Intangible Capital, Non-Rivalry, and Growth

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Introduction

Intangible assets are an important factor of production

> Examples: IT-related assets (software, data), intellectual property (patents, trademarks), organization capital (management processes)

Broad question: What is so special about intangible assets relative to physical assets?

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Goal: Write model emphasizing positive features of intangibles, examine implications for growth

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Institutions enforce exclusivity and therefore turn ideas into intangible assets (example: the patent system)

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Different types of intangible assets $\leftrightarrow$ different (ρ, δ)

Classifying intangibles

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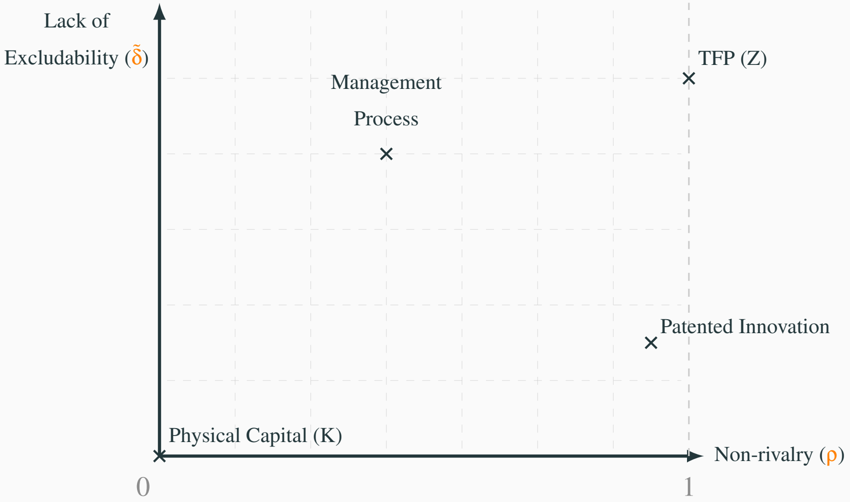

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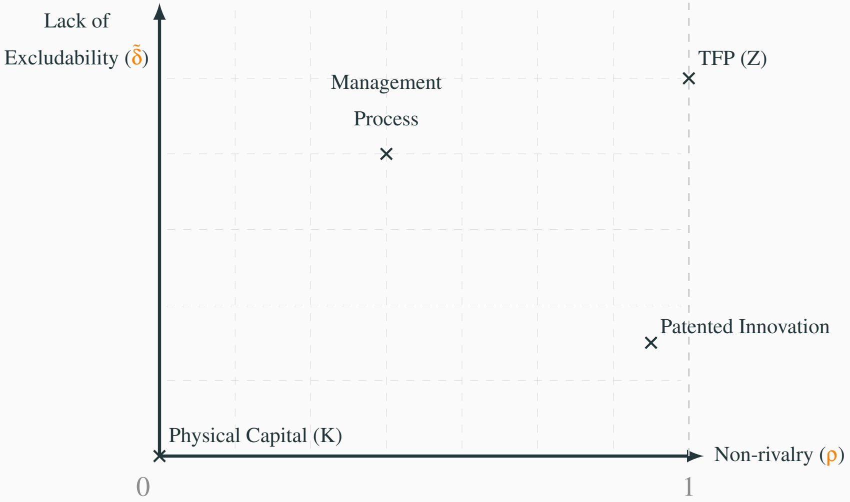

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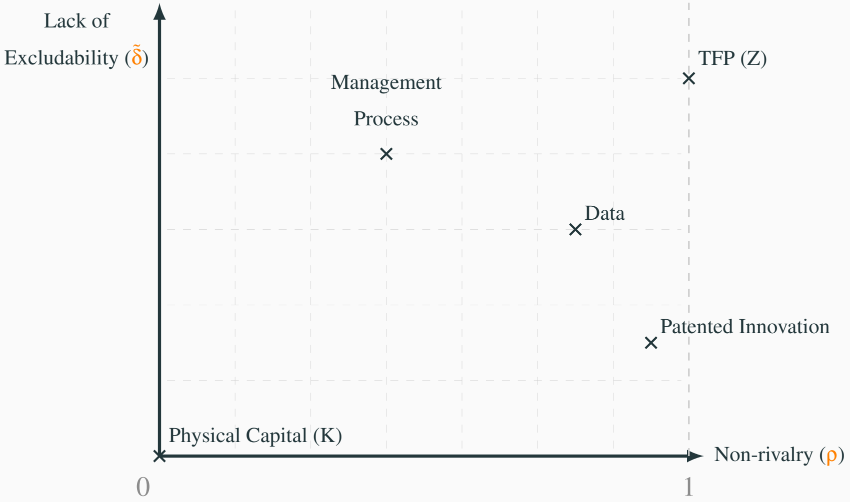

Contrast to Existing Approaches of Modeling Intangibles

1. Intangibles are just another type of capital, except hard to measure

   e.g. Hall (2001); Bhandari and McGrattan (2021)

2. Investment in intangibles allows firms to lower marginal cost

   e.g. Klette and Kortum (2004); De Ridder (2019)

