To Vaccinate or To Wait [§]

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ABSTRACT

This paper models individual decisions under irreversibility of the vaccine using real options. Vaccine irreversibility increases the value in waiting to vaccinate and postpone vaccination, even if vaccination has positive net gain. The waiting value magnifies the vaccination cost ex ante in a rational framework. In this framework, we analyze the difference between the reward at vaccination, or equivalent tax on non-vaccinated for increasing the likelihood to vaccinate. For individuals, any subsidy at vaccination to reduce the vaccine cost is more effective on the likelihood to vaccinate relative to any equivalent taxes imposed to increase the cost of no vaccination. The factors that add to the value in waiting and postpone vaccination are increase in the uncertainty about the disease, and likelihood for the expected cost of the infection to go down.

Keywords: Real option, Vaccination, Irreversibility, Waiting value

JEL classification code(s): D81, G13, I12.

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I. Introduction

"Leaders of Quebec, Canada, are planning to impose a tax on adults in the province who refuse to get vaccinated" (CBS News 2022). *"Instead of forcing, we need to go and see what's behind it for those who are not currently vaccinated" Dr. Yv Bonnier-Viger, public health director of the Gaspé region, Quebec* (The Canadian Press 2022).

This paper uses real option theory to analyze the optimal vaccination decision by individuals. Using real options theory allows us to take into account the irreversibility of vaccination, which leads to a value in waiting to vaccinate.

When costs related to a decision are irreversible, having a net positive immediate gain (benefits minus costs) from the decision itself is a necessary, but not sufficient, condition for the decision to be implemented. This holds within a rational framework. For example, an irreversible project with positive net present value (NPV) is not implemented immediately because individuals will seek to maximize NPV through optimal timing that accounts for the costs of lost reversibility and flexibility. The real option literature models similar decisions for corporations,[1] including decisions within the healthcare industry.[2] In all these models, conventional optimal decision models are challenged with real options if the decision is irreversible because the irreversibility removes the ability to gain in the future from changing the decision. Analogously, vaccination is irreversible for individuals. Vaccination has immediate irreversible costs while it saves individuals against uncertain sickness costs. Therefore, conventional optimal personal decisions also skew under lost options based on a rational framework. To the best of our knowledge, this paper is the first to extend the real option theory to individual vaccination decisions.

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[1] See Dixit and Pindyck (2012), Sick and Gamba (2010), and Li et al. (2022) for literature reviews.
[2] See, e.g., McLean and Magiera (2000), and Magiera and McLean (1996) for capital budgeting and program selection examples, Wernz, Gehrke, and Ball (2015) for operating room capacity, Park (2016) for vaccine stockpile, Shamsi, Torabi, and Shakouri (2018) for vaccine contracts, and Kudryavtsev and Trushin (2022) for pharmaceuticals.

Our rational model is in contrast to other writers who use distrust, ignorance, or other irrational behavior to explain vaccination rates that are lower than optimal (for examples, see the literature on vaccine hesitancy such as Dube et al., 2013 and MacDonald, 2015). While our paper does not discuss irrationality and does not rule out the contribution of irrationality, our finding raises the bar for labeling vaccination rates as being caused by as irrational behavior.

In this paper, first we present a basic model to show that the optimal vaccination decision changes because a vaccination cannot be reversed. This creates a value in waiting to vaccinate, also known as the value of lost reversibility, under uncertainty in our second basic model. The value in waiting is the present value of the gain from reversing the vaccination decision in the future. If the benefit of the vaccine is going to go below the cost of the vaccine in the future, this creates a value in waiting that, if it is large enough, can lead to non-vaccination at the present. In practical terms, the value in waiting model is relevant in cases where either a) the expected cost of vaccinating is expected to go down in the future, or b) the disease cost is expected to decrease in the future (we focus on the later). For example, the disease cost can go down through medical discoveries that provide less costly treatments (see, e.g., Triggle et al. 2020), herd immunity caused by others vaccinating (see, e.g., Pan, Ng, and Cheng 2021), or the avirulence of the pathogen (see, e.g., Alizon et al. 2009). When these circumstances drop the sickness costs below the vaccination cost, the expected net benefit of vaccination in the future turns negative, and a rational individual would wish to reverse the vaccination and recoup the costs of vaccinating (if vaccines were reversible like other economic decisions). Of course, vaccines are not reversible, so this irreversibility imposes a cost on individuals because they lose the value of the option to reverse their vaccination. Hence, for the individual to vaccinate immediately, the vaccination benefit not only must surpass its cost, but also must surpass the value of lost reversibility. Otherwise, it is optimal for individuals to wait even if the immediate vaccination gain (immediate benefit minus immediate cost) is positive.[5]

In this setting, we show that the value in waiting acts like a magnifier on the vaccination costs. The magnifier differentiates the vaccination costs from disease costs as the vaccination costs are relatively more certain over time than disease costs. This is in line with the lower effectiveness of offering lotteries at vaccination compared to certain cash rewards (Duch et al. 2021) because a lottery outcome is similar to probabilistic disease cost-saving and a cash reward is similar to vaccination costs that are known with more certainty. As another result, we find that a definite reward at vaccination is more effective in increasing the vaccination rate than a tax of the same amount on those who are not vaccinated by a certain date; both actions increase the net benefit of immediate vaccination, but the definite reward at vaccination also decreases the value in waiting. This result matches the empirical report by Fishman et al. (2022) that a $1,000 cash reward increases the likelihood of vaccination by 17% compared to 13% for an equivalent $1,000 tax on non-vaxxers.

Our third model is a continuous-time model. We evaluate it to report the effects of uncertainty on the value in waiting. The uncertainty increases the inaction area and the value in waiting. Moreover, when the uncertainty is high, we report that the effectiveness of both vaccination tax and rewards drop. Hence, reducing the uncertainty about vaccination benefits has a first-order effect on vaccination rates relative to other factors, such as the rate at which the vaccination benefit declines.

[5] This is in line with the report that 33%-39% of the participants without COVID-19 vaccination in the US responded that they would "wait and see" in May 2021 (Hamel et al. 2021).

Our model sets a higher bar for vaccination to be optimal at an individual level. Some earlier studies propose a positive gain (or benefit/cost ratio greater than 1) suffices for the vaccination justification (Hilbe, Nowak, and Sigmund 2013; Connolly and Reb 2003; Willems and Sanders 1981). The real option augments these studies with the condition that the gain must be larger than the lost waiting option; the benefit not only needs to cover the vaccine direct cost, but also must cover the lost options' value. This condition provides a rational explanation for the "inflated perceived vaccination cost" (Bhattacharyya and Bauch 2011) as an assumption required by some studies next to irrational explanations, such as rumors (Verelst, Willem, and Beutels 2016). Moreover, the extra compensation for the lost reversibility option of the vaccine is an example of a quantified measure for "payment for risk" suggested by Savulescu (2021).

This paper also presents the real option channel under certainty to distinguish how this channel adds to the regret theory. Regret theory discusses inflation in the perceived vaccination costs when "an agent may have inaccurate perceptions of the probabilities of states occurring or may have imperfect information about the efficacy of the vaccination technology" (Sadique et al. 2005). The regret theory fits under "confidence" in a "Four C" model for the vaccination decision (Betsch, Böhm, and Chapman 2015). In contrast, our models assume perfect information, and yet the vaccination cost is inflated with the value in waiting. Another difference between our model and regret theory is that we assume the individual makes a rational calculation to decide about vaccination, which locates our model under "calculation" for decision making about vaccine under "Four C" model (Betsch, Böhm, and Chapman 2015). Therefore, real options, such as the value in waiting, within a rational framework complement regret theory in explaining inflated vaccination costs. Consequently, real options also help to explain lower vaccination rates, particularly for newer diseases where the disease costs are more uncertain.

Finally, this paper contributes to individual health literature. In the healthcare literature for individuals, a few studies consider real options for treatment options, such as watchful waiting (see, e.g., Favato et al. 2013; Driffield and Smith 2007). They mostly consider limited discrete-time models and apply the real option theory to decisions made by doctors. We introduce continuous-time models to this literature and extend them to vaccination decisions by individuals. Some studies consider real options for corporate decisions in health care (see, e.g., Palmer and Smith 2000). We also expand these applications into individual decisions.

II. Model

II.A. Basic Model: Vaccine Irreversibility

We start by presenting a toy model (i.e., a deliberately simple model used to concisely demonstrate the concept) to show the basic idea of irreversibility and how it influences an individual's vaccination decision. The model shows the conditions under which there will be no vaccination due to irreversibility.

