Discount Rates: Recognizing Managerial Choice

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Presentation Overview

I. Introduction
   A. About the Author
   B. Motivation

II. Three Problems with Traditional Approach
   A. Managerial Choices and Real Options
   B. Bad Empirical Bases
   C. Incorrect Result

III. A Sequential Decision Problem Approach
   A. A Value Functional equation covering two periods
   B. One-Period Discount & Growth Rates
   C. Applications

IV. Conclusions

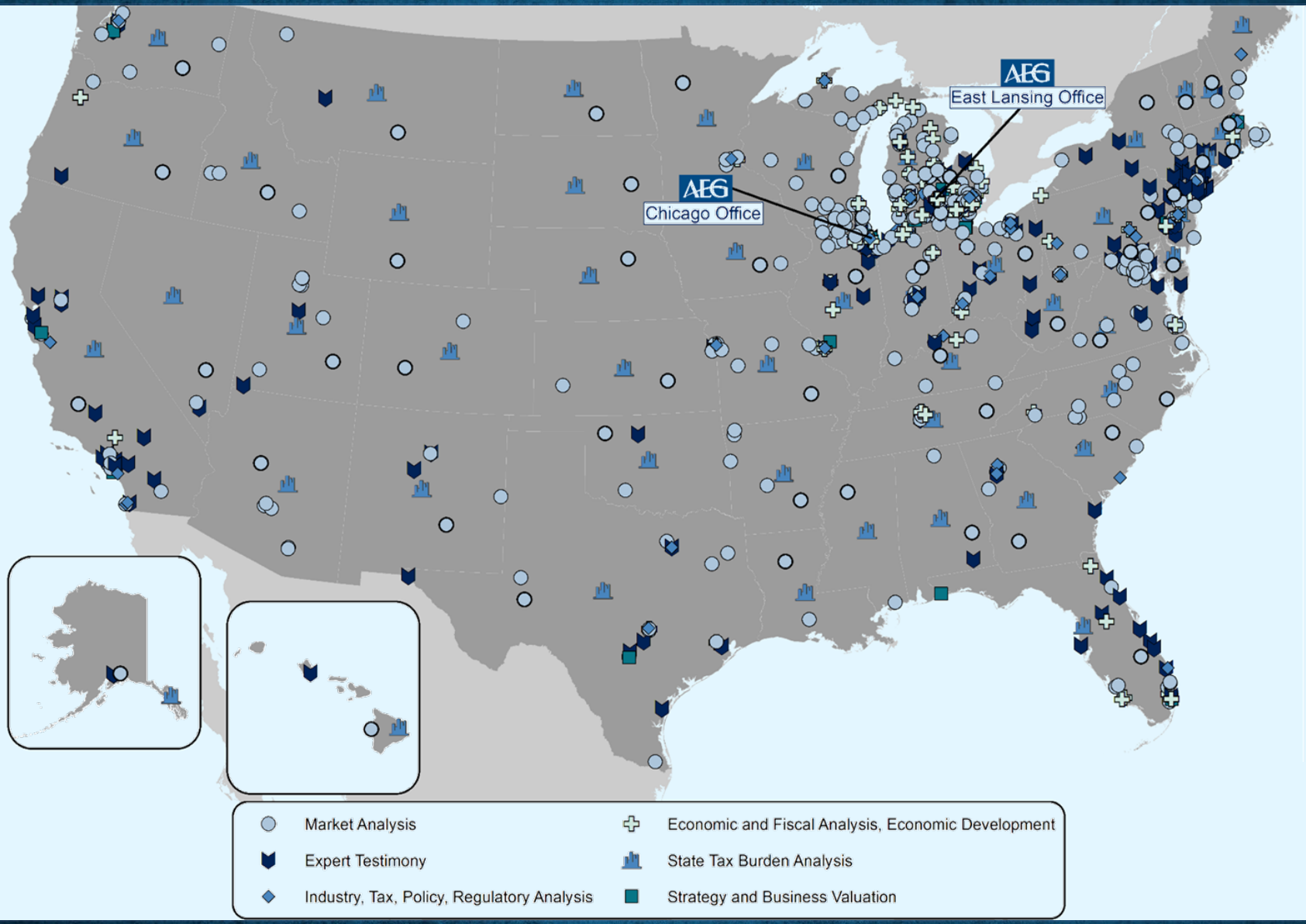
I. About the Author

Anderson Economic Group is a boutique consulting firm. We specialize in valuation, market analysis, and public policy.
Founded in 1996, we have offices in East Lansing and Chicago.

We advise some of the country's leading businesses, governments, and institutions.

About Anderson Economic Group

About Anderson Economic Group



Patrick L. Anderson

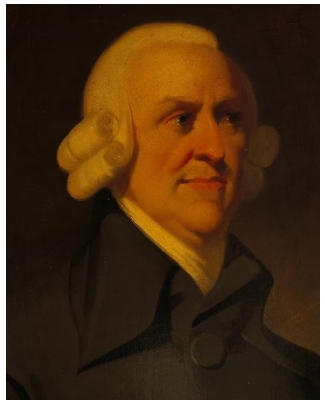
Mr. Anderson founded Anderson Economic Group in 1996, and currently serves as the firm's Principal and CEO.

