether spending for individual varieties produced by firm φ increases or decreases with the price of other varieties produced by the same firm depends on the relative size of ε and σ . When varieties are more substitutable within firms than across firms $(\varepsilon > \sigma)$, the demand for a given variety k is decreasing in $p_i(\varphi)$, and is hence reduced by lower prices for varieties other than variety k, thus capturing a demand cannibalization effect. Conversely, when varieties are more substitutable across firms than within firms ($\varepsilon < \sigma$), a lower firm-level price index $p_i(\varphi)$ disproportionately redirects demand towards all of firm φ 's varieties, thus creating a form of demand complementarity across a firm's varieties.

B. Manufacturing Production

Manufactured varieties are produced under increasing returns to scale and monopolistic competition. The variable φ used to index final-good firms also corresponds to their 'core' productivity, which firms only learn after incurring a fixed cost of entry.

After paying this fixed cost, each firm acquires blueprints to produce varieties of a final good. Although the firm could produce its varieties anywhere in the world, we assume that opening an assembly plant in a given country $k \in J$ entails a fixed overhead cost equal to f_k^a units of labor in country k. In equilibrium, firms therefore open a limited number of assembly plants (possibly a single one). We denote the optimal set of countries $k \in J$ for which firm φ has paid the associated fixed cost of assembly by $\mathcal{K}(\varphi) \subseteq J$, and refer to it as the firm's global assembly strategy.

For the time being, we assume that production of final-good varieties requires only local labor. The cost at which firm φ can manufacture in each location k is thus shaped by its core productivity, local wages in k, and a location-specific productivity parameter Z_k^a . Shipping final goods from

²Our Armington assumption can be micro-founded using an isomorphic set of equations that arises from a Ricardian model with production efficiency differences (see Antràs et al., 2022; Arkolakis et al., 2023). This implies that the parameter ε can also be interpreted as governing the heterogeneity in technology across plants for producing the same good.

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country k to country i also entails variable (iceberg) trade costs τ_{ki}^a . For now, we abstract from fixed costs of exporting.

As in Antràs et al. (2023), we assume that total manufacturing spending E_i and wages w_i in all countries are independent of the equilibrium in the manufacturing sector.

C. Interdependencies in the Intensive Margin

The model delivers a simple, closed-form solution for sales of an assembly plant in k to each market i (see Appendix A):

(3)
$$S_{ki}(\varphi) = \kappa \varphi^{\sigma-1} \xi_k^a (\tau_{ki}^a)^{1-\varepsilon} \times (\Psi_i(\varphi))^{\frac{\sigma-\varepsilon}{\varepsilon-1}} P_i^{\sigma-1} E_i,$$

where κ is a constant, $\xi_k^a \equiv (w_k/Z_k^a)^{1-\varepsilon}$ captures country k's assembly potential, and Ψ_i is given by

$$(4) \qquad \Psi_{i}\left(\varphi\right) \equiv \sum_{k' \in J} \mathcal{I}_{k'}^{a} \cdot \xi_{k'}^{a} \left(\tau_{k'i}^{a}\right)^{1-\varepsilon},$$

with $\mathcal{I}_{k'}^a$ taking a value of 1 when $k' \in \mathcal{K}(\varphi)$, and a value of zero otherwise.

Holding market demand $P_i^{\sigma-1}E_i$ and the firm's global assembly strategy constant, equation (3) indicates that an increase in the assembly potential ξ_k^a of country k increases sales of plants based in k to all countries $i \in J$. Reductions in the associated bilateral trade costs τ_{ki}^a generate analogous effects. These changes improve efficiency in plant k, which increases its sales.

Whether changes in ξ_k^a or τ_k^a generate positive or negative effects on the sales to country i of plants based in other countries $k' \neq k$ is less clear-cut and depends on the relative size of σ and ε . When varieties are more substitutable within firms than across firms $(\varepsilon > \sigma)$, cannibalization effects dominate and the sales of a particular plant k' of firm φ are decreased by improvements in the efficiency of other plants k producing firm φ varieties also sold in market i. Conversely, when varieties are more substitutable across firms than within firms ($\varepsilon < \sigma$), the opposite is true. In such a case, demand complementarities are dominant, and improvements in the efficiency of other plants k increase plant k''s sales in i.

