

A Method to Estimate Discrete Choice Models that is Robust to Consumer Search

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- Disentangling utility and information matters for:
 - Welfare
 - Product design
 - How more information would impact choices

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- But 90% of applied discrete choice models assume full info & rationality (survey of “top 5” papers since 2015)
- Even in cases where disentangling utility and information matters
 - 85% of papers that compute welfare assume full info

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 - “Weitzman” sequential search or simultaneous search
 - Behavioral models (e.g. “satisficing,” “search all goods ex-ante above a threshold”)

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 - Behavioral models (e.g. “satisficing,” “search all goods ex-ante above a threshold”)
- Can construct **tests for full information**
- We validate the approach in two settings:
 - Lab experiment
 - Observational data from Expedia

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- In our model, consumers observe attributes x for free, but need to search to uncover attributes z
- Examples of z attributes
 - Online purchases: things you see only if you click, like warranty info
 - Grocery: nutrition labels
 - Cars: miles per gallon

Literature

- **Theory literature:** Branco, Sun and Villas-Boas (2012), Ke, Shen and Villas-Boas (2016), Armstrong (2016), Gabaix (2019)
- **Empirical search:** Mehta, Rajiv and Srinivasan (2003), Kim, Albuquerque and Bronnenberg (2010), De los Santos, Hortaçsu and Wildenbeest (2012), Honka (2014), Honka and Chintagunta (2017), Honka, Hortaçsu and Vitorino (2017), Ursu (2018) estimate explicit search models
- **Dollarized attributes:** Abaluck and Gruber (2011), Alcott and Taubinsky (2015), Brown and Jeon (2020)
- **Consideration sets:** Often, these models involve search over goods rather than individual attributes (e.g. Roberts and Lattin 1991, Sovinsky, 2008; Conlon and Mortimer, 2013, Gaynor, Propper and Seiler, 2016, Abaluck and Adams, 2018)

Roadmap

- Model w/ linear utility and homogeneous preferences
- Prove key lemma and give intuition for full proof
- Extensions
 - Nonlinear utility
 - Random coefficients
 - Multiple hidden attributes
 - Endogeneity
 - Violations of visible utility assumption
- Estimation
- Empirical Validation
 - Experiment
 - Expedia Data

Baseline Model

$$U_{ij} = \alpha x_j + \beta z_j + \epsilon_{ij}, \quad \alpha, \beta > 0$$

- x_j : observed by econometrician and consumers
- z_j : observed by econometrician, but by consumers only if they search
- ϵ_{ij} : observed by consumer, but not econometrician
- So consumer i sees (x_j, ϵ_{ij}) and decides whether to learn z_j
- Finally, consumer i chooses the best product among those they searched

The Identification Problem

$$U_{ij} = \alpha x_j + \beta z_j + \epsilon_{ij}$$

- Let $s_j(\mathbf{x}, \mathbf{z})$ be the choice probability for good j
- Identification takes s_j as known and asks

Can we recover α and β ?

- With infinite data, we fully know s_j as a function of $\mathbf{x}, \mathbf{z}$

Standard Approach

$$U_{ij} = \alpha x_j + \beta z_j + \epsilon_{ij}$$

- With full information, $s_j = P(U_j \geq U_k \ \forall k)$ and thus

$$\frac{\partial s_j / \partial z_j}{\partial s_j / \partial x_j} = \frac{\beta}{\alpha}$$

Standard Approach

$$U_{ij} = \alpha x_j + \beta z_j + \epsilon_{ij}$$

- If consumers know z_j only for some goods, **attenuation bias**

$$\frac{\partial s_j / \partial z_j}{\partial s_j / \partial x_j} \leq \frac{\beta}{\alpha}$$

- Maybe s_j is insensitive to z_j because some consumers aren't aware of z_j , not because they don't like it

Our Approach

- Instead of looking at a ratio of first derivatives, we look at a ratio of second (cross) derivatives
- Let good 1 denote the good with the highest value of z in the choice set
- It turns out that

$$\frac{\partial^2 s_1 / \partial z_1 \partial z_j}{\partial^2 s_1 / \partial z_1 \partial x_j} = \frac{\beta}{\alpha}$$

for $j \neq 1$ holds no matter whether consumers search or not (for a class of search models)

Intuition for case with 2 goods

$$\begin{aligned}s_1 &= P(U_1 > U_2) \\ &\quad - P(U_1 > U_2, \text{only search 2}) \\ &\quad + P(U_1 < U_2, \text{only search 1})\end{aligned}$$

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- Third term drops out if consumers search in order of “visible utility”

Lemma

Assume consumer i searches in descending order of $VU_{ij} = \alpha x_j + \epsilon_{ij}$. Then, if they search good 1, they always choose the utility-maximizing good.

