The **Fiscal Channel** of **Quantitative Easing**[1]

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2. UAB & BSE | 3. BoE & LSE.

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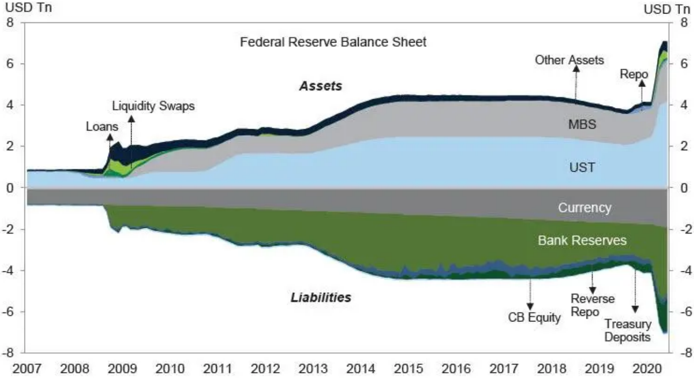
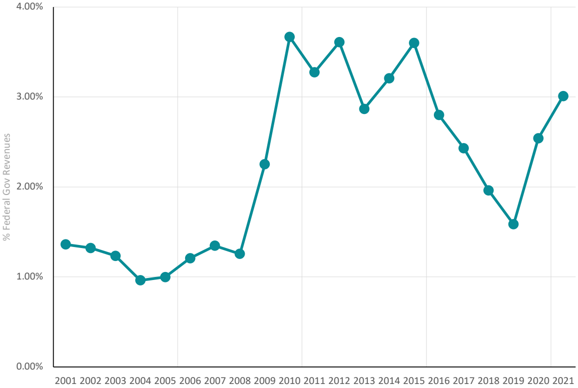
What is QE?

* Large scale **asset purchases** financed by **interest-paying reserves**.
* Goal: # long term interest rates) " (inflation, output).
* Massive expansion CB's **balance sheet**.

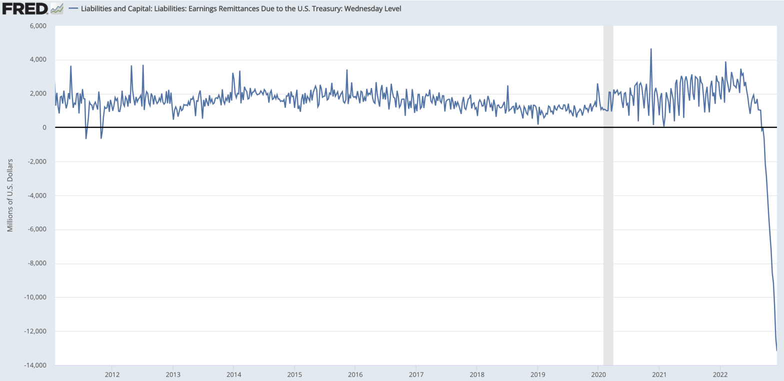


QE affects the Government´s budget via CB´s transfers



▸ Other fiscal consequences

QE can generate volatile remittances to the Government



This paper

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2. Transfers from households to the government are costly.

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The size of the CB balance sheet performs risk-shifting from private to government expenditures.

Does it matter how the Government uses these transfers?

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b) T adjustments) unchanged Deficits.

c) Income distribution is not altered.

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Key piece of QE's **Irrelevance theorems**.

This paper

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A rational Government would embark on **real resource reallocation**.

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1. Existing **policy** actions / proposals linking G to QE:
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   + Green corporate bonds programs (BoE, ECB).
   + Transmission Policy Instrument (ECB).

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2. **New uses of QE** aimed explicitly at reallocating resources
   + Green corporate bonds programs (BoE, ECB).
   + Transmission Policy Instrument (ECB).