Mr. Anderson has written over 100 published works, including the *Economics of Business Valuation* from Stanford University Press. Five of his articles have won awards for outstanding writing from the National Association of Business Economics.

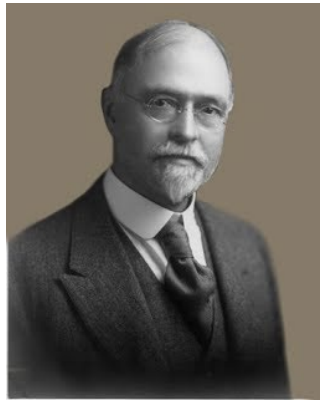
Motivation

Discount Rates

1. The "impatience and opportunity" theory of discount rates endures.
   - Adam Smith
   - Irving Fisher
2. Traditional empirical method is to consider historical average earnings on a portfolio of stock investments.
   - Similar principle: wage earnings over time.
   - The portfolio is consistent with the "opportunity" part of the theory.
3. Traditional use is in discounted cash flow (or earnings) models using NPV algorithm.



Adam Smith



Irving Fisher

Three Known Problems

1. Defective Model
   Standard discounted cash flow models fail to model businesses with managerial choices and real options.

2. Bad Empirical Basis
   Common empirical basis for business discount rates do not represent the actual discounting of most business investors and managers.

3. Incorrect Results
   We've known for over 100 years that standard DCF and the expected net present value rule incorrectly value options.

Goal of this Article

A. Recognize serious problems with the traditional approach and common empirical basis for discount rates.

B. Recommend an alternative that better represents actual decisions faced by workers and businesses and correctly values managerial choices and options.

C. Call for the reduction of *ad hoc* "methods."

Some limitations of this presentation: it has only a limited discussion of worker earnings; does not completely flesh out an alternative method; doesn't give a magic formula for discount rates; has only a few examples; presumes knowledge of functional equations and DCF methods; doesn't resolve "timeless debate" noted in literature; and demands thinking beyond what is taught in many standard texts.

Three Problems with the Traditional Approach

First Problem: Defective Model

Standard discounted cash flow models fail to model businesses with managerial choices and real options.

1. Business managers and individual employees regularly make choices on the size, structure, operations, pricing, and financing of a company.

2. Recognized "real options" are one subset of these decisions.

3. Standard DCF, with discount rates determined by aggregate returns on publicly-traded stocks, fail to account for this.

Second Problem: Bad Empirical Basis

Common empirical basis for business discount rates do not represent the actual discounting of most business investors and managers.

1. Asymmetric risks are the *rule*, not the exception.
2. Bankruptcy and business risk are a reality.
3. Most businesses are closely held and operate in incomplete markets.
4. Aggregate returns on publicly-traded stocks miss nearly all of these.

Third Problem: Incorrect Results

Standard DCF and the expected net present value rule incorrectly value options.

1. We have known for over 100 years that the expected-net-present-value method incorrectly values options.

2. *Most* businesses involve real options.

    - Many workers also have choices (options) such as additional effort, training, and education.

3. Therefore, most businesses will have their decisions and values incorrectly estimated using the standard method.

Traditional Approach: Problems

Problematic Results:

1. The value of options has been known for centuries.
See historical note.

2. Standard discount rates—and standard discounted cash flow models—systematically mis-price real options.
See example 1.

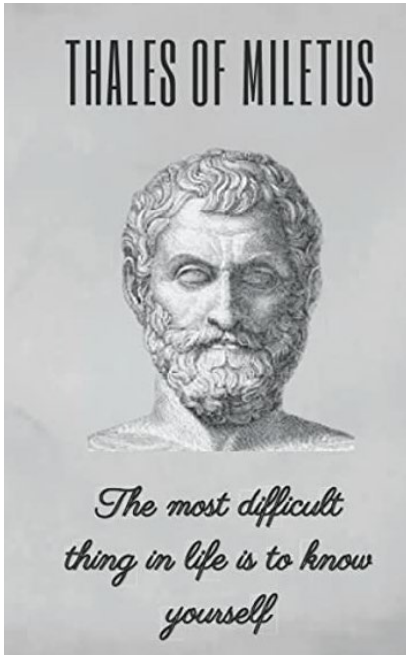
3. "CAPM" does not hold theoretically, or practically, for most firms. The literature does not curtail practitioners from asserting a wide variety of discount rates.

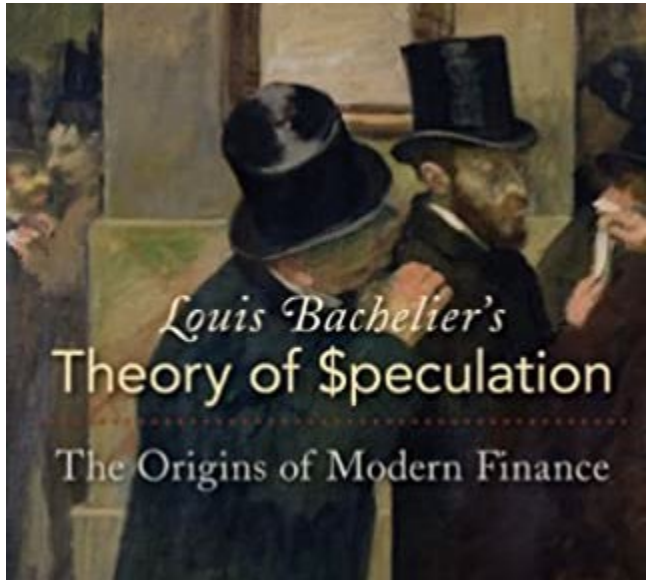
See example 2.