D. Interdependencies in the Extensive Margin

We now analyze the optimal set of countries in which a firm locates final-good assembly plants (i.e., its global assembly strategy $\mathcal{K}(\varphi) \subseteq J$). Starting from equation (3), using the optimal constant markup rule (see Appendix A), and aggregating across export platforms and their destination markets, firm profits (net of the initial entry cost) can be expressed as:

5)
$$\pi\left(\varphi\right) = \kappa_{\pi}\varphi^{\sigma-1} \sum_{i \in J} \left(\Psi_{i}\left(\varphi\right)\right)^{\frac{\sigma-1}{\varepsilon-1}} P_{i}^{\sigma-1} E_{i}$$

$$-\sum_{k \in J} \mathcal{I}_{k}^{a} \cdot w_{k} f_{k}^{a},$$

where κ_{π} is a constant and where $\Psi_{i}(\varphi)$ is defined in (4). Solving for the set $\mathcal{K}(\varphi)$ that maximizes equation (5) is a combinatorial problem, but regardless of its specific solution, we can characterize whether the various extensive margins of the firm are complements and substitutes.

Intuitively, when varieties are more substitutable within firms than across firm, the profitability of setting up an export platform in country k is reduced by the existence of assembly plants in other locations $k' \neq k$ because these other assembly plants cannibalize on plant k's sales (as shown in equation (3)). Similarly, setting up a new plant in k is less desirable, because this new plant would cannibalize sales from existing plants in other locations. These are canonical features of export-platform FDI models, but equation (5) demonstrates that when varieties are less substitutable within firms than across firms, the opposite is true, and extensive margin assembly decisions are complements.

Formally, we consider an increase in the assembly potential of a given plant k from ξ_k^a to $\hat{\xi}_k^a > \xi_k^a$. Denote by $\mathcal{I}^a = (\mathcal{I}_1^a, \dots, \mathcal{I}_J^a)$ and $\hat{\mathcal{I}}^a = (\hat{\mathcal{I}}_1^a, \dots, \hat{\mathcal{I}}_J^a)$ the optimal assembly decisions under ξ_k^a and $\hat{\xi}_k^a$, respectively. Denote by X_{-k} the vector X excluding element k. For vectors X and Y, we say that $X \geq Y$ if $X_i \geq Y_i$ for all i, and X > Y if $X \geq Y$ and $X_i > Y_i$ for some j. Given this

notation, we prove in Appendix A that:

PROPOSITION 1: Holding constant the market demand level $E_i P_i^{\sigma-1}$, an increase in the assembly potential of a given plant k from ξ_k^a to $\hat{\xi}_k^a > \xi_k^a$ leads to $\hat{\mathcal{I}}^a \geq \mathcal{I}^a$ whenever $\varepsilon \leq \sigma$, but it would not lead to $\hat{\mathcal{I}}_{-k}^a > \mathcal{I}_{-k}^a$ whenever $\varepsilon > \sigma$ and \mathcal{I}^a is a unique solution.

In words, whenever $\varepsilon > \sigma$, this baseline model cannot possibly feature complementarities in the extensive margin of global assembly.

II. Export-Platform FDI Model with Firm-Level Export Costs

We now assume that firms need to incur fixed marketing costs of f_i^x units of labor in country i to sell its varieties in country i. We use the superscript x to denote these fixed costs. We assume that these marketing costs are incurred at the firm-rather than the plant-level. In other words, this country-specific fixed cost allows the firm to sell in country i from all its assembly plants. We view this assumption as realistic in light of the fact that multinational firms often centralize their sales and marketing decisions to a specialized division. We denote the optimal set of countries $i \in J$ for which a firm with productivity φ has paid the associated fixed cost of marketing by $\Upsilon(\varphi) \subseteq J$, and refer to it as the firm's global marketing strategy.

