The Macroeconomics of Partial Irreversibility

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Motivation

- **Capital market characterized by a significant price wedge**
- **Purchase price > Resale price**
    - Evidence (Ramey and Shapiro, 02, Kermani and Ma, 22)
    - Significant and heterogenous across sectors
    - Asymmetric info, asset specificity, obsolescence, fees, taxes...
- **The price wedge renders investment partially irreversible**
- **How does partially irreversible investment affect...**

aggregate productivity? firms' market value? business cycles?

A parsimonious investment model

Technology, shocks, and frictions

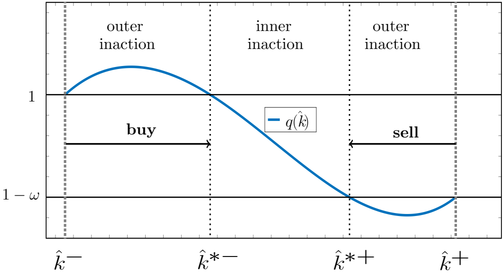
- **Production:** $y_s = u_s^{1-\alpha}\, k_s^{\alpha}, \quad \alpha < 1$
  - u, idiosyncratic productivity: $\mathrm{d}\log(u_s) = \mu\, \mathrm{d}s + \sigma\, \mathrm{d}W_s$
  - k, uncontrolled capital: $\mathrm{d}\log(k_s) = -\xi^k\, \mathrm{d}s$
- **Fixed adj. cost:** $\theta_s = \theta u_s$
  - Paid for each non-zero investment $\Delta k_s \neq 0$
- **Price wedge:** $p(\Delta k_s) = p\mathbb{1}_{\{\Delta k_s > 0\}} + p(1-\omega)\mathbb{1}_{\{\Delta k_s < 0\}}, \ \omega > 0$
  - Asymmetric linear cost $p(\Delta k_s)\Delta k_s$
- **Firm problem**

$$V(k_0, u_0) = \max_{\{T_h, \Delta k_{T_h}\}_{h=1}^{\infty}} \mathbb{E}_0 \left[\int_0^{\infty} e^{-\rho s} y_s\, \mathrm{d}s - \sum_{h=1}^{\infty} e^{-\rho T_h} \left(\underbrace{\theta_{T_h}}_{\text{fixed}} + \underbrace{p(\Delta k_{T_h})\, \Delta k_{T_h}}_{\text{wedge}} \right) \right]$$

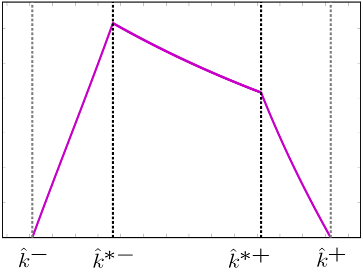
Policy and cross-sectional distribution

- **State:** capital-productivity ratio $\hat{k} \equiv \log(k/u)$
- **Policy:** $\mathcal{K} \equiv \{\hat{k}^- < \hat{k}^{*-} < \hat{k}^{*+} < \hat{k}^+\}$ and $q(\hat{k}) \equiv \frac{1}{p}\frac{\partial V(k,u)}{\partial k}$

Investment policy



Cross-section $g(\hat{k})$



- Marginal $q(\hat{k})$ is not a **sufficient statistic** for firm-level investment
- **Key: Two reset points** $\Longrightarrow$ Markov structure for adjustment sign

Three long-run macro outcomes

(1) Capital Allocation

Allocation $\equiv$ Dispersion of log marginal product

$$\mathbb{V}[\log mpk] \;=\; (1-\alpha)^2 \mathbb{V}[\hat{k}]$$

- Both investment frictions as a source of (efficient) dispersion
  - **Frictionless**: $\mathbb{V}[\hat{k}] \;=\; 0$
  - **With frictions**: $\mathbb{V}[\hat{k}] \;>\; 0$

(2) Capital Valuation

Aggregate q $\equiv$ weighted average of individual $q(\hat{k})$

$$q = \int q(\hat{k})\phi(\hat{k})g(\hat{k})d\hat{k}, \qquad \phi(\hat{k}) = \frac{e^{\hat{k}}}{\mathbb{E}[e^{\hat{k}}]}$$

- Investment frictions affect marginal valuations
  - **Frictionless**: $q = 1$
  - **With frictions**: $q \neq 1$
- Define the capital-loss function $\mathcal{P}(\hat{k})$ over $[\hat{k}^-, \hat{k}^+]$

$$\mathcal{P}(\hat{k}) \equiv \begin{cases} 0 & \text{for all} \quad \hat{k} \leq \hat{k}^{*-} \\ -\omega & \text{for all} \quad \hat{k} \geq \hat{k}^{*+} \end{cases}, \quad \mathcal{P}(\hat{k}) \in \mathbb{C}^2$$

(2) Capital Valuation (cont...)

