Talking Over Time - Dynamic Central Bank Communication

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\textsuperscript{1}The views expressed are solely the views of the author and do not necessarily reflect the views of the European Central Bank or the Eurosystem.
A new tradeoff in a dynamic world

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- Financial markets pay close attention (e.g. Rosa & Verga 2008)
- But households inattentive (e.g. Coibion et al. 2020)

- Misaligned preferences
- Financial markets care about future returns
- Central bank’s mandate: current inflation and employment

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- How to balance talking about today vs tomorrow?
- How clearly to talk?
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   - Dynamic Bayesian persuasion game

   ... vs. static benchmark: treat present & future as correlated in cross section
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     → Central bank sends signal to financial market

2. Findings: key role of prior beliefs
   - Direction of prior mean
     → talk more about the present
   - Tightness of prior variance
     → talk less clearly

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... vs. *static benchmark*: treat present & future as correlated in *cross section*
1. Global games


2. Bayesian persuasion

A DYNAMIC BAYESIAN PERSUASION GAME
Dynamic Bayesian persuasion game

- Sender (central bank, CB) and receiver (financial market, FM)
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- Two states $\theta_t, \theta_{t+1} \in \Theta$, known to CB, unknown to FM
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- **Persuasion**: CB sends signal $s_t \in S$ to FM
  
  $\rightarrow$ give FM info so FM takes action that maximizes CB’s payoff
The economic environment

\[ \theta_{t+1} = \rho \theta_t + \varepsilon_{t+1} \]

\[ \varepsilon_{t+1} \sim \mathcal{N}(0, \sigma_{\varepsilon}^2) \quad \sigma_{\varepsilon}^2 = 1 - \rho^2 \]
The economic environment

\[ \theta_{t+1} = \rho \theta_t + \varepsilon_{t+1} \]

“Future output”    “Current output”

\[ \varepsilon_{t+1} \sim \mathcal{N}(0, \sigma_{\varepsilon}^2) \quad \sigma_{\varepsilon}^2 = 1 - \rho^2 \]
Financial market and central bank payoffs

\[ L^F_{t}(I_t, \theta_{t+1}) = \mathbb{E}^F_{t}(I_t - \theta_{t+1})^2 \]

\[ L^{CB}(\{I_t, \theta_t\}_{t=0}^{\infty}) = \mathbb{E}^{CB}_{0} \sum_{t=0}^{\infty} \beta^t (I_t - b\theta_t)^2 \]
Financial market and central bank payoffs

\[ L_{t}^{FM}(I_{t}, \theta_{t+1}) = \mathbb{E}_{t}^{FM}(I_{t} - \theta_{t+1})^{2} \]

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Financial market (FM) sets investment (I) to maximize future returns
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Financial market (FM) sets investment (I) to maximize future returns

Central bank (CB) wants investment to track current output, with weight \( b \)

\( \beta \in (0, 1) \) CB’s discount factor
Information structure

\[ \mathcal{I}_t^{CB} = \{\theta_{t+1}, \theta_t, \ldots, \theta_0\}, \quad \mathcal{I}_t^{FM} = \{s_t, s_{t-1}, \ldots, s_0\} \]

where \( s_t \in S \) is a signal the CB sends the FM.
The central bank’s signal

\[ s_t = \theta_t + \frac{1}{\psi} \theta_{t+1} + v_t, \quad v_t \sim \mathcal{N}(0, \sigma_v^2) \]
Two dimensions of communication

\[ s_t = \theta_t + \frac{1}{\psi} \theta_{t+1} + \nu_t, \quad \nu_t \sim \mathcal{N}(0, \sigma_v^2) \]

\psi: how much the CB weights either state \(\rightarrow\) “targetedness”
Two dimensions of communication

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\( \psi \): how much the CB weights either state \( \rightarrow \) “targetedness”

- If \( \psi < 1 \) \( \rightarrow \) signal targeted toward tomorrow’s state
- If \( \psi > 1 \) \( \rightarrow \) signal targeted toward today’s state
- If \( \psi = 1 \) \( \rightarrow \) signal not targeted (“confounding”)

\( \sigma_v \): how much noise there is in the signal \( \rightarrow \) “precision”

- If \( \sigma_v = \infty \) \( \rightarrow \) signal perfectly imprecise
- If \( \sigma_v = 0 \) \( \rightarrow \) signal perfectly precise
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Perfect Bayesian equilibrium

Definition

Let $\mu_X(x)$ be the probability distribution of a variable $X$ induced by the FM’s beliefs. A Perfect Bayesian Equilibrium is an action rule $I_t$, belief system $\mu$ and a communication policy $(\psi^*, \sigma_v^*)$ such that

- $I_t = \arg\min L_{t}^{FM}(I_t, \theta_{t+1})$ s.t. $E_{t}^{FM}(\theta_{t+1}|s_t)$,

- $(\psi^*, \sigma_v^*) = \arg\min L_{t}^{CB}(\{I_t, \theta_t\}_{t=0}^{\infty})$ s.t. $E_{t}^{FM}(\theta_{t+1}|s_t) \forall t \geq 0$ and $s_t = \theta_t + \frac{1}{\psi} \theta_{t+1} + v_t$ with $v_t \sim N(0, \sigma_v^2)$,

