Tail risk and asset prices in the short-term*

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Abstract

We combine high-frequency stock returns with risk-neutralization to extract the daily common component of tail risks perceived by investors in the cross-section of firms. Our tail risk measure significantly predicts the equity premium and variance risk premium at short-horizons. Furthermore, a long-short portfolio built by sorting stocks on their recent exposure to tail risk generates abnormal returns with respect to standard factor models. Incorporating investors' preferences via risk-neutralization is fundamental to our findings: the predictive power of the physical tail risk is weaker and generally subsumed by its risk-neutral counterpart.

Keywords: Left tail risk, return predictability, factor models, risk-neutralization, high-frequency data.

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1 Introduction

Left tail risk is a pervasive feature of financial markets. As such, a large body of work has investigated its role in determining asset prices. Taken together, the empirical evidence indicates that compensation required by investors for bearing tail risk is fundamental to explain aggregate market risk premia and the cross-section of stock returns at relatively low frequencies (monthly or longer).[1] This evidence is based on a number of different tail measures. In particular, information can be extracted either from stock prices (e.g., Bali et al., 2009; Kelly and Jiang, 2014), reflecting risk under the physical or statistical measure under which prices are observed, or from option prices (e.g., Andersen et al., 2015; Bollerslev et al., 2015), capturing tail risk under the risk-neutral measure incorporating investors' preferences.

In this paper, we propose a new tail measure available at a daily frequency, which allows us to investigate the short-term effects of tail risk on asset prices. We first estimate the common tail risk component of a large cross-section of intra-day stock returns on day t, $\lambda_t^{\mathbb{P}}$, using the Hill (1975) power law estimator. This essentially adapts the tail index by Kelly and Jiang (2014) to a high-frequency environment. Then, we introduce a novel version of the Hill estimator, $\lambda_t^{\mathbb{Q}}$, that relies on risk-neutralized returns. More specifically, we apply a nonparametric adjustment to the cross-section of stock returns on day t where "bad" states of nature, represented by states of high marginal utility, are overweighted to reflect investors' compensation for risk. The dynamics of the physical and risk-neutral Hill estimators differ substantially as compensation for risk varies over time.

Our approach overcomes two main challenges. First, extreme events are infrequently observed by definition. This limits the information available from the time series of a single asset such as the market index. Second, option maturities are relatively long compared to intra-daily events, which makes it difficult to measure the tail risk specific to day t using option prices.[2] By using high-frequency data on one of the largest cross-

[1] The risk of extreme left tail shocks to fundamentals has also been an important ingredient in equilibrium models to help rationalize asset pricing stylized facts that are puzzling from the perspective of traditional macro-finance models (Barro, 2006; Gabaix, 2012; Wachter, 2013).

[2] Zero days-to-expiration (0DTE) option contracts are an exception (see, e.g, Bandi et al., 2023). However, as of now, the sample for which these options are available at a daily frequency (from May

sections of stock returns available, we are able to extract information about the level of tail risk at day t from the individual extreme events experienced by different stocks. Furthermore, our risk-neutralization allows to obtain a tail measure incorporating the economic valuation of tail risks by investors, which otherwise would only be possible using option prices (see, e.g., Aït-Sahalia and Lo, 2000).

Our empirical analysis is conducted considering each of our tail risk measures ($\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$) in order to assess the information content of investors' economic valuation of tail risk. Interestingly, both tail measures are weakly correlated with volatility-based measures. This is because the Hill estimator captures the thickness of the left tail after taking into account the effect of volatility. Moreover, $\lambda_t^{\mathbb{Q}}$ is always greater than $\lambda_t^{\mathbb{P}}$, which reflects the fact that investors are averse to downside risk. However, this difference varies over time and is particularly large during financial crises. In other words, investors are especially afraid of extreme negative events during periods of market distress.

We start by examining the short-term relation between the tail measures and the equity premium with one-day, one-week and one-month ahead daily predictive regressions. We find that the risk-neutral tail risk positively predicts excess market returns, with statistical significance for all horizons considered.[3] This is consistent with the idea that investors are averse to tail risk, such that they require higher returns to hold the market when tail risk increases. In contrast, $\lambda_t^{\mathbb{P}}$ only has predictive power for the one-month horizon. Even so, when both measures are included, $\lambda_t^{\mathbb{Q}}$ remains a significant predictor of market returns while $\lambda_t^{\mathbb{P}}$ becomes insignificant. We also consider a fully out-of-sample predictive exercise. Again, the risk-neutral tail risk consistently predicts excess market returns across all horizons, while its physical counterpart only contains information about the one-month horizon or longer. This indicates that only the economic perception of tail risk carries a premium in the short-term.

We also analyze the predictive power of the tail measures for the market variance risk premium (Bollerslev et al., 2009), which Bollerslev et al. (2015) show should be largely explained by investors' aversion to downside risk. Our risk-neutral tail measure strongly

2022 onwards) is still rather limited.

[3]This predictability is robust to different modeling choices and the inclusion of several controls.

predicts the variance risk premium with a positive sign across all horizons considered, meaning that investors demand a higher compensation for bearing market variance risk when tail risk is higher. The physical tail risk is also useful to predict the variance risk premium, except for the one-month horizon. When we include both of our tail measures in the predictive regressions, $\lambda_t^{\mathbb{P}}$ is still significant for one-day and one-week ahead, but offers little improvement in terms of R^2 relative to the highly significant $\lambda_t^{\mathbb{Q}}$. This provides further evidence that the economic valuation of tail risk by investors is an important determinant of market risk premia over short horizons.

We then investigate whether short-term tail risk is priced in the cross-section of stocks. To do so, for each of our tail measures, we build a long-short portfolio by sorting stocks each month on their recent exposure to the measure, based on contemporaneous daily regressions. The tail risk factor constructed from $\lambda_t^{\mathbb{Q}}$ generates statistically significant average returns that cannot be explained by standard factor models, where stocks with high exposure to tail risk have high hedging capacity and are thus highly priced, yielding subsequent low returns. In contrast, the tail factor associated with $\lambda_t^{\mathbb{P}}$ leads to insignificant spreads in returns. In other words, only the short-term exposure to tail risk as perceived by investors explains differences in expected returns across stocks.

The remainder of the paper is organized as follows. After a brief discussion of the related literature, Section 2 describes the methodology to construct our tail measures. Section 3 presents the data and the estimated tail measures, while Section 4 contains our empirical analysis. Section 5 concludes the paper. Lastly, Appendices A, B and C contain the main figures and tables, robustness results and variables definitions, respectively.

1.1 Related literature

Our paper is mainly related to an evolving literature investigating the effects of left tail risk and investors' compensation for such risk on financial markets. Bollerslev and Todorov (2011), Bollerslev et al. (2015) and Andersen et al. (2015, 2017) provide evidence that tail risk is an important determinant of the equity and variance risk premia using option-implied tail measures. Extracting information from observed stock prices, Bali et al. (2009), Kelly and Jiang (2014) and Almeida et al. (2017) show that tail risk strongly predicts future market returns and macroeconomic activity at monthly horizons or longer. Computing tail risk at the firm-level, Bali et al. (2014), Chabi-Yo et al. (2018) and Atilgan et al. (2020) document significant cross-sectional relations between tail risk and future stock returns. International evidence on the effects of tail risk beyond the U.S. market is provided by Andersen et al. (2020), Andersen et al. (2021) and Freire (2021). We contribute to this literature by proposing a novel method to estimate the tail risk specific to each day t and documenting new short-term return predictability for the aggregate market and the cross-section of stocks, with particular focus on the role of incorporating investors' preferences towards tail risk.[4]

The closest work to ours is by Kelly and Jiang (2014), who propose the Hill estimator to estimate the common tail risk component of a cross-section of stocks at a monthly frequency. We adapt their estimator to a daily frequency using intra-day stock returns and put forward a new version of the Hill estimator based on risk-neutralized returns. We use both physical and risk-neutral estimators to study the relation between tail risk (and its economic valuation) and risk premia at relatively short horizons. In this context, we find that the economic perception of tail risk, as opposed to the physical tail risk, is an important determinant of the equity premium, variance risk premium and the cross-section of stock returns.

Also closely related are Weller (2019) and Almeida et al. (2023). Weller (2019) exploits the cross-section of intra-day bid and ask quotes to estimate tail risk in real time and investigate predictability of jumps in high-frequency. We rely on the cross-section of intra-day returns and risk-neutralization to obtain the perception of tail risk and assess its role in determining risk premia over short horizons. Almeida et al. (2023) use high-frequency market returns to estimate an expected shortfall risk premium and show that it predicts risk premia one-day ahead. We, instead, extract information about left tail risks from intra-day extreme events of a large cross-section of stocks using the Hill estimator and document that it predicts market risk premia at different short horizons. We also

[4]For an early contribution on the role of taking economic valuation into account for computing risk measures, see Aït-Sahalia and Lo (2000).

investigate how short-term exposure to tail risk is priced in the cross-section of stocks using portfolio sorts.

Our paper is also related to the extensive literature identifying factors that are relevant to explain differences in the cross-section of stock returns, including Carhart (1997), Pástor and Stambaugh (2003), Fama and French (2015), among many others. Using our risk-neutral tail measure, we construct a tradable factor by sorting stocks based on their tail risk exposure calculated on a recent short window, which is possible given the daily availability of $\lambda_t^{\mathbb{Q}}$. This factor produces significant spreads in stock returns that cannot be explained by exposures to standard factors. Finally, it is worth noting that, in contrast to papers focusing on high-frequency factors and continuous-time asset pricing models (e.g., Aït-Sahalia et al., 2020; Bollerslev et al., 2016; Pelger, 2020), we build a factor at the usual monthly horizon by sorting stocks based on their exposure to a daily tail risk measure obtained from high-frequency returns.

2 Methodology

In this section, we describe our approach to estimate left tail risk at a daily frequency. Using a cross-section of intra-day stock returns, we first extract information about the common component of the tail risks of individual firms using the Hill estimator. Then, we introduce a new version of the Hill estimator that relies on risk-neutralized stock returns, thus taking into account the investors' perception of risk in the estimation.

2.1 Hill estimator

Extreme events in financial markets are rare by definition. This makes it challenging to construct an aggregate measure of tail risk relying on a single asset such as the market index, since informative observations for the tail are infrequent. To overcome this issue, we follow Kelly and Jiang (2014) by adopting a panel estimation approach capturing common tail behavior in the cross-section of individual stock returns. The identifying assumption is that the dynamics of the tail distributions of the firms are similar, so that extreme events in the cross-section allow us to extract the common component of their tail risk at each point in time.

More specifically, we assume that the left tail of the return distribution of asset i follows a power law structure.[5] That is, its day t conditional left tail distribution, defined as the set of extreme returns below some negative threshold u_t, obeys the following:

$$P(R^i_{t+1} < r | R^i_{t+1} < u_t \text{ and } \mathcal{F}_t) = \left(\frac{r}{u_t}\right)^{-a_i/\lambda_t}, \tag{1}$$

where $r < u_t < 0$ and $\mathcal{F}_t$ is the conditioning information set.[6] The parameter a_i/λ_t is the tail exponent which determines the shape of the tail distribution of asset i. The constant a_i may be different across assets in the cross-section, implying that they can have different levels of tail risk. However, their dynamics are driven by a common time-varying component, λ_t. The higher the λ_t, the thicker the stock returns' left tails and the higher the probabilities of extreme negative returns in the cross-section. Therefore, we refer to λ_t as our measure of aggregate tail risk.[7]

For each day t in our sample, we estimate the common tail risk component λ_t by applying the standard Hill (1975) power law estimator to the pooled cross-section of intra-day stock returns:[8]

$$\lambda^{\mathbb{P}}_t = \frac{1}{K_t}\sum_{k=1}^{K_t} \ln \frac{R_{k,t}}{u_t}, \tag{2}$$

where $R_{k,t}$ is the kth high-frequency return that is below the threshold u_t on day t, K_t is the total number of returns that fall below this threshold within day t and the superscript $\mathbb{P}$ denotes that returns are observed under the physical probability measure.[9] The threshold u_t represents an extreme quantile determining that the observed returns below u_t belong to the left tail and follow the power law structure. We define u_t to be the

[5] See Kelly and Jiang (2014) for a detailed motivation of the use of a power law structure to model the left tail distribution of returns. In sum, for a large class of heavy-tailed distributions, the left tail converges to a generalized power law distribution.

[6] $r < u_t < 0$ and $a_i/\lambda_t > 0$ guarantee that the probability $(r/u_t)^{-a_i/\lambda_t}$ is always between 0 and 1.

[7] In extreme value theory, the parameter λ_t is also often called the shape parameter, and its inverse $1/\lambda_t$ the tail index (see, e.g., Danielsson, 2011).

[8] While Kelly and Jiang (2014) use daily returns to estimate tail risk at a monthly frequency, we rely on intra-day returns to obtain the tail risk specific to day t.

[9] In the Hill formula, returns that fall below threshold u_t are treated as the first K_t entries of R_t. This is without loss of generality since in the pooled cross-section the elements of R_t are exchangeable.

first percentile of the return cross-section for each time period, which makes the threshold time-varying as the pooled intra-day return distribution changes from day to day.[10]

2.2 Risk-neutral Hill estimator

The Hill estimator extracts the common tail risk component from the pooled cross-section of returns observed under the physical probability measure, where all observations are deemed equally likely to happen. In that sense, $\lambda_t^{\mathbb{P}}$ does not incorporate the true risks that are perceived by investors in financial markets. In particular, if investors are risk averse, then "bad" states of the world where marginal utility is high should be overweighted to reflect compensation for risk. Since such states are precisely the ones that matter for the estimation of tail risk, the economic perception of the left tail of returns may be underestimated by the physical Hill estimator. Moreover, its dynamics will also differ from that captured by $\lambda_t^{\mathbb{P}}$, as compensation for risk demanded by investors varies over time depending on business conditions (see, e.g., Bliss and Panigirtzoglou, 2004).