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Contribution: Macro model w/ intangibles

Imperfect rivalry + imperfect excludability

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Benchmarks:

Physical Capital K: perfectly rival (Solow model, no growth)

TFP Z: perfectly non-rival (AK or Romer, perpetual growth)

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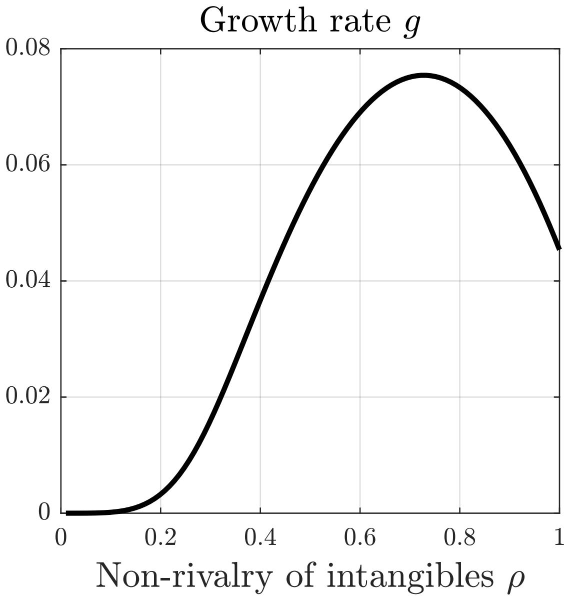
Findings: Non-monotonic (inverse U-shaped) relationship between non-rivalry and growth



Growth rate g
0.08
0.06
0.04
0.02
0
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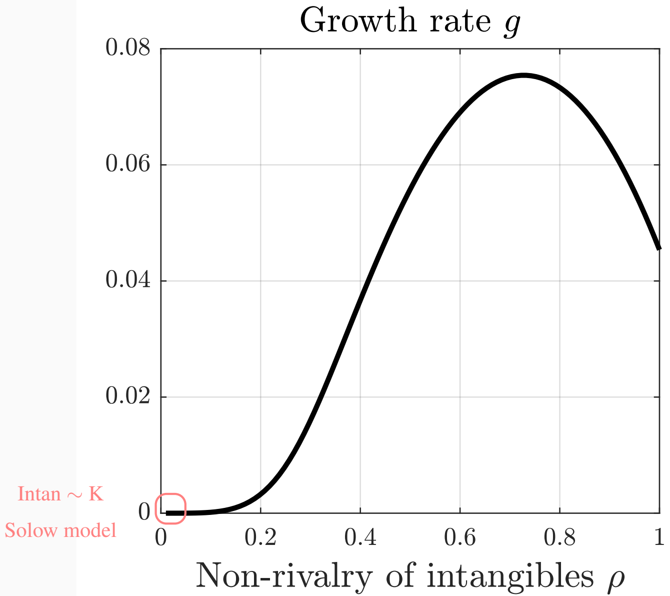
0.2
0.4
0.6
0.8
1
Non-rivalry of intangibles ρ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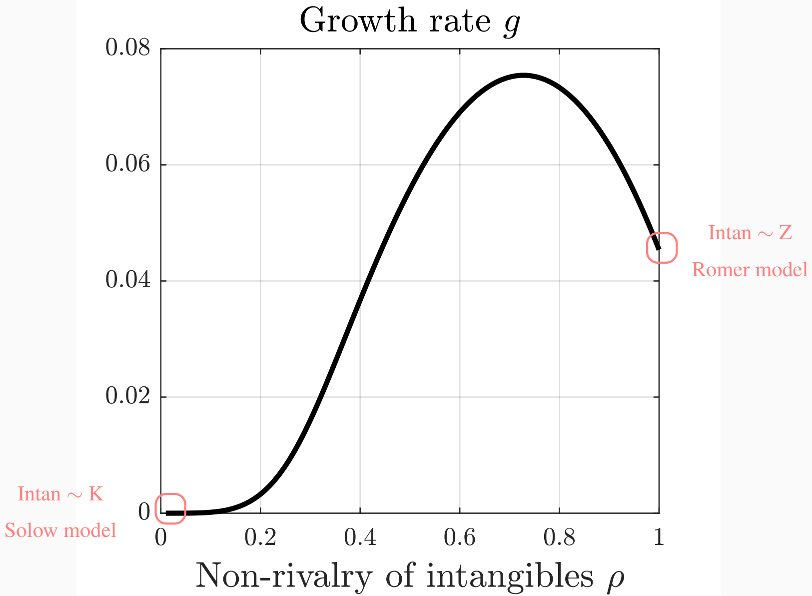
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$$\uparrow \rho$$