3. Evaluate fiscal effects of **raising interest** rates/ QT.

1.- QE with costly T and productive G

2.- QE with limited participation

The tool

The smallest possible model:

* A real and stochastic endowment economy.
* A representative investor.
* Rational Expectations.
* Incomplete markets: a risky asset $\{S; P; D\}$; a safe asset $\{B; 1/R; 1\}$.
* 2 periods t = 0, 1.
* Economic policy: $\{G; T; B; R; Q\}$.
* Tax cost function $H : T \to \mathbb{R}$, with $1 > H' > 0$ (Bohn, 1992).
* Welfare: $U = \mathrm{E}_0 \{u(C_0) + \ [y u(C_1) + (1 \ \ y) v(G_1)]\}$.

▸ Investor's problem

Equilibrium and Economic Policy

A **Competitive Equilibrium** is an asset price P, allocations $\{C_0; C_1; S; B^i\}$ and policies $\{G_0; G_1; T_0; T_1; B; R; QE\}$ that satisfy:

1. Investor's Euler Equations (2).
2. Investor's budget constraints (2).
3. Consolidated gov budget constraints (2).
4. Assets market clearing (2).

▸ Equations

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▸ Equations

* 12 endogenous variables; 8 equations.

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Economic policy needs to specify
4 variables out of $\{G_0; G_1; T_0; T_1; B; R; QE\}$.

* **Quantitative Easing**: $\{QE; B\} = \{Q; QPR\}$.

Unchanged fiscal policy

- **Institutional** framework: fiscal **support**; **passive** fiscal policy.
- Intertemporal Gov Budget Constraint:

$$\underbrace{Q\left(P \quad \frac{D_1}{R}\right)}_{\text{QE losses}} = \underbrace{\underbrace{T_0 + \frac{T_1}{R}}_{T} \quad \underbrace{G_0 \quad \frac{G_1}{R}}_{G}}_{\text{Primary Surplus } S} \tag{1}$$

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- Common assumption: lump-sum T; exogenous G.

$$S(Q; \;) = T(Q; \;) \qquad (2)$$

- WLOG suppose $f G_0; G_1; T_0; \boldsymbol{T_1} g = f0; 0; 0; - \underbrace{\boldsymbol{Q(D_1 - PR)}}_{\text{QE gains}} g$

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Is it the **best possible reaction**?

Exogenous G is not optimal

* Let $\{T_0; T_1; G_0; G_1\} = \{0; T; 0; G\}$... How should a **government** set (T, G)?

Exogenous G is not optimal

* Let $fT_0; T_1; G_0; G_1g = f0; T; 0; Gg$... How should a **government** set (T, G)?

$$\max_{fT;Gg} U = \left[yu(C_1) + (1 \quad y)v(G) \right] \tag{3}$$

$$\text{s.t. } C_1 = (1 \quad Q)D_1 + QRP \quad T \quad H(T)$$
$$G = Q(D_1 \quad RP) + T$$

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$$\max_{\{T;G\}} U = \left[y u(C_1) + (1 - y) v(G) \right] \qquad (3)$$

$$\text{s.t. } C_1 = (1 - Q)D_1 + QRP - T - H(T)$$
$$G = Q(D_1 - RP) + T$$

- **Optimality** condition:

$$\underbrace{y[u'(D_1 - X - T - H)(1 + H'(T))]}_{\text{Tax Mg cost}} = \underbrace{(1 - y)[v'(T + X)]}_{\text{Tax Mg gain}} \qquad (4)$$

with $\boldsymbol{X} = Q(D_1 - RP)$ being **QE gains** (CB transfers).

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$$\boldsymbol{G^* = g(Q)}$$

with $g^0(Q) > 0$ if $X > 0$

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X has an income effect as it lower the tax distortions.

Does it matter?

QE becomes **effective**
with an **optimal** fiscal reaction.

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- Goods market clearing:

$$\{C_0; C_1\} = \{D_0; D_1 \quad H(T) \qquad g(Q)\}$$

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- Goods market clearing:

$$\{C_0, C_1\} = \{D_0, D_1 - H(T) - g(Q)\}$$

- Stochastic Discount Factor

$$E_0\left[\beta\frac{u'(C_1)}{u'(C_0)}\right] = E_0\left[\beta\frac{u'[D_1 - H(T) - g(Q)]}{u'(D_0)}\right] \qquad (5)$$

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Cases:

1. $H^0 = 0$ or $y = 1$) Irrelevance.
2. $H^0 > 0$ and $y < 1$) **Relevance**.