Historical note and examples

Historical note:
Aristotle wrote that Thales of Miletus purchased options on the use of olive presses 2600 years ago.

Much later, Louis Bachelier described an option pricing formula that anticipated mathematical finance of today.





Thales of Miletus lived *circa* 585 BC. Bachelier's *Theory of Speculation* was published in 1900.

Example 1: Mispricing Real Options

An established business could face one of two situations:

A. *An option to expand operations with financing largely accomplished through mortgage and inventory borrowing.*

B. *Significant risk of the loss of a key customer, franchise, or license; serious counterparty risk, risk of law change or loss of key persons.*

Traditional discount rate and DCF methods *fail* in both situations.

Definition of "Fail": structurally produces an incorrect estimate.

Example 2

Exhibit A: Standard Indicators of Business Discount Rates

Returns on Publicly-Traded Equity Investments	Annual Rate (c)	*Source Notes*
"Large Cap" stock total returns, 1926-2020 (a)	10.20%	*SBBI 2021, Exhibit 6.1*
"Small-Cap" stock total returns, 1926-2020 (b)	12.10%	*SBBI 2021, Exhibit 7.1*
S&P 500 stock total returns, 1928-2021	9.67%	*Aswath Damodaran Website*
Indicators of The "Risk Free Rate"		
3-Month Treasury Bill Secondary Market Rate, Discount Basis, 1934-2022	3.44%	*Fred Series Name "TB3MS"*
Market Yield on U.S. Treasury Securities at 10-Year Constant Maturity, 1962-2022	7.20%	*Fred Series Name "DGS10"*
Market Yield on U.S. Treasury Securities at 30-Year Constant Maturity, 1977-2022	6.39%	*Fred Series Name "DGS30"*

Models for Estimating Discount Rates

Risk-free rate
CAPM; Risk-free rate plus Beta multiplied by Risk Premium
CAPM with Modifications
  Size premium
  Specific-company risk premium
  Franchisee and termination risk premium
  Variations on "beta" and levering
Fama-French Three-Factor and other Multi-Factor Models
  Risk Free rate
  Beta of Company
  Coefficient for Size Adjustment: Small Cap Less Big Cap
  Coefficient for Growth Adjustment: High Book Value to Market Cap Less Low Book Value to Market Cap
Numerous variations asserted to be a "build up" method

Example 2: Widely-Varying Estimated Discount Rates

Possible values calculated using "theories" in the literature, for the same business and using the same data:

- *9.6% ("CAPM" using S&P 500 long-term returns)*
- *13.6% ("CAPM" large-cap returns, beta=1)*
- *15.6% ("CAPM" as above, plus 2% small company risk premium)*
- *17.6% (… as above, plus illiquidity or lack-of-control premium)*
- *Any number between 9% and 17% (same inputs, varying "beta" by picking different stock market cohorts and time periods)*
- *Any number between 7% and 21% ("build up" method)*

A Sequential Decision Model Approach

Business as a Sequential Decision Problem

A model in which a manager, each time period:

1. Views the current state of the world;

2. Considers both the current state, and the potential future states;

3. Chooses a course of action that maximizes *value* given the current state and the feasible set of actions.

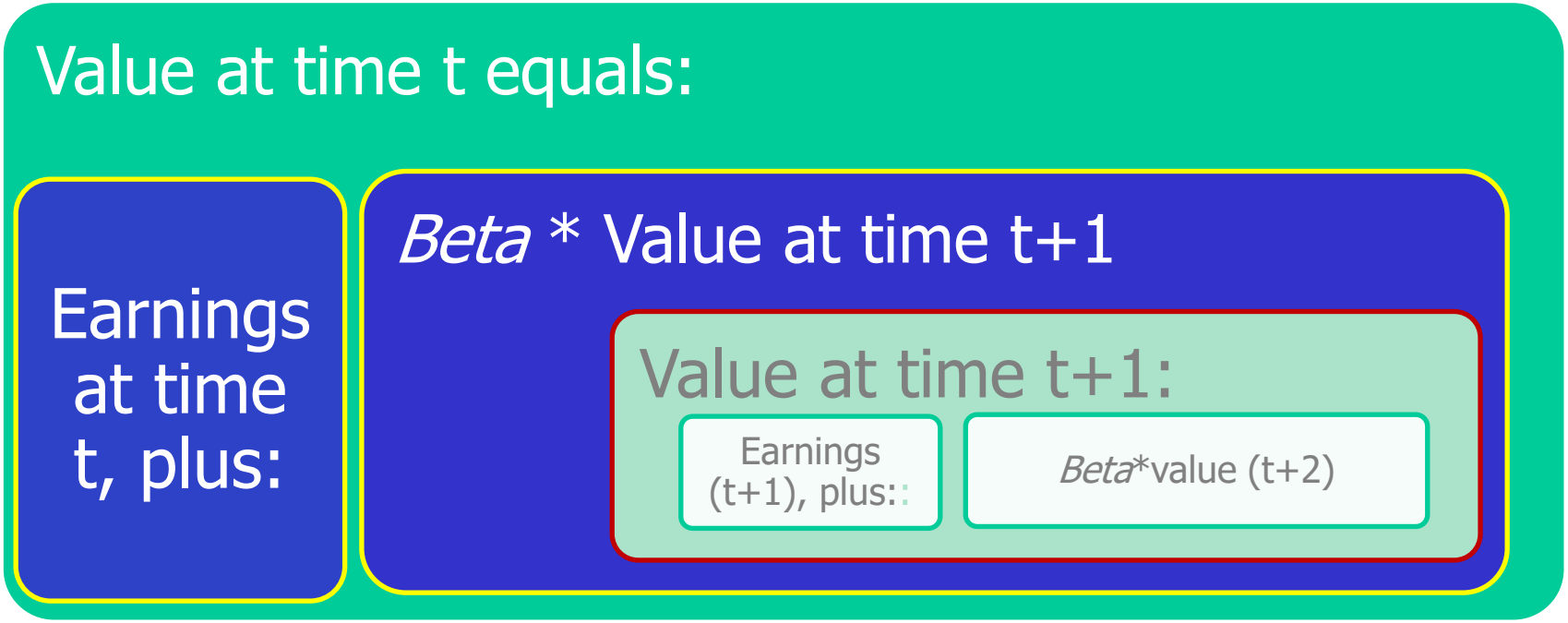
"Value" in this approach

1. "Value" is the sum of two parts:
   A. Current earnings; and
   B. Discounted future value, or "continuation value."

2. The manager maximizes the *sum of these,* and:
   - Considers the tension between current earnings and future value.
   - Can purchase or exercise real options when useful.

3. A "value functional" equation ("Bellman equation") represents this maximization.

Recursive Structure of Value Functional Equation



Notes: value is the maximization across a feasible set of actions;
beta is a factor that includes both discounting and growth over the next period.

Comparing Equations

Net Present Value Summation

$$NPV = \sum_{t=0}^{T} \beta^t C_t$$

$$where:$$

$$t = \text{time index},$$

$$\beta = \text{discount factor; } 0 < \beta < 1;$$

$$C_t = \text{cash flow at time } t.$$

Notes: the "cash flows" are typically earnings to shareholders; the discount rate is typically estimated a portfolio method (often with considerable interpretation), and is assumed to be steady over time; future earnings are often estimated using steady growth and cost assumptions; a "terminal value" may be calculated using these same assumptions.

Value Functional equation

$$V(s_t) = \max_{x \in \Gamma} \{ f(s,x) + \beta E[V(s_{t+1})] \};$$

where:

$V(s_t) \equiv$ value in state at time t;

$s_t \equiv$ state vector at time t;

$x \equiv$ action; $\Gamma \equiv$ feasible set of actions.

$f(s,x) \equiv$ reward function given state and action.

Notes: the control system literature often uses "reward" rather than "cash flow" to describe what would typically be earnings to shareholders; the reward function and future value can reflect real options; the Beta factor reflects one-period discounting and one-period trend growth; the "state" reflects current business conditions and is not assumed to be constant.

Convention for Expressing Discount and Growth Rates

$$\beta = \frac{1+g}{1+d}$$

where:

β = gross discount rate per period

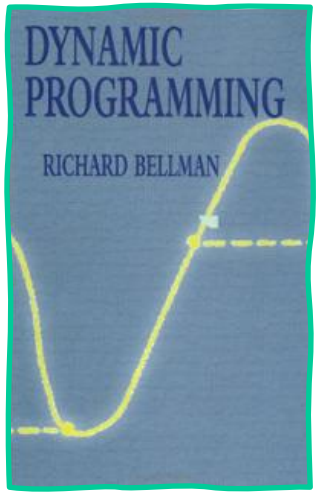
d = net discount rate per period

g = growth rate per period.

Notes: the Beta factor reflects one-period discounting and one-period trend growth. One-period discounting is not confined, theoretically or practically, to aggregate returns over time on business equities nor worker earnings.

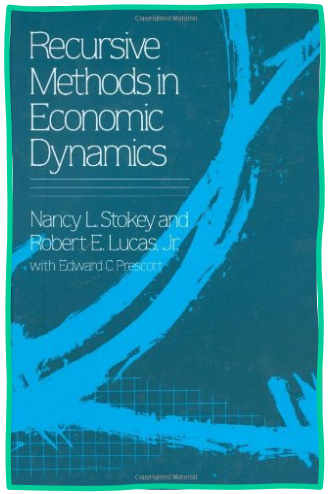
The theory behind recursive methods in Economics was developed from the 1950s through the 2010s.