- FM beliefs $E_{t}^{FM}$ come from $\mu \forall t$, and $\mu$ is consistent with Bayes’ rule:

$$
\mu_{\Theta|S=s}(\theta) = \frac{\mu_{S|\Theta=\theta}(s)\mu_{\Theta}(\theta)}{\mu_S(s)}
$$
OPTIMAL TARGETEDNESS
The graph shows the dynamic and static behavior of $\psi^*(\rho)$ over different values of $\rho$. The blue line represents $\psi^*_{static}$ and the red line represents $\psi^*_{dynamic}$. As $\rho$ increases, both functions initially rise and then decrease, with $\psi^*_{dynamic}$ reaching a peak at a lower $\rho$ compared to $\psi^*_{static}$. The dashed line at $\rho = 1$ indicates the boundary for the dynamic case, while the solid line at $\rho = 1$ represents the static case.
CB pushes against the prior

At time $t$, FM’s beliefs on $\theta_t$ informed by
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- Today’s signal: $s_t = \theta_t + \frac{1}{\psi} \theta_{t+1} + v_t$
CB pushes against the prior

At time $t$, FM’s beliefs on $\theta_t$ informed by

- Today’s signal: $s_t = \theta_t + \frac{1}{\psi} \theta_{t+1} + \nu_t$
- Prior beliefs: $s_{t-1} = \theta_{t-1} + \frac{1}{\psi} \theta_t + \nu_{t-1}$
OPTIMAL PRECISION
Optimal precision - a cross-section

\[ \psi = 0.2 \]

\[ \psi = 1 \]

\[ \psi = 1.8 \]

\[ \rho \]

\[ \sigma_{v, static}^{\rho} \]

\[ \sigma_{v, dynamic}^{\rho} \]
Tightness of priors

Prior variance: \[ \pi(\theta_T) := \mathbb{E}[(\theta_T - \theta_T|t-1)^2] \]

Posterior variance: \[ p(\theta_T, s_t) := \mathbb{E}[(\theta_T - \theta_T|t)^2] \]

\[ T = t, t + 1. \]
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Reduction in uncertainty (\( \approx \) mutual information):

\[ I(\theta_T, s_t) := \pi(\theta_T) - p(\theta_T, s_t) \]
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Reduction in uncertainty (≈ mutual information):

\[ I(\theta_T, s_t) := \pi(\theta_T) - p(\theta_T, s_t) \]

\( \leftrightarrow \) “Informativeness of the signal at time \( t \) about \( \theta_T \)”
Figure: Informativeness $I(\theta_{T}, s_{t})$ as a function of $\sigma_{v}$
Conclusion: CB pushes against priors

1. **Optimal communication policy** in a **static** and **dynamic** setting
Conclusion: CB pushes against priors

1. Optimal communication policy in a static and dynamic setting

2. Takeaway: relative to static communication, the dynamic policy is...
Conclusion: CB pushes against priors

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   - ... more targeted toward the present
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   • ... less precise
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   ... in order to correct direction and tightness of priors.
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3. A central bank following optimal static communication policy:
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   - behaves like discretionary policy
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   - ... more targeted toward the present
   - ... less precise

   ... in order to correct direction and tightness of priors.

3. A central bank following optimal static communication policy:
   - behaves like discretionary policy
     - ignores effect of current communication on future beliefs
The static analogue

Fundamental: \((\theta_1, \theta_2) \sim \mathcal{N}(0, \mathbf{V})\) with \(\mathbf{V} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}\),

Payoffs: \(\mathcal{L}^{FM}(I, \theta_2) = \mathbb{E}^{FM}(I - \theta_2)^2\),
\(\mathcal{L}^{CB}(I, \theta_1) = \mathbb{E}^{CB}(I - b\theta_1)^2\),

Info structure: \(\mathcal{I}^{CB} = \{\theta_1, \theta_2\}, \quad \mathcal{I}^{FM} = \{s\}\),

Signal: \(s = \theta_1 + \frac{1}{\psi} \theta_2 + v, \quad v \sim \mathcal{N}(0, \sigma_v^2)\).
Kalman filter

\[ x_{t+1} = hx_t + \eta \epsilon_{t+1} \]
\[ y_t = gx_t + v_t \]

\[ x_t = \begin{bmatrix} \theta_{t+1} \\ \theta_t \end{bmatrix}, \quad y_t = s_t, \quad h = \begin{bmatrix} \rho & 0 \\ 1 & 0 \end{bmatrix}, \quad g = \begin{bmatrix} \frac{1}{\psi} & 1 \end{bmatrix}, \]

\[ \eta = \begin{bmatrix} \sigma_{\epsilon} & 0 \\ 0 & 0 \end{bmatrix}, \quad \epsilon_t = \begin{bmatrix} \epsilon_t \end{bmatrix}, \quad Q = \eta \eta' = \begin{bmatrix} \sigma_{\epsilon}^2 & 0 \\ 0 & 0 \end{bmatrix}, \quad R = \sigma_v^2. \]

\[ m_1 = \rho - \kappa_1 (\frac{\rho}{\psi} + 1), \]

\[ m_2 = m_4 = \kappa_1, \]

\[ m_3 = \frac{\kappa_1}{\psi}, \]

where \( \kappa_1 \) is the first element of the 2 \( \times \) 1 Kalman gain and is given by

\[ \kappa_1 = \frac{\rho p_4 + \frac{1}{\psi} p_1}{p_4 + \frac{1}{\psi^2} p_1 + 2 \frac{\rho}{\psi} p_4 + \sigma_v^2}. \]
Figure: Optimal precision $\sigma^*$