Proof: Suppose that good 1 is searched and there is a good j with $U_{ij} > U_{i1}$. Since $z_1 > z_j$, it must be that $VU_{ij} > VU_{i1}$ and so j is searched and then chosen.

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- Assume i searches j iff $g_i(x_j, \epsilon_{ij}, \bar{u}) > 0$, where $\bar{u}$ is utility in hand
- Then, $\frac{\partial P(U_1 > U_2, \text{only search good 2})}{\partial z_1}$ depends on x_2 and z_2 only via U_2

► Proof

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▶ Proof
- So $\frac{\partial s_1}{\partial z_1}$ depends on x_2 and z_2 only via U_2 ($= \alpha x_j + \beta z_j + \epsilon_{ij}$)

$$\Rightarrow \frac{\partial^2 s_1 / \partial z_1 \partial z_2}{\partial^2 s_1 / \partial z_1 \partial x_2} = \frac{\beta}{\alpha}$$

▶ ID of α

Discussion

Assumption 1

Consumers search in descending order of $VU_{ij} = \alpha x_j + \epsilon_j$.

- Many microfoundations: e.g., Weitzman search with i.i.d. priors and search costs equal across goods; several simultaneous search models

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 - Extension: consumers draw inferences about z_j based on x_j

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 - Extension: consumers draw inferences about z_j based on x_j
 - Search costs differ across goods
 - Extension: observed search cost shifters that don't impact utility (e.g., rankings)
- This assumption has testable implications (bounds on choice probabilities)

Discussion

Assumption 2

Consumer i searches j iff $g_i(x_j, \epsilon_{ij}, \bar{u}) > 0$, where $\bar{u}$ is utility in hand.

- $g_i(x_j, \epsilon_{ij}, \bar{u}) = +\infty$ Full information
- $g_i(x_j, \epsilon_{ij}, \bar{u}) = \alpha x_j + \beta r_i + \epsilon_{ij} - \bar{u}$ Weitzman (1979) sequential search
- $g_i(x_j, \epsilon_{ij}, \bar{u}) = \tau_i - \bar{u}$ Satisficing
- $g_i(x_j, \epsilon_{ij}, \bar{u}) = \alpha x_j + \epsilon_{ij} - \tau_i'$ Search all goods above a threshold ex-ante

Note: The approach does not require one to know/specify g_i

Discussion

- Our assumptions are not without bite
- But they allow for a range of search protocols:
 - Sequential vs simultaneous search
 - Forward-looking vs myopic behavior
 - Biased vs unbiased beliefs

Taking Stock

$$\frac{\partial^2 s_1 / \partial z_1 \partial z_j}{\partial^2 s_1 / \partial z_1 \partial x_j} = \frac{\beta}{\alpha} \text{ for } j \neq 1$$

- This expression provides **robust identification of preferences**
 - It works both with full information models *and* if consumers search

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 - It works both with full information models *and* if consumers search
- **Our recommendation:** with enough data, compute $\frac{\partial^2 s_1 / \partial z_1 \partial z_j}{\partial^2 s_1 / \partial z_1 \partial x_j}$ instead of $\frac{\partial s_1 / \partial z_1}{\partial s_1 / \partial x_1}$

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- This expression provides **robust identification of preferences**
 - It works both with full information models *and* if consumers search
- **Our recommendation:** with enough data, compute $\frac{\partial^2 s_1 / \partial z_1 \partial z_j}{\partial^2 s_1 / \partial z_1 \partial x_j}$ instead of $\frac{\partial s_1 / \partial z_1}{\partial s_1 / \partial x_1}$
- Further, we can **test for full information** by comparing $\frac{\partial^2 s_1 / \partial z_1 \partial z_j}{\partial^2 s_1 / \partial z_1 \partial x_j}$ and $\frac{\partial s_1 / \partial z_1}{\partial s_1 / \partial x_1}$
 - In particular, given an x , can test whether another attribute is hidden or not

