

Goods Trade and Capital Investments in the Global Economy*

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We develop a new theoretical framework for modelling this interaction between goods trade and capital investments. Our framework rationalizes key features of the observed data. We allow for a large number of asymmetric countries connected by a network of trade and capital investments. We provide microfoundations for gravity equations for trade and capital investments. We use our framework to examine the following questions: Are capital investments complements or substitutes for goods trade? How much bigger are the gains from globalization when trade integration is combined with international capital liberalization? How is the impact of China's economic growth on its East Asian neighbors altered when capital is free to move across borders as well as goods? How much larger are the costs of Brexit for the United Kingdom when capital is free to reallocate internationally? How does this reallocation of capital affect the distributional consequences of trade disintegration? How much greater are the costs of international sanctions for targeting and targeted countries when restrictions on capital investments are combined with barriers to trade in goods? What are the global implications of a decoupling between China and the United States?

Keywords: Globalization, International Trade, Capital Investments

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1 Introduction

A key question at the heart of international economics is how do goods trade and capital investments interact with one another? This question is central to thinking about a whole host of issues. Are capital investments complements or substitutes for goods trade? How much bigger (or smaller) are the gains from globalization when trade integration is combined with international capital liberalization? How is the impact of China's economic growth on its East Asian neighbors altered when capital is free to move across borders as well as goods? How much larger (or smaller) are the costs of Brexit for the United Kingdom when capital is free to reallocate internationally? How does this reallocation of capital affect the distributional consequences of trade disintegration? How much greater are the costs of international sanctions for targeting and targeted countries when restrictions on capital investments are combined with barriers to trade in goods? What are the global implications of a decoupling between China and the United States?

We develop a new theoretical framework for modelling this interaction between goods trade and capital investments. Our framework accommodates a large number of countries that can differ from one another in size, productivity, and bilateral trade and capital market frictions. Despite the resulting rich asymmetries between countries and the high-dimensional state space, the model remains tractable and amenable analytical analysis. We show how to undertake transparent counterfactuals that depend on only a small number of structural parameters. Our framework is consistent with a number of key features of the observed data on trade and capital investments. First, we incorporate intra-temporal trade in goods in a way that is consistent with the observed gravity equation for bilateral international trade. Second, we include intra-temporal capital mobility, such that a gravity equation holds for bilateral international capital investments, as again observed empirically. Third, we allow for intertemporal trade through consumption-saving decisions, which is consistent with observed bilateral and multilateral current account imbalances across countries. Since our framework incorporates capital accumulation, trade and capital market integration not only affect the level of income per capita but also its rate of growth along the transition path to steady-state. We develop a many-country open-economy Ramsey model, which features goods trade, capital investments, and growth along the transition path to steady-state.

We use our theoretical framework to derive three main sets of results. First, we provide tractable microfoundations for the gravity equation in bilateral international capital investments. We show that this framework can be rationalized in terms of either idiosyncratic shocks to the productivity of investments or portfolio diversification. These two alternative microfoundations are isomorphic in terms of their predictions for bilateral international capital investments. Second, we derive sufficient statistics for the welfare gains from both international trade and international capital investments. In general, we show that these two sources of welfare gains interact

with one another, such the whole differs systematically from the sum of the parts. Third, we analyze how the incidence of productivity and trade costs shocks depends on both international trade and international capital linkages, using exact-hat algebra counterfactuals for the full non-linear model. Fourth, we derive analogous sufficient statistics for the first-order impact of productivity and trade cost shocks, and use these first-order sufficient statistics to understand the mechanisms through which international trade and international capital linkages interact with one another.

Our paper is related to a number of different strands of research in international economics. First, we build on research on quantitative trade models and sufficient statistics in international trade, including [Eaton and Kortum \(2002\)](#) and [Arkolakis et al. \(2012\)](#). Building on the matrix representation of comparative statics for productivity shocks and trade costs in the setting with international trade in goods in [Kleinman et al. \(2020\)](#) and [Kleinman et al. \(2021\)](#), we show how these comparative statics are fundamentally altered by the introduction of international capital investments. Our work also relates to the traditional literature in international trade on goods movements as an alternative to factor movements, including [Mundell \(1957\)](#) and [Markusen \(1983\)](#).

Second, we connect with wide research on the role of goods and financial markets in the international transmission of shocks, including [Jin \(2012\)](#), [Huo et al. \(2019\)](#), [Pellegrino et al. \(2021\)](#), [Jiang et al. \(2022\)](#). Third, our work relates to research on home bias and international risk diversification, including [Obstfeld \(1994\)](#), [Cole and Obstfeld \(1991\)](#), [Martin and Rey \(2004\)](#), [Mendoza et al. \(2009\)](#), [Chau \(2022\)](#), [Hu \(2022\)](#) and [Kucheryavyi \(2022\)](#). In particular, we build on research on international asset demand systems following [Kojen and Yogo \(2019\)](#) and [Kojen and Yogo \(2020\)](#). Fourth, our work relates to research on the intertemporal approach to the current account including [Obstfeld and Rogoff \(1996\)](#), [Reyes-Heroles \(2016\)](#), [Eaton et al. \(2016\)](#), and [Ju et al. \(2014\)](#).

Finally, our findings are related to the empirical research on the gravity equation. A large literature finds evidence of a gravity equation for international trade, as surveyed in [Head and Mayer \(2014\)](#). Although one might expect geographical distance to be less important for financial transactions than for physical shipments, international capital investments are also well described by a gravity, as found in [Portes and Rey \(2005\)](#). A wider body of research has explored the determinants of net and gross international capital investments between countries, including [Lucas \(1990\)](#), [Ohanian et al. \(2018\)](#), [Davis et al. \(2021\)](#), [Coppola et al. \(2021\)](#), and [Ohanian et al. \(2022\)](#).

The remainder of the paper is structured as follows. Section 2 reviews some key features of observed trade and capital investments data that are captured by our theoretical framework. Section 3 develops our theoretical framework. Section 4 summarizes our main quantitative results. Section 5 summarizes our conclusions.

2 Motivating Evidence

We begin in Subsection 2.1 by introducing our data sources and definitions. Subsection 2.2 shows that the gravity equation provides a good approximation not only to observed bilateral goods trade but also to observed bilateral capital investments. Subsection 2.3 shows that there are substantial two-way flows for both international trade and capital investments. Each of these features of the data emerges naturally from our theoretical framework developed below.

2.1 Data

We combine several sources of data on international trade and international capital investments.

COMTRADE Database COMTRADE reports values of bilateral trade between countries. Following [Feenstra et al. \(2005\)](#), we use the trade flows reported by the importing country whenever they are available, but use the corresponding exporter's report if the importer report is not available for a country pair. We augment these trade data with information on the bilateral distance between countries from the GEODIST dataset from CEPII. We use the bilateral distances between countries' largest cities weighted by the population of those cities as our baseline distance measure. We also use data on gross domestic product (GDP) from the Global Debt Database (converted to current price dollars). We measure expenditure on domestic goods as GDP minus total imports plus total exports.

Coordinated Portfolio Investment Survey Our measure for international capital investments comes from the Coordinated Portfolio Investment Survey (CPIS), which contains voluntary reports on international holdings of portfolio investment assets in the form of equity and investment fund shares, long-term debt securities, and short-term debt securities ([Josyula, 2018](#)). To construct total domestic holdings of equity, we subtract the equity held by foreign investors from total equity and add the equity investments in foreign nations. We measure total equity by total market capitalization of listed domestic companies, as reported by the World Bank. Likewise, we construct domestic debt holdings by subtracting foreign holdings of domestic debt securities from the nation's total debt and adding investment in foreign debt assets. We measure total debt from the debt-to-GDP ratio reported in the Global Debt Database.

Global Debt Database We obtain our measure of total debt from the Global Debt Database (GDD). This database covers the debt of nonfinancial sector -- both private and public -- for 190 countries dating back to the 1950s ([Mbaye et al., 2018](#)). It reports various measures of private and public debt as well as nominal GDP series. We use the broadest definition of debt: We include the

Table 1: Gravity Equation Regressions

	(1)	(2)	(3)	(4)
	Log		Log	
	Trade	Trade	Capital	Capital
	2012	2012	2012	2012
Log Distance	-1.053	-0.876	-1.426	-0.930
	(0.0844)	(0.0664)	(0.137)	(0.132)
Estimation	OLS	PPML	OLS	PPML
Origin FEs	Yes	Yes	Yes	Yes
Destination FEs	Yes	Yes	Yes	Yes
Observations	2103	2112	2112	2112
R-squared	0.849		0.827	
Pseudo R-squared		0.897		0.859

Note: Cross-section of origin and destination countries in 2012; all columns include origin and destination fixed effects (FEs); Columns (1)-(2) show results for bilateral trade; Columns (3)-(4) report results for bilateral capital investments; Columns (1) and (3) estimated in logs using ordinary least squares (OLS); Columns (2) and (4) estimated using the Poisson Pseudo Maximum Likelihood (PPML) estimator; standard errors clustered by origin and destination.

debt carried by the private sector, including households and nonfinancial corporations, and the public debt from the general public sector. We multiply the reported total debt-to-GDP ratio by nominal GDP in current prices to obtain the level of debt in current prices.