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Implications

$\uparrow$ profits, valuations, concentration

$\downarrow$ entry and investment

Roadmap

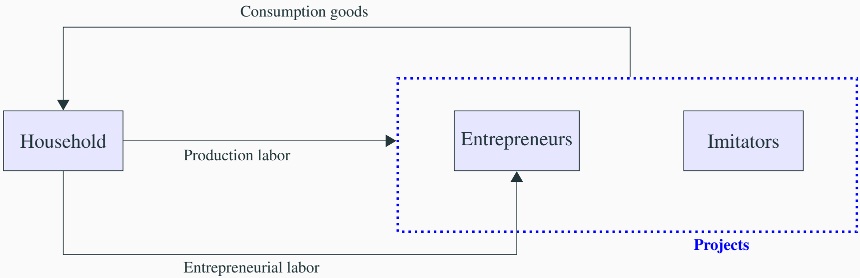
1. Economic Environment

2. The Effects of Non-rivalry on Growth

3. Model Implications

Economic environment

Overview


Consumption goods
Household
Production labor
Entrepreneurs
Imitators
Projects
Entrepreneurial labor

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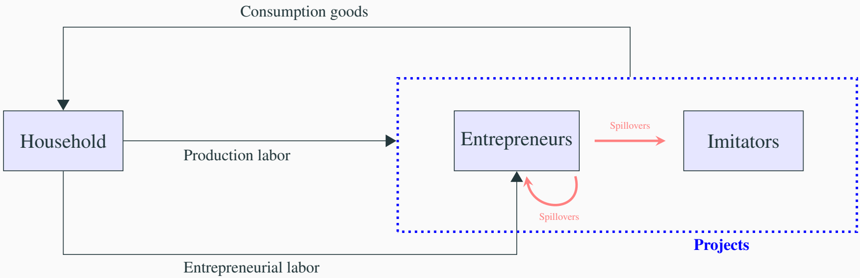

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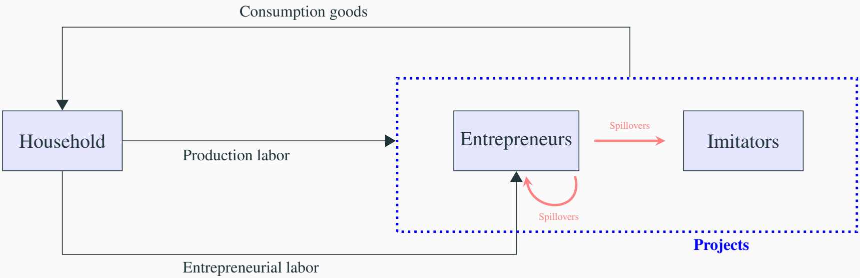

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Storing intangibles in a stream is an irreversible decision

Examples: Describing a production process to a local supervisor

Open a Starbucks in a new location

Stage 3: Hire workers

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Open a Starbucks in a new location

Stage 3: Hire workers

Labor is in fixed supply

No reallocation frictions

Allocating intangible capital within a project

$$\Pi(x_t, N_t) = \max_{\{N(s),L(s)\},L_t} \int_0^{x_t} N(s)^{1-\zeta} L(s)^{\zeta} ds - W_t L_t$$

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$$\Pi(x_t, N_t) \propto x_t^{\rho} N_t$$

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e.g. leasehold rights to airport gates

allocating a gate to a route makes it unavailable to other routes

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e.g. a patent for a steel alloy

using it in one mill does not reduce its availability to other mills

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$\Pi_t \propto x_t^{\rho} N_t$

if $\rho > 0$, N_t raises marginal returns to x_t

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Increasing scope $\Rightarrow$ owner captures a smaller share of a larger pie.