Extensions

- Nonlinear utility [▶ Go](#)
- Random Coefficients [▶ Go](#)
- Multidimensional z [▶ Go](#)
- Endogenous attributes [▶ Go](#)
- Violations of Assumption 1 [▶ Go](#)

Estimation

$$\frac{\partial^2 s_1 / \partial z_1 \partial z_j}{\partial^2 s_1 / \partial z_1 \partial x_j} = \frac{\beta}{\alpha} \text{ for } j \neq 1$$

- Key input: flexible estimator of demand function s_1

Estimation

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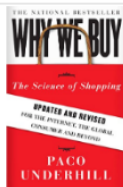
- Key input: flexible estimator of demand function s_1
- If J is not too large, can use nonparametric methods (e.g., Compiani, 2022)
- Develop a parametric method that estimates the cross-derivatives flexibly [▶ Go](#)
 - More scalable
 - Implementable in Stata

Experiment

- 93 participants each making 40 choices from sets of 3 ebooks
- They observe book covers, titles, authors, genre, ratings and prices for free
- Prices are randomized, \$11-\$15 (equally likely)
- Discounts are randomized \$0-\$10 (equally likely)
 - Shown for free in 10 choices
 - Hidden and costly to uncover in the remaining 30 choices

Sample Screen in Costly Info Treatment

Your balance : \$ 24.50 Cost to reveal a discount is \$0.50



Why We Buy: The Science of Shopping

Paco Underhill

Business

★★★★☆ (3.91/5 , 11474)

Initial Price: \$13

Discount: (\$0-\$10)

Final Price: \$13 - Discount



Watership Down

Richard Adams

Classics

★★★★☆ (4.07/5 , 340260)

Initial Price: \$12

Discount: - \$4

Final Price: \$8



The Soong Dynasty

Sterling Seagrave

History

★★★★☆ (3.96/5 , 532)

Initial Price: \$12

Discount: (\$0-\$10)

Final Price: \$12 - Discount

Results

	Standard approach	Our approach
Variable	Full Info	
Price	-0.386*** (0.038)	
Discount (-)	-0.376*** (0.020)	
Rating	0.591*** (0.190)	
Discount (-) / Price	0.986*** (0.093)	
N	930	

Results

Variable	Standard approach		Our approach
	Full Info	Costly Info	
Price	-0.386*** (0.038)	-0.302*** (0.018)	
Discount (-)	-0.376*** (0.020)	-0.206*** (0.009)	
Rating	0.591*** (0.190)	0.421*** (0.099)	
Discount (-) / Price	0.986*** (0.093)	0.683*** (0.044)	
<i>N</i>	930	2790	

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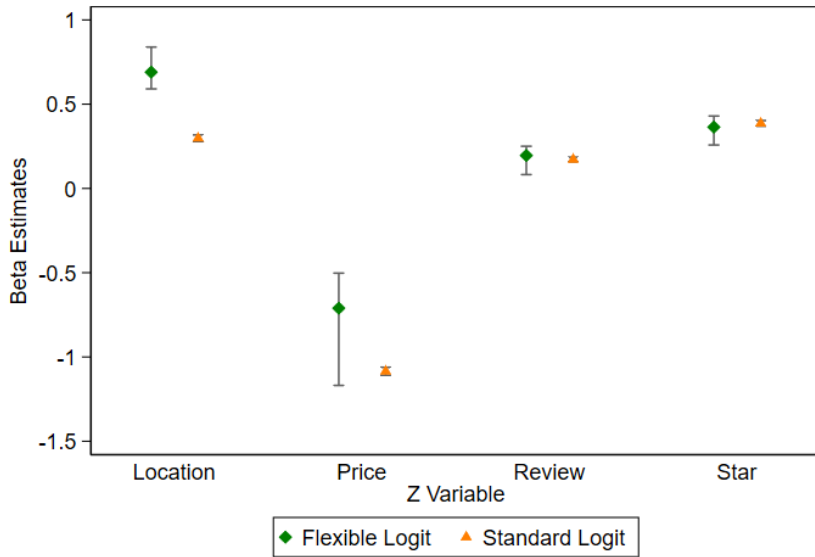
- Under $\epsilon_{ij} \sim$ extreme value assumption, we estimate a welfare gain from info of \$0.15 per choice
 - Close to \$0.18 per choice based on column 1
- ⇒ Can predict welfare gains *before* doing an information intervention

► Bounds

Expedia Data

- We see choice sets of hotels in a city
- People decide what hotels to choose given:
 - Price
 - Review score (sentiment)
 - Star rating
 - Location
- First three are visible in search results
- Location is visible only if you click through and look at a map

Expedia Estimation Results



Expedia: Counterfactuals

What if location was immediately visible to consumers?