1957



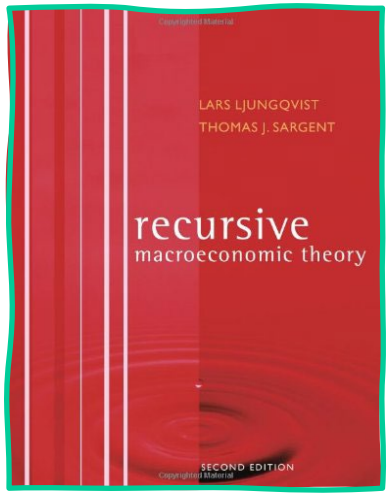
Richard Bellman
1983 Bellman Control Heritage Award

1989



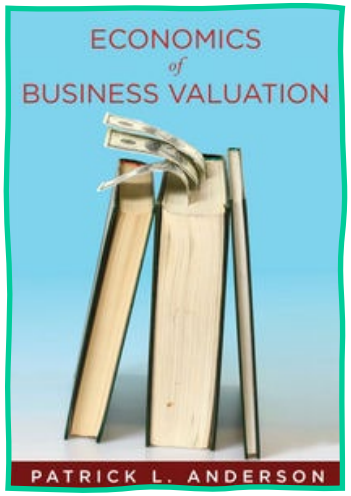
Robert Lucas, Jr.
1995 Nobel Prize in Economics

2004



Thomas Sargent
2011 Nobel Prize in Economics

2012



Patrick L. Anderson
Dues-paying NAFE Member

Sequential Decision Problem Approach: Advantages

1. Explicitly accounts for managerial choices.
2. Incorporates liquidity constraints and asymmetric risks.
3. Does not require improbable assumptions.
   (e.g., existence of complete markets, symmetric risks, representation by publicly-traded securities, existence of diversified portfolio, presumptions of well-established business and no risk of bankruptcy, etc.)
4. More accurately represents the decisions faced by most businesses, and also their value.

Sequential Decision Problem Approach: Disadvantages

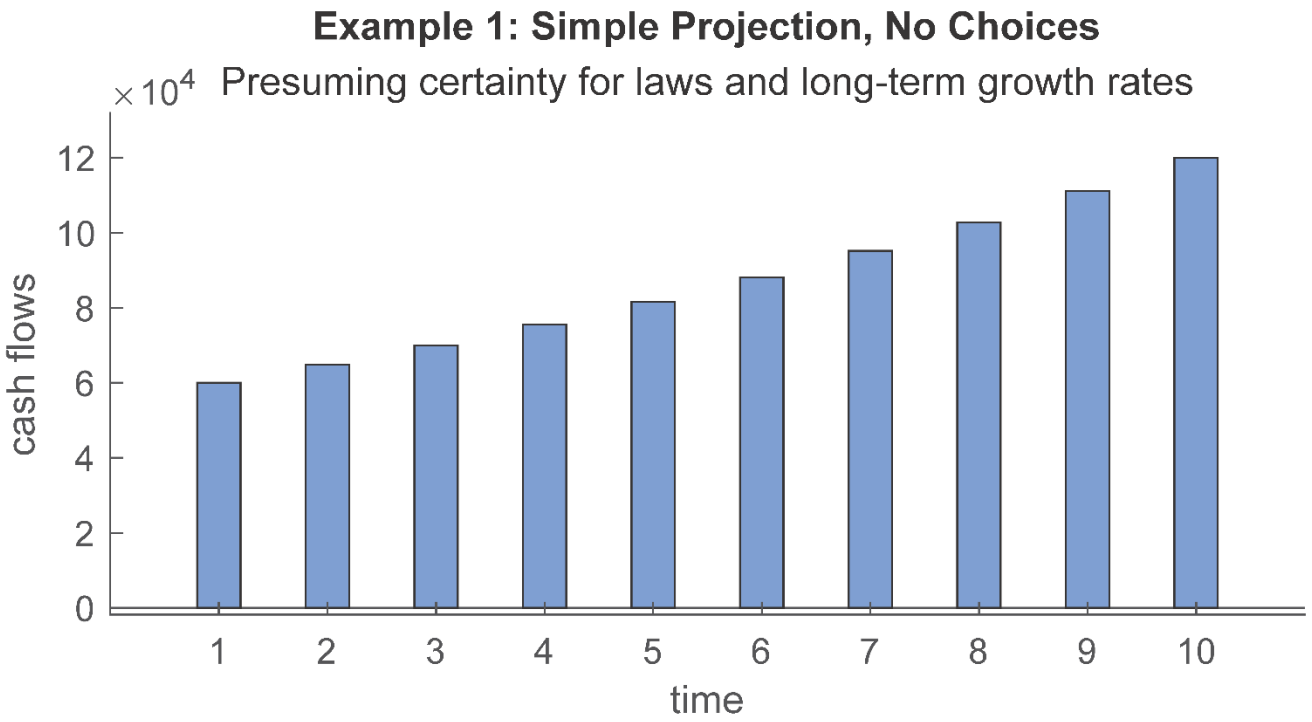
1. Novelty.
2. Compositional and computational difficulty.
3. Requires explicit specification of states, actions, and transition probabilities.
4. Relatively thin track record.
5. Still requires specifying a discount rate, for which aggregate stock market returns will probably not suffice.