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Cases:

1. $H' = 0$ or $\beta = 1 \Rightarrow$ Irrelevance.
2. $H' > 0$ and $\beta < 1 \Rightarrow$ **Relevance**.

Higher Q implies larger consumption volatility

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General Asset Price Inflation

Asset demand inelasticity makes QE relevant

$$g'(Q) = 0 \quad \Rightarrow \quad \frac{\partial C_0}{\partial Q} = 0; \quad \frac{\partial S}{\partial Q} = \quad 1$$

$$g'(Q) > 0 \quad \Rightarrow \quad \frac{\partial C_0}{\partial Q} < 0; \quad 0 > \frac{\partial S}{\partial Q} > \quad 1$$

▸ Some evidence ▸ Closed form

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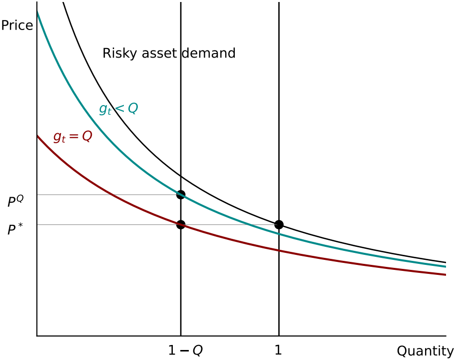

1.- QE with costly T and productive G

2.- QE with limited participation

Redistribution via QE

- 2 agents: investor & hand-to-mouth worker.
- Quantitative Easing: $\{QE, B\} = \{Q, QPR\}$
- Fiscal Policy: $\{T_0, T_1, G_0, G_1\} = \{0, QPR, 0, \boldsymbol{QD_1}\}$.
- Period 1 budget constraints:

$$\text{Investor:} \quad C_1 + T_1 = SD_1 + B \quad \Rightarrow \quad C_1 = D_1 - \boldsymbol{QD_1} \tag{6}$$

$$\text{Worker:} \quad C_1^w = W + G_1 \quad \Rightarrow \quad C_1^w = W_1 + \boldsymbol{QD_1} \tag{7}$$

- Asset price:

$$P^Q = \mathrm{E}_0\left[\frac{u'((1 - Q)D_1)}{u'(D_0)} D_1 \right] \tag{8}$$

Redistribution via τ

- Quantitative Easing: $f QE; Bg = f0; 0g$
- Fiscal Policy: $f T_0; T_1; G_0; G_1 g = f0; \quad D_1; 0; \quad D_1 g.$
- Period 1 budget constraints:

$$\text{Investor:} \quad C_1 + \quad D_1 = SD_1 + B \quad) \quad C_1 = D_1 - \boldsymbol{\tau D_1} \qquad (9)$$

$$\text{Worker:} \quad C_1^w = W + G_1 \quad) \quad C_1^w = W_1 + \boldsymbol{\tau D_1} \qquad (10)$$

- Asset price

$$P \quad = \mathrm{E}_0 \left[\frac{u^0((1 \quad)D_1)}{u^0(D_0)} (1 \quad) D_1 \right] \qquad (11)$$

- Relative asset price policy-wise:

$$\frac{P}{P^Q} = (1 \quad) \frac{\mathrm{E}_0 \left[\frac{u^0\left((1 \quad)D_1\right)}{u^0(D_0)} D_1 \right]}{\mathrm{E}_0 \left[\frac{u^0\left((1 \quad Q)D_1\right)}{u^0(D_0)} D_1 \right]} \quad) \quad \boldsymbol{\frac{P^\tau}{P^Q} = 1 - \tau} \quad \text{for} \quad = Q$$

$$+$$

QE: **Redistribution** with **higher** to asset prices.

What would the optimal redistribution be?

