How do Tom and Jerry Play? A Simple Application of Convex Analysis in Hide-and-Seek Games

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Abstract

We propose a simultaneous-move hide-and-seek game, where one player wins by matching the other player, while the other player wins by mismatching in a continuous space \( X \) in Euclidean space.

- A complete characterization of Type I Nash Equilibrium where the seeker plays a pure strategy, showing that the center of mass of the hider’s strategy coincides with the seeker's strategy at the center of the minimal cover ball.
- A characterization Type II Nash Equilibrium where the seeker plays a non-pure strategy, showing that the shape of \( X \) matters and the seeker will only allocate the probability weights along a straight line.
- Discussions of results under alternative settings.

These results can be applied to a large number of scenarios, characterizing the behavior of two players in a zero-sum game, where one player aims to maximize the distance between them, while the other aims to minimize it.

Introduction

Hide-and-seek is a well-known game, where one player aims to win by matching the other’s decision, while the other aims to win by mismatching. Typical hide-and-seek games take place every day in the real world:

- Animal society. In a jungle, predators hope to catch their prey, while prey always struggle to move away from predators’ territories.
- City management. Police officers hope to catch criminals, while criminals hope to stay as far away as possible from police officers.
- Race between innovation and imitation. Innovators hope to develop new methods, ideas or products, while imitators hope to mimic them.

Games in a space have attracted many game theorists for over a century.

- Hotelling model (Hotelling, 1929) studies how location affects duopoly competition, proposing early concepts linking games with space.
- Von Neumann (1953) studies the 2-dimension zero-sum hide-and-seek game with 2 players.
- Petrosjan (1993) discusses the hide-and-seek problem briefly based on triangles.
- Other different branches such as 3-player matching pennies games (Jordan, 1993; McCabe et al., 2000; Cao & Yang, 2014; Cao et al., 2019; among many others) or experiments (Crawford & Iriberri, 2007; among many others).

Model

Suppose there are two players, A (seeker) and B (hider). The territory is denoted as a compact convex set \( X \subseteq \mathbb{R}^n \). Player i’s pure strategy is a point \( x_i \in X \). Player i’s mixed strategy is a probability measure \( \sigma_i \in \Delta (X) \). Assume a \( 2\)-norm distance metric. The expected utility functions of the seeker A and hider B are denoted as

\[
U_A(\sigma_A, \sigma_B) = \int_{x \in X} -\|x - x_i\|_2 \, d\sigma_A \, d\sigma_B
\]

\[
U_B(\sigma_A, \sigma_B) = \int_{x \in X} -\|x - x_i\|_2 \, d\sigma_B \, d\sigma_A
\]

A strategy profile \((\sigma_A^*, \sigma_B^*)\) is a mixed Nash equilibrium if, and only if, for any mixed strategy \( \sigma_i \in \Delta (X) \), we have

\[
U_i(\sigma_i^*, \sigma_{-i}^*) \geq U_i(\sigma_i, \sigma_{-i})
\]

Definition (Minimal cover ball)

The ball \( B(x^*, r^-) \) is a minimal cover ball of a compact convex set \( X \subseteq \mathbb{R}^n \), if \( X \subseteq B(x^*, r^-) \) and for any ball \( B(x, r) \) with \( X \subseteq B(x, r) \), we have \( r^- \leq r \).

Table 1.

<table>
<thead>
<tr>
<th>Type I</th>
<th>Type II</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>continuum</td>
</tr>
<tr>
<td>✓ (acute triangle)</td>
<td>✓ (closed interval)</td>
</tr>
<tr>
<td>✗ (box)</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. The number of Type I and Type II Nash Equilibria. ✓ means a possible combination and ✗ means an impossible one. For each possible combination, an example where \( X \subseteq \mathbb{R}^2 \) or \( \mathbb{R}^3 \) is provided in the bracket.

Discussion

- Mathematical properties of minimal cover ball existance & uniqueness
- A convex optimization problem about the minimal cover ball
- Alternative settings
- When \( X \) is no longer a compact convex set in Euclidean space
- When \( X \) is a ball surface with the cosine distance metric

Conclusions

Many social, economic, political and military interactions between two parties with conflict of interest share the feature of a game between a distance-maximizing hider and a distance-minimizing seeker. In this paper, we formally characterize the Nash Equilibrium of a simultaneous-move version of such a hide-and-seek game with a commonly shared compact strategy space \( X \). Alongside this direction, many questions (e.g., the characterization of Nash Equilibrium under alternative settings) still remain open, which we leave for further exploration.

Key References


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