Applied Example 1: Theoretical Company with Near-Certain Cash Flows, and No managerial choices

- Company operates in the same place, in the same industry, with the same people.
- Company regularly reinvests in capital stock, worker training, advertising, R&D, etc.
- Company faces unchanging economic conditions, and stable competitive position, and no changes in law.
- Shareholders provide capital reliably and managers operate consistently.

Easy to discount this company's expected future earnings to a net present value, but…there aren't any companies like this.

An Alternative Approach



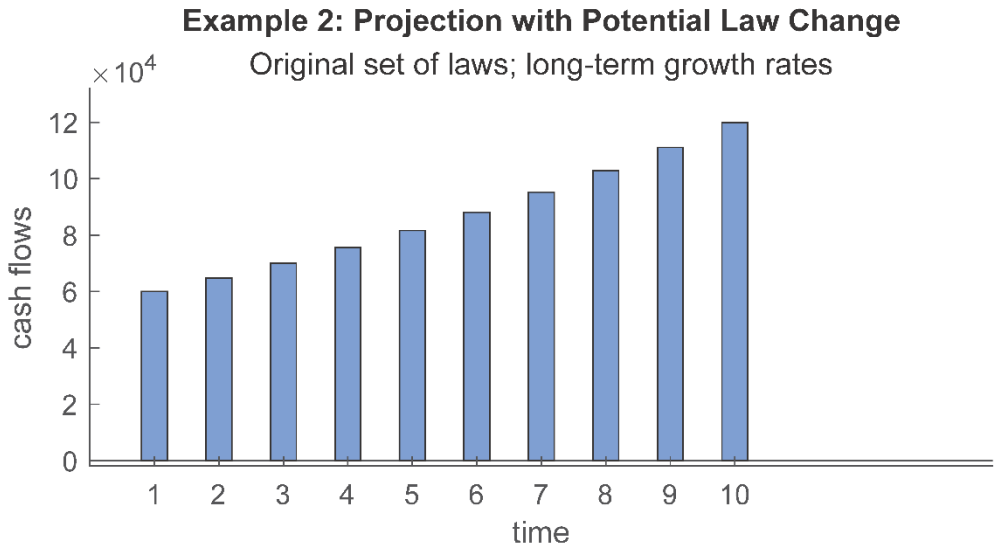
Applied Examples 2 and 3: A medium-sized company in the Real World

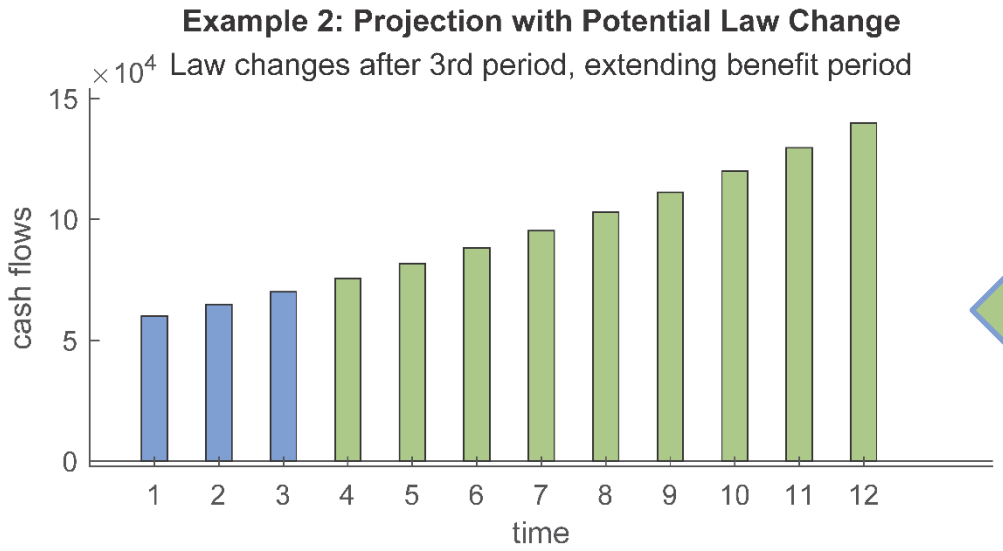
An actual medium-sized company whose managers and investors are aware that:

- Laws could change.
- Economy is certain to change.
  Competitors, management, technology, and shareholders also could change.
- The company has an opportunity to expand or contract, which it may want to do if laws or market conditions improve.

Our model must consider managerial choices and recognize uncertainty. The value functional method can do this.