- Optimal scope a function of strength of IP protection

Imperfect excludability and spillovers

Imitators appropriate streams owned by E

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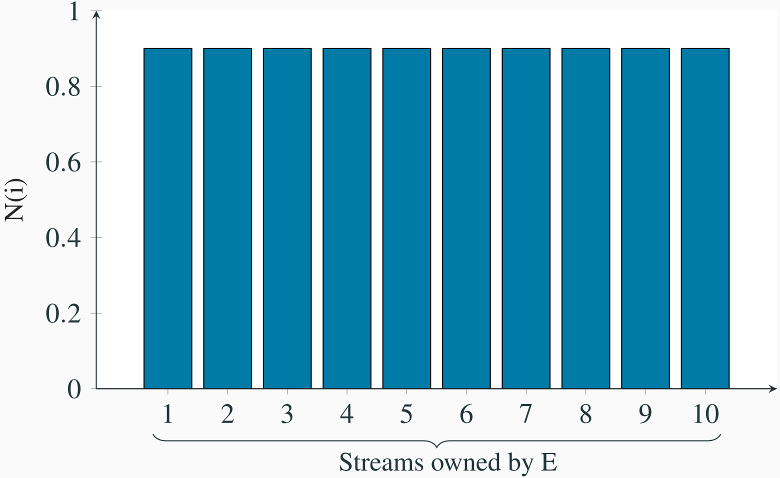
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N(i)
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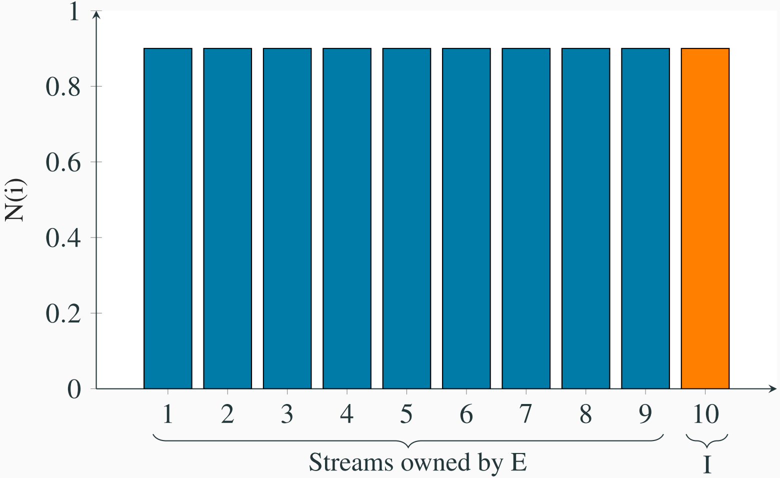
Streams owned by E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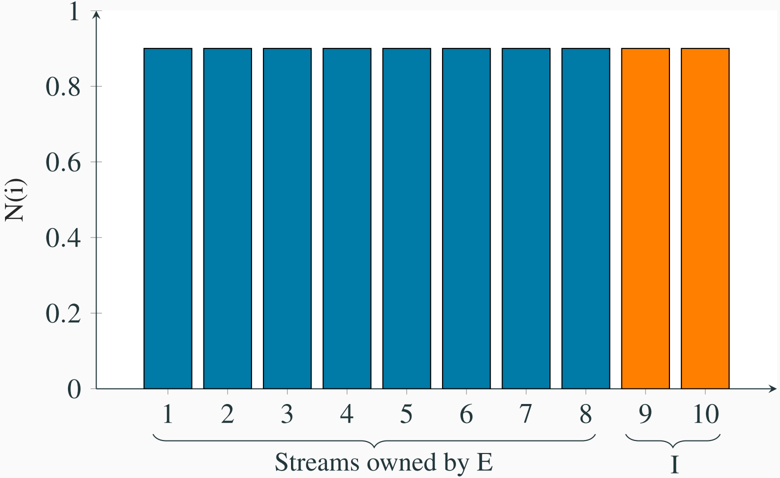

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Transforms N_t into S_t

N_t slowly leaks into S_t: $\quad N_0 = \left(N_t^{\frac{1}{1-\rho}} + S_t^{\frac{1}{1-\rho}} \right)^{1-\rho}$

Higher ρ accelerates spillovers

Rate of leakage (diffusion) of intangible rents to imitators function of ρ

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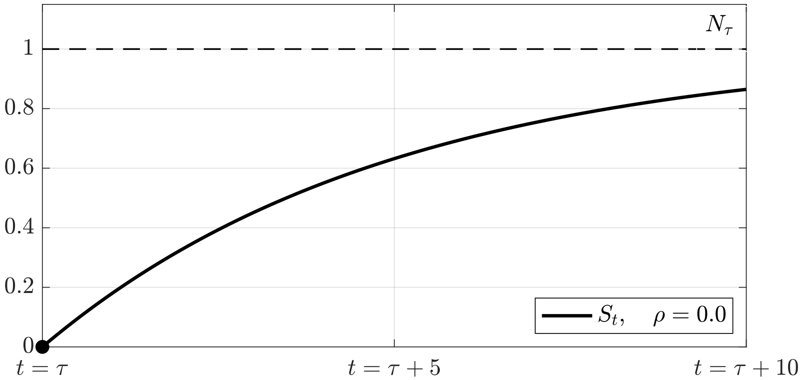

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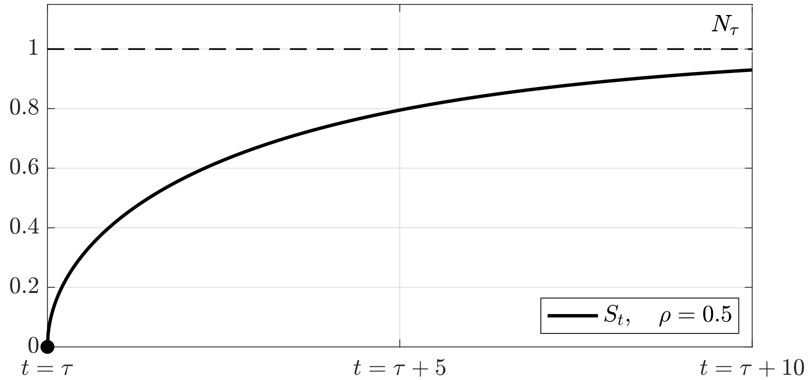

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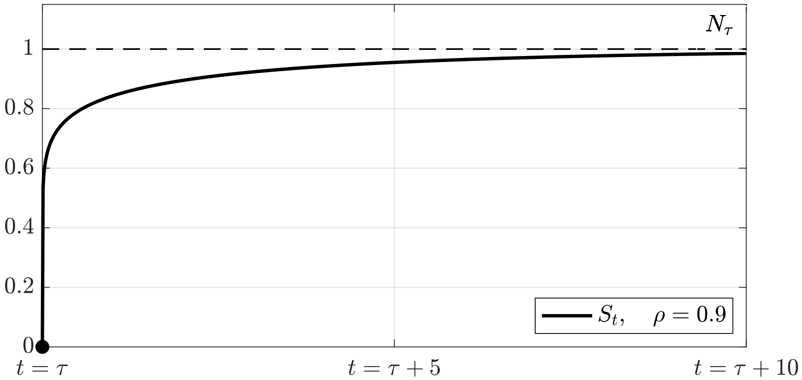

