# Hysteresis, the Big Push, and Technological Adoption 

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January, 2023

## Introduction

- Conventional view: GDP $=$ Trend + Cycle
- Trend $=f$ (supply side factors)
- Cycle $=f$ (demand shocks and monetary policy).
- Ideas $\rightarrow$ TFP.
- Alternative view
- Relationship between the cycle and the trend.
- Hysteresis: temporary shocks may have long run effects.
- Ideas $\rightarrow$ implementation process $\rightarrow$ TFP.


Figure: United States Real GDP and Linear Trend (Source: FRED)

## This paper

- Question: which supply-side factors determine if a temporary shock has permanent effects?
- Focus on the adoption and abandonment of a technology by firms with heterogeneous costs.
- Why not R\&D? supply-side scarring, relavance for non-R\&D intensive economies.
- Coordination failure in the share of adopters (multiple equilibria, Big Push).
- Equilibrium selection: history of shocks $\rightarrow$ hysteresis may appear.
- Key feature: if adoption threshold $>$ exit threshold $\rightarrow$ hysteresis.
- Typical in stochastic settings when there is an option value of waiting for a high draw.
"In 1899 and 1900, electric vehicles outsold all other types of cars"


Figure: Edison and electric car, 1913, (Source: National Museum of American History)
"[...] it would take more than 60 years before turbines of that size would be built [again]"


Figure: $\mathbf{1 0 0}$ kW turbine in Crimea 1931, (Source: Potter, 2020)

## Results

Hysteresis depends on:

- Elasticity of substitution between input varieties.
- More sustituability/markups $\rightarrow$ more hysteresis.
- Barriers to adoption.
- Higher entry costs $\rightarrow$ more hysteresis.
- Shock size.
- Larger shocks $\rightarrow$ more hysteresis.

On the possibility of reverse hysteresis (positive shocks $\rightarrow$ positive long-run effect):

- Complementarity in adoption vs. decreasing profits for late adopters.
- It is relatively more expensive to adopt the higher the adoption share (holding costs constant).
- Reverse hysteresis more likely in less technologically advanced economies.
- Friedman's (1964) plucking model more likely in economies near/on the technological frontier.


## Related Literature

- Relationship between cycle and trend: Cerra, Fatás, \& Saxena (forthcoming); Elfsbacka Schmöller, 2022; Garga \& Singh, 2020; Anzoategui, Comin, Gertler, \& Martinez, 2019; Bianchi, Kung, \& Morales, 2019; Benigno \& Fornaro, 2018; Aguiar \& Gopinath, 2007;
- Hysteresis non-linearity: Jordá, Singh, \& Taylor (2021); Amador (2022); Aikman, Drehmann, Juselius, \& Xing, (2022).
- Hysteresis is asymmetric (no reverse hysteresis).
- Hysteresis seems to be driven by the largest episodes.
- Output fluctuations as coordination failures: Rosenstein-Rodan (1943); Murphy et al., (1989); Ciccone (2002); Diamond (1982); Kiyotaki (1988); Cooper \& John (1988); Durlauf (1991); Fajgelbaum, Schaal, \& Taschereau-Dumouchel (2017); Schaal \& Taschereau-Dumouchel (2018), Choi \& Shim (2022).


## Contribution

- Parsimonious supply side model of hysteresis consistent with empirical evidence on asymmetry and non-linearity.
- Supply-side explanation for Friedman's (1964) plucking model.


## Model

- Time is discrete and goes on forever.
- The economy:

1. Representative household.

- $\mathbb{E} \sum_{t=0}^{\infty} \rho^{t} U\left(C_{t}\right) ; U\left(C_{t}\right)=\ln C_{t} ; P_{t} C_{t} \leq W_{t}+\Pi_{t}$.

2. Final good sector.

- $Y_{t}=\left(\iint_{j, k} \frac{\sigma-1}{\frac{\sigma-k, t}{\sigma}} \mathrm{~d} j \mathrm{~d} k\right)^{\frac{\sigma}{\sigma-1}} ;(j, k) ; P_{t}=\left(\iint_{j, k} p_{j, k, t}^{1-\sigma} \mathrm{d} j \mathrm{~d} k\right)^{\frac{1}{1-\sigma}}$.