In order to incorporate investors' compensation for risk in the estimation of left tail risk, we propose a new version of the Hill estimator coupled with risk-neutralization. The idea is to tilt the physical measure such that systematic risk in the cross-section of stock returns is corrected for. This is possible by weighting observations with a pricing kernel, or stochastic discount factor (SDF), that correctly prices systematic factors of high-frequency stock returns. Motivated by Kozak et al. (2020), we rely on principal component analysis (PCA) to identify the systematic factors. They show that the absence of near-arbitrage opportunities implies that the SDF can be represented as a function of a few dominant principal components (PCs) of returns. This is also consistent with Pelger (2020), who uses PCA to document the presence of at least five systematic factors explaining the intra-day returns of individual stocks.

More specifically, we consider as the systematic factors the top-seven PCs driving most of the intra-day return variation on day t. Results are similar for alternative specifications such as 5 or 10 PCs, as shown in Appendix B. In following Ait-Sahalia and Xiu (2017)

[10] Results are similar if we define u_t to be the percentile 0.5 or 5 instead, as seen in Appendix B.

and Pelger (2020), the factor loadings Λ_t are obtained as the eigenvectors associated with the seven largest eigenvalues of the realized covariance $R_t^\intercal R_t$, where R_t denotes the panel matrix of the high-frequency log-returns of the stocks. The matrix of intra-day factor returns is then given by $F_t = R_t \Lambda_t$.

Let the vector $F_{n,t}$ denote the return over the n-th intra-daily time interval on day t of the top-seven PCs of stock returns on day t. We work with an SDF that satisfies the Euler equations for the systematic factors:

$$\frac{1}{N} \sum_{n=1}^{N} m_{n,t} F_{n,t} = 0, \tag{3}$$

where 0 is a conformable vector of zeros and N denotes the total number of intra-daily observations. We normalize the mean of the SDF to be one ($\frac{1}{N} \sum_{n=1}^{N} m_{n,t} = 1$).[11] The pricing kernel tilts the physical measure $1/N$ to produce risk-neutral probabilities $m_{n,t}/N$ that overweight states with high marginal utility to reflect higher compensation for risk demanded by risk averse investors. That is, the SDF corrects for risk by risk-neutralizing assets returns with $\tilde{R}^i_{n,t} = m_{n,t} R^i_{n,t}$. We discuss how to identify the pricing kernel in the next subsection.

To derive the risk-neutral Hill estimator $\lambda_t^{\mathbb{Q}}$, we posit that the left tail of the risk-neutral return distribution of each asset i in the cross-section also follows a power law structure. The estimator is then obtained by using the pooled cross-section of risk-neutralized returns in equation (2):

$$\lambda_t^{\mathbb{Q}} = \frac{1}{\tilde{K}_t} \sum_{k=1}^{\tilde{K}_t} \ln \frac{\tilde{R}_{k,t}}{u_t}. \tag{4}$$

Due to the risk-neutralization, negative stock returns observed during states of high (low) marginal utility get properly overweighted (downweighted) by values of the pricing kernel above (below) its mean one, reflecting compensation for risk. The difference between $\lambda_t^{\mathbb{Q}}$ and $\lambda_t^{\mathbb{P}}$ captures the additional tail thickness coming from investors' risk preferences

[11] This implies an implicit gross risk-free rate of one such that we can treat the net stock returns in the cross-section as excess returns.

towards extreme negative events.

2.3 Risk-neutralization

The exact distortion of the physical measure, or correction for systematic risk in the cross-section of stocks, depends on the pricing kernel considered.[12] Besides correctly pricing the factor returns, there are two important properties that the SDF must satisfy. First, it must be nonnegative in order to be consistent with no-arbitrage. This guarantees that the tilted risk-neutral probabilities $m_{n,t}/N$ constitute a proper probability measure. Second, it should incorporate information about higher moments of the return distribution. This is important for modeling tail risk, since investors' aversion to downside risk is related to negative skewness aversion (see, e.g., Schneider and Trojani, 2015).

We follow the nonparametric approach developed by Almeida and Garcia (2017) to obtain a nonlinear pricing kernel satisfying the properties above. Their method consists in estimating SDFs minimizing a family of discrepancy loss functions (Cressie and Read, 1984) subject to correctly pricing a set of returns. This approach is a generalization of Hansen and Jagannathan (1991), who show how to obtain a minimum variance SDF from data on asset returns. Almeida and Garcia (2017) consider more general loss functions that take into account higher moments and imply nonnegative SDFs. Adapted to our context, the minimum discrepancy problem is given by:

$$\begin{aligned} &\min_{\{m_{1,t},\dots,m_{N,t}\}} \frac{1}{N}\sum_{n=1}^{N} \frac{m_{n,t}^{\gamma+1}-1}{\gamma(\gamma+1)}, \\ &\text{s.t. } \frac{1}{N}\sum_{n=1}^{N} m_{n,t}F_{n,t}=0,\ \frac{1}{N}\sum_{n=1}^{N} m_{n,t}=1,\ m_{n,t}\geq 0\ \ \forall n, \end{aligned} \tag{5}$$

where the parameter $\gamma \in \mathbb{R}$ indexes the convex loss function in the Cressie and Read (1984) discrepancy family. This family captures as particular cases several loss functions in the literature, such as the Hansen and Jagannathan (1991) quadratic loss function when $\gamma = 1$ and the Kullback Leibler Information Criterion adopted by Stutzer (1995)

[12] We consider the realistic case of an incomplete market, where there exists an infinity of pricing kernels that correctly price the systematic factors under no-arbitrage.

when $\gamma \to 0$.

Under the assumption of no-arbitrage in the observed sample, Almeida and Garcia (2017) show that solving (5) is equivalent to solving the simpler dual problem below, for $\gamma < 0$:[13]

$$\lambda_\gamma^* = \arg\max_{\lambda \in \Lambda_\gamma} \frac{1}{N} \sum_{n=1}^{N} -\frac{1}{\gamma+1} \left(1 - \gamma \lambda F_{n,t}\right)^{\left(\frac{\gamma+1}{\gamma}\right)}, \tag{6}$$

where $\Lambda_\gamma = \{\lambda \in \mathbb{R}^7 : \text{for } n = 1, ..., N, \ (1 - \gamma \lambda F_{n,t}) > 0\}$. The minimum discrepancy SDF can then be recovered from the first-order condition of (6) with respect to the row-vector λ, evaluated at λ_γ^*:

$$m_{\gamma,n,t}^* = \left(1 - \gamma \lambda_\gamma^* F_{n,t}\right)^{\frac{1}{\gamma}}, \ \ n = 1, ..., N. \tag{7}$$

The dual problem can be economically interpreted as an optimal portfolio problem for an investor maximizing hyperbolic absolute risk aversion (HARA) utility, where $\lambda_\gamma^* F_{n,t}$ is the endogenous optimal portfolio of the systematic factors. The SDF $m_{\gamma,n,t}^*$ is the marginal utility of the investor and will be higher for "bad" states of nature represented by negative realizations of the optimal portfolio of the factors.

For each γ, the solution λ_γ^* of the dual problem (6) leads to a different minimum discrepancy SDF. While by construction they all correctly price the systematic factor returns, they do so by representing distinct risk preferences. In particular, Almeida and Freire (2022) show that positive absolute prudence (Kimball, 1990), which is related to aversion to downside risk and a convex marginal utility, is captured by $\gamma < 1$. Moreover, the smaller the γ, the more aversion to downside risk is embedded in the SDF, where the pricing kernel gets more convex, putting more weight on extreme negative observations of the optimal portfolio returns.[14] They also show that, for extreme negative γs (usually below -5), the constrained maximization in the dual problem (6) may not have a solution. In order to successfully identify a pricing kernel capturing aversion to downside risk, we

[13] For $\gamma > 0$, the problem is unconstrained with an indicator function in the objective function: $\frac{1}{N}\sum_{n=1}^{N} -\frac{1}{\gamma+1}\left(1-\gamma\lambda F_{n,t}\right)^{\left(\frac{\gamma+1}{\gamma}\right)} I_{\Lambda_\gamma(F_{n,t})}(\lambda)$, where $\Lambda_\gamma(F_{n,t}) = \{\lambda \in \mathbb{R}^7 : (1-\gamma\lambda F_{n,t}) > 0\}$ and $I_A(x) = 1$ if $x \in A$, and 0 otherwise. For $\gamma \to 0$, the problem is unconstrained and the objective function is exponential: $\frac{1}{N}\sum_{n=1}^{N} -e^{-\lambda F_{n,t}}$.

[14] Since the mean of the pricing kernel continues to be the same, this means that less weight is given to intermediary return observations.

choose the one associated with $\gamma = -3$ to calculate the risk-neutral Hill estimator.[15]

3 Data description and implementation details

3.1 Data

Our sample consists of 5-minute returns for an unbalanced panel of 875 stocks between January 1996 and December 2022. The data is obtained from Refinitiv Tick History. We sample the 5-minute returns using the previous tick interpolation so that each stock has 78 intraday observations on a given day. In addition, we incorporate the overnight return, yielding a total of 79 observations per day.[16] We apply standard filters and cleaning procedures to the data.[17] In doing so, on a given day, we remove any stocks for which the number of non-zero returns is smaller than 35 observations.[18] To illustrate the impact of this filter, Figure 1 plots the average number of stocks per year with at least 35 non-zero returns per day. As can be seen, this filter has a bigger impact during the years prior to the decimalization of the stocks (April 9, 2001), whereas after this period the filter mostly eliminates stocks around holidays.

Throughout the paper, we use data on market returns, risk factors, and uncertainty measures. The popular five factors of Fama and French (2015), the momentum factor and the risk-free rate are obtained from Kenneth French's website. The liquidity factor of Pástor and Stambaugh (2003) is available from Lubos Pastor's website. The VIX index is obtained from the Chicago Board Options Exchange (CBOE), while the left tail variation (LTV) and the right tail variation (RTV) proposed by Bollerslev et al. (2015) are computed by ourselves.[19] The LTV and RTV capture, respectively, the option-implied risk-neutral expectation of return volatility stemming from large negative and

[15]Considering pricing kernels minimizing loss functions indexed by alternative γs associated with aversion to downside risk (such as -2) leads to similar results, as shown in Appendix B.

[16]Results are similar without including the overnight return, as can be seen in Appendix B.

[17]To formally exclude the impact of microstructure noise, we have performed the Hausman tests for microstructure noise and first-order serial correlation of Aït-Sahalia and Xiu (2019), for each stock and each day. The tests reject any significant presence of microstructure noise and first-order serial autocorrelation in the returns.

[18]Although this filter mitigates the impact of zero returns (see, e.g., Bandi et al., 2020, 2017), our results are robust to setting the threshold for non-zero returns to 20 or 50 observations.

[19]We thank Viktor Todorov for making his code available on his tailindex website.

positive price jumps. Using high-frequency market returns sampled every 5 minutes obtained from TickData Inc, we compute measures of the realized variance (RV), realized skewness (RSK), realized kurtosis (RK) and jump variation (JV) of the S&P 500 index (Amaya et al., 2015; Andersen et al., 2001, 2003; Barndorff-Nielsen and Shephard, 2004). Using daily market returns, we further calculate the reversal (REV) of Jegadeesh (1990) and Lehmann (1990), momentum (MoM) of Jegadeesh and Titman (1993) and maximum (Max) and minimum (Min) daily return (Bali et al., 2011) for the market observed over the previous week.

We also compute the variance risk premium (VRP) as the difference between the risk-neutral and physical expectations of the market return variance (e.g. Bekaert and Hoerova, 2014; Bollerslev et al., 2009). We define the VRP on day t as the squared VIX index (scaled to the monthly level) minus the physical conditional expected value of the future monthly realized variance computed using the HAR model of Corsi (2009). Appendix C contains the detailed definitions of the variables we use.

Finally, for our portfolio analysis, we obtain the returns and prices for all ordinary common stocks (share code 10 and 11) from the Center for Research and Security Prices (CRSP) for the period spanning January 1996 to December 2022. In line with the literature, we remove all penny stocks with prices less than five dollars. In Appendix B.4, we show that our results are robust to an even more conservative filter further removing relatively illiquid stocks with market capitalization smaller than the tenth percentile.

3.2 Principal components and risk-neutral estimates

As described in Section 2, for a given day t in our sample, we first extract the top-seven PCs explaining most of the variation in the high-frequency panel of stock returns. The PCs are themselves returns of portfolios of the original stocks. Figure 2 plots the average over each day of our sample of the percentage of variance explained by each of the PCs. The first PC (PC1) explains nearly 30% of the variation in the stock returns. In our data, PC1 is always a level factor with long positions of similar magnitude across stocks. In other words, as it is usually the case, PC1 can be interpreted as a market factor. The remaining PCs are long-short portfolios of the original stocks. The top-seven PCs together explain around 60% of the return variation across the stocks returns.

Then, we estimate the SDF for each day t using the 78 intra-day returns and overnight return of the top-seven PCs. Given that PC1 can be seen as the market factor, we impose the economic restriction of a 5% lower bound on the annualized equity premium, following Almeida and Freire (2022).[20] While results are similar compared to those where this restriction is not imposed, we keep it because it is economically sound to consider a lower bound on the equity premium (Campbell and Thompson, 2008; Martin, 2017; Pettenuzzo et al., 2014). This restriction is only imposed for the estimation of the SDF. For the remaining PCs, we do not impose any restrictions.

To illustrate how the SDF distorts the physical measure, Figure 3 plots the estimated risk-neutral probabilities ($m_{\gamma,n,t}/N$) for various values of γ and the physical probabilities ($1/N$) for a random day in our sample. The observed patterns are representative of other dates. As can be seen, the risk-neutral measures give more probability weight to negative returns of the optimal portfolio of PCs and less weight to positive returns compared to the physical measure. This reflects agents' risk aversion: investors require more compensation (i.e., the SDF is higher) for "bad" states of the world. In the estimation of tail risk, this is such that negative stock returns, observed during intra-daily intervals for which the optimal portfolio of systematic factors experiences negative (positive) returns, get overweighted (downweighted) to reflect more (less) compensation for risk. The relative compensation for risk in the left tail of the optimal portfolio returns depends, in turn, on the aversion to downside risk. The smaller the γ, the more averse to downside risk (or, equivalently, the more prudent) is the investor and the greater are the weights to negative returns under the risk-neutral measure. As previously mentioned, we use the SDF associated with $\gamma = -3$ for the estimation of the risk-neutral Hill estimator.

