Optimal Time-Consistent Debt Policies

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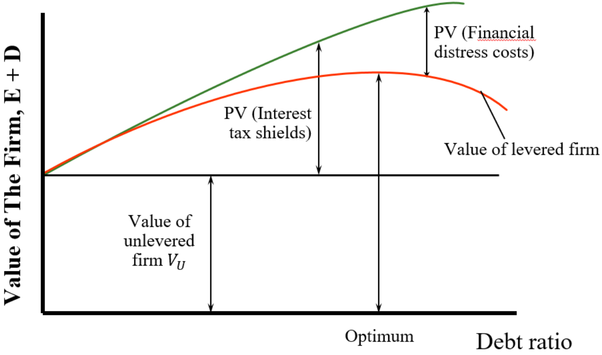
Static Trade-off Theory of Leverage



Common dynamic interpretation:

- Firms should actively manage liabilities to stay at the target.

Dynamic Inconsistency

Recent research emphasizes **dynamic inconsistency** of this policy:

- Reducing leverage is not in the ex-post interest of equity holders due to the leverage ratchet effect (Admati, DeMarzo, Hellwig, and Pfleiderer 2018).
  - A form of debt overhang.
- "Optimal dynamic capital structure choice is fundamentally a problem of commitment" Peter DeMarzo's AFA 2019 Presidential Address.

DeMarzo and He (JF, 2021)

- DeMarzo and He (2021) study leverage dynamics without commitment in a diffusion continuous-time trade-off model.
- Solve for the unique **Markov Perfect Equilibrium**.
- Very different implications from the static trade-off theory:
  - The firm continuously issues debt and never does repurchases.
  - No abrupt rebalancing to a “target” leverage.
  - Equityholders do not realize any tax benefits of new debt issuances.
  - Firms are indifferent in the debt maturity choice.
- Mechanism: equityholders can’t commit not to borrow in the future, which decreases the debt price today, and dissipates tax benefits from the debt issuance.

This Paper

A different solution concept: Optimal Time-Consistent Debt Policy

- Suppose equityholders announce a debt policy at t=0: How much debt the firm will issue/repurchase in each state.
- An announcement is **time-consistent (credible)** if equityholders do not want to deviate from the announcement ex-post.
  - If a deviation occurs, creditors no longer trust equityholders, and the DeMarzo-He Markov perfect equilibrium is played.
- What is the optimal time-consistent debt policy?
  - Looking for the best Subgame (non-Markov) Perfect Equilibrium (similar to most cooperative eqm in the repeated prisoners dilemma).

Technical innovation: introduce downward jumps to cash flows.

Combination of these assumptions gives new insights into the role of commitment in leverage dynamics.

Preview of Results

1. Leverage dynamics without commitment:
   - Firm targets a certain leverage ratio and compensates **small** shocks by debt issuance/repurchase to stay at the target.
   - After a **sufficiently large** negative shock, firm waits until cash flows recover or it goes bankrupt.
   - Bridge static trade-off theory and implications of leverage ratchet effect.
2. Large repurchases of debt are most costly in terms of equity holders' incentives to stick to the policy.
   - Time-consistency requirement restricts the max size of credible repurchases, hence, lowers optimal leverage.
   - This affects comparative statics (e.g., wrt volatility of shocks).
3. Interior optimal debt maturity.
   - Trade-off between tax benefits and commitment to delever when incentives from reputation are insufficient.

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Model: Environment

DeMarzo-He (2021) debt issuance game with different solution concept and downward jumps.

Preferences: equity/debt holders are risk-neutral with common discount rate r.

Assets in place:

- cash flows Y_t follow GBM with downward jumps:

$$\frac{\mathrm{d}Y_t}{Y_{t-}} = \underbrace{\hat{\mu}\mathrm{d}t + \sigma \mathrm{d}Z_t}_{\text{diffusion}} + \underbrace{\mathrm{d}\left(\sum_{i=1}^{N_t}(S_i - 1)\right)}_{\text{downward jumps}},$$

- Jumps arrive at a Poisson intensity λ and exponentially distributed;
- Expected jump size $\mathbb{E}[S_i - 1] = \zeta < 0$.