It should be clear that the introduction of firm-level fixed export costs has no bearing on equation (3) capturing sales of an assembly plant in k to each market i, except for the fact that the equation now only applies to destination markets i in the firm's global marketing strategy (i.e., $i \in \Upsilon(\varphi)$). Holding market demand $E_i P_i^{\sigma-1}$ and the firm's extensive-margin strategies constant, whether an increase in the assembly potential ξ_k^a of country k increases or decreases sales of plants based in $k' \neq k$ continues to depend only on the relative size of σ and ε , with $\varepsilon > \sigma$ leading to complementarity.

Profits net of entry costs are also given by an expression almost identical to that in equation (5), namely:

(6)

$$\pi\left(\varphi\right) = \kappa_{\pi}\varphi^{\sigma-1} \sum_{i \in J} \mathcal{I}_{i}^{x} \left(\Psi_{i}\left(\varphi\right)\right)^{\frac{\sigma-1}{\sigma-1}} P_{i}^{\sigma-1} E_{i}$$

$$-\sum_{i \in J} \mathcal{I}_{i}^{x} \cdot w_{i} f_{i}^{x} - \sum_{k \in J} \mathcal{I}_{k}^{a} \cdot w_{k} f_{k}^{a},$$

Despite these strong similarities with our starting model of export-platform FDI, the presence of firm-level fixed costs of exporting carries important implications for the nature of the interdependencies across the assembly plants of a firm. More specifically, denote by \mathcal{I}^x and $\hat{\mathcal{I}}^x$ the optimal exporting decisions under ξ_k^a and $\hat{\xi}_k^a$. We show in Appendix A that:

PROPOSITION 2: With firm-level fixed costs of exporting, holding constant the market demand level $E_i P_i^{\sigma-1}$, an increase in the assembly potential of a given plant k from ξ_k^a to $\hat{\xi}_k^a > \xi_k^a$ leads to $\hat{\mathcal{I}}^a \geq \mathcal{I}^a$ and $\hat{\mathcal{I}}^x \geq \mathcal{I}^x$ whenever $\varepsilon \leq \sigma$, and it may lead to $\hat{\mathcal{I}}_{-k}^a > \mathcal{I}_{-k}^a$ and $\hat{\mathcal{I}}^x > \mathcal{I}^x$ even when $\varepsilon > \sigma$.

This result implies that the model generates complementarities across assembly locations for a wider range of parameter values than our baseline model without fixed costs of exporting. In fact, complementarities may arise even when varieties are more substitutable within firms than across firms.

The intuition for this result is as follows. An increase in ξ_k^a necessarily increases the profits associated with sales emanating from that plant k. This increase in profitability may lead firm φ to activate export destinations that were not profitable before the increase in ξ_k^a . Crucially, because plants in other potential assembly locations $k' \neq k$ would benefit from the activation of such an export destination, this induced change in the firm-level extensive margin of exports may well increase the profitability of activating these other potential assembly locations k', especially when cannibalization effects are small (i.e., when $\varepsilon - \sigma$ is small).

As we show in Appendix A, the fact that the fixed costs of exporting are incurred at the firm-level is crucial for these VOL. VOL NO. ISSUE EXPORT-PLATFORM FDI 5

results: if these fixed costs were incurred at the plant-level, we would revert back to the result similar to the one in Proposition 1, and complementarities could not arise when $\varepsilon > \sigma$.

III. Export-Platform FDI Model with Firm-Level Sourcing Costs

We finally relax the assumption that final goods are only produced with labor and introduce tradable intermediate inputs. Following our approach for preferences, we assume that inputs sourced from different countries are imperfect substitutes, with a constant elasticity of substitution ρ . Intermediates are produced worldwide by a competitive fringe of suppliers that sells its products at marginal cost. All intermediates are produced with labor under a linear technology delivering Z_j^s units of output per unit of labor. Shipping intermediates from country j to country k entails ice
berg trade costs τ_{jk}^s . As a result, the cost at which firms producing in k can procure inputs from country j is given by $\tau_{ik}^s w_j/Z_i^s$.