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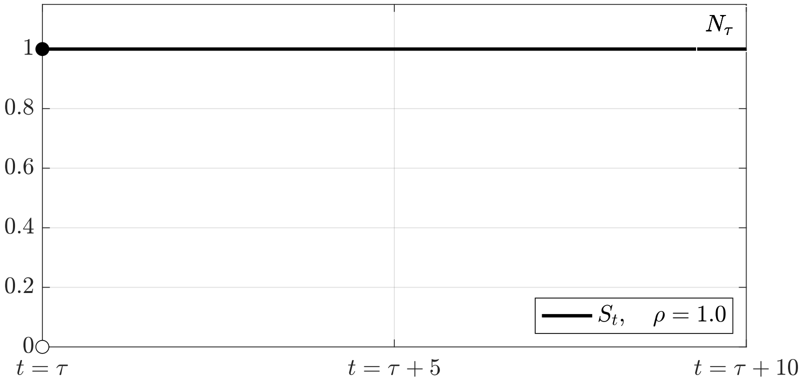

New projects

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Growth

Each time t a measure of new projects in being created.

- Measure of new projects a function of agents who choose to be entrepreneurs $L_{e,t}$

Aggregate output

$$Y_t = \frac{\Lambda_t}{1-\zeta}\int_0^t \underbrace{\nu S_\tau L_{e,\tau}}_{\substack{\text{new} \\ \text{intangibles}}} \underbrace{x_\tau^{\rho_\tau}}_{\text{scale}} d\tau$$

Labor markets and equilibrium

Free-entry $$V_t^e(x_t, N_t) = W_t$$

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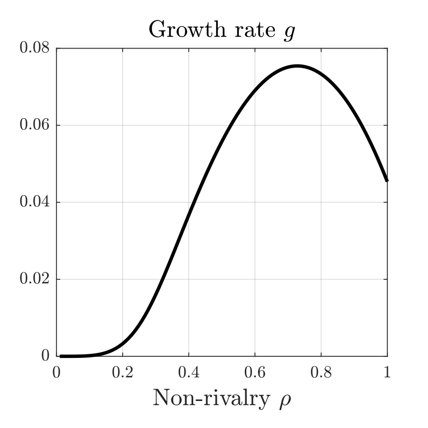
Result 1 (Balanced growth path)

For any $\rho \in [0,1]$, if ν is sufficiently high, there exists a unique equilibrium where $(x_t, L_{e,t})$ are constant and $(\bar{S}_t, N_t)$ grow at the same constant rate g.

The Effects of Non-Rivalry

The effects of non-rivalry



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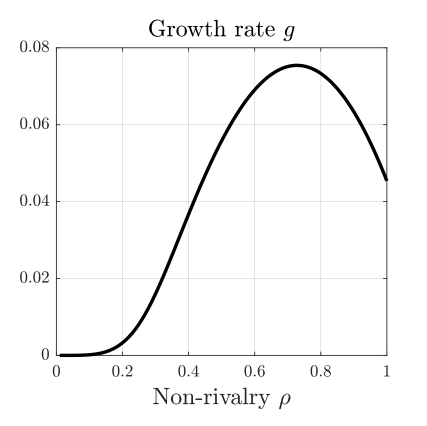

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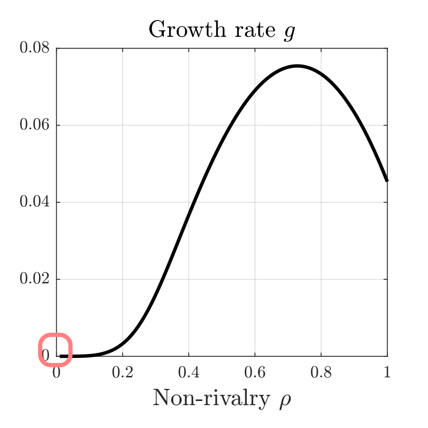

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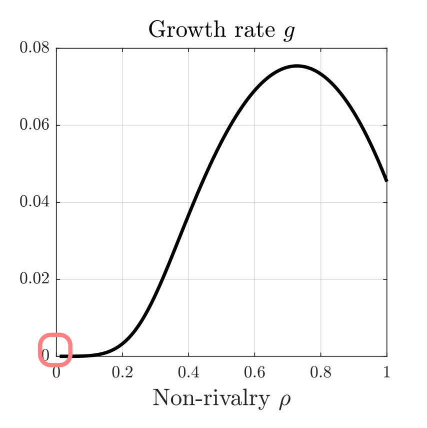