Costs and benefits of capital accumulation

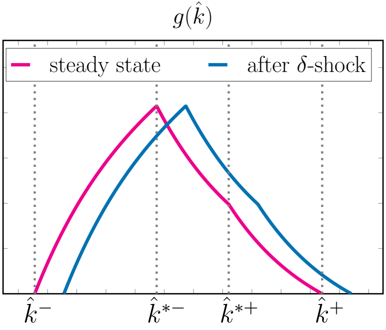
$$q = \frac{1}{r}\left(\underbrace{\frac{\alpha\hat{Y}}{p\hat{K}} + \frac{\sigma^2}{2} - \nu}_{\text{Productivity}} - \underbrace{\mathbb{E}\left[\frac{1}{\mathrm{d}s}\mathbb{E}_s\left[\mathrm{d}(\mathcal{P}(\hat{k}_s)\phi(\hat{k}_s))\right]\right]}_{\text{Irreversibility}(>0)} \right)$$

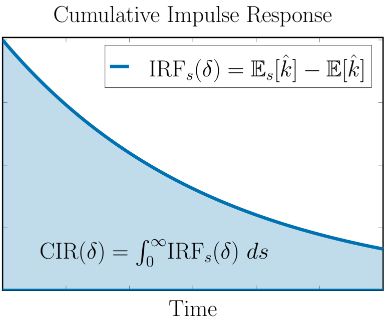
$$\text{where} \quad \frac{\hat{Y}}{\hat{K}} \approx \exp\left\{-(1-\alpha)\left(\mathbb{E}[\hat{k}] + \frac{\alpha}{2}\mathbb{V}[\hat{k}]\right)\right\}$$

- q **is monotonic!** Sufficient statistic for aggregate investment
- **Productivity**
  - Volatility ($\sigma^2 \Uparrow$, $q \Uparrow$) + Dispersion ($\mathbb{V}[\hat{k}] \Uparrow$, $q \Downarrow$)
- **Irreversibility**
  - Capital losses “amortized” across inaction periods ($q \Downarrow$)

(3) Capital Fluctuations

- MIT aggregate shock $\delta > 0$ reduces all firms' productivity
- Transitional dynamics of aggregate capital (fixed r, SOE)





- Investment frictions as a source of persistence
  - **Frictionless:** CIR $= 0$
  - **With frictions:** CIR > 0

(3) Capital Fluctuations (cont...)

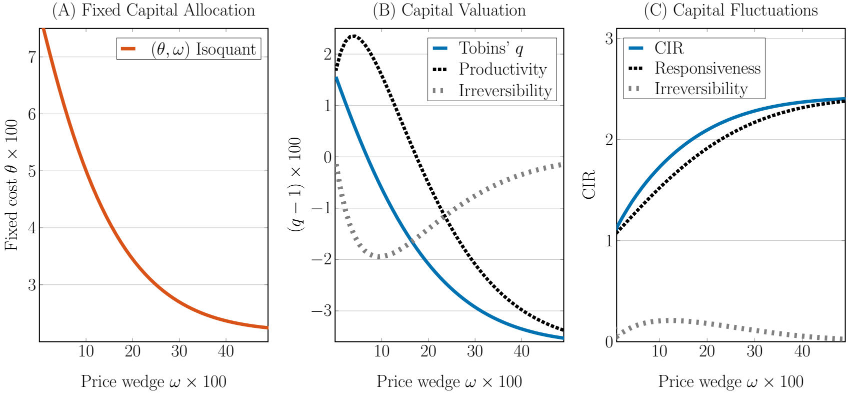
Persistence of capital fluctuations

$$\frac{\mathrm{CIR}(\delta)}{\delta} = \underbrace{\frac{\mathbb{V}[\hat{k}]}{\sigma^2} + \frac{\nu \mathbb{C}ov[\hat{k}, a]}{\sigma^2}}_{\text{Responsiveness}} - \underbrace{\mathbb{E}\left[\frac{1}{\mathrm{d}s}\mathbb{E}_s[\mathrm{d}(\mathcal{M}(\hat{k}_s)\hat{k}_s)]\right]}_{\text{Irreversibility}<0} + o(\delta)$$

- **Responsiveness**
  - Dispersion ($\mathbb{V}[\hat{k}] \Uparrow$, CIR $\Uparrow$) + Asymmetry (If $\mathbb{C}ov[\hat{k}, a] > 0$, CIR $\Uparrow$)
- **Irreversibility**
  - Deviations from steady-state mean: $\mathcal{M}(\hat{k})$ (similar to $\mathcal{P}(\hat{k})$)
  - One large adjustment is "amortized" across periods (CIR $\Uparrow$)

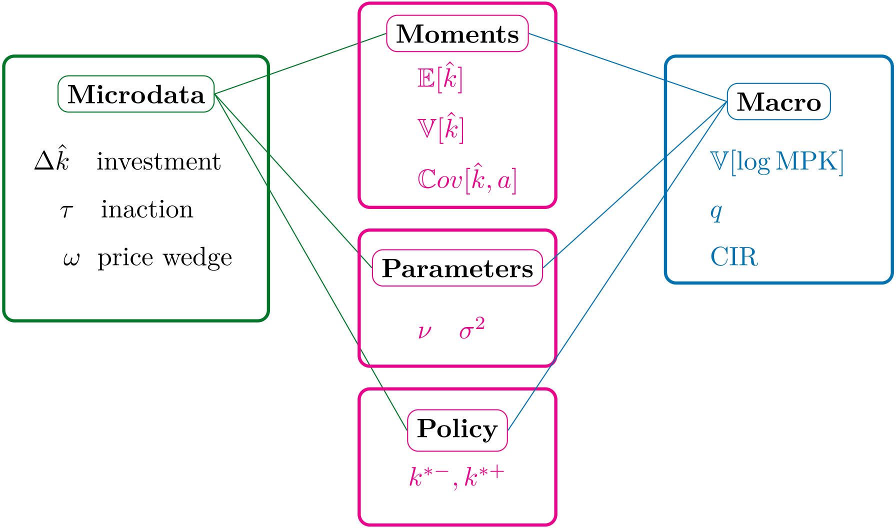
Nature of frictions matters for macro

- Consider isoquant (θ, ω) that delivers constant $\mathbb{V}[\hat{k}]$
- Opposite macro implications:
  - Price wedge ω dominates: $q \Downarrow$ and CIR $\Uparrow$
  - Fixed cost θ dominates: $q \Uparrow$ and CIR $\Downarrow$