Let us consider a two-period model where a risk-neutral individual[6] faces a disease with an option to vaccinate. Vaccination has an irreversible fixed cost V. The individual can opt to vaccinate today (time 0) or opt to vaccinate next time (time 1). By construction, vaccination is not desirable at time 1 because we choose parameters such that the potential disease costs at time 1 will be less costly than the vaccination at time 1. Thus, the decision at time 0 is the only decision, which results in a decision to either to vaccinate at time 0 or to postpone to period 1 and, thus, not to vaccinate. Therefore, we only focus on the vaccination decision at time 0 (see Figure 2).. The

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[6] We assume risk-neutrality for simplicity. Adding risk-aversion will strengthen our results since a risk-averse utility reduces the immediate vaccination benefit relative to waiting more than a risk-neutral utility in both sides of Equation (1).

time difference between the two periods is t, the individual applies an annualized discount rate r to any future costs or benefits, and all the parameters are known with certainty. The vaccination

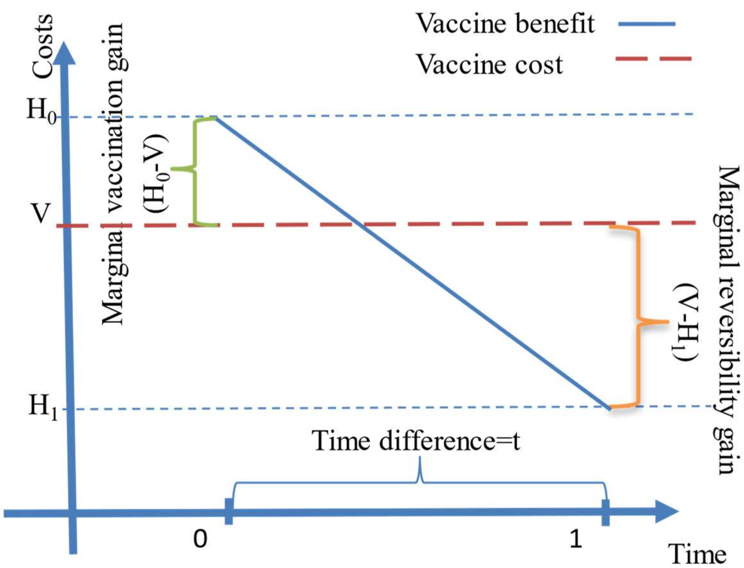
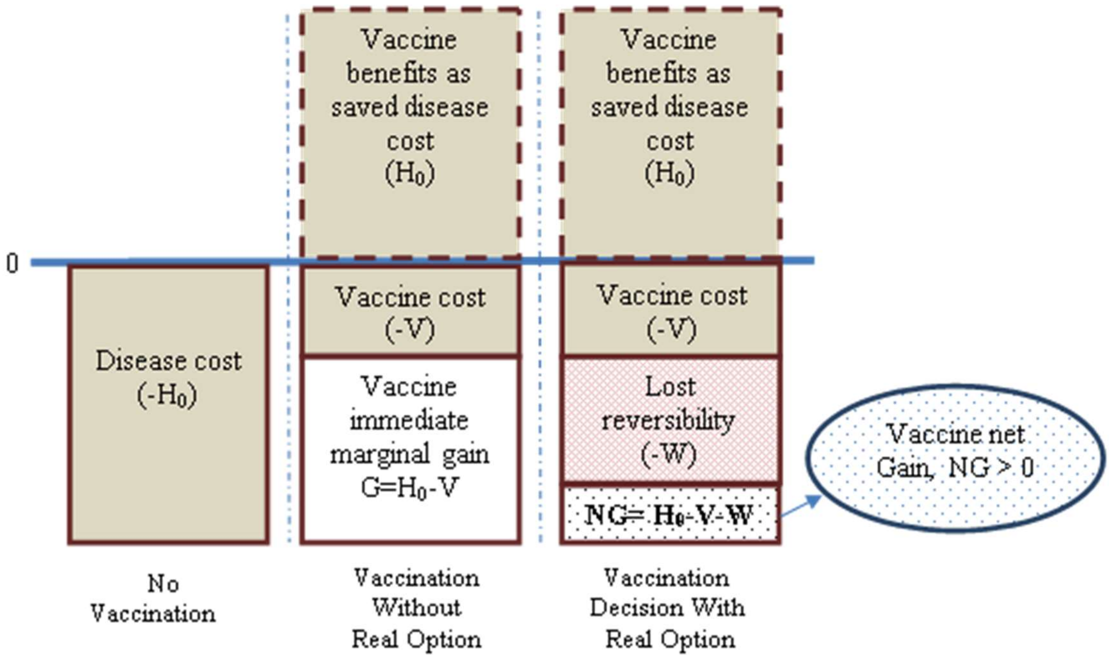


Figure 1- The comparison between immediate vaccination gain at time 0 and reversibility gain in the future once the benefit falls below the cost. Vaccination is optimal if the gain is larger than the value in waiting (or reversibility value): $H_0 - V > e^{-rt}(V - H_1)$

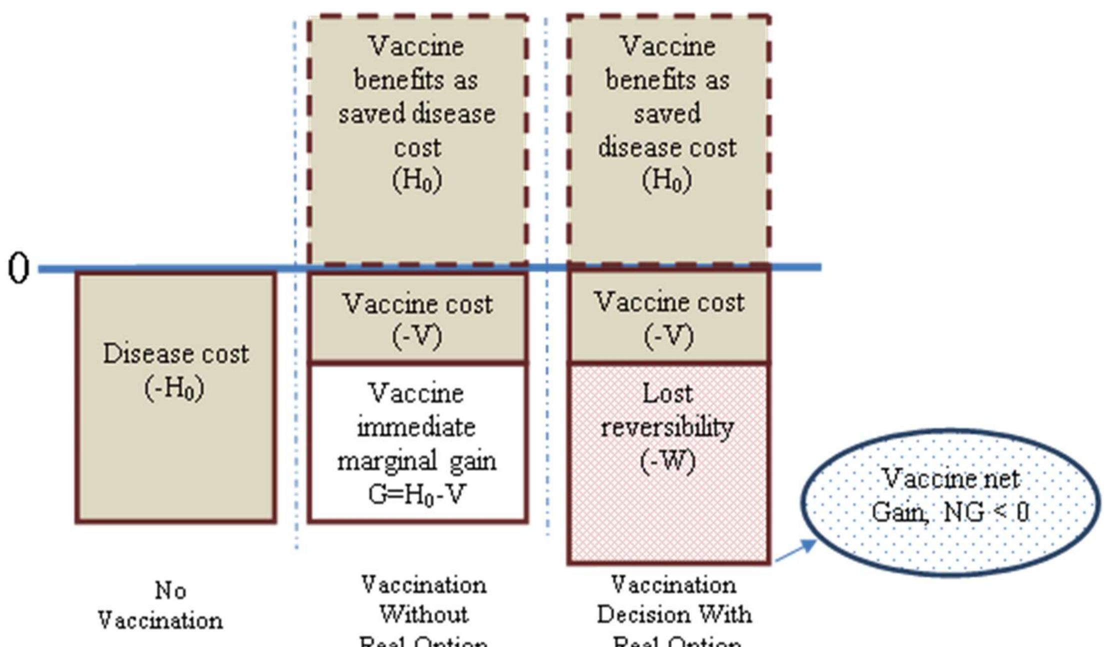
cost includes the private disutility of the vaccine, such as a vaccine-related sickness and side-effects, the price of the vaccine, and lost wages while getting the vaccine. Similar to Pan, Ng, and Cheng (2021), any subsidy paid to the individual at vaccination reduces this cost because the subsidy is tied directly to the decision to vaccinate (just like paying the vaccination cost). Appendix A defines all the parameters.

Without vaccination, the individual faces disease-related costs. The disease's expected infection cost (-H) is the infection probability times the infection costs (such as hospitalization expenses, lost wages, and the private disutility of infection (that is, the individual's aversion to feeling sick)). For example, one disease may have a low infection cost with a high probability of infection while another disease may have a high infection cost with a low probability of infection. Therefore, on the positive side, the vaccination benefit H for the individual is to save the expected costs associated with the disease. The benefits are time-varying where H_i is the total benefit associated with period I (i=0, 1) and h_i is the benefit associated with being vaccinated at the moment of the time period i. Any hypothetical tax on non-vaxxers increases the disease costs because a tax is tied to being unvaccinated in each period similar to the disease costs, which are saved in each period that the individual is vaccinated. For simplicity, similar to the literature (see, e.g., Chen and Toxvaerd (2014)), the vaccine is assumed to provide complete and permanent immunity.[7] Therefore, receiving immunity in time 0 provides full immunity in both time 0 and time 1, meaning that $H_0 = h_0 + h_1 e^{-rt}$ (i.e. the full benefits of immunity at i=0 and the discounted value of the benefits at i=1) and $H_1 = h_1$ (i.e. the full benefits of immunity at i=1). The vaccine acts like a cost-switching mechanism from facing disease costs to facing vaccination costs. Hence, the vaccination immediate marginal gain (G) is the vaccination benefit (i.e., saving the expected disease cost) minus vaccination cost (G=H-V). Our approach for costs and benefits of vaccination follows the typical approach in the literature (see, e.g., Park 2016).

[7] It also shows what model parameters represent if vaccine is partially effective. If the vaccine does not create full immunity, H measures the marginal decline in diseases costs, such as the expected infection cost without vaccination minus the expected infection cost with vaccination.



Error! Reference source not found.A-In this case, immediate vaccination is optimal with the real option because the immediate gain is larger than the lost reversibility: $H_0 - V > W$, making the vaccine net gain ($H_0 - V - W$) positive.