Zero recovery at liquidation.

Model: Actions

Debt policy: Equity holders can issue/repurchase debt at any time without costs.

- F_t face value of outstanding debt of infinite maturity;
- dF_t current debt issuance/repurchase;
- in the paper, analyze finite maturity debt and optimal debt maturity.

Default policy: Equity holders optimally default at stopping time τ_b.

Equity holders cannot commit to $\Sigma \equiv \{F_t\}_{t \geq 0}$ and τ_b.

Timing within $[t, t + dt]$: i) Cash flow realization Y_t; ii) default or coupon/principal payment; iii) debt issuance/rep; iv) debt price p_t is set.

Model: Payoffs

Fix a Markov debt policy Σ and default stopping time τ_b (states F_{t-} and $y_t = Y_t / F_{t-}$).

- Equity value per \$1 of debt $e\left(y|\Sigma\right) \equiv$

$$\mathbb{E}\left\{ \int_t^{\tau_b} e^{-r(s-t)} \left[\underbrace{(1-\pi)(y_s - c)ds}_{\text{net income}} + \underbrace{p(y_s|\Sigma)df_s}_{\substack{\text{issuance revenue or} \\ \text{repurchase cost}}} \right] \middle| y_t = y \right\}.$$

- Debt price

$$p\left(y|\Sigma\right) \equiv \mathbb{E}\left\{ \int_t^{\tau_b} e^{-r(s-t)} \underbrace{c}_{\text{coupon}} ds \middle| y_t = y \right\}.$$

- Firm value: $W(Y, F|\Sigma) \equiv F\left(e\left(\frac{Y}{F}|\Sigma\right) + p\left(\frac{Y}{F}|\Sigma\right)\right)$.

Max firm value at $t = 0$ is $W(\Sigma) \equiv \max_{F_0 \geq 0} F_0\left(e\left(\frac{Y_0}{F_0}|\Sigma\right) + p\left(\frac{Y_0}{F_0}|\Sigma\right)\right)$.

Time-Consistent Debt Policies

$$\max_{\Sigma \in \mathbb{S}} W(\Sigma) \text{ s.t. credibility constraints:}$$

$$e\left(y|\Sigma\right) \geq e_m\left(y\right) \underbrace{\forall\ y \in \mathcal{R}(\Sigma)}_{\text{all states reachable under } \Sigma}$$

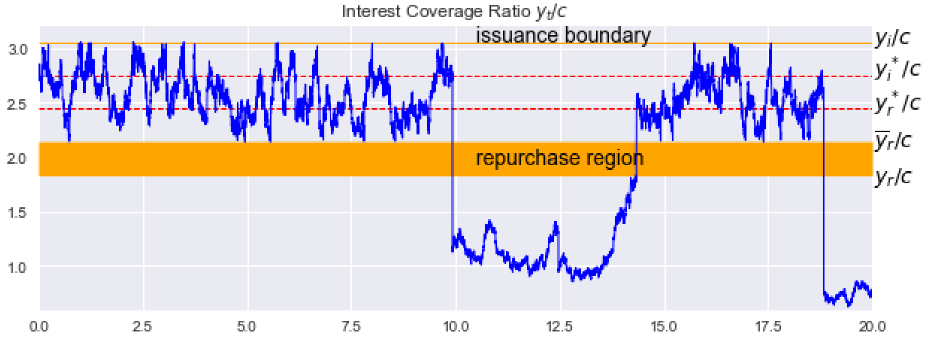
Policy Σ is **credible (time-consistent)** if supported by the grim-trigger strategies with threat of MPE reversal:

- equity holders announce policy Σ at $t = 0$ and promise to follow it;
- as long as they stick to Σ, bond holders expect them to do so in the future and price debt accordingly at $p(\cdot|\Sigma)$;
- if a deviation occurs, the play switches to the MPE (debt is priced according to $p_m(\cdot)$ and equity value is $e_m(\cdot)$);
- MPE is the most severe credible punishment;
- policy Σ is credible if deviations are not profitable in any state.