Mappings from investment microdata

Strategy: From micro to macro



Output: Macro outcomes

- External/estimated parameters for Chile 1980-2011

ρ	μ	α	ω	ξ^k	σ^2
0.066	0.033	0.5	0.15	0.08	0.055

- Irreversibility **lowers valuation** and **slows down fluctuations**

Investment Policy		**Capital Allocation**	
Reset capitals $(\hat{k}^{*+} - \hat{k}^{*-})$	0.57	$\mathbb{V}[\log mpk]$	0.024
Exogenous price wedge	0.33		
Endogenous response	0.24		

Capital Valuation		**Capital Fluctuations**	
Aggregate q	1.06	CIR	3.07
Productivity	1.09	Responsiveness	2.29
Irreversibility	-0.03	Irreversibility	0.78

Application: Corporate income tax

Corporate income tax t^c and deductions ξ^d

- Pay t^c on cashflow net of deductions and capital losses

$$\pi_s = \underbrace{(1-t^c)\, u_s^{1-\alpha} k_s^{\alpha}}_{\text{after-tax revenue}} + \underbrace{t^c \xi^d d_s}_{\text{deductions}} - \underbrace{t^c \omega p \Delta k_s \mathbb{1}_{\{\Delta k_s < 0\}}}_{\text{capital losses}}$$

where deductions d_s evolve as:

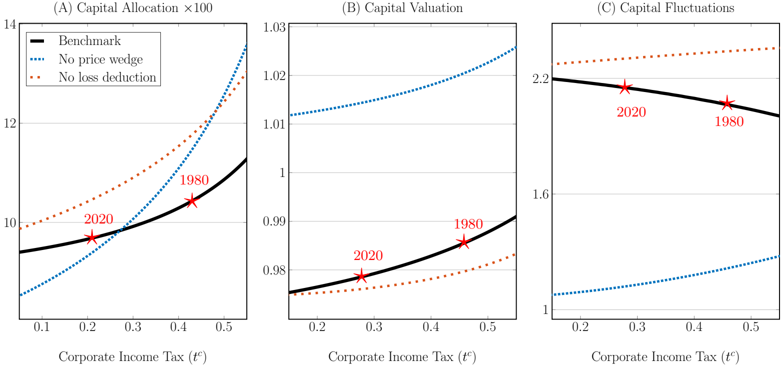
$$\log d_s = \log d_0 - \xi^d s + \sum_{h: T_h \leq s} \left(1 + \frac{\theta_{T_h} + p\Delta k_{T_h}}{d_{T_h^-}}\right)$$

  - Fixed costs are capitalized

- Let $z \equiv \frac{\xi^d}{r+\xi^d} < 1$, then **after-tax effective frictions**
  - ⋆ Fixed cost: $\tilde{\theta} \propto \left(\frac{1-t^c z}{1-t^c}\right)^{\frac{1}{1-\alpha}} \theta$, increasing in t^c
  - ⋆ Price wedge: $\tilde{\omega} \propto (1-t^c)\, \omega$, decreasing in t^c

Corporate income tax cut

- From 42% in 1980 to 25% in 2020 (average OECD)
- Increases importance of price wedge
  1. Decreases $\mathbb{V}[\log mpk]$ (lower frictions, improves allocation)
  2. Decreases q (higher capital loses)
  3. Increases CIR (slower propagation)



Backup material

Backup

A1. Contributions

Contributions

- **Investment irreversibility**
  - Sargent (1980), Abel and Eberly (94, 96), Bertola and Caballero (94), Dixit and Pindyck (94), Veracierto (02), Kahn and Thomas (13), Lanteri (18), Lanteri, Medina and Tan (2020), Fang (2022)

> We characterize effect on capital allocation, valuation, and fluctuations

- **Role of micro-level frictions for macro dynamics**
  - Caballero and Engel (99, 07), Alvarez and Lippi (14), Alvarez, Le Bihan and Lippi (16), Baley and Blanco (21)

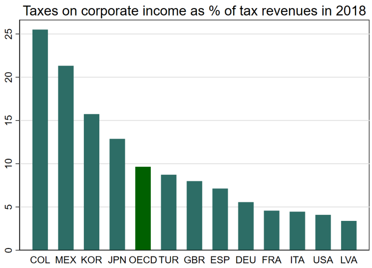
> We derive sufficient statistics for the effects of irreversibility

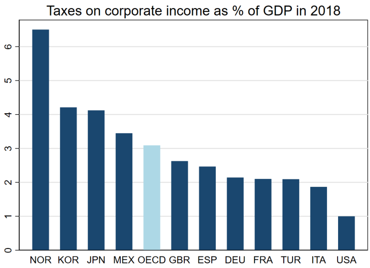
- **Interaction of investment frictions and corporate taxes**
  - Altug, Demers and Demers (09), Gourio and Miao (10), Miao and Wang (14), Miao (19), Chen, *et. al.* (19), Winberry (21)

