Picking Your Patients:

Selective Admissions in the Nursing Home Industry

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Abstract

This paper uses patient-level administrative data on skilled nursing facilities in California to estimate a structural model of selective admissions in the nursing home industry. I exploit within-facility covariation between occupancy and admitted patient characteristics to distinguish admission patterns attributable to selective admission practices from heterogeneous patient preferences. In spite of anti-discrimination laws, I find strong evidence of selective admission practices that disproportionately harm Medicaid-eligible patients with lengthy anticipated stays. Counterfactual simulations show that enforcing a prohibition on selective admissions would increase access for these residents at the cost of crowding out short-stay non-Medicaid patients from their preferred facilities. I simulate two additional policies intended to mitigate selective admissions: raising the Medicaid reimbursement rate and expanding capacity. I find the latter to be less costly and more effective than the former.

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1 Introduction

Policymakers and academics have long been concerned with inequality in access to quality healthcare. Even with equal geographic access, inequality can persist when providers discriminate in picking their patients. This paper uses patient-level administrative data on residents of skilled nursing facilities (SNFs) to study the role of selective admission practices in the nursing home industry.The United States has more than 15,000 nursing homes with approximately 1.4 million residents at any given time (National Center for Health Statistics, 2017). The quality of these facilities varies substantially (Castle and Ferguson, 2010), and patients may not all have access to the same quality of care. Patients' rights activists (Carlson, 2005), policymakers (Senate, 1984; Wright, 2002), and academics (Gruenberg and Willemain, 1982; Ettner, 1993; Uili, 1995; Ching et al., 2015) have often accused SNFs of discriminating against patients who are poorer or sicker in admissions. If true, these practices have important implications for the health outcomes of these patients and may violate federal and state laws prohibiting Medicaid and disability discrimination.[1]

A number of factors incentivize selective admission practices in the nursing home industry. First, reimbursement rates can vary substantially depending on the payment source.[2] For example, while facilities are legally obligated to offer the same type and quality of care to their Medicaid and private-pay residents, private-pay rates are nearly 30% higher than Medicaid rates on average. Second, private and Medicaid reimbursements are frequently per-diems that do not adjust for additional care required by high-needs patients. Third, capacity constraints imply an opportunity cost to admissions.[3] These opportunity costs can be substantial since some residents are expected to remain at the facility for years, during which time their beds cannot be allocated to admit more profitable patients.

In order to measure the prevalence and impacts of selective admission practices, I estimate a structural model of admissions in the nursing home industry that incorporates both residents' preferences and facilities' admission policies. Specifically, I model nursing homes as multiproduct firms (Sullivan, 2017; Wollmann, 2018) facing capacity constraints (Ryan, 2012; Ching et al., 2015; Kalouptsidi, 2014, 2018) that dynamically adjust the types of patients they are willing to admit. Each facility considers each arriving patient and decides whether to offer her admission based on her characteristics, the facility's current census of residents, and the facility's beliefs about prospective residents that will arrive in the future. Patients then choose and are admitted to their preferred facility offering admission.

The primary empirical challenge is that only realized admissions are observed. Since realized admissions are determined by both facilities' admission policies and patients' preferences, it is difficult to distinguish admission patterns attributable to selective admissions from those attributable to heterogeneous patient preferences. For example, though dual-eligibles—i.e., residents eligible for Medicaid in addition to Medicare—are admitted to lower quality facilities (Rahman et al., 2014a,b), this does not necessarily imply Medicaid discrimination since the disparity could

[1] Being unable to access facilities that perform well on quality metrics such as staffing ratios and deficiencies is expected to result in worse health outcomes along a number of dimensions, including rates of mortality, infection, re-hospitalization, and discharge to community (Schnelle et al., 2004; Backhaus et al., 2014; Lin, 2014; Foster and Lee, 2015; Friedrich and Hackmann, 2021; Einav et al., 2022).

[2] One form of selection on payment source is a provider's decision about which insurance plans to accept. Recent literature has explored the relationship between networks, plan choice, and negotiated rates (Ho, 2006, 2009; Ho and Lee, 2019; Shepard, 2022). The vast majority of SNFs, however, accept all three primary payers: Medicare, Medicaid, and out-of-pocket. This paper examines selective admissions in the context of patients whose reimbursements are nominally accepted by the provider.

[3] The median facility occupancy in my sample is 88%, and my estimates suggest many facilities would quickly reach capacity if they admitted all applicants. Firms may also face increasing marginal costs due to challenges in adjusting staffing levels and other inputs. Such diseconomies of scale would raise the effective cost of admitting residents similarly to capacity constraints.

be also be explained by dual-eligibles having weaker preferences for quality metrics. Prior work modeling selective admissions typically assumes its prevalence under a specific form: that dual-eligibles can only choose from facilities with capacity remaining after all other patients have been admitted (Nyman, 1985; Gertler, 1992; Ching et al., 2015).

My model incorporates selective admissions as a source of unobserved variation in residents' choice sets (Crawford et al., 2021). Unlike many other settings with availability variation (Hickman and Mortimer, 2016), the set of facilities willing to admit a patient is never observed, and facilities' willingness to admit particular patients is plausibly correlated with those patients' preferences.[4] Agarwal (2015) addresses a similar issue in estimating a model of the medical residency match using only data on observed matches. While the dynamic and decentralized nature of admissions in the nursing home industry precludes the methods in Agarwal (2015), it also provides an alternative source of identification: facilities facing capacity constraints or increasing marginal costs are expected to increase the selectivity of their admission practices as their census of residents increases (Greenlees et al., 1982). I therefore exploit within-facility covariation between facility census and admitted resident characteristics to identify selective admission practices. The key identifying assumption is that transitory variation in a facility's census affects the facility's preferences over patients but not patients' preferences over facilities. In fact, Agarwal and Somaini (2022) show how in general, demand under latent choice constraints is identified given some instruments affecting only choice sets—e.g., transitory variation in occupancy—and some affecting only consumers' preferences—e.g., distance to the facility.

I first provide evidence that the composition of realized admissions covaries with facility occupancy: as facilities become more full, their newly admitted residents are less likely to be Medicaid-eligible, have long anticipated stays, or require uncompensated care. This suggests that admission policies discriminate against residents with these characteristics. I then use machine learning methods to aggregate residents' many observable characteristics into a univariate desirability score for each resident.

Finally, I estimate a structural model of admissions. As in the reduced form, the key challenge is that only realized admissions are observed. Estimating demand without accounting for selective admissions may incorrectly infer that patients dislike facilities that are turning them away. Moreover, estimating dynamic admission rules separately from demand using common conditional choice probability (CCP) methods (Hotz and Miller, 1993; Hotz et al., 1994; Pakes et al., 2007; Bajari et al., 2007; Arcidiacono et al., 2016) is precluded by the fact that facilities' actions (admission rules) are not observed. Therefore, I simultaneously estimate resident preference parameters and facilities' admission policies via maximum likelihood, where admission policies determine residents' choice sets, and resident preferences determine the realized admissions from within those sets. Then, in a second stage, I estimate the structural profitability parameters underlying facilities' admission policies by matching moments in the data to those implied by optimal admission policies (Rust, 1987). These second-stage estimates enable computing facilities' optimal dynamic admission policies in counterfactual simulations. As in the reduced form, selective admissions in both stages of the structural

[4] Prior work on product availability has primarily focused on assortment variation (Tenn and Yun, 2008; Bruno and Vilcassim, 2008; Draganska and Klapper, 2011; Shah et al., 2015; Musalem, 2015) and stock-outs (Anupindi et al., 1998; Musalem et al., 2010; Conlon and Mortimer, 2013) in retail settings. Recent innovations have considered persistently unobserved choice sets such as Abaluck and Adams-Prassl (2021) that leverages violations of Slutsky symmetry, Barseghyan et al. (2021a) that leverages excluded regressors, Barseghyan et al. (2021b) that assumes only a minimum choice set size and employs a conditional moment inequality approach, and He et al. (2021) that leverages stability and exclusion restrictions in matching markets. Most notably, Agarwal and Somaini (2022) formalizes and generalizes a similar identification argument in this paper to a broad class of two-sided markets.

estimation are identified from the relationship between facilities' censuses and realized admissions.

My structural estimates indicate that 19% of California's SNF residents are not admitted to their first-choice facility with available beds. Medicaid-eligible residents with very long anticipated stays fare particularly poorly and are denied admission to their most-preferred facility nearly 50% of the time. Due to outsize length of their stays, care for these residents constitutes a large fraction of all care in the industry. Correspondingly, 40% of all care-days in the industry are spent at facilities other than the resident's first choice. While this suggests significant admissions discrimination against Medicaid-eligible residents, I also find that Medicaid-eligible residents have weaker preferences for licensed nurse staffing levels, meaning that heterogeneous preferences also contribute to admission disparities. However, this preference disparity is much smaller than when demand is estimated under the assumption that selective admissions do not occur. Therefore, failing to account for selective admission practices leads the econometrician to significantly misestimate both demand elasticities and welfare measures.

I use my estimated model to simulate the effects of three policies intended to improve access to high-quality facilities: mandating that facilities practice "first come, first served" (FCFS) admissions, raising the Medicaid reimbursement rate, and increasing capacity at facilities with excess demand from Medicaid-eligible residents. Each of these relates to policies that have been considered or implemented in various states.[5] My simulations indicate that FCFS admissions would lead to welfare gains that residents value equivalently to .20 hours (1.1 standard deviations) of registered nurse (RN) care per day on average. These gains accrue primarily to Medicaid-eligible residents with lengthy anticipated stays, who experience increased access under FCFS, and come partially at the expense of non-Medicaid short-stay patients, who are sometimes crowded out of their preferred facilities under FCFS. Raising the Medicaid reimbursement rate has similar distributional effects: access for Medicaid-eligible residents improves slightly but does so largely at the expense of non-Medicaid residents. Targeted capacity increases, on the other hand, benefit both groups. I find that adding 800 beds (an 11% increase) to facilities with excess demand in the San Diego area leads to welfare gains for both Medicaid (.13 RN hours per day) and non-Medicaid residents (.06 RN hours per day) and is substantially more cost-effective than raising the Medicaid reimbursement rate. The most important caveat to these counterfactual simulations is that they hold facilities' characteristics fixed. Raising Medicaid reimbursement rates, for example, may achieve welfare gains not captured by the counterfactuals if facilities to increase their quality to attract Medicaid patients (Hackmann, 2019).

While this paper specifically examines nursing homes, concerns about providers picking their patients are not limited to the nursing home industry. Providers may face similar incentives in other parts of the healthcare industry, especially those in which patient desirability is heterogeneous and providers face capacity constraints or diseconomies of scale. For example, previous work has expressed similar concerns about specialty hospitals (Cram et al., 2008) and dialysis clinics (Desai et al., 2009). The trend toward fully and partially capitated systems, such as Accountable Care Organizations, may also exacerbate financial incentives for providers to pick their patients (Watnick et al., 2012).

[5]Examples include FCFS admissions (e.g., Connecticut General Statutes 19a-533), raising Medicaid reimbursement rates (e.g. California Assembly Bill No. 1629), limiting private-pay rates (e.g., Minnesota Statutes 256B.47 and North Dakota Century Code 50-24.4), and regulating facility capacities through Certificate of Need (CON) laws (e.g., Connecticut Statutes 19a-638 and Rhode Island General Laws 23-17-44) and bed-buybacks (e.g., Oregon Revised Statutes 410.070 and Louisiana Revised Statutes §40:2116). While CON laws and bed-buybacks reduce capacity, counterfactual simulations of increasing capacity may still be informative about the impacts of these policies.

Finally, an increasing reliance of patients, insurers, and regulators on outcome-based quality metrics may incentivize providers to pick their patients in order to boost these metrics (Dranove et al., 2003). Occupancy rates and other sources of variation in providers' policies may help to identify related models of providers picking their patients in these other healthcare markets. Moreover, the broader method of explicitly modeling suppliers' decisions to participate in demanders' choice sets, and of exploiting variation in suppliers' optimality conditions to identify suppliers' policies, may be extended to other contexts and industries with endogenous unobserved choice sets.[6]

The remainder of the paper is organized as follows. Section 2 provides background on the nursing home industry. Section 3 describes the data used in my analysis, and Section 4 provides preliminary evidence of selective admissions. Section 5 describes my model, and Section 6 gives my structural estimation procedure. Sections 7 and 8 respectively present model estimates and counterfactual simulations. Section 9 concludes.

2 Industry Background

This paper examines Medicare-certified skilled nursing facilities, which are certified by the Centers for Medicare and Medicaid Services (CMS) to provide a broad range of care, including skilled nursing care, specialized rehabilitative services, medically-related social services, and treatment for the mentally ill and mentally retarded (42 U.S.C. §1395i). Because SNFs are equipped to provide such a broad range of care, they often serve a census of residents with highly varied care requirements and lengths of stay.

Most SNF residents are initially admitted in order to receive rehabilitative and other therapy care following a discharge from an acute care facility. The majority of these residents are discharged to the community after receiving short-term rehabilitative therapy related to their preceding hospital stay. However, some continue to require long term care and continue to reside at the facility in order to receive daily nursing care and additional therapy care as needed.

The remainder of this section provides background on reimbursements in the industry, details facilities' incentives to pick their patients, and describes the history of regulations against selective admission practices.

2.1 Reimbursement

There are three primary sources of reimbursement for nursing home care: Medicare, private-pay, and Medicaid.

Medicare only reimburses SNF care for Medicare enrollees with a physician-certified need in relation to a recent acute care hospital stay lasting at least three inpatient days.[7] Medicare covers 100% of the first 20 days of qualifying care, requires coinsurance after 20 days ($119 per day in 2006), and is capped at 100 days. Medicare reimbursements are adjusted based on both patient care requirements and the cost of inputs in the facility's market. Because Medicare only covers post-acute care, residents covered by Medicare tend to require rehab and therapy care that is costly to provide and is therefore reimbursed at a high rate.

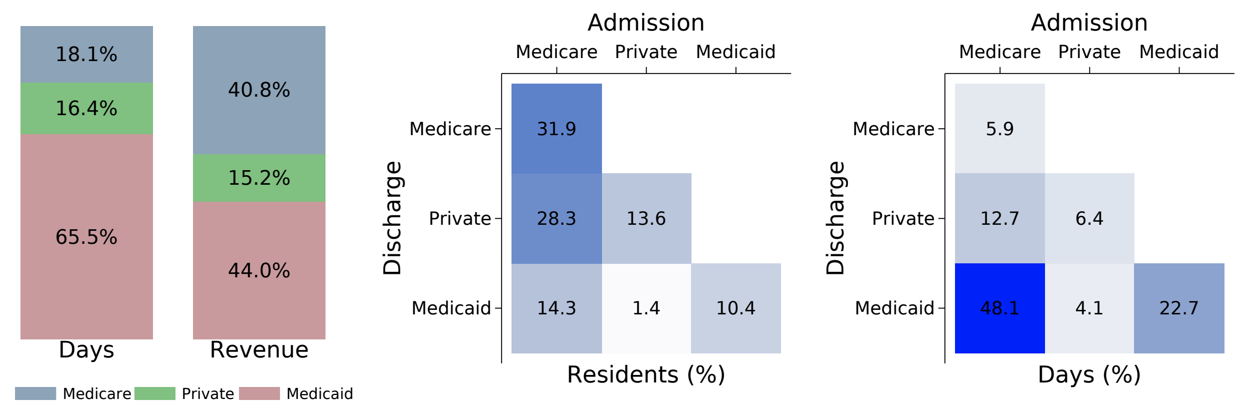
[6] Many salient examples involve discrimination, including transportation (Ge et al., 2020), lending (Ladd, 1998; Blanchard et al., 2008; Hanson et al., 2016), employment (Heckman, 1998; Bertrand and Mullainathan, 2004), and real estate (Kain and Quigley, 1972; Page, 1995; Ahmed and Hammarstedt, 2008). In all cases, the suppliers' optimality condition must reflect the incentives and institutional details specific to the industry.

[7] For additional details, see https://www.medicare.gov/coverage/skilled-nursing-facility-snf-care.

Figure 1: Descriptive Statistics for Payer Sources

(a) Percentages by Payer Source

(b) Payer Source Transitions between Admission and Discharge



Notes. Figure 1a gives the percentage of residents and days covered by each payer source. The left panel of Figure 1b gives the joint distribution of admission and discharge payment sources. The right panel of 1b weights this distribution by residents' length of stay.

Residents who are ineligible or have exhausted their Medicare coverage and are not yet eligible for Medicaid—i.e. have not exhausted personal financial resources—are "private pay." Long term care insurance is uncommon (Brown and Finkelstein, 2007, 2009, 2011), so most private pay residents pay out of pocket. Private rates are per-diem and are usually lower than Medicare reimbursement rates, though the care required by private pay patients tends to be less intensive than the post-acute care covered by Medicare. Private rates are typically not adjusted based on a patient's care requirements, though they do vary within a facility based on whether a resident's room is private or shared.

Once a resident over 65 has exhausted her private financial resources, she may qualify for Medicaid, which will cover her SNF care indefinitely without copayment or coinsurance. so long as she continues to require a level of care that cannot reasonably be obtained outside of a skilled nursing facility.[8] The vast majority of facilities in my sample (94.3%) are Medicaid-certified and are therefore legally obligated to accept Medicaid reimbursements. Medicaid reimbursement rates in California are not adjusted according to individual residents' care requirements and are almost always lower than private-pay and Medicare rates (Figures 1a and B.1). This generates a strong incentive to discriminate against Medicaid-eligible residents, which I discuss further in Section 2.2.

While only 26.0% of residents ever utilize Medicaid coverage (Figure 1b), a large majority of SNF care (65.5% of days) is reimbursed by Medicaid (Figure 1a). This is both because Medicaid-eligible residents tend to have longer stays and because long stays are more likely to exhaust private resources and result in Medicaid eligibility. Figure 1b details the transitions that residents make between their admission and discharge payer sources. Nearly 58% percent of days are from residents who transition between payer sources during their stay, most commonly starting their stay Medicare and ending it on Medicaid.[9]

[8] Typically this means needing skilled nursing care at a level not provided by alternatives such as outpatient or home health care.

[9] Note that 29.1% of residents in the sample are enrolled in Medicare Advantage. Insurers participating in Medicare Advantage are not required to submit claims data to CMS. Therefore, I impute the part of Medicare Advantage enrollees' stays that were covered by their Medicare Advantage plan. I impute these values using predictions based on resident assessment data. I train the predictive function using machine learning methods on data for Traditional Medicare enrollees since Medicare Advantage plans are required to cover SNF care when Traditional Medicare would have.

2.2 Incentives to Discriminate

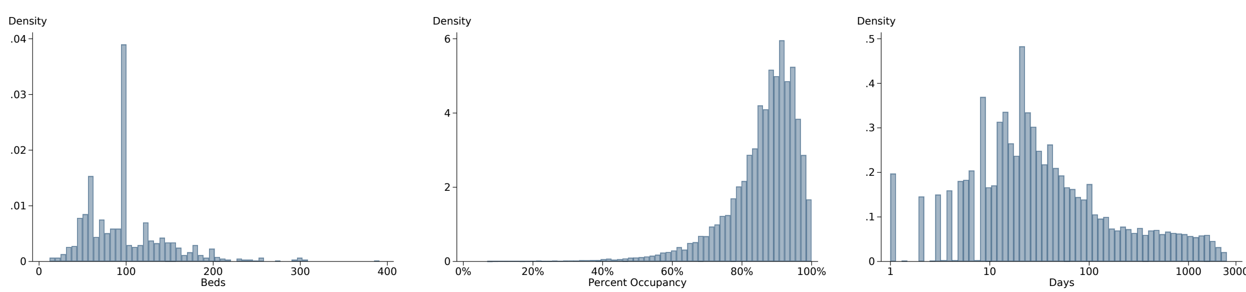
Nursing homes face a number of incentives to pick their patients. First, as discussed above, reimbursement rates can vary substantially by payment source. In particular, Medicaid reimbursement rates tend to be substantially lower than either the private rate or the Medicare reimbursement rate. While residents are entitled to the same care on Medicaid as private pay, Figure 1a shows that on a per-day basis, private pay reimbursements are 38% higher than Medicaid reimbursements. The comparison to Medicare is even more stark, as the per-day reimbursements are 236% higher than Medicaid. While this premium is partially offset by the additional rehab and therapy care required during the Medicare-eligible part of residents' stay, Medicare margins are reported to be very high and are perceived by the industry and regulators cross-subsidize Medicaid and even private pay care (MedPAC, 2021).

The result of the large disparity in reimbursement rates is that facilities may have strong financial incentives to screen for current and future Medicaid eligibility. In fact, some facilities require prospective residents to submit financial documentation that can be used to forecast when a resident is likely to become eligible for Medicaid based on an anticipated spend-down of assets on care. Facilities can even purchase commercial "admission analysis" software to assist in making these projections.[10]

In addition to variation in reimbursements, there may also be substantial uncompensated variation in the cost of providing care to different patients. For example, patients with behavioral problems or diagnoses such as dementia or obesity may require more staff time and facility resources than other residents. In California, only Medicare reimbursements are adjusted according to a patient's anticipated resource utilization. Facilities receive no additional compensation for high-needs patients on days not covered by Medicare, and even Medicare reimbursements may not perfectly insure the facility against heterogeneous care requirements. Legal prohibitions (42 CFR §483.10(a)(2)) and organizational challenges also prevent facilities from achieving higher margins by providing low-reimbursement patients lower quality care (Grabowski et al., 2008).