A firm must incur a country-specific fixed cost $w_j f_j^s$ to source inputs from a particular country j. Although this assumption is similar to Antràs et al. (2017), a crucial distinction here is that the fixed cost grants all of the firm's assembly plants $k \in \mathcal{K}(\varphi)$ access to inputs from that country. We denote the set of countries for which firm φ has paid the fixed costs of sourcing by $\mathcal{J}(\varphi) \subseteq J$ and refer to it as the firm's global sourcing strategy.

The overall marginal cost for firm φ to produce units of the final-good variety in country k is given by

$$c\left(\varphi,k\right) = \frac{\left(\xi_{k}^{a}\right)^{\frac{1-\alpha}{1-\varepsilon}}\left(\sum_{j\in\mathcal{J}\left(\varphi\right)}\xi_{j}^{s}\left(\tau_{jk}^{s}\right)^{1-\rho}\right)^{\frac{\alpha}{1-\rho}}}{\varphi},$$

where $1 - \alpha$ is the labor share in final-good production, $\xi_k^a \equiv \left(w_k/Z_k^a\right)^{1-\varepsilon}$ is country k's assembly potential, and $\xi_j^s \equiv \left(w_j/Z_j^s\right)^{1-\rho}$ is country j's sourcing potential.

Invoking constant-markup pricing, one can show that the sales of an assembly plant in k to each market i (we ignore fixed costs

of exporting in this section) are given by

(7)
$$S_{ki}(\varphi) = \kappa \varphi^{\sigma-1} \left(\xi_k^a\right)^{1-\alpha} \left(\tau_{ki}^a\right)^{1-\varepsilon} \times \left(\Theta_k(\varphi)\right)^{\frac{\alpha(\varepsilon-1)}{\rho-1}} \left(\Lambda_i(\varphi)\right)^{\frac{\sigma-\varepsilon}{\varepsilon-1}} E_i P_i^{\sigma-1},$$

where κ is a constant and $\xi_k^a \equiv (w_k/Z_k^a)^{1-\varepsilon}$ is again country k's assembly potential. The term $\Theta_k(\varphi)$ is plant k's sourcing capability (see Antràs et al., 2017)), and is given by

$$\Theta_k\left(\varphi\right) \equiv \sum_{i \in J} \mathcal{I}_j^s \cdot \xi_j^s \left(\tau_{jk}^s\right)^{1-\rho},$$

where \mathcal{I}_{j}^{s} is an indicator function that takes a value of 1 if the firm activate country j as a source of inputs, and 0 otherwise. Finally, the term $\Lambda_{i}(\varphi)$ is

$$\Lambda_{i}\left(\varphi\right) \equiv \sum_{k' \in J} \mathcal{I}_{k'}^{a} \cdot \left(\xi_{k'}^{a}\right)^{1-\alpha} \times \left(\tau_{k'i}^{a}\right)^{1-\varepsilon} \left(\Theta_{k'}\left(\varphi\right)\right)^{\frac{\alpha(\varepsilon-1)}{\rho-1}}.$$

The empirical complementarities in global sourcing documented in Antràs et al. (2017) lead us to impose:

Assumption 1:
$$\alpha(\varepsilon - 1) \ge \rho - 1$$
.

Equation (7) is significantly more involved than its counterpart (3) in our baseline model, but holding market demand $E_i P_i^{\sigma-1}$ and the firm's extensive-margin strategies constant, whether an increase in the assembly potential ξ_k^a of country k increases or decreases sales of plants based in $k' \neq k$ continues to be shaped solely by the relative size of σ and ε , with $\varepsilon > \sigma$ leading to cannibalization and $\varepsilon < \sigma$ leading to complementarity.

Profits are now given by

$$\pi\left(\varphi\right) = \kappa_{\pi} \varphi^{\sigma-1} \sum_{i \in J} E_{i} P_{i}^{\sigma-1} \times \left(\Lambda_{i}\left(\varphi\right)\right)^{\frac{\sigma-1}{\varepsilon-1}}$$
$$- \sum_{k \in J} \mathcal{I}_{k}^{a} \cdot w_{k} f_{k}^{a} - \sum_{j \in J} \mathcal{I}_{j}^{s} \cdot w_{j} f_{j}^{s},$$

for some constant κ_{π} .