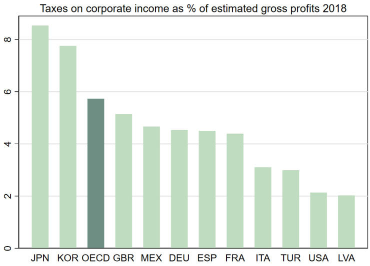
> We reduce complex interactions to rescaling of frictions

A2. Importance of Corporate Taxes

⋆ Importance of Corporate Taxes

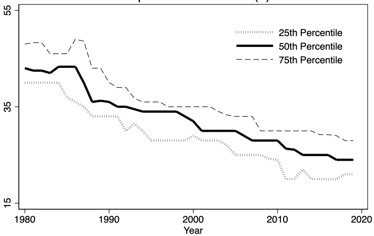






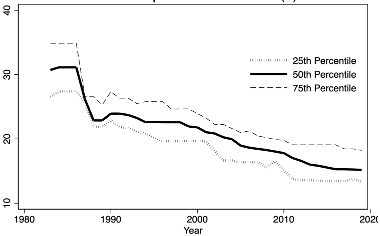
⋆ General decreasing trends

Corporate Income Tax (t^c)



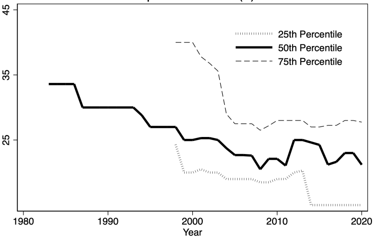
Source: Vegh and Vuletin (2015)

PDV of Depreciation Allowance (t^d)



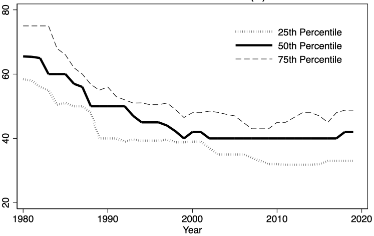
Source: Tax Foundation

Capital Gains Tax (t^g)



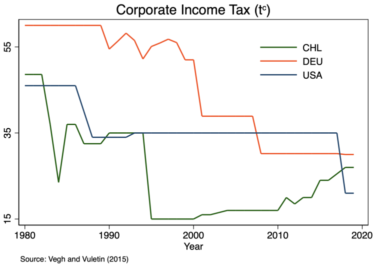
Sources: Bertels and Jenderny (2015), Dell (2007), E&Y (2021), Ireland Revenue, Spengel et al.(2019), Tax Foundation, Tax Policy Center

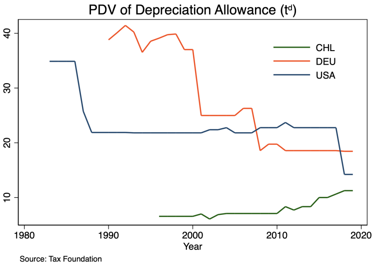
Personal Income Tax (t^p)

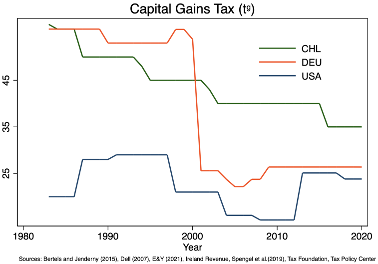


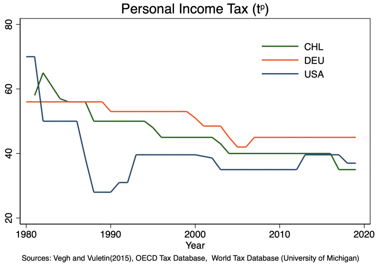
Sources: Vegh and Vuletin (2015), OECD Tax Database, World Tax Database (University of Michigan)

⋆ Very persistent reforms at country-level









A3. General Hazard Model

Asymmetric Generalized Hazard Model

- **Adjustment technology**:

$$\theta_s = \Theta(i_s, \mathrm{d}N_s^-, \mathrm{d}N_s^+, \vartheta_s^-, \vartheta_s^+) u_s$$

$$\Theta(i, \mathrm{d}N^+, \mathrm{d}N^-, \vartheta^-, \vartheta^+) = \begin{cases} 0 & \text{if} \quad i = 0 \\ \bar{\theta}^+(1 - \mathrm{d}N) \ + \ \mathrm{d}N\vartheta^+ & \text{if} \quad i < 0 \\ \bar{\theta}^-(1 - \mathrm{d}N) \ + \ \mathrm{d}N\vartheta^- & \text{if} \quad i > 0. \end{cases}$$

  - $N_t^{\pm} \sim Poisson(\lambda^{\pm})$, $\vartheta^{\pm} \sim_{i.i.d.} J^{\pm}(\varphi)$, $\mathrm{Supp}(\vartheta^{\pm}) = [0, \bar{\theta}^{\pm}]$
  - Models of adjustment:
    - **Standard Ss model**: $\lambda = 0$ and $\bar{\theta}^+ = \bar{\theta}^-$
      Sheshinski and Weiss (77)
    - **Bernoulli fixed costs:** Free adj. opportunity $\vartheta^+ = \vartheta^- = 0$
      Baley and Blanco (21)
    - **Generalized hazard**: $\vartheta^+ = \vartheta^-$
      Caballero and Engel (93)