[20]More specifically, for each day t, we impose that the average return of PC1 is at least 5% above the risk-free rate, in annualized terms. That is, we shift the mean of PC1 to the lower bound when the bound is binding.

3.3 Tail risk estimates

We estimate the tail risk measures $\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$ as detailed in Section 2 using the set of intra-day return observations of all stocks available for each day t. The upper panels of Figure 4 plot their one-month moving averages, for ease of exposition. The measures share some similarities. In particular, neither $\lambda_t^{\mathbb{P}}$ nor $\lambda_t^{\mathbb{Q}}$ behave like usual volatility-based risk measures. In fact, the physical tail risk even decreases during the global financial crisis of 2008. To understand this pattern, the left lower panel of Figure 4 reports the time-varying threshold u_t (in absolute value), that determines where the left tail begins in the Hill estimator. As can be seen, u_t resembles a volatility measure, peaking during periods of market distress. The tail risk measures $\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$ can thus be thought of as capturing the thickness of the left tail after taking into account the effect of volatility. As Kelly and Jiang (2014) note, a fixed percentile (of 1% of the return cross-section of time t) is used to define u_t exactly for this reason: if volatility increases but the shape of the return left tail is unchanged, an increase of the threshold (in absolute value) absorbs the effect of volatility changes and leaves estimates of the tail exponent unaffected.[21]

Even though the tail risk measures $\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$ display similarities, they are still fundamentally different. The right lower panel of Figure 4 plots the difference between the risk-neutral and physical tail measures. As would be expected, $\lambda_t^{\mathbb{Q}}$ is always above $\lambda_t^{\mathbb{P}}$, reflecting the fact that investors are averse to downside risk. However, the additional thickness of the left tail coming from risk compensation required by investors varies substantially over time. In particular, $\lambda_t^{\mathbb{Q}} - \lambda_t^{\mathbb{P}}$ tends to peak during crisis periods. This suggests that, even though financial crises are not necessarily associated with higher physical tail risk, they are associated with a higher compensation demanded to bear tail risk. In other words, investors are especially afraid of extreme negative events during periods of market distress.

Table 1 reports summary statistics of and correlations among $\lambda_t^{\mathbb{P}}$, $\lambda_t^{\mathbb{Q}}$, $|u_t|$ and realized and option-implied measures of return variation. The results in the table afford two

[21]In unreported tests, we calculate $\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$ with a constant threshold $u_t = u$ and find that both measures behave like volatility-type measures. This indicates that defining u_t as a fixed percentile of the return cross-section is instrumental to isolate the effects of volatility from the shape of the left tail.

main conclusions. First, the tail measures have a relatively small correlation of 30.8%, reinforcing that they capture substantially distinct information over time. Second, both $\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$ are weakly correlated with and less persistent than volatility risk measures, whereas (the absolute value of) u_t has a strong positive correlation with these measures and is highly persistent. This is consistent with the fact that the time-varying threshold u_t controls for the effect of volatility in the calculation of $\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$.

4 Empirical results

This section provides empirical evidence of the information content of our tail risk measures for asset prices in the short-term. We document the predictive power of the tail measures in forecasting the equity premium and the variance risk premium. In particular, we consider one-day ($h = 1$), one-week ($h = 5$), and one-month ($h = 22$) ahead daily predictive regressions. In addition, we investigate how tail risk is priced in the cross-section of stocks. To do so, we construct monthly long-short portfolios by sorting stocks on their recent exposure to the tail risk measures.

4.1 Predicting the equity premium

There is extensive empirical evidence that, at relatively low frequencies (monthly or longer), measures of tail risk strongly predict future excess market returns, with a positive relation (see, e.g., Almeida et al., 2017; Bollerslev et al., 2015; Kelly and Jiang, 2014). This is consistent with the idea that investors are averse to tail risk, such that they require a higher return to hold the market when tail risk increases. To shed light on this relation in the short-term, we investigate whether and how our tail risk measures predict the equity premium at short horizons with daily regressions.

Table 2 contains our main results for predicting the equity premium.[22] There is a positive relation between $\lambda_t^{\mathbb{P}}$ and the equity premium. However, this relation is statistically significant only at the one-month horizon. That is, investors require a higher return

[22]The reported coefficients of the predictive regressions are scaled to be interpreted as the effect of a one standard deviation increase in the regressor on future excess market returns.

to hold the market after a tail risk shock, but this compensation appears only over the horizon of one month. While $\lambda_t^{\mathbb{Q}}$ is also positively related to future market returns, this relation is statistically significant across all horizons and associated with substantially higher R^2's. This indicates that investors immediately require compensation to hold the market following an increase in their perception of tail risk, where such compensation persists for over a month. To further assess the role of risk-neutralization in predicting the equity premium, we consider predictive regressions including both the risk-neutral and physical tail risk measures. Across all horizons, $\lambda_t^{\mathbb{Q}}$ remains a significant predictor, while $\lambda_t^{\mathbb{P}}$ has no predictive power. In other words, the information about future market returns contained in the physical tail risk is subsumed by its risk-neutral counterpart.

In Appendix B.3, we show that our predictability results for the equity premium are robust to controlling for several alternative predictors. In fact, the risk-neutral tail risk is the strongest predictor of future excess market returns among the controls. The only variable with comparable predictive power is the VRP. The inclusion of the VRP drives out the effect of $\lambda_t^{\mathbb{Q}}$ at the one-day horizon, while the opposite happens at the one-week horizon. For the one-month ahead regressions, both predictors remain significant. This indicates that the variance risk premium helps explain variation in future excess market returns not only at lower frequencies (Bollerslev et al., 2009), but also in the short-term.

We also investigate whether the predictive power of tail risk holds in a fully out-of-sample exercise. For either $\lambda_t^{\mathbb{Q}}$ or $\lambda_t^{\mathbb{P}}$, we estimate predictive regressions for market returns in a previous window and use it to predict one-day, one-week, one-month or one-quarter ahead in the future. We consider an expanding window scheme with initial window size of two years, re-estimating the model each day t. We evaluate predictive power via the $R^2_{oos} = 1 - \sum_t (R_t - \widehat{R}_t)^2 / \sum_t (R_t - B_t)^2$, where $\widehat{R}_t$ is the model's predicted return in period t and B_t, the benchmark, is either the average excess return through time t (Panel A), the average excess market return over the whole sample (Panel B), or set to zero (Panel C).[23]

[23] The average excess market return is commonly considered as the benchmark in the literature (see, e.g., Campbell and Thompson, 2008). However, if the historical mean is a poor predictor of future returns over short horizons, this could inflate our measures of R^2_{oos}. This is why we also include: a benchmark of zero, such that the R^2_{oos} simply reflects the relative variation explained by using tail risk as predictor; and the average market return over the whole sample, which is a tougher benchmark as it uses information from the future.

We assess statistical significance based on the Clark and West (2007) statistic, where the null hypothesis is that the regressor has no predictive power. Table 3 reports the out-of-sample results. A positive R^2_{oos} means that the predictor outperforms the benchmark. As can be seen, $\lambda_t^{\mathbb{P}}$ only consistently beats the benchmarks for longer horizons over a month in the future. In contrast, $\lambda_t^{\mathbb{Q}}$ yields positive and statistically significant R^2_{oos}'s for nearly all horizons regardless of the benchmark. This reinforces the strong predictive power for the equity premium afforded by the risk-neutral tail risk.

In sum, our results indicate that the economic perception of tail risk is an important determinant of the equity premium in the short-term. Investors require a significantly higher market return following an increase in their perception of tail risk. This effect persists over time, is robust to controlling for several alternative predictors and holds out-of-sample. Importantly, accounting for investors' aversion to downside risk in computing tail risk provides fundamental information about the equity premium that is not contained in the physical tail risk measure.

4.2 Predicting the variance risk premium

The variation of volatility is often associated with time-varying economic uncertainty. In particular, the variance risk premium (VRP) captures investors' compensation for variance risk and is usually regarded as a proxy for aggregate risk aversion (see, e.g., Bekaert et al., 2013; Campbell and Cochrane, 1999). Bollerslev et al. (2015) show that a large fraction of the variance risk premium comes from compensation demanded by investors for bearing left tail risk. Motivated by that, we examine the predictive relation between our tail risk measures and the VRP at short horizons, with particular focus on the role of incorporating investors' aversion to downside risk with risk-neutralization.

Table 4 reports the main predictability results for the variance risk premium. Both $\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$ positively predict the VRP, consistent with the idea that investors require a higher compensation to bear variance risk when tail risk increases. While the statistical significance of $\lambda_t^{\mathbb{P}}$ mostly survives the inclusion of $\lambda_t^{\mathbb{Q}}$ in the predictive regressions, the predictive power of the risk-neutral tail measure is stronger, as indicated by higher t-

statistics and individual R^2's. Moreover, at the one-month horizon, only $\lambda_t^{\mathbb{Q}}$ is significant. This indicates that, again, the investors' perception of tail risk plays a prominent role in explaining market risk premia in the short-term. In Appendix B.3, we show that these results are robust to controlling for a number of alternative predictors given by realized and option-implied measures of continuous and jump variation. Together with $\lambda_t^{\mathbb{Q}}$, the strongest predictor is the LTV from Bollerslev et al. (2015), which captures the expected volatility stemming for negative price jumps implied by short-dated options. In other words, the perception of tail risk in the cross-section of firms offers complementary information to LTV about the future VRP.

In sum, we document that our risk-neutral tail measure contains strong predictive power for the variance risk premium in the short-term. Investors require a higher compensation to bear variance risk when their perception of tail risk increases. These effects are robust to several measures of volatility, jump risk and the LTV of Bollerslev et al. (2015). In contrast, the relation between physical tail risk and future VRP is weaker and often statistically insignificant.

4.3 Predicting the cross-section of stock returns

So far, we have shown that the economic perception of tail risk by investors is an important determinant of aggregate market risk premia at short-horizons. This section investigates whether recent exposure to tail risk is priced in the cross-section of stock returns through portfolio sorts. To do so, at the end of each month in our CRSP sample, we measure the insurance value of the stocks with daily regressions over a short previous window of 24 months, i.e., we estimate contemporaneous betas with respect to our tail measures: $R_{i,t} = \mu_i + \beta_i TR_{i,t}$, where $TR_{i,t} \in \{\lambda_t^{\mathbb{P}}, \lambda_t^{\mathbb{Q}}\}$.[24] Then, we form equally-weighted portfolios over the next month by sorting the stocks into portfolios using quintile breakpoints calculated based on the given sorting variable.

The two panels of Table 5 report the results for $\lambda_t^{\mathbb{P}}$ and $\lambda_t^{\mathbb{Q}}$, respectively. In addition to the average returns of the quintile portfolios, we also report the portfolios alphas (i.e.,

[24]Our results are robust to different estimation windows for the betas, such as 18 or 36 months.

intercepts) from regressions of portfolio excess returns on the Fama-French three and five factors as well as extended models controlling for momentum (Carhart, 1997) and liquidity (Pástor and Stambaugh, 2003) factors. The last two columns report the average returns and alphas of the high minus low zero net investment portfolio and associated t-statistics, which are estimated using Newey-West robust standard errors. Panel C presents the p-values from various tests of the monotonicity (Patton and Timmermann, 2010) of average returns across the five quintile portfolios reported in Panels A–B. All tests have a null hypothesis of a flat pattern (no relation). While the MR Up and MR Down tests have alternative hypotheses of an increasing and decreasing pattern, the MR test alternative hypothesis is unrestricted. The tests are estimated using 10,000 bootstrap replications and a block length equal to 10 months.

Several conclusions can be drawn from these results. First, stocks that are more positively related to tail risk in the short-term earn lower returns. This is economically sound, as stocks with high β_i provide hedging opportunities against tail risk and are thus highly priced, yielding subsequent low returns. This relation is monotonic across quintile portfolios for $\lambda_t^{\mathbb{Q}}$, which is formally confirmed by the rejection of the flat pattern using the MR and MR Down tests. In contrast, a flat pattern cannot be rejected for $\lambda_t^{\mathbb{P}}$. In fact, in this case it is clear that the average return decreases from the low quintile to the fourth one, but then increases for the high quintile.

Second, exposure to physical tail risk generates insignificant average returns for the high minus low portfolio. On the other hand, the return spreads associated with exposures to risk-neutral tail risk are both statistically and economically significant, where the corresponding high minus low strategies earn an average monthly return of -0.86%. This shows that recent exposure to tail risk as perceived by investors is priced in the cross-section. To further illustrate, Figure 5 plots the cumulative returns of the quintile portfolios based on $\lambda_t^{\mathbb{Q}}$. As can be seen, the robust profitability of (selling) the high minus low strategy is mainly driven by the stocks that have high expected returns because they do not help hedge against tail risk.

Third, the average high minus low returns of the tail factor in Table 5 survive after controlling for standard factor models in the literature. The alphas with respect to the market and the Fama-French size and value factors are largely of the same magnitude and significance of the original average returns. The additional profitability, investment, momentum and liquidity factors hold some explanatory power to the short-term tail risk premium, reducing (in absolute value) the average return to -0.573%. Even so, this still roughly amounts to a 7% annualized (absolute) alpha, which is strongly significant with a t-statistics of 3. This suggests that our risk-neutral tail factor captures risk premium that is not reflected in firms' exposures to these popular factors in the literature.

In sum, we find that the investors' perception of tail risk in the short-term is strongly priced in the cross-section of stocks. High minus low portfolios based on the recent exposure to tail risk generate statistically and economically significant average returns, even after controlling for standard factor models. The information content of risk-neutralization beyond that contained in physical tail risk is especially relevant for these findings.