2.2 Gravity Equations for Trade and Investment

We begin by estimating gravity equations for both bilateral trade and capital investments. We consider the following gravity equation specification:

$$X_{ni} = \eta_n \mu_i \text{dist}_{ni}^\delta u_{ni}, \quad (1)$$

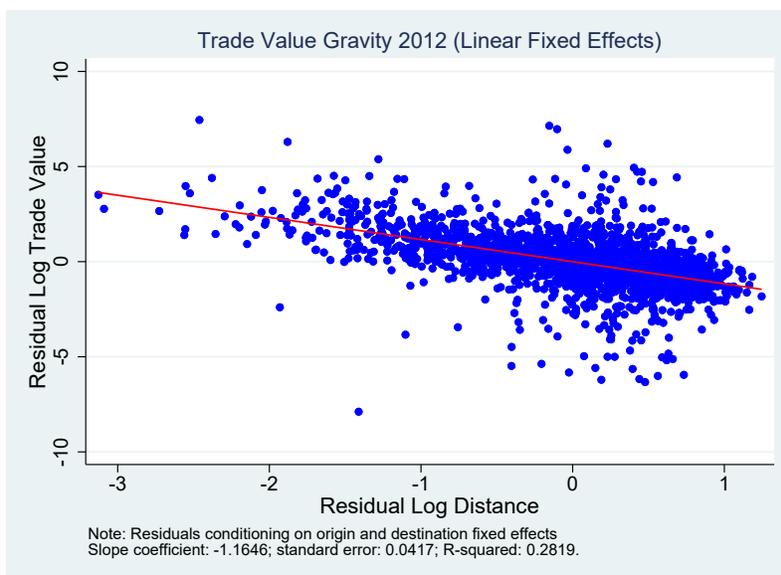
where X_{ni} is the trade or capital flow from origin i to destination n ; η_n is a destination fixed effect; μ_i is an origin fixed effect; dist_{ni} is bilateral distance; and u_{ni} is a stochastic error. We report standard errors clustered by both origin and destination.

In Column (1) of Table 1, we report the results of taking logs in equation (1) and estimating this gravity equation for international trade using ordinary least squares (OLS) with origin and destination fixed effects. In line with existing evidence, we find a negative and highly significant relationship between bilateral trade and distance, with an elasticity of around minus one, and a regression R-squared of close to 85 percent. We next show that these findings are not sensitive to the dropping of zeros when we take logs. In Column (2), we demonstrate the same pattern of results if we estimate this gravity equation in levels using the Poisson Pseudo Maximum Likelihood (PPML) estimator, as in Santos Silva and Tenreyro (2006) and Head and Mayer (2014). Again

we find a negative and highly statistically significant coefficient on bilateral distance that is only marginally smaller than that in Column (1).

In Column (3), we estimate this same gravity equation for international capital investments. Although capital investments are not subject to transportation costs in the way that goods trade is, we again find a negative and highly statistically significant coefficient on distance, and a regression R-squared of around 80 percent. Indeed, the estimated elasticity for capital investments is if anything larger in absolute magnitude than for goods trade. In Column (4), we show that we find the same pattern of results using the Poisson Pseudo Maximum Likelihood (PPML) estimator.

Figure 1: Conditional Correlation Between Bilateral Trade and Bilateral Distance



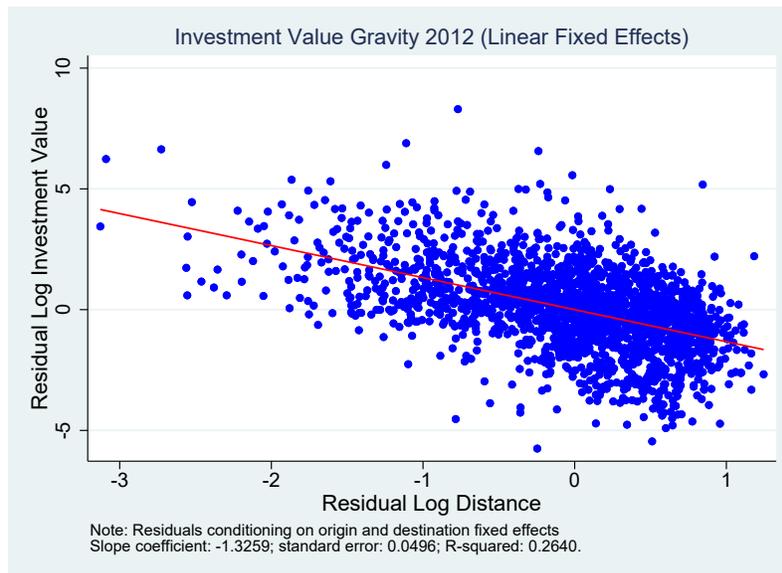
Note: Cross-section of origin and destination countries in 2012; residual log trade and residual log distance are residuals from OLS regressions of log trade and log distance on origin and destination fixed effects, respectively; Blue dots correspond to origin-destination pairs; Red solid line shows the linear fit between these two residuals.

While Table 1 provides overall evidence on the explanatory power of the gravity equation for both of these international transactions, it does not reveal the relative importance of bilateral distance and the fixed effects for this explanatory power. To separate out the explanatory power of bilateral distance, we use the Frisch-Waugh-Lovell Theorem. We first run two separate OLS regressions of log values and log distance on origin and destination fixed effects, and generate the residuals. We next regress these two residuals against one another. In Figures 1 and 2, we display these conditional correlations between bilateral values and distance, for goods trade and capital investments respectively. In both cases, we find negative and highly statistically significant relationships, with a regression R-squared for the conditional correlation of around 0.25. Therefore, even after removing the origin and destination fixed effects, we find that bilateral distance has as much explanatory power for capital investments as for goods trade.

The theoretical framework that we develop below jointly rationalizes these gravity equation

relationships for both goods trade and capital investments. This framework also highlights that these two gravity equation relationships interact systematically with one another to affect both welfare and the counterfactual impact of changes in productivities and trade policies.

Figure 2: Conditional Correlation Between Bilateral Capital Investments and Bilateral Distance



Note: Cross-section of origin and destination countries in 2012; residual log capital investments and residual log distance are residuals from OLS regressions of log capital investments and distance on origin and destination fixed effects, respectively; Blue dots correspond to origin-destination pairs; Red solid line shows the linear fit between these two residuals.

2.3 Two-Way Trade and Investments

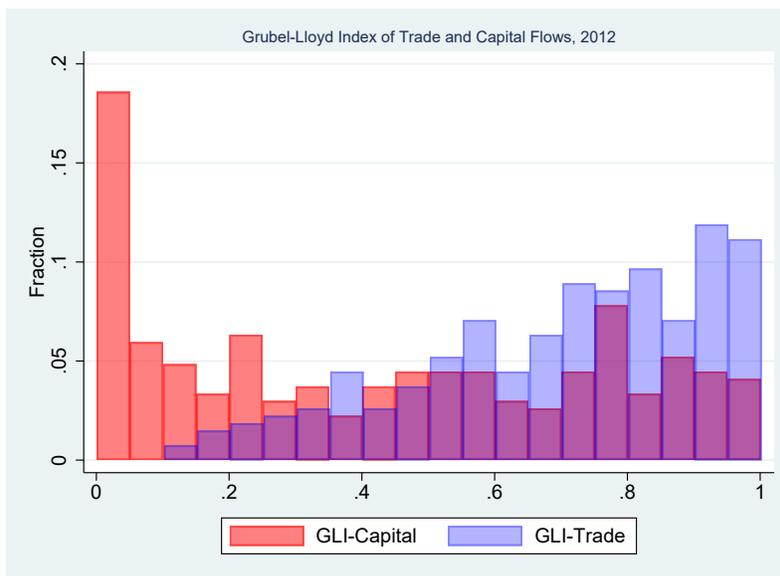
We next provide evidence on the extent to which these bilateral goods trade and capital investments are two-way versus one-way. In particular, we compute the following Grubel-Lloyd index for bilateral values (X_{ni}) between each origin i and destination n :

$$GLI_{ni} = \frac{X_{ni} + X_{in} - |X_{ni} - X_{in}|}{X_{ni} + X_{in}}. \quad (2)$$

This Grubel-Lloyd index is bounded between zero and one, with larger values implying more two-way interactions (Grubel and Lloyd 1971). In the limiting case in which bilateral values are perfectly balanced ($X_{ni} = X_{in}$), this index is equal to one. In special case in which bilateral values are perfectly imbalanced ($X_{ni} > X_{in} = 0$ or $X_{in} > X_{ni} = 0$), it takes the value zero.