Figure 2: Capacities and Occupancy Rates

(a) Capacities (b) Occupancy Rates (c) Length of Stay



Facilities also face dynamic incentives to pick their patients. Most directly, each facility's capacity is constrained by its number of CMS-certified beds, so each admitted patient reduces the capacity available to admit more profitable

[10] For example, American HealthTech's Admission Analysis module advertises being able "to project total cost of care per resident, as well as how much each will generate in revenue..." See https://www.healthtech.net/admissions-analysis.

patients in the future.[11] Figure 2a and 2b give the capacities and occupancy rates of the SNFs in my sample. The average capacity is approximately 100 beds, and the median occupancy rate is 88%. The industry's high occupancy rates suggest that many facilities could quickly reach their capacity constraint if they were to admit additional residents. Facilities facing excess demand will therefore want to ration their capacity by turning away less desirable residents.

Furthermore, length of stay can vary substantially across residents. While most stays are brief, some residents stay at the facility for years. Figure 2c shows the distribution of stay lengths in my sample. While most stays are relatively brief—the 50th percentile resident stays just 25 days—the standard deviation of their lengths is large (333 days). I find in Section 3 that this variation is broadly predictable using only information that was plausibly observable at admission. Facilities practicing selective admissions may therefore be particularly wary of admitting low-margin residents predicted to stay for a very long time, especially since involuntarily discharging residents without cause is illegal (42 CFR §483.15(c)).[12]

In addition to capacity constraints, facilities may also face diseconomies of scale. The primary input cost for nursing homes is staffing, and meeting demand above what long-term staff are equipped to handle may bear additional cost. For example, nursing cost report data from California indicate that facilities pay an average wage premium of 37% when hiring temporary registered nurses (RNs).[13] Temporary staff may also be less productive or require additional training in order to be effective. Such increasing marginal costs may lead facilities to ration admissions even when capacity constraints are unlikely to bind in the future.

2.3 History and Regulation

State and Federal governments have passed a large number of regulations restricting selective admission practices on the basis of either disability or payer-status.

Section 504 of the Rehabilitation Act of 1973 prohibits nursing homes receiving either Medicare or Medicaid reimbursements from discriminating in admissions on the basis of disability.[14] Wagner v. Fair Acres Geriatric Center, 859 F. Supp. 776 (E.D. Pa. 1994), upheld these protections by finding that Fair Acres Geriatric Center violated the Rehabilitation Act when it denied Wagner admission on the basis of the behavioral difficulties resulting from Wagner's Alzheimer's.[15] The Americans with Disabilities Act of 1990, which defines disability to include "a physical or mental impairment that substantially limits one or more major life activities" (42 U.S. Code § 12102 (1)(A)), extended protection against disability discrimination to places of "public accommodation," including all nursing homes (42 U.S. Code §12181 (7)(F)).[16] Furthermore, courts have held that nursing homes constitute "dwellings" under the

[11]Capacity adjustments are rare, as they require re-certification of the facility by CMS.

[12]Recent evidence from Hackmann and Pohl (2018) suggests at least some misconduct in discharges: Medicaid residents are approximately 1 percentage point more likely to be discharged during high-occupancy weeks than low-occupancy ones.

[13]This premium is respectively 34% and 23% when weighting by a facility's reported full time and part time hours.

[14]The Act charges that individuals with disabilities not be "subjected to discrimination under any program or activity receiving Federal financial assistance or under any program or activity conducted by any Executive agency or by the United States Postal Service" (29 U.S.C. §794 (a)).

[15]However, refusing admission based on disability is legal if the patient's care requirements exceed the ability of the facility: Grubbs v. Medical Facilities of America, Inc., 879 F. Supp. 588 (W.D. Va. 1995) held that Medical Facilities of America was legally justified in denying readmission to Grubbs, whose worsening condition required care that exceeded what Medical Facilities of America's nursing facilities were certified to provide.

[16]The Act lists "hospital, or other service establishment" as examples of places of public accommodation, and the 1994 Supplement to the Americans with Disabilities Act Title III Technical Assistance Manual states that "nursing homes are expressly covered in the title III regulation as social service center establishments."

Fair Housing Act (see Carlson (2012), Section IV), and therefore inquiries into a patient's disability when determining admission may be subject to additional scrutiny and regulation under the Fair Housing Act (24 CFR 100.202 (C)).[17]

There have also been multiple attempts to restrict and prohibit admissions discrimination against Medicaid-eligible patients. Section 1909(d)(1) of the 1977 Social Security Amendments made it a felony to solicit or receive payment in addition to Medicaid or Medicare reimbursements as a condition of admission or continued care. Despite this prohibition, discrimination against Medicaid patients continued. As Chairman of the Special Committee on Aging, Senator John Heinz observed the following in his opening remarks of Senate Hearing 98-1091 on "Discrimination Against the Poor and Disabled in Nursing Homes" (Senate, 1984):

> Findings of a recent committee investigation show that in some areas of this country, up to 80 percent of what are called federally certified nursing homes are reported to actively discriminate against medicaid beneficiaries in their admission practices. These acts of discrimination are a flagrant violation of U.S. law.

A common avenue for this discrimination was "duration of stay clauses," which required prospective residents to contractually agree to forego enrollment in Medicaid for a period of time as a condition of admission. In response to these practices, the 1987 Nursing Home Reform Amendments explicitly prohibited facilities from requiring residents to waive rights to Medicare or Medicaid benefits (42 CFR 483.15 (a)). Furthermore, the Reform Amendments required that nursing homes provide a standardized admission agreement that more clearly notified prospective residents of their rights (Ambrogi, 1990). Many states, including California (SB 1061, 1997), passed laws standardizing admission agreements to this effect (see CA Health & Safety Code §1599.60-84).[18] As a result, all California admissions agreements contain the following clause:

> You should be aware that no facility that participates in the Medi-Cal program may require any resident to remain in private pay status for any period of time before converting to Medi-Cal coverage. Nor, as a condition of admission or continued stay in such a facility, may the facility require oral or written assurance from a resident that he or she is not eligible for, or will not apply for, Medicare or Medi-Cal benefits.

Selective admission practices have received less regulatory attention since the Reform Amendments, however evidence suggests that such practices may have continued in forms less discernable by policymakers. In response to complaints made to the Health Care Finance Administration and the Office of Civil Rights, the Health and Human Services Office of the Inspector General (HHS OIG) investigated the degree to which the use of financial screening led to defacto Medicaid discrimination in 1999. HHS OIG found that while 70% of hospital discharge planners surveyed reported that nursing homes denied access for financial reasons either somewhat or very often, less than 4% of Medicaid officials surveyed agreed (Brown, 2000).[19] In spite of this, the report concludes that "financial screening may cause access problems for some Medicaid beneficiaries, but these problems do not appear to be widespread." Financial

[17] See Carlson (2012) for the basis and implications of applying the Fair Housing Act "'no inquiry" rule to SNFs.

[18] In addition to regulating admission agreements, many states have passed laws directly regulating admission practices. Connecticut began requiring nursing homes to serve residents on a first come, first serve basis in 1980, and Ohio prohibited admission discrimination against Medicaid eligible patients until 80% of a facility's census is on Medicaid in 1983.

[19] Hospital discharge planners are hospital employees responsible for helping patients plan their discharge. One role of discharge planners is to help patients find a skilled nursing facility.

screening continues today, and patients' rights groups continue to claim that these and other practices constitute illegal Medicaid discrimination.[20]

The conflicting responses in the HHS OIG report and the continued concern of patients' rights activists in spite of existing legal protections suggest that the prevalence of discriminatory admission practices after 1987 are not well understood. A primary contribution of this paper is to identify and quantify the form, severity, and impacts of continued selective practices.

3 Data

I combine a number of administrative datasets from the Centers for Medicare & Medicaid Services (CMS) to construct a panel of resident stays at California SNFs between 2004 and 2006.

Resident Data Resident assessments from the nursing home Minimum Data Set (MDS) form the base of this panel. Federal law (42 CFR §483.20) requires nursing homes to complete assessments of each resident at regular intervals, starting with admission and ending at discharge. These assessments collect a wide range of demographic and clinical information that nursing staff use to develop a care plan and that are reported to CMS for inclusion in the MDS. CMS uses these data to construct publicly available quality metrics and to determine Medicare reimbursements.

I augment the MDS with Medicare and Medicaid claims and enrollment data that identify payment sources throughout a resident's stay.[21] I identify the days of each stay that were reimbursed by Medicare using Medicare SNF claims from the Medicare Provider and Analysis Review (MedPAR) files.[22] Using acute care claims from MedPAR, I am also able to identify the likely hospital from which residents were discharged to a SNF for 98% of stays reimbursed by Traditional Medicare. I use three different data sources to identify Medicaid reimbursements: the Medicaid Analytic eXtract Long Term Care (MAX LT), Medicare-Medicaid Linked Enrollee Analytic Data Source (MMLEADS), and Beneficiary Summary File (BSF). The MAX LT consists of Medicaid long term care claims data, and the MMLEADS and BSF provide monthly Medicaid eligibility data.[23] In addition to information on Medicare and Medicaid enrollment, the BSF also provides 9-digit zip codes for all Medicare eligible individuals. I use these in conjunction with prior residence data from the MDS to geocode patients' prior residences.[24]

When modeling facilities' admissions decisions, it is important to consider only information plausibly observable by the facility at the time the admission decision was made. Therefore, I characterize residents based on the first

[20] See, for example, this California Advocates for Nursing Home Reform newsletter.

[21] Combining multiple CMS datasets requires resolving inconsistencies between the datasets. I have largely based my construction of an aggregated panel on the Residential History File (Intrator et al., 2011), which resolves inconsistencies by preferencing claims data over assessment data. A full description of the data cleaning process can be obtained from the author on request.

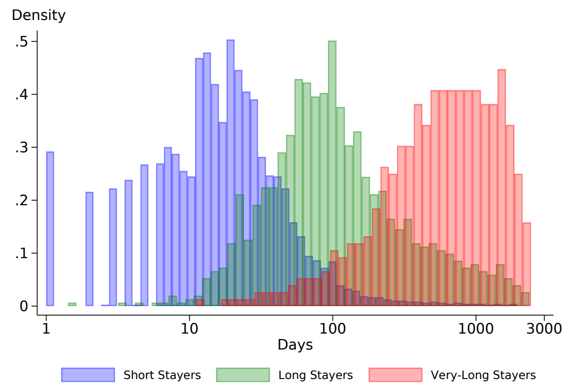
[22] Since Medicare Advantage plans are not required to report their claims for inclusion in MedPAR, I use MDS data to impute Medicare Advantage coverage for Medicare Advantage enrollees' stays. Specifically, I use machine learning methods (Appendix H) to fit a function that predicts Traditional Medicare enrollees coverage using MDS, and use this function to impute coverage for Medicare Advantage enrollees. The key assumption underlying this imputation method is that the relationship between MDS variables and qualification for coverage under Medicare and Medicare Advantage is the same.

[23] I combine all three sources because Medicaid Managed Care plans do not always report their claims to CMS (Cheh, 2011). In such cases, Medicaid enrollees are still likely to appear as dual-eligibles in the MMLEADS and BSF. Additionally, I do not have Medicaid claims after 2007.

[24] When prior residence is absent in both the BSF and MDS, I use the location of the hospital that discharged the resident to a SNF as an imputed location of prior residence.

available data—typically their MDS admission assessment—as the closest available approximation to what the facility could have considered in making their decision.

Figure 3: Length of Stay Distribution by Predicted Stayer-Type



Notes. Short, long, and very-long stayers respectively constitute the 1-70, 70-90, and 90-100 percentiles of predicted length of stay.

Anticipated Length of Stay As discussed in Section 2.2, facilities may be particularly concerned with a resident's length of stay when making admissions decisions. Since facilities cannot perfectly anticipate when a patient will be discharged, they use must form beliefs about her probable length of stay using other observed characteristics. To approximate this, I train a regularized and cross-validated machine learning model (see Appendix H) that predicts the natural logarithm of residents' length of stay using residents' first available assessment data. I find that 73.51 % of variation is predictable out-of-sample. In much of my analysis, I coarsen these predictions by binning patients into short-stay (bottom 70 percentile), long-stay (70-90) percentile, and very-long-stay (top 10 percentile) based on their anticipated length of stay.[25] Figure 3 shows that realized lengths of stay vary substantially across these groups.

Facility Characteristics I also utilize a number of publicly available datasets on nursing facilities in my analysis. First, I compile facility characteristics from the Online Survey, Certification and Reporting (OSCAR) database and Brown University's LTCFocus database.[26] I augment these with Medicaid reimbursement rates and cost data from California's Medicaid Cost Reports.[27] Because private price data are not surveyed directly, I infer private prices using the revenue and quantity data from the cost reports (Huang and Hirth, 2016).

I provide additional details summarizing the data in the Appendix. Table B.1 summarizes the data sources, and

[25] Industry lingo typically segregates residents into short- and long-stay. I deviate from this slightly because the residents I categorize as very-long-stay typically reside at the facility for much longer than long-stay residents. Correspondingly, the financial implications of admitting long-stay and very-long-stay residents are likely substantially different, and, in fact, I do estimate in both my reduced form and structural models that admission policies are much less favorable for very-long-stay residents.

[26] Data available on LTCFocus.org are provided by The Shaping Long Term Care in America Project at the Brown University Center for Gerontology and Healthcare Research, which is funded in part by the National Institute on Aging (1P01AG027296).

[27] The reimbursement rate and cost report data were collected from the California Department for Health Care Services and Office of Statewide Health Planning and Development websites, respectively.

Tables B.2 and B.3 give summary statistics for the resident and facility-level data, respectively.[28] Figures B.3a and B.3b map the distribution of residents and facilities at the state level and in the Los Angeles area. These maps reflect the geographic granularity of these data that plays a significant role in my analysis since most patients are admitted to SNFs that are close to their prior residence (Figure B.2).

4 Preliminary Evidence of Selective Admission Practices

This section provides reduced form evidence of selective admission practices by examining how the composition of a facility's admitted patients covaries with its occupancy. If facilities tend to become more selective as they become more full, then we expect patient characteristics associated with admission primarily when facilities have low occupancy to be undesirable characteristics. We similarly expect residents whose characteristics are associated with admission when facilities are relatively less full to be less desirable residents. Section 4.1 develops this intuition in a model with one facility and one possible discharge rate for patients. Section 4.2 then presents my reduced form estimates. I first show the relationship between admitted patient characteristics and facility occupancy. Then, I use machine learning methods to aggregate residents' many characteristics into a univariate desirability score for each resident.

4.1 Model: One Facility and One Discharge Rate

The following model gives intuition for why we expect a facility facing capacity constraints to be more selective as it becomes more full. Consider a lone nursing home with a capacity of b beds. Prospective residents arrive according to a Poisson process with rate parameter λ^A and have a profitability Π_i drawn from a known distribution F. Upon arrival, each patient instantaneously applies for admission at the facility, and the facility instantaneously determines whether to admit the patient. If the facility chooses to admit the patient, the facility receives an upfront payoff Π_i, and the patient resides at the facility until she is discharged according to an exponential process with discharge parameter λ^D. If a patient arrives when the facility is full, the patient is automatically rejected.

Fix a probability space and filtration $\mathcal{F}_t$ representing the arrival, admission, and discharge processes. The facility chooses an adapted admission plan $\{a_\tau\}$, where $a_\tau \subseteq \mathbb{R}$ is the set of profitabilities that would qualify a patient for admission at time τ.In other words, a resident i arriving at τ is admitted if and only if the facility has an available bed and $\Pi_i \in a_\tau$. At each time t, the admission plan maximizes the present discounted value of future admitted Π_i:

$$\mathbb{E}\left[\sum_{\{i:\tau_i^A \geq t, N_{\tau_i^A} < b, \Pi_i \in a_{\tau_i^A}\}} \exp\left(\rho(\tau_i^A - t)\right)\Pi_i \,\middle|\, \mathcal{F}_t\right], \tag{1}$$

where ρ is the discount rate, N_τ is the facility's level of occupancy at τ, and τ_i^A is the time that i arrived.

Since arrivals and discharges are memoryless and residents are indistinguishable after admission, the facility need only condition its policy on its current level of occupancy. Furthermore, because the facility should always prefer to

[28] I exclude facilities from my analysis that do not file cost reports as SNFs in California since these facilities are likely to be specialized facilities, such as sub-acute care facilities or institutions specializing in mental diseases.

admit more profitable residents to less profitable ones, optimal admission policies must be of the form $a_\tau = [\underline{\Pi}_\tau, \infty)$ for some threshold profitability $\underline{\Pi}_\tau$. Together, these imply that the facility's policy function is a threshold rule for each level of occupancy, where arriving residents are only admitted if their profitability exceeds the threshold rule for the facility's current level of occupancy. I denote this rule by a function $\underline{\Pi} : \{0, 1, 2, \ldots, b-1\} \to \mathbb{R}$. Then the facility's value function is:

$$V(N_t) = \max_{\underline{\Pi}} \mathbb{E}\left[\sum_{\{i: \tau_i^A \geq t, N_{\tau_i^A} < b, \Pi_i \geq \underline{\Pi}(N_{\tau_i^A})\}} \exp\left(\rho(\tau_i^A - t)\right) \Pi_i \,\middle|\, N_t \right]. \tag{2}$$

The corresponding Bellman equation and optimal policies are:

$$\rho V(N_t) = N_t \lambda^D \left(V(N_t - 1) - V(N_t)\right) + \lambda^A \max_{\underline{\Pi}} \int_{\underline{\Pi}}^{\infty} \left(\Pi + V(N_t + 1) - V(N_t)\right) dF(\Pi), \tag{3}$$

$$\underline{\Pi}(N_t) := V(N_t) - V(N_t + 1). \tag{4}$$

The Bellman equation (3) is intuitive: $N_t\lambda^D$ is the arrival rate of a discharge, and $V(N_t - 1) - V(N_t)$ is the change in the value function if a resident is discharged. Similarly, λ^A is the arrival rate of a new prospective resident, Π is the payoff received if the resident is admitted, and $V(N_t + 1) - V(N_t)$ is the opportunity cost of admitting the resident. I generalize this model in Section 5 and Appendix A.

Theorem 1. *Threshold rule $\underline{\Pi}$ is increasing in N_t.*

Proof. See Appendix A.2. □

Theorem 1 indicates that the facility's threshold rule $\underline{\Pi}$ is increasing in occupancy. Intuitively, as the facility's occupancy increases, the likelihood that the facility will reach capacity and need to reject future profitable residents increases. Since the facility admits an increasingly censored distribution of residents as it becomes more full, we expect the average profitability of admitted residents to rise as the facility becomes more full. Correspondingly, the set of N_t such that $\Pi > \underline{\Pi}(N_t)$ is nested and increasing in Π. It follows that the average N_t at which residents with profitability Π are admitted is weakly increasing in Π. Therefore, residents whose characteristics are associated with admission when facilities are relatively less full can be inferred to be less desirable. This intuition underlies both the reduced form and structural estimation procedures in this paper.

4.2 Reduced Form Evidence

The primary empirical challenge in this paper is that we only observe realized admissions, which reflect both residents' preferences over facilities as well as selective admissions practices. In particular, the fact that some types of low-margin residents, such as dual-eligibles, tend to receive care at lower quality facilities even when controlling for geographic availability need not reflect selective admissions. Instead, it may simply reflect that these patients have weaker preferences for some dimensions of quality.

Therefore, I use a more conservative test of selective admissions by looking for evidence that facilities are dynamically adjusting their admission policies as they become more or less full. Insofar as transitory variation in occupancies do not affect patients' preferences for facilities, this represents evidence of selective admissions that is robust to any amount of heterogeneity in patient preferences.

Formally, I identify selective admissions based on within-facility covariation between facility occupancy and admitted patient characteristics. This approach requires two important identifying restrictions. The first is that the composition of residents seeking admission at the facility does not evolve over time in a way that is systematically correlated with the facility's occupancy.[29] I consider potential sources of such correlations in Appendix C.1 and find little cause for concern. One such possibility is that the residents arriving to the market in need of skilled nursing care covaries with facility occupancy. Fortunately, I find that any such correlations are small relative to my estimated results. Another possibility is that if the occupancy of facilities and their competitors are highly correlated, then the composition of residents a facility admits could covary with occupancy due to changes in residual demand from competitors being full. I likewise find that correlations between competitors' occupancies are very small and unlikely to introduce significant bias.

The second is that I assume that resident preferences for facilities are not affected by transitory variation in occupancies. This assumption is not testable but is plausible given that transitory variation in occupancy is difficult for prospective residents to observe. So much so, in fact, that facilities are often accused of implementing selective admissions by lying to undesirable prospective residents by falsely claiming that all beds are occupied (Kowalczyk and Arsenault, 2020). Moreover, violations of this assumption are likely to attenuate rather than strengthen my results, since the residents I find to be most desirable—short-stay Medicare residents—are also the residents that most plausibly prefer lower occupancy rates. First, these patients disproportionately require rehab and therapy care, which may suffer in quality from congestion when resources are spread thin due to a high current occupancy rate. Second, congestion at admission is more likely to be perceived as purely transitory and of negligible consequence by residents with longer anticipated stays that expect to experience the long-run distribution of occupancies during their stay.