5 Conclusion

In this paper, we introduce a new tail risk measure at a daily frequency by combining high-frequency returns of a cross-section of stocks with a risk-neutralization algorithm. We use our measure to shed light on the effects of tail risk on asset prices at short-horizons and investigate to what extent these effects depend on information coming from the physical measure, under which asset prices are observed, and the risk-neutral measure, which incorporates investors' preferences. We find that the economic valuation of tail risk is an important determinant of the equity premium and the variance risk premium at horizons up to a month. In addition, short-term exposure to tail risk is priced in the cross-section of stocks. A tradable tail factor built by sorting stocks on their recent exposure to tail risk produces significant spreads in stock returns that cannot be explained by standard factor models. Incorporating investors' preferences in the estimation of tail risk is fundamental to our findings: the predictability afforded by the physical tail risk is weaker and generally subsumed by its risk-neutral counterpart.

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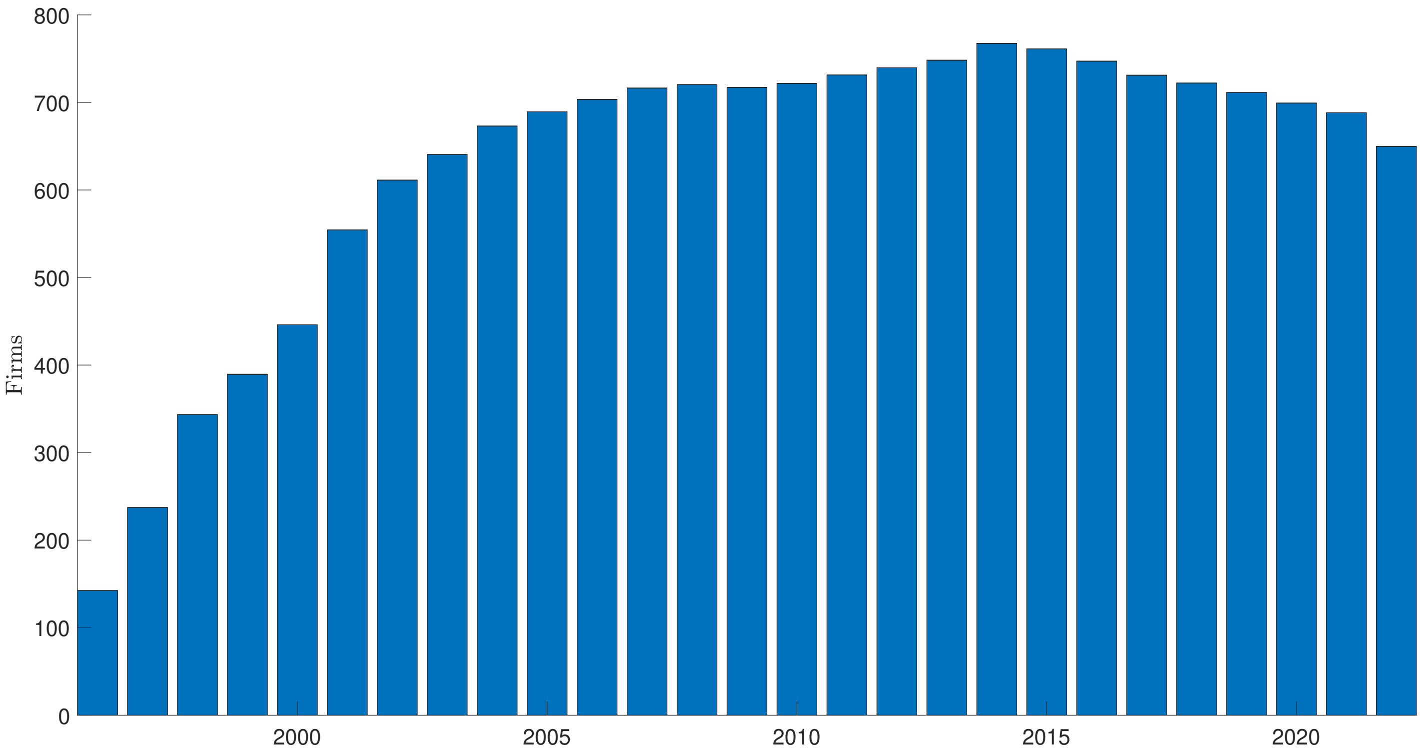
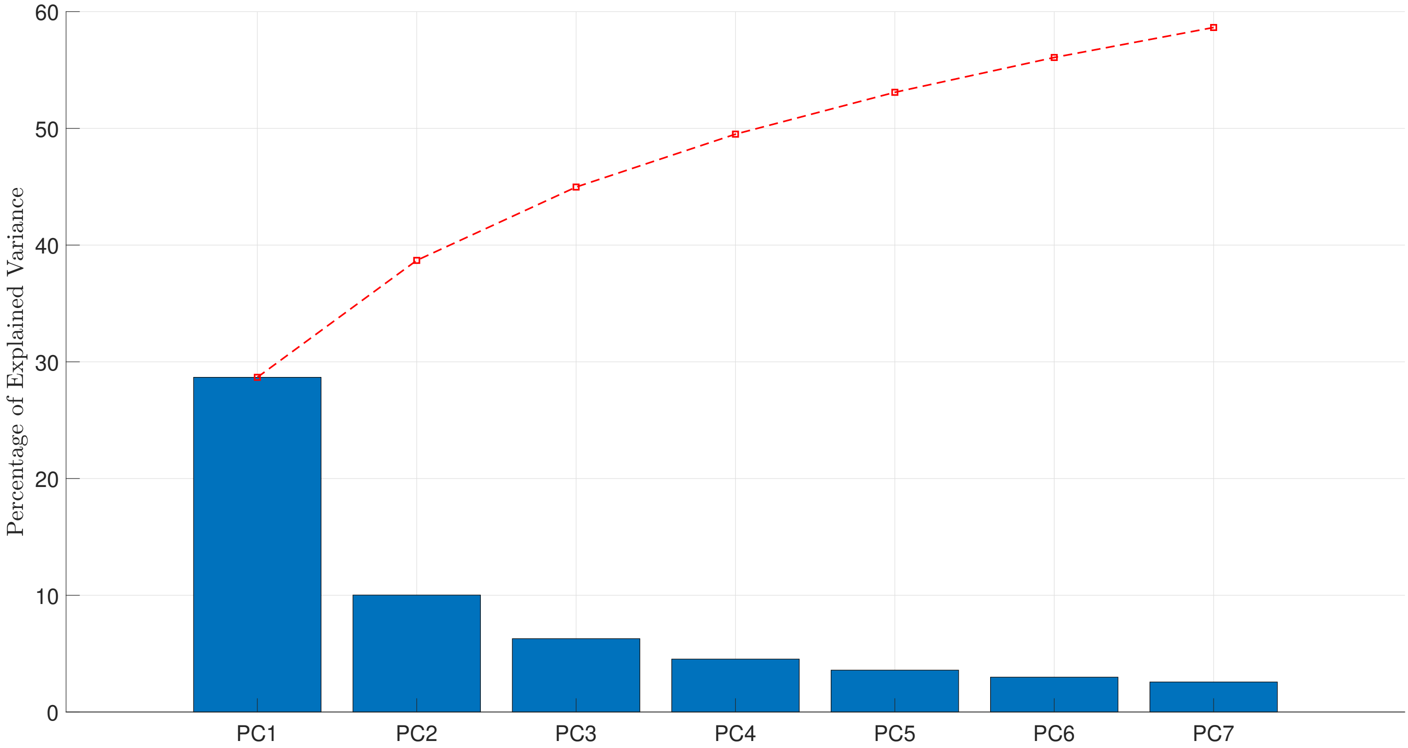
A Figures and tables

Figure 1: Yearly average number of stocks



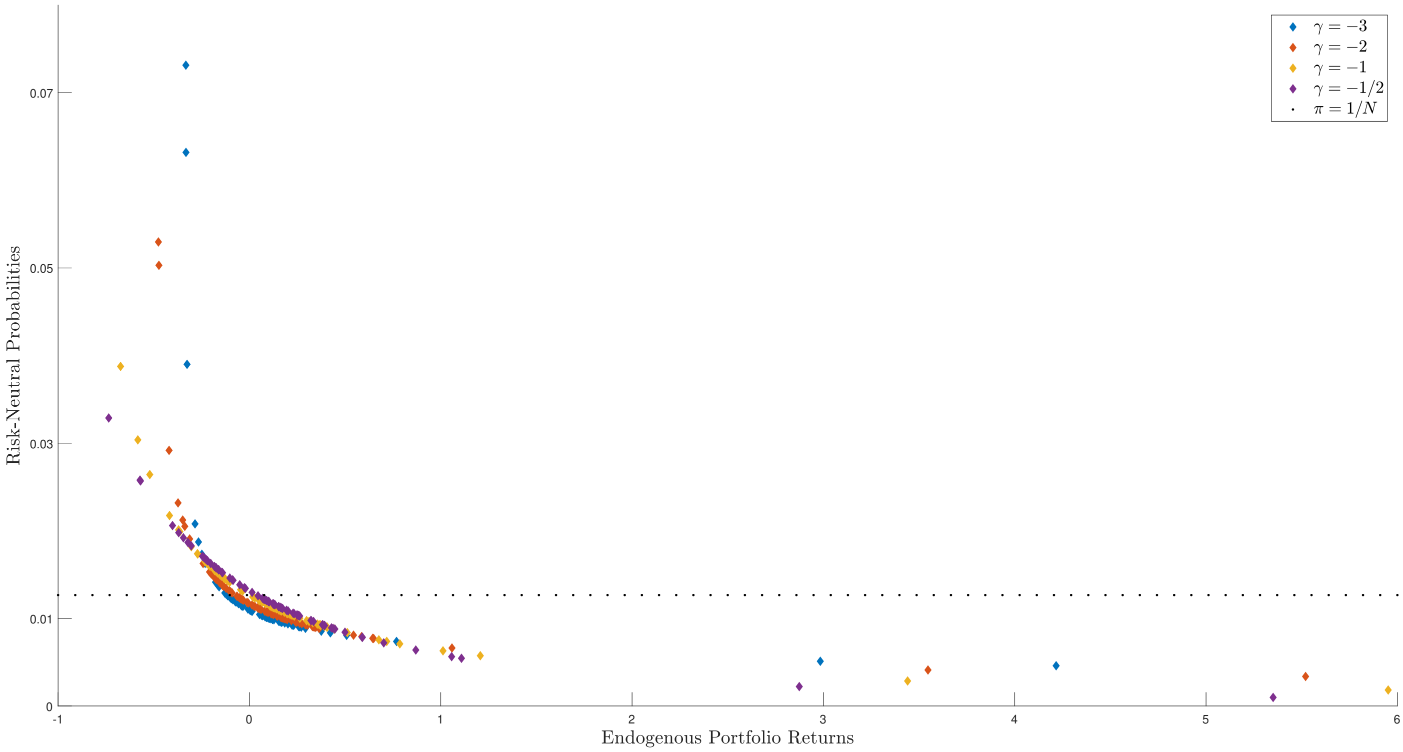
Note: The figure depicts the yearly average number of stocks used to compute the tail risk measures. These stocks have at least 35 non-zero returns per day.

Figure 2: Explained variation of principal components



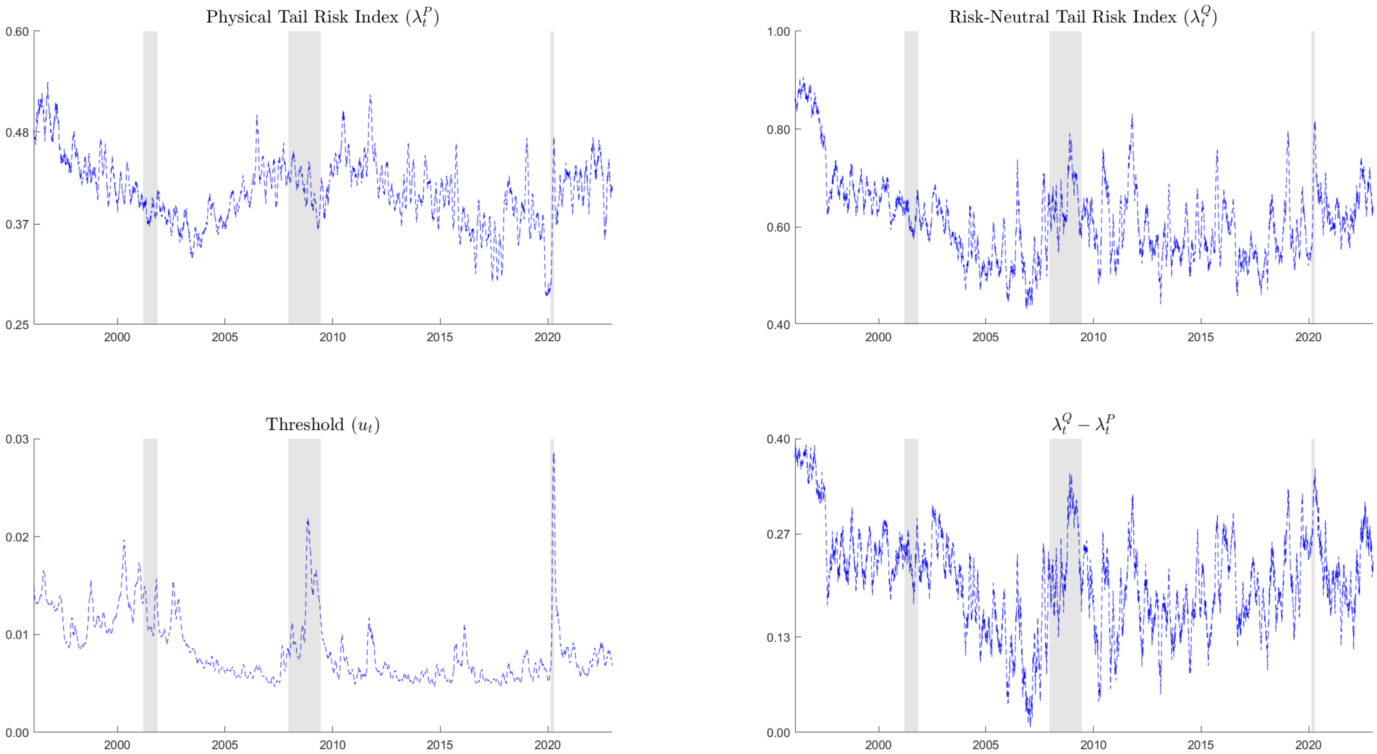
Note: The figure depicts the time series average of the percentage of explained variance in the panel of intra-day stocks returns by the top-seven PCs (in blue bars) and the accumulated percentage of explained variance (in red). The sample ranges from January 1996 to December 2022.