Figure 3 displays a histogram of the distribution of values of the Grubel-Lloyd index across each bilateral origin-destination pair in 2012 for both goods trade and capital investments. We find the distribution for goods trade is shifted to the right relative to that for capital investments. Therefore, goods trade is more two-way than capital investments. Nevertheless, we observe many

Figure 3: Grubel-Lloyd Index of Two-way Versus One-Way Interactions



Note: histogram of Grubel-Lloyd indexes for origin-destination pairs of countries, as defined in equation (2) in the main text; values for goods trade in blue; values for capital investments in red; a Grubel-Lloyd index of 1 corresponds to perfectly balanced values in both directions ($X_{ni} = X_{in}$); a Grubel-Lloyd index of 0 corresponds to perfectly imbalanced values ($X_{ni} > X_{in} = 0$ or $X_{in} > X_{ni} = 0$).

bilateral pairs with substantial two-way capital investments, with a substantial mass of the distribution for capital closer to perfectly balanced (one) than perfectly imbalanced (zero).

Our theoretical framework below yields sharp predictions for both gross and net values of goods trade and capital investments. Furthermore, this framework implies that bilateral trade deficits and bilateral capital investments are systematically related to one another. Other things equal, a foreign country that is an important trade partner for the home country is also an attractive investment destination, consistent with the positive correlation between these two bilateral interactions in the data.

3 Theoretical Framework

We consider an economy that consists of many countries indexed by $n \in \{1, \dots, N\}$. Time is discrete and is indexed by $t \in \{1, \dots, \infty\}$. Each country supplies a differentiated good that is produced using labor and capital under constant returns to scale. Markets are perfectly competitive. The representative agent in each country is endowed with a mass ℓ_n of labor.

At the beginning of each period t , the representative agent in each country n inherits a stock of wealth (k_{nt}) that is measured (and can be accumulated) in terms of its own consumption bundle. Wealth (k_{nt}) is the aggregation of the capital invested in each country (k_{nit}). Investments in each country are subject to idiosyncratic productivities and capital market frictions. At the beginning

of period t , the representative agent chooses the investment allocation across countries. At the beginning of period $t + 1$, investment returns are realized, depreciation occurs, and investment is again allocated across countries. We assume that agents have perfect foresight for all aggregate variables.

3.1 Intertemporal Problem

The representative consumer in each country chooses current consumption and investments in each location to maximize their intertemporal utility. We assume that intertemporal utility take the constant relative risk aversion (CRRA) form:

$$u_{nt} = \sum_{s=0}^{\infty} \beta^{t+s} \left(\prod_{u=0}^s \phi_{nt+u} \right) \frac{c_{nt+s}^{1-1/\psi}}{1-1/\psi} \quad (3)$$

where β is the discount rate; ϕ_{nt+s} is a discount factor shock at time $t + s$ that introduces a wedge into consumer Euler equations and helps to match the fluctuations in trade imbalances observed in the data; c_{nt} is a consumption index that depends on the consumption of the goods produced by each country; and ψ is the intertemporal elasticity of substitution.

The representative consumer's period-by-period budget constraint requires that the value of consumption in period t plus the value of the future capital stock in period $t + 1$ is equal to capital income from period t net of depreciation plus labor income:

$$\text{s.t. } p_{nt}c_{nt} + p_{nt} \sum_{i=1}^N k_{nit+1} = (p_{nt}(1 - \delta) + v_{nt}) \sum_{i=1}^N k_{nit} + w_{nt}\ell_n, \quad (4)$$

where p_{nt} is the price index dual to the consumption index; δ is the rate of depreciation; v_{nt} is the realized return to capital, which in equilibrium is the same across investment locations i ($v_{nit} = v_{nt}$); and w_{nt} is the wage.

Given an investment of a unit of the consumption bundle at the beginning of period $t - 1$, the representative consumer receives $(1 - \delta)$ units of the consumption bundle back at the beginning of period t and a return from the investment of v_{nt} units of the numeraire. Therefore, the gross nominal return from the investment made at the beginning of period $t - 1$ is:

$$R_{nt}^{nom} = \frac{p_{nt}(1 - \delta) + v_{nt}}{p_{nt-1}}. \quad (5)$$

Dividing through by the rate of inflation, the corresponding gross real return to the investment is:

$$R_{nt} = \frac{R_{nt}^{nom}}{p_{nt}/p_{nt-1}} = 1 - \delta + \frac{v_{nt}}{p_{nt}}. \quad (6)$$

Denoting country n 's total capital wealth at period t by $k_{nt} \equiv \sum_{i=1}^N k_{nit}$, we can re-write the period-by-period budget constraint (4) as:

$$c_{nt} + k_{nt+1} = R_{nt}k_{nt} + \frac{w_{nt}\ell_n}{p_{nt}}. \quad (7)$$

Using this representation, the consumer's utility maximization problem can be solved in two stages. First, the consumer chooses how much to consume this period and how much to invest for the next period. Second, the consumer chooses how much of this overall investment to allocate to each country. From equations (3) and (7), the first of these two decisions for consumption-saving takes the same form as in Angeletos (2007). Therefore, optimal consumption and saving are linear functions of current period wealth:

$$c_{nt} = \varsigma_{nt} \left(R_{nt}k_{nt} + \frac{w_{nt}\ell_n}{p_{nt}} + h_{nt} \right), \quad (8)$$

where $h_{nt} \equiv \sum_{s=1}^{\infty} \frac{w_{nt+s}\ell_{nt+s}/p_{nt+s}}{\prod_{u=1}^s R_{nt+u}}$ is the present discounted value of labor income measured in consumption units, and the saving rate $(1 - \varsigma_{nt})$ is defined recursively as:

$$\varsigma_{nt}^{-1} = 1 + \beta^\psi \phi_{nt+1}^\psi R_{nt+1}^{\psi-1} \varsigma_{nt+1}^{-1}, \quad (9)$$

as shown in Section A.4 of the online appendix.

3.2 Intra-temporal Capital Allocation

We now turn to the second capital allocation decision. We assume that each unit of capital is subject to an idiosyncratic productivity shock for each of the possible countries i to which it can be allocated (α_{nit}). Investments also face capital market frictions, such that $\kappa_{nit} \geq 1$ units of capital from source country n must be allocated to host country i in order for one unit to be available for production, where $\kappa_{nnt} = 1$ and $\kappa_{nit} > 1$ for $n \neq i$.

Therefore, each unit of capital allocated from source n to host i becomes $\alpha_{nit}/\kappa_{nit}$ efficiency units that can be used for production, where each efficiency unit earns a rental rate r_{it} . The realized rate of return in country n from allocating one unit of capital to country i is thus:

$$v_{nit} = \frac{\alpha_{nit}r_{it}}{\kappa_{nit}}. \quad (10)$$

We assume that these idiosyncratic shocks to the productivity of capital are drawn independently across source and host countries from the following Fréchet distribution:

$$F_{nit}(\alpha) = e^{-\alpha^{-\epsilon}}, \quad \epsilon > 1, \quad (11)$$

where we normalize the scale parameter to one, because it enters the model isomorphically with financial frictions (κ_{nit}). The shape parameter (ϵ) controls the dispersion of idiosyncratic productivity shocks, and regulates the sensitivity of the capital allocation to rates of return relative to idiosyncratic productivity shocks.

A first key implication of our extreme value specification for idiosyncratic productivity is that the share of capital from source country n that is allocated to host country i satisfies the following gravity equation:

$$b_{nit} = \frac{k_{nit}}{k_{nt}} = \frac{(r_{it}/\kappa_{nit})^\epsilon}{\sum_{h=1}^N (r_{ht}/\kappa_{nht})^\epsilon}. \quad (12)$$

Therefore, bilateral capital investments (k_{nit}) are decreasing in bilateral capital market frictions (κ_{nit}), as determined for example by bilateral distance. But these bilateral capital investments (k_{nit}) also depend on capital market frictions with other locations (“multilateral resistance”), as captured by the term in the denominator. This specification rationalizes a number of observed features of international capital investments, as discussed for example in [Obstfeld and Rogoff \(2000\)](#). First, it is consistent with the evidence above and other empirical findings that international capital investments are well approximated by a gravity equation (e.g., [Portes and Rey 2005](#)). Second, it is in line with empirical findings of home bias in international capital investments (e.g., [French and Poterba 1991](#)), because capital market frictions abroad are greater than those at home ($\kappa_{nit} > \kappa_{nnt}$ for $n \neq i$). Third, it provides a natural explanation for empirical findings of limited capital flows from rich to poor countries (e.g., [Lucas 1990](#)), because even if poor countries offer higher rental rates (higher r_{it}), they may have higher capital market frictions (higher κ_{nit}). We refer to the parameter ϵ as the capital investment elasticity, because it determines the elasticity of these investments to capital market frictions, and plays a similar role in capital markets as the trade elasticity (θ) does in goods markets.

A second key implication of our extreme value distribution for idiosyncratic productivity is that the expected return to capital owned by source country n is the same across all host countries i and given by:

$$v_{nit} = v_{nt} = \gamma \left[\sum_{h=1}^N (r_{ht}/\kappa_{nht})^\epsilon \right]^{\frac{1}{\epsilon}}, \quad \gamma \equiv \Gamma \left(\frac{\epsilon - 1}{\epsilon} \right), \quad (13)$$

where $\Gamma(\cdot)$ is the Gamma function.