Error! Reference source not found.B- In this case, immediate vaccination would be optimal if the lost reversibility is not considered. However, the existence of the lost reversibility causes the optimal decision to be waiting because the immediate gain is smaller than the lost reversibility: $H_0 - V < W$, and the vaccine net gain ($NG = H_0 - V - W$) is negative.

Figure 3- The comparison between immediate vaccination gain at time 1 and taking into account the cost of lost reversibility W as $e^{-rt}(V - H_1)$ in the future.

The vaccination decision at time 1 is not straightforward, even if the immediate benefit is larger than vaccine cost ($V<H_0$).[8] A positive immediate gain is not sufficient to justify vaccination because there is value lost due to irreversibility (or, value gain in inaction); the individual compares the vaccination gain today against the lost reversibility. **Error! Reference source not found.**-A shows the situation that leads to vaccination with and without reversibility and real option consideration where both situations result in vaccination. However, **Error! Reference source not found.**-B shows when the value of lost reversibility is high enough that when it is considered, it changes the decision from vaccination to not vaccinating (aka, waiting until t=1).

The value of reversibility (W) measures the present value from hypothetically reversing the vaccination in the next period ($W = e^{-rt}(V - H_1)^+$) when the vaccine benefit is smaller than the vaccination cost, if the individual waits. We drop the plus sign for brevity. W measures the lost reversibility due to vaccination and not waiting.. The (assumed) reversibility of the decision works like an insurance contract and is analogous to a put option in financial markets. A put option pays the holder of the option if the stock price is below the exercise price at the exercise date. The put option's payoff is the difference between the exercise price and stock price if the difference is positive. Analogously, the vaccination cost is like the exercise price, the benefit is like the stock price, and the difference is like the option payoff. *Ex post*, reversibility has a positive benefit when the vaccination benefit drops below the vaccination cost in period 1. This analogy between an option contract and a real-life situation is called a real option. A rational expected value model that does not use real options assumes that the decision is reversible and therefore the individual gains the present value of W (in fact, it is not necessary to calculate this in an expected value model because of this assumption). However, in reality, if an individual vaccinates at t=0 then he is unable

[8] If the vaccination benefit today is higher than its cost, and the vaccination benefit is about to remain above the cost ($V<H_0$, $V<H_1$), there is no value in reversibility and immediate vaccination is optimal just because the vaccination has positive net immediate gain ($G= H_0- V >0$).

to11owever11atede at t=1, and therefore he loses the benefit of gaining W in the future. Therefore, the real option theory enables us to model the value in vaccine reversibility.

It is rationally optimal for the individual to vaccinate only if the gain G is larger than W that compensates for the lost reversibility:

$$G > W > 0 \Rightarrow H_0 - V > e^{-rt}(V - H_1) > 0$$
$$\Rightarrow H_0 > V\,(1 + e^{-rt}) - e^{-r}\,H_1 \qquad (1)$$

where the term $(1 + e^{-r}\ )$ in $V\,(1 + e^{-r}\ )$ magnifies the effective vaccination cost V by including the lost option. Appendix B presents a formal mathematical model for the intuitive results. Hypothetically, if the individual vaccinates today (at time 0) and reverses the decision at time 1, they make $V - H_1$ at time 1 from reversing the decision. However, the discounted value of reversing the decision at time 1 is lost if the individual vaccinates in addition to the vaccination cost paid. Therefore, in order for the individual to choose vaccination, the benefit needs to exceed the vaccination cost plus the lost reversibility as in the last line of the equation. For example, one outcome of the model is that if the interest rate is 0 at its lowest value, vaccine cost V must be larger than the average vaccine benefit ($\frac{H_0+H_1}{2}$) or else the individual will not vaccinate.

With considering the lost reversibility, the decision depends on the true net gain (NG) instead of making vaccination decisions based on the immediate gain G. Vaccination is feasible only if the true net gain is positive:

$$\text{NG} = G - W = H_0 - \left[V\,(1 + e^{-rt}) - e^{-rt}H_1\right] > 0,$$
$$\frac{\Delta NG}{\Delta V} = -(1 + e^{-rt}), \quad \frac{\Delta NG}{\Delta \text{H}_0} = 1\,, \quad -\Delta V = \Delta \text{H}_0 \Rightarrow \frac{\Delta NG}{\Delta V} > \frac{\Delta NG}{\Delta \text{H}_0} \qquad (2)$$

where $-\Delta V = \Delta \text{H}_0$ considers the hypothetical scenario in which a marginal decrease in the vaccination cost is equivalent to the marginal increase in the benefit. The model shows that a marginal decrease in the vaccination cost (e.g., reward at vaccination) has more effect on the vaccination true net gain compared to an equivalent-sized marginal increase in the benefit (e.g., saving tax on non-vaxxers by vaccinating). Moreover, *ceteris paribus*, there are four variables under this model that increase the likelihood to wait rather than immediately vaccinate due to declining net gain: an increase in fixed vaccination cost ($\frac{\partial NG}{\partial V} < 0$), a decrease in the current benefits of vaccinating ($\frac{\partial NG}{\partial \mathrm{H}_0} > 0$), a decrease in the future benefits of vaccinating ($\frac{\partial NG}{\partial \mathrm{H}_1} > 0$), and a decrease in the discount rate ($\frac{\partial NG}{\partial \mathrm{r}} > 0$).

In sum, this basic model shows that the decision to vaccinate considers lost reversibility on the top of the immediate gain. An observer who ignores the lost reversibility may witness a seemingly puzzling decision by an observed individual where the immediate gain from vaccination is positive and, yet the individual does not vaccinate (see **Error! Reference source not found.**-B). This is because the total benefit must compensate for both vaccine cost *and* lost reversibility to justify vaccination. Next section further analyzes this setting under uncertainty.

II.B. Basic Model with Uncertainty

In this model, we add uncertainty and lost abandonment to the earlier irreversibility model in a basic binomial setup where the vaccination benefit is both time-varying and uncertain. We show that uncertainty magnifies the lost reversibility and abandonment in form of the value in waiting. Like the earlier model, we have two periods with constant vaccination cost V. Time 0 is today with vaccination benefit $H_0>V$ so that it is feasible to vaccinate today; the vaccination has positive immediate gain ($G=H_0-V>0$). At time 1 with time difference t, the vaccination benefit (H_1) can go up ($H_{u1}> H_0$) or down ($H_{d1}<V$).[9] The real probability of up (down) is p,$(1-p)$.

[9] If the benefit in the down branch is higher than the vaccination cost, then waiting is not feasible at all. In Appendix C, we present analysis of the conditions for the option to lead to waiting.

We have:

$$H_0 = h_0 + e^{(-t\rho)}E(H), \quad E(H) = H_{u1}p + (1-p)H_{1d},$$

$$u = \frac{H_{u1}}{H_0} > 1, d = \frac{H_{d1}}{H_0} < 1 \tag{3}$$

where $\rho > r$ is the risk-adjusted discount rate for the uncertain benefits. When the risk of the benefits increases (e.g., the range increases while the expected value remains the same), ρ increases. The setting is like a binomial tree (see Figure 4 for the decision tree). We apply the classic risk-neutral valuation (Cox, Ross, and Rubinstein 1979; Hull 2018, Chapter 10) on the