Figure 3: Minimum discrepancy risk-neutral probabilities



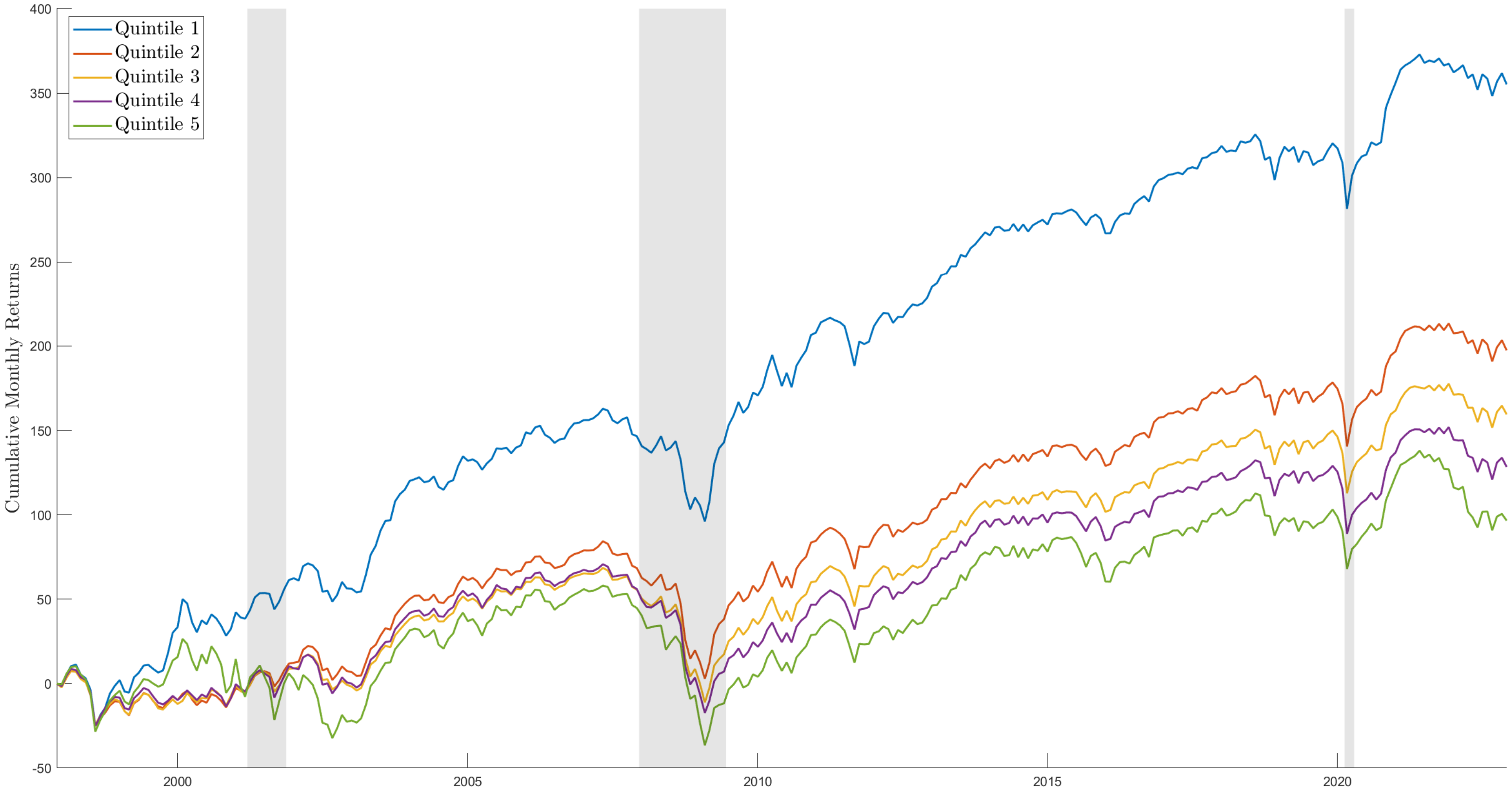
Note: The figure depicts the minimum discrepancy risk-neutral probabilities for various values of γ and the physical measure ($\pi = 1/N$) for the 79 intra-daily endogenous portfolio returns ($\lambda^*_\gamma F_{n,t}$) for a random day in our sample.

Figure 4: Tail risk index measures



Note: The figure plots, in the upper panels, the 1-month moving average of the physical and risk-neutral tail risk indices. In the bottom panels, the corresponding moving averages for the (absolute value of the) threshold and the difference between risk-neutral and physical tail risk are depicted. Shaded areas depict the NBER recession dates. The sample ranges from January 1996 to December 2022.

Figure 5: Cumulative monthly quintile portfolio returns formed by sorting on $\lambda_t^{\mathbb{Q}}$



Note: The figure depicts the cumulative monthly returns for each quintile portfolio formed by sorting on the risk-neutral tail risk measure ($\lambda_t^{\mathbb{Q}}$). Shaded areas depict NBER recession dates. The sample ranges from January 1998 to December 2022.

Table 1: Summary Statistics

	$\lambda_t^{\mathbb{P}}$	$\lambda_t^{\mathbb{Q}}$	$\lvert u_t \rvert$	RV_t	JV_t	LTV_t	RTV_t
Panel A: Summary Statistics							
Mean	0.409	0.615	0.855	1.060	0.073	2.599	0.005
Median	0.407	0.617	0.721	0.522	0.019	1.682	0.001
5th percentile	0.300	0.325	0.466	0.087	0.000	0.532	0.000
95th percentile	0.521	0.893	1.575	3.309	0.286	7.315	0.021
St. Dev.	0.069	0.169	0.425	2.204	0.250	3.210	0.017
Panel B: Correlations and AR(1) Coefficients							
$\lambda_t^{\mathbb{P}}$	0.395	0.342	0.046	0.119	0.061	0.109	−0.107
$\lambda_t^{\mathbb{Q}}$		0.308	0.117	0.151	0.105	0.131	−0.071
$\lvert u_t \rvert$			0.761	0.699	0.357	0.431	−0.023
RV_t				0.683	0.527	0.575	−0.026
JV_t					0.116	0.512	−0.006
LTV_t						0.874	−0.038
RTV_t							0.560

Note: The table reports, in Panel A, the mean, median, 5th and 95th percentiles, and the standard deviation of the physical tail risk index ($\lambda_t^{\mathbb{P}}$), the risk-neutral tail risk index ($\lambda_t^{\mathbb{Q}}$), the absolute value of the threshold ($|u_t|$), the realized variance (RV_t) estimated using 5-min intraday returns, the Jump Variation (JV_t) estimated as the difference between the RV and the bipower variation of Barndorff-Nielsen and Shephard (2004), the left tail variation (LTV_t), and the right tail variation (RTV_t) of Bollerslev et al. (2015). Panel B reports in the off-diagonal the correlation of each pair of variables and in the main diagonal the AR(1) coefficient.

Table 2: Predicting excess market returns

	One-day ($h = 1$)			One-week ($h = 5$)			One-month ($h = 22$)		
$\lambda_t^{\mathbb{P}}$	0.015		0.004	0.058		0.015	0.367**		0.236
	(0.910)		(0.266)	(1.365)		(0.362)	(2.556)		(1.597)
$\lambda_t^{\mathbb{Q}}$		0.032**	0.030*		0.134***	0.129***		0.473***	0.395***
		(2.003)	(1.838)		(3.189)	(3.091)		(4.360)	(3.625)
R^2	0.014	0.066	0.068	0.051	0.270	0.273	0.528	0.879	1.073

Note: The table reports the regression coefficients and robust t-statistics (in parentheses) of daily predictive regressions for excess returns over one-day ($h = 1$), one-week ($h = 5$), and one-month ($h = 22$) horizons. For forecasting horizons larger than 1 day, we compound the daily return from $t + 1$ to $t + h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. We denote with *, **, and *** significance at the 10%, 5% and 1% level, respectively. The R^2 is the OLS R-squared reported in percentage points. The sample ranges from January 1996 to December 2022.

Table 3: Out-of-sample excess market return predictability

	$\lambda_t^{\mathbb{P}}$	$\lambda_t^{\mathbb{Q}}$
Panel A: Benchmark $\widehat{\mu}_t$		
One-day ($h = 1$)	-0.033	$0.029^{\star\star}$
One-week ($h = 5$)	-0.003	$0.213^{\star\star\star}$
One-month ($h = 22$)	$0.473^{\star\star\star}$	$0.846^{\star\star\star}$
One-quarter ($h = 66$)	$1.223^{\star\star\star}$	$1.590^{\star\star\star}$
Panel B: Benchmark $\widehat{\mu}$		
One-day ($h = 1$)	-0.093	-0.032
One-week ($h = 5$)	-0.155	$0.062^{\star\star\star}$
One-month ($h = 22$)	-0.259	$0.117^{\star\star\star}$
One-quarter ($h = 66$)	$0.167^{\star\star\star}$	$0.538^{\star\star\star}$
Panel C: Benchmark $\widehat{\mu} = 0$		
One-day ($h = 1$)	-0.040	$0.021^{\star}$
One-week ($h = 5$)	$0.129^{\star\star}$	$0.345^{\star\star\star}$
One-month ($h = 22$)	$1.050^{\star\star\star}$	$1.420^{\star\star\star}$
One-quarter ($h = 66$)	$2.921^{\star\star\star}$	$3.282^{\star\star\star}$

Note: The table reports the out-of-sample R-squared, R^2_{oos}, computed as $R^2_{oos} = 1 - \sum_t (R_t - \widehat{R}_t)^2 / \sum_t (R_t - B)^2$. $\widehat{R}_t$ is the model's predicted return in period t and B, the benchmark, is the average excess market return through t (Panel A), the historical average excess market return (Panel B), or it is set to zero (Panel C). We also denote with *, **, *** significance at the 10%, 5%, 1% level based on the Clark and West (2007) statistic, whose null hypothesis is that the additional regressor has no predictive power. The models are estimated on expanding windows, with an initial window size of 528 days (equivalent to approximately 24 months).

Table 4: Predicting variance risk premium (VRP)

	One-day ($h = 1$)			One-week ($h = 5$)			One-month ($h = 22$)		
$\lambda_t^{\mathbb{P}}$	2.161***		1.261***	7.917***		4.326**	9.255		0.086
	(5.680)		(3.714)	(3.451)		(2.168)	(0.758)		(0.007)
$\lambda_t^{\mathbb{Q}}$		3.862***	3.541***		15.222***	14.122***		36.076***	36.055***
		(9.075)	(8.913)		(5.815)	(5.850)		(3.666)	(3.791)
R^2	0.991	3.163	3.479	0.648	2.396	2.577	0.076	1.160	1.160

Note: The table reports regression coefficient and robust t-statistics (in parentheses) of daily predictive regressions for the variance risk premium (VRP) over one-day ($h = 1$), one-week ($h = 5$), and one-month ($h = 22$) horizons. For forecasting horizons larger than 1 day, we aggregate the variance risk premium from $t + 1$ to $t + h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. We denote with *, **, and *** significance at the 10%, 5% and 1% level, respectively. The R^2 is the OLS R-squared reported in percentage points. The sample ranges from January 1998 to December 2022.

Table 5: Monthly sorted portfolios

	Low	2	3	4	High	High-Low	t-stat
Panel A: λ_t^P							
Average Return	0.807	0.603	0.567	0.495	0.650	−0.157	−0.988
CAPM alpha	0.073	0.002	0.010	−0.068	0.021	−0.052	−0.334
FF3 alpha	0.000	−0.079	−0.073	−0.157	−0.071	−0.071	−0.517
FF5 alpha	0.110	−0.134	−0.173	−0.237	−0.053	−0.163	−1.135
FF5 + Mom alpha	0.131	−0.109	−0.152	−0.207	−0.006	−0.138	−0.940
FF5 + Mom + Liq alpha	0.103	−0.126	−0.168	−0.217	−0.005	−0.108	−0.716
Panel B: λ_t^Q							
Average Return	1.184	0.658	0.532	0.428	0.322	−0.862	−5.321
CAPM alpha	0.518	0.077	−0.034	−0.156	−0.366	−0.884	−5.012
FF3 alpha	0.422	−0.013	−0.116	−0.240	−0.432	−0.854	−5.128
FF5 alpha	0.405	−0.123	−0.216	−0.298	−0.253	−0.657	−4.034
FF5 + Mom alpha	0.425	−0.099	−0.197	−0.270	−0.201	−0.627	−3.378
FF5 + Mom + Liq alpha	0.398	−0.128	−0.215	−0.292	−0.176	−0.573	−2.937

Panel C: Monotonicity Test	MR	MR Up	MR Down
Avg. return $\lambda_t^{\mathbb{P}}$	0.829	0.259	0.109
Avg. return $\lambda_t^{\mathbb{Q}}$	**0.005**	0.931	**0.001**

Note: The table reports the results of univariate portfolio analyses of the relation between the tail risk measures and the cross-section of returns. Monthly portfolios are formed by sorting the CRSP stocks with share code 10 and 11, excluding those with a share price below \$5, into portfolios using quintile breakpoints calculated based on the given sort variable. The table also reports portfolio alphas from regression of portfolio excess return using the Fama-French three and five factors as well as extended models controlling for momentum (Carhart, 1997) and liquidity (Pástor and Stambaugh, 2003) factors. Returns and alphas are in percentage terms. The last two columns report the high minus low zero net investment portfolio and associated t-statistics, which are estimated using Newey-West robust standard errors with a lag length equal to 5. Panel C presents the p-values from various tests of the monotonicity (Patton and Timmermann, 2010) of average returns across the 5 quintile portfolios reported in Panels A–B. All tests have a null hypothesis of a flat pattern (no relation). While the MR Up and MR Down tests have alternative hypotheses of an increasing and decreasing pattern, the MR test is unrestricted. Bold p-values indicate significance at the 5% or better. The tests are estimated using 10,0000 bootstrap replications and a block length equal to 10 months. The sample ranges from January 1998 to December 2022.