* Fiscal authority problem:

$$\max_{\{T;G;G^W\}} \omega U^I + (1 \quad \omega)U^W \tag{12}$$

$$s.t.\ Competitive\ Equilibrium,\ given\ asset\ prices$$

with

$$U^I = \mathrm{E}_0\left\{u(C_0) + \quad \left[yu(C_1) + (1 \quad y)v(G)\right]\right\}$$

$$U^W = \mathrm{E}_0\left\{u(C_0^W) + \quad \left[yu(C_1^W) + (1 \quad y)v(G^W)\right]\right\}$$

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s.t. Competitive Equilibrium, given asset prices

with

$$U^I = E_0\left\{u(C_0) + \quad \left[y u(C_1) + (1 \quad y) v(G)\right]\right\}$$

$$U^W = E_0\left\{u(C_0^W) + \quad \left[y u(C_1^W) + (1 \quad y) v(G^W)\right]\right\}$$

* **Optimality** conditions:

$$\underbrace{y E_0[u^0(D_1 \quad X \quad T \quad H)(1 + H^0(T))]}_{\text{Tax Mg cost}} = \underbrace{(1 \quad y)E_0[v^0(G)]}_{\text{Tax Mg gain}} \qquad (13)$$

$$! E_0[v^0(G)] = (1 \quad !)E_0[v^0(G^W)] \qquad (14)$$

An example

* Optimal taxes (from investor):

$$T \quad = aD_1 + (b \quad 1)X + yG^W \tag{15}$$

* Optimal investor preferred-G:

$$G \quad = \,!\,(aD_1 + bX) \tag{16}$$

* Optimal worker preferred-G:

$$G^W \quad = (\overline{1 \quad !})(aD_1 + bX) \tag{17}$$

with

$$x = \frac{X}{! + (1 \quad y)(1 \quad !)}$$

* Equivalent to a tax on dividends...

$$= \frac{(\overline{1 \quad !})bX}{D_1} \tag{18}$$

* ... except asset prices are higher under QE

$$\frac{P}{P^Q} = 1 \tag{19}$$

Conclusions

1. Important effects of QE on the **fiscal space**.
2. How this additional fiscal space is managed is key to determine the overall QE effects.
3. Literature: "**unchanged** fiscal policy" but... **not optimal** in relevant environments.
4. With optimal fiscal reaction: **redistribute** real resources.

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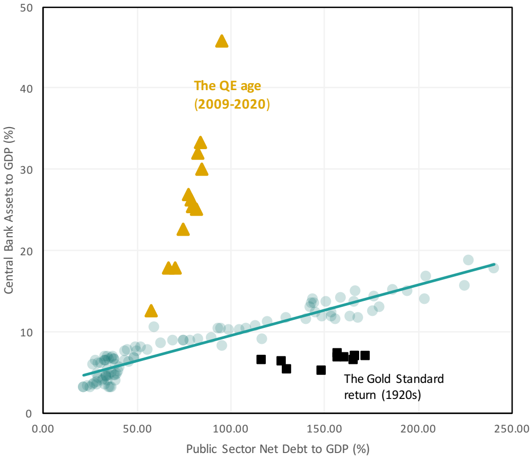
The **Fiscal Channel**.

5. Looking backward: how relevant was it?
6. Looking forward: **new uses** of QE exploiting it?

Additional Info

New Monetary Policy



Back

QE influences the fiscal space decisively

Real government's budget constraint:

$$G_t + \sum_{m=1}^{M} B_{t \quad m;t} = \sum_{m=1}^{M} \frac{1 + \quad_{t+m}}{1 + i_{t+m}} B_{t+1;t+1+m} + T_t + X_t \tag{20}$$

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QE's Direct Effect:

$$G_t + \sum_{m=1}^{M} B_{t \quad m;t} = \sum_{m=1}^{M} \frac{1 + \quad_{t+m}}{\underbrace{\# \ 1 + i_{t+m}}_{\text{Interests}}} B_{t+1;t+1+m} + T_t + \overbrace{'' \ X_t}^{\text{CB's Remittances}} \tag{21}$$

