Good Rents versus Bad Rents: R&D Misallocation and Growth

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The views presented here should not be interpreted as reflecting the views of the Federal Reserve Bank of San Francisco or the Board of Governors of the Federal Reserve System.
Question and motivation

A much-studied topic is whether the economy devotes sufficient resources to R&D overall.

Less studied is the question of whether a given amount of R&D resources are efficiently allocated across firms.

We investigate a potentially importance source of R&D misallocation, namely heterogeneity in knowledge spillovers.
Motivating facts from French manufacturing

A firm’s size is positively associated with both its quantity per unit of inputs (its TFPQ) and its average unit price – each when controlling for the other variable

Meager within-firm growth in TFPQ

Firms are much more likely to patent product innovations than process innovations
What we do

Develop a model of endogenous growth where markup dispersion arises from **two sources**: differences in process efficiency and the step size of quality innovations.

Process efficiency is exogenous and permanent while quality grows through creative destruction.

Knowledge spillovers for quality innovations.

We calibrate the model to French manufacturing data and compare the decentralized equilibrium with the social planner’s solution.
What we find

Compared to the decentralized equilibrium, the planner wants to reallocate research effort toward firms with big quality steps

Infer small differences in firm-level process efficiencies and large differences in firm-level quality step sizes from price and revenue productivity data in French manufacturing

~ 30 basis points lower growth in the decentralized equilibrium than socially optimal

Planner also wishes to undo the static misallocation, but this has a small level effect
Roadmap

Theoretical framework

Decentralized equilibrium

Planner’s solution

Calibration and quantitative results
Preferences and resource constraint

Representative household with preferences

\[ U_0 = \sum_{t=0}^{\infty} \beta^t \log C_t \]

Final output is produced using a Cobb-Douglas aggregator of intermediate goods

\[ Y = \exp \left( \int_{0}^{1} \log \left[ q(i) y(i) \right] di \right), \]

where \( q(i) \) denotes the quality of good \( i \) and \( y(i) \) its quantity.
Preferences and resource constraint

Final output can be used for consumption or to cover production overhead

\[ Y = C + O \]

Fixed \( Z \) units of R&D labor

Fixed \( L \) units of production labor

Our focus is on the allocation of R&D across firms, not total R&D
Intermediate goods production

Fixed $J$ intermediate good producers

Firms differ in two primitive dimensions

- their level of process efficiency $\varphi_j$
- the step size of their quality improvements $\gamma_j$

R&D leads to endogenously evolving differences in product-specific quality across firms

Firms endogenously differ in the number of products they produce
Process efficiency

Firm $j$ can produce variety $i$ at quality $q(i,j)$ using production labor

$$y(i,j) = \varphi_j \cdot l(i,j)$$

Binary process efficiency types: high $\varphi_H$ and low $\varphi_L$

High/Low process efficiency ratio: $\Delta \equiv \frac{\varphi_H}{\varphi_L} > 1$
Quality step sizes

Firm $j$ has the knowledge to produce at quality $q(i, j)$ in line $i \in [0, 1]$

$v_z$ units of research labor increases the quality of a randomly drawn line by factor $\gamma_j > 1$

Binary step size types: big $\gamma_B$ and small $\gamma_S$

High/Low quality ratio: $\Gamma \equiv \frac{\gamma_B}{\gamma_S} > 1$
Firm types and market shares

With the two dimensions of binary heterogeneity — high (H) vs. low (L) process efficiency and big (B) vs. small (S) step sizes — we have 4 firm types $k \in \{HB, HS, LB, LS\}$

$\phi_k \equiv$ the fraction of firms of type $k$

$S_k \equiv$ the share of lines operated by firms of type $k$
Boundary of the firm

Per-period overhead cost for firm $j$ “active” in $n(j)$ product lines (expressed in final goods)

$$
\psi_o \cdot \frac{1}{2} n(j)^2 \cdot Y
$$

Convexity yields a well-defined boundary of the firm in the decentralized equilibrium and a non-trivial planner’s solution

Aggregate resources used for overhead are given by

$$
O = \sum_j \psi_o \cdot \frac{1}{2} n(j)^2 \cdot Y
$$
We analyse the Balanced Growth Path (BGP) in which quantities grow at constant rates.

The growth rate is the product-weighted geometric mean of the step sizes raised to $Z/\psi_z$:

$$1 + \bar{g} = \left( \prod_k \gamma_k \right)^{Z/\psi_z}$$

We will compare the planner's $\bar{S}_k$ values with the decentralized equilibrium values.
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Market structure

Both the final goods market and the labor market are competitive.

There is Bertrand competition within each intermediate product line \( i \in [0, 1] \)

We assume \( \gamma_S > \Delta \equiv \frac{\varphi_H}{\varphi_L} \) so the highest quality producer is always active in each line.

In line \( i \) the leading firm \( j(i) \) chooses the markup

\[
\mu(i, j(i), j'(i)) = \frac{\varphi(j(i))}{\varphi(j'(i))} \cdot \frac{q(i, j(i))}{q(i, j'(i))}
\]

where \( j'(i) \) indexes the next highest quality firm.
Firm innovation and value

Each period a firm loses $\frac{Z}{\psi_z} n$ of its $n$ products to creative destruction.