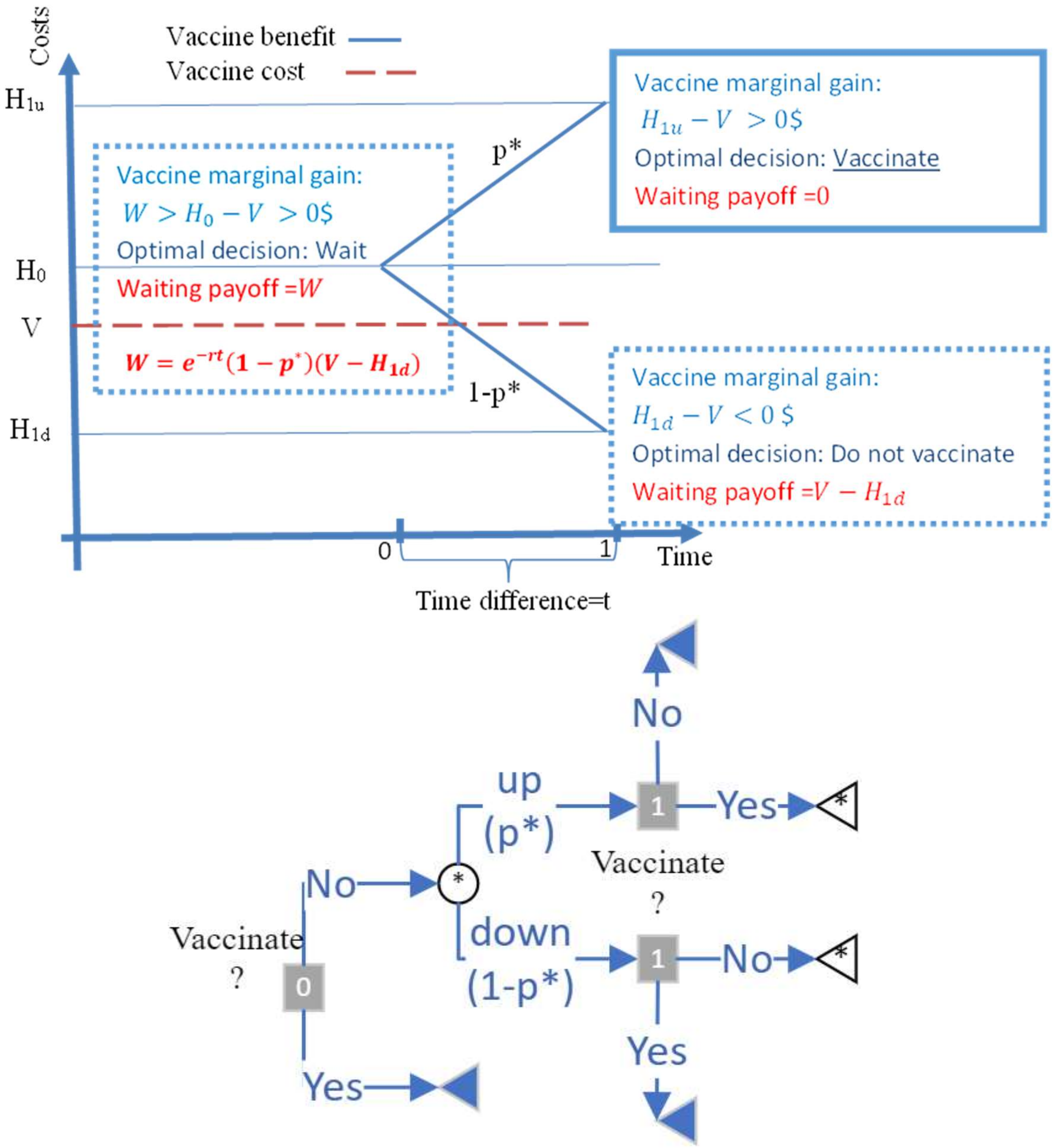


Figure 4- A basic model with uncertainty about the vaccine benefits and waiting to vaccinate: p* shows the risk-neutral probability. For the top figure, each box represents an outcome at time 0 (today) or the next period (time 1) in the binomial tree. The individual will not vaccinate today or tomorrow if the vaccine has negative marginal gain (boxes with dashed borders) and only vaccinates if the marginal gain increases substantially (the box with solid border). In each box, the top lines in blue show the marginal value of the vaccination. The bottom lines in red show the payoff from the option to reverse the vaccination benefit. The figure at the bottom shows the decision-tree diagram where the shapes with * show the optimal branches.

reversibility option (W). The technique uses valuation of the option relative to the benefit's value with a no-arbitrage trading strategy. We define the risk-neutral probability on the tree and use the risk-free rate to discount the payoff from the waiting to value the option at time 0. Since the option is lost after time 0, the total vaccination gain must exceed the lost option ($G>W$) in order for the model to be informative about vaccination decisions, so we have:

$$H_0 - V = G > W = e^{-rt}(1-p^*)(V - H_{0d}d),\ \ p^* = \frac{e^{rt}-d}{u-d}$$
$$\Rightarrow H_0 > V + W: H_0 = E^*(H) = p^* H_{1u} + (1-p^*)H_{1d} \quad (4)$$

Otherwise (i.e., $G < W$), Figure 4 shows that it is optimal for the individual to wait and only vaccinate if the marginal vaccine gain increases in the upper branch. This result means that the decision to vaccinate not only needs to have a net positive immediate marginal gain, but it also must exceed the lost reversibility option which is like a put option. We define Δ as the equal

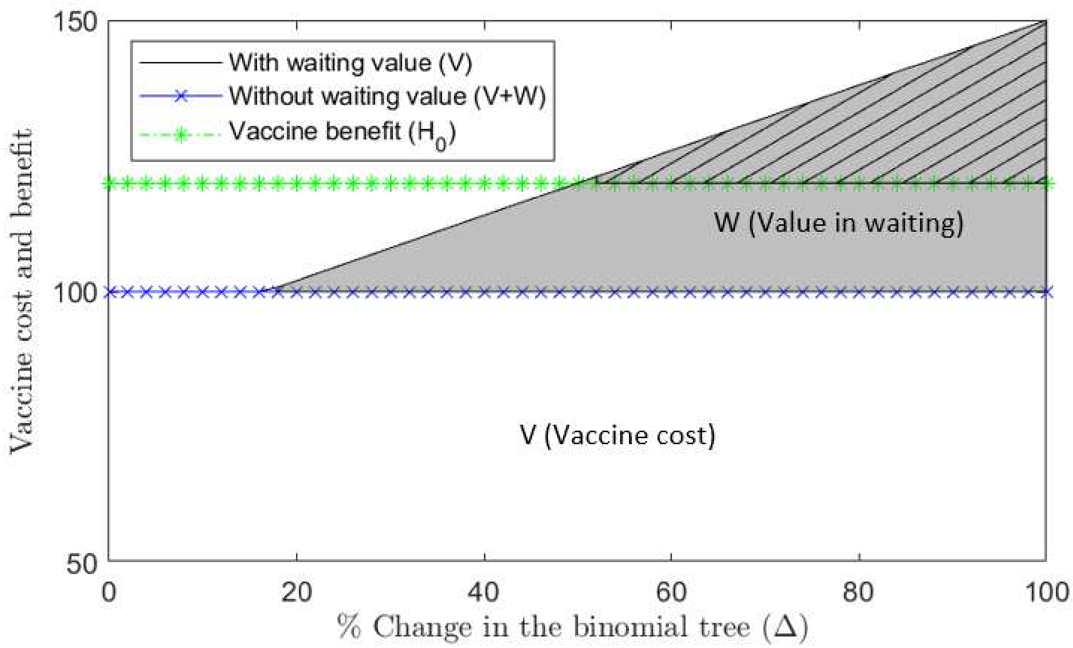


Figure 5- Vaccine benefit and cost with/out the waiting option: Y-axis shows the cost and benefit. X-axis shows the relative size of the change (increase or decrease) in the vaccination benefit in the binomial branches ($\Delta = H_{1u}/H_0 - 1 = 1 - H_{1d}/H_0$). Asterisked line shows the vaccine benefit. Crossed line shows the vaccine cost. Black line shows the vaccine cost plus the waiting value. Gray area shows the vaccine cost topped up with the value in waiting. We set risk free rate r at 0, vaccine cost V at \$100, vaccine benefit H_0 at \$120, and period length t at 1 year. For the changes (Δ) above 50% (the patterned gray area), the vaccine cost plus the waiting value (black line) is higher than the benefit (asterisked line), which means inaction (waiting) is optimal.

percentage change (increase or decrease) that the benefit will have depending on the outcome from the upper or lower branch in the binomial ($u = 1 + \Delta$, and $d = 1 - \Delta$). Since Δ represents equal (in)decrease in the benefits without changing the expected value, its change represents risk. For simplicity, we set the risk free rate at 0. Figure 5 shows how an increase in the risk after a certain level makes inaction (i.e., waiting) optimal. For example, if $H_{1d} = 0$ (Δ=100%), the value of lost reversibility is half of the vaccination cost ($V/2$) where the optimal decision is waiting and only vaccinating if the disease gets worse in the binomial upper branch as in Figure 4.

II.C. Continuous-Time Model with Uncertainty

To consider the uncertainty involved with the disease, we move to a continuous-time setting. For simplicity, we assume that the individual faces the potential threat of disease forever (perpetual exposure).[10] Once infected, the individual continuously pays disease cost h (cost per unit of time, e.g., daily hospitalization cost) that lasts during infection period τ.[11] We assume the disease cost in time follows Geometric Brownian Motion (GBM)

$$\frac{dh}{h} = (r - \delta)dt + \sigma dB, \qquad E(h_{s+u}|h_s) = e^{(r-\delta)u}h_s \tag{5}$$

where σ is uncertainty, and B is Standard Brownian motion. δ is the adjustment to the growth of the expected disease costs (high δ means declining expected disease costs). For example, if the disease reproduction number (R0) is greater (less) than 1, the chance of infection increases (decreases), and a high (low) δ can capture its dynamic. The expected infection cost H_s is the

[10] Limited-time disease exposure require numerical derivation, instead of closed-form solutions, similar to an option with limited maturity, but it does not change our results.

[11] In Appendix D, we present an alternative explanation for the expected disease cost to follow GBM. Alternative random processes, such as mean-reverting processes as in Park (2016), does not change the model inferences, but requires more extensive numerical calculations.

probability of infection π times the present value of the total disease costs during the infection period:

$$\mathrm{H_s} = \pi \times \mathrm{E}\left(\int_{s}^{\mathrm{s}+\tau} e^{-r(u-\mathrm{s})} h_{\mathrm{u}}\, du | h_s\right) = \frac{\left(1 - e^{-\delta\tau}\right)}{\delta} \pi \mathrm{h}_s$$
$$\frac{dH_s}{H_s} = \frac{dh_s}{h_s} = (\mathrm{r} - \delta)dt + \sigma dB \tag{6}$$

Once the vaccine is administered, the expected infection cost drops by the vaccine effectiveness (ϵ) times *H*. This change in infection costs represents the benefit of vaccination because vaccination saves this cost. An effectiveness equal to 1 means that the vaccine gives full immunity to the individual and make the vaccine benefit equal to *H*.