The most straightforward test of selective admissions is to use facility occupancy as a covariate in predicting the characteristics of admitted residents:

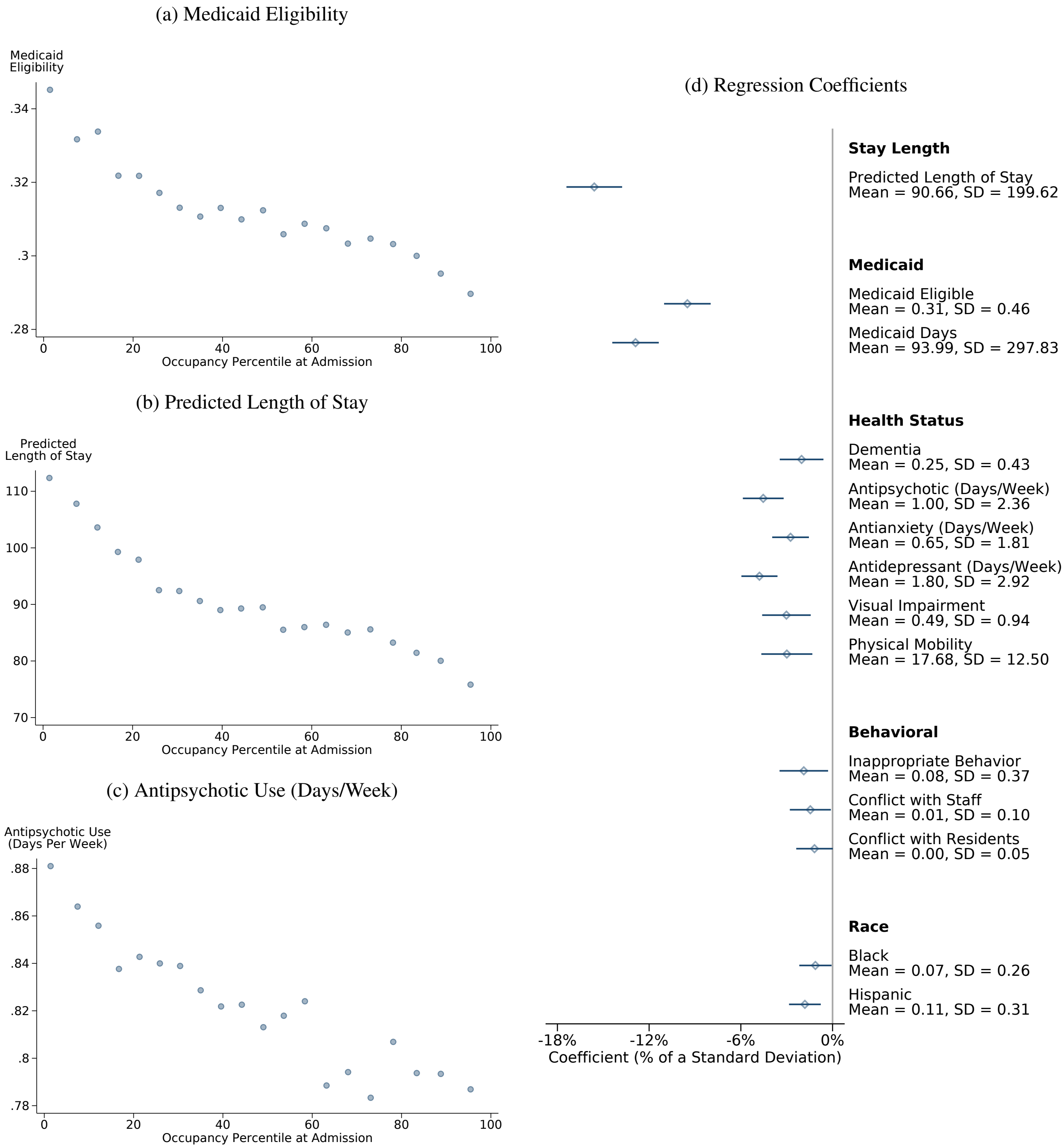
$$y_i = \beta_{j(i)} + \beta^{occ} occ^{\%le}_{j(i)t(i)} + \beta^h h_i + \epsilon_i, \tag{5}$$

where y_i is a given characteristic of interest of resident i admitted to facility j at time t, $occ^{\%le}_{jt}$ is facility j's percentile occupancy *within its own distribution* at time t, and h_i are other resident characteristics for which we wish to control. Under the null-hypothesis that facilities are not picking their patients, we expect $\beta^{occ} = 0$ for all characteristics. Note that since occupancy percentile is computed within a facility's own distribution of occupancy, β^{occ} is identified exclusively from *within-facility* covariation between occupancy and admitted patient characteristics.[30]

[29] In the toy model above, this is satisfied by the assumption that $F(\Pi)$ is unchanging.

[30] Likewise, the inclusion of facility fixed effects also ensures that all comparisons are within-facility. Correspondingly, the results are very similar with and without fixed effects.

Figure 4: Relationship Between Admitted Patient Characteristics and Percentile Occupancy



Notes. All regressions and binscatters include facility fixed effects and controls for oth independent variables. Short, long, and very-long stayers respectively constitute the 1-70, 70-90, and 90-100 percentiles of predicted length of stay. Estimates for Figure 4d are also shown in Table C.2.

Figure 4 presents reduced form evidence of selective admissions. Figures 4a, 4b, and 4c respectively show binscatters for example characteristics that are anticipated to be undesirable: residents' Medicaid eligibility at admission, anticipated length of stay, and use of antipsychotics, a pharmaceutical suggestive of challenging care requirements. All three show clear negative relationships within-facility between occupancy percentile and admitted patient characteristics, suggesting that facilities are not only practicing selective admissions but are covarying their admission policies over time with the opportunity cost of new admissions. The implied within-facility variation in admission policies is quite substantial: relative to their highest occupancy days, on average the same facilities on their lowest occupancy days appear to be admitting residents that are approximately 19% more likely to be on Medicaid, are anticipated to stay 36% longer, and require 12% more antipsychotics.

Figure 4d shows the regression estimates (equation (5)) for length of stay, medicaid utilization, additional health characteristics, indicators of behavioral problems, and race. For ease of interpretation, β^{occ} is scaled by the standard deviations of y_i. The estimates suggest that facilities' admitted residents are less likely to have these characteristics as the opportunity cost of admissions increases. While all estimates are negative, the relationship is most substantial for Medicaid utilization and predicted length of stay. That disabilities and behavioral problems do not show as substantial a relationship likely reflects that these factors are not individually as important to facilities as whether a prospective resident is Medicaid-eligible or anticipated to occupy a bed for a long time.[31] Note that Figure 4d does not utilize additional controls. In Appendix C.2 I control for Medicaid eligibility, Medicaid days, and length of stay—our dimensions with clearest evidence of selective admissions—and find that the negative coefficients for health characteristics, behavioral issues, and race persist but are sometimes smaller or have larger standard errors.

It is important to emphasize that these binscatters and regressions in Figure 4 do not capture the full extent of selective admissions. Rather, they capture the average extent to which selective admissions vary over time due to within-facility variation in the opportunity cost of admissions. In particular, the extent to which facilities are practicing selective admissions on their least-full days in the sample would not be captured. In fact, some facilities—such as those with persistent excess demand from highly profitable patients—may be consistently discriminating against low-margin patients without varying their admission policies much over time due to transitory variation in occupancy. These facilities may be the most selective in their admissions but would not contribute to a nonzero estimate of β^{occ}. In this way, rejecting $\beta^{occ} = 0$ is a very conservative test for selective admissions, as it detects only evidence of strategic variation in misconduct rather than trying to attribute baseline admission inequality to selective admissions.

In later sections, we leverage a structural model in conjunction with this same identifying variation in order to estimate admissions policies. In estimating these admission policies, we can infer the level of selective admissions that facilities are employing even in their least full days in the sample. Moreover, insofar as these admission policies are generated by structural primitives, we can simulate how they would change under counterfactual circumstances.

4.3 Machine Learning Resident Desireability

While the estimates in Section 4.2 do provide evidence of selective admission practices, they do not coherently aggregate the many characteristics that may affect a resident's desirability. To accomplish this, I flip the regressions in (5) and instead predict facility occupancy at admission using individual characteristics. The intuition for this regression is that it approximates the answer to the question: "How full is facility j, on average, when it admits a resident with characteristics c_i?" If facilities admit an increasingly desirable distribution of residents as they become more full, then

[31] Alternatively, it may also reflect facilities being less willing to discriminate along these dimensions, as the legal precedent more clearly establishes legal precedent that doing so is illegal.

the answer to this question is monotonic in the desirability of residents with characteristics c_i.[32]

I use gradient boosted decision trees described in Appendix H to fit $\mathbb{E}\left[occ_{jt}^{\%le}|a_{ij}, c_i\right]$, where a_{ij} signifies that resident i was admitted at facility j and c_i are the characteristics of resident i.[33] This machine learning method allows me to approximate the conditional expectation as a flexible nonlinear function that minimizes the square of prediction error out of sample. I allow the model to condition flexibly on the admitting facility and hundreds of resident characteristics that were likely observable at time of admission. To reduce over-fitting, I regularize the model's complexity to maximize its cross-validated out-of-sample fit. This procedure is analogous to cross-validating the penalty parameter in a LASSO or Ridge regression and allows me to use a high dimensional input without over-fitting.

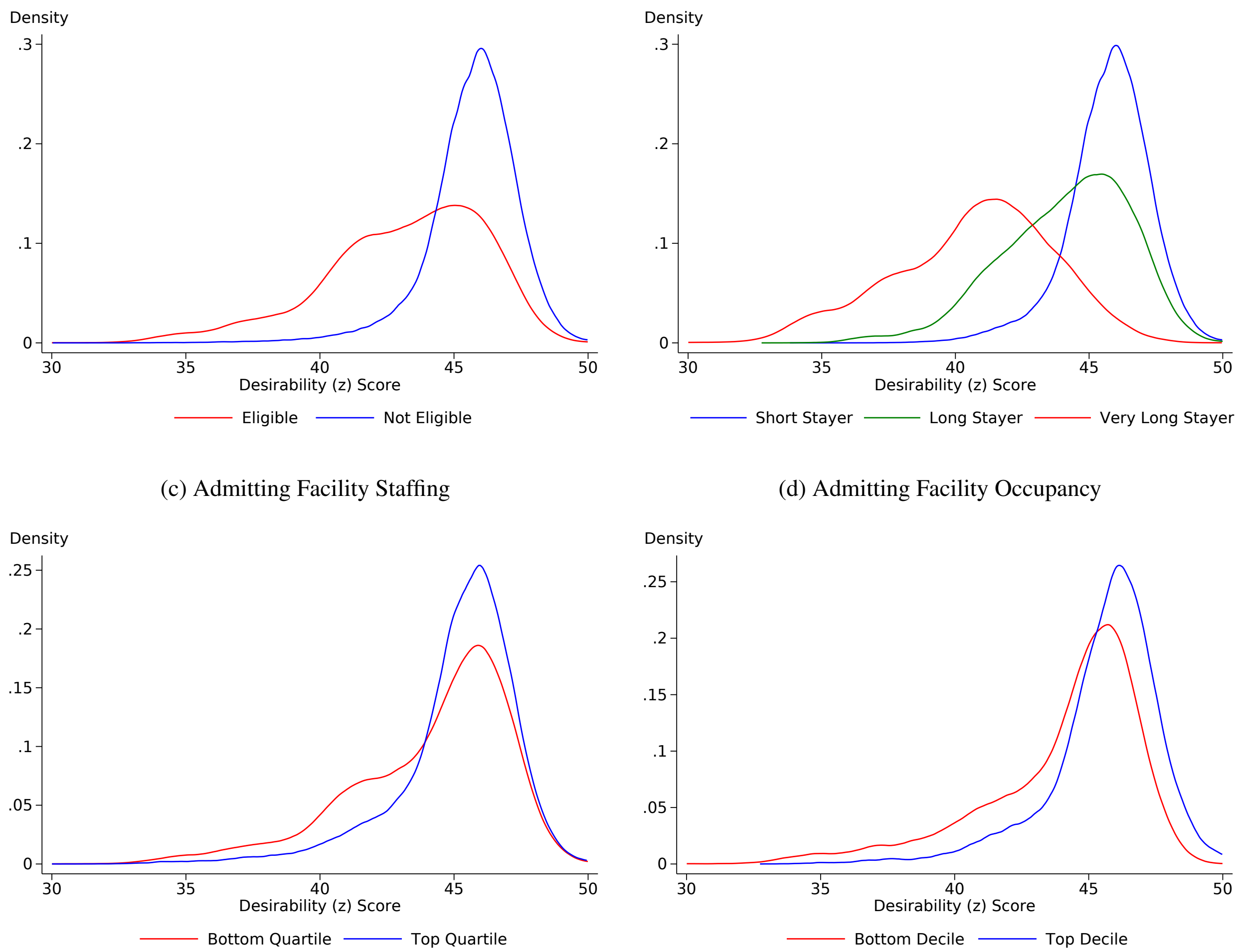
Figure 5: Distribution of Desirability Scores by Subgroup

(a) Medicaid Eligibility

(b) Stayer Type

(c) Admitting Facility Staffing

(d) Admitting Facility Occupancy



Notes. Short, long, and very-long stayers respectively constitute the 1-70, 70-90, and 90-100 percentiles of predicted length of stay.

[32] The assumptions in Subsection 4.1 guarantee an increasing relationship between profitability and occupancy rate. When allowing for multiple discharge rates or interactions between resident profitability, however, this increasing relationship is plausible but not guaranteed since multiple census compositions can have the same level of occupancy. Therefore, in interpreting the results in this section, I implicitly assume that the realized distribution of censuses and interactions between resident profitabilities are sufficiently well-behaved that this intuitive relationship between occupancy and selectivity holds on average.

[33] The results are qualitatively similar if using a linear regression: Medicaid utilization, lengthy anticipated stays, and high care requirements are associated with lower expected occupancy at admission.

To construct a "desirability score" (denoted z_i) for each resident i that is comparable across residents, I take the average of i's predicted occupancy percentile at admission for each facility in California:

$$z_i := \frac{1}{|J^{CA}|} \sum_{j \in J^{CA}} \hat{\mathbb{E}} \left[occ_{jt}^{\%le} | a_{ij}, c_i \right] \tag{6}$$

where $\hat{\mathbb{E}} \left[occ_{jt}^{\%le} | a_{ij}, c_i \right]$ is the predicted occupancy percentile for i admitted at j, and J^{CA} includes all California facilities. Residents with higher z_i scores are those who facilities tend to agree are more desirable, and those with lower z_i scores are those who facilities tend to agree are less desirable. Note that since occupancy percentile is uniformly distributed between 0 and 100, a score of $z_i = 50$ is the theoretical upper bound for a desirability score, as it suggests that a resident is always offered admission at all facilities regardless of occupancy level.

Figure 5 shows how the distribution desirability scores z_i varies with different characteristics. Figures 5a and 5b respectively show that Medicaid eligible residents and residents with longer anticipated stays tend to have much lower desirability scores than their non-Medicaid and short-stay counterparts. Figure 5c shows that both low- and high-staffing facilities admit high-desirability residents but that low-desirability residents are admitted primarily at low-staffing facilities. Figure 5d shows that the machine learning model has predictive power: low-desirability patients are much more likely to be admitted in low occupancy states than high occupancy ones.[34] I project these desirability scores onto a linear model of resident characteristics in Appendix C.3 in order to provide additional intuition and interpretation of the patterns discerned by the machine learning algorithm.

Figure 5 suggests that the machine learning desirability score z_i captures much of the important variation in resident desirability. Therefore, I leverage desirability score as a key resident characteristic to reduce the dimensionality of my structural model in Sections 5 and 6.

5 Theoretical Model

In this section, I present a model of selective admissions in the nursing home industry that informs my estimation procedure in Section 6. I first provide an overview of the matching process and then expand on the resident choice problem and the facility admission control problem. Finally, I define an equilibrium concept for beliefs and strategies.

5.1 Overview of the Matching Process

I model the industry through the following decentralized matching process. Patients requiring skilled nursing care arrive to the market according to a Poisson process with rate parameter λ^A. The characteristics of these residents (c_i) are distributed according to a known distribution F_c and include:

1. loc_i: location of the patient's prior residence.

[34] The out of sample R^2 for the predictive function is .25.

2. J_i: The set of all facilities that are within 20 kilometers of loc_i.[35]

3. $\{u_{ij}\}_{j \in J_i}$: The resident's utility from admission at each facility.

4. $\{\Pi_{ij}\}_{j \in J_i}$: The resident-specific payoff that each facility j expects to receive from admitting i.[36]

5. ϕ_i: The resident's stayer-type governing the resident's exponential discharge rate, which we denote $\lambda^D_{j\phi_i}$ for each facility j.[37] In my empirical exercise, I allow ϕ_i to be one of $\Phi = \{\text{short, long, very long}\}$, and I assign ϕ_i using information that was likely observable at admission. (See Section 3 and Figure 3.)

Upon i's arrival to the market, all facilities in J_i instantaneously observe c_i and independently decide whether or not to offer admission to i. In offering admission to resident i, facility j is choosing to be in i's choice set, which I denote $\mathcal{O}_i$.[38] Resident i considers all $j' \in \mathcal{O}_i$ and is admitted to her preferred facility $j \in \mathcal{O}_i$:[39]

$$j = \underset{j' \in \mathcal{O}_i}{\arg\max}\, u_{ij'}. \tag{7}$$

It is important to note that "admitted" in this context means that a resident matches with and actually receives care at a facility. While this terminology is common in the healthcare literature, it differs from uses in other contexts, such as college admissions, where "admitted" signifies an offer to attend.

This model aims to reflect the process by which many residents are actually matched with a SNF. The process by which an acute care patient is discharged to a SNF often involves a hospital discharge planner assessing which facilities are likely to accept the patient. Historically, these determinations were made by a combination of the discharge planner's knowledge of local facilities and facilities' responses to referrals. Electronic systems have increasingly allowed discharge planners to quickly solicit expressions of interest in admitting a patient from a large number of local SNFs. Healthcare information technology company Allscripts describes one such system in a February 2008 press release (accessed October 2018):

> "Allscripts enables a detailed electronic referral to be sent to selected locations across its proprietary network of 90,000 providers, and interested facilities reply, often within minutes. The patient and family then receive a discharge packet that details their facility choices, making their decision more transparent and informed."

This set of facilities that express interest in the resident are represented by $\mathcal{O}_i$ in the model.

[35] In my empirical estimation, I limit potential admissions to facilities within 20 kilometers for computational reasons. The vast majority of residents in the sample (87.4%) have an identified location of prior residence within 20 kilometers of their chosen facility. I treat the arrival and admission of residents who either do not have an identified location of prior residence or who have a location of prior residence that is outside of the 20km radius as an exogenous Poisson process for each facility.

[36] I assume that $\{\Pi_{ij}\}_{j \in J_i}$ are distributed with finite expectation and unbounded support.

[37] An exponential discharge process implicitly assumes that facilities do not learn about a resident's likely length of stay after admission, including advance notice of discharge, and that facilities cannot affect a resident's rate of discharge. While there are strong legal protections against early discharge, recent work has shown some evidence that providers may speed up discharge rates when more full: (Hackmann and Pohl, 2018) find that Medicaid residents are approximately 1 percentage point more likely to be discharged during high-occupancy weeks than low-occupancy ones. Such control over discharge rates could reduce facilities' incentives to pick their patients.

[38] $\mathcal{O}_i \subseteq J_i$ is analogous to a consideration set in the marketing and inattention literatures (Goeree, 2008). A recent paper that explicitly models choice sets as being determined by suppliers is (Dubois and Sæthre, 2020).

[39] If $\mathcal{O}_i = \emptyset$, then i is not admitted to any facility.

Another natural way to model the matching process in the nursing home industry is for each arriving resident to instantaneously and iteratively submit applications to facilities in decreasing order of her preference until she finds a facility willing to admit her. If facilities' willingness to admit a resident are unaffected by knowing whether other facilities wished to admit her, then the matches yielded by this algorithm will be identical to those yielded by the matching algorithm described in this section. Assumption 1 in the next section satisfies this condition, so it is equivalent to consider the theoretical model as reflecting resident i iteratively applying to facilities until she finds one willing to admit her.

5.2 Facility Admission Control Problem

In this section, I present the facility's admission control problem and optimal admission policy. I model facilities as profit maximizers that observe the characteristics c_i of each arriving local resident requiring care and determine whether to make the resident an offer of admission based on her characteristics, the facility's own current census of residents, and the facility's beliefs about potential residents it may admit in the future.

5.2.1 Assumptions

I make two key assumptions that greatly simplify the optimal control problem:

Assumption 1. *Facility j conditions its belief about the probability that resident i will accept an offer of admission only on characteristics c_i.*[40] *I denote this belief by $\mu_j(c_i) := P(u_{ij} \geq u_{ij'}\ \forall j' \in \mathcal{O}_i | c_i)$ and give its equilibrium condition in Subsection 5.3.*

Assumption 2. *Facility j receives a lump-sum payoff Π_{ij} when admitting resident i and continuous flow payoff $\Psi_j(N_{jt})$ based on its current census counts $N_{jt} := (N_{jt\phi})_{\phi \in \Phi}$, where $N_{jt\phi}$ denotes the number of residents of stayer-type ϕ residing at j at time t.*

Assumption 1 allows facility j to condition its belief that i will accept an offer of admission on i's characteristics and a general notion of competitors' admission rules (see Subsection 5.3). However, it notably does not allow j to adjust its beliefs based on the day-to-day variation in competitors' admission rules due to day-to-day variation in competitors' censuses. There are reasons to believe that this approximates reality. First, it is implausible that facilities know the exact current census of other local facilities since facilities are not required to disclose this information publicly. In my interviews with nursing staff and facility managers, none have suggested that they track the censuses of other facilities in an attempt to anticipate competitors' future admission policies. Furthermore, most J_i are large: approximately 50 facilities on average. Therefore, weak or negative correlation in the selectivity of other facilities in J_i may attenuate discernible variation in the probability that offers made by j will be accepted.[41]

[40] Note that this restriction need only apply to j's beliefs about i that may arrive in the future. This is because once i has arrived, the probability that this specific i will accept admission is not relevant to whether facility j would like to admit her.