A firm hires $\psi_z x$ R&D labor in a given period to take over $x$ new lines.

$$V_{k,0} = \max_{\{x_t, n_{t+1}\}_{t=0}^\infty} \sum_{t=0}^\infty Y_t \left[ \pi_k(n_t, h_t) - x_t \cdot \psi_z \cdot \frac{w_{z,t}}{Y_t} \right] \prod_{s=0}^t \left( \frac{1}{1 + r_s} \right)$$

subject to

$$n_{t+1} = n_t \cdot (1 - Z/\psi_Z) + x_t, \quad \forall t.$$ 

$$\pi_k(n, h) = n \cdot h \cdot \left( 1 - \frac{\varphi_H}{\varphi_k \cdot \gamma_k} \right) + n \cdot (1 - h) \cdot \left( 1 - \frac{\varphi_L}{\varphi_k \cdot \gamma_k} \right) - \frac{1}{2} \cdot \psi_o \cdot n^2$$
The BGP of the decentralized equilibrium

Along the BGP we have \( x_k = \frac{Z}{\psi_z} n_k \) and \( h = \bar{S}_{HB} + \bar{S}_{HS} \equiv \bar{S} \)

From the firm’s FOC for R&D, any two \( n_k \) and \( n_{k'} \) satisfy

\[
n_k - n_{k'} = \frac{1}{\psi_o} \left[ \bar{S} \cdot \varphi_H \cdot \left( \frac{1}{\varphi_{k'} \cdot \gamma_{k'}} - \frac{1}{\varphi_k \cdot \gamma_k} \right) + (1 - \bar{S}) \cdot \varphi_L \cdot \left( \frac{1}{\varphi_{k'} \cdot \gamma_{k'}} - \frac{1}{\varphi_k \cdot \gamma_k} \right) \right]
\]

- A firm’s size depends **only** on the average markup across its products: \( \mu_k \propto \varphi_k \cdot \gamma_k \)
- Firms with higher markups are larger (more products, sales, and production labor)
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The social planner maximizes household utility

\[
\max \left\{ C_t, Q_{t+1}, \{ n_{k,t+1}, x_{k,t} \}_{t=0}^{\infty} \right\} \sum_{t=0}^{\infty} \beta^t \log(C_t)
\]

subject to resource constraints and the laws of motion

The planner allocates research labor across firms to raise quality levels on randomly drawn product lines and to (dynamically) distribute products across firms
Features of the BGP of the planner’s solution

The planner equalizes production labor across product lines: $L = l(i)$

Any two $n_k$ and $n_{k'}$ satisfy

$$n_k - n_{k'} = \frac{1 - \bar{o}}{\psi_o} \left[ \log \left( \frac{\varphi_k}{\varphi_{k'}} \right) + \left( 1 + \frac{Z/\psi_Z}{1/\beta - 1} \right) \log \left( \frac{\gamma_k}{\gamma_{k'}} \right) \right]$$

where $\bar{o}$ is the steady state fraction of output devoted to overhead

The planner puts greater weight on the $\gamma$ differences because of spillovers
Example: no markup heterogeneity across firms

Two types of firms: LB and HS and $\Delta = \Gamma$

Decentralized equilibrium: $\mu_{LB} = \mu_{HS}$ and hence same firm size

Planner’s solution: LB firms are bigger

- higher growth rate
- lower average process efficiency
- higher overhead and lower consumption share of output
- higher allocative efficiency (decentralized equilibrium still has markup dispersion across products)
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Datasets

- A representative sample of French manufacturing firms over 2012–2019 with
  - Firm-level value added, wage bill, and asset values (FARE)
  - Firm-level hours worked (DADS)
  - Product-level prices and quantities (EAP)

- 128,485 firm-year observations and 32,641 firms
Properties of the BGP in the decentralized equilibrium

• High $\varphi_j$ and big $\gamma_j$ firms have higher markups and higher TFPR levels

$$\text{TFPR}_j \equiv \frac{\text{Revenue}_j}{\text{Inputs}_j} \propto \gamma_j \cdot \varphi_j$$

• Big $\gamma_j$ firms have higher sales-weighted average prices (not quality adjusted)

$$p_j \propto \gamma_j$$

• High $\varphi_j$ firms have lower marginal cost (not quality adjusted) and higher TFPQ levels

$$\text{TFPQ}_j \equiv \frac{\text{TFPR}_j}{p_j} \propto \varphi_j$$
## Calibration results