Under Assumption 1, whenever $\varepsilon \leq \sigma$, profits continue to feature increasing differences in $(\mathcal{I}_k^a, \mathcal{I}_{k'}^a)$ for $k, k' \in \{1, ..., J\}$ and

 $k \neq k'$. Denote by \mathcal{I}^s and $\hat{\mathcal{I}}^s$ the optimal sourcing decisions under ξ_k^a and $\hat{\xi}_k^a$. In Appendix A, we show that:

PROPOSITION 3: With firm-level fixed costs of sourcing, under Assumption 1, and holding constant the market demand level $E_i P_i^{\sigma-1}$, an increase in the assembly potential of a given plant k from ξ_k^a to $\hat{\xi}_k^a > \xi_k^a$ leads to $\hat{\mathcal{I}}^a \geq \mathcal{I}^a$ and $\hat{\mathcal{I}}^s \geq \mathcal{I}^s$ whenever $\varepsilon \leq \sigma$, and it may lead to $\hat{\mathcal{I}}_{-k}^a > \mathcal{I}_{-k}^a$ and $\hat{\mathcal{I}}^s > \mathcal{I}^s$ even when $\varepsilon > \sigma$.

As in the case of firm-level fixed export costs, the presence of firm-level fixed costs of sourcing again widens the range of parameter values for which assembly locations are complements. Intuitively, an increase in ξ_k^a increases the profits associated with sales emanating from plant k, which in turn increases the marginal benefit of investing in a larger sourcing capability $\Theta_k(\varphi)$ for that plant. But because all of the firm's plants benefit from the activation of input sources, the sourcing capability $\Theta_{k'}(\varphi)$ of other potential plants tends to be enhanced, and necessarily so under Assumption 1. This larger sourcing capability may in turn increase the profitability of activating these other potential assembly locations k', especially when cannibalization effects are small (i.e., when $\varepsilon - \sigma$ is small).

Importantly, if fixed costs of sourcing were at the plant- rather than the firm-level, this complementarity force would disappear, and the result similar to the one in Proposition 1 would apply again (see Appendix A).

IV. Conclusion: Additional Sources of Complementarity

We identify two features – firm-level fixed costs of exporting and of sourcing – that increase the range of parameter values for which global assembly strategies are complements. These enhanced complementarities are mediated by scale effects,

so other mechanisms that generate larger profit changes for larger firms are likely to produce similar results. Three potential candidates come to mind: first, firm-level investments in R&D that enhance firm productivity; second, alternative demand systems that generate lower demand elasticities (and thus larger markups) for large firms; and third, oligopolistic settings in which markups increase in a firm's market share.

REFERENCES

- Antràs, Pol, Evgenii Fadeev, Teresa C. Fort, and Felix Tintelnot, "Global Sourcing and Multinational Activity: A Unified Approach," Working Paper 30450, NBER 2022.
- -, -, -, and -, "Exporting, Global Sourcing, and Multinational Activity: Theory and Evidence from the United States," 2023.
- Antràs, Pol, Teresa C. Fort, and Felix Tintelnot, "The Margins of Global Sourcing: Theory and Evidence from US Firms," *American Economic Review*, 2017, 107 (9), 2514–2564.
- Arkolakis, Costas, Fabian Eckert, and Rowan Shi, "Combinatorial Discrete Choice," Working Paper, 2023.
- Garetto, Stefania, Lindsay Oldenski, and Natalia Ramondo, "Multinational Expansion in Time and Space," *NBER* Working Paper 25804, 2019.
- Helpman, Elhanan, Marc Melitz, and Stephen Yeaple, "Export Versus FDI with Heterogeneous Firms," American Economic Review, 2004, 94 (1), 300–316.
- **Tintelnot, Felix**, "Global Production with Export Platforms," *Quarterly Journal of Economics*, 2017, 132 (1), 157–209.

³Profits also feature increasing differences in $\left(\mathcal{I}_{j}^{s},\mathcal{I}_{j'}^{s}\right)$ for $j,j'\in\{1,...,J\}$ and $j\neq j'$ and in $\left(\mathcal{I}_{k}^{a},\mathcal{I}_{j}^{s}\right)$ for $k,j\in\{1,...,J\}$