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QE's Indirect Effect:

$$\underbrace{\# \ G_t}_{\text{Spending}} + \overbrace{\# \sum_{m=1}^{M} B_t \quad {}_{m;t}}^{\text{debt real repayments}} = \sum_{m=1}^{M} \frac{\overbrace{'' \ 1 + \quad {}_{t+m}}^{\textit{Inflation}}}{1 + i_{t+m}} B_{t+1;t+1+m} + \underbrace{'' \ T_t}_{\text{Taxes}} + X_t \qquad (22)$$

Back

Investor's program

$$\max_{\{C_0;C_1;S;B\}} U = \mathrm{E}_0 \{ u(C_0) + \;\; [y u(C_1) + (1 \;\;\; y) v(G)] \} \tag{23}$$

s.t.

$$C_0 + PS + \frac{B}{R} + T_0 + H(T_0) = (P + D_0) S_{\;\;1}$$

$$C_1 + T_1 + H(T_1) = D_1 S + B$$

Back

Equilibrium

A **Competitive Equilibrium** is a vector of prices $f P; R g$, allocations $f C_0; C_1; S; B^i g$ and policies $f G_0; G_1; T_0; T_1; B; Q g$ such that:

1. Investor's Euler Equations are satisfied:

$$P = \mathrm{E}_0 \left[y \frac{u^0(C_1)}{u^0(C_0)} D_1 \right] \tag{24}$$

$$\frac{1}{R} = \mathrm{E}_0 \left[y \frac{u^0(C_1)}{u^0(C_0)} \right] \tag{25}$$

2. Investor's budget constraints:

$$C_0 + PS + \frac{B}{R} + T_0 + H(T_0) = (P + D_0)S \quad _1 \tag{26}$$

$$C_1 + T_1 + H(T_1) = D_1 S + B \tag{27}$$

3. Consolidated gov budget constraints

$$G_0 + QP = \frac{B}{R} + T_0 \tag{28}$$

$$G_1 + B = T_1 + QD_1 \tag{29}$$

4. Assets market clearing:

$$S + Q = 1; \quad B^i = B \tag{30}$$

▸ Back

Exogenous G is not optimal: proof (I)

* **Optimality** condition:

$$\underbrace{y\,\mathrm{E}_0[u^0(D_1 \quad X \quad T \quad H(T))(1 + H^0(T))]}_{\text{Tax Mg cost}} = \underbrace{(1 \quad y)\mathrm{E}_0[v^0(T + X)]}_{\text{Tax Mg gain}} \tag{31}$$

* To simplify, assume H involves no uncertainty. Then, without QE (i.e. $X = 0$):

$$\frac{(1 \quad y)}{y(1 + H^0(T))} = \frac{\mathrm{E}_0[u^0(D_1 \quad T \quad H(T))]}{\mathrm{E}_0[v^0(T)]} \tag{32}$$

* Now, consider $X > 0$. Call T the new tax level. If all the adjustment goes through taxes, $T = T \quad X$. That implies $''\ C_1$ since $C_1 = D_1 \quad X \quad (T \quad X) \quad H(T \quad X)$ and $H^0 > 0$.
* $''\ C_1$ implies $\#\ u^0(\ )$ by the concavity of u. Then,

$$\frac{(1 \quad y)}{y(1 + H^0(T))} = \frac{\mathrm{E}_0[u^0(D_1 \quad T \quad H(T \quad X)]}{\mathrm{E}_0[v^0(T)]} < \frac{\mathrm{E}_0[u^0(D_1 \quad T \quad H(T))]}{\mathrm{E}_0[v^0(T)]} = \frac{(1 \quad y)}{y(1 + H^0(T))} \tag{33}$$

which is a contradiction. Hence, $T = T \quad X$ cannot be optimal.