<table>
<thead>
<tr>
<th>Targets</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Dispersion in firm-level prices, $Var_j(\log \hat{p})$</td>
<td>1.74</td>
<td>1.72</td>
</tr>
<tr>
<td>2. Dispersion in firm-level TFPQ, $Var_j(\log \hat{TFPQ})$</td>
<td>1.88</td>
<td>1.86</td>
</tr>
<tr>
<td>3. Dispersion in firm-level TFPR, $Var_j(\log \hat{TFPR})$</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>4. Within-firm dispersion in product prices, $Var_{j(i)}(\log \hat{p}_{i,j})$</td>
<td>0.98</td>
<td>0.97</td>
</tr>
<tr>
<td>5. Dispersion in firm sales shares (StDev/Mean)</td>
<td>4.97</td>
<td>1.28</td>
</tr>
<tr>
<td>6. Skewness in firm sales shares (Median/Mean)</td>
<td>0.19</td>
<td>0.21</td>
</tr>
<tr>
<td>7. Semi-elasticity of firm employment share wrt firm price, $\hat{\beta}_{l,p}$</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>8. Semi-elasticity of firm employment share wrt firm TFPQ, $\hat{\beta}_{l,TFPQ}$</td>
<td>-0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>9. Aggregate price-cost markup ratio</td>
<td>1.50</td>
<td>1.48</td>
</tr>
<tr>
<td>10. Productivity growth rate (ppt year)</td>
<td>2.3</td>
<td>2.3</td>
</tr>
<tr>
<td>11. Interest rate (ppt/year)</td>
<td>5.2</td>
<td>5.2</td>
</tr>
</tbody>
</table>
Parameter values

- $\phi_{HB}$: Share of firms with high process efficiency and big step size, 0.01
- $\phi_{HS}$: Share of firms with high process efficiency and small step size, 0.28
- $\phi_{LB}$: Share of firms with low process efficiency and big step size, 0.20
- $\phi_{LS}$: Share of firms with low process efficiency and small step size, 0.51
- $\gamma_S$: Small step size, 1.30
- $\gamma_B$: Big step size, 1.56
- $\Gamma \equiv \frac{\gamma_B}{\gamma_S}$: Step size gap, 1.20
- $\Delta \equiv \frac{\varphi_H}{\varphi_L}$: Process efficiency gap, 1.02
- $\psi_o/J$: Overhead cost, 0.04
- $\beta$: Discount factor, 0.97
- $\psi_Z/Z$: R&D cost relative to R&D labor, 18.4
Planner reallocates toward big step size firms

Recall growth is given by

\[ 1 + \tilde{g} = \left( \prod_k \frac{\bar{S}_k}{\gamma_k^z} \right)^{\frac{Z}{\psi z}} \]

Product shares (in ppt)

<table>
<thead>
<tr>
<th></th>
<th>$\bar{S}_{HB}$</th>
<th>$\bar{S}_{HS}$</th>
<th>$\bar{S}_{LB}$</th>
<th>$\bar{S}_{LS}$</th>
<th>high proc. eff.</th>
<th>big step size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decentralized</td>
<td>3.8</td>
<td>16.6</td>
<td>68.8</td>
<td>10.9</td>
<td>20.4</td>
<td>72.5</td>
</tr>
<tr>
<td>Planner</td>
<td>5.2</td>
<td>0.0</td>
<td>94.8</td>
<td>0.0</td>
<td>5.2</td>
<td>100.0</td>
</tr>
<tr>
<td>Difference</td>
<td>+1.4</td>
<td>−16.6</td>
<td>+26.0</td>
<td>−10.9</td>
<td>−15.2</td>
<td>+27.5</td>
</tr>
</tbody>
</table>

Biggest growth effect comes from reallocation of R&D towards firms with low $\varphi$ but big $\gamma$
Welfare along the BGP

\[ \frac{1}{1 - \beta} \left( \log(C_0) + \frac{\beta}{1 - \beta} \cdot \log(\bar{g}) \right) \]

- \( C_0 \equiv C_t (1 + \bar{g})^{-t} \) is the detrended consumption level along the BGP

- \( C_0 = (1 - o) \cdot Q_0 \cdot \Phi \cdot \mathcal{M} \)
  - \( o \) is the fraction of output used for overhead
  - \( \Phi \) is the geometric average of process efficiency
  - \( \mathcal{M} \) is misallocation of labor across lines due to markup dispersion
  - \( Q_0 \) is the initial geometric mean quality level
Welfare gains along the BGP mostly come from eliminating R&D misallocation

Planner has \( \sim 30 \text{ bps} \) higher growth rate: \( \bar{g}^P = 2.6\% \) vs. \( \bar{g}^D = 2.3\% \)

Planner has similar detrended consumption \( C_0 \equiv C_t (1 + \bar{g})^{-t} \)

\[
\frac{C_0^P}{C_0^D} = \frac{1 - o^P}{1 - o^D} \cdot \frac{\Phi^P}{\Phi^D} \cdot \frac{M^P}{M^D} \cdot \frac{Q^P}{Q^D}
\]

\[
1.004 \quad 0.995 \quad 0.997 \quad 1.003 \quad 1.051
\]
Concluding remarks

We studied R&D allocation in an economy where markup heterogeneity arose from differences in the step size of quality innovations and process efficiency across firms

- Calibrated the model using data on French manufacturing firms
- Planner tilted innovation toward big quality step firms to enhance growth

Next steps:

- Analyze the effects of actual policies (e.g., size-dependent R&D subsidies)
- Incorporate the impact of transition dynamics on welfare gains for SP vs. DE