Intuitively, host countries can differ in terms of the rental rate for capital (r_{it}). But host countries with higher rental rates for capital (r_{it}) attract investments with lower realizations for idiosyncratic productivity (α_{nit}), such that the expected return to capital conditional on investing in a host country is the same across all possible host countries ($v_{nit} = v_{nt}$ for all i). With a continuous measure of units of capital, this common expected return equals the realized return for each

host country. Along the transition path to steady-state, this expected return to capital can differ across source countries ($v_{nt} \neq v_{it}$), if some source countries have better access to investments in host countries ($\kappa_{nht} \neq \kappa_{iht}$ for some h). In steady-state equilibrium, we show below that the expected return to capital is also equalized across source countries.

Finally, we can solve explicitly for the average productivity of capital from source country n in host country i conditional on investment occurring ($\bar{\alpha}_{nit}$), which is monotonically decreasing in the share of capital from source country n invested in host country i (b_{nit}):

$$\bar{\alpha}_{nit} = \gamma b_{nit}^{-\frac{1}{\epsilon}}, \quad (14)$$

Therefore, each source country faces a downward-sloping marginal efficiency of investment schedule, as in Keynes (1935), such that it experiences diminishing marginal returns from allocating a larger share of its investments to a given host country, where the rate of diminishing returns is determined by the dispersion of idiosyncratic productivity shocks (ϵ).

Using the above expression for the average productivity of capital in equation (14), capital payments can be either written in terms of productivity-adjusted of effective units of capital (\tilde{k}_{it}) or in terms of capital investments (k_{nit}):

$$r_{it}\tilde{k}_{it} = \sum_{n=1}^N v_{nt}k_{nit}, \quad \tilde{k}_{it} = \sum_{n=1}^N \bar{\alpha}_{nit}k_{nit}. \quad (15)$$

3.3 Consumption, Production and Trade

The consumption index takes the same form as in the Armington model of international trade and is defined over consumption of the varieties produced by each country i (c_{nit}):

$$c_{nt} = \left[\sum_{i=1}^N (c_{nit})^{\frac{\theta}{\theta+1}} \right]^{\frac{\theta+1}{\theta}}, \quad \theta = \sigma - 1, \quad \sigma > 1, \quad (16)$$

where $\theta = \sigma - 1$ is the trade elasticity and $\sigma > 1$ is the elasticity of substitution between varieties.

Using the properties of CES demand, the share of importer n 's expenditure on exporter i takes the conventional form:

$$s_{nit} = \frac{p_{nit}^{-\theta}}{\sum_{h=1}^N p_{nht}^{-\theta}}. \quad (17)$$

Each country's variety is produced using labor and capital according to a constant returns to scale production technology. Production occurs under conditions of perfect competition. Varieties can be traded between locations subject to iceberg variable trade costs, where $\tau_{nit} \geq 1$ units of a variety must be shipped from country i in order for one unit to arrive in country n , where $\tau_{nnt} = 1$ and $\tau_{nit} > 1$ for $n \neq i$.

Profit maximization and zero profits imply that the price to a consumer in country n of sourcing the variety supplied by country i is given by:

$$p_{nit} = \frac{\tau_{nit} w_{it}^\lambda r_{it}^{1-\lambda}}{z_{it}}, \quad 0 < \lambda < 1, \quad (18)$$

where w_{it} is the wage; r_{it} corresponds to the rental rate per effective unit of capital; and z_{it} denotes country productivity.

Substituting the equilibrium pricing rule (18) into the CES expenditure share (17), the model is also consistent with empirical findings that international trade goods is well approximated by a gravity equation. The price index (p_{nt}) dual to the consumption index (c_{nt}) is given by:

$$p_{nt} = \left[\sum_{i=1}^N p_{nit}^{-\theta} \right]^{-\frac{1}{\theta}}. \quad (19)$$

Applying Shephard's Lemma to the unit cost function, total payments for the capital used in country i are proportional to the total wagebill in that country:

$$\sum_{n=1}^N v_{nt} k_{nit} = r_{it} \tilde{k}_{it} = \frac{1-\lambda}{\lambda} w_{it} \ell_{it}. \quad (20)$$

3.4 Market Clearing

Goods market clearing requires the country i 's income, and hence its factor payments, are equal to expenditure on the goods produced by it:

$$\left(w_{it} \ell_{it} + \sum_{h=1}^N v_{iht} k_{iht} \right) = \sum_{n=1}^N s_{nit} [p_{nt} c_{nt} + p_{nt} k_{nt+1} - p_{nt} (1-\delta) k_{nt}]. \quad (21)$$

Using the period-by-period budget constraint (4) and our expression for factor payments in equation (20) above, we can rewrite this equality between income and expenditure as follows:

$$w_{it} \ell_{it} = \lambda \sum_{n=1}^N s_{nit} [v_{nt} k_{nt} + w_{nt} \ell_n]. \quad (22)$$

We choose world GDP as our numeraire, such that:

$$\begin{aligned} 1 &= \sum_{i=1}^N \left(w_{it} \ell_i + \sum_{n=1}^N v_{nit} k_{nit} \right), \\ &= \frac{1}{\lambda} \sum_{i=1}^N w_{it} \ell_i. \end{aligned} \quad (23)$$

3.5 Balance of Payments

We now characterize the relationship between each country's trade balance and the change in its international asset position. The financial account (FA_{it}) is defined as the increase in foreign assets in country i minus the increase in country i 's assets abroad:

$$FA_{it} = \underbrace{\left(\sum_{n=1}^N p_{nt} k_{nit+1} - \sum_{n=1}^N p_{nt} k_{nit} \right)}_{\text{Increase in foreign assets in country } i} - \underbrace{(p_{it} k_{it+1} - p_{it} k_{it})}_{\text{Increase in country } i\text{'s assets abroad}}. \quad (24)$$

Trade balance (TB_{it}) corresponds to the difference between the value of goods produced in a country and the value of goods used in that country:

$$TB_{it} = \underbrace{w_{it} \ell_i + \sum_{n=1}^N v_{nt} k_{nit}}_{\text{Value of goods produced}} - \underbrace{\left(p_{it} c_{it} + \sum_{n=1}^N p_{nt} k_{nit+1} - (1 - \delta) \sum_{n=1}^N p_{nt} k_{nit} \right)}_{\text{Value of goods used in the country}}. \quad (25)$$

Net investment income (NII_{it}) is the difference between income receipts from assets owned by country i minus income payments on foreign-owned assets used in country i :

$$NII_{it} = \underbrace{(R_{it}^{Nom} - 1) p_{it-1} k_{it}}_{\text{Income receipts from assets owned}} - \underbrace{\sum_{n=1}^N (R_{nt}^{Nom} - 1) p_{it-1} k_{nit}}_{\text{Income payments to foreign-owned assets}}. \quad (26)$$

Combining these definitions in equations (24)-(26), we confirm that the conventional balance of payments accounting identity holds:

$$CA_{it} = TB_{it} + NII_{it} = -FA_{it}. \quad (27)$$

3.6 General Equilibrium

Given the state variables $\{k_{nt}\}_{n=1}^N$, the equilibrium endogenous variables in the static trade and capital allocation bloc of the model $\{w_{nt}, r_{nt}, s_{nt}, v_{nt}, b_{nt}\}_{n=1}^N$ are determined as the solution to the following system of equations:

$$s_{nit} = \frac{(\tau_{nit} w_{it}^\lambda r_{it}^{1-\lambda} / z_{it})^{-\theta}}{\sum_h (\tau_{nht} w_{ht}^\lambda r_{ht}^{1-\lambda} / z_{ht})^{-\theta}}, \quad (28)$$

$$w_{it} \ell_i = \lambda \sum_{n=1}^N s_{nit} (v_{nt} k_{nt} + w_{nt} \ell_n), \quad (29)$$

$$b_{nit} = \frac{(r_{it} / \kappa_{nit})^\epsilon}{\sum_h (r_{ht} / \kappa_{nht})^\epsilon}, \quad (30)$$

$$v_{nt} = \gamma \left[\sum_{h=1}^N (r_{ht}/\kappa_{nht})^\epsilon \right]^{1/\epsilon}, \quad (31)$$

$$\sum_n v_{nt} b_{nit} k_{nt} = \frac{1-\lambda}{\lambda} w_{it} \ell_i, \quad (32)$$

along with the choice of numeraire:

$$\frac{1}{\lambda} \sum_i w_{it} \ell_i = 1. \quad (33)$$

The evolution of the state variables $\{k_{nt}\}_{n=1}^N$ over time is determined by optimal consumption-saving decisions according the following dynamic bloc of equations:

$$k_{nt+1} = (1 - \varsigma_{nt}) \left(R_{nt} k_{nt} + \frac{w_{nt} \ell_n}{p_{nt}} + h_{nt} \right) - h_{nt}, \quad (34)$$

$$h_{nt} \equiv \sum_{s=1}^{\infty} \frac{w_{nt+s} \ell_{nt+s} / p_{nt+s}}{\prod_{u=1}^s R_{nt+u}}, \quad (35)$$

$$p_{nt} \equiv \left[\sum_i (\tau_{nit} w_{it}^\lambda r_{it}^{1-\lambda} / z_{it})^{-\theta} \right]^{-1/\theta}, \quad (36)$$

where ς_{nt} is defined recursively as

$$\varsigma_{nt}^{-1} = 1 + \beta^\psi \phi_{nt+1}^\psi R_{nt+1}^{\psi-1} \varsigma_{nt+1}^{-1}. \quad (37)$$

3.7 Steady-state Equilibrium

The steady-state equilibrium of the model is characterized by time-invariant values of the state variables $\{k_n^*\}_{n=1}^N$ and the other endogenous variables of the model $\{w_n^*, r_n^*, s_{ni}^*, v_{nt}^*, b_{ni}^*\}_{n=1}^N$, given time-invariant values of country fundamentals $\{\ell_n, z_n\}_{n=1}^N$ and $\{\tau_{ni}, \kappa_{ni}\}_{n,i=1}^N$, where we denote the steady-state values of variables by an asterisk.