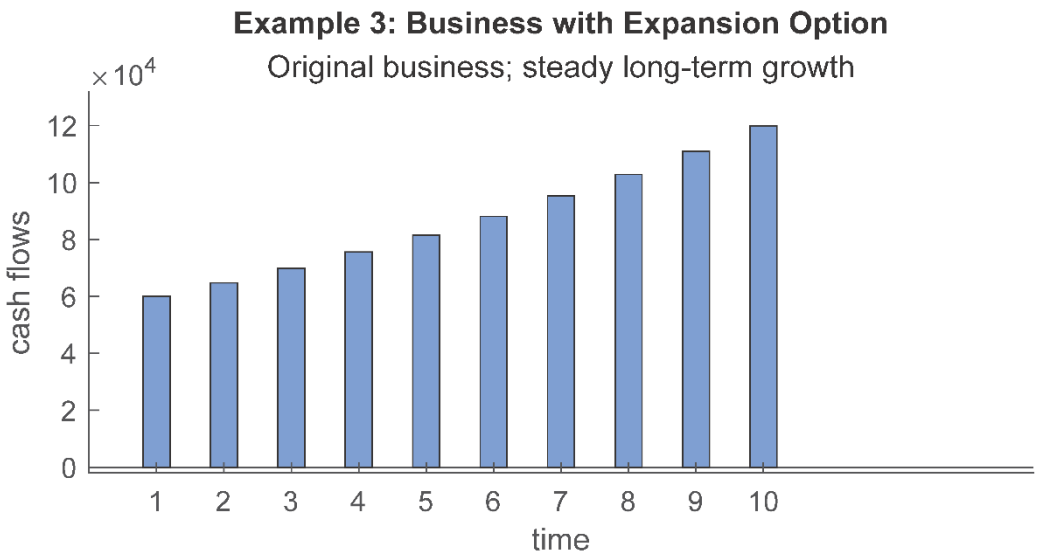
An Alternative Approach

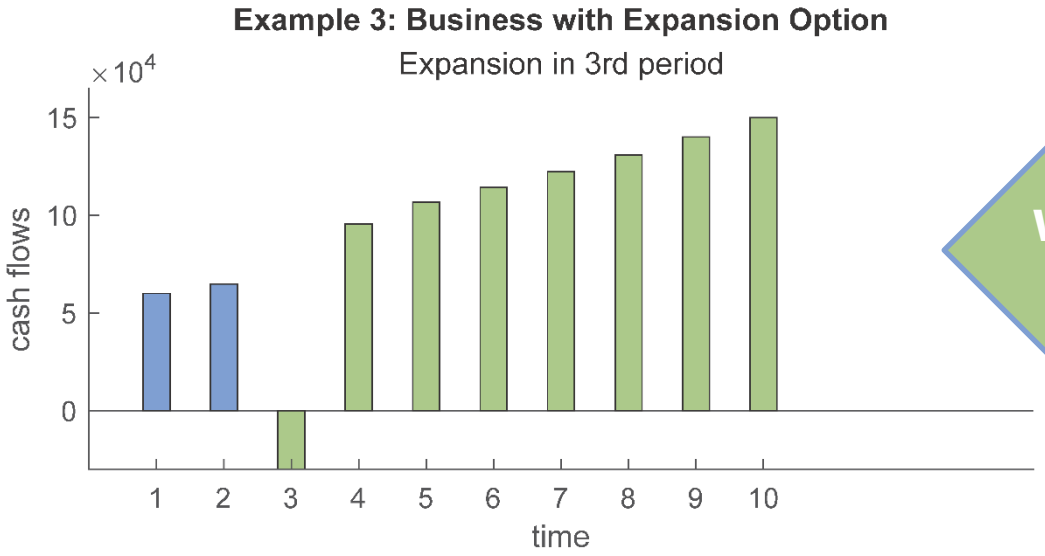




What if the law changes?

An Alternative Approach





What if business expands?

Conclusions

Example 3: Discount rates for a company in the real world, with expansion option.

Medium-sized company that could rely on mortgage financing for construction of new facility, existing lines of credit for financing of inventory, and can expand.

1. Value functional method: Discount factor of 8% to 10%.
   Here, the one-period discounting factor relevant to the business operation is based on the financing costs for inventory and construction, as well as the underlying equity risk. A weighted mix of 12% for equity and 6% for construction and inventory financing costs would yield an 8% to 10% discount factor.

2. Traditional method: 11% to 16%.
   A modified "CAPM" method could produce equity discount rates of anywhere from 11% to 14%. Adding a premium for small or closely-held firms could bring that to 16%. A "build up" method could produce any number between about 9% and 18%.

   See article. Note that full costs of expansion option must be considered, and that traditional DCF model cannot natively incorporate the real option.

Conclusions

1. Ubiquitous "Portfolio" Theory and Discounted Cash Flow Method

1. Introduced by Markowitz in the 1950s; extended by CAPM in 1970s; further extended by "factor models."

2. Assumes that investors:
   - Consider risk and return on a portfolio of investments; can readily diversify; and
   - That these investments are a good proxy for the wealth of households.

3. Largely ignores real options, bankruptcy and counterparty risk, and incomplete markets.

2. Big Problems with Portfolio Theory and Practice of DCF

1. Defective Model
   Standard DCF models fail to model businesses with managerial choices and real options.

2. Bad Empirical Basis
   Common empirical basis for business discount rates does not represent the actual discounting of most business investors and managers.

3. Incorrect Results
   Standard DCF and the expected net present value rule incorrectly value options.

…and

2. Big Problems…for Forensic Economists

4. Bad Practice:
Literature does not curtail practitioners from asserting a wide variety of discount rates, nor using *ad hoc* "methods."

