

# A Theory of Economic Sanctions as Terms-of-Trade Manipulation

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How can a country design economic sanctions to maximize their economic cost to the sanctioned country at the lowest cost to the sanctioner? I consider this problem from the perspective of international trade and draw a close connection between trade restrictions as economic sanctions and trade restrictions as terms-of-trade manipulation. This connection has several useful implications for sanction design: Small sanctions increase welfare in the sanctioning country. Sanctions target the same goods as terms-of-trade manipulation. Sanctions ignore elasticities of demand and supply in the sanctioning country. Sanctions treat imports and exports asymmetrically.

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# 1 Introduction

Many countries, including the U.S. and members of the E.U., have recently imposed severe economic sanctions on Russia in response to its invasion of Ukraine. Although many of these sanctions are financial in nature—freezing Russian assets or preventing financial transactions—others directly restrict international trade.<sup>1</sup> How can these trade restrictions be designed so as to achieve their aims at limited cost to the sanctioning countries?

A vast literature on sanction design has addressed this question, but seldom from a purely economic standpoint. One strand of this literature compiles detailed case studies and, more recently, comprehensive databases that emphasize the institutional context of successful and unsuccessful historical sanctions (Hufbauer et al., 1990; Pape, 1997; Felbermayr et al., 2020; Demena et al., 2021). Other research considers how sanctions should target particular “strategic” goods—such as arms used to wage war or technology that may contribute to future military power—or particular actors in positions of political power (Førland, 1991; Cortright et al., 2002). Yet another focus is on how sanctions shape the game-theoretic interactions among nation states and within them, between ruling elites and regular citizens (Eaton and Engers, 1992; Morgan and Bapat, 2003; Baliga and Sjöström, 2022).

In contrast to this literature, I take a canonical economic perspective, asking what restrictions on international trade maximize the economic cost to a sanctioned country at the lowest cost to the sanctioning country.<sup>2</sup> While new to the academic literature, this economic lens is already evident in the approach and indeed language of many policymakers:

*“We’ve intentionally scoped our sanctions to deliver severe impact on the Russian economy while minimizing the cost to the U.S., as well as our Allies and partners.”*

– U.S. Deputy N.E.C. Director Daleep Singh, February 24, 2022

This framing allows me to draw on ideas from the optimal tariff literature and shed new light on the question of sanction design. My main result shows that optimal trade taxes as sanctions bear a striking resemblance to optimal trade taxes as terms-of-trade manipulation:

$$t_k = (1 - \lambda^F) t_k^{\text{ToT}},$$

where  $t_k$  is the optimal ad-valorem import tariff or export subsidies on good  $k$  from the perspective of sanctions, and where  $\lambda^F$  is a measure of a sanctioning country’s willingness

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<sup>1</sup>This includes bans on commodity trade in various sectors, such as energy, transport, and technology, as well as bans on intertemporal trade in the form of limitations on lending to sanctioned Russian individuals.

<sup>2</sup>I take this objective of sanctions as given and do not evaluate their ethical or geopolitical foundations. These are important and extensively studied topics in their own right.

to pay for economic welfare in the sanctioned country (negative in the case of sanctions). When the sanctioning country is willing to incur economic losses at home in order to impose economic costs in the sanctioned country—so that  $\lambda^F < 0$ —sanctions target the same goods as pure terms of trade manipulation but simply increase in intensity.<sup>3</sup>

At the heart of this result is a well-known observation about the foreign welfare effects of domestic trade taxes: In simple neoclassical settings, domestic trade taxes affect foreign welfare only through effects on its terms of trade. Since foreign terms of trade are simply the negative of domestic terms of trade, two of the effects that one might have expected to influence sanction design—domestic terms of trade and foreign welfare—are in fact proportional to one another. Therefore, optimal sanctions simply balance domestic terms of trade against inefficiencies stemming from the wedge between domestic and world prices, like standard unilateral trade taxes, but with different weights that account for the sanctioning motive.

This insight leads to four broad lessons for optimal sanction design. I focus these lessons on the case where sanctions take the form of simple, linear trade taxes (import tariffs and export taxes), as I show that linear taxes can implement the same allocations as more complex instruments such as price caps and quotas.

1. Starting from near free trade, weak enough sanctions *improve* the welfare of the sanctioning country. More severe sanctions come at a cost to the sanctioning country.
2. Optimal trade taxes as sanctions target the same goods as trade taxes as terms-of-trade manipulation—typically using higher taxes on goods that the sanctioned country supplies or demands inelastically—but with potentially higher tax rates.
3. Optimal trade taxes as sanctions do not depend on domestic elasticities of demand for imports or supply of exports.
4. In a leading case, optimal sanctions treat imports and exports asymmetrically. As sanctions become more severe, each good a sanctioning country exports is embargoed, one-by-one. For each good a sanctioning country imports, an embargo in that good is efficient only if sanctions are so severe as to cause complete autarky.

While simple, these lessons are novel to the literature on economic sanctions and, in several cases, have direct policy relevance.

I develop these ideas in a benchmark neoclassical model. In the model, two countries—a sanctioning country and a sanctioned country—each contain a representative household and firm. Markets are competitive. Whereas the sanctioned country engages in free trade, the sanctioning country may impose a general form of trade sanctions.

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<sup>3</sup>It is worth noting that were  $1 + t_k$  proportional to  $1 + t_k^{\text{ToT}}$ , then changing the constant of proportionality would have no real effect, by Lerner symmetry. However, the formulation above has non-neutral effects, even cutting off trade entirely for negative enough  $\lambda^F$ .

My main result characterizes the sanctions that trace out the frontier of sanctioner-sanctionee utilities, therefore characterizing how the sanctioner can reduce welfare in the sanctioned country at the least economic cost to itself. Notably, this result and the lessons that follow from it apply to all points on this frontier; they do not require taking a stance on one’s preferred severity of sanctions.<sup>4</sup> Finally, I discuss robustness to the presence of multiple sanctioning countries and “neutral” third-party countries.

**Literature** To the best of my knowledge, this paper provides the first general analysis of optimal sanction design from the perspective of international trade theory. Other research and indeed practice has emphasized that sanctions should target goods that a sanctioned country demands inelastically—particularly bottleneck inputs—but typically in the context of “strategic” political or military goods rather than economic welfare (Førland, 1991). In emphasizing terms-of-trade manipulation, I build on an idea dating back to Mill (1844) and first formalized by Johnson (1951).<sup>5</sup> While I study competitive markets and the case of two countries, other work posted after the first draft of this paper has considered how sanction design can account for foreign monopoly power and passive third-party countries (Gros, 2022).

More broadly, this paper is part of a quickly growing literature that studies the design and potential effects of sanctions in the context of Russia’s invasion of Ukraine. Bachmann et al. (2022) and related works use semi-structural models to predict the economic cost to European countries of a full embargo on imports of Russian energy goods (Baqae et al., 2022; Berger et al., 2022). Others have called for tariffs on imports as a way to inflict economic damage on Russia at lower cost to sanctioning countries (Hausmann, 2022; Chaney et al., 2022).<sup>6</sup> I seek to clarify in which goods it is most efficient to restrict trade and in what cases embargoes are optimal.

## 2 Benchmark Two Country Trade Model

I consider a competitive world economy with two countries, Home ( $i = H$ ) and Foreign ( $i = F$ ), and many goods  $k \in K$ . In order to set aside redistributive concerns and focus my analysis, I assume each country contains a representative household and a representative

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<sup>4</sup>I use the word “optimal” to refer to any point on this frontier.