[41] Appendix C.1 estimates the relationship between local facilities' censuses to be relatively weak. However, Assumption 1 might still be concerning in concentrated markets in which stochastic variation in the policy of one facility may have non-trivial impacts on the likelihood that offers from competing facilities are accepted.

Assumption 2 imposes that facility profits can be decomposed into resident-specific components, Π_{ij}, and a census component, $\Psi_j(N_{jt})$. The resident-specific components encompass the variation in residents' profitabilities based on their health and payment characteristics and does not vary with current census. I parameterize Π_{ij} in Section 6.

For ease of exposition, I represent Π_{ij} as a lump-sum payment. This is without significant loss of generality: I show in Appendix F.1 that if facilities were to receive resident-specific flow payoffs instead of lump-sums, then facilities' admission policies are identical to those in a translated game with lump-sums. Specifically, if facilities were to receive flow payoff $\pi_{ij}(t-\tau_i^A)$ at time t for resident i who arrived at τ_i^A, the facility's optimal policies are identical to the case in which it instead received upfront payoffs Π_{ij} equal to the i's expected admission-discounted π_{ij}:

$$\Pi_{ij} := \mathbb{E}_{\tau_i^D-\tau_i^A}\left[\int_0^{\tau_i^D-\tau_i^A} \exp\left(-\rho\tau\right)\pi_{ij}(\tau)d\tau\right], \tag{8}$$

where τ_i^D is i's time of discharge, and ρ is the discount rate. Intuitively, this follows from facilities' risk neutrality, that discharges are exogenous, and the assumption that the π_{ij} flow payoff neither affects nor is affected by any other current or future residents. Together, these imply that a facility's incentive to admit a resident is the same whether it expects to receive a flow payoff or an equivalent value lump-sum payoff discounted to the time of admission.

Assumption 2 restricts that all non-separable payoffs occur only through N_{jt}, the census count of the different stayer-types. This prohibits, for example, that particularly sick patients increase the marginal costs of other patients. On the other hand, $\Psi_j(N_{jt})$ may encompass or approximate a number of plausible interactions such as diseconomies of scale.[42] I parameterize $\Psi_j(N_{jt})$ in Section 6.

5.2.2 Admission Control Problem

Facility j chooses an adapted admission plan $\{a_{j\tau}\}$, where j offers admission to resident i arriving at τ if and only if j has a bed available—i.e. $\|N_{j\tau_i^A}\|_1 < b_j$—and i's characteristics satisfy j's admission rule—i.e. $c_i \in a_{j\tau}$. At each time t, the facility's admission plan maximizes the present discounted value of future admitted Π_{ij}:

$$\mathbb{E}\left[\sum_{i\in\mathcal{A}_j^{\geq t}(\{a_{j\tau}\})} \exp\left(\rho(\tau_i^A-t)\right)\Pi_{ij} + \int_t^\infty \exp(-\rho\tau)\Psi_j(N_{j\tau})d\tau \mid \mathcal{F}_t^j\right], \tag{9}$$

where $\mathcal{A}_j^{\geq t}(\{a_{j\tau}\})$ is the set of all patients admitted on or after t given admission plan $\{a_{j\tau}\}$, τ_i^A denotes the time that i arrives to the market, $\mathcal{F}_t^j$ is a filtration representing j's information, and the expectation is taken given j's beliefs satisfying Assumption 1.[43] I show in Appendix A.1 that Assumptions 1 and 2 imply that the optimal admission policy can be characterized by threshold profitabilities for each stayer-type, $\{\underline{\Pi}_{j\phi}\}_{\phi\in\Phi}$, that vary only with the facility's current census counts N_{jt}. In other words, facility j offers admission to i arriving at time t if and only if $\Pi_{ij} \geq$

[42] For example, if short-stay residents are likely to disproportionately consume resources required for rehab care and long-stay and very-long-stay residents are more likely to consume resources required for long-term care, then $\Psi_j(N_{jt})$ could represent increasing marginal costs in these respective resources through linearly separable increasing marginal costs in the number of short-stayers and the number of long- and very-long-stayers.

[43] Formally, $\mathcal{A}_j^{\geq t}(\{a_{j\tau}\}) := \{i : \tau_i^A \geq t, \|N_{j\tau_i^A}\|_1 < b_j, c_i \in a_{j\tau_i^A}, u_{ij} \geq u_{ij'}\ \forall j' \in \mathcal{O}_i\}$

$\underline{\Pi}_{j\phi_i}(N_{jt})$.[44] Theorem 2 gives these threshold policies explicitly.

Theorem 2. *Facility j offers admission to resident i if and only if:*

$$\Pi_{ij} \geq \underline{\Pi}_{j\phi_i}(N_{jt}) := V_j(N_{jt}) - V_j(N_{jt} + 1^{\phi_i}), \tag{10}$$

where V_j is the facility's value function, 1^ϕ denotes a unit-vector increment of element ϕ (i.e. adding one more resident of type ϕ), and $\underline{\Pi}_{j\phi_i}(N_{jt}) := V_j(N_{jt}) - V_j(N_{jt} + 1^{\phi_i})$ is the opportunity cost of admitting i and allowing her to occupy a bed. Formally, the facility's corresponding Bellman equation is:

$$\begin{aligned}\rho V_j(N_{jt}) = \Psi_j(N_{jt}) &+ \sum_\phi \lambda^D_{j\phi} N_{jt\phi} \left(V_j(N_{jt} - 1^\phi) - V_j(N_{jt})\right) \\ &+ \sum_\phi \lambda^A_{j\phi} \int \max\left\{0, \Pi + V_j(N_{jt} + 1^\phi) - V_j(N_{jt})\right\} dF^\Pi_{j\phi}(\Pi),\end{aligned} \tag{11}$$

where $\lambda^A_{j\phi} := \lambda^A P(j \in J_i, \phi_i = \phi)$ and $F^\Pi_{j\phi}(\Pi) := \int_{\{c_i:\Pi_{ij} \leq \Pi\}} \mu_j(c_i) dF_c(c_i | j \in J_i, \phi_i = \phi)$.

Proof. See Appendix A.3. □

Theorem 2 is extremely intuitive to interpret: equation (10) implies that facility j admits resident i if and only if Π_{ij}, the profit that resident-specific profit that i generates at j, exceeds $V_j(N_{jt}) - V_j(N_{jt} + 1^{\phi_i})$, the opportunity cost of admitting j and allowing her to take up a bed (i.e. increment $N_{jt\phi_i}$). Equation (11) then gives the facility's Bellman equation, where the three terms on the right hand side respectively represent census-specific profits, potential future discharges, and potential future admissions.

The relative simplicity of Theorem (2) is central to my estimation procedure. In particular, the optimal admission policy (10) does not vary with other facilities' censuses, the facility's own history, or anything about the facility's current census other than the vector of counts of different stayer-types, N_{jt}. I have emphasized Assumptions 1 and 2 in this section as they underlie this result. Without Assumption 1, optimal policies would need to incorporate anticipated dynamic variation in other facilities' policies, and without Assumption 2, optimal policies might need to incorporate additional characteristics and even arrival times of current residents. I discuss the implications of relaxing these assumptions further in Appendix F.2 and find that relaxing either assumption even slightly leads to an explosion in the size of facilities' policy-relevant state spaces. Even while imposing the Assumptions 1 and 2, the state spaces for each facility is quite large: 176,851 states for a typical 100 bed facility and 585,276 for a 150 bed facility.

5.3 Equilibrium Concept and Beliefs

I define an equilibrium to be a set of strategies $\{\underline{\Pi}_{j\phi}\}$ and beliefs $\{\mu_j\}$ such that for each facility j, $\{\underline{\Pi}_{j\phi}\}_\phi$ satisfies optimality under Theorem 2 given μ_j, and μ_j satisfies the following condition:

$$\mu_j(c_i) = \int \mathbf{1}\left\{u_{ij} \geq u_{ij'}\ \forall j' \in \mathcal{O}_{i-j}\right\} dP\left(\mathcal{O}_{i-j}\right), \tag{12}$$

[44] In terms of (9): $a_{j\tau} = \{c_i : \Pi_{ij} \geq \underline{\Pi}_{j\phi}(N_{jt})\}$.

$$\mathcal{O}_{i-j} := \mathcal{O}_i \setminus \{j\} \tag{13}$$

where $P\left(\mathcal{O}_{i-j}\right)$ is the long-run joint distribution of offers made by other facilities implied by $\{\underline{\Pi}_{j\phi}\}$, and the indicator is for whether i would accept an offer of admission from j given that j's others offers are $\mathcal{O}_{i-j}$.[45] Intuitively, equation (12) imposes a consistency requirement on the beliefs in Assumption 1. Assumption 1 implies that facilities behave as if playing a single-agent game in which the probability of a resident with characteristics c_i is willing to accept an offer of admission is constant, and (12) requires that these beliefs be consistent with the unconditional admission policies being applied by j's competitors. Thus, facility j knows the unconditional likelihood that a resident with characteristics c_i will accept an offer of admission but does not condition that belief on, for example, day-to-day variation in the census (and therefore admissions policies) of j's competitors.

6 Structural Estimation

In this section, I present my procedure to estimate a structural model of admissions in the nursing home industry. The key challenge to estimation is that only realized admissions are observed. First, this entails estimating demand with unobserved choice set restrictions. In particular, I cannot infer that a patient's admitting facility was her most preferred local facility since she may have preferred a different facility to which she was denied admission.[46] Second, this also complicates estimating facilities' admission policies because I do not directly observe facilities' actions—i.e. whether they offer admission to a given resident—which precludes common conditional choice probability methods to directly estimate firm policies from observed actions.

I address the fact that choice sets are not observed by explicitly modeling patients' choice sets as being determined by facilities' admission policies. In a first stage, I parameterize facilities' threshold rules as functions of facilities' characteristics and current census counts so that a facility offers a patient admission if the patient's expected profitability exceeds the facility's threshold at its current census. I then estimate demand simultaneously with these non-structural admission policies: given a patient's set of admission offers, I let the patient's choice probabilities be distributed according to a logit demand model in which the patient's preferences for each facility are functions of the facility's characteristics, its distance from the patient's home, and the amount the resident expects to pay out of pocket at the facility. Preferences over characteristics are allowed to differ for Medicaid-eligible patients so that disparate admission patterns for dual-eligibles may potentially be attributed to heterogeneous preferences.

I estimate this first stage via maximum likelihood, where the likelihood of an observed admission is computed by integrating the patient's logit choice probabilities over the distribution of choice sets that may have been available to her given her characteristics and the current censuses of her local facilities.[47] As in Section 4, the key restriction allowing

[45]The $P\left(\mathcal{O}_{i-j}\right)$ corresponding to $\{\underline{\Pi}_{j\phi}\}$ is unique. This follows from the fact that unbounded support for Π_{ij} and nonzero discharge rates imply that the Markov process characterizing the joint evolution of all facilities' states given $\{\underline{\Pi}_{j\phi}\}$ has a unique recurrent class.

[46]Availability variation has been addressed in the marketing and economics literatures (see Hickman and Mortimer (2016)), and my model of unobserved choice set variation mirrors these approaches. Two unusual features of this setting are that choice sets are never observed and that they are are plausibly correlated with the preferences of individual demanders. Therefore, common methods to estimate the product availability process in a first stage are not feasible.

[47]Note that even given the parameterizations of facilities' threshold rules and residents' profitabilities, I allow for uncertainty in residents' choice sets by allowing a component of each resident's profitability to be unobserved by the econometrician. The probability of each choice set is then

patient preferences and selective admissions to be separately identified is that current facility censuses are excluded from residents' preferences. Because admission policies vary with current census counts but patient preferences do not, the relationship between facilities' censuses and realized admission patterns identifies facilities' admission policies.

While this first stage identifies both residents' preference parameters and facilities' admission policies, the estimated admissions policies are non-structural.[48] Therefore, the first stage estimates cannot be used to simulate how facilities would change their threshold rules in response to counterfactual policies. Only the first of my three counterfactuals—the strict enforcement of first come, first served admission policies—can be simulated using these first-stage estimates since admission policies under this counterfactual are trivial.

I therefore estimate the structural profitability parameters underlying facilities' admission policies in a second stage. In my chosen parameterization, these parameters characterize the baseline profitabilities of different stayer-types. I estimate the second stage by matching admission patterns implied by optimal policies computed from Theorem 2 to moments in the data.[49] These structural parameters allow me to simulate how facilities would adjust their admission policies in response to counterfactual government policies by recomputing admission policies and beliefs that satisfy the equilibrium optimality and belief conditions given in Section 5.3.

6.1 First Stage

In the first stage, I jointly estimate resident preferences and facility admission policies via maximum likelihood. I denote by $\boldsymbol{X}_i$ the set of observed data related to resident i and the facilities in J_i. These observed data include the resident's health and demographic characteristics (h_i), stayer-type ($\phi_i \in \{\text{short, long, very-long}\}$), and desirability score (z_i) from Section 4.3, as well as the characteristics and states of all of the resident's local facilities ($\{(X_j, N_{jt})\}_{j\in J_i}$).

I parameterize resident i's indirect utility for each facility $j \in J_i$ by:

$$u_{ij} = X_j\beta_i^X + f(D_{ij}, \beta^D) - \alpha\mathbb{E}[OoP_{ij}] + \xi_j + \epsilon_{ij}^u, \tag{14}$$

$$\beta_i^X = \beta^X + h_i\beta^{Xh}, \tag{15}$$

where X_j are the characteristics of facility j, D_{ij} is the distance from i's prior residence to facility j, $\mathbb{E}[OoP_{ij}]$ is the admission-discounted average daily out-of-pocket payment that i expects to make at j, ξ_j is facility quality unobserved to the econometrician, and ϵ_{ij}^u is Type-I Extreme Value distributed. Rather than placing a parametric distribution on ξ_j, I include facility fixed effects capturing both observed and unobserved quality:

$$Q_j = X_j\beta^X + \xi_j. \tag{16}$$

While this greatly increases the computational burden of estimation, it allows me to capture unobserved quality and

computed using the parametric distribution of this unobserved component of profitability.

[48] This can be thought of as analogous to the first-stage of a conditional choice probability estimator.

[49] In order to reduce the computational burden of the second stage, I hold fixed first-stage estimates of residents' preferences, the relative profitabilities of residents within each stayer-type, and facilities' beliefs about the arrival process of patients willing to accept an offer of admission.

helps the model to match aggregate shares for each facility.

While this preference model does not include random coefficients, it does allow preferences over facilities' characteristics to vary with patients' characteristics at admission. In particular, I allow Medicaid-eligible residents to have different preferences over facilities' staffing ratios of registered nurses and licensed practical nurses. This allows the model to incorporate some preference heterogeneity between low- and high-margin patients. Most notably, it admits the possibility that disparities in the quality of facilities where Medicaid-eligible residents receive care may simply reflect that these residents are less sensitive to quality, either due to weaker structural preferences or fewer resources to obtain and leverage quality information. Most importantly, equation (14) does not allow preferences over a facility's census at time of admission. This is the key exclusion restriction that allows me to use within-facility covariation between facility census and admitted patient characteristics to identify selective admission practices.

Given a choice set $\mathcal{O}_i$, I denote the admission probability of resident i at facility $j \in J_i$ by $s_{ij}^{\mathcal{O}_i}$:

$$s_{ij}^{\mathcal{O}_i} = \begin{cases} \frac{\exp\left(X_j \beta_i^X + f(D_{ij}, \beta^D) - \alpha \mathbb{E}[OoP_{ij}] + \xi_j\right)}{\sum_{j' \in \mathcal{O}_i} \exp\left(X_{j'} \beta_i^X + f(D_{ij'}, \beta^D) - \alpha \mathbb{E}[OoP_{ij'}] + \xi_{j'}\right)} & \text{if } j \in \mathcal{O}_i \\ 0 & \text{if } j \notin \mathcal{O}_i. \end{cases} \tag{17}$$

If $\mathcal{O}_i$ were observed, the likelihood of observing resident i admitted at facility j would be $s_{ij}^{\mathcal{O}_i}$. However, since $\mathcal{O}_i$ is not observed, the econometrician must integrate over the likelihood of all possible $\mathcal{O}_i$. Denote i's realized admission at j by a_{ij}. Then the econometrician's likelihood of observing resident i admitted at facility j is:

$$P\left(a_{ij} | \theta_1, \boldsymbol{X}_i\right) = \sum_{\mathcal{O} \in 2^{J_i} \setminus \emptyset} P\left(\mathcal{O}_i = \mathcal{O} | \theta_1, \boldsymbol{X}_i\right) s_{ij}^{\mathcal{O}}, \tag{18}$$

where 2^{J_i} denotes the set of all subsets of J_i, θ_1 are the parameters to be estimated governing residents' preferences and facilities' admission policies, and $P\left(\mathcal{O}_i = \mathcal{O} | \theta_1, \boldsymbol{X}_i\right)$ is the probability that the resident's choice set is $\mathcal{O}_i$.

Since residents' choice sets are determined by facilities' admission policies, $P\left(\mathcal{O}_i = \mathcal{O} | \theta_1, \boldsymbol{X}_i\right)$ can be expressed in terms of the admission policies of the facilities in J_i. In doing this, it is helpful to first scale Π_{ij} so that it is more easily interpreted and compared across stayer-types.

Definition 6.1. *Define $\tilde{\pi}_{ij}$ to be the average admission-discounted resident-specific flow profit from i at j:*

$$\tilde{\pi}_{ij} := \left(\rho + \lambda_{j\phi_i}^D\right) \Pi_{ij}. \tag{19}$$

In other words, $\tilde{\pi}_{ij}$ is the unique constant flow payoff that, if received throughout resident i's stay, is expected to yield an admission discounted payoff of Π_{ij}. Because $\tilde{\pi}_{ij}$ is scaled in terms of flows, it is more easily comparable across stayer-types than Π_{ij}. If the unit of time is days, then $\tilde{\pi}_{ij}$ is an average discounted daily resident-specific profitability. By Theorem 2, facility j offers admission to resident i arriving at time t if and only if:

$$\tilde{\pi}_{ij} \geq \underline{\tilde{\pi}}_{j\phi_i}(N_{jt}) := \left(\rho + \lambda_{j\phi_i}^D\right) \underline{\Pi}_{j\phi_i}(N_{jt}). \tag{20}$$

Since neither $\tilde{\pi}_{ij}$ nor $\underline{\tilde{\pi}}_{j\phi_i}(N_{jt})$ are observed directly, I parameterize them:[50]

$$\tilde{\pi}_{ij} = \tilde{\pi}(z_i, \phi_i; \gamma_\pi) - \epsilon_i^\pi, \qquad \underline{\tilde{\pi}}_{j\phi}(N_{jt}) = \underline{\tilde{\pi}}(N_{jt}, X_j, \phi; \gamma_{\underline{\pi}}), \tag{21}$$

where ϵ_i^π follows a standard normal distribution. It is important to note that in this first stage, $\underline{\tilde{\pi}}_{j\phi}(N_{jt})$ is not the solution to facilities' admission control problems as in Theorem 2. This is because it is too computationally expensive solve for each facility's optimal policies 2 as a nested part of estimating preference parameters. The specific parameterization I use for $\tilde{\pi}$ is a polynomial in z_i and $z_i^{\%le}$ with intercepts in ϕ_i. Similarly, I parameterize $\underline{\tilde{\pi}}$ to include intercepts in ϕ, a polynomial in $\mathbb{E}[z_i|a_{ij}, N_{jt}]$, and the difference between the average z_i of j's residents and the average z_i of j's local community. I provide additional details on these parameterizations in Appendix I. Note that a consequence of parameterization (21) is that $\tilde{\pi}_{ij}$ does not vary across facilities since z_i, ϕ_i, and ϵ_i^π do not vary across facilities.[51] Therefore, variation in i's admission offers at different facilities derives entirely from variation across facilities in current threshold rules $\underline{\tilde{\pi}}_{j\phi_i}(N_{jt})$.