B Robustness

B.1 Predicting excess market returns

Table 6: Risk-neutral tail risk measures estimated using the first 5 PCs

	One-day ($h = 1$)			One-week ($h = 5$)			One-month ($h = 22$)		
$\lambda_t^{\mathbb{P}}$	0.015		0.010	0.058		0.030	0.367**		0.233
	(0.910)		(0.593)	(1.365)		(0.709)	(2.556)		(1.574)
$\lambda_t^{\mathbb{Q}}$		0.017	0.014		0.094**	0.084*		0.480***	0.403***
		(1.044)	(0.794)		(2.182)	(1.949)		(4.658)	(3.891)
R^2	0.014	0.019	0.024	0.051	0.133	0.145	0.528	0.905	1.093

Note: The table reports the regression coefficients and robust t-statistics (in parentheses) of daily predictive regressions for excess returns over one-day ($h = 1$), one-week ($h = 5$), and one-month ($h = 22$) horizons. For forecasting horizons larger than 1 day, we compound the daily return from $t + 1$ to $t + h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. We denote with *, **, and *** significance at the 10%, 5% and 1% level, respectively. The R^2 is the OLS R-squared reported in percentage points. The sample ranges from January 1996 to December 2022.

Table 7: Risk-neutral tail risk measures estimated using the first 10 PCs

	One-day ($h = 1$)			One-week ($h = 5$)			One-month ($h = 22$)		
$\lambda_t^{\mathbb{P}}$	0.015		0.005	0.058		0.011	0.367**		0.215
	(0.910)		(0.291)	(1.365)		(0.263)	(2.556)		(1.484)
$\lambda_t^{\mathbb{Q}}$		0.028*	0.027		0.135***	0.131***		0.499***	0.422***
		(1.674)	(1.475)		(3.208)	(3.165)		(4.485)	(3.901)
R^2	0.014	0.053	0.054	0.051	0.274	0.276	0.528	0.978	1.136

Note: The table reports the regression coefficients and robust t-statistics (in parentheses) of daily predictive regressions for excess returns over one-day ($h = 1$), one-week ($h = 5$), and one-month ($h = 22$) horizons. For forecasting horizons larger than 1 day, we compound the daily return from $t + 1$ to $t + h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. We denote with *, **, and *** significance at the 10%, 5% and 1% level, respectively. The R^2 is the OLS R-squared reported in percentage points. The sample ranges from January 1996 to December 2022.

Table 8: Tail risk measures estimated using the first 7 PCs and without overnight returns

	One-day ($h = 1$)			One-week ($h = 5$)			One-month ($h = 22$)		
$\lambda_t^{\mathbb{P}}$	0.020		0.005	0.062		0.014	$0.289^{\star}$		0.164
	(1.298)		(0.300)	(1.331)		(0.280)	(1.790)		(0.936)
$\lambda_t^{\mathbb{Q}}$		$0.039^{\star\star}$	$0.037^{\star\star}$		$0.125^{\star\star\star}$	$0.119^{\star\star\star}$		$0.374^{\star\star\star}$	$0.308^{\star\star}$
		(2.492)	(2.122)		(3.039)	(2.816)		(3.030)	(2.297)
R^2	0.026	0.099	0.101	0.057	0.235	0.237	0.327	0.550	0.638

Note: The table reports the regression coefficients and robust t-statistics (in parentheses) of daily predictive regressions for excess returns over one-day ($h = 1$), one-week ($h = 5$), and one-month ($h = 22$) horizons. For forecasting horizons larger than 1 day, we compound the daily return from $t + 1$ to $t + h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. We denote with *, **, and *** significance at the 10%, 5% and 1% level, respectively. The R^2 is the OLS R-squared reported in percentage points. The sample ranges from January 1996 to December 2022.

Table 9: Risk-neutral tail risk measures estimated using the first 7 PCs and $\gamma = -2$

	One-day ($h = 1$)			One-week ($h = 5$)			One-month ($h = 22$)		
$\lambda_t^{\mathbb{P}}$	0.015		0.007	0.058		0.013	$0.367^{\star\star}$		0.232
	(0.910)		(0.435)	(1.365)		(0.302)	(2.556)		(1.577)
$\lambda_t^{\mathbb{Q}}$		0.023	0.020		$0.129^{\star\star\star}$	$0.125^{\star\star\star}$		$0.456^{\star\star\star}$	$0.372^{\star\star\star}$
		(1.435)	(1.214)		(2.996)	(2.920)		(4.069)	(3.353)
R^2	0.014	0.033	0.036	0.051	0.252	0.255	0.528	0.816	0.999

Note: The table reports the regression coefficients and robust t-statistics (in parentheses) of daily predictive regressions for excess returns over one-day ($h = 1$), one-week ($h = 5$), and one-month ($h = 22$) horizons. For forecasting horizons larger than 1 day, we compound the daily return from $t + 1$ to $t + h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. We denote with *, **, and *** significance at the 10%, 5% and 1% level, respectively. The R^2 is the OLS R-squared reported in percentage points. The sample ranges from January 1996 to December 2022.

Table 10: Tail risk measures estimated by setting the threshold (u_t) to 0.5%

	One-day ($h = 1$)			One-week ($h = 5$)			One-month ($h = 22$)		
$\lambda_t^{\mathbb{P}}$	0.013		0.000	0.101**		0.036	0.473***		0.308**
	(0.860)		(0.017)	(2.390)		(0.829)	(3.505)		(2.132)
$\lambda_t^{\mathbb{Q}}$		0.028*	0.027		0.158***	0.141***		0.499***	0.357***
		(1.743)	(1.552)		(3.767)	(3.310)		(4.600)	(3.152)
R^2	0.011	0.050	0.050	0.154	0.375	0.390	0.876	0.978	1.270

Note: The table reports the regression coefficients and robust t-statistics (in parentheses) of daily predictive regressions for excess returns over one-day ($h = 1$), one-week ($h = 5$), and one-month ($h = 22$) horizons. For forecasting horizons larger than 1 day, we compound the daily return from $t + 1$ to $t + h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. We denote with *, **, and *** significance at the 10%, 5% and 1% level, respectively. The R^2 is the OLS R-squared reported in percentage points. The risk-neutral measure is estimated using the first 7 PCs and $\gamma = -3$. The sample ranges from January 1996 to December 2022.

Table 11: Tail risk measures estimated by setting the threshold (u_t) to 5%

	One-day ($h = 1$)			One-week ($h = 5$)			One-month ($h = 22$)		
$\lambda_t^{\mathbb{P}}$	0.029		0.028	0.077		0.061	0.191		0.110
	(1.398)		(1.363)	(1.460)		(1.167)	(1.449)		(0.803)
$\lambda_t^{\mathbb{Q}}$		0.012	0.003		0.070*	0.051		0.300***	0.267**
		(0.739)	(0.222)		(1.654)	(1.265)		(2.759)	(2.399)
R^2	0.057	0.010	0.058	0.088	0.074	0.124	0.144	0.354	0.396

Note: The table reports the regression coefficients and robust t-statistics (in parentheses) of daily predictive regressions for excess returns over one-day ($h = 1$), one-week ($h = 5$), and one-month ($h = 22$) horizons. For forecasting horizons larger than 1 day, we compound the daily return from $t + 1$ to $t + h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. We denote with *, **, and *** significance at the 10%, 5% and 1% level, respectively. The R^2 is the OLS R-squared reported in percentage points. The risk-neutral measure is estimated using the first 7 PCs and $\gamma = -3$. The sample ranges from January 1996 to December 2022.

B.2 Predicting variance risk premium

Table 12: Risk-neutral tail risk measures estimated using the first 5 PCs

	One-day ($h = 1$)			One-week ($h = 5$)			One-month ($h = 22$)		
$\lambda_t^{\mathbb{P}}$	2.161***		1.307***	7.917***		4.371**	9.255		−0.119
	(5.680)		(3.815)	(3.451)		(2.171)	(0.758)		(−0.010)
$\lambda_t^{\mathbb{Q}}$		3.645***	3.308***		14.859***	13.731***		36.264***	36.295***
		(8.701)	(8.441)		(5.799)	(5.806)		(3.718)	(3.842)
R^2	0.991	2.823	3.161	0.648	2.286	2.470	0.076	1.174	1.174

Note: The table reports regression coefficient and robust t-statistics (in parentheses) of daily predictive regressions for the variance risk premium (VRP) over one-day ($h = 1$), one-week ($h = 5$), and one-month ($h = 22$) horizons. For forecasting horizons larger than 1 day, we aggregate the variance risk premium from $t + 1$ to $t + h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. We denote with ⋆, ⋆⋆, and ⋆⋆⋆ significance at the 10%, 5% and 1% level, respectively. The R^2 is the OLS R-squared reported in percentage points. The sample ranges from January 1998 to December 2022.

Table 13: Risk-neutral tail risk measures estimated using the first 10 PCs

	One-day ($h = 1$)			One-week ($h = 5$)			One-month ($h = 22$)		
$\lambda_t^{\mathbb{P}}$	2.161***		1.032***	7.917***		3.360*	9.255		−3.114
	(5.680)		(3.032)	(3.451)		(1.708)	(0.758)		(−0.271)
$\lambda_t^{\mathbb{Q}}$		4.251***	3.957***		16.917***	15.960***		42.434***	43.322***
		(9.985)	(9.875)		(6.366)	(6.507)		(3.963)	(4.372)
R^2	0.991	3.842	4.050	0.648	2.966	3.074	0.076	1.609	1.617

Note: The table reports regression coefficient and robust t-statistics (in parentheses) of daily predictive regressions for the variance risk premium (VRP) over one-day ($h = 1$), one-week ($h = 5$), and one-month ($h = 22$) horizons. For forecasting horizons larger than 1 day, we aggregate the variance risk premium from $t + 1$ to $t + h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. We denote with ⋆, ⋆⋆, and ⋆⋆⋆ significance at the 10%, 5% and 1% level, respectively. The R^2 is the OLS R-squared reported in percentage points. The sample ranges from January 1998 to December 2022.

Table 14: Tail risk measures estimated using the first 7 PCs and without overnight returns

	One-day ($h = 1$)			One-week ($h = 5$)			One-month ($h = 22$)		
$\lambda_t^{\mathbb{P}}$	3.037***		1.679***	11.498***		6.134***	24.975*		9.617
	(8.898)		(5.032)	(5.114)		(2.753)	(1.944)		(0.736)
$\lambda_t^{\mathbb{Q}}$		4.573***	4.005***		17.895***	15.817***		48.550***	45.294***
		(12.484)	(10.855)		(7.952)	(6.942)		(4.509)	(4.102)
R^2	1.955	4.438	4.967	1.365	3.312	3.656	0.555	2.101	2.174

Note: The table reports regression coefficient and robust t-statistics (in parentheses) of daily predictive regressions for the variance risk premium (VRP) over one-day ($h = 1$), one-week ($h = 5$), and one-month ($h = 22$) horizons. For forecasting horizons larger than 1 day, we aggregate the variance risk premium from $t + 1$ to $t + h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. We denote with *, **, and *** significance at the 10%, 5% and 1% level, respectively. The R^2 is the OLS R-squared reported in percentage points. The sample ranges from January 1998 to December 2022.

Table 15: Risk-neutral tail risk measures estimated using the first 7 PCs and $\gamma = -2$

	One-day ($h = 1$)			One-week ($h = 5$)			One-month ($h = 22$)		
$\lambda_t^{\mathbb{P}}$	2.161***		1.071***	7.917***		3.578*	9.255		−2.545
	(5.680)		(3.175)	(3.451)		(1.811)	(0.758)		(−0.213)
$\lambda_t^{\mathbb{Q}}$		4.006***	3.690***		15.745***	14.689***		39.195***	39.947***
		(9.745)	(9.706)		(6.095)	(6.206)		(3.723)	(3.929)
R^2	0.991	3.408	3.631	0.648	2.566	2.687	0.076	1.371	1.376

Note: The table reports regression coefficient and robust t-statistics (in parentheses) of daily predictive regressions for the variance risk premium (VRP) over one-day ($h = 1$), one-week ($h = 5$), and one-month ($h = 22$) horizons. For forecasting horizons larger than 1 day, we aggregate the variance risk premium from $t + 1$ to $t + h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. We denote with *, **, and *** significance at the 10%, 5% and 1% level, respectively. The R^2 is the OLS R-squared reported in percentage points. The sample ranges from January 1998 to December 2022.

Table 16: Tail risk measures estimated by setting the threshold (u_t) to 0.5%

	One-day ($h = 1$)			One-week ($h = 5$)			One-month ($h = 22$)		
$\lambda_t^{\mathbb{P}}$	3.208***		1.621***	12.375***		5.930***	21.335*		2.538
	(7.999)		(4.747)	(5.063)		(2.983)	(1.795)		(0.219)
$\lambda_t^{\mathbb{Q}}$		4.616***	3.966***		18.484***	16.107***		47.989***	46.972***
		(10.040)	(9.284)		(6.813)	(6.684)		(4.610)	(4.701)
R^2	2.185	4.516	4.985	1.585	3.530	3.836	0.406	2.051	2.056

Note: The table reports regression coefficient and robust t-statistics (in parentheses) of daily predictive regressions for the variance risk premium (VRP) over one-day ($h = 1$), one-week ($h = 5$), and one-month ($h = 22$) horizons. For forecasting horizons larger than 1 day, we aggregate the variance risk premium from $t + 1$ to $t + h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. We denote with *, **, and *** significance at the 10%, 5% and 1% level, respectively. The R^2 is the OLS R-squared reported in percentage points. The risk-neutral tail risk is estimated using the first 7 PCs and $\gamma = -3$. The sample ranges from January 1998 to December 2022.