<sup>5</sup>For a modern treatment, see Dixit (1985). More recent contributions have studied terms-of-trade manipulation in models that microfound Foreign’s demand system using models of Ricardian trade, fixed costs of exporting, and dynamic substitution (Costinot et al., 2015, 2016, 2014)

<sup>6</sup>More indirectly related are several papers that study the implications of financial sanctions and the financial implications of sanctions in general (Bianchi and Sosa-Padilla, 2022; Itskhoki and Mukhin, 2022; Lorenzoni and Werning, 2022).

firm. Foreign engages in free trade. Home sets import tariffs and export taxes, and it rebates revenues lump-sum.

**Prices** Goods are traded between countries at world prices  $p$ . Households and firms within each country  $i$  trade at domestic prices  $q^i$ . Foreign domestic prices are equal to world prices. Home domestic prices differ from world prices by ad-valorem trade taxes  $t(x^F)$ , which may in general depend on Foreign's net exports  $x_k^F$  of each good  $k$ . The quantity-dependence of taxes allows us to nest quantity restrictions (e.g. prohibitive tariffs once imports cross a threshold) and price caps (e.g. export subsidies that adjust in order to hold prices fixed).

$$q_k^F = p_k \quad \text{and} \quad q_k^H = p_k(1 + t_k(x^F)). \quad (1)$$

A positive tax  $t_k$  corresponds to a tariff if Home imports  $k$  and an export subsidy if Home exports  $k$ .

**Households** In each country  $i$ , a representative household chooses net consumption  $c^i$  to maximize utility  $V^i$  derived from a utility function  $u^i(\cdot)$  subject to a budget constraint.<sup>7</sup> The household owns the profits  $\pi^i$  of domestic firms and receives a lump-sum transfer  $T^i$ .

$$V^i = u^i(c^i) \quad \text{and} \quad c^i \in \arg \max_{c \in \mathbb{R}^K} u^i(c) \quad \text{s.t.} \quad q^i \cdot c \leq \pi^i + T^i. \quad (2)$$

**Firms** In each country  $i$ , a representative firm chooses net output  $y^i$  to maximize profits  $\pi^i$  subject to a production frontier  $G^i(y^i) \leq 0$ .

$$\pi^i = q^i \cdot y^i \quad \text{and} \quad y^i \in \arg \max_{y \in \mathbb{R}^K} q^i \cdot y \quad \text{s.t.} \quad G^i(y) \leq 0. \quad (3)$$

**Market clearing** Each country  $i$  has net exports  $x^i$  equal to the difference between its production and consumption and, globally, markets clear.

$$x^i = y^i - c^i \quad \text{and} \quad x^i + x^{-i} = 0. \quad (4)$$

**Governments** Foreign has no taxes. Home levies trade taxes  $t(\cdot)$  and rebates revenues with a lump-sum transfer  $T^H$  in order to maintain budget balance.

$$T^F = 0 \quad \text{and} \quad T^H = - \sum_{k \in K} p_k t_k(x^F) x_k^H. \quad (5)$$

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<sup>7</sup>I impose standard regularity conditions on utility functions and production frontiers in Appendix B.

### 3 Designing economic sanctions

I now present my main theoretical result—a formal characterization of optimal economic sanctions—before turning to several practical takeaways for sanction design.

#### 3.1 Optimal economic sanctions

My main result studies trade policies that are optimal the sense that they implement an extremal point in the set of feasible Home and Foreign welfares.

Formally, I define the set of utility profiles  $\mathcal{V}$  implementable with (possibly quantity-dependent) Home trade taxes as

$$\mathcal{V} \equiv \left\{ (V^H, V^F) \mid \text{there exist quantity-dependent taxes } t : \mathbb{R}^K \rightarrow \mathbb{R}^K \text{ and a trade equilibrium at taxes } t \text{ that achieves household utilities } (V^H, V^F) \right\}. \quad (6)$$

A natural question is whether quantity restrictions or price caps—in general, quantity-dependent taxes—ever allow Home to implement utility profiles that it cannot implement with linear taxes. The following lemma clarifies that this is not the case: linear taxes are sufficient.

**Lemma 1.** *Linear Home trade taxes can implement any utility profile implementable with quantity-dependent Home trade taxes, i.e. all of  $\mathcal{V}$ .*

This basic result relies on perfect information on the part of the planner and perfect competition. While instruments such as price caps or “autarky threats” can dominate linear taxes when these assumptions fail,<sup>8</sup> this result allows me to narrow my focus to linear taxes for the remainder of the paper. This in mind, I denote trade taxes simply by  $t \in \mathbb{R}^K$ .

Moving toward optimal policy, I now consider a generalization of  $\mathcal{V}$ ’s Pareto frontier: the set of points  $\bar{\mathcal{V}}$  in  $\mathcal{V}$  that are extremal in either the positive- or negative-utility directions:

$$\bar{\mathcal{V}} \equiv \left\{ (V^H, V^F) \in \mathcal{V} \mid \text{there exist “welfare signs” } s^H, s^F \in \{-1, 1\} \text{ such that} \right. \\ \left. \text{for all } (\tilde{V}^H, \tilde{V}^F) \in \mathcal{V}, s^H V^H \geq s^H \tilde{V}^H \text{ and } s^F V^F \geq s^F \tilde{V}^F \right\} \quad (7)$$

Figure 1 depicts  $\mathcal{V}$  and  $\bar{\mathcal{V}}$ . The green line traces the first-best Pareto frontier, whereas the red line traces the Pareto frontier of  $\mathcal{V}$ , i.e. the subset of  $\bar{\mathcal{V}}$  corresponding to  $s^H = s^F = 1$ .  $\mathcal{V}$  lies within the first-best Pareto frontier, since trade taxes are distortionary; the two intersect only when trade taxes are set to zero. The figure notes Home and Foreign welfare at autarky

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<sup>8</sup>For example, see Weitzman (1974); Baron and Myerson (1982).

(“A”), at free trade (“FT”), and at the level of Home tariffs that maximizes Home welfare by optimally manipulating Home’s terms of trade (“ToT”). As long as there is non-zero trade and Foreign’s supply of exports and demand for imports are not perfectly elastic, there is scope for such manipulation, so that Home can make itself better off than under free trade and make Foreign worse off (Johnson, 1951).

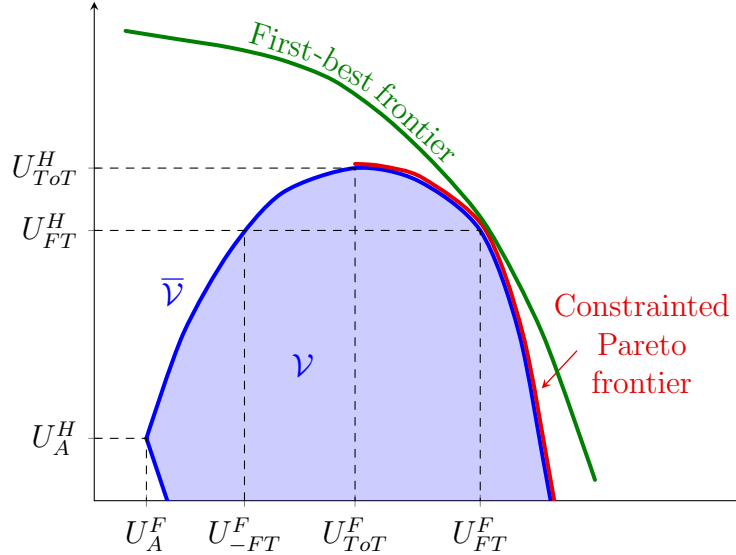


Figure 1

My main result characterizes the trade taxes that implement the extremal points  $\bar{\mathcal{V}}$  of the set of implementable utilities. In doing so, I describe the sanctions that are efficient in the sense that they reduce Foreign welfare to any given level at the lowest welfare cost to Home.