Another implication of each resident's ϵ_i^π being shared by all facilities is that the set of possible choice sets is much smaller than $2^{|J_i|}$. To see this, order facilities in J_i by decreasing willingness to admit residents of stayer-type ϕ_i:

$$\underline{\tilde{\pi}}(N_{1t}, X_1, \phi_i; \gamma_{\underline{\pi}}) \leq \underline{\tilde{\pi}}(N_{2t}, X_2, \phi_i; \gamma_{\underline{\pi}}) \leq .... \leq \underline{\tilde{\pi}}(N_{|J_i|t}, X_{|J_i|}, \phi_i; \gamma_{\underline{\pi}}) \tag{22}$$

Then all possible choice sets are of the form $[k] := \{1, 2, ..., k\}$.[52] By Theorem 2:[53]

$$\begin{aligned} P(\mathcal{O}_i = [k]|\theta_1, \boldsymbol{X}_i) &= P\left(\tilde{\pi}_{ij} \geq \underline{\tilde{\pi}}_{j\phi_i}(N_{jt}) \;\forall\; j \in [k], \tilde{\pi}_{ij} < \underline{\tilde{\pi}}_{j\phi_i}(N_{jt}) \;\forall\; j \notin [k]|\theta_1, \boldsymbol{X}_i\right) \\ &= P\left(\tilde{\pi}(z_i, \phi_i; \gamma_\pi) - \underline{\tilde{\pi}}(N_{kt}, X_k, \phi_i; \gamma_{\underline{\pi}}) \geq \epsilon_i^\pi \geq \tilde{\pi}(z_i, \phi_i; \gamma_\pi) - \underline{\tilde{\pi}}(N_{k+1t}, X_{k+1}, \phi_i; \gamma_{\underline{\pi}})\right) \\ &= F_{\epsilon^\pi}\left(\tilde{\pi}(z_i, \phi_i; \gamma_\pi) - \underline{\tilde{\pi}}(N_{kt}, X_k, \phi_i; \gamma_{\underline{\pi}})\right) - F_{\epsilon^\pi}\left(\tilde{\pi}(z_i, \phi_i; \gamma_\pi) - \underline{\tilde{\pi}}(N_{k+1t}, X_{k+1}, \phi_i; \gamma_{\underline{\pi}})\right). \end{aligned} \tag{25}$$

Therefore the log-likelihood of observing resident i admitted at j is:[54]

$$\log\left(P\left(a_{ij}|\theta_1, \boldsymbol{X}_i\right)\right) = \log\left(\sum_{k=j}^{|J_i|} s_{ij}^{[k]} P\left(\mathcal{O}_i = [k]|\gamma_\pi, \gamma_{\underline{\pi}}, \boldsymbol{X}_i\right)\right), \tag{27}$$

where $\theta_1 = \{\gamma_\pi, \gamma_{\underline{\pi}}, \beta^D, \beta^{Xh}, \alpha, \{Q_j\}_j\}$, F_{ϵ^π} is the CDF of the standard normal distribution, and the relevant terms are substituted from (23), (24), and (25).[55] The parameters $\{\beta^D, \beta^{Xh}, \alpha, \{Q_j\}_j\}$ characterize residents' preferences,

[50] While it would be ideal to parameterize $\tilde{\pi}_{ij}$ and $\underline{\tilde{\pi}}_{j\phi_i}(N_{jt})$ with facility-specific functions, this is both computationally infeasible and requires a large sample for each facility.

[51] While this is a consequence of my chosen parameterizations, it is not a requirement of my estimation procedure.

[52] It is straightforward to verify that if (20) holds for k—i.e. if k offers admission to i—then (20) holds for $k' < k$. This relies primarily on the assumption that ϵ_i^π is not facility-specific.

[53] For completeness:

$$P(\mathcal{O}_i = \emptyset|\theta_1, \boldsymbol{X}_i) = 1 - F_\epsilon\left(\tilde{\pi}(z_i, \phi_i; \gamma_\pi) - \underline{\tilde{\pi}}(N_{1t}, X_1, \phi_i)\right), \tag{23}$$
$$P(\mathcal{O}_i = [|J_i|]|\theta_1, \boldsymbol{X}_i) = F_\epsilon\left(\tilde{\pi}(z_i, \phi_i; \gamma_\pi) - \underline{\tilde{\pi}}(N_{|J_i|t}, X_{|J_i|}, \phi_i)\right). \tag{24}$$

[54] In estimation, the econometrician must additionally condition on i being observed in the data (i.e., on $\mathcal{O}_i \neq \emptyset$):

$$\log\left(P\left(a_{ij}|\theta_1, \boldsymbol{X}_i, \mathcal{O}_i \neq \emptyset\right)\right) = \log\left(P\left(a_{ij}|\gamma_\pi, \gamma_{\underline{\pi}}, \boldsymbol{X}_i\right)\right) - \log\left(P\left(\mathcal{O}_i = \emptyset|\gamma_\pi, \gamma_{\underline{\pi}}, \boldsymbol{X}_i\right)\right). \tag{26}$$

[55] The ϕ-specific intercept terms in $\tilde{\pi}(z_i, \phi_i; \gamma_\pi)$ and $\underline{\tilde{\pi}}(N_{jt}, X_j, \phi; \gamma_{\underline{\pi}})$ are not identified since only their difference, $\tilde{\pi}(z_i, \phi_i; \gamma_\pi)$ −

γ_π characterizes the relative profitabilities of residents within each stayer-type, and $\gamma_{\underline{\pi}}$ characterizes facilities' admission thresholds.[56]

6.2 Second Stage

While the first-stage estimates do characterize facilities' admission policies, they are not sufficient to simulate how facilities would adjust their threshold rules in response to counterfactual government policies. Doing so requires computing threshold rules as the solution to facilities' admission control problems as in Theorem 2. I therefore estimate the remaining primitives underlying facilities' admission control problems in a second stage by matching admission patterns implied by optimal admission policies to moments in the data.

The structural parameters I estimate in the second stage are ψ in $\Psi_j(N_{jt}) = N_{jt}'\psi$. In this parameterization of $\Psi_j(N_{jt})$, the vector ψ gives a baseline profitability for each stayer-type. Intuitively, where the γ_π in the first stage identified the relative profitabilities of residents within each stayer-type, ψ identifies the levels of profitability relative to not admitting the patient.[57] Note that this specification includes neither heterogeneity in Ψ across facilities nor diseconomies of scale. Therefore, selective admission practices under this specification primarily reflect the opportunity costs to admissions that are due to facilities' capacity constraints. Optimal threshold rules will vary across facilities by capacity constraints and the arrival process of residents willing to accept an offer of admission. They will also vary within facility over time with census counts.

I compute facilities' optimal admission policies via policy function iteration. Computing optimal threshold rules given V_j is direct from equation (20). On the other hand, computing V_j given admission policies $\{\tilde{\underline{\pi}}_{j\phi}\}_\phi$ requires integrating over $\{F^{\Pi}_{j\phi}\}_\phi$, the facility's beliefs about the profitability distribution of arriving residents willing to accept an offer of admission. In order to reduce the computational burden of the estimation procedure, I simulate beliefs $\{\hat{F}^{\Pi}_{j\phi}\}_\phi$ using first-stage estimates of residents' preferences and other facilities' admission policies.[58] Additional details are provided in Appendix J. After substituting $\{\hat{F}^{\Pi}_{j\phi}\}_\phi$ into equation (11), V_j can be solved in terms of $\{\tilde{\underline{\pi}}_{j\phi}\}_\phi$ as the solution to a sparse banded system of equations.[59]

Denote the optimal admission thresholds solving facility j's admission control problem by $\tilde{\underline{\pi}}_j(N_{jt}; \psi, \hat{\theta}_1)$. The realized admission probabilities implied by the model are:

$$P\left(a_{ij}|\psi, \hat{\theta}_1, \boldsymbol{X}_i\right) = \sum_{k=j}^{|J_i|} s_{ij}^{[k]} P\left(\mathcal{O}_i = [k]|\psi, \hat{\theta}_1, \boldsymbol{X}_i\right), \tag{28}$$

$$P\left(\mathcal{O}_i = [k]|\psi, \hat{\theta}_1, \boldsymbol{X}_i\right) = F_{\epsilon^\pi}\left(\tilde{\pi}(z_i, \phi_i; \hat{\gamma}_\pi) - \tilde{\underline{\pi}}_k(N_{jt}; \psi, \hat{\theta}_1)\right) - F_{\epsilon^\pi}\left(\tilde{\pi}(z_i, \phi_i; \hat{\gamma}_\pi) - \tilde{\underline{\pi}}_k(N_{jt}; \psi, \hat{\theta}_1)\right). \tag{29}$$

$\tilde{\underline{\pi}}(N_{jt}, X_j, \phi_i; \gamma_{\underline{\pi}})$, appears in the likelihood.

[56] Heuristically, the preference parameters are identified off of admission patterns for patients with similar predicted choice sets, while the admission policies are identified off the relationship between admission patterns and facilities' censuses.

[57] In particular, ψ identifies the intercept terms that were not identified in the first stage. See footnote 55.

[58] Note that because beliefs $\hat{F}^{\Pi}_{j\phi}$ are fixed throughout the second stage estimation procedure, there is no guarantee that $\hat{F}^{\Pi}_{j\phi}$ implied by our first stage policy estimates is the same as that implied by our second stage policy estimates. This is often the case for two-step estimators, and ensuring internal consistency would require recomputing the beliefs and optimal policies as a fixed point during the estimation procedure, which is computationally prohibitive.

[59] Solving (11) for V_j also requires $\{\lambda^A_{j\phi}\}$ and $\{\lambda^D_{j\phi}\}$, which I estimate via maximum likelihood prior to the second stage.

I estimate ψ by matching the following moments in the data:[60]

$$\mathbb{E}\left[\mathbf{1}\left\{a \leq occ_{jt}^{\%le} \leq b, \phi_i = \phi\right\}\right], \tag{30}$$

$$\mathbb{E}\left[\tilde{\pi}(z_i, \phi_i; \hat{\gamma}_\pi)\mathbf{1}\left\{a \leq occ_{jt}^{\%le} \leq b, \phi_i = \phi\right\}\right], \tag{31}$$

for each $\phi \in \Phi = \{\text{short, long, very-long}\}$ and $(a, b) \in \{(0, \frac{1}{3}), (\frac{1}{3}, \frac{2}{3}), (\frac{2}{3}, 1)\}$, where $occ_{jt}^{\%le}$ is facility j's percentile occupancy within its own occupancy distribution.[61] In other words, I match the degree to which residents of different stayer-types and levels of profitability are admitted to facilities in low, middle, or high occupancy states.

Due to the substantial computational requirements of computing facilities' optimal policies (see Sections 5 and F.2), I estimate the second stage only on the San Diego area. I chose the San Diego area for its moderate size and geographic separation from other markets (see Figure B.3a). The region has 67 facilities and approximately 50 thousand arriving residents in my sample.

7 Estimates

7.1 Demand Estimates

Table 1 presents the preference parameter estimates. Column 1 gives the estimates from the main specification. These estimates indicate that residents are highly sensitive to the distance of a facility from their previous residence: the average elasticity of demand with respect to distance is 4.15%. This high degree of sensitivity is consistent with the prior literature estimating demand models for both nursing homes (Rahman et al., 2014a; Hackmann, 2019) and hospitals (Tay, 2003; Ho, 2006; Ho and Pakes, 2014; Gowrisankaran et al., 2015; Shepard, 2022). Residents may prefer closer facilities because of their likely proximity to the resident's family, friends, and community. It is also plausible that residents are less aware of distant facilities.[62]

The estimates also suggest that while all residents prefer facilities with higher nursing staff ratios, these preferences are weaker for Medicaid-eligible residents. The average own-staffing elasticity of demand for Registered Nurses (RNs) and Licensed Practical Nurses (LPNs) are respectively .91% and 1.21% for non-Medicaid residents and .59% and .65% for Medicaid residents. This estimated preference disparity can be interpreted in a number of ways. First, Medicaid-eligible residents may truly care less about staffing ratios because their longer stays and differing disease diagnoses may benefit less from high staffing ratios than residents who primarily utilize short-term rehab and therapy care. Alternatively, it may be that Medicaid-eligible residents have similar preferences to non-Medicaid residents but are less able to identify and apply to high-staffing facilities. For example, admission assessments from the MDS data indicate that Medicaid-eligible residents are less likely to have present and responsible family members, which suggests that these residents may have less assistance in selecting a facility. Lastly, if the model does not fully capture

[60] As with the first stage, the econometrician must also condition i being observed in the data (i.e., on $\mathcal{O}_i \neq \emptyset$).

[61] There are 18 moments in total, six for each of the three stayer-types.

[62] For example, the default for CMS' Nursing Home Compare tool is to sort by distance. Because awareness is unobserved, my model cannot distinguish true preference over distance from a lack of awareness and attributes both to preferences.

Table 1: Preference Parameter Estimates

	Main (1)	No Preference Heterogeneity (2)	No Selective Admission (3)
Distance (km)	-0.36	-0.36	-0.34
	(0.0014)	(0.0014)	(0.0013)
Distance-squared	0.0069	0.0069	0.0064
	(0.00007)	(0.00007)	(0.00007)
Out of Pocket ($100)	-0.05	-0.0086	-0.13
	(0.01)	(0.01)	(0.01)
RN Hours	3.45	3.30	2.98
	(0.34)	(0.33)	(0.27)
RN Hours X Medicaid Eligible	-1.05		-1.83
	(0.026)		(0.025)
LPN Hours	1.71	1.48	1.18
	(0.25)	(0.24)	(0.19)
LPN Hours X Medicaid Eligible	-0.76		-1.10
	(0.02)		(0.02)
For Profit	0.22	0.20	0.34
	(0.17)	(0.16)	(0.13)
Facility Fixed Effects	X	X	X
N	485,065	485,065	485,065

Notes. Mean preference terms (RN Hours, LPN Hours, and For Profit) are by regressing facility characteristics on estimated facility fixed effects. Standard errors are in parenthesis.

the extent of admissions discrimination against Medicaid-eligible residents, then estimates of Medicaid residents' preference for staffing are likely to be biased downward. This is clearest when contrasting the demand estimates from the main specification (Column 1) to those from a demand model without selective admission practices (Column 3). The estimated average staffing elasticities of demand for RNs and LPNs are substantially lower when failing to account for selective admissions, especially for Medicaid-eligible residents: respectively .74% and .81% for non-Medicaid residents and just .3% and .06% for Medicaid-eligible residents.

I estimate price sensitivities to be very small: the average elasticity of demand with respect to out-of-pocket expenditure is just .015%. Because I include facility fixed effects in the estimation, price sensitivity is identified from variation in residents' expected out-of-pocket payments at the same facility. Therefore, while these low estimates may indicate minuscule price sensitivity, they may also indicate that my model of out-of-pocket expenditure does not align with the resident's anticipated out-of-pocket expenditure. They may also reflect poor measurement of private pay rates, poor forecasting of out-of-pocket expenditure by residents, or indifference due to a perception that private pay is a purely transitory state before utilizing Medicaid. I discuss the computation of expected out-of-pocket costs and potential concerns with identifying price sensitivity in Appendix K.

7.2 Supply Estimates

Figure 6 summarizes the estimates from the first stage. Figure 6a depicts the estimated relationship between individual profitability and the desirability scores computed in Section 4.3. As expected, residents' individual-specific profitability is generally increasing in desirability score. Note that the scale is in terms of standard deviations of the normally distributed error representing the unobserved component of resident-specific profitability (equation 21), of which one standard deviation approximately corresponds to \$48.71 (see Section L).[63] I have also underlay the distribution of desirability scores for each stayer-type to show the regions of the curve most relevant for its profitability.

Figures 6b and 6c show the average relationship between offer probabilities and occupancies implied by facilities' estimated admission policies for different types of patients.[64] Consistent with the reduced form evidence, Figure 6b shows that the probability Medicaid-eligible patients are offered admission is both lower in the baseline and more sensitive to the opportunity cost of admission. On average, facilities are estimated to be just under 90% likely to offer admission to a Medicaid-eligible resident that arrives on the facility's least-full day. This drops to nearly 70% on facilities' most full days. On the other hand, while offer probabilities for non-Medicaid residents do decrease with occupancy, they are consistently over 90% on average. Likewise, we see a similar pattern with length of stay: the estimates suggest that facilities are much less likely to offer admission to prospective residents with longer anticipated stays and that the likelihood of receiving an offer is especially sensitive to occupancy for very-long-stay residents.

Figures 6d and 6e combine the estimated admission policies with the demand model to compute the model-implied probability of realized admissions and compares these with the data. Both suggest that the model matches the empirical patterns well. Ultimately, it is these patterns in the data that identifies the selective admissions component of the model.

[63] It is also worth noting that the intercepts in Figure 6a are not identified (see Footnote 55).

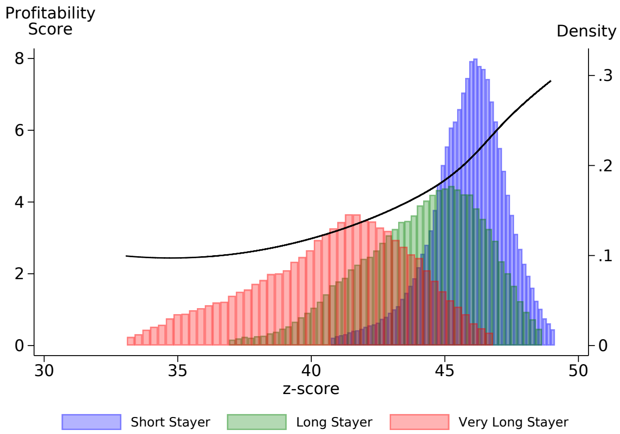
[64] Note that this relationship is not identical for all facilities: the figures average over all opportunities to offer admission and therefore represent a weighted average of all facilities' policies.

Likewise, Appendix Figure D.1 shows that the second stage model shows a similar decreasing pattern to the first stage, and it also matches the data quite well, though less precisely given the smaller sample for the second stage and the restriction requiring second-stage policies to be computed as solutions to each facility's Bellman equation.

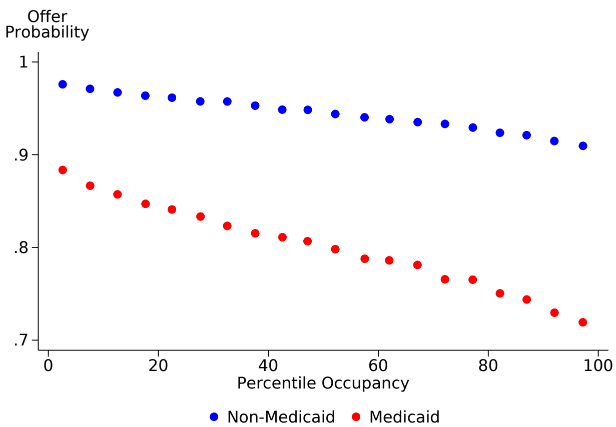
Figure 7 visualizes the extent to which selective admissions prevent patients from accessing their most preferred facilities. A large majority of patients (81%) are admitted to their first choice facility, suggesting that selective admissions do not limit the choice sets for most residents. The 19% of residents who are not admitted to their most-preferred facilities are disproportionately Medicaid-enrollees with very long anticipated stays. In fact, just 51% of Medicaid very-long-stay residents are admitted to their first choice facility. Moreover, because these residents tend to have lengthy stays, the impacts of selective admissions are much larger when measured by days rather than residents: 40% of resident days are spent at facilities other than the resident's first choice. Especially when viewed through this lens, selective admissions have a fairly dramatic effect on the allocation of care in the industry.

Figure 6: First Stage Estimates, Offer Probabilities, and Comparison to Data

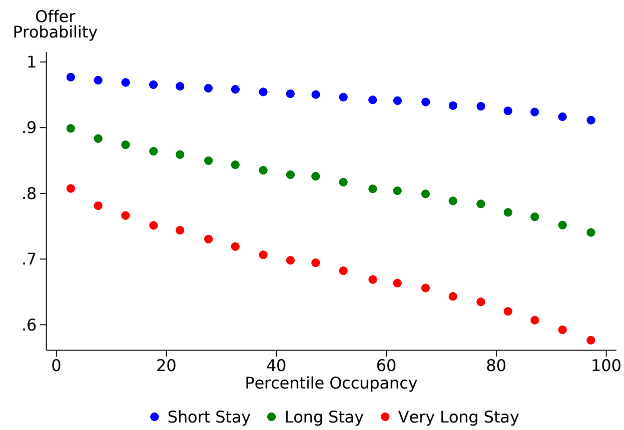
(a) Profitability ($\tilde{\pi}$) and Desirability (z) Scores, by Stayer-Type



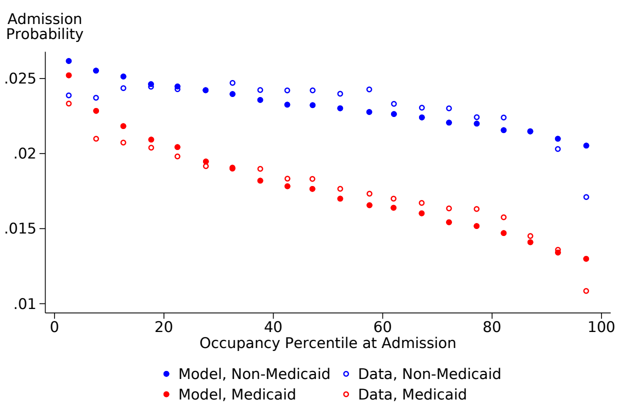
(b) Offer Probabilities, by Medicaid Eligibility