Table 17: Tail risk measures estimated by setting the threshold (u_t) to 5%

	One-day ($h = 1$)			One-week ($h = 5$)			One-month ($h = 22$)		
$\lambda_t^{\mathbb{P}}$	1.210***		0.365	3.670		0.356	9.940		1.357
	(2.633)		(0.812)	(1.195)		(0.118)	(0.955)		(0.136)
$\lambda_t^{\mathbb{Q}}$		2.917***	2.807***		11.115***	11.008***		28.922***	28.514***
		(8.181)	(8.338)		(5.204)	(5.536)		(3.093)	(3.207)
R^2	0.312	1.812	1.837	0.140	1.282	1.283	0.088	0.748	0.750

Note: The table reports regression coefficient and robust t-statistics (in parentheses) of daily predictive regressions for the variance risk premium (VRP) over one-day ($h = 1$), one-week ($h = 5$), and one-month ($h = 22$) horizons. For forecasting horizons larger than 1 day, we aggregate the variance risk premium from $t + 1$ to $t + h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. We denote with *, **, and *** significance at the 10%, 5% and 1% level, respectively. The R^2 is the OLS R-squared reported in percentage points. The risk-neutral tail risk is estimated using the first 7 PCs and $\gamma = -3$. The sample ranges from January 1998 to December 2022.

B.3 Predictive regressions with controls

Table 18: One-day ahead predictive excess market return regressions

	$\lambda_t^{\mathbb{P}}$	$\lambda_t^{\mathbb{Q}}$	REV	MOM	RK	RSK	RV	MAX	MIN	VRP	LTV	RTV	R^2
I.I	0.012												0.010
	(0.680)												
I.II	0.003		$-0.0766^{\star\star}$										0.379
	(0.183)		(−2.364)										
I.III	0.012			0.004									0.011
	(0.644)			(0.204)									
I.IV	0.013				0.013								0.021
	(0.690)				(0.873)								
I.V	0.012					−0.003							0.010
	(0.647)					(−0.213)							
I.VI	0.002						0.077						0.389
	(0.125)						(1.248)						
I.VII	0.015							−0.031					0.071
	(0.835)							(−0.935)					
I.VIII	0.007								−0.031				0.069
	(0.397)								(−0.894)				
I.IX	0.002									$0.095^{\star\star}$			0.584
	(0.128)									(2.477)			
I.X	0.006										0.045		0.135
	(0.312)										(1.163)		
I.XI	0.012											−0.003	0.011
	(0.658)											(−0.263)	
I.XII	0.002		−0.047	0.012	0.007	0.005	0.090	−0.024	0.077	$0.079^{\star}$	−0.006	−0.001	1.047
	(0.106)		(−0.796)	(0.547)	(0.426)	(0.311)	(0.933)	(−0.349)	(1.327)	(1.768)	(−0.123)	(−0.059)	
II.I		$0.041^{\star\star}$											0.107
		(2.231)											
II.II		0.028	$-0.072^{\star\star}$										0.427
		(1.611)	(−2.244)										
II.III		$0.041^{\star\star}$		0.009									0.112
		(2.283)		(0.424)									
II.IV		$0.041^{\star\star}$			0.013								0.119
		(2.246)			(0.912)								
II.V		$0.041^{\star\star}$				−0.002							0.108
		(2.217)				(−0.175)							
II.VI		0.021					0.072						0.416
		(1.043)					(1.114)						
II.VII		$0.048^{\star\star\star}$						−0.039					0.201
		(2.633)						(−1.161)					
II.VIII		$0.034^{\star}$							−0.021				0.132
		(1.911)							(−0.590)				
II.IX		0.022								$0.091^{\star\star}$			0.614
		(1.212)								(2.338)			
II.X		$0.031^{\star}$									0.038		0.192
		(1.851)									(0.972)		
II.XI		$0.041^{\star\star}$										−0.002	0.108
		(2.224)										(−0.179)	
II.XII		0.028	−0.046	0.012	0.007	0.006	0.089	−0.026	0.082	$0.079^{\star}$	−0.008	0.000	1.091
		(1.558)	(−0.780)	(0.532)	(0.401)	(0.408)	(0.916)	(−0.382)	(1.411)	(1.755)	(−0.159)	(0.001)	
III.I	0.002	$0.040^{\star\star}$											0.107
	(0.107)	(2.223)											
III.II	−0.004	$0.029^{\star}$	$-0.072^{\star\star}$										0.428
	(−0.199)	(1.675)	(−2.243)										
III.III	0.001	$0.041^{\star\star}$		0.009									0.112
	(0.036)	(2.306)		(0.415)									
III.IV	0.002	$0.041^{\star\star}$			0.013								0.119
	(0.114)	(2.237)			(0.913)								
III.V	0.002	$0.040^{\star\star}$				−0.002							0.108
	(0.087)	(2.218)				(−0.158)							
III.VI	−0.003	0.022					0.072						0.417
	(−0.148)	(1.098)					(1.114)						
III.VII	0.003	$0.047^{\star\star}$						−0.039					0.201
	(0.194)	(2.597)						(−1.165)					
III.VIII	0.000	$0.034^{\star}$							−0.021				0.132
	(0.004)	(1.922)							(−0.588)				
III.IX	−0.003	0.023								$0.091^{\star\star}$			0.615
	(−0.175)	(1.271)								(2.338)			
III.X	−0.002	$0.032^{\star}$									0.038		0.192
	(−0.091)	(1.863)									(0.974)		
III.XI	0.002	$0.040^{\star\star}$										−0.002	0.108
	(0.095)	(2.219)										(−0.166)	
III.XII	−0.004	0.029	−0.046	0.013	0.007	0.005	0.089	−0.025	0.082	$0.079^{\star}$	−0.008	0.000	1.092
	(−0.202)	(1.611)	(−0.783)	(0.558)	(0.403)	(0.369)	(0.916)	(−0.376)	(1.408)	(1.755)	(−0.155)	(−0.019)	

Note: The table reports in three panels the one-day ahead daily predictive regressions coefficient and robust t-statistics (in parentheses) for the excess market returns. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. We denote with *, **, and *** significance at the 10%, 5% and 1% level, respectively. The R^2 is the OLS R-squared reported in percentage points. The sample ranges from January 1998 to December 2021.

Table 19: One-week ahead predictive excess market return regressions

	$\lambda_t^{\mathbb{P}}$	$\lambda_t^{\mathbb{Q}}$	REV	MOM	RK	RSK	RV	MAX	MIN	VRP	LTV	RTV	R^2
I.I	0.049												0.036
	(1.015)												
I.II	0.024		−0.211**										0.695
	(0.507)		(−2.346)										
I.III	0.046			0.026									0.046
	(0.943)			(0.334)									
I.IV	0.049				−0.007								0.037
	(1.014)				(−0.215)								
I.V	0.044					−0.032							0.051
	(0.899)					(−1.093)							
I.VI	0.035						0.104						0.197
	(0.771)						(0.795)						
I.VII	0.056							−0.080					0.132
	(1.180)							(−0.805)					
I.VIII	0.037								−0.071				0.111
	(0.779)								(−0.546)				
I.IX	0.026									0.214			0.716
	(0.535)									(1.580)			
I.X	0.045										0.029		0.048
	(0.946)										(0.212)		
I.XI	0.046											−0.025	0.045
	(0.967)											(−0.635)	
I.XII	0.021		−0.262*	0.045	−0.005	−0.003	0.081	0.098	0.217	0.214	−0.101	−0.016	1.298
	(0.425)		(−1.758)	(0.520)	(−0.153)	(−0.084)	(0.6430	(0.726)	(1.1410	(1.327)	(−0.674)	(−0.394)	
II.I		0.125**											0.235
		(2.575)											
II.II		0.089**	−0.197**										0.803
		(1.989)	(−2.221)										
II.III		0.128***		0.041									0.260
		(2.679)		(0.527)									
II.IV		0.125***			−0.005								0.235
		(2.578)			(−0.164)								
II.V		0.123**				−0.031							0.249
		(2.525)				(−1.100)							
II.VI		0.103**					0.081						0.327
		(2.459)					(0.611)						
II.VII		0.145***						−0.103					0.388
		(3.307)						(−1.039)					
II.VIII		0.111**							−0.041				0.257
		(2.426)							(−0.305)				
II.IX		0.084*								0.199			0.808
		(1.776)								(1.472)			
II.X		0.124***									0.004		0.235
		(2.915)									(0.031)		
II.XI		0.124**										−0.023	0.242
		(2.558)										(−0.583)	
II.XII		0.104**	−0.259*	0.046	−0.007	0.000	0.077	0.091	0.236	0.212	−0.107	−0.014	1.435
		(2.202)	(−1.734)	(0.537)	(−0.193)	(0.014)	(0.605)	(0.680)	(1.229)	(1.322)	(−0.708)	(−0.349)	
III.I	0.018	0.120***											0.239
	(0.385)	(2.583)											
III.II	0.003	0.089**	−0.197**										0.803
	(0.056)	(2.038)	(−2.217)										
III.III	0.012	0.125***		0.039									0.262
	(0.260)	(2.768)		(0.503)									
III.IV	0.018	0.120***			−0.005								0.239
	(0.384)	(2.585)			(−0.160)								
III.V	0.014	0.120**				−0.030							0.252
	(0.288)	(2.564)				(−1.008)							
III.VI	0.013	0.100**					0.080						0.329
	(0.278)	(2.438)					(0.605)						
III.VII	0.022	0.139***						−0.104					0.395
	(0.474)	(3.295)						(−1.048)					
III.VIII	0.014	0.108**							−0.040				0.260
	(0.308)	(2.436)							(−0.294)				
III.IX	0.007	0.082*								0.199			0.808
	(0.141)	(1.841)								(1.464)			
III.X	0.018	0.120***									0.003		0.239
	(0.379)	(2.907)									(0.019)		
III.XI	0.016	0.119**										−0.021	0.246
	(0.343)	(2.571)										(−0.551)	
III.XII	0.000	0.104**	−0.259*	0.046	−0.007	0.000	0.077	0.091	0.236	0.212	−0.107	−0.014	1.435
	(0.007)	(2.301)	(−1.732)	(0.530)	(−0.193)	(0.016)	(0.605)	(0.675)	(1.229)	(1.321)	(−0.708)	(−0.350)	

Note: The table reports in three panels the one-week ahead daily predictive regressions coefficient and robust t-statistics (in parentheses) for the excess market returns. The market return is the compounded return from $t+1$ to $t+h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. We denote with *, **, and *** significance at the 10%, 5% and 1% level, respectively. The R^2 is the OLS R-squared reported in percentage points. The sample ranges from January 1998 to December 2021.

Table 20: One-month ahead predictive excess market return regressions

	$\lambda_t^{\mathbb{P}}$	$\lambda_t^{\mathbb{Q}}$	REV	MOM	RK	RSK	RV	MAX	MIN	VRP	LTV	RTV	R^2
I.I	0.371**												0.539
	(2.394)												
I.II	0.346**		−0.213										0.714
	(2.179)		(−1.301)										
I.III	0.359**			0.099									0.576
	(2.329)			(0.319)									
I.IV	0.370**				−0.046								0.547
	(2.390)				(−0.610)								
I.V	0.361**					−0.065							0.555
	(2.273)					(−1.066)							
I.VI	0.342**						0.220						0.726
	(2.199)						(0.602)						
I.VII	0.360**							0.126					0.601
	(2.352)							(0.366)					
I.VIII	0.338**								−0.195				0.684
	(2.042)								(−0.527)				
I.IX	0.294*									0.730**			2.602
	(1.838)									(2.484)			
I.X	0.350**										0.139		0.613
	(2.203)										(0.405)		
I.XI	0.381**											0.099	0.577
	(2.450)											(0.958)	
I.XII	0.284*		−0.185	0.311	−0.035	−0.043	0.009	0.497*	0.342	1.023**	−0.390	0.141	3.562
	(1.717)		(−0.576)	(1.119)	(−0.481)	(−0.624)	(0.035)	(1.670)	(0.758)	(2.536)	(−1.182)	(1.434)	
II.I		0.409***											0.656
		(3.505)											
II.II		0.376***	−0.187										0.788
		(3.323)	(−1.173)										
II.III		0.421***		0.172									0.771
		(3.796)		(0.563)									
II.IV		0.408***			−0.044								0.664
		(3.516)			(−0.586)								
II.V		0.404***				−0.096							0.692
		(3.432)				(−1.716)							
II.VI		0.364***					0.167						0.757
		(3.708)					(0.455)						
II.VII		0.393***						0.082					0.681
		(4.255)						(0.240)					
II.VIII		0.366***							−0.132				0.717
		(3.307)							(−0.348)				
II.IX		0.265**								0.706**			2.530
		(2.160)								(2.402)			
II.X		0.385***									0.095		0.689
		(3.774)									(0.281)		
II.XI		0.414***										0.084	0.683
		(3.572)										(0.832)	
II.XII		0.325***	−0.190	0.354	−0.037	−0.066	−0.004	0.502*	0.383	1.017**	−0.393	0.130	3.639
		(2.954)	(−0.592)	(1.278)	(−0.505)	(−1.022)	(−0.017)	(1.695)	(0.838)	(2.542)	(−1.207)	(1.345)	
III.I	0.284	0.336***											0.951
	(1.809)	(2.892)											
III.II	0.271*	0.309***	−0.166										1.055
	(1.693)	(2.772)	(−1.031)										
III.III	0.264*	0.350***		0.135									1.020
	(1.722)	(3.320)		(0.441)									
III.IV	0.284*	0.335***			−0.041								0.957
	(1.806)	(2.898)			(−0.551)								
III.V	0.275*	0.334***				−0.059							0.964
	(1.718)	(2.876)				(−0.960)							
III.VI	0.274*	0.298***					0.149						1.031
	(1.735)	(3.057)					(0.409)						
III.VII	0.281*	0.323***						0.071					0.970
	(1.791)	(3.456)						(0.209)					
III.VIII	0.274*	0.303***							−0.107				0.990
	(1.676)	(2.927)							(−0.279)				
III.IX	0.245	0.204*								0.693**			2.748
	(1.539)	(1.731)								(2.343)			
III.X	0.277*	0.320***									0.069		0.968
	(1.729)	(3.184)									(0.205)		
III.XI	0.294*	0.339***										0.109	0.997
	(1.865)	(2.938)										(1.080)	
III.XII	0.229	0.279***	−0.177	0.313	−0.039	−0.034	−0.004	0.479	0.392	1.017***	−0.405	0.146	3.817
	(1.406)	(2.709)	(−0.553)	(1.126)	(−0.532)	(−0.495)	(−0.014)	(1.614)	(0.857)	(2.551)	(−1.249)	(1.495)	

Note: The table reports in three panels the one-month ahead daily predictive regressions coefficient and robust t-statistics (in parentheses) for the excess market returns. The market return is the compounded return from $t+1$ to $t+h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. We denote with *, **, and *** significance at the 10%, 5% and 1% level, respectively. The R^2 is the OLS R-squared reported in percentage points. The sample ranges from January 1998 to December 2021.