Given constant population in each country (ℓ_n), diminishing marginal returns to capital accumulation in the production technology implies a steady-state capital stock (k_n^*) and capital-labor (k_n^*/ℓ_n), as in the traditional Solow-Swan Model. Unlike the Solow-Swan model, the saving rate here is endogenously determined as the solution to a forward-looking consumption-saving problem. As a result, the steady-state gross real return to capital (R_n^*) and the steady-state saving rate (ς_n^*) are inversely related to discount factor (β):

$$R_n^* = \frac{1}{\beta}, \quad (38)$$

$$\varsigma_n^* = 1 - \beta. \quad (39)$$

This common steady-state value of the gross real return to capital (R_n^*) implies that the steady-state realized real return to capital (v_n^*/p_n^*) is the same across all countries:

$$\frac{v_n^*}{p_n^*} = \beta^{-1} - 1 + \delta. \quad (40)$$

Given time-invariant country fundamentals $\{\ell_n, z_n\}_{n=1}^N$ and $\{\tau_{ni}, \kappa_{ni}\}_{n,i=1}^N$, and the steady-state capital stocks $\{k_n^*\}_{n=1}^N$, we can represent the static trade and capital allocation bloc of the model using two vectors for labor income ($q_n^* \equiv w_n^* \ell_n$) and capital income ($\zeta_n^* \equiv v_n^* k_n^*$); two matrices of expenditure shares ($S_{nh}^* \equiv [s_{nh}^*]$) and income shares ($T_{in}^* \equiv \frac{S_{ni}^*(v_n^* k_n^* + w_n^* \ell_n)}{\sum_{h=1}^N S_{hi}^*(v_h^* k_h^* + w_h^* \ell_h)}$); and two matrices of capital investment shares ($B_{ni}^* \equiv [b_{ni}^*]$) and capital income shares ($X_{in}^* \equiv \frac{v_n^* b_{ni}^* k_n^*}{\sum_{h=1}^N v_h^* b_{hi}^* k_h^*}$). For the two trade matrices, S_{ni}^* is the expenditure share of importer n on exporter i , while T_{in}^* is the income share of exporter i from importer n . For the two capital matrices, B_{ni}^* is the share of source country n 's investments in host country i , while $X_{in}^* \equiv \frac{v_n^* b_{ni}^* k_n^*}{\sum_{h=1}^N v_h^* b_{hi}^* k_h^*}$ is the share of income from host country i in source country n . The order of subscripts switches between the expenditure and income share matrices (S_{ni}^* versus T_{in}^*), and between the capital investment and income share matrices (B_{ni}^* versus X_{in}^*), because n and i correspond to rows and columns of these matrices, such that columns of S_{ni}^* and rows of T_{in}^* are exporters.

From the static trade and capital allocation bloc of the model in equations (28)-(32), we can derive the following relationships between the labor and capital income vectors (q^* , ζ^*) and these trade and investment share matrices (S^* , T^* , B^* , X^*), where we use bold math font to denote vectors or matrices:

$$\begin{aligned} w_i^* \ell_i &= \lambda \sum_{n=1}^N S_{ni}^* (v_n^* k_n^* + w_n^* \ell_n), & q^{*'} &= \lambda (\zeta^* + q^*)' S^*, \\ \frac{1}{\lambda} w_i^* \ell_i T_{in}^* &= S_{ni}^* (v_n^* k_n^* + w_n^* \ell_n), & q^{*'} T^* &= \lambda (q^* + \zeta^*)', \\ \sum_{n=1}^N v_n^* b_{ni}^* k_n^* &= \frac{1-\lambda}{\lambda} w_i^* \ell_i, & \zeta^{*'} B^* &= \frac{1-\lambda}{\lambda} q^{*'}, \\ X_{in}^* \equiv \frac{v_n^* b_{ni}^* k_n^*}{\sum_{h=1}^N v_h^* b_{hi}^* k_h^*} &= \frac{v_n^* b_{ni}^* k_n^*}{\frac{1-\lambda}{\lambda} w_i^* \ell_i}, & \zeta^{*'} &= \frac{1-\lambda}{\lambda} q^{*'} X^*. \end{aligned}$$

Therefore, we can recover steady-state $\{q^*, \zeta^*, S^*, T^*, B^*, X^*\}$ if we know one of the steady-state trade matrices $\{S^*, T^*\}$ and one of the steady-state capital matrices $\{B^*, X^*\}$.

The steady-state gross real return to capital (R_n^*) and the steady-state saving rate (ζ_n^*) are inversely related to the discount factor (β):

$$R_n^* = \frac{1}{\beta}, \quad \zeta_n^* = 1 - \beta. \quad (41)$$

This common gross real return to capital (R_n^*) in turn implies a common steady-state realized real return to capital (v_n^*/p_n^*):

$$\frac{v_n^*}{p_n^*} = \beta^{-1} - 1 + \delta. \quad (42)$$

3.8 Transition Dynamics

A key property of the model is that there is gradual convergence towards steady-state in response to shocks, such as trade or capital market integration, because of consumption smoothing. Therefore, trade and capital market integration affect the economy's growth rate along the transition path to steady-state, and their welfare impact differs systematically from their comparative steady-state impact.

We now solve in closed-form for the economy's transition dynamics up to first-order in response to shocks to country fundamentals, which include productivity $\{z_i\}$ and trade and capital market frictions $\{\tau_{ni}, \kappa_{ni}\}$. We suppose that we observe the economy somewhere along the transition path to an unobserved steady-state with time-invariant fundamentals. We suppose that we observe labor and capital income (q, ζ) and the trade and investment share matrices (S, T, B, X) for this initial equilibrium on the transition path. We derive a closed-form expression for the evolution of the endogenous variables of the model in response to a shock to fundamentals in terms of these observed variables.

Linearization We begin by linearizing the model around the initial unobserved steady-state equilibrium. We totally differentiate the system of equations for general equilibrium (28)-(37) around the initial steady-state. We use a tilde above a variable to denote a log deviation around the initial steady-state, such that $\tilde{x} \equiv \ln x_t - \ln x^*$. We define measures of incoming and outgoing trade and capital friction shocks, which aggregate bilateral changes across partner countries, using initial trade and investment share weights: $\tilde{\tau}_{nt}^{in} \equiv \sum_{i=1}^N S_{nit} \tilde{\tau}_{nit}$, $\tilde{\tau}_{it}^{out} \equiv \sum_{n=1}^N T_{int} \tilde{\tau}_{nit}$, $\tilde{\kappa}_{nt}^{out} \equiv \sum_{i=1}^N B_{nit} \tilde{\kappa}_{nit}$, and $\tilde{\kappa}_{it}^{in} \equiv \sum_{n=1}^N X_{int} \tilde{\kappa}_{nit}$.

Using this notation, the evolution of the endogenous variables of the model along the economy's transition path is determined by the following system of equations. Goods market clearing implies:

$$\tilde{w}_{it} = \sum_{n=1}^N T_{ni} \left[\tilde{S}_{nit} + \mu_{nt} (\tilde{v}_{nt} + \tilde{k}_{nt}) + (1 - \mu_{nt}) \tilde{w}_{nt} \right]. \quad (43)$$

The consumption price index satisfies:

$$\tilde{p}_{nt} \equiv \sum_i S_{ni} (\tilde{\tau}_{nit} + \lambda \tilde{w}_{it} + (1 - \lambda) \tilde{r}_{it} - \tilde{z}_{it}) \quad (44)$$

The expected return to capital is:

$$\tilde{v}_{nt} = \sum_{h=1}^N B_{nh} (\tilde{r}_{ht} - \tilde{\kappa}_{nht}). \quad (45)$$

The relationship between capital and labor payments implies:

$$\tilde{r}_{it} = \sum_{n=1}^N X_{in} \left[(1 - 1/\epsilon) \tilde{B}_{nit} + \tilde{k}_{nt} - \tilde{\kappa}_{nit} \right] = \tilde{w}_{it}. \quad (46)$$

Our choice of numeraire requires:

$$\sum_{n=1}^N q_n \tilde{w}_{nt} = 0. \quad (47)$$

Human capital evolves according to:

$$\tilde{h}_{nt} = \frac{1 - \beta}{\beta} \sum_{s=1}^{\infty} \beta^s (\tilde{w}_{nt+s} - \tilde{p}_{nt+s}) - \frac{1}{\beta} \sum_{s=1}^{\infty} \tilde{R}_{nt+s}. \quad (48)$$

Physical capital evolves according to:

$$\xi_n \tilde{k}_{nt+1} = -\frac{1 - \beta}{\beta} \tilde{\zeta}_n + \xi_n (\tilde{R}_{nt} + \tilde{k}_{nt}) + (1 - \xi_n) (1 - \beta) (\tilde{w}_{nt} - \tilde{p}_{nt} - \tilde{h}_{nt}) \quad (49)$$

The saving rate satisfies

$$-\tilde{\zeta}_n = \beta (\psi \tilde{\phi}_{nt+1} + (\psi - 1) \tilde{R}_{nt+1} - \tilde{\zeta}_{nt+1}) \quad (50)$$

where recall $\xi_n \equiv \frac{k_n}{k_n + h_n}$.