	Status quo	Counterfactual
Average Value for the Transacted Item		
Price (\$)	143.52	136.90
Stars	3.47	3.42
Review Score	4.03	4.03
Chain	0.64	0.64
Promotion	0.40	0.40
Position	4.55	4.91
Location Score	3.32	3.40
Welfare Difference per Choice (\$)	-	1.93

Conclusion

- We propose a way to estimate preferences that is robust to consumers searching
- Uses:
 - Predict impact of info
 - More reliable welfare analyses
 - Test for full info
- Future research:
 - Theory: can any of our sufficient conditions be relaxed?
 - Full-fledged empirical analyses



Why Estimate the Full Search Model?

- Search cost distribution is needed to do counterfactuals that reduce search costs
 - But, for “zero search costs” counterfactuals, our approach suffices
- Search model primitives are needed to compute welfare improvement from eliminating search frictions
 - Caveat: Survey data or data on time spent searching might be more reliable than (parametric) search cost estimates
- Full structural model might offer a parsimonious way to estimate preferences when our identification results are hard to apply directly in estimation
 - E.g. unknown functional form for utility, random coefficients

◀ Return

Why does this work with full information?

$$\frac{\partial^2 s_1 / \partial z_1 \partial z_2}{\partial^2 s_1 / \partial z_1 \partial x_2} = \frac{\partial^2 P(U_1 > U_2) / \partial z_1 \partial z_2}{\partial^2 P(U_1 > U_2) / \partial z_1 \partial x_2} = \frac{\partial^2 P(U_1 > U_2) / \cancel{\partial U_1 \partial U_2}}{\cancel{\partial^2 P(U_1 > U_2) / \partial U_1 \partial U_2}} \cdot \frac{\beta}{\alpha}$$

- This asks: how does the marginal response of choice probabilities to z_1 change with attributes of rival goods?
- In standard models, attributes of rival goods impact choices via utility for rival goods
- And they impact utility in proportion to $\frac{\beta}{\alpha}$

◀ Return

Extension: Nonlinear Utility

- It generalizes to nonlinear utility functions

$$U_{ij} = v(x_j, z_j) + \epsilon_{ij}$$

- Without loss, can write: $U_{ij} = a_{ij}(x_j) + b_{ij}(x_j, z_j)$ where $b_{ij}(x_j, 0) = 0$. Define $VU_{ij} = a_{ij}(x_j)$.
- Can show that

$$\frac{\partial^2 s_1 / \partial z_1 \partial z_2}{\partial^2 s_1 / \partial z_1 \partial x_2} = \frac{\partial v(x_2, z_2) / \partial z_2}{\partial v(x_2, z_2) / \partial x_2}$$

- By differentiating more times with respect to x_2 and z_2 , one can recover the entire function v to arbitrary accuracy

◀ Return

Extension: Random Coefficients

- We can also allow for random coefficients. Suppose

$$U_{ij} = \alpha_i x_j + \beta_i z_j + \epsilon_{ij}$$

- If (α_i, β_i) take a finite number of values $[\alpha_1, \dots, \alpha_{K_\alpha}]$ and $[\beta_1, \dots, \beta_{K_\beta}]$ we can recover the corresponding $K_\alpha K_\beta$ probabilities using derivatives of the form

$$\frac{\partial^{n+1} s_1}{\partial z_1 \partial z_2^{\bar{n}} \partial x_2^{n-\bar{n}}} \quad \bar{n} = 0, \dots, n$$

◀ Return

Extension: Endogeneity

- Can accommodate endogenous attributes (e.g. price p_j)

$$U_{ij} = \alpha x_j + \beta_i z_j + \gamma_i p_j + \xi_j + \epsilon_{ij} \quad \xi \not\perp \mathbf{p}$$

- Idea: show that our class of search models satisfies the sufficient conditions for nonparametric identification of demand in Berry and Haile (2014)
 - See also Berry, Gandhi and Haile (2013)
- Once the demand system

$$\mathbf{s} = \sigma(\mathbf{x}, \mathbf{z}, \mathbf{p}, \xi)$$

is identified, we can effectively control for ξ and apply our results

◀ Return

Extension: Multidimensional z

- Our argument extends to multidimensional z provided all the z are revealed conditional on search
- The relative weights attached to the z components can be recovered from ratios of first-derivatives
- Our argument then applies to the new z -index, an appropriately weighted combination of the z 's