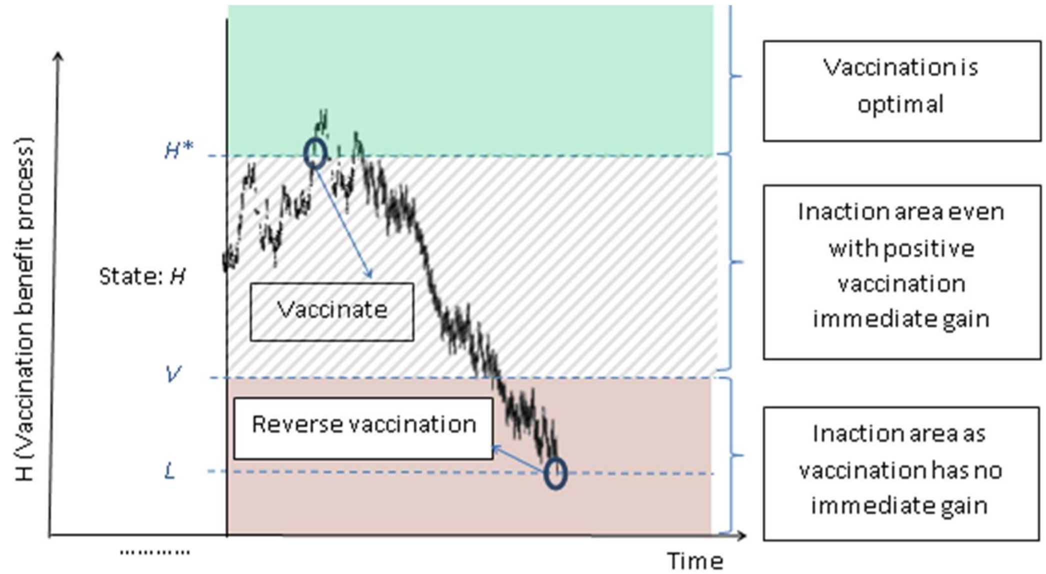


Figure 6-A-The decision steps depending on the vaccination benefit state: The patterned inaction area is analogous to the same area in Figure 5.

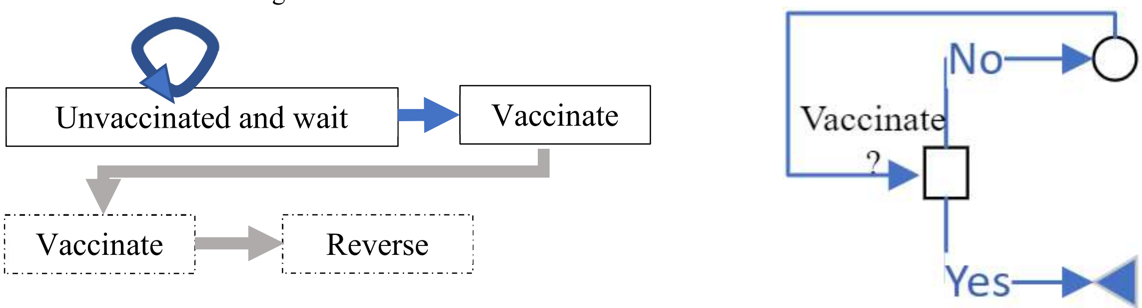


Figure 6-B-The influence diagram for the vaccination decision (blue) and hypothetical vaccination reversibility (gray). The decision-tree diagram is on the right.

Figure 6- The decision steps depending on the vaccination benefit state, and its influence diagram for the vaccination decision

Following the literature, we assume that the individual "faces a trade-off between the probabilistic benefit of vaccination and the deterministic cost of vaccination" (Lim and Zhang 2020). Similar to the binomial model in the previous section, the individual waits to vaccinate unless the benefit of the vaccination is larger than the vaccination cost plus the lost reversibility option. To evaluate lost reversibility, first we consider the hypothetical scenario that we do have the option to reverse the vaccination decision. Of course this option does not actually exist in real life, but we display this model here as a first step in order to make it clear that reversibility has value.

The graph in Figure 6-A displays two decisions that the individual would optimally like to make if the option to reverse existed: first we determine the optimal level to reverse vaccination and the reversibility option, assuming vaccination is done. Then, we move back in time and find the optimal strategy to vaccinate given the optimal reversing strategy ex ante. Figure 6-B shows the influence diagram of the individual's decision with the hypothetical scenario where there is the option to reverse the vaccination (for more influence diagram examples, see Sick and Gamba (2010)). An influence diagram is a common way of determining the decision of whether a project should be started or shut down (e.g., reverse).

The decision to reverse happens when the benefits fall significantly to L, which is below the vaccination cost V. For the benefits below V and above L ($L<H<V$) for which the gain is negative ($H-V<0$), it is not yet optimal to reverse the vaccination decision due to the possibility of the benefits rising again. This reversing is similar to classical project decisions; for example, one can imagine an oil well that has a cost to shut down when oil prices are below the unit production cost. Even if the oil well is losing money because the prices are below the unit production cost, there is a price range at which it is optimal to maintain the current course, i.e., if you are currently producing oil, you do not stop (see, e.g., Smith and McCardle 1998). Analogously, for some benefit levels below the vaccine cost, it is not optimal to reverse vaccination even if vaccine benefits are below its immediate cost.

Next, we value the option to reverse the vaccination, which we will then use as the value of lost reversibility. To find the option value, we characterize the optimal benefit at which the individual would reverse the vaccination decision. The option value is like an American put option, and the optimal benefit is like the optimal exercise threshold of the put option. The put option value can be derived based on classical Black-Scholes partial differential equation (PDE). The PDE is the result of solving Hamilton–Jacobi–Bellman (HJB) equation or using the Martingale approach as described in Appendix D (Dixit and Pindyck 2012, Chapter 4). The PDE, the solution for the option value, and the optimal reversing point L are:

$$\frac{1}{2}\,\sigma^2\,H^2\,W_{HH} + (r-\delta)HW_H - rW = 0, \qquad W_H = \frac{\partial W}{\partial H},$$

$$W(H=L) = V - \epsilon L, \qquad W = (V-\epsilon L)\left(\frac{H}{L}\right)^{-b}, \qquad L = \frac{b}{\epsilon(1+b)}V \tag{7}$$

$$b = \frac{\sqrt{(a^2+2r)}+a}{\sigma}, \qquad a = \frac{r-\delta-0.5\sigma^2}{\sigma}$$

where the option value is from the general solution of the PDE, which is derived by the boundary condition at the reversing point, and the optimal reversing point is derived from the smooth-pasting condition applied to the option value (see, e.g., Dixit 2013 for the technique). The outcome is that the optimal reversing point is smaller than the vaccination cost. For any values between the vaccination cost and the optimal point, reversing is not optimal because the chance that the vaccine benefit increases due to uncertainty prevents the decision to reverse the vaccine.

Finally, we determine the optimal vaccination decision (see Appendix E). The individual chooses vaccination if the benefit level (H^*) exceeds the vaccination cost plus its lost reversibility, which satisfies the following equation:

$$H^* - V - (1-\epsilon)H^* - (V-\epsilon L)\left(\frac{H^*}{L}\right)^{-b} = 0 \Rightarrow H^* = \frac{V + (V-\epsilon L)\left(\frac{H^*}{L}\right)^{-b}}{\epsilon} \tag{8}$$

We solve this equation numerically for optimal benefit level because it does not have a closed-form solution. Yet, the formula has some intuitive results. For example, as vaccine effectiveness approaches to zero, the individual will not vaccinate because the required benefit (H^*) in order to vaccinate grows close to infinity,. If the current vaccination benefit H is above H^*, individuals immediately vaccinate without waiting. But, if the current vaccination benefit H is below H^*, the individual waits to vaccinate until the benefit reaches the required level.

The model here is similar to asset-replacement models in operation management. Asset replacement models are about optimal replacement of an asset that continuously produces operation costs. The firm decides when to incur a fixed cost to replace the asset with a new asset that has lower operating costs (see, e.g., Dobbs (2004), and Adkins and Paxson (2011)). Analogous to these models, the fixed replacement cost is like the fixed vaccine cost, and the drop in the operation cost is like the drop in the infection cost due to the immunity provided by the vaccine. However, our model has some different features: we include the lost reversibility of the vaccine, and we sum up the continuous exposure to the infection cost within the present value of the vaccine benefit.

The probability that the random process for *H* hits the optimal level before time *T* is:

$$P = N(A) + e^{C} N(Z), \qquad x = \ln\left(\frac{H}{H^*}\right)$$
$$A = \frac{x + (r - \delta - 0.5\sigma^2)T}{\sigma\sqrt{T}}, Z = \frac{x - (r - \delta - 0.5\sigma^2)T}{\sigma\sqrt{T}}, C = -\frac{2x(r - \delta - 0.5\sigma^2)}{\sigma^2} \qquad (9)$$

where *N(.)* is standard cumulative normal distribution.

III. Sensitivity Analysis

In this section, we investigate the implications of the model. We aim to find the most critical factors in the vaccination decision.