Exogenous G is not optimal: proof (II)

* Consider now all the adjustment going through G. Then, $T = T$. By the concavity of u and v

$$\frac{(1 \quad y)}{y(1 + H^0(T))} = \frac{\mathrm{E}_0[u^0(D_1 \quad T \quad H(T) \quad X]}{\mathrm{E}_0[v^0(T + X)]} > \frac{\mathrm{E}_0[u^0(D_1 \quad T \quad H(T))]}{\mathrm{E}_0[v^0(T)]} = \frac{(1 \quad y)}{y(1 + H^0(T))} \tag{34}$$

which is another contradiction. Then, no tax adjustment cannot be optimal either.

* Hence, the optimal T must lie somewhere in the middle, that is,

$$1 < \frac{@T}{@X} < 0 \tag{35}$$

and then,

$$0 < \frac{@G}{@X} < 1 \tag{36}$$

since $\frac{\partial G}{\partial X} = 1 + \frac{\partial T}{\partial X}$. That completes the proof.

▸ Back

Exogenous G is not optimal: an example

* No uncertainty.
* $u(\ ) = v(\ ) = ln(\ )$.
* Tax adjustment cost $H(T) = \quad T$.
* Then,

$$\boldsymbol{G^*} = \boldsymbol{aD_1} + \boldsymbol{bX} = \bar{g}(Q) \tag{37}$$

with $a = \frac{1 \quad y}{1+} > 0$ and $b = 1 \quad \frac{1+ \quad y}{1+} > 0$.

* Two tax policies:

$$T = \begin{cases} aD_1 - \boldsymbol{X} & \text{if } \textbf{unchanged} \text{ FP} \\ aD_1 + \underbrace{(\boldsymbol{b} - \boldsymbol{1})}_{<1} X & \text{if } \textbf{optimal} \text{ FP} \end{cases}$$

+

With costly taxes it is **optimal**
to digest X with a **T-G combination**.

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Equilibrium taxes

Government:

$$G_0 + QP = \frac{B}{R} + T_0$$

$$0 + QP = QP + T_0 \quad) \quad \mathbf{T_0 = 0}$$

$$G_1 + B = T_1 + QD_1$$

$$G(Q) + QPR = T_1 + QD_1 \quad) \quad \mathbf{T_1 = G(Q) + Q(RP - D_1)}$$

Investor (taxes T_t that leave BC unchanged):

$$C_0 + PS + \frac{B}{R} + T_0 = (P + D_0)S \quad 1$$

$$C_0 + P(1 \quad Q) + QP + T_0 = (P + D_0) \quad) \quad \mathbf{T_0^* = 0}$$

$$C_1 + T_1 = D_1 S + B$$

$$C_1 + T_1 = D_1(1 \quad Q) + QPR \quad) \quad \mathbf{T_1^* = Q(PR - D_1)}$$

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Closed form solutions

* A particular reaction function

$$G_t = G(Q; \ ) = (Q \quad g)D_t$$

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* Stock equilibrium price

$$P = \mathrm{E}_0\left[\frac{C_0 + xG_0}{C_1 + xG_1} D_1\right] = \frac{D_0}{1 \quad (1 \quad x)(Q \quad g)} \tag{38}$$

since $C_1 + xG_1 = D_1[1 \quad (1 \quad x)(Q \quad g)]$.

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* Stock demand

$$S = \frac{1}{P(1 + \ )}\left(D_0 + [\ (1 \quad Q) + x(g \quad Q) \quad g]P \right) \tag{39}$$

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QE and precautionary savings (I)

* QE pass-through to taxes is a random variable $g_t \quad N(Q;q)$.
* Expected full pass-through (irrelevance), but some fiscal risk.
* Bond's Euler Equation

$$u^0(P_0(1 \quad S_0 \quad Q) + D_0) = R\mathrm{E}_0[u^0(D_1(S_0 + g_1)] \tag{40}$$

* With convex marginal utility

$$\mathrm{E}_0[u^0(D_1(S_0 + g_1)] > \underbrace{\mathrm{E}_0[u^0(D_1(S_0 + \mathrm{E}_0(g_1))]}_{\text{With QE}} = \underbrace{\mathrm{E}_0[u^0(D_1 S_0)]}_{\text{Without QE}} \tag{41}$$