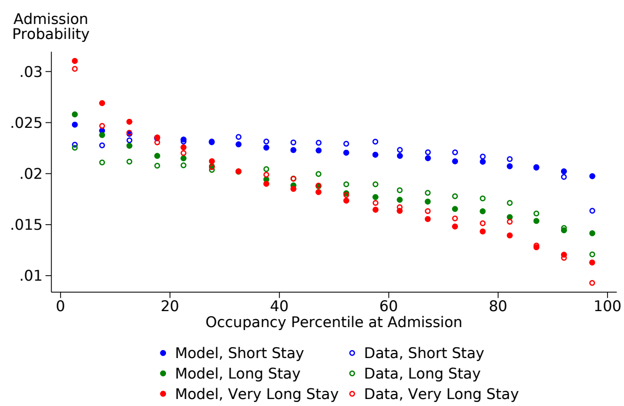
(c) Offer Probabilities, by Stayer-Type



(d) Admission Likelihood, by Medicaid Eligibility

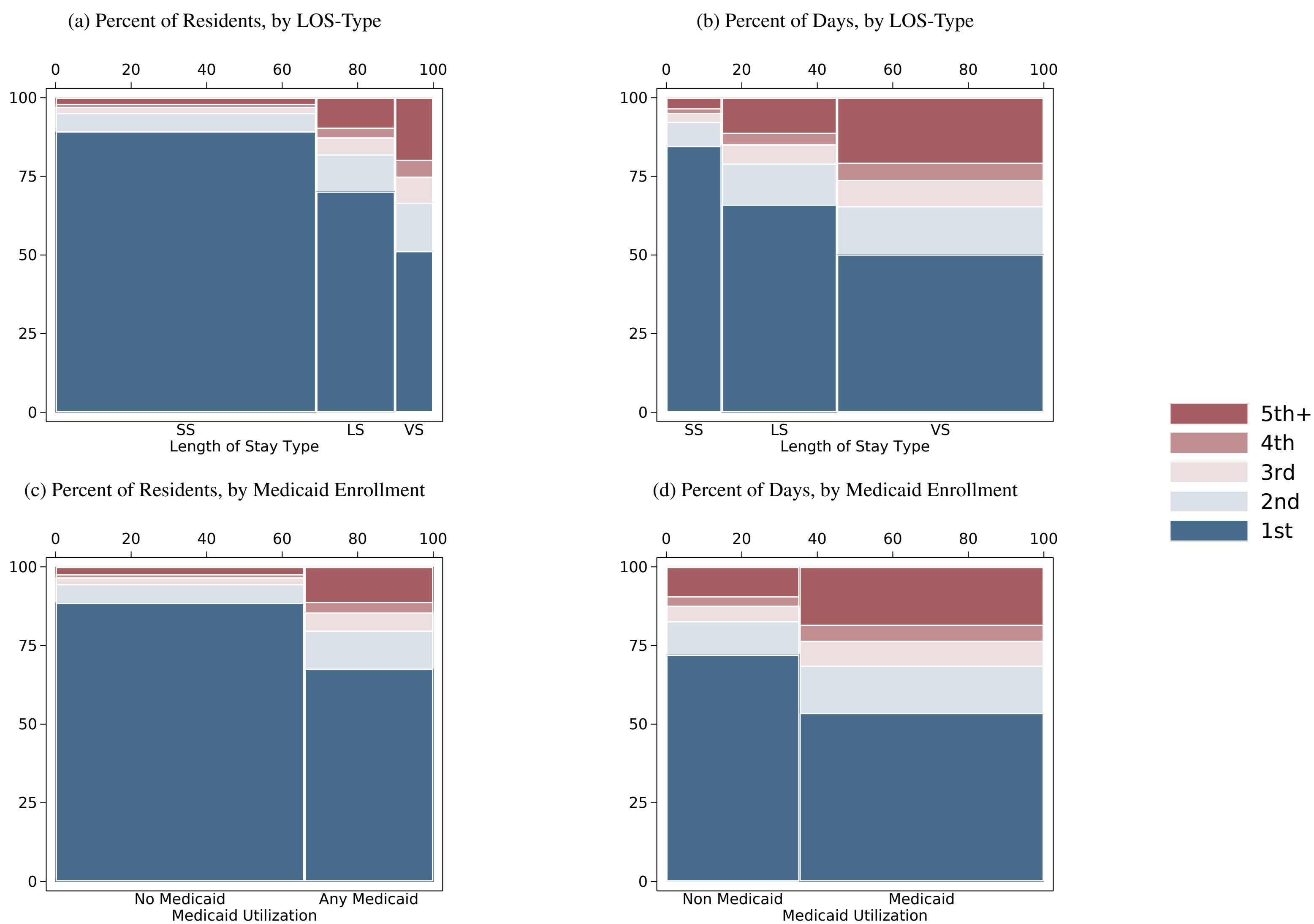


(e) Admission Likelihood, by Stayer-Type



Notes. The intercepts of $\tilde{\pi}$ are not identified.

Figure 7: Preference Rank of Admitting Facility



8 Counterfactuals

In this section, I present the simulated impact of three counterfactual policies intended to prevent or mitigate selective admission practices. The first is to prevent selective admissions by strictly enforcing that all facilities adhere to "first come, first serve" (FCFS) admission policies. Because admission policies are fixed by the regulator in this counterfactual, I am able to simulate the policy impacts for all of California using only preference parameter estimates. The second is to reduce incentives to discriminate against Medicaid-eligible residents by raising the Medicaid reimbursement rate. The third counterfactual is to increase capacity at facilities in a way that is targeted to benefit Medicaid-eligible residents. Because the latter two counterfactuals require computing counterfactual admission policies, I only simulate their impacts for the San Diego area. This reduces the computational burden, as well as restricts them to the region in which structural policy function were estimated.

It is important to note that all counterfactuals presented in this section hold facility characteristics fixed, including price, staffing, and unobserved measures of quality. While I do not allow facilities to adjust their characteristics, I present some evidence that suggests how facilities might adjust their staffing and other quality metrics if able in response counterfactual policies.

8.1 Counterfactual 1: First Come, First Serve

I model first come, first serve (FCFS) as an enforced prohibition on selective admissions, whereby facilities with vacancies must offer admission to all arriving residents. Under this rule, residents are admitted to their first choice facility with an available bed. I use the preference estimates from the first stage to simulate the realized admissions that would occur under FCFS.[65]

Figure 8 summarizes the impacts of enforcing FCFS by contrasting resident utilities under FCFS with those under selective admissions and the first-best case in which all patients are admitted to their most-preferred facility. Figure 8a shows the distribution of utilities for all residents, which suggest that most residents are actually better off under selective admissions than FCFS. Intuitively, this reflects the the fact the burden of selective admissions falls disproportionately on just a fraction of patients, with the majority receiving care at their first choice facility under selective admissions (see Section 7.2).

Therefore, to the extent that FCFS increases access to desirable facilities for low-margin patients, it simultaneously reduces access for high-margin patients by increasing the likelihood that desirability facilities will be at their capacity constraint.[66] This crowding-out effect is particularly strong because the patients whose access is most improved by FCFS tend to be very-long-stay patients who will occupy their bed for a long time. Figures 8c and 8d show the distributional impacts clearly: non-Medicaid short-stay patients were already almost achieving their first-best utility under selective admissions and are made worse off under FCFS, while Medicaid-eligible very-long-stay residents are

[65] Specifically, I simulate the arrival of 100 years of residents and allow these residents to be admitted at their most preferred facility with available occupancy at their time of arrival. I draw observable resident characteristics from the data and simulate each resident's idiosyncratic preferences, length of stay, and unobserved profitability parametrically. I initialize facilities to their mean census counts in the data and burn the first three years of simulation.

[66] This is because under selective admissions, even highly desirable facilities would rarely be entirely full, as they would intentionally reduce admissions as they near capacity to reduce the likelihood of having to turn away highly profitable patients.

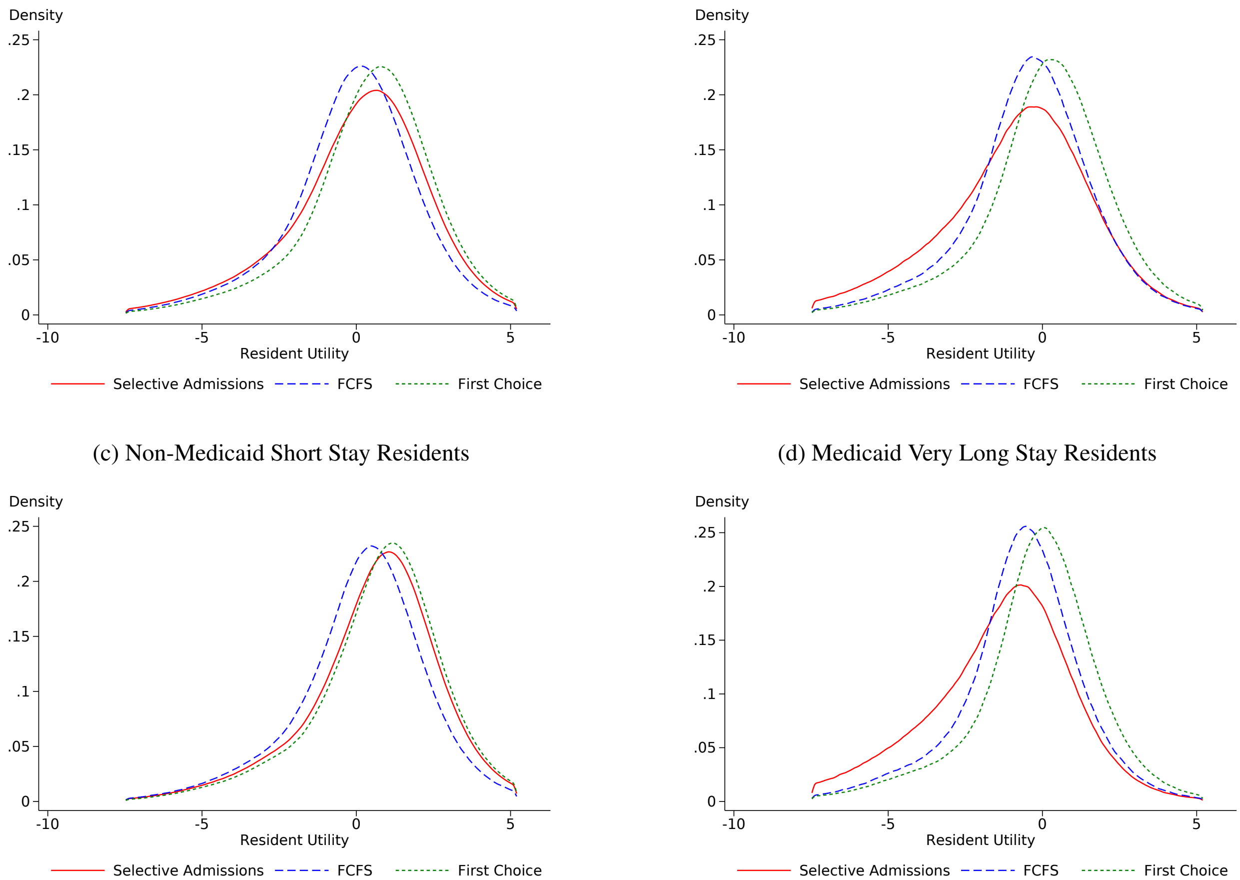
Figure 8: Counterfactual Utility under FCFS

(a) All Residents

(b) All Residents, weighted by LOS

(c) Non-Medicaid Short Stay Residents

(d) Medicaid Very Long Stay Residents



substantially better off under FCFS. These findings are further corroborated by comparing the preference ranks of patients' admitting facilities (i.e., realized matches) under FCFS (Figure E.1) to under selective admissions (Figure 7). Relative to the latter, the former shows significantly increased access for Medicaid-eligible and very-long-stay residents but substantially reduced access for short-stay residents.

While the majority of residents are worse off under FCFS, Figure 8b shows that when accounting for residents' differing lengths of stay, overall welfare is greater under FCFS. This is because residents who benefit from increased access under FCFS are very-long-stay residents whose care constitute a majority of care-days. That there are welfare gains under FCFS—rather than just a distributional transfer between residents—is largely because capacity at high-quality facilities is more heavily utilized under FCFS since high-quality facilities cannot hold beds vacant in hope of more profitable future residents. On average, consumers value enforcing FCFS equivalently to an additional 0.20 hours (1.1 standard deviations) of registered nurse care per day.[67] The cost to hire this additional care at the average California RN wage and benefits ($38.81) would be $7.68 per resident-day.

[67] Medicaid and non-Medicaid residents respectively value enforcing FCFS at 0.26 and 0.05 hours of RN care per day. Since the demand model allows different preferences for RN staffing for Medicaid and non-Medicaid residents, I apply the corresponding sensitivity in translating the welfare impacts to RN hours for each group.

Finally, it is worth reiterating that facilities do not adjust their quality under this counterfactual. Insofar as FCFS reduces facilities' ability to allocate their limited capacity to the most profitable part of their residual demand curve, it could reduce facilities' incentive to invest in quality as a means to increase demand. Figure E.2 suggests this is the case: average admitted patient profitability is less strongly increasing in facility quality and staffing levels under FCFS than selective admissions. Therefore, if facilities are able to adjust their characteristics, high-quality facilities are likely to reduce their expenditure on staffing and quality under FCFS.

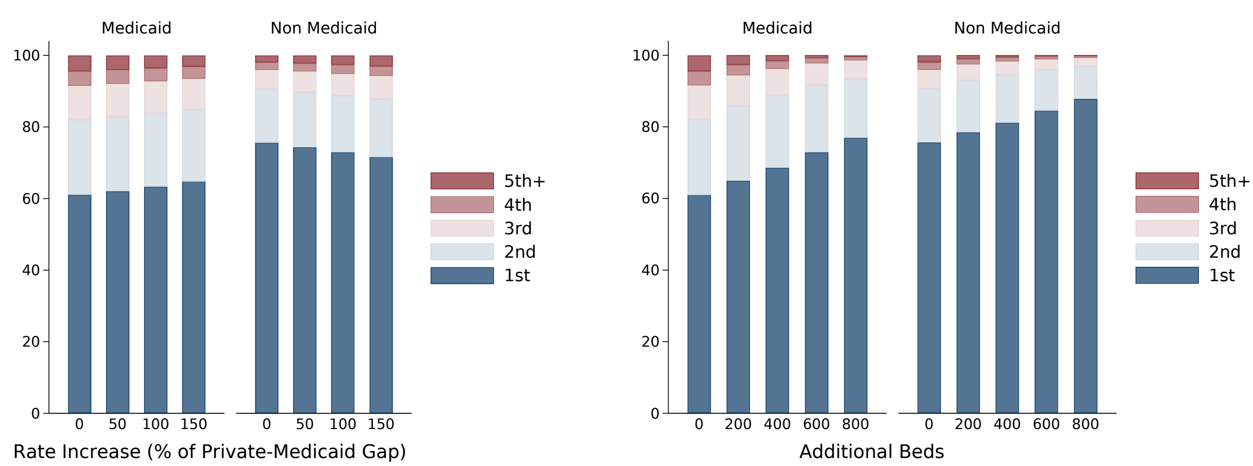
8.2 Simulating Counterfactual Beliefs

Simulating the remaining counterfactuals requires computing facilities' equilibrium optimal admission policies under the counterfactual. To accomplish this, I simulate 100 years of resident arrivals to San Diego and compute counterfactual policies and beliefs jointly as a fixed point in the simulation. I initialize facility policies to be FCFS and compute facilities' beliefs about the distribution of residents willing to accept an offer of admission based on these policies.[68] I then iteratively update facilities' optimal admission policies based on beliefs and beliefs based on policies until I reach a fixed point.[69] The simulated admissions given these fixed point beliefs and policies represent the admissions (i.e., matches between residents and facilities) that would occur under the counterfactual.

Figure 9: Counterfactual: Expanding Capacity

(a) Raising the Medicaid Reimbursement Rate

(b) Expanding Capacity



8.3 Counterfactual 2: Raising the Medicaid Reimbursement Rate

The average difference between private-pay and Medicaid rates is \$37.26. In order to model the impact of increasing the Medicaid reimbursement rate, I simulate counterfactuals that increase Medicaid reimbursements by half the \$37.26 gap, the whole gap, and 150% of the gap. In implementing these counterfactuals, I represent the rate increases

[68]I initialize each facility in the simulation at its empirical mean census and burn the first three years of simulated arrivals in computing its beliefs.

[69]I consider the algorithm to have converged when facilities' updated policies disagree with the previous policies on fewer than 5 in 10,000 "applicants" (i.e., individuals willing to accept an offer of admission). Note that there is no theoretical guarantee that this equilibrium is unique.

by increasing the daily average admission-discounted profitability ($\tilde{\pi}_{ij}$) of residents in proportion to the admission-discounted fraction of their stay reimbursed by Medicaid. In other words, I represent the increase in the Medicaid reimbursement rate as an increase in the profitability of Medicaid residents in proportion to the fraction of their stay reimbursed by Medicaid. While Section 6 parameterizes $\tilde{\pi}_{ij}$ in terms of a desirability score z_i that incorporates information about a resident's anticipated payer sources, it does not identify the structural relationship between daily reimbursement rates and $\tilde{\pi}_{ij}$. Therefore, additional assumptions are required to model raising the Medicaid reimbursement rate by raising $\tilde{\pi}_{ij}$ for residents on Medicaid. Appendix L estimates the structural relationship between reimbursement rates and $\tilde{\pi}_{ij}$ by attributing to reimbursement rates the variation in $\tilde{\pi}_{ij}$ that is correlated with a resident's anticipated payer source after controlling for the resident's health characteristics and stayer-type.[70]

Figure 9a summarizes the extent to which access to residents' most desired facilities is affected by raising the Medicaid reimbursement rate. The rate increases only slightly improve access for Medicaid patients, and those gains occur largely at the expense of crowding out non-Medicaid patients. Counterfactual utilities and admitting facility staffing levels show similar patterns (Figure E.3). The simulations suggest that raising the Medicaid reimbursement rate is not very cost effective: while closing the gap with private rates costs \$37.26 per Medicaid day, Medicaid residents value the increased access equivalently to just .02 hours (\$0.72) of RN care per day.

Multiple factors contribute to the inefficiency of raising the Medicaid reimbursement rate. First, even though Medicaid-eligible residents are some of the worst-affected by selective admissions, the majority are still admitted to their preferred facility under selective admissions (Figure 7). Second, capacity constraints severely limit the degree to which access can be increased. Even when Medicaid residents are treated equitably under FCFS, capacity constraints prevent 36% of residents utilizing Medicaid from being admitted at their preferred facility. Third, my estimates suggest that facilities find residents on Medicare to be the most desirable. Raising the Medicaid rate to match the private rate may not increase Medicaid residents' desirability sufficiently for high-quality facilities to shift their allocation of beds to Medicaid away from Medicare.

It is important to reiterate that these counterfactual simulations hold facility characteristics fixed. It is also possible that facilities would respond to an increase in the Medicaid reimbursement rate by increasing quality in order to attract Medicaid residents (Hackmann, 2019). As such, these results are best interpreted as showing that raising the Medicaid reimbursement rates is a very expensive way to improve Medicaid residents' access to existing high quality facilities. However, it may still be worthwhile if it induces facilities to increase their quality.

8.4 Counterfactual 3: Increasing Capacity

The final counterfactuals I simulate are to expand capacities at nursing homes with excess demand from Medicaid-eligible patients that would significantly benefit from access to the facility.[71] Figure 9b summarizes the extent to which adding 200, 400, 600, and 800 beds to the San Diego area's existing stock of 7,283 beds increases access to residents'

[70] The relationship between health characteristics and profitability may be different when residents are on Medicare than when they are on private pay or Medicaid. This is both because Medicare reimbursement rates are intensity adjusted and because the rehab and therapy care typically received on Medicare is different than the long term care typically received by Medicaid and private pay patients. In order to address this, I allow the impact of health characteristics on $\tilde{\pi}_{ij}$ to differ depending on the share of a resident's stay that is is reimbursed by Medicare.

[71] I enumerate the algorithm I use for allocating beds in Appendix M.

most desired facilities. The estimates suggest that expanding capacity substantially improves access for Medicaid-eligible patients. Moreover, in contrast with raising the Medicaid reimbursement rate, increasing capacity increases access for non-Medicaid residents as well. Counterfactual utilities and admitting facility staffing levels show similar patterns (Figure E.4).

The welfare gain from expanding capacity by 800 beds is valued equivalently to .13 RN hours per day by Medicaid-eligible residents and .06 RN hours per day by non-Medicaid residents.[72] Providing this equivalent RN care at the average hourly wage plus benefits in San Diego ($36.20) would cost $27,384 per day: $4.78 and $2.16 per resident-day for Medicaid and non-Medicaid residents, respectively.[73]

Cost reports suggest an approximate cost of expanding a facility by one bed of $14.43/day.[74] By this metric, adding 800 beds to San Diego facilities would cost $11,544 per day, which is approximately $2.60 per day of Medicaid-covered SNF care. This is both less costly than raising the Medicaid reimbursement rate and results in larger welfare gains for both Medicaid and non-Medicaid residents. Moreover, the cost of expanding capacity is lower than the cost of hiring the utility-equivalent hours of additional RN care.

9 Conclusion

Whether healthcare providers pick their patients has important implications for academics and policymakers concerned with healthcare inequality. In spite of existing anti-discrimination laws, this paper finds evidence of selective admission practices in the nursing home industry along a number of dimensions, including Medicaid eligibility, anticipated length of stay, and care requirements. I estimate that 19% of residents in California are unable to gain admission at their first choice facility. These residents are disproportionately very-long-stay residents on Medicaid, and their care constitutes 40% of all resident days in the state. Counterfactual simulations indicate that prohibiting selective admission practices benefits these disadvantaged residents by increasing their access to quality facilities, though the gains are partially offset by the crowding out of other residents. Simulations also suggest that raising the Medicaid reimbursement rate is a costly way to increase access for Medicaid-eligible residents at existing high-quality facilities. Moreover, the small benefits are largely at the expense of non-Medicaid residents. Targeted capacity increases, on the other hand, are more cost-effective and benefit both Medicaid and non-Medicaid residents.

There are a number of potential avenues for future related work. Most notably, my counterfactual simulations hold fixed facility characteristics such as staffing ratios. Endogenizing facilities' choice of characteristics (Hackmann, 2019)

[72] While the capacity expansions are targeted at facilities with excess demand from Medicaid-eligible residents, many of the added beds are still allocated by facilities to non-Medicaid residents. Furthermore, vacancy chains allow more than just the individual occupying the new vacancy to benefit. If the additional beds were reserved for Medicaid eligible residents, the impacts are expected to favor Medicaid eligible residents more heavily.

[73] It's worth noting that the welfare gains of increasing capacity do not derive primarily from additional RN care (Figure E.4b). In fact, hours of RN care received by residents on Medicaid increases by just 4.9% when adding 800 beds. Instead, the simulations suggest that gains derive largely from residents being admitted at facilities with higher unobserved quality and for which the resident has greater idiosyncratic preference. Since higher RN staffing has been shown to improve health outcomes (Schnelle et al., 2004; Friedrich and Hackmann, 2021), policymakers may be particularly interested in adjusting capacity to increase the amount of RN care received by residents. Figure E.5 shows the impact of targeting capacity expansions based on RN staffing. In spite of this targeting, the improvements are still small, which suggests that residents' strong preferences for characteristics other than quality may pose a significant obstacle to using increased capacity to improve the amount of RN care received by residents.