First, we show that not all positive vaccination gains (H>V) result in immediate vaccination. For calibration purposes, we set the risk-free rate at 6%, which is slightly above the historical average of 4% (see Lotfaliei 2018) to remain parsimonious. A lower risk-free rate increases the future values and makes the delay more likely. We assume vaccine effectiveness is at 100% for brevity. We set vaccination cost at $100 as the reference point, which is scalable. The vaccination benefit is at $120, 20 points above the vaccination benefit so that the immediate vaccination is positive. Both numbers are scalable. We check the probability of vaccination in half a year (T=0.5). We set the benefit uncertainty at 0.45 and the benefit decrease rate at 0.03. For the calibrated values above, the optimal vaccination benefit level is about $135 (see Figure 7). For vaccination benefits above $100 and below $135, the vaccination has positive immediate gain, yet the individual waits and does not vaccinate because the net gain is negative.

To avoid the current levels for these factors influencing our inferences, we report the sensitivity of the results for both the uncertainty and the decline rate. Figure 8 shows increasing optimal benefit-cost ratio for the vaccination, and, thus, increasing optimal benefit level to vaccinate when uncertainty increases. The uncertainty increases the likelihood that the individual considers the lost reversibility of the vaccination in the future. This in turn increases the value in waiting to vaccinate, and the optimal benefit threshold. Figure 9 reports the same effect for the vaccination benefit decline rate. When the benefit declines faster, the optimal threshold to vaccinate increase with the hopes that the disease cost becomes so low that there is no need for

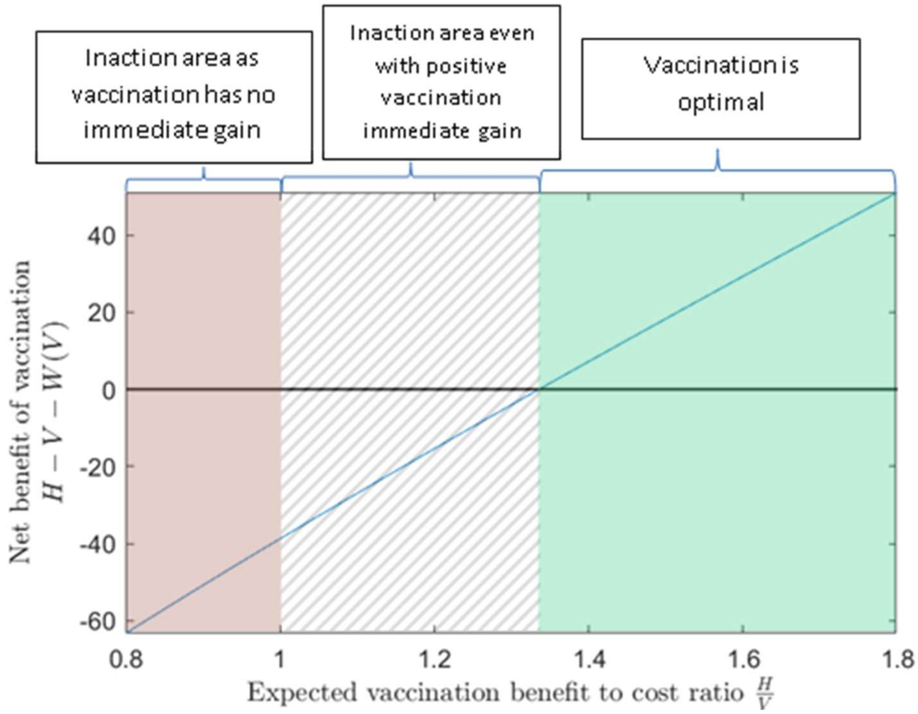


Figure 7- The net vaccination gain as function of the vaccination benefit to cost ratio: Due to the value in waiting to vaccinate the benefit-cost ratio should exceed 1.35 for the vaccination to be optimal (green area on the right). For the ratio below 1.35 and above 1 (patterned area in the middle) inaction is optimal because the vaccine benefit does not exceed the lost reversibility plus the vaccine cost. The patterned inaction area is analogous to the same area in Figure 5. For the ratio below 1, the benefit does not even exceed the vaccine cost (red area on the left). The uncertainty is 0.45. The benefit decrease rate is 0.03. The risk-free rate is 0.06. The vaccination cost is $100 and is scalable.

vaccination. Both uncertainty and the decline rate reduce the probability of the vaccination (see Figure 11 and Figure 10) because they raise the bar for the optimal benefit of the vaccination. The

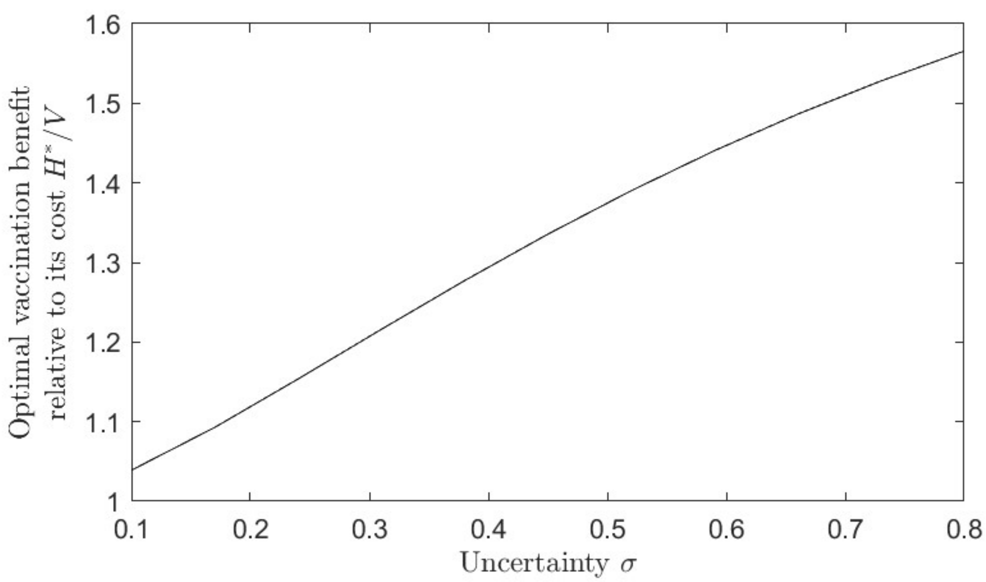


Figure 8- The optimal vaccination benefit-cost ratio increases as the vaccination benefit uncertainty increases: Due to the value in waiting to vaccinate the benefit-cost ratio should exceed 1. The benefit decline rate is 0.03. The risk-free rate is 0.06. The vaccination cost is $100 and is scalable.

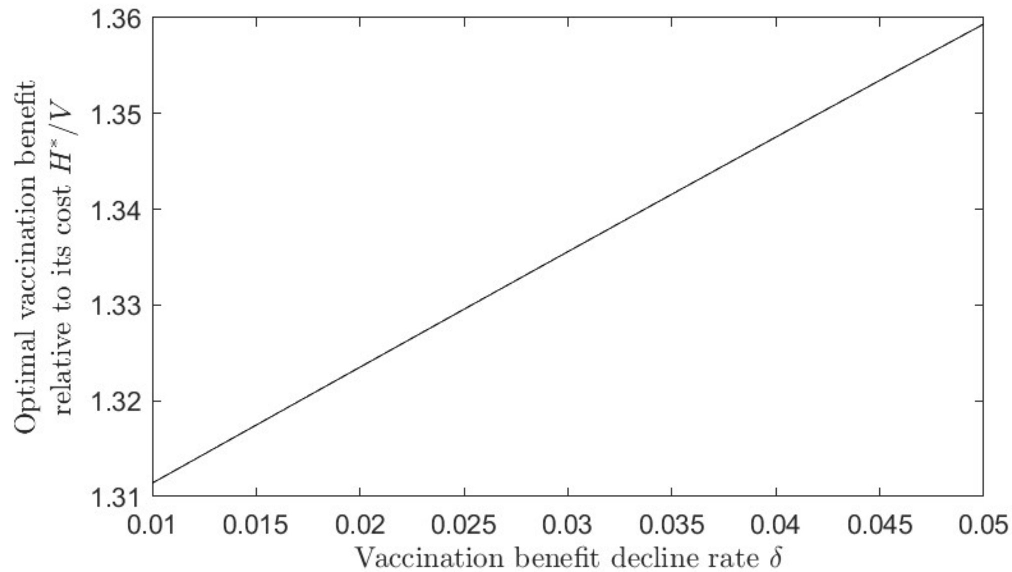


Figure 9- The optimal vaccination benefit-cost ratio increases as the vaccination benefit decline rate increases: Due to the value in waiting to vaccinate the benefit-cost ratio should exceed 1. The uncertainty is 0.45. The risk-free rate is 0.06. The vaccination cost is $100 and is scalable.

model-generated probabilities show the capability of the model to produce a wide range of reported vaccination rates, such as 52% for seasonal flu (CDC 2021), or 82% for Covid-19 (CDC 2022). The probability range is wider for uncertainty (40%) relative to the decline rate (10%). Therefore, the decision to delay is much more sensitive to uncertainty than to the decline rate.