Table 21: One-day ahead predictive variance risk premium regressions

	$\lambda_t^{\mathbb{P}}$	$\lambda_t^{\mathbb{Q}}$	RV	JV	LTV	RTV	R^2
I.I	2.072***						0.900
	(5.131)						
I.II	1.088***		7.572***				12.788
	(3.119)		(5.267)				
I.III	1.961***			2.838***			2.595
	(4.941)			(2.745)			
I.IV	0.655**				9.242***		18.395
	(2.040)				(7.226)		
I.V	1.980***					−0.902***	1.070
	(4.905)					(−3.575)	
I.VI	0.432		3.994**	−1.252	7.206***	−0.639***	20.184
	(1.394)		(2.440)	(−1.188)	(5.411)	(−2.979)	
II.I		3.852***					3.113
		(8.584)					
II.II		1.911***	7.200***				13.255
		(4.808)	(4.936)				
II.III		3.558***		2.496**			4.408
		(7.992)		(2.520)			
II.IV		1.594***			8.939***		18.806
		(4.367)			(6.936)		
II.V		3.801***				−0.884***	3.277
		(8.485)				(−3.873)	
II.VI		1.145***	3.801**	−1.236*	7.095***	−0.627***	20.397
		(3.162)	(2.304)	(−1.177)	(5.335)	(−3.028)	
III.I	1.152***	3.554***					3.372
	(3.219)	(8.519)					
III.II	0.691**	1.744***	7.155***				13.347
	(2.128)	(4.602)	(4.907)				
III.III	1.130***	3.267***		2.487**			4.658
	(3.202)	(7.902)		(2.513)			
III.IV	0.312	1.520***			8.910***		18.825
	(1.018)	(4.281)			(6.907)		
III.V	1.079***	3.528***				−0.792***	3.503
	(3.010)	(8.457)				(−3.591)	
III.VI	0.195	1.100***	3.795**	−1.231	7.079***	−0.610***	20.404
	(0.651)	(3.100)	(2.300)	(−1.172)	(5.319)	(−2.963)	

Note: The table reports in three panels the one-day ahead predictive regression coefficients and robust t-statistics (in parentheses) for the market variance risk premium. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. The R^2 is the OLS R-squared reported in percentage points. The controls are defined in Appendix C. The Sample ranges from January 1998 to December 2021.

Table 22: One-week ahead predictive variance risk premium regressions

	$\lambda_t^{\mathbb{P}}$	$\lambda_t^{\mathbb{Q}}$	RV	JV	LTV	RTV	R^2
I.I	7.480***						0.572
	(3.053)						
I.II	4.099**		26.030***				7.418
	(2.034)		(3.381)				
I.III	7.083***			10.226**			1.644
	(2.977)			(2.032)			
I.IV	1.638				38.113***		15.072
	(0.825)				(3.555)		
I.V	7.015***					−4.599***	0.787
	(2.878)					(−2.946)	
I.VI	0.998		7.447	−3.276	34.516***	−3.473**	15.481
	(0.513)		(1.075)	(−0.736)	(3.412)	(−2.514)	
II.I		15.198***					2.361
		(5.484)					
II.II		8.664***	24.235***				7.961
		(4.613)	(3.120)				
II.III		14.156***		8.840*			3.153
		(5.450)		(1.863)			
II.IV		5.882**			36.878***		15.376
		(2.379)			(3.379)		
II.V		14.942***				−4.451***	2.565
		(5.423)				(−3.056)	
II.VI		5.052**	6.527	−3.177	33.923***	−3.334**	15.709
		(2.056)	(0.932)	(−0.721)	(3.326)	(−2.491)	
III.I	3.801*	14.215***					2.499
	(1.792)	(5.580)					
III.II	2.249	8.122***	24.091***				8.009
	(1.158)	(4.514)	(3.109)				
III.III	3.722*	13.197***		8.808*			3.285
	(1.777)	(5.539)		(1.865)			
III.IV	0.330	5.805**			36.847***		15.377
	(0.181)	(2.452)			(3.372)		
III.V	3.417	14.076***				−4.158**	2.675
	(1.613)	(5.540)				(−2.924)	
III.VI	−0.097	5.074**	6.530	−3.180	33.931***	−3.342**	15.709
	−(0.054)	(2.138)	(0.933)	(−0.723)	(3.322)	(−2.496)	

Note: The table reports in three panels the one-week ahead predictive regression coefficients and robust t-statistics (in parentheses) for the market variance risk premium. The weekly variance risk premium is estimated as the aggregate from $t+1$ to $t+h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. The R^2 is the OLS R-squared reported in percentage points. The controls are defined in Appendix C. The Sample ranges from January 1998 to December 2021.

Table 23: One-month ahead predictive variance risk premium regressions

	$\lambda_t^{\mathbb{P}}$	$\lambda_t^{\mathbb{Q}}$	RV	JV	LTV	RTV	R^2
I.I	7.118						0.044
	(0.550)						
I.II	4.140		22.923				0.500
	(0.331)		(0.819)				
I.III	5.851			32.569*			0.979
	(0.449)			(1.806)			
I.IV	−9.767				110.150***		10.446
	(−0.786)				(2.920)		
I.V	5.228					−18.680***	0.350
	(0.402)					(−2.617)	
I.VI	−7.834		−83.020***	34.209*	149.250***	−15.092**	13.716
	(−0.665)		(−2.834)	(1.711)	(4.018)	(−2.288)	
II.I		35.936***					1.134
		(3.437)					
II.II		31.920***	14.897				1.316
		(4.275)	(0.542)				
II.III		32.521***		28.982*			1.865
		(3.316)		(1.712)			
II.IV		9.067			106.364***		10.432
		(1.130)			(2.816)		
II.V		34.948***				−17.211***	1.395
		(3.376)				(−2.651)	
II.VI		17.668**	−87.124***	34.949*	145.911***	−13.547**	13.914
		(2.524)	(−2.959)	(1.767)	(3.952)	(−2.209)	
III.I	−2.339	36.541***					1.138
	(−0.184)	(3.619)					
III.II	−3.313	32.719***	15.111				1.325
	(−0.263)	(4.392)	(0.551)				
III.III	−2.600	33.190***		29.004*			1.870
	(−0.204)	(3.594)		(1.711)			
III.IV	−12.471	11.996			107.534***		10.558
	(−1.019)	(1.620)			(2.837)		
III.V	−3.962	35.953***				−17.551***	1.408
	(−0.310)	(3.587)				(−2.626)	
III.VI	−12.249	20.461***	−86.718***	34.596*	146.892***	−14.565**	14.034
	(−1.042)	(3.032)	(−2.969)	(1.759)	(3.961)	(−2.282)	

Note: The table reports in three panels the one-week ahead predictive regression coefficients and robust t-statistics (in parentheses) for the market variance risk premium. The monthly variance risk premium is estimated as the aggregate from $t+1$ to $t+h$. We compute the t-statistics using Newey-West robust standard errors with a lag length equal to h. The R^2 is the OLS R-squared reported in percentage points. The controls are defined in Appendix C. The Sample ranges from January 1998 to December 2021.

B.4 Portfolio sorts

Table 24: Monthly sorted portfolios after removing stocks with market cap smaller than the 10th percentile

	Low	2	3	4	High	High-Low	t-stat
Panel A: $\lambda_t^{\mathbb{P}}$							
Average Return	0.890	0.685	0.613	0.580	0.789	−0.101	−0.583
CAPM alpha	0.087	0.039	0.019	−0.020	0.119	0.033	0.198
FF3 alpha	0.020	−0.039	−0.061	−0.105	0.034	0.014	0.099
FF5 alpha	0.129	−0.097	−0.164	−0.176	0.089	−0.040	−0.258
FF5 + Mom alpha	0.147	−0.079	−0.144	−0.147	0.133	−0.014	−0.085
FF5 + Mom + Liq alpha	0.122	−0.093	−0.165	−0.158	0.134	0.012	0.077
Panel B: $\lambda_t^{\mathbb{Q}}$							
Average Return	1.243	0.727	0.608	0.550	0.429	−0.814	−4.235
CAPM alpha	0.503	0.100	0.006	−0.062	−0.302	−0.806	−3.781
FF3 alpha	0.408	0.013	−0.071	−0.141	−0.358	−0.766	−3.955
FF5 alpha	0.375	−0.093	−0.178	−0.181	−0.142	−0.517	−2.802
FF5 + Mom alpha	0.387	−0.071	−0.163	−0.148	−0.093	−0.480	−2.302
FF5 + Mom + Liq alpha	0.353	−0.098	−0.188	−0.166	−0.060	−0.414	−1.905

Panel C: Monotonicity Test	MR	MR Up	MR Down
Avg. return $\lambda_t^{\mathbb{P}}$	0.924	0.134	0.215
Avg. return $\lambda_t^{\mathbb{Q}}$	**0.005**	0.923	**0.000**

Note: The table reports the results of univariate portfolio analyses of the relation between the tail risk measures and the cross-section of returns. Monthly portfolios are formed by sorting the CRSP stocks with share code 10 and 11, excluding those with a share price below \$5 and a market cap smaller than the 10th percentile based on the ME breakpoints obtained from French's website, into portfolios using quintile breakpoints calculated based on the given sort variable. The table also reports portfolio alphas from regression of portfolio excess return using the Fama-French three and five factors as well as extended models controlling for momentum (Carhart, 1997) and liquidity (Pástor and Stambaugh, 2003) factors. Returns and alphas are in percentage terms. The last two columns report the high minus low zero net investment portfolio and associated t-statistics, which are estimated using Newey-West robust standard errors with a lag length equal to 5. Panel C presents the p-values from various tests of the monotonicity (Patton and Timmermann, 2010) of average returns across the 5 quintile portfolios reported in Panels A–B. All tests have a null hypothesis of a flat pattern (no relation). While the MR Up and MR Down tests have alternative hypotheses of an increasing and decreasing pattern, the MR test is unrestricted. Bold p-values indicate significance at the 5% or better. The tests are estimated using 10,0000 bootstrap replications and a block length equal to 10 months. The sample ranges from January 1998 to December 2022.

C Variables definitions

- Reversal (REV): following Jegadeesh (1990) and Lehmann (1990), the short-term reversal variable is defined as the weekly market return over the previous week from day $t-4$ to day t.

- Momentum (MOM): following Jegadeesh and Titman (1993), the momentum variable at the end of day t is defined as the compound gross market return from day $t-252$ through day $t-21$, skipping the short-term reversal month.

- Variance Risk Premium (VRP): we compute the variance risk premium as a short position in a variance swap, namely, as the difference between risk-neutral and physical expectations of the variance of market returns (e.g. Bekaert and Hoerova, 2014; Bollerslev et al., 2009):

$$VRP_t = VIX_t^2 - \mathbb{E}_t\left[RV_{t+1}\right],$$

where VIX_t is the CBOE volatility index scaled at the monthly level, and RV_{t+1} is the SPY realized variance measured over the next month (22 trading days) using 5-minute returns. We compute the physical conditional expected value of the future realized variance using the heterogeneous autoregressive (HAR) model of Corsi (2009) based on a rolling window of 500 days.

- Maximum daily return (Max): the Max variable is defined as the largest total daily market return observed over the previous week (see Bali et al., 2011).

- Minimum daily return (Min): the Min variable is defined as the smallest total daily market return observed over the previous week (see Bali et al., 2011).

- Realized Variance (RV): the realized variance is defined as the sum of the intraday squared returns (e.g., Andersen et al., 2001, 2003):

$$RV_t = \sum_{n=1}^{N} R_{n,t}^2,$$

where $R_{n,t}$ denotes the log-return on the S&P 500 index over the n-th intra-daily time interval on day t.

- Realized Skewness (RSK): the RSK is the ex-post daily realized skewness based on intra-day market returns standardized by the realized variance (e.g., Amaya et al., 2015):

$$RSK_t = \frac{\sqrt{N}\sum_{n=1}^{N} R_{n,t}^3}{RV_t^{3/2}}.$$

- Realized Kurtosis (RK): the RK is the ex-post daily realized kurtosis based on intra-day market returns standardized by the variance (e.g., Amaya et al., 2015):

$$RSK_t = \frac{N\sum_{n=1}^{N} R_{n,t}^4}{RV_t^2}.$$

- Jump Variation (JV): the jump variation is defined as the difference between the RV and a consistent measure of the integrated variance, such as the bipower variation (BV) of Barndorff-Nielsen and Shephard (2004):

$$JV_t = \max\left(RV_t - BV_t, 0\right),$$

where $BV_t = \pi/2(N/(N-1))\sum_{n=2}^{N} |R_{n,t}||R_{n-1,t}|$.

- Left Tail Variation (LTV): the left tail variation proposed by Bollerslev et al. (2015) is an option implied measure of short-horizon downside tail risk obtained from short-dated out-of-the-money put options.

- Right Tail Variation (RTV): the right tail variation proposed by Bollerslev et al. (2015) is an option implied measure of short-horizon upside tail risk obtained from short-dated out-of-the-money call options.