For expositional convenience, we focus here on the simplest form of fundamental shock, such that agents at time $t = 0$ learn about a one-time permanent shock to fundamentals from time $t = 1$ onwards, such that $\tilde{\mathbf{f}}_t = \tilde{\mathbf{f}}$ for $t \geq 1$.

Static bloc of equations The static bloc of equations for trade and production can be reduced to the following matrix system of equations for wages and rental rates $\{\tilde{w}_t, \tilde{r}_t\}$ as functions of the state variables $\{\tilde{\mathbf{k}}_t\}$ and shocks $\{\tilde{z}_t, \tilde{\kappa}_t^{in}, \tilde{\kappa}_t^{out}, \tilde{\tau}_t^{in}, \tilde{\tau}_t^{out}\}$:

$$\begin{bmatrix} \tilde{w}_t \\ \tilde{r}_t \end{bmatrix} = \begin{bmatrix} \mathbf{L}^w \\ \mathbf{L}^r \end{bmatrix} \tilde{\mathbf{k}}_t + \begin{bmatrix} \mathbf{M}^w \\ \mathbf{M}^r \end{bmatrix} \tilde{\mathbf{f}}, \quad (51)$$

where

$$\tilde{\mathbf{f}} \equiv [\tilde{z} \quad \tilde{\kappa}^{in} \quad \tilde{\kappa}^{out} \quad \tilde{\tau}^{in} \quad \tilde{\tau}^{out}]'.$$

Dynamic bloc of equations The dynamic bloc of equations for the evolution of the state variables can be reduced to the following matrix system of second-order difference equations:

$$\Psi \tilde{k}_{t+2} = \Gamma \tilde{k}_{t+1} + \Theta \tilde{k} + \Pi \tilde{f}, \quad (52)$$

where the matrices $\{\Psi, \Gamma, \Theta, \Pi\}$ depend on the trade and capital share matrices $\{S, T, B, X\}$ and structural parameters $\{\psi, \beta, \sigma, \theta, \epsilon, \lambda, \delta\}$.

We can solve this second-order difference equation using the method of undetermined coefficients following Uhlig (1999) to obtain a closed-form solution for the evolution of the state variables along the transition path $\{\tilde{k}_t\}_{t=1}^{\infty}$. Given this solution for the state variables, we can recover the transition path of all the other endogenous variables of the model. Although for expositional simplicity, we focus on this one-time permanent shock, analogous closed-form solutions can be derived for any future sequence of fundamental shocks.

Note that in this second-order difference equation (52), there are no terms in the change in the trade and investment share matrices, because these terms are second-order in the underlying Taylor-series expansion, involving interactions between the shocks to fundamentals and the resulting changes in trade and investment shares. As we consider first-order changes in fundamentals, these second-order, non-linear terms drop out of the linearization. Therefore, we can write the trade and investment share matrices with no time subscript (S, T, B, M) for first-order changes in fundamentals. In our empirical analysis below, we show that we find similar results from our spectral analysis whether we use the observed trade and investment share matrices or the implied steady-state matrices.

3.9 Properties of General Equilibrium

We now highlight three distinctive ways in which trade and capital investments interact in the model to shape the impact of trade and capital market integration. In Subsection 3.9.1, we contrast the predictions of our framework for the impact of opening the closed economy on real wages with those from conventional static trade models. In Subsection 3.9.2, we show that trade integration influences the rate of return to capital accumulation in our framework. Therefore, there are dynamic welfare gains from trade in our framework, because trade affects the economy's growth rate along the transition path to steady-state, and its steady-state capital-labor ratio. In Subsection 3.9.3, we show that capital market integration acts like an improvement in the productivity of the investment technology, which also affects the the economy's growth rate along the transition path to steady-state, and its steady-state capital-labor ratio.

3.9.1 Real Wage Gains from Trade

We begin by characterizing the impact of trade integration on the static real wage in our framework. Using the relationship between capital and labor payments (20), the expenditure share (28), and the price index (36), we can express the relative increase in the real wage between the open economy (superscript T) and the closed economy (superscript A) in terms of the relative productivity-adjusted capital-labor ratio (\tilde{k}_{it}/ℓ_i) in the open and closed economies, the share of capital in production costs ($1 - \lambda$), the domestic expenditure share (s_{nnt}^T) in the open economy, and the trade elasticity (θ):

$$\frac{w_{nt}^T/p_{nt}^T}{w_{nt}^A/p_{nt}^A} = \left(\frac{\tilde{k}_{it}^T/\ell_i^T}{\tilde{k}_{it}^A/\ell_i^A} \right)^{1-\lambda} \left(\frac{1}{s_{nnt}^T} \right)^{\frac{1}{\theta}}, \quad (53)$$

where recall that the tilde denotes the productivity-adjustment for effective units of capital, such that $\tilde{k}_{it} = \gamma \sum_{n=1}^N b_{nit}^{-1/\epsilon} k_{nit}$.

If the productivity-adjusted capital labor ratio were held constant in the open and closed economies ($\tilde{k}_{it}^T/\ell_i^T = \tilde{k}_{it}^A/\ell_i^A$), the first term in equation (53) would equal one, and the domestic trade share and trade elasticity would be sufficient statistics for the impact of trade on the real wage, as in conventional static trade models. More generally, the impact of trade integration on welfare differs systematically from these conventional static trade models for three reasons. First, income in our framework is the sum of labor and capital income, which implies that the impact of trade on the static real wage does not fully capture its impact on static real income. Second, trade integration affects the rate of return to capital accumulation, which implies that the productivity-adjusted capital-labor ratio (\tilde{k}_{it}/ℓ_{it}) in general differs between the open and closed economies, as examined further in the next subsection. Third, since trade integration affects the rate of return to capital accumulation, there are dynamic welfare gains from trade in our framework, such that the opening of the closed economy to trade affects the economy's rate of growth along the transition path to steady-state. As a result, the impact of trade on welfare differs systematically from its comparative steady-state impact, because of these transition dynamics.

3.9.2 Trade Integration and Capital Accumulation

We now show that trade integration affects the rate of return to capital accumulation and hence the steady-state capital-labor ratio. From the realized return to capital (13), the capital investment allocation (12), and the steady-state solution for the realized return to capital (40), the steady-state rental rate (r_n^*) can be expressed in terms of the steady-state consumption price index (p_n^*), the domestic capital investment share (b_{nn}^*) and parameters:

$$r_n^* = \frac{v_n^*}{\gamma/\kappa_{nn}} (b_{nn}^*)^{\frac{1}{\epsilon}} = \frac{[\beta^{-1} - 1 + \delta] p_n^*}{\gamma/\kappa_{nn}} (b_{nn}^*)^{\frac{1}{\epsilon}}. \quad (54)$$

From the relationship between factor payments (20), the steady-state productivity-adjusted capital-labor ratio (\tilde{k}_n^*/ℓ_n) is increasing the steady-state wage-rental ratio (w_n^*/r_n^*). Substituting for the steady-state rental rate (54) in this relationship between factor payments, we obtain the following expression for the steady-state capital-labor ratio (\tilde{k}_n^*/ℓ_n):

$$\frac{\tilde{k}_n^*}{\ell_n} = \frac{1 - \lambda}{\lambda} \frac{\gamma/\kappa_{nn}}{[\beta^{-1} - 1 + \delta] (b_{nn}^*)^{\frac{1}{\epsilon}}} \frac{w_n^*}{p_n^*}. \quad (55)$$

Therefore, other things equal, trade integration that raises steady-state real labor income (w_n^*/p_n^*) raises the steady-state capital-labor ratio (k_n^*/ℓ_n). Intuitively, trade integration lowers the consumption price index through the conventional static gains from trade, which raises the rate of return to capital accumulation, and hence increases the steady-state capital labor ratio. Evaluating the real wage gains in equation (53) in steady-state, and using equation (55) to substitute for the steady-state capital-labor ratio (\tilde{k}_n^*/ℓ_n), we obtain the following expression for the real wage gain from the opening of the closed economy:

$$\frac{w_{nt}^{T^*}/p_{nt}^{T^*}}{w_{nt}^{A^*}/p_{nt}^{A^*}} = \left(\frac{1}{b_{nn}^{T^*}} \right)^{\frac{1}{\lambda\epsilon}} \left(\frac{1}{s_{nnt}^{T^*}} \right)^{\frac{1}{\lambda\theta}}, \quad (56)$$

where we have used $b_{nn}^{A^*} = s_{nnt}^{A^*} = 1$.