Default boundary y_b is pinned down by $e\left(y_b\right) = 0$ and $e'\left(y_b\right) = 0$.

Debt Policies Σ in Class $\mathbb{S}$

- Typical policy in class $\mathbb{S}$:



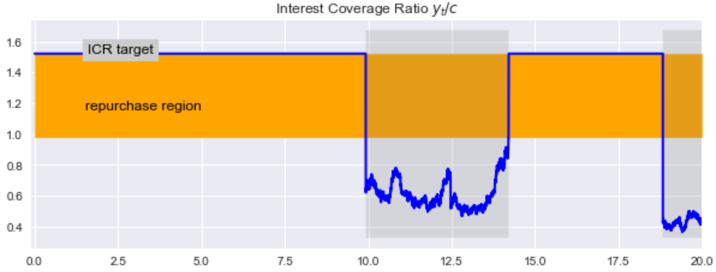
- A limitation of our analysis is that class $\mathbb{S}$ is potentially restrictive.
  - captures most debt policies analyzed in the literature.

Targeted ICR Policy

Main Result: Targeted ICR policy is an optimal time-consistent debt policy.

Targeted ICR policy is limit as $y_i = \overline{y}_r$. Two regimes:

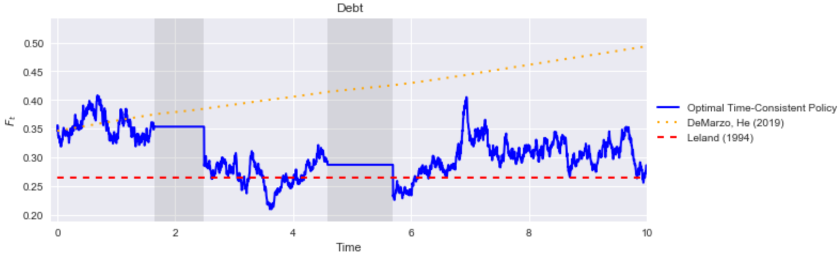
1. Stable regime: the firm sticks to **ICR target**
   - positive/small negative shocks to Y compensated by issuances/repurchases;
2. Distress regime: after large negative shocks, the firm temporarily doesn't manage liabilities.
   - either defaults or gets back to the stable regime.



- Bridge static trade-off theory and implications of the leverage ratchet effect.

Implication 1: Leverage Dynamics

Debt Dynamics



Implication 1: Leverage Dynamics

Interest Coverage Ratio

Implication 1: Leverage Dynamics

Debt Price

Implication 2: Effect of Credibility Constraints

- "Commitment optimum:" equity holders can commit to debt policy, but not default policy:

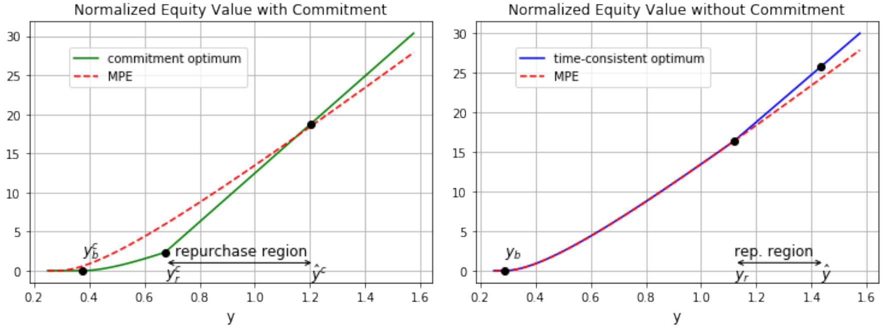
$$\max_{\Sigma \in \mathbb{S}} W(\Sigma)$$
$$\text{s.t.} \underbrace{e(y|\Sigma) \geq 0, \forall\, y \in \mathcal{R}(\Sigma)}_{\text{limited liability constraints}} \quad (\text{LL})$$

- CC are replaced by LL constraints.
  - Interpretation: commitment to liquidate the firm whenever equity holders deviate from Σ.
- Optimal default boundary still pinned down by $e(y|\Sigma) = 0$ and $e'(y|\Sigma) = 0$.