5. Perverse Incentives:
Skilled economists can often adjust or modify the traditional methods, or rely on market prices…
but there are few barriers for unskilled or unethical practitioners.

3. Sequential Decision Problem Approach

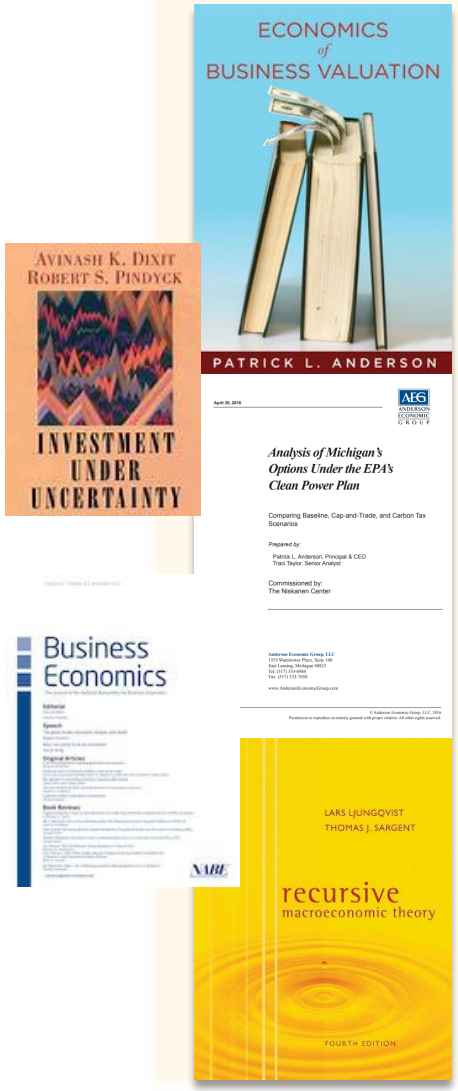
An alternative theory of valuing a business, originated by Anderson (2012); rooted in Bellman (1957), Lucas (1978), & Dixit & Pindyck (1994).

1. Presumes active entrepreneur rather than passive investor.

2. Natively handles real options.

3. Value functional equation breaks value into two parts, with single-period discounting.

4. There are published applications.



Some Demonstrated Applications:

- Hiring decisions when laws may change [*Bus. Econ*. 2014]
- Business location decisions [*Bus. Econ*. 2019]
- Consumer demand when laws or technology change [Niskanen Center 2016]
- Pricing real options [Dixit & Pindyck 1994]
- Estimating business value [Anderson 2012]
- Fiscal and monetary policy [Ljundqvist & Sargent 2018]

See references section of article.

4. Work is Needed.

1. This is a novel approach.
    - It involves mathematics not commonly taught until recently.
    - The number of published examples is relatively small.
2. There is a need to develop:
    - Applications for personal decisions and individual wage losses
    - Additional examples involving privately held businesses
    - Derivation of matching discount and growth series
    - Additional tests on market decisions and market prices

Questions?

Additional Reading

The Economics of Business Valuation: Toward a Value Functional Approach (Stanford University Press, 2012);

Solutions Manual for The Economics of Business Valuation (AEG, 2013)

"Business strategy and firm location decisions: testing traditional and modern methods, *Business Economics,* 2019.

"Persistent Unemployment and Policy Uncertainty," *Business Economics*, 49, pp. 2–20 (2014).

"The Value of Private Businesses in the United States," Business Economics, 44(2):87-108 (April 2009).

Recursive Macroeconomic Theory, 4th edition, (MIT, 2018)

Links to some of these publications can be found at: https://www.AndersonEconomicGroup.com.

Thank you!

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Appendix

Matching Principle

- Matching Principle.
  Both traditional and the alternative approaches require current earnings and future value be modeled with consistent assumptions regarding discounting, growth, and earnings (revenues and costs, including for real options).

Changing growth and discount rates

- When conditions change, discount and growth rates will not remain stable over time.

Asymmetric Risks & Real Options

- Value functional method superior to traditional models where:
  - Significant managerial choices and real options exist
  - Asymmetric risks are possible and multiple courses of action may be considered
- Traditional net present value methods applied to a single projected time series are not well suited to these more complex scenarios

Stochastic Discount Factor

$$m_{t+1} \equiv \beta \frac{u'(c_{t+1})}{u'(c_t)} \text{ stochastic discount factor or pricing kernel;}$$

where :

$\beta =$ subjective discount factor, and

$\frac{u'(c_{t+1})}{u'(c_t)} =$ ratio of marginal utilities.

Basic Pricing Equation, incorporating stochastic discount factor

$$p_t = E(m_{t+1} x_{t+1})$$

where:

$p_t =$ price of asset with payoff x_{t+1}

$m \equiv \beta \dfrac{u'(c_{t+1})}{u'(c_t)}$ stochastic discount factor (pricing kernel)

$x_{t+1} \equiv$ payoff at time $t+1$

This functional equation is recursive.

"To understand recursion, you must first understand recursion…"