Finally, we analyze how vaccination probabilitiy is affected by hypothetical public policies to either punish no vaccination or reward vaccination. We consider a onetime tax on individuals who do not vaccinate immediately, which increases the vaccination benefit by Δ between 0 and $10. As an alternative, we consider an equal-sized reward at vaccination, which reduces the vaccination cost by the same amount. Figure 12 shows the increasing probability of vaccination as the result of both policies.25oweverer, a reward at vaccination is more effective in increasing the probability of the vaccination relative to the same amount as a tax on non-vaxxers. Under both cases, the uncertainty reduces the effectiveness of both policies by flattening the probability of vaccination in the graphs. Therefore, reducing uncertainty for the individuals (e.g., through information dissemination, education, and reduction in contradictory advice) seems to be the most critical factor in promoting vaccination (i.e., increasing the probability of vaccination) based on our model. The next critical factor in promoting vaccination is the certain cost of vaccination, which can be reduced by a vaccination reward.

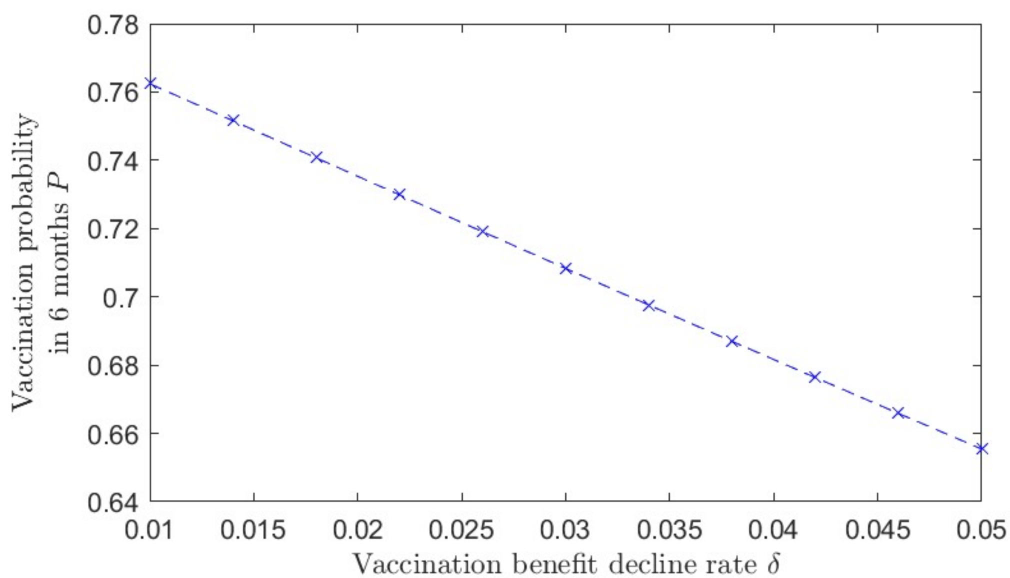


Figure 10- The probability of vaccination decreases as the vaccination benefit decline rate increases: Due to the value in waiting to vaccinate the benefit-cost ratio should exceed 1. The uncertainty is 0.45. The risk-free rate is 0.06. The vaccination cost is $100, vaccination benefit is $120, and both are scalable.

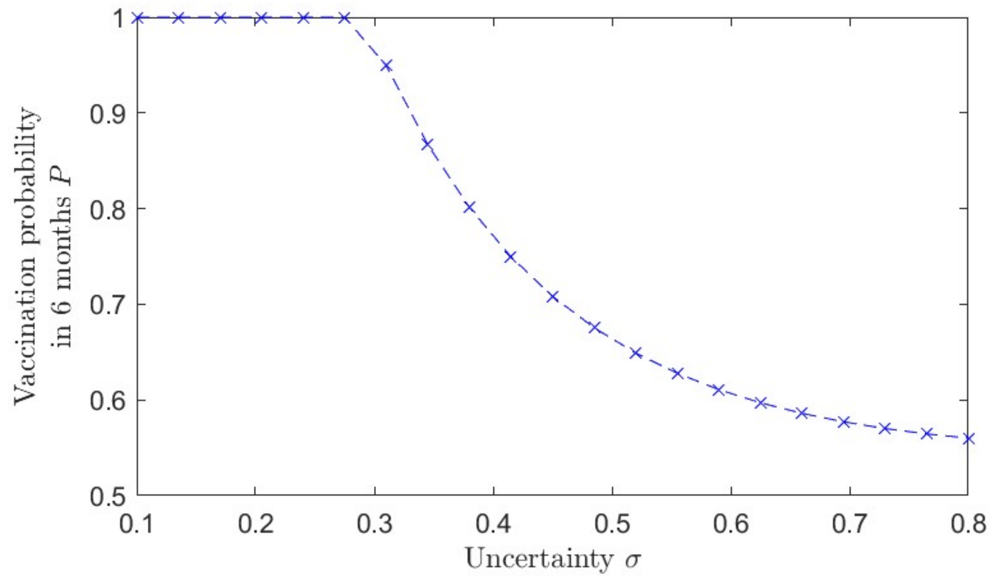


Figure 11- The probability of vaccination decreases as the vaccination benefit uncertainty increases: Due to the value in waiting to vaccinate the benefit-cost ratio should exceed 1. The benefit decline rate is 0.03. The risk-free rate is 0.06. The vaccination cost is \$100, vaccination benefit is \$120, and both are scalable.

This may seem like a difficult result to accept. After all, is there really a difference between calling it a reward versus calling it a reverse tax? The difference is its effect on the value of reversibility. The benefits of vaccination in the future no longer include avoidance of the tax penalty. Therefore, the value of reversibility no longer accounts for the punitive one-time tax penalty. The reward for vaccination, on the other hand, may still be available in the future in some states of the world, including in some states of the world where the individual would desire to reverse vaccination (if such a thing were possible, which of course is not the case).

IV. Conclusion

This paper shows that the value in waiting to vaccinate delays vaccination, even if vaccination has a positive immediate gain for individuals. The value in waiting measures the irreversibility of the vaccination. We calculate the value in waiting using the real option theory. Real option theory in health care mostly addresses investment decision (de Cássia Rocha, Gonçalves, and Lawryshyn 2020). In this paper, we extend the real option theory to model individual vaccination decisions.

Our model shows that uncertainty about the vaccination benefit is a strong factor in decreasing the vaccination rate. Therefore, a very effective policy is to target reducing the uncertainty about the vaccination benefits. Among two other policy options (either increase vaccination benefit by exerting a tax on no vaccination or rewarding vaccination with a subsidy), the later is more effective. However, the effectiveness of both policies on increasing the

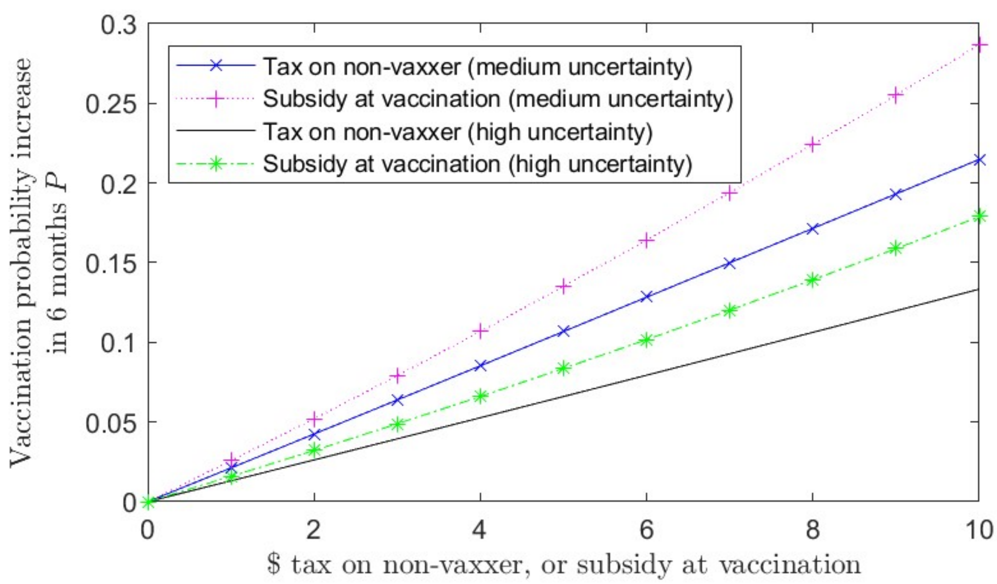


Figure 12- The probability of vaccination increases as a reward (or subsidy) is offered at vaccination or tax is exerted on no vaccination: The medium uncertainty is 0.45, and high uncertainty is 0.7. The risk-free rate is 0.06. The benefit decline rate is 0.03. The vaccination cost is \$100, vaccination benefit is \$120, and both are scalable.

vaccination rate declines if uncertainty is high. This shows why uncertainty is the most critical factor.