As in the case of pure terms-of-trade manipulation, a key object in this characterization is Foreign’s inverse net export supply curve,  $p(x^F)$ , which specifies world prices as a function of Foreign’s net exports.<sup>9</sup> The formal statement of the result relies on a weak regularity condition on this curve.<sup>10</sup>

**Assumption 1.** *At any non-zero level of foreign exports  $x^F \neq 0$  that satisfies trade balance,*

1. *The market value of Foreign’s existing net exports varies to first order in those net exports. That is,  $p(x^F) \cdot x^F \neq 0$ .*

<sup>9</sup>Formally, one may take  $p(\cdot)$  to be any differentiable function  $\mathbb{R}^K \rightarrow \mathbb{R}^K$  such that at all  $x$ ,  $p(x) \propto -V_x^F(x)$ , where  $V^F(x) \equiv \max_c u^F(c)$  s.t.  $G^F(c+x) \leq 0$ . Lemma 4 in Appendix C.2 validates this definition by showing  $p \propto -V^F(x^F)$  in any equilibrium.

<sup>10</sup>It also uses three more standard and purely technical assumptions that I accordingly relegate to Appendix B.

2. *There exist goods  $j, k \in K$  such that additional net exports of  $j$  cause Foreign's existing, total net exports to weakly appreciate and additional net exports of  $k$  cause them to weakly depreciate. That is,  $\frac{dp(x^F)}{dx_j^F} \cdot x^F \geq 0$  and  $\frac{dp(x^F)}{dx_k^F} \cdot x^F \leq 0$ .*

For typical preference specifications, one may take  $j$  to be any good Foreign exports and  $k$  to be any good Foreign imports: An increase in exports of  $j$  moves Foreign sellers up their supply curve, pushing up prices and improving Foreign terms of trade, whereas an increase in net exports of  $k$ —i.e. a decrease in imports—moves Foreign buyers up their demand curve, pushing up prices and worsening Foreign terms of trade.

**Theorem 1.** *At any point in the set  $\bar{\mathcal{V}}$  except autarky, trade taxes are, up to Lerner symmetry,<sup>11</sup> given by*

$$t_k = (1 - \lambda^F) \sum_{j \in K} \sigma_{k,j}^F, \quad \text{where} \quad \sigma_{k,j}^F = \frac{x_j^F}{p_k(x^F)} \frac{dp_j(x^F)}{dx_k^F} \quad (8)$$

and  $\lambda^F$  is a measure Home's willingness to pay for welfare in Foreign.<sup>1213</sup>

Theorem 1 nests the well known cases of free trade, which is optimal for a global planner with no redistributive preference across countries ( $\lambda^F = 1$ ), and full terms-of-trade manipulation, which is optimal for a purely self-interested Home planner ( $\lambda^F = 0$ ). It shows that, surprisingly, optimal trade taxes at any point on the implementable frontier  $\bar{\mathcal{V}}$  have the same structure as those that implement full terms-of-trade manipulation. The only difference is in the weight they place on the world price elasticities  $\sigma_{k,j}^F$ .

The economic intuition behind this stark result is that two seemingly distinct motives for trade taxes—manipulating Home's terms of trade and making Foreign worse (or better) off—are in fact two sides of the same coin. On one hand, the terms-of-trade manipulation motive encourages Home to artificially restrict trade when doing so lowers (raises) the import (export) prices it faces on the world market, *so that its net exports appreciate*. On the other hand, the first-order welfare loss to Foreign from any trade restriction is—by the envelope theorem—proportional to the *depreciation of its net exports*. So trade taxes as terms-of-trade manipulation seek to appreciate Home's terms of trade, whereas trade taxes as sanctions seek to depreciate Foreign's terms of trade.

<sup>11</sup>It is well known that for any equilibrium with trade taxes  $t$  and any  $\kappa > 0$ , there is an equivalent equilibrium with trade taxes  $t'$  given by  $1 + t'_k = \kappa(1 + t_k)$  (Lerner, 1936; Costinot and Werning, 2019).

<sup>12</sup>More formally, Assumption 1 implies there exists a positive bundle of goods  $x^* > 0$  in the direction of which foreign's net exports neither appreciate nor depreciate—i.e.  $x^F \cdot p_x(x^F) \cdot x^* = 0$ . Then  $\lambda^F$  is how many units of  $x^*$  one would have to give to  $H$  per unit of  $x^*$  one takes from  $F$  in order to move along the boundary.

<sup>13</sup>Since  $p_j(F) = V_x^F(x^F)$ ,  $\sigma_{k,j}^F$  can be written as  $\frac{x_j^F}{p_k(x^F)} \frac{dp_k(x^F)}{dx_j^F}$  by the symmetry of second derivatives. This cross-elasticity representation is perhaps more elegant, but obscures the economic mechanism.



### 3.2 Lessons for sanction design

The characterization of Section 3.1 has four useful lessons for sanction design.

The first lesson is apparent simply from the formal setup of the sanction design problem, as represented in Figure 1.

**Lesson 1.** *Starting from any extremal point between free trade and full terms-of-trade manipulation, Home can impose a welfare loss on Foreign while increasing its own welfare.*

In other words, if Home initially does not engage in full terms-of-trade manipulation—for instance because of a free trade agreement—then imposing sanctions can actually make it better off.<sup>14</sup> For example, suppose Home is initially in free trade, so that it achieves welfare  $U_{FT}^H$  in the figure. Home can reduce foreign welfare to  $U_{ToT}^F < U_{FT}^F$  while strictly increasing its own welfare, and can reduce foreign welfare further, to  $U_{-FT}^F < U_{ToT}^F$ , before Home is worse off than under free trade.

The remaining three lessons are applications of my main result, Theorem 1. For simplicity, I state each of these lessons in the special case where Foreign’s net export supply curve has constant elasticities, and I discuss its robustness.

To begin, note that, while the *level* of trade taxes depends on the point in  $\bar{\mathcal{V}}$  they implement—i.e. on the severity of sanctions—Theorem 1 implies that the *ratio* of trade taxes on any two goods is invariant.

**Lesson 2.** *Trade taxes as sanctions target the same goods as do trade taxes as terms-of-trade manipulation—generally using higher taxes on goods that Foreign demands or supplies inelastically—just with higher tax levels.*<sup>15</sup>

In other words, to the extent that Foreign elasticities do not depend on trade taxes, one may design sanctions by simply scaling up any pre-existing trade taxes that lie along the efficient frontier.

For example, imagine that Foreign demand is quasilinear and constant-elasticity, exporting gas with an elasticity  $1/\sigma_g$ , exporting oil with an elasticity  $1/\sigma_o$ , and importing a numeraire good:

$$u^F(c_g, c_o, c_n) = -k_g(-c_g)^{1+\sigma_g} - k_o(-c_o)^{1+\sigma_o} + c_n, \quad (9)$$

Moreover, suppose Foreign has no ability to transform these goods, i.e. its technology is one-to-one in each good. It is well known that, taking the numeraire good to be untaxed, Home’s

<sup>14</sup>Of course, this observation breaks down if Foreign retaliates with tariffs of its own.