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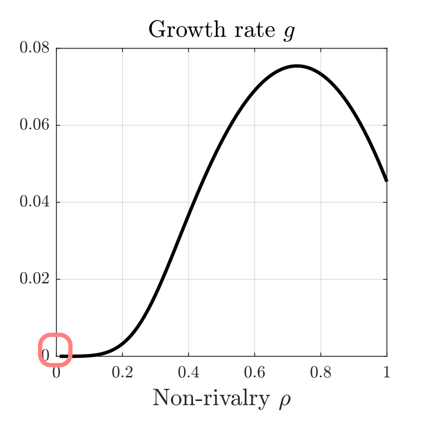

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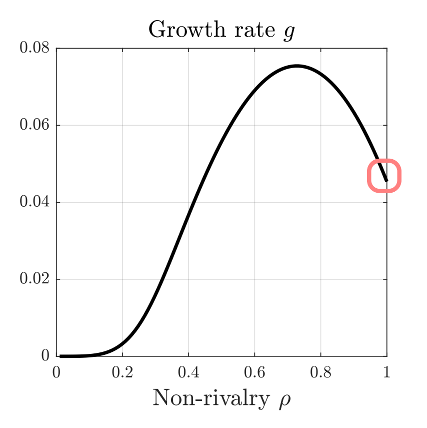

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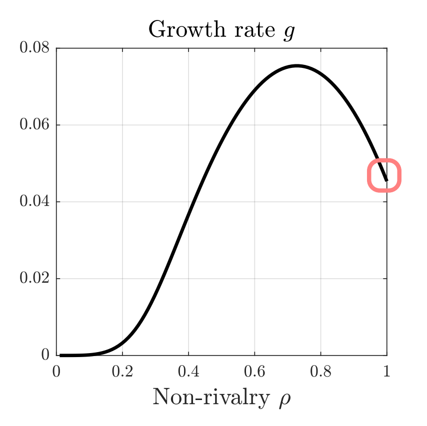

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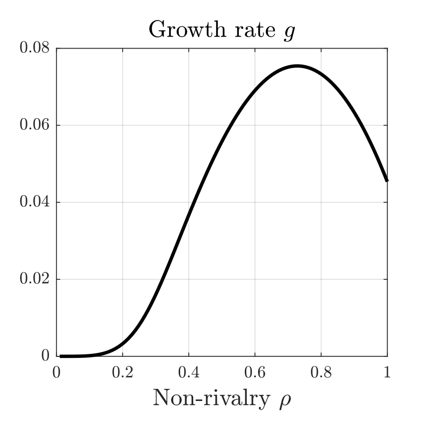

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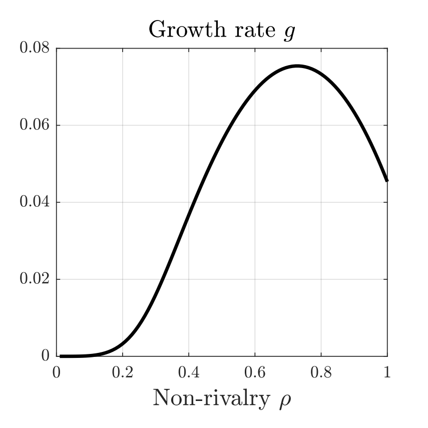

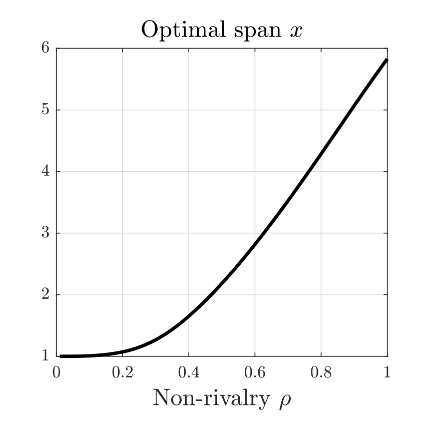

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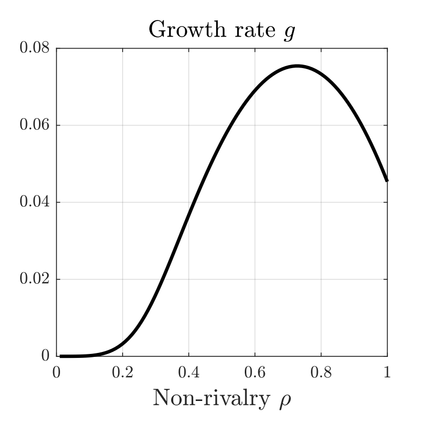