←

Asymmetric Generalized Hazard Model

- **Firm Problem**:

$$V(k_0, u_0) \;=\; \max_{\{T_h, i_{T_h}\}_{h=1}^{\infty}} \mathbb{E}\left[\int_0^{\infty} e^{-\rho s}\pi_s \,\mathrm{d}s - \sum_{h=1}^{\infty} e^{-\rho T_h}\left(\theta_{T_h} + p\left(i_{T_h}\right) i_{T_h}\right)\right]$$

- **Hazard rate of adjustment** $\Lambda(\hat{k})$: Adjustment prob. $\Lambda(\hat{k})\,\mathrm{d}t$

1. $\Lambda(\hat{k}) = 0$ for all $\hat{k} \in (\hat{k}^{*-}, \hat{k}^{*+})$
2. $\Lambda(\hat{k})$ weakly increasing in $|\hat{k} - \frac{\hat{k}^{*-}+\hat{k}^{*+}}{2}|$
3. If $J^{-}(0) > 0$, then $\Lambda(\hat{k}) \geq \lambda^{-} J^{-}(0)$ in $(\hat{k}^{-}, \hat{k}^{*-})$
4. If $J^{+}(0) > 0$, then $\Lambda(\hat{k}) \geq \lambda^{+} J^{+}(0)$ in $(\hat{k}^{*+}, \hat{k}^{+})$

←

A4. Firm policy and HJB

Sufficient optimality conditions

←

- Let $r \equiv \rho - \mu - \sigma^2/2$ and $\nu \equiv \mu + \xi^d$
- $v(\hat{k})$ and the optimal policy $\{\hat{k}^-, \hat{k}^{*-}, \hat{k}^{*+}, \hat{k}^+\}$ satisfy:

1. **HJB**:

$$rv(\hat{k}) \;=\; Ae^{\alpha\hat{k}} \;-\; \nu v'(\hat{k}) \;+\; \frac{\sigma^2}{2}v''(\hat{k})$$

2. **Value-matching:**

$$\begin{aligned} v(\hat{k}^-) \;&=\; v(\hat{k}^{*-}) \;-\; \theta \;+\; p^{buy}(e^{\hat{k}^-} - e^{\hat{k}^{*-}}) \\ v(\hat{k}^+) \;&=\; v(\hat{k}^{*+}) \;-\; \theta \;+\; p^{sell}(e^{\hat{k}^+} - e^{\hat{k}^{*+}}) \end{aligned}$$

3. **Optimality and smooth-pasting:**

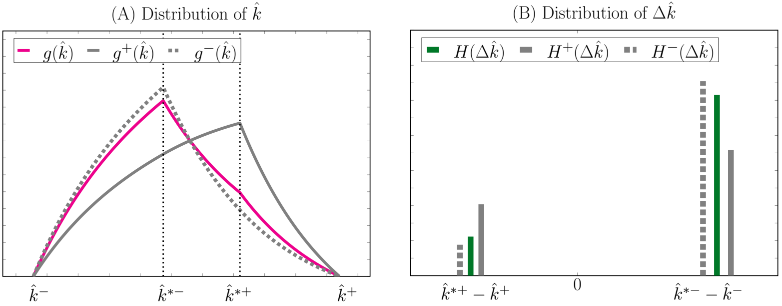
$$\begin{aligned} v'(\hat{k}) \;&=\; p^{buy}e^{\hat{k}}, \quad \hat{k} \in \left\{\hat{k}^-, \hat{k}^{*-}\right\} \\ v'(\hat{k}) \;&=\; p^{sell}e^{\hat{k}}, \quad \hat{k} \in \left\{\hat{k}^{*+}, \hat{k}^+\right\} \end{aligned}$$

A5. Distributions and KFE

Cross-sectional distributions

←

- **Distribution of capital-productivity ratio** $g(\hat{k})$
  - Conditional on last reset point: $g^{\pm}(\hat{k})(\hat{k})$
  - Expectations in cross-section: $\mathbb{E}, \mathbb{E}^{\pm}$
- **Distribution of investment** $H(\Delta\hat{k}, \tau)$
  - Conditional on last reset point: $H^{\pm}(\Delta\hat{k})$
  - Expectations of adjusters: $\overline{\mathbb{E}}, \overline{\mathbb{E}}^{\pm}$



Characterizing cross-sectional distribution

←

- Characterizing $g(\hat{k}) \in \mathbb{C}$
  - KFE: $$0 = \nu \frac{\mathrm{d}g(\hat{k})}{\mathrm{d}\hat{k}} + \frac{\sigma^2}{2}\frac{\mathrm{d}^2 g(\hat{k})}{\mathrm{d}\hat{k}^2}, \qquad \forall \hat{k} \in \left(\hat{k}^-, \hat{k}^+\right) / \{\hat{k}^{*-}, \hat{k}^{*+}\}$$
  - Border conditions: $$0 = g(\hat{k}^-) = g(\hat{k}^+) \quad ; \quad \int_{\hat{k}^-}^{\hat{k}^+} g(\hat{k})\,\mathrm{d}\hat{k} = 1$$
  - Irreversibility
$$\underbrace{\frac{\sigma^2}{2} \lim_{\hat{k}\downarrow\hat{k}^-} g'(\hat{k})}_{\text{freq. with } \Delta\hat{k}>0} = \underbrace{\frac{\sigma^2}{2}\left[\lim_{\hat{k}\uparrow\hat{k}^{*-}} g'(\hat{k}) - \lim_{\hat{k}\downarrow\hat{k}^{*-}} g'(\hat{k})\right]}_{\text{discontinuity due to entry}}$$