<sup>15</sup>In the general case where Foreign’s elasticities are not constant, the ratio of trade taxes can vary to the extent that the ratio of the relevant Foreign elasticities does.

optimal tariffs for terms-of-trade manipulation are simply equal to each good’s inverse supply elasticity:  $t_g = \sigma_g$ ,  $t_o = \sigma_o$ . According to Theorem 1, Home’s optimal tariffs as sanctions are simply reweighted to  $t_g = (1 - \lambda^F)\sigma_g$ ,  $t_o = (1 - \lambda^F)\sigma_o$ , where  $\lambda^F$  is Home’s willingness to pay for a transfer of the numeraire to Foreign (negative in the case of sanctions). Notably, the ratio of Home’s tariff on gas relative to oil is invariant to the harshness of the sanction.

Another key implication of Theorem 1 is that Home elasticities only affect the design of its sanction policy insofar as they affect its willingness to trade off Home and Foreign welfare.

**Lesson 3.** *Given Home’s willingness to pay for welfare in Foreign,  $\lambda^F$ , its optimal trade taxes only depend on Foreign elasticities. In particular, Home elasticities are irrelevant.*<sup>16</sup>

For example, Home need not avoid tariffs that raise import prices in sectors away from which Home cannot easily substitute, such as natural gas.

Figure 2 illustrates the irrelevance of Home elasticities in an elementary example of a single import market. Any small increase in Home’s import tariff reduces Foreign producer surplus by the reduction in price of Foreign’s inframarginal exports. At the same time, it increases Home surplus—which is the sum of consumer surplus and tariff revenue—by the amount lost in producer surplus minus the reduction in quantity traded at the existing tariff level.

$$d(\text{PS}) = p \cdot Q \cdot d \log p \quad \text{and} \quad d(\text{CS} + \text{Rev}) = -d(\text{PS}) + t \cdot p \cdot Q \cdot \underbrace{d \log Q}_{\varepsilon^S \cdot d \log p}, \quad (10)$$

where here I have noted that the change in world prices and quantities are related by the elasticity of Foreign’s export supply,  $\varepsilon^S$ . Although Home’s elasticity of demand does impact the change in world price, this change affects both Home and Foreign surplus proportionately. It therefore drops out of any determination of whether taxes are set optimally—i.e. whether the two terms balance one another, given some weight on Foreign relative to Home surplus. Notably, the change in Home’s domestic price is irrelevant for Home surplus conditional on the world price, provided Home can redistribute tax revenue frictionlessly. It is important to note that domestic elasticities *do* factor into the welfare impacts of optimal sanctions, in both Home and Foreign. I discuss this topic briefly in Appendix A.

Finally, Theorem 1 implies that—at least in the constant-elasticity case—optimal sanctions treat imports and exports in an asymmetric way.

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<sup>16</sup>In the general case where Foreign’s elasticities depend on the quantity of its net exports, Home elasticities play an *indirect* role by determining the point at which Foreign’s supply elasticities are evaluated.

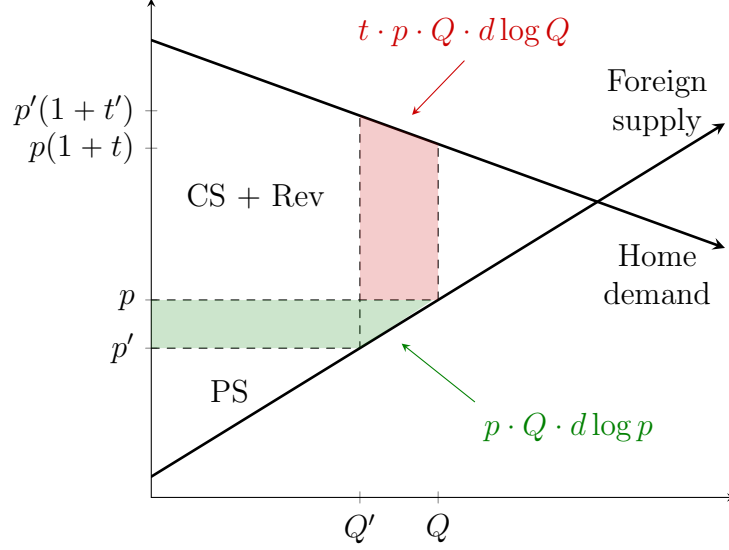


Figure 2

**Lesson 4.** For any good (typically an export) whose net exports worsen Home's terms of trade, sufficiently strong sanctions should embargo (i.e. infinitely tax) that export.<sup>17</sup> For any good (typically an import) whose net exports improve Home's terms of trade, Home should not embargo that good unless it engages in complete autarky.<sup>18</sup>

To understand the mathematical origin of this result, consider a sequence of increasingly severe sanctions—i.e.  $\lambda^F$  becoming more and more negative. Under constant elasticities, the ad-valorem tariff on a typical import gradually rises to infinity, whereas the ad-valorem export subsidy on a typical export falls to negative infinity. However, these trade taxes affect importers and exporters differently: The number of importers will fall as tariffs rise, but may only reach zero in the limit. By contrast, no one will export a good once its export subsidy reaches  $-1$ , so that the post-tax price is negative. So it has the same effect as an embargo.

To see why this asymmetry is optimal from the perspective of a sanction designer, consider the asymmetry in a trade tax's welfare impact on sellers and buyers. For a seller, the value of trading a unit is its profit, which is at most the price it receives. For a buyer, the value of trade is the difference between the price paid and the buyer's valuation, which—for an inframarginal buyer—can be unboundedly large. When Foreign's net export supply curve has constant elasticities, this logic implies Home import tariffs face diminishing returns in their ability to move Foreign prices, whereas Home export taxes do not. As a result, Home imposes more severe sanctions on its exports, even to the point of fully embargoing them.

<sup>17</sup>Formally, a good  $k$ 's marginal net exports worsen (improve) Home's terms of trade if  $\sum_{j \in K} \sigma_{j,k}^F < (>) 0$ .

<sup>18</sup>Of course, this asymmetry only has bite if there are more than two goods; with just one import and one export, an embargo on exports implies complete autarky, by trade balance.

Lesson 4 is somewhat more dependent on the assumption of constant elasticities than are Lessons 1–2, but not completely special to that case. For example, suppose one maintains quasilinearity but allows for variable elasticities. Then in the case of Home exports, harsh enough sanctions completely cut off the export of any given good unless Foreign demand for that good is (or converges to being) completely elastic at some quantity, for example because Foreign has a choke price. In the case of Home imports, sanctions embargo an import before full autarky only if either (a) Foreign supply is infinitely elastic at some quantity or (b) Home demand has a choke price.

## 4 Extensions

So far, I have abstracted from many salient features of reality, including the existence of “neutral” third-party countries and the presence of multiple sanctioning countries. I briefly discuss these issues below.