3. Intermediate goods sector (continuum of varieties).

- Final good used for consumption and to pay for the cost of adopting and operating a modern technology.
- Resources used in adoption are not used for any other purpose.


## Intermediate goods producer

- Unit mass of firms indexed by adoption/abandonment thresholds $(j, k)$.
- Varieties produced by monopolist choosing traditional $(\mathcal{T})$ or a modern $(\mathcal{M})$ technology.
- Production function $y_{j, k, t}=A_{j, k, t} l_{j, k, t}$
- Productivity: $A_{j, k, t}=e^{a_{t}} u_{j, k, t}$
- Shock: $a_{t}=\psi a_{t-1}+\varepsilon_{t}^{a}$,
- If in the $\mathcal{T}$-sector, then $u_{j, k, t}=1$
- If in the $\mathcal{M}$-sector then $u_{j, k, t}=\mathcal{A}>1$.


## Partial equilibrium

- Firm quantity and pricing decisions are independent of $j$ and $k$.
- Proposition 1: Prices, $p_{j, k, t}^{i}$, and quantities, $y_{j, k, t}^{i}$, will be symmetric across all firms in each type $i \in\{\mathcal{T}, \mathcal{M}\}$ :
- $\phi_{t}^{\mathcal{T}}=\frac{W_{t}}{e^{e_{t}}} ; \phi_{t}^{\mathcal{M}}=\frac{\phi_{t}^{\mathcal{T}}}{\mathcal{A}} ; y_{t}^{\mathcal{T}}=\left(\frac{\sigma}{\sigma-1} \phi_{t}^{\mathcal{T}}\right)^{-\sigma} Y_{t} ; y_{t}^{\mathcal{M}}=\left(\frac{\sigma}{\sigma-1} \frac{\phi_{t}^{\mathcal{T}}}{\mathcal{A}}\right)^{-\sigma} Y_{t}$
- Gross profits in the $\mathcal{M}$-sector will be a linear function of profits in the $\mathcal{T}$-sector.
- $\pi_{t}^{\mathcal{T}}=\left(\frac{1}{\sigma-1}\right)\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\left(\frac{1}{\phi_{t}^{\tau}}\right)^{\sigma-1} Y_{t}$
- $\pi_{t}^{\mathcal{M}}=\mathcal{A}^{\sigma-1} \pi_{t}^{\mathcal{T}}$
- Proposition 2: Let the final good, $Y_{t}$, be the numéraire ( $\left.P_{t}=1 \forall t\right)$. Given a share of firms in the modern sector, $m_{t}$, and the optimal choices of prices and quantities for firms in each sector, it is possible to derive functions for aggregate quantities (in terms of $m_{t}$ and $a_{t}$ ).


## Intermediate goods producer



Adoption operator:
$\gamma_{j, k, t}= \begin{cases}1 & \text { if } \gamma_{j, k, t-1}=0, \text { and } \pi_{t}^{\mathcal{T}}>j \\ 1 & \text { if } \gamma_{j, k, t-1}=1, \text { and } \pi_{t}^{\mathcal{T}}>k \\ 0 & \text { if } \gamma_{j, k, t-1}=0, \text { and } \pi_{t}^{\mathcal{T}}<j \\ 0 & \text { if } \gamma_{j, k, t-1}=1, \text { and } \pi_{t}^{\mathcal{T}}<k\end{cases}$