- Characterizing $g^{\pm}(\hat{k}) \in \mathbb{C}$
  - KFE: $$0 = \nu \frac{\mathrm{d}g^{\pm}(\hat{k})}{\mathrm{d}\hat{k}} + \frac{\sigma^2}{2}\frac{\mathrm{d}^2 g^{\pm}(\hat{k})}{\mathrm{d}\hat{k}^2}, \qquad \forall \hat{k} \in \left(\hat{k}^-, \hat{k}^+\right) / \{\hat{k}^{*^{\pm}}\}$$
  - Border conditions: $$0 = g(\hat{k}^-) = g(\hat{k}^+) \quad ; \quad \int_{\hat{k}^-}^{\hat{k}^+} g(\hat{k})\,\mathrm{d}\hat{k} = 1$$

Steady-state distribution of firms

←

- $g(\hat{k})$: firms' distribution
- $H^{\pm}(\Delta\hat{k},\tau)$: firms' distribution conditional on last reset $\hat{k}^{\pm}$
- $g^{\pm}(\hat{k})$: firms' distribution conditional on last reset $\hat{k}^{\pm}$
- $\mathcal{N}^{+}$ & $\mathcal{N}^{-}$: frequency of of $\Delta\hat{k} < 0$ and $\Delta\hat{k} > 0$
- Bayes' law

$$
\begin{aligned}
H(\Delta\hat{k},\tau) &= \frac{\mathcal{N}^{-}}{\mathcal{N}} H^{-}(\Delta\hat{k},\tau) + \frac{\mathcal{N}^{+}}{\mathcal{N}} H^{+}(\Delta\hat{k},\tau) \\
g(\hat{k}) &= \frac{\mathcal{N}^{-}}{\mathcal{N}} \frac{\overline{\mathbb{E}}^{-}[\tau]}{\overline{\mathbb{E}}[\tau]}\, g^{-}(\hat{k}) + \frac{\mathcal{N}^{+}}{\mathcal{N}} \frac{\overline{\mathbb{E}}^{+}[\tau]}{\overline{\mathbb{E}}[\tau]}\, g^{+}(\hat{k})
\end{aligned}
$$

A6. Measuring Misallocation

Measuring cross-sectional moments with microdata

- **Challenge:** $g(\hat{k})$ is not observed
- Let $\hat{k}^*(\Delta\hat{k})$ and $\hat{k}_\tau(\Delta\hat{k})$ be given by

$$\begin{aligned}
\hat{k}^*(\Delta\hat{k}) &= \begin{cases} \hat{k}^{*-} & \text{if} \quad \Delta\hat{k} > 0 \\ \hat{k}^{*+} & \text{if} \quad \Delta\hat{k} < 0, \end{cases} \\
\hat{k}_\tau(\Delta\hat{k}) &= \hat{k}^*(\Delta\hat{k}) - \Delta\hat{k}.
\end{aligned}$$

- Two steps:
  1. Obtain ν, σ, $\hat{k}^{*-}$, $\hat{k}^{*+}$
  2. Obtain $\mathbb{E}[\hat{k}]$ and $\mathbb{V}[\hat{k}]$

←

Step 1

Let $\Phi(\nu, \sigma^2) \equiv \log\left(\alpha A/(r + \alpha\nu - \alpha^2\sigma^2/2)\right)$. Then

$$\nu = \frac{\overline{\mathbb{E}}[\Delta\hat{k}]}{\overline{\mathbb{E}}[\tau]}, \quad \sigma^2 = \frac{\overline{\mathbb{E}}[(\hat{k}_\tau + \nu\tau)^2] - \overline{\mathbb{E}}[(\hat{k}^*)^2]}{\overline{\mathbb{E}}[\tau]}$$

$$\hat{k}^{*-} = \frac{1}{1-\alpha}\left[\Phi(\nu, \sigma^2) - \log(p^{buy}) + \log\left(\frac{1 - \overline{\mathbb{E}}^-\left[e^{-\hat{r}\tau + \alpha\left(\hat{k}_\tau - \hat{k}^{*+}\right)}\right]}{1 - \overline{\mathbb{E}}^-\left[\frac{p(\Delta\hat{k})}{p^{\text{buy}}} e^{-\hat{r}\tau + \hat{k}_\tau - \hat{k}^{*+}}\right]}\right)\right]$$

$$\hat{k}^{*+} = \frac{1}{1-\alpha}\left[\Phi(\nu, \sigma^2) - \log(p^{sell}) + \log\left(\frac{1 - \overline{\mathbb{E}}^+\left[e^{-\hat{r}\tau + \alpha\left(\hat{k}_\tau - \hat{k}^{*-}\right)}\right]}{1 - \overline{\mathbb{E}}^+\left[\frac{p(\Delta\hat{k})}{p^{\text{sell}}} e^{-\hat{r}\tau + \hat{k}_\tau - \hat{k}^{*-}}\right]}\right)\right]$$