### 4.1 Many non-sanctioning countries

The model studied above assumes there is a single foreign country. It therefore does not speak to salient questions such as “How should a country design sanctions taking into account its desire to avoid harming a third country?”<sup>19</sup>

One may incorporate these considerations into the model described above by assuming there are many foreign countries. A slightly modified version of Theorem 1 carries over:

$$t_k = \sum_{j \in K} (1 - \lambda_j^F) \sigma_{k,j}^F, \quad \text{where} \quad \lambda_j^F \equiv \mathbb{E}_i[\lambda^i] (1 + |F| \cdot \text{corr}_i[\lambda^i, x_j^i]) \quad (11)$$

where  $\sigma_{k,j}^F$  is an inverse elasticity of *aggregate* foreign net export supply,  $|F|$  is the number of foreign countries, and  $\mathbb{E}_i$  and  $\text{corr}_i$  are unweighted across countries. Intuitively, the correlation between  $\lambda^i$  (a “welfare weight” for foreign country  $i$ ) and  $x_j^i$  captures the extent to which good  $j$  is exported by countries whose welfare Home would like to increase. Home’s optimal trade taxes target relatively higher world prices of goods for which this correlation is high.

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<sup>19</sup>A related concern is that one might like to harm the leaders of Foreign but not its average citizens. If one abstracts away from taxes within Foreign, one can apply (11) while treating Foreign as two separate countries, one consisting of leaders and one consisting of average citizens.

## 4.2 Many sanctioning countries

The model studied above assumes there is a single sanctioning country, as well as a single household within this country. However, in practice, sanctions are typically imposed by several coordinating countries, and—as in the case of EU oil and gas sanctions on Russia—the incidence of welfare impacts among these countries and their citizens is a hotly debated topic.

One notable feature of Theorem 1 is that it extends to such environments, provided that the designer of trade taxes has rich enough policy instruments. A simple example is the case where there are no domestic distortions and a planner can ensure that all Home countries / households face the same prices, can make lump-sum transfers across them, and can still impose trade taxes between Home countries and the foreign country. More generally, Theorem 1 applies as policy is set by a planner whose objective depends only on the quantity of its net exports and not on their prices.<sup>20</sup> Costinot and Werning (2018) provide an example of such an economy without lump-sum taxation.

Although Theorem 1 still holds when there are many sanctioning countries, it is important to note that Foreign’s inverse net export supply elasticities  $\sigma_{k,j}^F$  are endogenous to whether sanctioning countries coordinate with one another. Intuitively, a single sanctioning country in isolation faces low Foreign inverse elasticities, because in response to, say, import tariffs, Foreign can simply sell to other third-party countries rather than lowering its prices. Conversely, Foreign cannot substitute between buyers if all other countries impose tariffs on its exports, so it must lower prices by more. Sturm et al. (2022) study optimal trade sanctions in a model that explicitly considers Foreign’s ability to substitute to third-party countries.

## 5 Conclusion

I have characterized the trade taxes that trace out the extremal frontier of Home and Foreign utilities in a benchmark neoclassical model—in other words, optimal trade sanctions.<sup>21</sup> This characterization has several useful implications for sanction design, and I have discussed how these lessons carry over to more general settings.

These lessons are as follows: First, in the absence of retaliation, weak enough sanctions actually improve the welfare of a sanctioning country. Second, optimal trade taxes as sanc-

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<sup>20</sup>This implies that Home welfare is given by a function  $V^H(x^H)$ , to which the proof of Theorem 1 then applies. A caveat is that one must still impose technical assumptions analogous to those discussed in Appendix B.

<sup>21</sup>I have justified, my focus on trade taxes—as opposed to other instruments such as price caps or quotas—by showing that they can implement the same allocations as these more complex policies.

tions target the same goods as trade taxes as terms-of-trade manipulation, namely those whose restriction causes the sanctioner’s terms of trade to appreciate. Third, the relative magnitudes of trade taxes on different goods do not depend on any features of the domestic economy. Fourth—in a leading case—sanctions should treat imports and exports asymmetrically, completely cutting off particular exports when sanctions become harsh enough but only embargoing imports in the case of total autarky.

One potentially-important limitation of my analysis is that it abstracts away from distortions in the Foreign economy, including Foreign trade taxes. In general, “loading onto” such distortions provides an additional motive in sanction design. For example, sanctions can reduce Foreign welfare by cutting Home’s imports of goods on which Foreign imposes large export taxes, as these export taxes imply that Foreign’s marginal cost of production is already lower than the world price. Sturm (2022) and Gros (2022) provide starting points for this analysis in two special cases.

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# Appendix

## A Taxonomy of trade taxes' welfare impacts

In general, the welfare impacts of trade taxes depend on the full demand systems of Home and Foreign. However, the main ideas can be illustrated by considering four cases of an elementary supply and demand diagram. The diagrams in Figure A1 depict the market for a good that Home imports from Foreign under a tariff, in the extremal cases of perfectly elastic demand (top left), perfectly inelastic demand (top right), perfectly elastic supply (bottom left), and perfectly inelastic supply (bottom right).

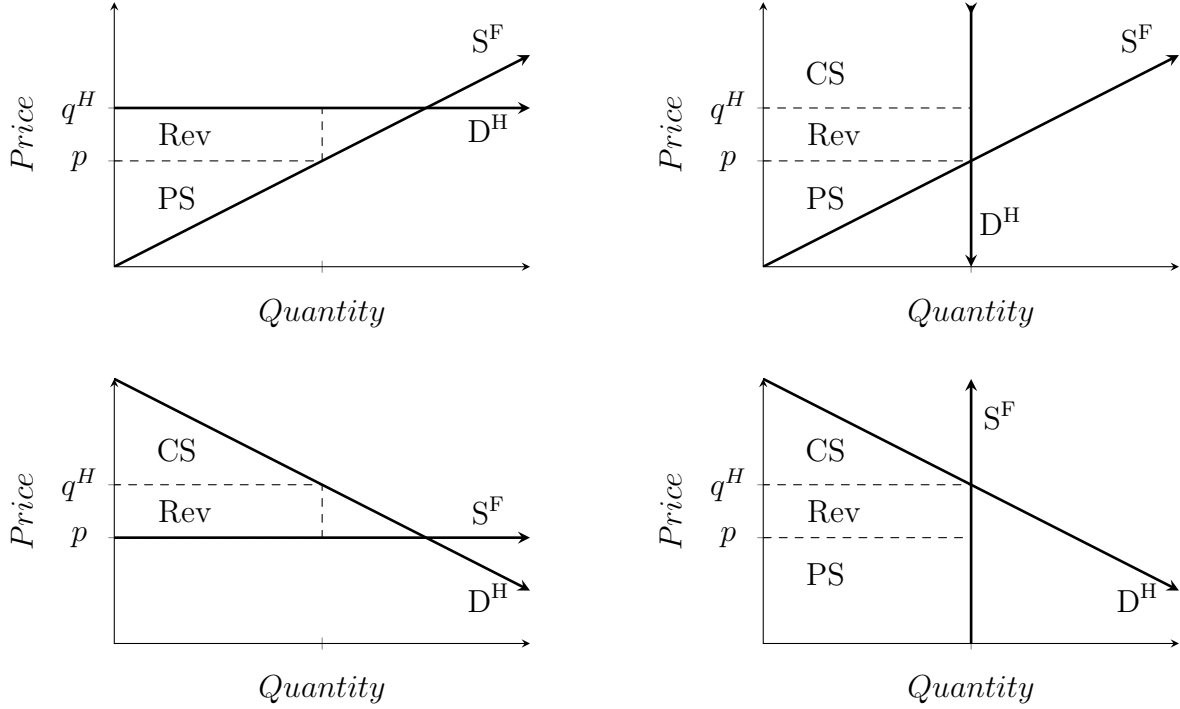


Figure A1

Each of these four extremal cases corresponds to its own stark set of implications for the welfare impacts of tariffs. If Home demand is perfectly elastic, then its consumers experience no gains from trade and tariffs simply trade off revenue maximization against foreign welfare. If instead Home demand is perfectly inelastic, then changes in tariffs simply transfer inframarginal dollars between Home consumer surplus and Home tax revenue. If Foreign supply is perfectly elastic, then all surplus is Home surplus, so tariffs should simply maximize aggregate surplus—i.e. be set equal to zero. If Foreign supply is instead perfectly inelastic, then changes in tariffs simply transfer inframarginal dollars between Foreign producer surplus and Home tax revenue.