[74] I compute this approximation using facilities' per-bed lease expenditures and gross property, plant, and equipment (PP&E) as a metric for the cost of adding an additional bed. I use an interest rate of 5% to amortize the PP&E over a 30 year depreciation period.

in context of their dynamic optimal control problems could provide further insights into the impacts of the policies I study. Likewise, given the significant welfare gains of additional capacity at high-quality facilities, further research on how reimbursements and other policy levers affect capacity investment, entry, and exit could prove important.

While this paper directly examines nursing homes, concerns about providers picking their patients extend to many parts of the healthcare industry, such as primary care, specialist care, and emergency care. The methods I use to model and identify selective admissions may be extended to these contexts and even to non-healthcare markets with discrimination or unobserved endogenous choice sets.

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A Model Appendix

A.1 The Optimal Control Problem

At each time t, the facility chooses an adapted admission plan $\{a_\tau\}_{\tau \geq t}$ to maximize:

$$\mathbb{E}\left[\sum_{\{i:\tau_i^A \geq t, c_i \in a_{\tau_i^A}, \|N_{j\tau_i^A}\|_1 < b_j, u_{ij} \geq u_{ij'}\ \forall j' \in \mathcal{O}_i\}} \exp\left(\rho(\tau_i^A - t)\right)\Pi_{ij} + \int_t^\infty \exp(-\rho\tau)\Psi_j(N_{j\tau})d\tau \mid \mathcal{F}_t^j\right], \tag{32}$$

where $\mathcal{F}_t^j$ is a filtration representing j's information, b_j is the total number of beds at the facility, and the expectation is taken with respect to j's beliefs.[75]

Assumptions 1 and 2 imply that facilities condition their admission policies only on N_{jt}. Assumption 1 implies that facilities' perceive the arrival of future prospective residents to be independent of the facility's history, and Assumption 2 and memoryless discharge processes imply that all current residents of a given stayer-type ϕ are indistinguishable in

[75]Note that the measure with respect to which the expectation is taken must satisfy Assumption 1.

their effect on j's future payoffs. Therefore, the facility perceives no benefit to conditioning its strategy on anything other than N_{jt}.

The same arguments imply that the facility only considers ϕ and Π_{ij} when admitting a resident. Furthermore, because the facility always prefers higher Π_{ij} given ϕ_i, admission policies are thresholds $\underline{\Pi}_\tau \in \mathbb{R}^{|\Phi|}$, where i is offered admission only if $\Pi_{ij} \geq \underline{\Pi}_{\tau\phi_i}$.

Together, these imply that the facility's admission policy is a vector of threshold rules that vary by census counts N_{jt}. I denote this policy by functions $\{\underline{\Pi}_{j\phi}\}_\phi$, where $\underline{\Pi}_{j\phi} : \{N : N \in \mathbb{Z}^{|\Phi|}_{\geq 0}, \|N\|_1 < b_j\} \to \mathbb{R}$. Thus, the facility chooses $\{\underline{\Pi}_{j\phi}\}_\phi$ to maximize:

$$\mathbb{E}\left[\sum_{\{i:\tau_i^A \geq t, \Pi_{ij} \geq \underline{\Pi}_{j\phi_i}(N_{j\tau_i^A}), \|N_{j\tau_i^A}\|_1 < b_j, u_{ij} \geq u_{ij'}\ \forall j' \in \mathcal{O}_i\}} \exp\left(\rho(\tau_i^A - t)\right)\Pi_{ij} + \int_t^\infty \exp(-\rho\tau)\Psi_j(N_{j\tau})d\tau \mid \mathcal{F}_t^j\right]. \tag{33}$$

A.2 Proof of Theorem 1

Towards induction, assume that $V(N-1) - V(N) \leq V(N) - V(N+1)$. Compare (3) evaluated at N and $N+1$. Note that $\rho V(N) \geq \rho V(N+1)$, and by the inductive hypothesis:[76]

$$N\lambda^D \left(V(N-1) - V(N)\right) \leq (N+1)\lambda^D \left(V(N) - V(N+1)\right). \tag{34}$$

Therefore, it must be that:

$$\lambda^A \int_{V(N)-V(N+1)}^\infty \left(\Pi + V(N+1) - V(N)\right) dF(\Pi) \geq \lambda^A \int_{V(N+1)-V(N+2)}^\infty \left(\Pi + V(N+2) - V(N+1)\right) dF(\Pi), \tag{35}$$

which holds only if $V(N+1) - V(N+2) \geq V(N) - V(N+1)$, proving the inductive step. The base case holds by the same argument where $V(0) - V(1) > 0$ is used in lieu of the inductive hypothesis.

A.3 Proof of Theorem 2

Without loss of generality, I exclude the facility index j. I derive the continuous time Bellman equation and optimal policies as the limit of the analogous discrete time problem with vanishingly small discrete time intervals Δt. For

[76] That V is decreasing follows from the fact that any admission plan by a facility in $N+1$ can be mimicked by a facility in N to yield at least the same value.

small Δt, $\exp(-\rho\Delta t) \approx 1 - \rho\Delta t$. Then, since the problem is stationary:[77]

$$\begin{aligned} V(N_t) &= \max_{a_t} \left\{ \Delta t \Psi(N_t) + (1-\rho\Delta t)\mathbb{E}\left[\sum_{\{i: i \in A_t, c_i \in a_t\}} \Pi_{ij} + V(N_{t+\Delta t}) | \mathcal{F}_t^j \right] \right\} \\ \implies \rho V(N_t)\Delta t &= \max_{a_t} \left\{ \Delta t \Psi(N_t) + (1-\rho\Delta t)\mathbb{E}\left[\sum_{\{i: i \in A_t, c_i \in a_t\}} \Pi_{ij} + V(N_{t+\Delta t}) - V(N_t) | \mathcal{F}_t^j \right] \right\}, \end{aligned} \tag{36}$$

where A_t is the set of residents that arrive to the market at t and are willing and able to accept an offer of admission from the facility. Applying the Poisson arrival processes and the exponential discharge rates:[78]

$$\begin{aligned} \mathbb{E}\left[V(N_{t+\Delta t}) - V(N_t) | \mathcal{F}_t^j \right] = &\sum_\phi \left(\lambda_\phi^A \Delta t + o(\Delta t) \right) \int \mathbf{1}\{\Pi_{ij} \in a_t\} \mu(c_i) \left(\Pi_{ij} + V(N_t + 1^\phi) - V(N_t) \right) dF(c_i | \phi_i = \phi) \\ &+ \sum_\phi N_{t\phi} \left(\lambda_\phi^D \Delta t + o(\Delta t) \right) \left(V(N_t - 1^\phi) - V_j(N_t) \right) + o(\Delta t), \end{aligned} \tag{37}$$

where $o(\Delta t)$ are terms such that $\lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t} = 0$. Substituting (37) into (36), dividing both sides by Δt, and taking $\Delta t \to 0$ yields the result:

$$\begin{aligned} \rho V(N_t) = &\Psi(N_t) + \sum_\phi \lambda_\phi^D N_{t\phi} \left(V(N_t - 1^\phi) - V(N_t) \right) \\ &+ \sum_\phi \lambda_\phi^A \max_{a_t} \int \mathbf{1}\{\Pi_{ij} \in a_t\} \left(\Pi_{ij} + V(N_t + 1^\phi) - V(N_t) \right) \mu(c_i) dF_c\left(c_i | j \in J_i, \phi_i = \phi \right) \\ &= \Psi(N_{jt}) + \sum_\phi \lambda_\phi^D N_{t\phi} \left(V(N_t - 1^\phi) - V(N_t) \right) \\ &+ \sum_\phi \lambda_\phi^A \int \max\left\{ 0, \Pi_{ij} + V(N_t + 1^\phi) - V(N_t) \right\} \mu(c_i) dF_c\left(c_i | j \in J_i, \phi_i = \phi \right), \\ &= \Psi(N_{jt}) + \sum_\phi \lambda_\phi^D N_{t\phi} \left(V(N_t - 1^\phi) - V(N_t) \right) \\ &+ \sum_\phi \lambda_\phi^A \int \max\left\{ 0, \Pi + V(N_t + 1^\phi) - V(N_t) \right\} dF_{j\phi}^\Pi(\Pi), \end{aligned} \tag{38}$$

$$F_{j\phi}^\Pi(\Pi) := \int_{\{c_i : \Pi_{ij} \le \Pi\}} \mu_j(c_i) dF_c\left(c_i | j \in J_i, \phi_i = \phi \right). \tag{39}$$

It is clear from the maximand in (38) that $a_t = \{c_i : \Pi_{ij} \geq V(N_t) - V(N_t + 1^\phi)\}$ up to a measure-zero set.

[77] When $N_t = b$, the capacity constraint can be enforced by imposing that either of A_t or a_t is empty.

[78] When $N_{t\phi} = 0$, define $N_{t\phi}\left(\lambda_\phi^D \Delta t + o(\Delta t) \right) \left(V(N_t - 1^\phi) - V_j(N_t) \right)$ to be 0 since $V(N_t - 1^\phi)$ is not defined.

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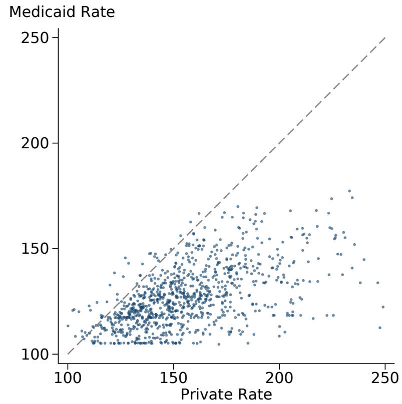
Online Appendix

These appendices contain material for the Online Appendix.

B Additional Tables and Figures on the Industry and Data

Figure B.1: Private and Medicaid Rates



Notes. Private rates are calculated as the total revenue from private sources divided by the total number of private-pay days. 4.53% of private prices are above $250.

Figure B.2: Distance to Admitting Facility

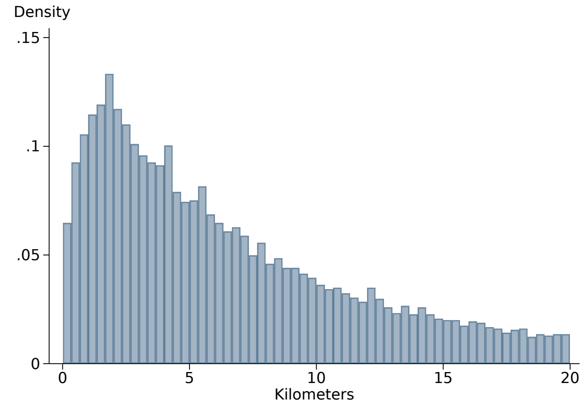


Table B.1: Data Sources

Data Source	**Years**	**Relevant Data**
MDS	1999-2010	Demographics, health status, entry and discharge dates
MedPAR	2004-2010	Medicare claims, hospital source, entry and discharge dates
MAX LT	2004-2007	Medicaid claims, entry and discharge dates
MMLEADS	2008-2009	Medicaid eligibility
BSF	2004-2010	Dual eligibility, zip code
OSCAR	2004-2007	Facility characteristics
LTCFocus	2004-2007	Facility characteristics
Medi-Cal Data	2004-2007	Facility cost data, private rates

Table B.2: Resident Summary Statistics

	Mean	SD	N
# Local Facilities	46.56	44.25	555,166
Length of Stay	143.48	332.76	555,166
Medicare Days	18.78	35.69	555,166
Private Days	23.58	123.21	555,166
Medicaid Days	93.99	297.83	555,166
Dual-Eligible (Start)	0.31	0.46	555,166
ADL Score	2.29	0.76	454,354
Dementia	0.25	0.43	391,608
Weight	149.75	38.97	555,166

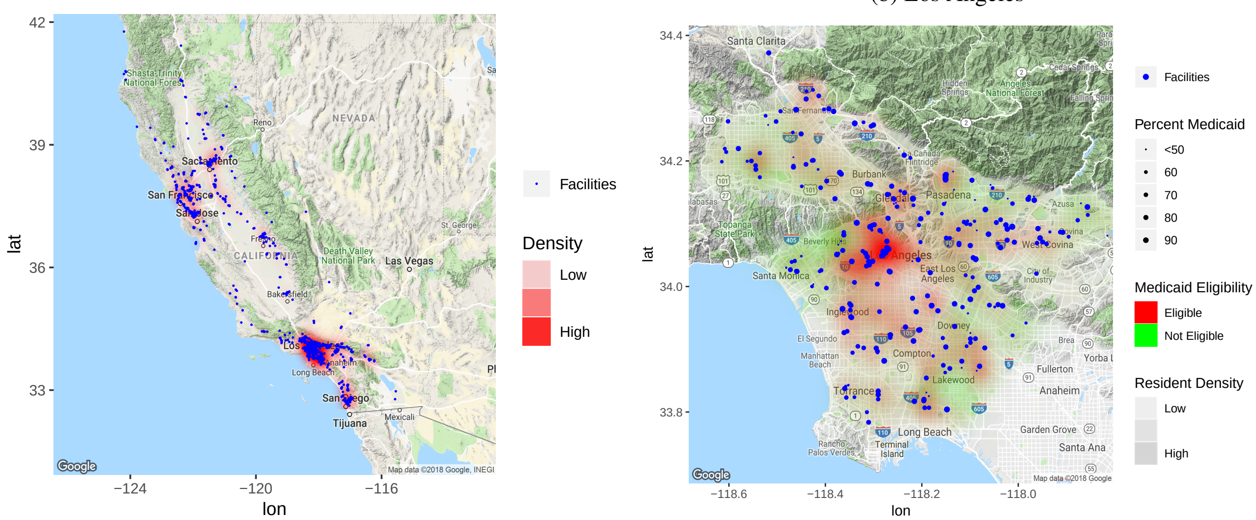
Table B.3: Facility Summary Statistics

	Mean	SD	N
Beds	100.58	47.95	971
% Occupancy	87.31	9.25	971
% Medicaid	63.41	23.99	971
% Medicare	12.26	10.71	971
RNs Per Bed-Day	0.29	0.19	971
LPNs Per Bed-Day	0.72	0.23	971
CNAs Per Bed-Day	2.46	0.50	971
Case Mix Index	1.04	0.10	970

Figure B.3: Maps of Facilities and Residents

(a) California

(b) Los Angeles



C Online Appendix on Preliminary Evidence

C.1 Testing Whether Arriving Patient Composition Covaries with Occupancy

In this section, I examine whether the residents seeking admission at each facility evolve over time in a way that is systematically correlated with the facility's occupancy.

One such possibility is that the residents arriving to the market in need of skilled nursing care covaries with facility occupancy. For example, it might be that there is both significant seasonality in occupancy and the characteristics of patients requiring care. Figure C.1 suggests that seasonality in occupancy is not dramatic: the average occupancy percentile does not deviate substantially from 50 in systematic way. Additionally, Table C.3 gives analogous regressions to Figure 4d and Table C.2 but also includes resident-facility pairs where the resident arrived locally to the facility but was admitted to a different facility. While these estimates do indicate that a facility's occupancy is slightly correlated with characteristics of arriving residents, the estimated coefficients are very small, especially when compared to the coefficient estimates in Table C.2. This suggests that the covariance between facility occupancy and admitted patient

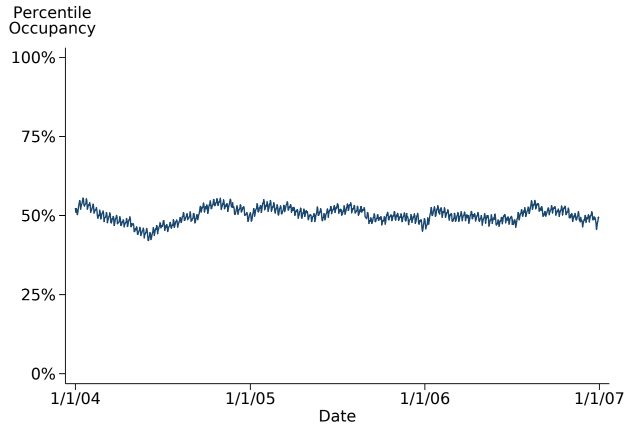


Figure C.1: Seasonality in Occupancy

characteristics that we identify in Figure 4d and Table C.2 is not driven by variation in the distribution of arriving patient characteristics.

Another possibility is that if the occupancy of facilities and their competitors are highly correlated, then the composition of residents a facility admits could covary with occupancy due to changes in residual demand due to competitors being full. Table C.1 gives estimates from regressing a facility's occupancy percentile on the occupancy percentiles of its local competitors. I estimate only a small positive relationship that is strongest in highly concentrated markets. For example, the occupancy percentile of a facility with the average number of competitors within 5km is only expected to increase by 1.72 between its geographically closest competitor's least full day and most full day in the sample. Similarly, a one standard deviation increase in the average occupancy percentile of a facility's closest 5 competitors corresponds to a .97 percentile increase in the facility's occupancy percentile. Insofar as variation in competitors' occupancies reflects variation in competitors' admission policies, the small magnitudes of the coefficients in Table C.1 suggest that the covariance between facility occupancy and admitted patient characteristics is unlikely to driven by simultaneous variation in competitors' policies.

Table C.1: Relationship Between Local Facility Occupancies

		(1)	(2)	(3)	(4)	(5)	(6)
$occ^{\%le}_{j't}$ of closest j'		0.0286	0.0187	0.0223			
		(0.00149)	(0.00113)	(0.00149)			
	X # fac. in 5km	-0.000777	-0.000241	-0.000816			
		(0.000179)	(0.0000891)	(0.000179)			
Mean $occ^{\%le}_{j't}$ of closest 5 j'					0.152	0.0767	0.128
					(0.00316)	(0.00219)	(0.00317)
	X # fac. in 5km				-0.00829	-0.000562	-0.00896
					(0.000392)	(0.0000995)	(0.000392)
	Facility FEs	Y	N	Y	Y	N	Y
	Date FEs	N	Y	Y	N	Y	Y
	N	1,045,151	1,045,151	1,045,151	1,054,363	1,054,363	1,054,363
	R-squared	0.00153	0.00689	0.00785	0.00267	0.00775	0.00821

Notes. Standard errors in parenthesis. The standard deviation of the mean occupancy percentile of a facility's closest 5 competitors is .134.

Table C.2: Relationship Between Occupancy and Admitted Patient Characteristics

	Predicted Length of Stay	Medicaid		Health Status						Behavioral			Race	
		Medicaid Eligible	Medicaid Days	Dementia	Antipsychotics Days/Week	Antianxiety Days/Week	Antidepressants Days/Week	Visual Impairment	Physical Mobility	Inappropriate Behavior	Conflict with Staff	Conflict with Residents	Black	Hispanic
Occupancy Percentile	-31.12	-0.0440	-38.39	-0.00872	-0.107	-0.0497	-0.139	-0.0284	-0.374	-0.00690	-0.00150	-0.000562	-0.00295	-0.00565
	(1.853)	(0.0036)	(2.309)	(0.00314)	(0.016)	(0.0111)	(0.0175)	(0.00763)	(0.106)	(0.00297)	(0.000705)	(0.00029)	(0.00141)	(0.00165)
FE: Facility	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Facility Clusters	971	971	971	971	971	971	971	971	971	971	971	971	971	971
N Observations	555,166	555,166	555,166	391,608	454,307	454,298	454,311	446,204	555,166	452,640	390,545	390,545	551,054	551,054
R^2	0.121	0.247	0.087	0.052	0.128	0.023	0.020	0.089	0.122	0.063	0.057	0.062	0.195	0.128
Mean	90.665	0.312	93.995	0.246	1.004	0.654	1.797	0.489	17.678	0.076	0.011	0.002	0.074	0.109
Std. Dev.	199.617	0.463	297.830	0.431	2.356	1.808	2.916	0.943	12.497	0.367	0.103	0.048	0.263	0.312

Notes. Standard errors in parenthesis. Occupancy percentiles are scaled between 0 and 1.

Table C.3: Relationship Between Occupancy and Arriving Patient Characteristics

	Predicted Length of Stay	Medicaid		Health Status						Behavioral			Race	
		Medicaid Eligible	Medicaid Days	Dementia	Antipsychotics Days/Week	Antianxiety Days/Week	Antidepressants Days/Week	Visual Impairment	Physical Mobility	Inappropriate Behavior	Conflict with Staff	Conflict with Residents	Black	Hispanic
Occupancy Percentile	-2.235	-0.00363	-3.037	-0.00145	-0.0105	-0.00411	-0.00499	-0.00173	-0.0351	-0.000784	-0.0000369	-0.0000705	-0.000446	-0.000108
	(0.191)	(0.000427)	(0.278)	(0.000475)	(0.00255)	(0.00178)	(0.00288)	(0.00102)	(0.0116)	(0.000403)	(0.0000919)	(0.0000513)	(0.000304)	(0.000333)
FE: Facility	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Individual Clusters	555,166	555,166	555,166	391,608	454,307	454,298	454,311	446,204	555,166	452,640	390,545	390,545	551,054	551,054
N Observations	25,846,693	25,846,693	25,846,693	18,131,496	20,898,928	20,898,746	20,898,927	20,415,421	25,846,693	20,807,841	18,065,021	18,065,021	25,627,127	25,627,127
R^2	0.011	0.053	0.009	0.003	0.009	0.003	0.003	0.008	0.014	0.002	0.003	0.000	0.058	0.031
Mean	99.831	0.377	101.419	0.257	1.190	0.612	1.694	0.575	17.735	0.090	0.007	0.002	0.125	0.152
Std. Dev.	209.964	0.485	303.379	0.437	2.529	1.773	2.869	0.996	12.745	0.397	0.086	0.048	0.331	0.359

Notes. Standard errors in parenthesis. Occupancy percentiles are scaled between 0 and 1.