- Drift =adjustment size $\times$ frequency of adjustment
- Volatility =quadratic size without trend $\times$ frequency of adjustment
- $\Phi(\cdot)$ = profitability to user cost $/p^{sell}, p^{buy}$ = cost of investment
- Last term= PDV marginal profits over expected resale value

←

Step 2

$$\mathbb{E}[\hat{k}] = \overline{\mathbb{E}}\left[\overline{\mathbb{E}}\left[\underbrace{\left(\frac{\hat{k}^* + \hat{k}_\tau}{2}\right)}_{\text{midpoint start-finish}} \overbrace{\left(\frac{\hat{k}^* - \hat{k}_\tau}{\overline{\mathbb{E}}[\Delta\hat{k}]}\right)}^{\text{renewal weight}} \middle| \Delta\hat{k}\right]\right] + \underbrace{\frac{\sigma^2}{2\nu}}_{\text{accum. drift correction}},$$

$$\mathbb{V}[\hat{k}] = \overline{\mathbb{E}}\left[\overline{\mathbb{E}}\left[\underbrace{\left((\hat{k}^* - \mathbb{E}[\hat{k}])(\hat{k}_\tau - \mathbb{E}[\hat{k}]) + \frac{(\hat{k}^* - \hat{k}_\tau)^2}{3}\right)}_{\text{distance start-finish}} \underbrace{\left(\frac{\hat{k}^* - \hat{k}_\tau}{\overline{\mathbb{E}}[\Delta\hat{k}]}\right)}_{\text{renewal weight}} \middle| \Delta\hat{k}\right]\right]$$

←

A7. CIR and cumulative deviations

CIR

- Let $\mathcal{M}(\hat{k})$ be equal to

$$\mathcal{M}(\hat{k}) = \begin{cases} \mathcal{M}^{buy} & \text{if } \hat{k} \in [\hat{k}^-, \hat{k}^{*-}] \\ \mathcal{M}^{sell} & \text{if } \hat{k} \in [\hat{k}^{*+}, \hat{k}^+], \end{cases}$$

$$\mathcal{M}^{buy} \equiv (\mathbb{E}^-[\hat{k}] - \mathbb{E}[\hat{k}])\overline{\mathbb{E}}^-[\tau] \, \frac{\mathbb{E}[\mathbb{P}^+]}{\mathbb{P}^{-+}} < 0,$$

$$\mathcal{M}^{sell} \equiv (\mathbb{E}^+[\hat{k}] - \mathbb{E}[\hat{k}])\overline{\mathbb{E}}^+[\tau] \, \frac{\mathbb{E}[\mathbb{P}^-]}{\mathbb{P}^{+-}} > 0.$$

  - $\mathbb{E}[\mathbb{P}^+] \equiv \Pr[\Delta\hat{k}' < 0]$ and $\mathbb{P}^{-+} \equiv \Pr[\Delta\hat{k}' < 0|\Delta\hat{k} > 0]$

- $\mathbb{C}ov[\hat{k}, a]$ can be obtained as

$$\mathbb{C}ov[\hat{k}, a] = \frac{1}{2\nu}\left(\mathbb{V}[\hat{k}] - \frac{\overline{\mathbb{E}}[(\hat{k}_\tau - \mathbb{E}[\hat{k}])^2\tau]}{\overline{\mathbb{E}}[\tau]} + \frac{\sigma^2}{2}\frac{\overline{\mathbb{E}}[\tau]}{2}(1 + \overline{\mathbb{CV}}^2[\tau])\right),$$

←

A7. Taxes

Personal income tax t^p and capital gains tax t^p

- Equity held by a stockholder, with access to risk-less bond return ρ

$$\textbf{No-arbitrage:}\quad \underbrace{(1-t^p)\rho\,\mathrm{d}s}_{\text{bond return}} \;=\; \underbrace{(1-t^g)\frac{\mathbb{E}[\mathrm{d}P_s]}{P_s}}_{\text{capital gains}} \;+\; \underbrace{(1-t^p)\frac{D_s}{P_s}\,\mathrm{d}s}_{\text{dividends}}$$

  - P_s price per share, 1 share (normalization)
  - D_s dividend per share

- Let V_0 be the firm's market value:

$$V_0 \;=\; P_0 \;=\; \frac{1-t^p}{1-t^g}\,\mathbb{E}_0\left[\int_0^\infty e^{-\frac{1-t^p}{1-t^g}\rho s} D_s\,\mathrm{d}s\right]$$

  - Firm maximizes cum-dividends market value of equity P_0
  - Uses stockholder's discount $(1-t^p)/(1-t^g)\rho$

- **Dividend policy**: tax capitalization view

$$D_s\,\mathrm{d}s \;=\; \pi_s\,\mathrm{d}s \;-\; [\theta_s \;+\; p(\Delta k_s)\Delta k_s]\,\mathbb{D}(\Delta k_s \neq 0), \quad \mathbb{D} \sim \mathit{Dirac}$$

←

A7. Two cases: Additional Material

Two benchmark cases

- Study macro outcomes under two polar cases
  1. Symmetry: $\nu \to 0$ and $\tilde{p}^{buy} = -\tilde{p}^{sell} = \tilde{p}$
  2. Small idiosyncratic shocks: $\sigma \to 0$
- Why?
  - Isolate the role of each friction
  - Characterize analytically macro elasticities to taxes