While considered above for the case of Home imports, one may easily verify that the same ideas apply to the case of Home exports.

## B Regularity conditions

I impose mild regularity conditions on preferences, production frontiers, and the relationship between them.

**Assumption 2.** *The utility function  $u^i(\cdot)$  of each country  $i \in \{H, F\}$  is twice continuously differentiable, strictly concave, and is always increasing to first order in at least one good.<sup>22</sup>*

**Assumption 3.** *The production frontier  $G^i(\cdot)$  of each country  $i \in \{H, F\}$  is twice continuously differentiable, weakly convex, and increasing to first order in all goods.<sup>23</sup>*

**Assumption 4.** *For  $i \in \{H, F\}$ , utility  $u^i$  and the production frontier  $G^i$  satisfy the following:*

- *For all  $x$ , there exists  $c_x^* \in \mathbb{R}^K$  and  $d_x > 0$  such that (a)  $G^i(c_x^* + x) \leq 0$  and (b) for all  $c \notin B_{d_x}(0)$  satisfying  $G^i(c + x) \leq 0$ ,  $u^i(c) < u^i(c_x^*)$ .*
- *For any  $c, y \in \mathbb{R}^K$ , there exists  $k \in K$  with  $u_{c_k}^i(c) > 0$  such that the  $K \times K$  matrix*

$$\left[ \frac{d}{dc} \left( \frac{u_{c-k}^i(c)}{u_{c_k}^i(c)} \right) - \frac{d}{dy} \left( \frac{G_{y-k}^i(y)}{G_{y_k}^i(y)} \right), G_y^i(y) \right] \quad (\text{A1})$$

*has full rank.*

The first part of Assumption 4 ensures that, conditional on any level of net exports, there exists a welfare maximizing level of domestic consumption. The second part ensures that this consumption level is continuously differentiable in net exports. It is easy to verify that these conditions hold in simple examples such as additive, CES, quasi-linear consumption preferences and linear production.

More formally, for  $i \in \{H, F\}$ , define a country's highest possible level of welfare given net exports  $x^i$  as

$$V^i(x^i) = \max_{y \in \mathbb{R}^K} u^i(y - x^i) \quad \text{s.t.} \quad G^i(y) \leq 0. \quad (\text{A2})$$

Assumptions 2, 3, and 4 allow us to characterize  $V^i(\cdot)$ .

**Lemma 2.** *For all  $i \in \{H, F\}$ ,  $V^i(\cdot)$  is well defined, twice continuously differentiable, and, for all  $x^i \in \mathbb{R}^K$ , satisfies  $V_x^i(x^i) = -u^i(y^i(x^i) - x^i) \propto -G_y^i(y^i(x^i)) \ll 0$  for some  $\phi_{x^i}^i > 0$ , where  $y^i(x^i)$  is the (unique) solution to (A2).*

A second useful lemma connects this characterization to equilibrium outcomes.

**Lemma 3.** *In any trade equilibrium in which a country  $i \in \{H, F\}$  has net exports  $x^i$ ,  $i$  has production  $y^i$  equal to the unique solution of (A2) defined in Lemma 2,  $y^i(x^i)$ , and domestic prices  $q^i \gg 0$  proportional to both  $G_y^i(y^i(x^i))$  and  $u^i(y^i(x^i) - x^i)$ .*

<sup>22</sup>That is, for all  $c \in \mathbb{R}^K$ , there exists  $k \in K$  such that  $u_{c_k}^i(c) > 0$ .

<sup>23</sup>That is, for all  $y \in \mathbb{R}^K$ ,  $G_y^i(y) \gg 0$ .

## C Proofs

### C.1 Proof of Lemma 1

Consider any equilibrium with quantity-dependent trade taxes  $t(\cdot)$ , prices  $p = q^F$  and  $q^H$ , exports  $x^H$  and  $x^F$ , production  $y^H$  and  $y^F$ , consumption  $c^H$  and  $c^F$ , and transfers  $T^F = 0$  and  $T^H$ .

Let linear taxes  $\tilde{t}$  be given by  $\tilde{t} = t(x^F)$ . It follows immediately from the equilibrium conditions that the same prices, exports, production, consumption, and transfers are an equilibrium with linear taxes  $\tilde{t}$ .

### C.2 Proof of Theorem 1

I begin with a simple characterization of the set of implementable utilities.

**Lemma 4.**

$$\mathcal{V} = \tilde{\mathcal{V}} \equiv \{(V^H(x), V^F(-x)) \in \mathbb{R}^{\{H,F\}} \mid x \in \mathbb{R}^K, V_x^F(-x) \cdot x = 0\}. \quad (\text{A3})$$

Moreover, in any equilibrium that implements some  $(V^H, V^F) \in \mathcal{V}$ ,  $V_x^i(x^i) \propto -p$ , for  $i \in \{H, F\}$ .

Intuitively, each country achieves the highest possible utility *given* its net exports—there are no domestic distortions, and Home has a full set of trade instruments to insulate prices in the domestic economy from the world prices that its trade policy may directly affect. However, Home is constrained to choose a level of net exports consistent with trade balance. The interesting part of Home's problem is that it has some flexibility in the way it achieves trade balance, since it can move world prices by shifting exports (note that  $V_x^F(x^F) \propto -p$  by the envelope theorem).

#### Proof of Theorem 1

Fix any point  $(V^H, V^F)$  in  $\tilde{\mathcal{V}}$  and let  $p = q^F$ ,  $x^H$ , and  $x^F = -x^H$  be world prices and Home and Foreign net exports in any equilibrium that implements  $(V^H, V^F)$ . Note that by goods market clearing and the Foreign household's budget constraint,  $p \cdot x^H = 0$ . By Lemma 4, (a)  $(V^H, V^F)$  is also in the extremal set of  $\tilde{\mathcal{V}}$  and (b)  $V_x^F(x^F) \cdot x^H = p \cdot x^H = 0$ . So, for some  $s^H, s^F \in \{-1, 1\}$   $x^H$  solves:

$$x^H \in \arg \max_{x \in \mathbb{R}^K} s^H V^H(x) \quad \text{s.t.} \quad s^F V^F(-x) \geq s^F V^F \quad \text{and} \quad V_x^F(-x) \cdot x = 0. \quad (\text{A4})$$

In order to show that (A4) admits a first order condition in terms of Lagrange multipliers, we note that, by Lemma 2, the objective and constraints are continuously differentiable and now proceed to argue that the Jacobians of its constraints are linearly independent at  $x^H$ . First, by Lemma 3,  $V_x^i(x^i) = -\gamma^i(q^i)^T < 0$  for some  $\gamma^i > 0$ , for  $i \in \{H, F\}$ . Second  $\frac{d}{dx}(V_x^F(-x) \cdot x) = x^T V_{xx}^F(-x) + V_x^F(-x) \neq 0$ , where by Assumption 1, (a)  $x^T V_{xx}^F(-x) \neq 0$  and (b) for some  $j, k \in K$ ,  $x^T V_{xx_j}^F(-x) \leq 0$  and  $x^T V_{xx_k}^F(-x) \geq 0$ . Note that (a) and (b) imply  $x^T V_{xx}^F(-x)$  is linearly independent of  $V_x^F(-x)$  since  $V_x^F(-x) > 0$ . The two constraints' Jacobians are therefore linearly independent.