Implication 2: Effect of Credibility Constraints

CC restrict max size of credible repurchases $\Longrightarrow$ lower the optimal leverage target.

1. CC start to bind for the largest repurchase, but are slack for smaller repurchases.
2. If equity holders could commit to debt (but not default) policy, then
   - they would choose larger repurchases and larger leverage target than in the optimal time-consistent policy;
   - but larger repurchases would violate CC.



Implication 3: Effect of Brownian Shocks

Slack Credibility Constraints

- In stable regime, Brownian volatility does not matter much, because the firm compensates Brownian shocks.
- In distress regime, higher volatility increases the option value of waiting for equity $\Longrightarrow$ lower default threshold and higher optimal leverage.

The firm's target leverage ratio is **higher** when σ is higher.

Slack Credibility Constraints

σ	target ICR	at target		in distress	
		leverage ratio	credit spread	median leverage ratio	median credit spread
10%	1.16	**42%**	94 bps	89%	313 bps
25%	1.11	**44%**	97 bps	93%	837 bps
40%	1.01	**46%**	110 bps	93%	1257 bps

Implication 3: Effect of Brownian Shocks

Binding Credibility Constraints

The effect of binding credibility constraint:

- An increase in Brownian volatility increases the value of equity upon deviation.
- Hence, the maximal credible repurchase amount is smaller $\Longrightarrow$ riskier debt.

Thus, the firm's target leverage ratio is **lower** when σ is higher.

Binding Credibility Constraints

σ	target ICR	at target		in distress	
		leverage ratio	credit spread	median leverage ratio	median credit spread
10%	1.54	**30%**	27 bps	86%	246 bps
25%	1.73	**27%**	23 bps	80%	466 bps
40%	2.17	**22%**	26 bps	65%	598 bps

Implication 4: Effect of Poisson Shocks

Larger downward jumps

- optimal target leverage ratio decreases, because of more risky cash flows.

Analogous results for more frequent jumps.

Exp. jump size	ICR target	at target		in distress	
		leverage ratio	credit spread	median leverage ratio	median credit spread
−15%	0.47	**39%**	13 bps	97%	962 bps
−20%	1.73	**27%**	23 bps	80%	466 bps
−25%	4.44	**17%**	38 bps	57%	323 bps

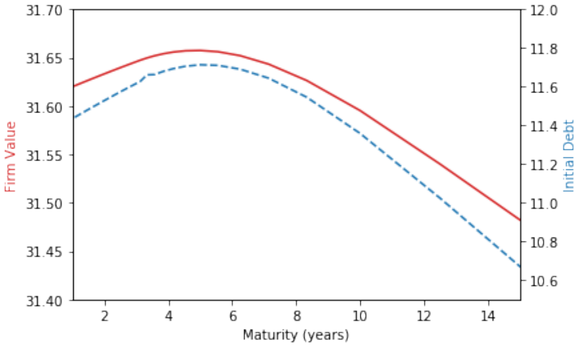
Implication 5: Interior Optimal Maturity

Debt matures at rate ξ and equity holders can choose ξ at $t = 0$.

- Coupon c is set so that newly issued debt is priced at par.

Optimal debt maturity is interior:

- Principal payments are not tax deductible $\implies$ debt with longer maturity better exploits tax benefits.
- Shorter maturity commits to reduce the debt burden in the distress regime (when repurchases are not credible).



Implication 5: Interior Optimal Maturity

Firm responds differently to volatility risk and jump risk

σ	maturity	leverage ratio at target	ζ	maturity	leverage ratio at target
5%	13.2 years	37%	-10%	26.3 years	53%
25%	4.9 years	37%	-15%	4.9 years	37%
40%	2.3 years	37%	-20%	2.4 years	26%

Summary

Optimal time-consistent debt policies in the standard trade-off theory.

Main implications:

1. Optimal leverage dynamics that combines elements of static trade-off theory and implications of leverage ratchet effect.
2. Large repurchases can cause credibility constraints to bind, which restricts leverage and affects comparative statics.
3. Interior optimal debt maturity.