Future research can combine game theory where there is free-riding possibility between vaxxers and non-vaxxers due to herd immunity (e.g., Bhattacharyya and Bauch 2011) with the real option. Pan, Ng, and Cheng (2021) show that free-riders benefit from increasing uncertainty about their own vaccination among the rest of the population so that they can free ride on others' vaccinations. Including real options in these games may exacerbate the use of this strategy; since more uncertainty means lower probability of vaccination, it is possible that increasing uncertainty through more cheap talk may send a stronger no-vaccination signal to others. The model is extendable into rationally explaining other medical decisions: e.g., health providers may delay the transfer of the critically ill patients from emergency room to intensive cares due to high uncertainty about the patients' condition in addition to some of the classical reasons like delayed x-rays (see, e.g., Teklie et al. 2021). Patients may rationally delay their treatment (e.g., going to emergency room) due to uncertainty about their own condition. If patients facing irreversible treatment costs are more uncertain about their health condition, or the quality of the health services, they may be more likely to wait and only seek treatment when their condition gets worse even if the immediate treatment would improve their conditions net of costs (see, e.g., for heart diseases as in Moser et al. 2006).[12]

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[12] To the argument that"…each of us to weigh the value of those services (their benefits and harms) against the cost we would have to pay and would ensure that, both for the individual and for society, the value of health care would be worth its cost" (Eddy 1991) we can add "and worth the value in waiting to vaccinate."

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Appendices

Appendix A- List of the variables and their description

Table 1- List and description of the model variables

$-H$	Present value of the disease costs	r	Risk-free interest rate
H	Vaccine benefit saving disease cost ($-(-H)$)	$-V$	Irreversible vaccination cost
G	Immediate vaccination gain (H-V)	δ	Adjustment to disease cost growth rate
W	Reversibility option value for vaccination	H_i	Vaccine benefits at time i in discrete model
NG	Vaccination true net gain (G-W)	H^*	Optimal benefit level for vaccination
σ	Vaccine benefit-change standard deviation, volatility	P	Probability of vaccination (vaccine benefit reaching optimal benefit)
ϵ	Vaccine effectiveness, 0=no effectiveness, 1= 100% effectiveness	B	Standard Brownian motion
$H_{u1,d1}$	Vaccine benefit in upper (u1), and lower (d1) of the binomial tree	p	Real probability of vaccine benefit increase in binomial model
u, d	(In)decrease rate for vaccine benefit in binomial tree respectively	p^*	Risk-neutral probability of vaccine benefit increase in binomial model
ρ	Risk-adjusted discount rate for the uncertain benefits	Δ	Equal percentage change (increase or decrease) for benefit H on binomial tree

Appendix B- Vaccination Decision

Appendix B-1- Partially effective vaccine

An individual without vaccination faces expected infection cost as the probability of infection times infection costs:

$$-[A \times P_i + (1 - P_i) \times 0] \tag{10}$$

where A is infection cost, and P_i is probability of infection without vaccination. With partially effective vaccine, the individual faces lower expected infection cost plus vaccination cost:

$$-[V + A_v \times P_{iv} + (1 - P_{iv}) \times 0] \tag{11}$$

where V is the vaccine cost, A_v is infection cost for a vaccinated individual, and P_{iv} is the probability of infection for a vaccinated individual. The marginal immediate gain for vaccination is:

$$A \times P_i - [V +] = [A_v \times P_{iv} - A_v \times P_{iv}] - V = H - V, \qquad [A_v \times P_{iv} - A_v \times P_{iv}] = H \tag{12}$$

where *H* measures the marginal decline in diseases costs.

Appendix B-2- A basic optimization model

The individual considers the following decision for vaccination:

$$v = \underset{v=\{0,1\}}{\operatorname{argmin}}[f(v,H)], \quad f = (1-v)H + v\big([1-\epsilon]H + V + W(H)\big) \tag{13}$$

where v is 0 when the decision is not to vaccinate and 1 otherwise. The minimization is to reduce the cost that the individual faces where the vaccination decision acts like a switch from exposure to disease costs to exposure to vaccine-related costs. The solution to the minimization problem is:

$$v = \begin{cases} 0, & H < H^* \\ 1, & H \geq H^* \end{cases}, \quad H^* = V + (1-\epsilon)H^* + W(H^*) \tag{14}$$

Appendix C- Feasibility Condition for Binomial Model

For the risk-neutral probability in Equation (4), we have:

$$r, t\colon\ 0 < p^* = \frac{H_0 e^{rt} - H_{1d}}{H_{1u} - H_{1d}} < 1 \tag{15}$$

We focus on the upper boundary for the risk-neutral probability. For the upper boundary to hold, we must have:

$$p^* < 1 \ \Rightarrow h_0 < e^{-rt}H_{1u} - e^{-\rho t}H_{1d} - e^{-\rho t}E(H) \tag{16}$$

In the special case that the lower branch of the binomial hits its lower boundary ($H_{1d} = 0$), we get:

$$\begin{gathered} h_0 < H_{1u}(e^{-rt} - pe^{-\rho t}) = UB, \\ r < \rho, p < 1 \Rightarrow e^{-rt} > pe^{-\rho t} \end{gathered} \tag{17}$$

UB is the upper boundary for h_0 such that waiting is optimal. For values of h_0 above UB, immediate vaccination is optimal. Since the upper boundary of h_0 is larger than 0, there are ranges of feasible values for h_0 under which the waiting is optimal.

Appendix D- Alternative Explanation for Vaccine Benefit Process

Appendix D-1- GBM process

As an alternative explanation for the vaccine benefit to follow GBM, we present the following setting. The individual is within a population and the infected population size (IP) follow GBM:

$$\frac{d\mathrm{IP}}{\mathrm{IP}} = \mathrm{a}_1 dt + \sigma_1 dB_1 \tag{18}$$

The individual estimates the probability of infection naively as the infected population size divided by total population. Hence, the expected infection cost as the probability of infection times infection treatment cost is:

$$\mathrm{H_s} = \frac{\mathrm{IP}}{\mathrm{TP}} \times C \tag{19}$$

where TP is total population, *C* is infection treatment cost. The infection treatment cost *C* also follows GBM with a positive correlation (ρ) with the infected population size:

$$\frac{dC}{C} = a_2 dt + \sigma_2 dB_2, \quad E(dB_1 dB_2) = \rho dt, \quad \rho > 0 \tag{20}$$

For simplicity, we assume the treatment cost positively correlates with the infected population because, e.g., an increase in the infected population reduces the treatment capacity of the medical system and increases the treatment costs. Then, the expected infection cost still follows GBM, and we have:

$$\frac{dH_s}{H_s} = \frac{d\mathrm{IP}}{\mathrm{IP}} + \frac{dC}{C} + \gamma\sigma\rho dt = (r - \delta)dt + \sigma dB$$
$$\delta = r - (a_1 + a_2 + \gamma\sigma\rho), \quad \sigma = \sqrt{\sigma_1^2 + \sigma_1^2 + 2\rho\sigma_1\sigma_2} \tag{21}$$

Appendix D-2- HJB derivation of pricing PDE

The HJB is the dynamic programming approach instead of the martingale approach to formulate and solve the optimal stopping decision. The optimal stopping decision in our setting is the time that the individual wishes to reverse the vaccination decision. Following Dixit and Pindyck (2012, Chapter 4), the value function is:

$$W(H_0, 0) = \max_{I=1,0}\{e^{-rt}E[W(H_t, t\,)|H_0, I]\} \Rightarrow rW(H_t, t)dt = \max_{I=1,0}\{E[dW]\} \tag{22}$$

where *I* is 1 if reverse decision is executed, and 0 otherwise. The equation on the right is the results of taking derivative from the equation on the left.

Appendix E- Valuation PDE and Solution

The opportunity cost measured by the discount rate *r* when time changes by *dt* matches the change in the value of the reversibility for the individual:

$$rdt = \frac{E(dW)}{W} \tag{23}$$

Since the option is infinitely lived, its value W does not directly depend on time ($W_t=dW/dt=0$). In asset pricing, the same term appears based on the no-arbitrage argument, i.e., the expected return of any asset with value *W*, including the reversing option, must match their opportunity cost measured by the discount rate *r* when time changes. The expected value of the change in *W* follows Ito's lemma as the function of the random variable H:

$$\begin{gathered} E(dW) = \frac{1}{2}\,\sigma^2\,H^2\,W_{HH} + (r-\delta)HW_H \\ rdt = \frac{E(dW)}{W} \Rightarrow \frac{1}{2}\,\sigma^2\,H^2\,W_{HH} + (r-\delta)HW_H - rW = 0 \end{gathered} \tag{24}$$

The general solution to the PDE is:

$$W = AH^{-b} + BH^{b_2}, \quad y = -b, b_2 : \frac{1}{2}\,\sigma^2\,(\mathrm{y}-1)\mathrm{y} + (r-\delta)\mathrm{y} - r = 0 \tag{25}$$

where y represents the roots of the characteristic quadratic of the PDE. A and B are constants determined by the boundary conditions of the value function. For the option to reverse, the value when H approaches infinity is 0 because there is no value in reversing vaccination for very high vaccine benefits, and, thus, B is 0. A is determined by the boundary condition and L is determined by the smooth pasting condition:

$$
\begin{aligned}
&W(H = L) = W(L): AL^{-b} = V - \epsilon L \Rightarrow A = \frac{V - \epsilon L}{L^{-b}} \\
&W_H(H = L) = W_L(L): -bAL^{-b-1} = -\epsilon \Rightarrow L = \frac{b}{\epsilon(1 + b)} V
\end{aligned}
\tag{26}
$$