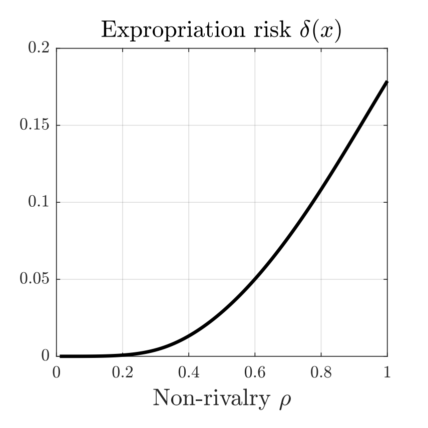

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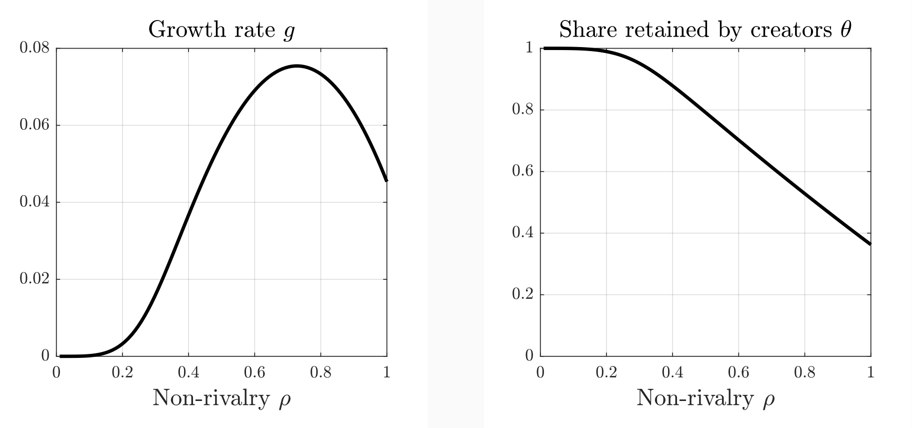

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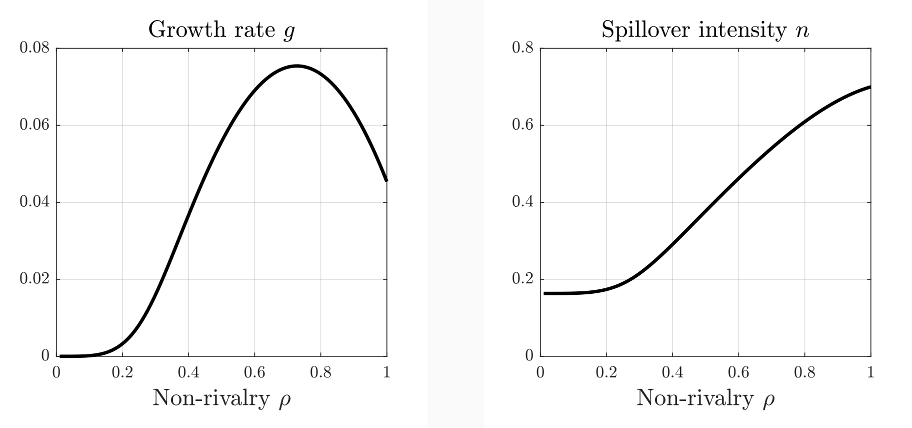

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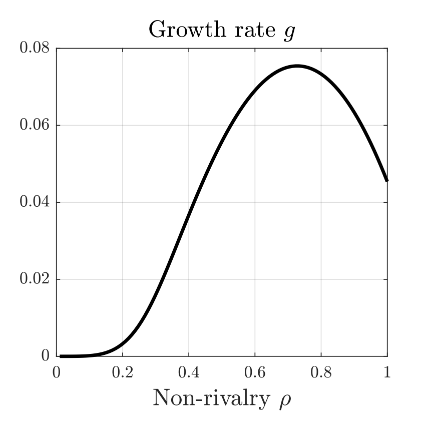

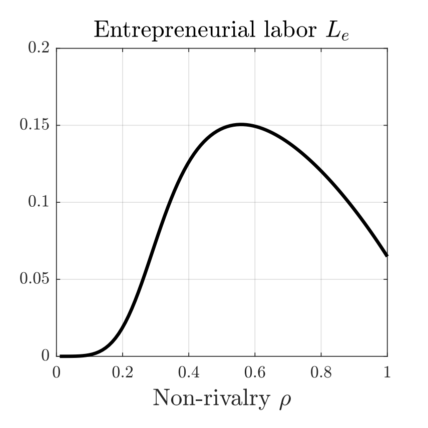

Model Implications

Returns to capital and Tobin's Q

$$V_t = \underbrace{V_t^e}_{\text{creators}} + \underbrace{(1-\theta)V_t}_{\text{imitators}}$$

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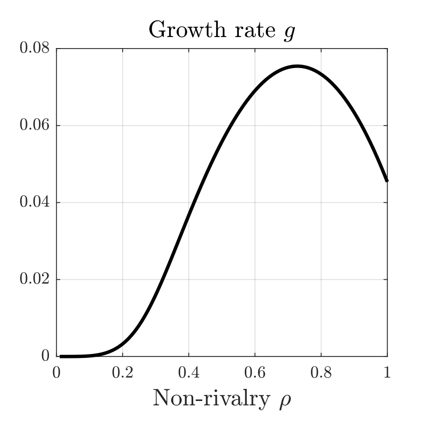
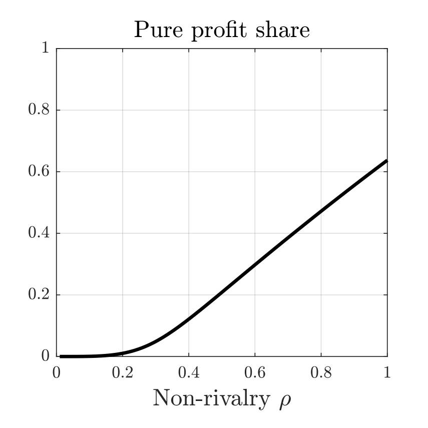
$$Q_t^e \equiv \frac{V_t^e}{p_{N,t}\overline{N}_{tot,t}} = 1$$