Therefore, we find that endogenous capital accumulation magnifies the impact of trade integration on the steady-state real wage. The exponent on the domestic trade share is no longer $1/\theta$, as in conventional trade models, but rather $1/(\lambda\theta)$. Since $0 < s_{nnt}^{T^*} < 1$ and $0 < \lambda < 1 < \theta$, this implies a larger steady-state real wage gain from opening the closed economy for a given steady-state open-economy domestic trade share ($s_{nnt}^{T^*}$). Intuitively, the capital accumulation induced by the opening of goods trade raises the marginal product of labor in a similar way as an increase in productivity, thereby raising the real wage.

In contrast to conventional trade models, the real wage gain from the opening of the closed economy also depends on the extent of capital market integration (as captured by $b_{nn}^{T^*}$). Just as the domestic trade share ($s_{nnt}^{T^*}$) is sufficient statistic for goods market integration in conventional trade models, the domestic investment shares ($b_{nn}^{T^*}$) is a sufficient statistic for capital market integration in our framework. Additionally, we find that goods and capital market integration interact with one another. First, the real wage gain ($(w_{nt}^{T^*}/p_{nt}^{T^*}) / (w_{nt}^{A^*}/p_{nt}^{A^*})$) from a given level of goods market integration (given $s_{nnt}^{T^*}$) in equation (56) is larger for more open capital markets (lower $b_{nn}^{T^*}$). Second, the real wage gain ($(w_{nt}^{T^*}/p_{nt}^{T^*}) / (w_{nt}^{A^*}/p_{nt}^{A^*})$) from a given level of trade integration (given $s_{nnt}^{T^*}$) in equation (56) depends only a capital market parameter (the labor share (λ)).

3.9.3 Capital Market Integration and Capital Accumulation

We now examine further the way in which capital market integration affects the rate of return to capital accumulation and hence the steady-state capital-labor ratio. From equations (13), (12), and (40), the steady-state realized real return to capital (v_n^*/p_n^*) is equalized across countries, but the steady-state real rental rate (r_n^*/p_n^*) can differ depending on the productivity of investments (a_n) and capital market frictions (κ_{ni} for all i):

$$\frac{v_n^*}{p_n^*} = \beta^{-1} - 1 + \delta = \gamma \frac{r_n^*/\kappa_{nn}}{p_n^* (b_{nn}^*)^{\frac{1}{\epsilon}}} \quad (57)$$

Under autarky, the steady-state domestic investment share is equal to one ($b_{nn}^{A*} = 1$). In contrast, under capital market integration, the steady-state domestic investment share is strictly less than one ($0 < b_{nn}^{T*} < 1$). Therefore, other things equal, the steady-state real rental rate (r_n^*/p_n^*) is lower in the open economy than in the closed economy, in order for the right-hand side of equation (57) to equal the unchanged value of the left-hand side ($v_n^*/p_n^* = \beta^{-1} - 1 + \delta$). Intuitively, capital market integration acts like an improvement in investment productivity, because capitalists gain access to another set of draws for idiosyncratic productivity for each foreign host country, which increases the average productivity of the investments that they choose to undertake in equilibrium. This increased average productivity of investment raises the rate of return to capital accumulation, which leads to a higher steady-state capital stock, and hence a lower steady-state real rental rate.

4 Quantitative Results

We illustrate these theoretical predictions quantitatively using Brexit as a first empirical application. We start with the observed data for 2015 before the Brexit vote. We next shock trade frictions, capital market frictions or both sets of frictions between the U.K. and E.U. countries. We solve for the transition path of the endogenous variables in each country in response to this shock to trade and/or capital frictions.

As in conventional static trade models, higher trade frictions between the U.K. and the E.U. lead to cross-substitution effects, which increase trade domestically and with other nations. Both the U.K. and its E.U. trade partners experience a reduction in consumption and flow utility as a result of these higher trade frictions. In contrast, third nations can potentially enjoy higher consumption and flow utility, if for example they benefit from increased demand for their products in E.U. markets following the reduction in the competitiveness of U.K. suppliers as a result of the higher trade frictions.

In contrast to conventional static trade models, these higher trade frictions also make the U.K. a less attractive investment destination, because of the reduction in its market access to the

European Union. This reduction in capital accumulation leads to slower economic growth along the transition path to steady-state and a lower steady-state capital-labor ratio and level of income per capita. Increases in trade frictions thus lead to dynamic welfare losses in the form of lower growth along the transition path to steady-state. As a result of these transition dynamics, the welfare impact of these changes in trade frictions is no longer equal to its comparative steady-state impact.

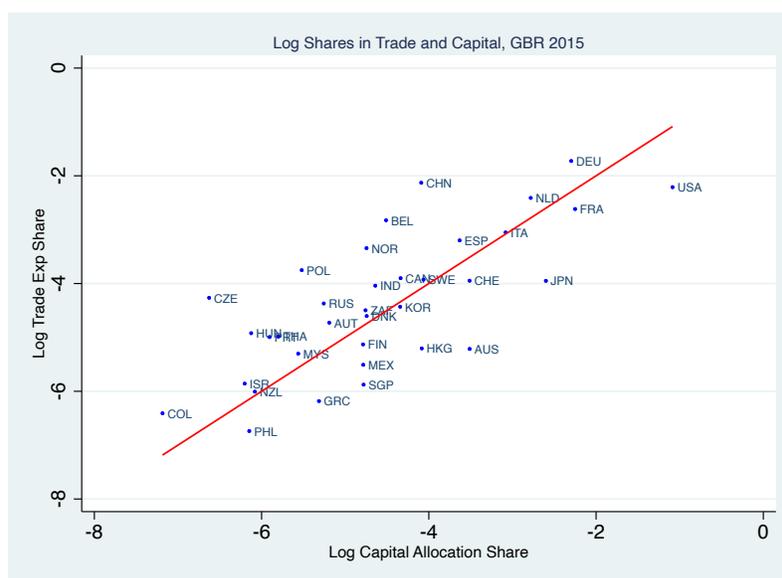
Within our framework, higher capital market frictions between the U.K. and the E.U. also reduce capital accumulation within the UK, as domestic investors experience diminished access to investment opportunities in the European Union. In contrast, third nations again can potentially benefit as for example investments that otherwise would have occurred in the U.K. relocate to other nations, such as the United States. Furthermore, these two forms of international integration interact with one another, such that the welfare costs of increased trade frictions are greater in a more open capital market, and the welfare costs of higher capital market frictions are larger in more open goods markets.

Parameterization To illustrate the quantitative magnitude of this interaction between trade and capital investments, we consider standard values for model parameters from the existing empirical literature. We assume a value of the discount rate equal to $\beta = 0.9$. We assume a value of the intertemporal elasticity of substitution of $\psi = 2$. We assume a value for the depreciation rate of $\delta = 0.05$. We assume a labor share of $(1 - \lambda) = 0.5$. We assume a value of the trade elasticity of $\theta = 5$, which lies in the center of the range from 2-12 considered in [Eaton and Kortum \(2002\)](#), and is the baseline value assumed in [Costinot and Rodríguez-Clare \(2014\)](#). We assume a value of the investment elasticity of $\epsilon = 2$, which is in the middle of the range of values estimated in [Kojen and Yogo \(2020\)](#).

We implement our quantitative analysis as follows. We observe the expenditure share (\mathbf{S}), income share (\mathbf{T}) and capital investment share (\mathbf{B}) matrices in 2015 immediately before the Brexit vote. We assume that the world economy is close to steady-state equilibrium in this year and solve for the implied values of the capital income share (\mathbf{X}) matrix, the labor income vector (\mathbf{q}) and the capital income vector ($\mathbf{\zeta}$). Starting from this initial equilibrium, we consider a small change in trade frictions (τ_{ni}), capital frictions (κ_{ni}) or both frictions between the U.K. and E.U. countries, and use our closed-form solution for the economy's transition path from the linearized model to solve for the elasticity of consumption in each country in each year with respect to this shock.

Trade and Investment Shares In Figure 4, we display the expenditure share (\mathbf{S}) and investment share (\mathbf{B}) matrices for the United Kingdom in 2015. We label each of the other countries by their three-letter International Organization for Standardization (ISO) code. We show the

Figure 4: Expenditure Shares (S) and Investment Shares (B) for the United Kingdom in 2015



Note: Vertical axis shows the share of UK expenditure on each importer (S_{UKi}); horizontal axis shows the share of UK capital invested in each host country (B_{UKi}); both shares sum to one across countries; three letter codes are country ISO codes; red line shows the 45 degree line.

45-degree line by the solid red line. We find a strong, positive and statistically significant relationship between a country's share of the U.K.'s expenditure on goods trade and its share of the U.K.'s investments. This strong positive correlation between trade and investment by itself suggests that these two margins of participation in the international economy are likely to interact systematically with one another.