CASE 1: $\nu \to 0$ and $\tilde{p}^{buy} = -\tilde{p}^{sell} = \tilde{p}$

- **Only fixed costs**: $x^{*+} = x^{*-} = 0$ and $\bar{x} = \left(\frac{6\tilde{\theta}\sigma^2}{\alpha(1-\alpha)}\right)^{1/4}$

$$\mathbb{V}[\hat{k}] = \bar{x}^2/6; \qquad q = 1 - \frac{\tilde{\mathcal{U}}}{\tilde{r}}\frac{\alpha(1-\alpha)}{2}\mathbb{V}[\hat{k}]; \qquad \text{CIR} = \frac{1}{\sigma^2}\mathbb{V}[\hat{k}]$$

  - Lower t^c, decreases $\tilde{\theta}$
  - $\mathbb{V}[\hat{k}]$ and CIR fall, q increases if $\rho > \sigma^2$

- **Both frictions:** marginal increase of smaller friction has no effect

$$\left.\frac{\mathrm{d}M}{\mathrm{d}\tilde{\theta}}\right|_{\tilde{\theta}=0,\,\tilde{p}>0} = 0, \text{ for } M \in \{\mathbb{V}[\hat{k}], q, \text{CIR}\}.$$

CASE 2: $\sigma \to 0$

- Partial irreversibility has no effect
- Indifference curve for relevant steady-state moment

$$\mathbb{E}[x]\sqrt{\mathbb{V}[x]} = -\frac{\tilde{r}\tilde{\theta}}{\sqrt{12}\alpha(1-\alpha)}; \qquad \frac{\mathbb{E}[x]}{\mathbb{V}[x]+\mathbb{E}[x]^2} = -\left(\frac{\tilde{r}}{\nu} + \frac{\alpha+1}{2}\right),$$

- Macro outcomes

$$q = 1 - \frac{\tilde{U}}{\tilde{r}}(1-\alpha)\left(\mathbb{E}[x] + \frac{\alpha}{2}\mathbb{V}[x]\right); \qquad \text{CIR} = 0.$$

  - Lower t^c, decreases $\tilde{\theta}$
  - $\mathbb{V}[\hat{k}]$ and $|\mathbb{E}[x]|$ fall, ambiguous effect on q

A10. Observability

Parameters and policy from microdata ←

- Use $\overline{\mathbb{E}}[\cdot]$ to denote expectations conditional on adjustment
- Assume for simplicity $\hat{k}^{\pm} = \mathbb{E}[\hat{k}]$
- We recover **stochastic process** (ν, σ^2) as:

$$\nu = \frac{\overline{\mathbb{E}}[\Delta \hat{k}]}{\overline{\mathbb{E}}[\tau]} \quad ; \quad \sigma^2 = \frac{\overline{\mathbb{E}}[(\nu\tau - \Delta\hat{k})^2]}{\overline{\mathbb{E}}[\tau]}$$

  - Drift = frequency × average of investment
  - Volatility = frequency × dispersion of investment

- We recover the **reset capital** $\hat{k}^*$ as:

$$\hat{k}^* = \frac{1}{1-\alpha}\left[\Phi + \log\left(\frac{1 - \overline{\mathbb{E}}\left[e^{-\hat{r}\tau - \alpha\Delta\hat{k}}\right]}{1 - \overline{\mathbb{E}}\left[e^{-\hat{r}\tau - \Delta\hat{k}}\right]}\right)\right]$$

where $\Phi \equiv \log\left(\frac{\alpha(1-t^c)}{(1-t^d)p(\hat{r}+\alpha\nu-\alpha^2\sigma^2/2)}\right)$

Cross-sectional moments from microdata ←

- We recover **cross-sectional moments** as:

$$\mathbb{E}[\hat{k}] = \hat{k}^* + \frac{1}{2\nu}\left(\sigma^2 - \frac{\overline{\mathbb{E}}[\Delta\hat{k}^2]}{\overline{\mathbb{E}}[\tau]}\right)$$

$$\mathbb{V}[\hat{k}] = \frac{(\hat{k}^* - \mathbb{E}[\hat{k}])^3 - \overline{\mathbb{E}}\left[\left(\hat{k}_\tau - \mathbb{E}[\hat{k}]\right)^3\right]}{3\overline{\mathbb{E}}[\Delta\hat{k}]}$$

$$\mathbb{C}ov[\hat{k}, a] = \frac{1}{2\nu}\left[\mathbb{V}[\hat{k}] - \frac{\overline{\mathbb{E}}[\tau\hat{k}_\tau^2]}{\overline{\mathbb{E}}[\tau]} + \frac{\sigma^2}{2}\overline{\mathbb{E}}[\tau]\left(1 + \overline{\mathbb{CV}}^2[\tau]\right)\right]$$

where $\hat{k}_\tau = \hat{k}^* + \Delta\hat{k}$

- Intuition for $\mathbb{V}[\hat{k}]$:
  - If $\hat{k}^* = \mathbb{E}[\hat{k}]$: $\quad \mathbb{V}[\hat{k}] = (1/3)\underbrace{\overline{\mathbb{E}}[\Delta k]^2}_{\text{size}} \underbrace{\overline{\mathbb{E}}\left[\left(\Delta k/\overline{\mathbb{E}}[\Delta k]\right)^3\right]}_{\text{dispersion}}$
  - Large investments $\Longrightarrow$ Signals large $\hat{k}$
  - Dispersed investments $\Longrightarrow$ Large $\hat{k}$ more representative