$$Q_t \equiv \frac{V_t}{p_{N,t}\overline{N}_{tot,t}} = \frac{1}{\theta} > 1$$

Returns to capital and valuations





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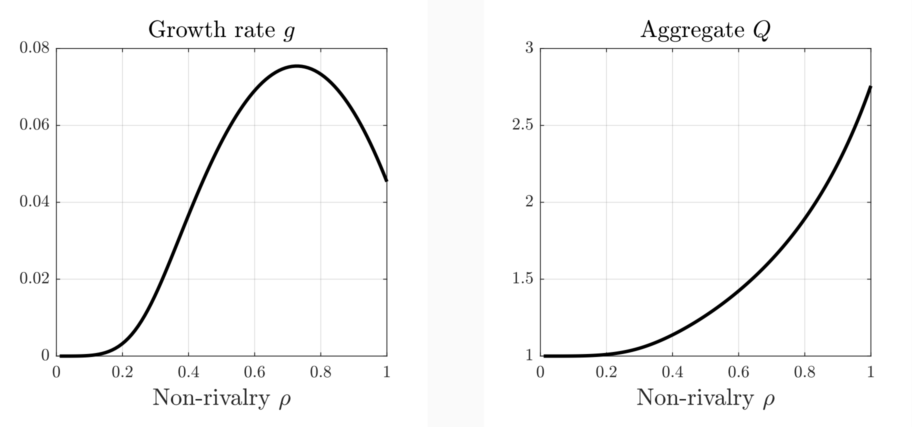

Concentration

Sales share for project i

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Stronger spillovers (n) makes the relative size of new projects larger

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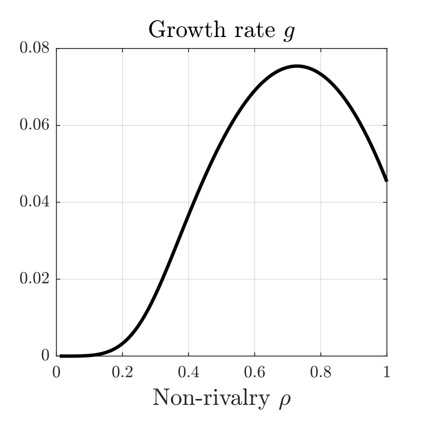
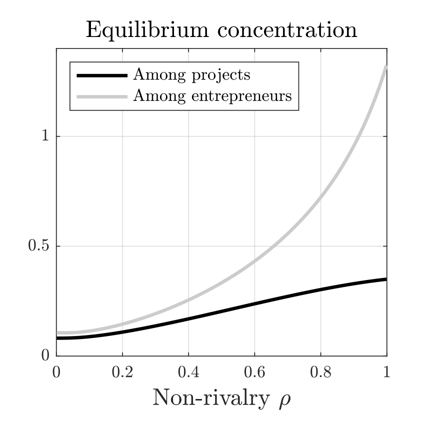
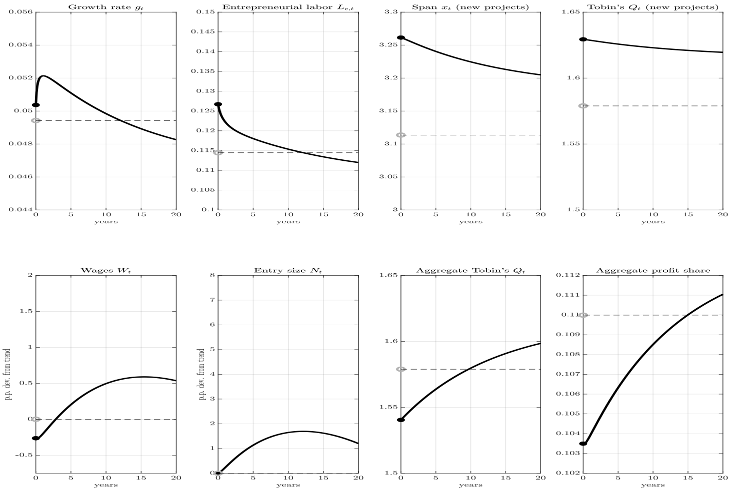




IRF: Increase in ρ, high ρ case



Conclusion

Q: Intangibles are imperfectly rival within firms. Does that matter for growth?

- Scale + spillovers to new firms vs. spillovers to imitators
- Non-monotonic relationship btw. ρ and growth

Next:

- Estimation of (ρ, δ)
- Implications of non-rivalry for capital structure and for firm boundaries