There therefore exist  $\mu^H = s^H, \mu^F, \phi \in \mathbb{R}$  such that

$$\mu^H V_x^H(x^H) - \mu^F V_x^F(-x^H) - \phi (V_x^F(-x^H) - (x^H)^T V_{xx}^F(-x^H)) = 0. \quad (\text{A5})$$

As in Footnote 9, let  $p(\cdot)$  be any differentiable function that satisfies  $-p(\tilde{x})^T \propto V_x^F(\tilde{x})$ . Note this implies that  $V_x^F(\tilde{x}) = -f(\tilde{x})p(\tilde{x})$  for some strictly positive, locally differentiable, scalar-valued function  $f$ , and  $(x^H)^T V_{xx}^F(x^F) = -f(x^F)(x^H)^T p_x(x^F) - f_x(x^F)(x^H)^T p(x^F) = -f(x^F)(x^H)^T p_x(x^F)$  since  $V_x^F(x^F) \cdot x^H = 0$ . Using  $x^F = -x^H$  and  $q_k^H = p_k^H(1 + t_k)$ , substituting, observing that  $p_k(x^F) = \gamma^F/f(x^F) \cdot p_k$ , and taking a transpose, we can rewrite (A5) as, for all  $k \in K$ ,

$$\mu^H \gamma^H(1 + t_k) = \mu^F \gamma^F + \phi \gamma^F \left( 1 + \frac{(x^F)^T p_{x_k}(x^F)}{p_k(x^F)} \right). \quad (\text{A6})$$

By Assumption 1, take  $j, k \in K$  such that  $(x^F)^T V_{xx_j}^F(x^F) \leq 0$  and  $(x^F)^T V_{xx_k}^F(x^F) \geq 0$ . Then there exist  $\alpha_j, \alpha_k \geq 0$ , not both zero, such that for  $x^* = \alpha_j \hat{e}_j + \alpha_k \hat{e}_k$ ,  $(x^F)^T V_{xx}^F(x^F) \cdot x^* = 0$ . (A5) then implies

$$\mu^H V_x^H(x^H) \cdot x^* = (\mu^F + \phi) V_x^F(x^F) \cdot x^*, \quad (\text{A7})$$

where note all multiplicative terms are non-zero, since  $\mu^H \neq 0$  and that  $V_x^i(x^i) > 0$  for  $i \in \{H, F\}$  implies  $\mu^F + \phi \neq 0$ . Plugging (A7) into (A6) and rearranging implies

$$1 + t_k = \underbrace{\frac{\gamma^F/(V_x^F(x^F) \cdot x^*)}{\gamma^H/(V_x^H(x^H) \cdot x^*)}}_{>0} \left( 1 + \left( 1 - \frac{\mu^F V_x^F(x^F) \cdot x^*}{\mu^H V_x^H(x^F) \cdot x^*} \right) \frac{(x^F)^T p_{x_k}(x^F)}{p_k(x^F)} \right). \quad (\text{A8})$$

Defining  $\lambda^F \equiv (\mu^F V_x^F(x^F) \cdot x^*)/(\mu^H V_x^H(x^F) \cdot x^*)$  completes the proof.  $\square$

# Online Appendix to “A Theory of Economic Sanctions as Terms-of-Trade Manipulation”

John Sturm

## Proof of Lemma 2

First, fixing any  $x^i \in \mathbb{R}^K$ , we argue  $V^i(x^i)$  is well defined. By Assumption 4, there exists  $d_x > 0$  such that for all  $y \notin B_{d_x}(x^i)$  satisfying  $G^i(y) \leq 0$ ,  $u^i(y - x^i) \leq \max_{\tilde{y} \in \overline{B_{d_x}(x^i)}, G^i(y - x^i) \leq 0} u^i(\tilde{y} - x^i)$ , where note that the latter exists since  $u^i$  is continuous by Assumption 2 and  $\overline{B_{d_x}(x^i)}$  is compact (here we have used  $G^i$ 's continuity, from Assumption 3).  $V^i(x^i)$  therefore has the same value as if its definition in (A2) restricts to the domain  $\overline{B_{d_x}(x^i)}$ . That restricted problem has a solution by the extreme value theorem, so  $V^i(x^i)$  is well defined. We moreover note that—since by Assumptions 2 and 3  $u^i$  is strictly concave and  $G^i$  is weakly convex—this solution is achieved by a unique level of production that we denote by  $y^i(x^i)$ .

Second, fixing any  $x^i \in \mathbb{R}^K$ , we argue  $y^i(\cdot)$  is twice continuously differentiable at  $x^i$ . To see this first note that since (A2) is a convex optimization problem with a continuously differentiable objective and constraint, there exists a Lagrange multiplier  $\lambda(x^i) > 0$  such that

$$u_c^i(y^i(x^i) - x^i) = \lambda(x^i)G_y^i(y^i(x^i)) \quad \text{and} \quad G^i(y^i(x^i)) = 0. \quad (\text{A9})$$

Here, we have used that the constraint must bind since, by Assumptions 2 and 3, there exists a good  $k \in K$  for which  $u_{c_k}^i(y^i(x^i) - x^i) > 0$  and  $G_{y_k}^i(y^i(x^i)) > 0$ . Letting  $k \in K$  be the good referred to in the second part of Assumption 4, (A9) implies

$$\frac{u_{c-k}^i(y^i(x^i) - x^i)}{u_{c_k}^i(y^i(x^i) - x^i)} - \frac{G_{y-k}^i(y^i(x^i))}{G_{y_k}^i(y^i(x^i))} = 0 \quad \text{and} \quad G^i(y^i(x^i)) = 0, \quad (\text{A10})$$

and by the second part of Assumption 4, the  $K \times K$  matrix

$$\mathcal{M}(x^i)^T = \left[ \frac{d}{dc} \left( \frac{u_{c-k}^i(y^i(x^i) - x^i)}{u_{c_k}^i(y^i(x^i) - x^i)} \right) - \frac{d}{dy} \left( \frac{G_{y-k}^i(y^i(x^i))}{G_{y_k}^i(y^i(x^i))} \right), G_y^i(y^i(x^i)) \right] \quad (\text{A11})$$

is full rank. By the implicit function theorem and—from Assumptions 2 and 3—the twice continuous differentiability of  $u^i(\cdot)$  and  $G^i(\cdot)$ ,  $y^i(x^i)$  is locally continuously differentiable and

$$y_x^i(x^i) = \mathcal{M}(x^i)^{-1} \left[ \frac{d}{dx^i} \Big|_{y^i=y^i(x^i)} \left( \frac{u_{c-k}^i(y^i - x^i)}{u_{c_k}^i(y^i - x^i)} \right), 0 \right]. \quad (\text{A12})$$