* Marginal benefits of savings go up for precautionary motives:

$$\text{QE:} \quad " \, \mathrm{E}_0[u^0(D_1(S_0 + g_1)] \quad) \quad " \, S_0$$

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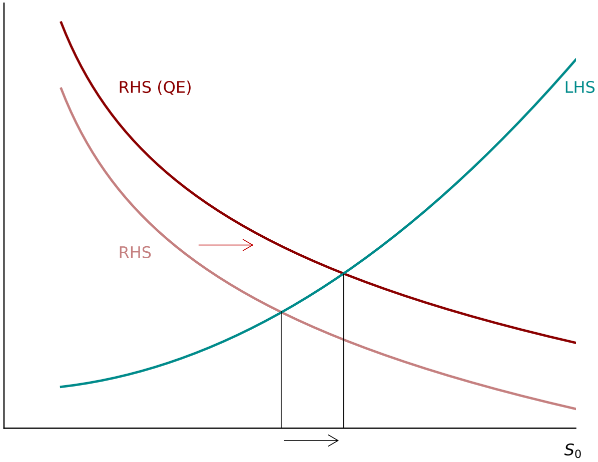
QE and precautionary savings (II)



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Euler Equations and asset pricing

* 2 readings of the Euler Equation
  1. Consumption theory: given interest rates) use EE to determine $f C_t; C_{t+1} g$
  2. Asset pricing: given a consumption path) use EE to determine P_t.

* Most QE literature, goes via 1. E.g. Harrison, 2017:

  Long rate equation

$$E_t R^1_{L;t+1} = \hat{R}_t \qquad _t$$

  Consumption Euler Equation

$$\hat{c}_t = E_t \hat{c}_{t+1} \qquad \left[\frac{1}{1 +} \hat{R}_t + \frac{}{1 +} E_t R^1_{L;t+1} \quad E_t \quad _{t+1} \right] \qquad (42)$$

* QE: $" \ _t \ ) \ \# E_t R^1_{L;t+1} \ ) \ " \hat{c}_t$
* We take the Asset pricing reading of EEs.
* Question: $_t = 0 \ ) \ E_t R^1_{L;t+1} = \hat{R}_t$. However, without log-lin

$$E_t R^1_{L;t+1} = R_t \qquad \frac{Cov_t[u^0(C_{t+1}); R^1_{L;t+1}]}{E_t[u^0(C_{t+1})]}$$

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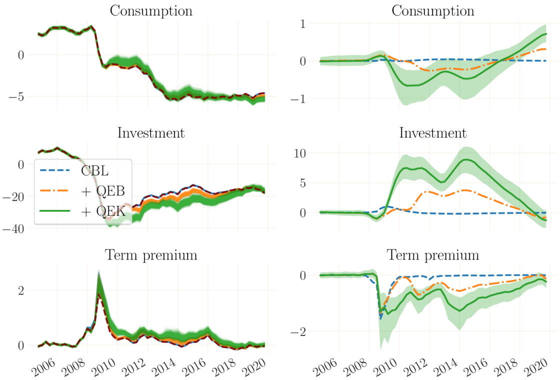
QE effects on consumption

* Boehl et al., 2021: # aggregate consumption = 0.7%.



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2 agents economy

* Investor's problem:

$$\max_{\{C_0;C_1;S_0\}} \mathrm{E}_0[log(C_0) + \quad log(C_1)] \tag{43}$$

s.t.

$$C_0 + P_0 S_0 = (P_0 + D_0) S_{\ \ 1} \tag{44}$$

$$C_1 = (1 \qquad) D_1 S_0 \tag{45}$$

* Worker's problem

$$\max_{\{C_0^w;C_1^w\}} \mathrm{E}_0[log(C_0^w) + \quad log(C_1^w)] \tag{46}$$

s.t.

$$C_0^w = W_0 \tag{47}$$

$$C_1^w = W_1 + M \tag{48}$$

* Government: $M = \quad D_1$
* Market clearing: $C_t + C_t^w = D_t + W_t;\ S_t = S_{\ \ 1} = 1.$

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