Trade Frictions Only We begin by considering a small change in trade frictions alone between the U.K. and E.U. countries

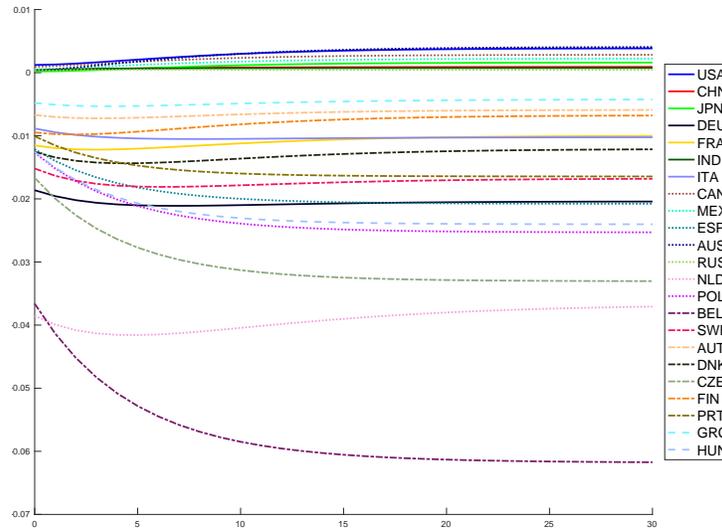
Capital frictions only

Both trade and capital frictions

5 Conclusions

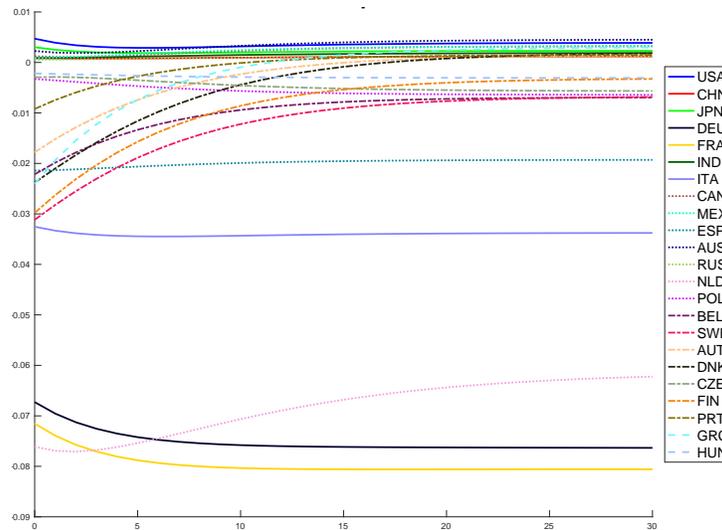
A key question at the heart of international economics is how do goods trade and capital investments interact with one another? This question is central to thinking about a whole host of issues. Are capital investments complements or substitutes for goods trade? How much bigger are the gains from globalization when trade integration is combined with international capital liberalization? How is the impact of China's economic growth on its East Asian neighbors al-

Figure 5: Elasticity of Country Consumption with Respect to an Increase in Trade Frictions Between the UK and EU Countries



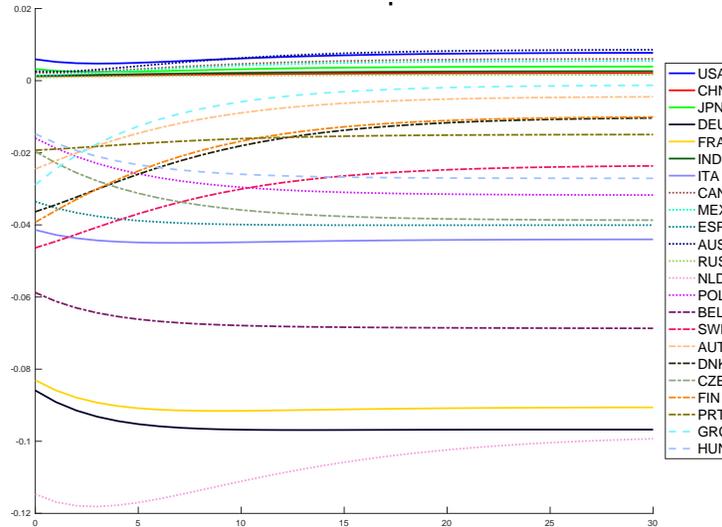
Note: Elasticity of country consumption with respect to an increase in trade costs between the UK and EU countries in each year along the transition path in the linearized model; vertical axis shows proportional change in consumption with respect to a small change in trade costs; horizontal axis shows years; three letter codes in legend are country ISO codes.

Figure 6: Elasticity of Country Consumption with Respect to an Increase in Capital Frictions Between the UK and EU Countries



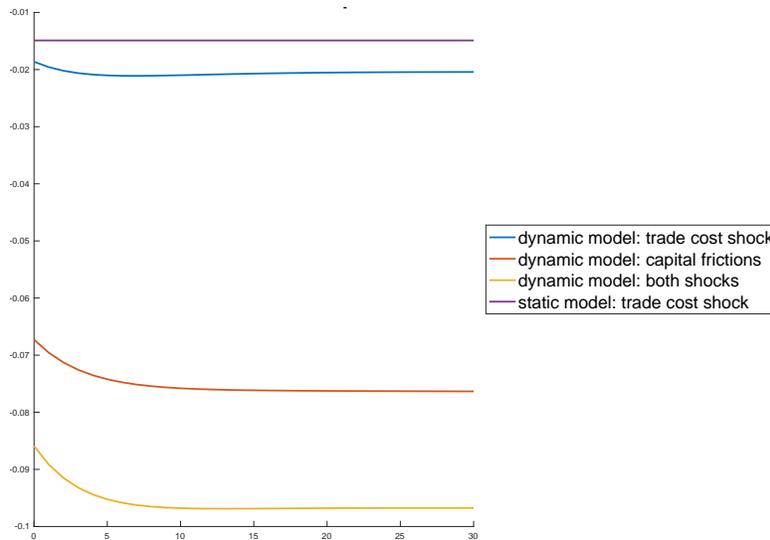
Note: Elasticity of country consumption with respect to an increase in capital frictions between the UK and EU countries in each year along the transition path in the linearized model; vertical axis shows proportional change in consumption with respect to a small change in trade costs; horizontal axis shows years; three letter codes in legend are country ISO codes.

Figure 7: Elasticity of Country Consumption with Respect to an Increase in both Trade and Capital Frictions Between the UK and EU Countries



Note: Elasticity of country consumption with respect to an increase in capital frictions between the UK and EU countries in each year along the transition path in the linearized model; vertical axis shows proportional change in consumption with respect to a small change in trade costs; horizontal axis shows years; three letter codes in legend are country ISO codes.

Figure 8: Elasticity of Germany's Consumption with Respect to an Increase in Trade Frictions, Capital Frictions and Both Frictions Between the UK and EU Countries



Note: Elasticity of Germany's consumption with respect to an increase in trade frictions, capital frictions and both frictions between the UK and EU countries in each year along the transition path in the linearized model; vertical axis shows proportional change in consumption with respect to a small change in trade costs; horizontal axis shows years; three letter codes in legend are country ISO codes.

tered when capital is free to move across borders as well as goods? How much larger are the costs of Brexit for the United Kingdom when capital is free to reallocate internationally? How does this reallocation of capital affect the distributional consequences of trade disintegration? How much greater are the costs of international sanctions for targeting and targeted countries when restrictions on capital investments are combined with barriers to goods trade?

We develop a new theoretical framework for modelling this interaction between goods trade and capital investments. Our framework accommodates a large number of countries that can differ from one another in terms of their size, productivity, and the geography trade costs and capital market frictions. Despite these rich asymmetries and high-dimensional state space, the model remains tractable and amenable analytical analysis, and permits transparent counterfactuals that depend on only a small number of structural parameters. Our framework is consistent with a new of key features of the observed data. First, we incorporate intra-temporal trade in goods in a way that is consistent with the observed gravity equation for bilateral international trade. Second, we include intra-temporal capital mobility such that a gravity equation holds for bilateral international capital investments, as again observed empirically. Third, we allow for intertemporal trade through consumption-saving decisions, which is consistent with observed bilateral and multilateral current account imbalances across countries.

We use our theoretical framework to derive three main sets of results. First, we provide tractable microfoundations for the gravity equation in bilateral international capital investments. We show that this framework can be rationalized in terms of either a Keynesian marginal efficiency of capital schedule or in terms of portfolio diversification. These two alternative microfoundations are isomorphic in terms of their predictions for bilateral international capital investments. Second, we derive sufficient statistics for the welfare gains from both international trade and international capital investments. In general, we show that these two sources of welfare gains interact with one another, such the whole differs systematically from the sum of the parts. Third, we analyze how the incidence of productivity and trade costs shocks depends on both international trade and international capital linkages, using exact-hat algebra counterfactuals for the full non-linear model. Fourth, we derive analogous sufficient statistics for the first-order impact of productivity and trade cost shocks, and use these first-order sufficient statistics to understand the mechanisms through which international trade and international capital linkages interact with one another.

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