Third, fixing any  $x^i \in \mathbb{R}^K$ , we argue  $V^i(\cdot)$  is twice continuously differentiable at  $x^i$ . Fixing any  $\epsilon > 0$ , we first note that since  $y^i(x^i)$  is continuous, there exists  $\delta > 0$  such that  $y^i(\tilde{x}^i) \in \overline{B_\epsilon}(y^i(x^i))$  when  $\tilde{x}^i \in B_\delta(x^i)$ . So for  $\tilde{x}^i \in B_\delta(x^i)$ ,

$$V^i(\tilde{x}^i) = \max_{y^i \in B_\epsilon(y^i(x^i)), G^i(y^i) \leq 0} u^i(y^i - \tilde{x}^i). \quad (\text{A13})$$

Since, by Assumption 2,  $u^i(\cdot)$  is twice continuously differentiable, the function  $u^i(y^i + x^i)$  is has a bounded Jacobian in  $y^i$  on the (compact) domain of (A13) is therefore equidifferentiable. Since the feasible set in (A13) is non-empty, Theorem 3 of Milgrom and Segal (2002) implies that  $V^i(\cdot)$  is differentiable at  $x^i$ —and moreover  $V_x^i(x^i) = \frac{d}{dx^i}|_{y^i=y^i(x^i)}u^i(y^i - x^i) = -u_c^i(y^i(x^i) - x^i)$ —if and only if  $u_c^i(y^i(x^i) - x^i)$  is continuous in  $x^i$ . Indeed, this follows from that  $u_c^i(\cdot)$  and  $y^i(x^i)$  are both continuous (by Assumption 2 and shown above, respectively). Note that  $V_x^i(x^i) = -u_c^i(y^i(x^i) - x^i)$  implies  $V^i(\cdot)$  is in fact twice continuously differentiable in  $x^i$ , since  $u_c^i(\cdot)$  and  $y^i(x^i)$  are both continuously differentiable (again, by Assumption 2 and shown above, respectively).

Fourth and finally, fixing any  $x^i \in \mathbb{R}^K$ , we argue  $V_x^i(x^i) \propto -G_y^i(y^i(x^i) - x^i) \ll 0$ . We have already shown (a)  $V_x^i(x^i) = -u_c^i(y^i(x^i) - x^i)$  and (b) there exists  $\lambda > 0$  such that  $u_c^i(y^i(x^i) - x^i) = \lambda G_y^i(y^i(x^i))$ . So the desired claim follows from that, by Assumption 3,  $G_y^i(y^i(x^i)) \gg 0$ .  $\square$

### Proof of Lemma 3

By domestic market clearing, consumption in  $i$  satisfies  $c^i = y^i - x^i$ . Since, by Assumption 2,  $u^i(\cdot)$  is non-satiated, there exists at least one good  $k \in K$  with a strictly positive price  $q_k^i > 0$ . Firm optimization and the fact that, by Assumption 3,  $G^i(\cdot)$  is increasing in all goods then implies all prices are strictly positive, i.e.  $q^i \gg 0$ . Moreover since, by Assumption 3,  $G^i(\cdot)$  is continuously differentiable, convex, and increasing,  $G_y^i(y^i) \propto q^i$  and  $G^i(y^i) = 0$ . Similarly, since, by Assumption 2,  $u^i(\cdot)$  is continuously differentiable, concave, and increasing,  $u_y^i(c^i = y^i - x^i) \propto q^i$ . Since—as argued in the proof of Lemma 2—the constraint in (A2) always binds, concavity and Lagrangian sufficiency then implies that  $y^F$  solves (A2). Lemma 2 then implies that  $y^F = y^F(x^F)$ .  $\square$

### Proof of Lemma 4

First, we claim that  $\mathcal{V} \subset \tilde{\mathcal{V}}$ . To see this, take any point  $(V^H, V^F) \in \mathcal{V}$  and let  $(x^i)_{i \in \{H, F\}}$ ,  $y^F$ , and  $q^F$  be net exports, Foreign production, and Foreign prices, respectively, in any equilibrium that implements it. Since by international goods market clearing  $x^F = -x^H$ , it suffices to show that  $V_x^F(x^F) \cdot x^F = 0$ .

To see this, we first use that, by Lemma 3,  $y^F = y^F(x^F)$  for the function  $y^F(\cdot)$  described in Lemma 2, and  $q^F \propto G_y^F(y^F)$ . So Lemma 2 implies  $V^F(x^F) \propto -G_y^F(y^F)$ . The fact that  $V_x^F(x^F) \cdot x^F = 0$  then follows from (a)  $q^F \propto G_y^F(y^F)$  and (b) by domestic market clearing and the household budget constraint,  $q^F \cdot x^F = q^F \cdot (y^F - c^F) = \pi^F - q^F \cdot c^F = 0$ .<sup>24</sup>

Second, we claim that  $\tilde{\mathcal{V}} \subset \mathcal{V}$ . To see this, take any  $(\tilde{V}^H, \tilde{V}^F) \in \tilde{\mathcal{V}}$  and let  $x \in \mathbb{R}^K$  be the corresponding vector satisfying  $V^H(x) = \tilde{V}^H$ ,  $V^H(x) = \tilde{V}^H$ , and  $V_x^F(-x) \cdot x = 0$ . It suffices to construct an equilibrium  $(p, (q^i, c^i, y^i, x^i, V^i, \pi^i, T^i)_{i \in \{H, F\}})$  with  $V^i = \tilde{V}^i$  for  $i \in \{H, F\}$ . To this end, we define, for all  $i \in \{H, F\}$ ,

$$\begin{aligned} x^H &= x, & x^F &= -x^H, & p &= q^F = -V_x^F(x^F), & q^H &= -V_x^H(x^H), & V^i &= \tilde{V}^i \\ T^i &= (p - q^i) \cdot x^i, & y^i &= y^i(x^i), & c^i &= y^i - x^i & \pi^i &= q^i \cdot y^i. \end{aligned} \quad (\text{A14})$$

where  $y^i(x^i)$  is as in Lemma 2. It remains to verify that (A14) satisfies equilibrium conditions

<sup>24</sup>Here I have used that, by Assumption 2, households are non-satiated.

(1)–(5) at trade taxes  $t = q^H - p$ . Of these, all of these except for the optimality of (a) production and (b) consumption are immediate from our construction. (a) Since  $G^i$  is concave by Assumption 3,  $q^i \gg 0$  by Lemma 2, and  $G_y^i(y^i) \gg 0$  by Assumption 3, it suffices for production optimality to show that  $G^i(y^i) = 0$  and  $q^i \propto G_y^i(y^i)$ . The former follows from the definition of  $V^i(x^i)$  in (A2), given that, by Assumptions 2 and 3, consumption utility is increasing in some good that is produceable on the margin; the latter follows from that, by construction and Lemma 2,  $q^i = -V_x^i(x^i) = \kappa^i G_y^i(y^i)$  for some  $\kappa^i > 0$ . (b) Since, by Assumption 2,  $u^i$  is non-satiated and concave, it suffices for consumption optimality to show that  $u_c^i(c^i) \propto q^i$  and that  $q^i \cdot c^i = \pi^i + T^i$ . The former follows from Lemma 2 and  $q^i$ 's definition. The latter follows from the definition of  $T^i$  and the fact that  $p \cdot x^i = V_x^F(x^F) \cdot x^i = 0$ .  $\square$