◀ Return

Extension: Violations of Assumption 1

- Can allow for non-utility factors affecting search (e.g., rank on Amazon page)
- Assume i searches goods in decreasing order of $m(VU_{ij}, r_j)$, where m is increasing in both arguments
- Then if i searches good 1, i maximizes utility among all goods with higher rank than 1
- Can modify our argument to show identification of preferences

◀ Return

Extension: Violations of Assumption 1 (cont'd)

- Also, can allow for consumers to form expectations on z_j conditional on x_j

$$u_{ij} = \alpha x_j + \beta E(z_j | x_j) + \beta \underbrace{(z_j - E(z_j | x_j))}_{\tilde{z}_j} + \epsilon_{ij}$$

- If $E(z_j | x_j) = \gamma_0 + \gamma_1 x_j$, can recover $\frac{\beta}{\alpha + \beta \gamma_1}$ as above
- Given that $\alpha + \beta \gamma_1$ is identified by $\frac{\partial s_1}{\partial x_1}$, we recover β as well

◀ Return

Bounds from Visible Utility Assumption

- Good j cannot be chosen if an alternative good has higher VU_{ij} and higher U_{ij} :

$$s_j(\mathbf{x}, \mathbf{z}) \leq 1 - P(U_{ik} \geq U_{ij} \text{ and } VU_{ik} \geq VU_{ij} \text{ for some } k) \quad (1)$$

- Probability of choosing good j is at least as large as the probability that good j maximizes both utility and visible utility:

$$s_j(\mathbf{x}, \mathbf{z}) \geq P(U_{ij} \geq U_{ik} \text{ and } VU_{ij} \geq VU_{ik} \text{ for all } k) \quad (2)$$

◀ Return

Formal Argument

$$\begin{aligned} & P(\{U_1 > U_2\} \cap \{\text{only search good 2}\}) \\ = & P(\{U_1 > U_2\} \cap \{VU_2 > VU_1\} \cap \{g_i(x_1, \epsilon_1, U_2) < 0\}) \\ = & P(\{U_1 > U_2\} \cap \{g_i(x_1, \epsilon_1, U_2) < 0\}) \\ & - P(\{U_1 > U_2\} \cap \{VU_1 > VU_2\} \cap \{g_i(x_1, \epsilon_1, U_2) < 0\}) \\ = & \textcolor{blue}{P(\{U_1 > U_2\} \cap \{g_i(x_1, \epsilon_1, U_2) < 0\})} \\ & - \textcolor{red}{P(\{VU_1 > VU_2\} \cap \{g_i(x_1, \epsilon_1, U_2) < 0\})} \end{aligned}$$

- **Second term** does not depend on $z_1 \Rightarrow$ cancels out when differentiating wrt z_1
- **First term** only depends on x_2 and z_2 via U_2

◀ Return

Why is this enough?

$$\frac{\partial^2 s_1 / \partial z_1 \partial z_2}{\partial^2 s_1 / \partial z_1 \partial x_2} = \frac{\beta}{\alpha}$$

- If $z_j = z \forall j$, consumers always maximize utility (good with largest VU_j is searched and is utility-maximizing)
- Thus, conditional on $z_j = z \forall j$, seeing how s_j varies with $\mathbf{x}$ identifies α by standard arguments
- Together with identification of $\frac{\beta}{\alpha}$, we fully recover preferences

◀ Return

“Flexible Logit”

- Let attributes of rival goods enter the utility of good 1

$$\begin{aligned}\tilde{u}_{i1} &= x_1 a + z_1 b \\ &+ \sum_{k \neq 1} (w_{z1k} \gamma_{zk} z_k + w_{x1k} \gamma_{xk} x_k + w_{z2k} \delta_{zk} z_k z_1 + w_{x2k} \delta_{xk} x_k z_1) + \epsilon_{i1}\end{aligned}$$

- Estimate this model using standard packages, then obtain second derivatives and compute estimate of $\frac{\beta}{\alpha}$
- Similar to Taylor expansion, but in practice performs better than polynomial approximation

◀ Return

Visible Utility Bounds