## How the optimal thresholds are obtained for each firm?

- $\tilde{\pi}_{t}=\gamma_{j, k, t}\left(\mathcal{A}^{\sigma-1} \pi_{t}^{\mathcal{T}}-\gamma_{j, k, t} \gamma_{j, k, t-1} k^{c}-\gamma_{j, k, t}\left(1-\gamma_{j, k, t-1}\right) j^{c}\right)+\left(1-\gamma_{j, k, t}\right) \pi_{t}^{\mathcal{T}}$
- $V_{0}=E_{0}\left\{\sum_{t=1}^{\infty} \beta^{t} \tilde{\pi}_{t}\left(a_{t}, m_{t}, \gamma_{j, k, t} \mid \gamma_{j, k, t-1}\right)\right\}$
- $\max _{\gamma_{j, k, t}} \tilde{\pi}_{t}+\beta E\left\{V_{t+1}\left(a_{t+1}, m_{t+1}, \gamma_{j, k, t+1} \mid \gamma_{j, k, t}\right)\right\}$
- Optimal threshold $j$ :
$\tilde{\pi}_{t}\left(\gamma_{j, k, t}=0\right)+\beta E\left\{V_{t+1}\left(a_{t+1}, m_{t+1} \mid \gamma_{j, k, t+1}=\gamma_{j, k, t}=0\right)\right\}=E\left\{V_{t}\left(a_{t}, m_{t} \mid \gamma_{j, k, t}=1\right)\right\}$
- Optimal threshold $k$ :
$\tilde{\pi}_{t}\left(\gamma_{j, k, t}=1\right)+\beta E\left\{V_{t+1}\left(a_{t+1}, m_{t+1} \mid \gamma_{j, k, t+1}=\gamma_{j, k, t}=1\right)\right\}=E\left\{V_{t}\left(a_{t}, m_{t} \mid \gamma_{j, k, t}=0\right)\right\}$
- Assumption: firms expect $\pi_{t}^{\mathcal{T}}$ to behave as a random walk bounded in the long-run with i.i.d. disturbances.
- Expectations are derived from simulations to obtain $j$ and $k$.


## How the optimal thresholds are obtained for each firm？




## Equilibrium

Definition 1: An equilibrium is $C_{t}, L_{t} ; \gamma_{j, k, t}, y_{j, k, t}^{i}, l_{j, k, t}^{i}, i \in\{\mathcal{M}, \mathcal{T}\} ; p_{j, k, t}^{\mathcal{T}}, p_{j, k, t}^{\mathcal{M}}$, $P_{t}\left(a^{t}\right), W_{t}$; and $m_{t}$, such that;

1. The household maximizes utility.
2. All intermediate producers maximize their profits net of technological costs.
3. The final good producer solves its problem.
4. Prices clear all markets.
5. $m_{t}$ satisfies:

- $m_{t}=\int_{j \geq k} \int_{k} \delta(j, k) \gamma_{j, k, t} \mathrm{~d} j d k$.
- where $\delta(j, k)$ is the joint density of $j$ and $k$.
- and; $\gamma_{j, k, t}= \begin{cases}1 & \text { if } u_{j, k, t}=\mathcal{A} \\ 0 & \text { if } u_{j, k, t}=1\end{cases}$


## What if $j=k \forall(j, k)$ ?

Figure: Single steady state equilibrium when $j=k$


## What if $j \geq k$ ?

- Multiple steady state equilibria (intuition: two CDFs instead of one).
- Path dependence.
- Possibility of hysteresis (graphic proof).


## What if $j \geq k$ ?

Figure: The $j \geq k$ half plane $(t=0)$


## What if $j \geq k$ ?

Figure: $(t=1)$


## What if $j \geq k$ ?

Figure: $(t=2)$


## Computational exercises

1. Explore how different parameters affect hysteresis.

- Barriers to entry (cost).
- Shock size.
- Distance to the technological frontier.
- Elasticity of substitution/markups.

2. Asymmetries in hysteresis effects (reverse hysteresis and Friedman's plucking model).

## Hysteresis depends on entry costs $j^{c}$



Baseline


Wage $W_{t}$




30\% higher entry costs


Wage $W_{t}$


## Hysteresis depends on shock size and adoption share



Hysteresis depends on productivity of new technology


## Hysteresis depends on markups



## Hysteresis depends on elasticity of substitution (markups)

Figure: Response of $m_{t}$ to a $-15 \%$ shock for different values of $\sigma$ (markup $\frac{\sigma}{\sigma-1}$ )


## Conclusion

- Theoretical supply-side framework for hysteresis.
- Relevant for drops in trend and for non-R\&D intensive economies.
- Defines a set of conditions for the possibility of reverse hysteresis.
- Novel equilibrium selection criterion based on history of shocks.
- Key mechanism based on heterogeneous adoption and abandonment thresholds.
- Results:
- Hysteresis depends non-linearly on the size of the shock.
- Hysteresis depends on market power (markups, entry costs).
- Reverse hysteresis more likely in economies far away from the technological frontier.
- Friedman's plucking model more likely for economies on the technological frontier.