C.2 Controlling for Other Resident Characteristics

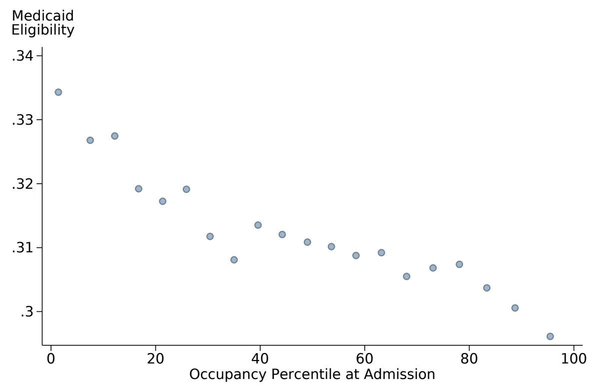
Figure C.2 presents the same relationships as in Figure 4 with additional controls. In particular I control for predicted length of stay in all regressions—except when predicted length of stay is the outcome variable—and for Medicaid eligibility and Medicaid days—except when one is the outcome variable. In general these estimates suggest similar relationships to Figure 4 but are sometimes smaller or have larger standard errors.

C.3 Projecting Desirability Scores Onto a Linear Model

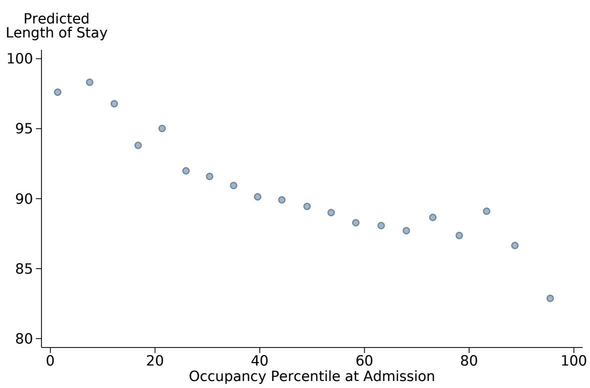
Table C.4 gives the estimates of a regression of resident characteristics on desirability scores. These coefficient estimates must be interpreted cautiously because they are not causal and because they are a linear approximation to a highly nonlinear function. Still, Table C.4 provides valuable insight into the correlations being exploited by the machine learning algorithm. The estimates agree with Figure 5 that Medicaid residents and residents with longer anticipated stays are less desirable, and they further suggest that Medicare coverage is more desirable than private pay. The positive coefficient on Case Mix Index (CMI) are explained by the fact that Medicare reimbursements are determined using CMI. I estimate a negative coefficient on DRG weights likely because the additional care requirements associated with higher DRG weights are not compensated by Medicare except through correlation with CMI. Additionally, consistent with Table C.2, I find small negative coefficients on health characteristics associated with uncompensated care requirements. Finally, I find that even controlling for the aforementioned, black and Hispanic residents have lower expected desirability scores.

Figure C.2: Relationship Between Admitted Patient Characteristics and Percentile Occupancy with Controls

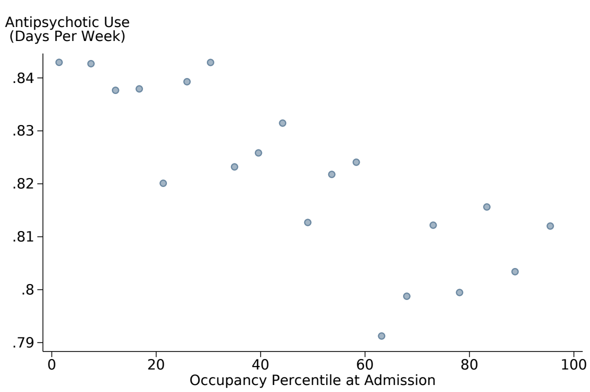
(a) Medicaid Eligibility



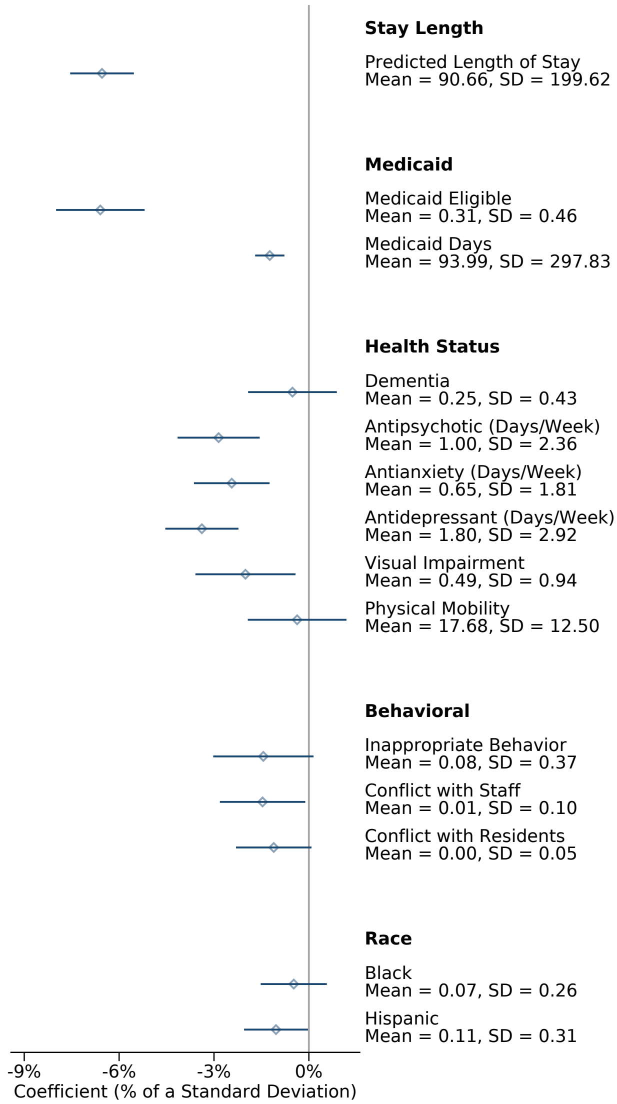
(b) Predicted Length of Stay



(c) Antipsychotic Use (Days/Week)



(d) Regression Coefficients



Notes. All regressions and binscatters include facility fixed effects. Regressions on health characteristics, behavioral issues, and race include controls for anticipated length of stay, Medicaid eligibility, and Medicaid days. Regressions on anticipated length of stay include controls for Medicaid eligibility and Medicaid days. Regressions on Medicaid eligibility and Medicaid days include controls for length of stay. Short, long, and very-long stayers respectively constitute the 1-70, 70-90, and 90-100 percentiles of predicted length of stay.

Table C.4: Regression on Desirability (z) Scores

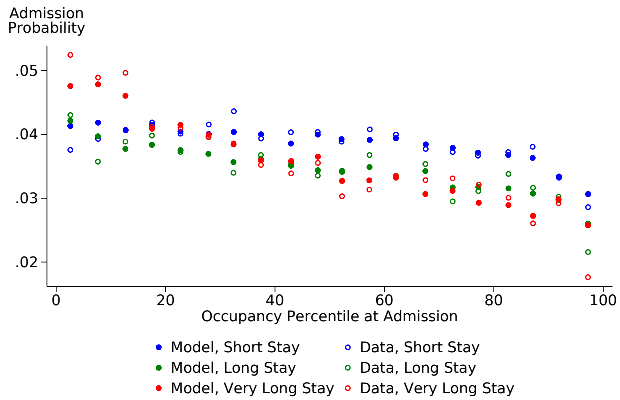
Share of Stay:	Medicare	43.71
		(0.0488)
	Private Pay	43.03
		(0.0488)
	Medicaid	41.36
		(0.0498)
Stayer Type:	Long Stayer	-0.455
		(0.00772)
	Short Stayer	-3.034
		(0.0147)
Race:	Native American	2.331
		(0.0573)
	Asian	2.415
		(0.0486)
	Black	1.892
		(0.0485)
	Hispanic	2.167
		(0.0482)
	White	2.426
		(0.0477)
Resource Utilization:	Case Mix Index	0.0124
		(0.000480)
	DRG Weight	-0.531
		(0.0135)
Health:	Visual Impairment (0-4)	-0.118
		(0.00313)
	Diabetes	-0.0560
		(0.00611)
	Chewing Problems	-0.0307
		(0.00692)
	Varied Mental Function	-0.0993
		(0.00765)
	Antipsychotics (days/week)	-0.0730
		(0.00139)
	Antienxiety (days/week)	-0.0853
		(0.00160)
	Antidepressants (days/week)	-0.0456
		(0.000985)
	Socially Inappropriate Behavior (0-3)	-0.164
		(0.00899)
	Conflict with Staff	-0.176
		(0.0310)
	Conflict with Residents	-0.508
		(0.0827)
	Body Mass Index	-0.00841
		(0.000492)
	Underweight	-0.0777
		(0.00935)
	N	555,170
	R^2	0.567

Notes. Standard errors in parenthesis. Regression includes addition controls for whether each characteristic is observed for each resident. Shares of stay are the expected present-discounted share of a resident's stay reimbursed by a given source. See Appendix L.

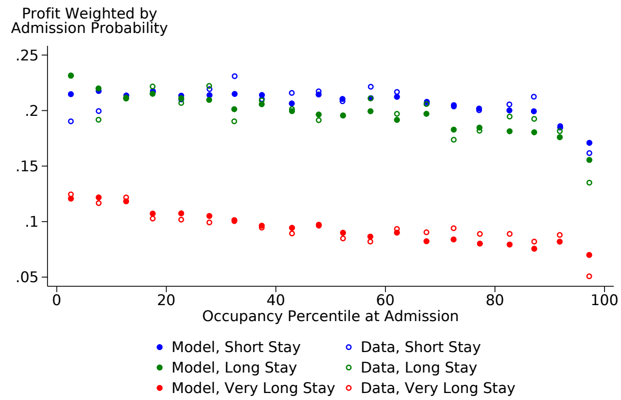
D Additional Tables and Figures From Estimation

Figure D.1: Second Stage Model and Data Comparison

(a) Admission Probability by Stayer-Type



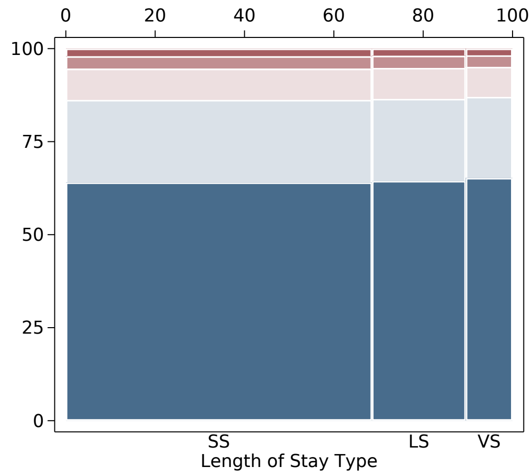
(b) Admission Profit by Stayer-Type



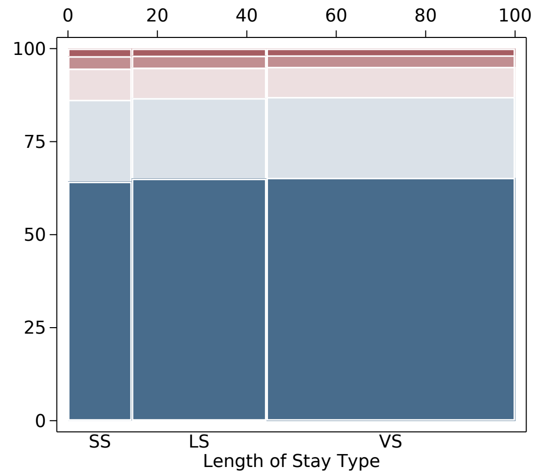
E Additional Tables and Figures From Counterfactuals

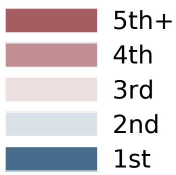
Figure E.1: Preference Rank of Admitting Facility under FCFS

(a) Percent of Residents, by LOS-Type

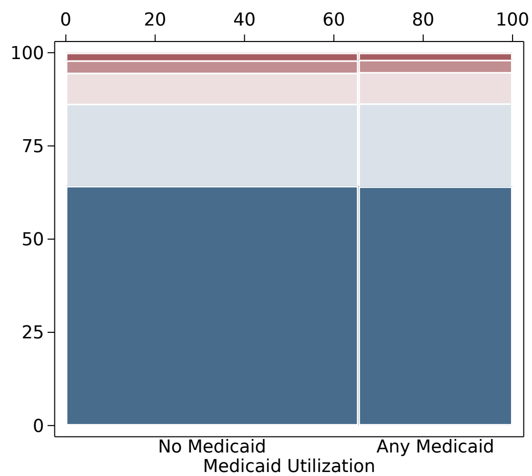


(b) Percent of Days, by LOS-Type





(c) Percent of Residents, by Medicaid Eligibility



(d) Percent of Days, by Medicaid Eligibility

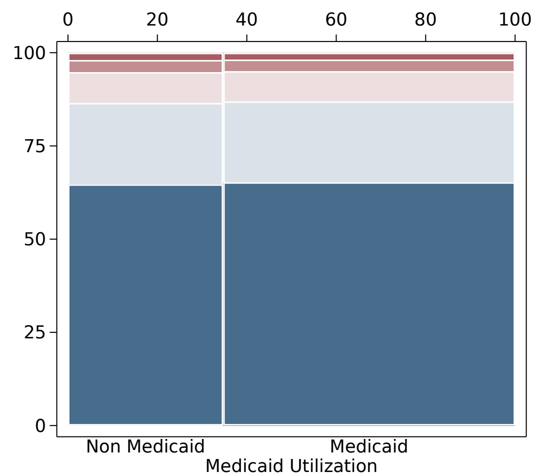
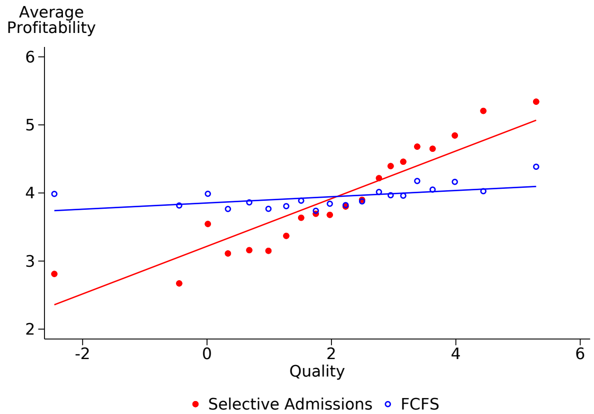


Figure E.2: Average Resident Profitability Under Selective Admissions and FCFS

(a) Quality



(b) Registered Nurse Hours

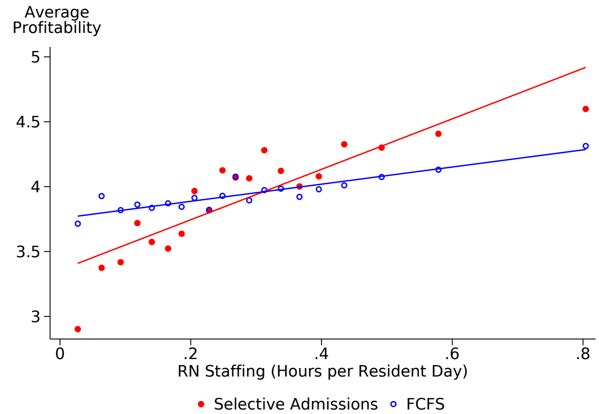
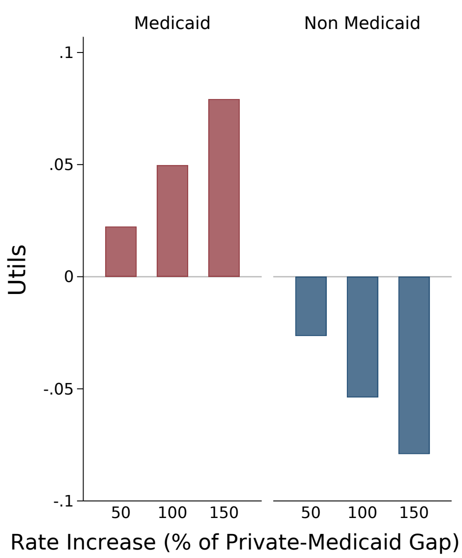


Figure E.3: Counterfactual: Raising Medicaid Reimbursements

(a) Mean Utility Relative to First Choice



(b) Registered Nurse Staffing

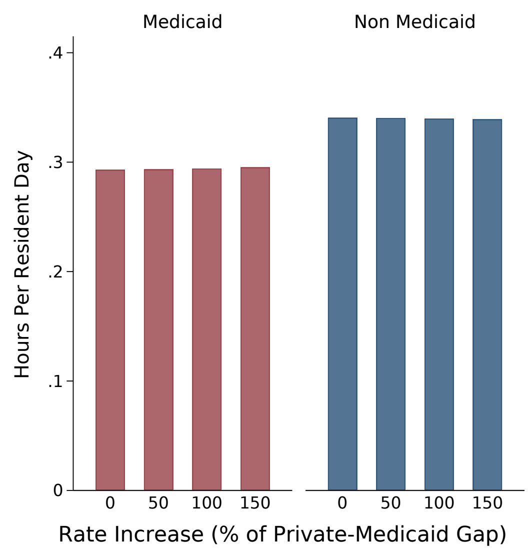
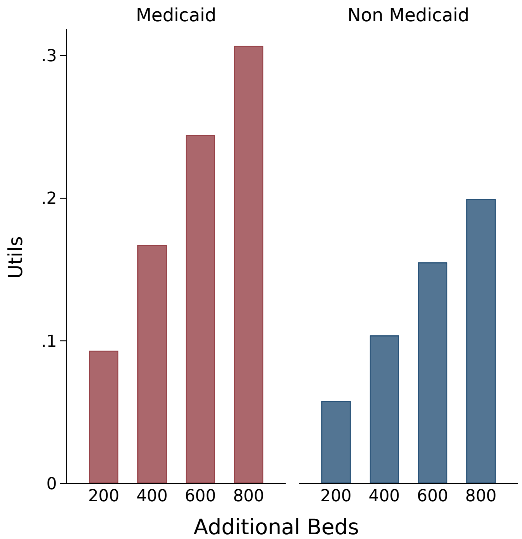


Figure E.4: Counterfactual: Expanding Capacity

(a) Mean Utility Relative to First Choice



(b) Mean Registered Nurse Staffing

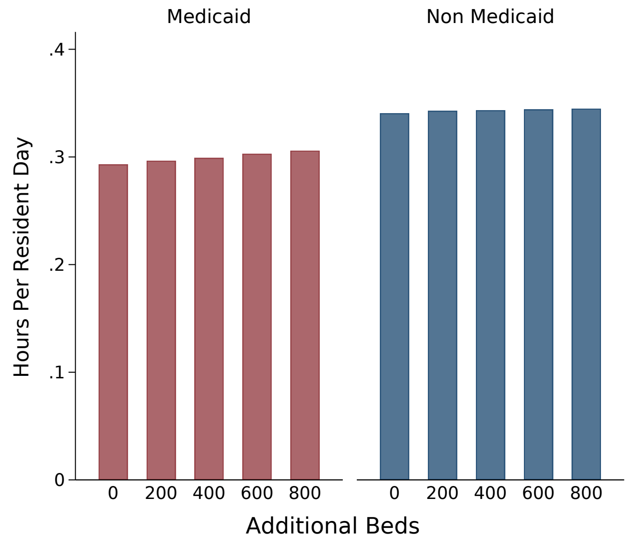
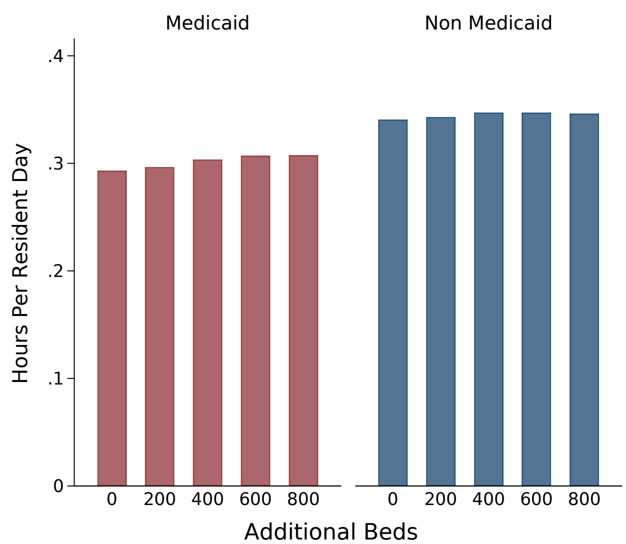


Figure E.5: Targeting Capacity Expansion to Raise Medicaid Resident Nurse Staffing



F Additional Theoretical Results

F.1 Relationship Between Continuous and Upfront Payoffs

Theorem 3. *The admission policies given continuous flow payoffs* $\pi_{ij}(t-\tau_i^A)$ *and upfront payoffs* Π_{ij} *satisfying* (8) *are identical.*

Proof of Theorem 3. Without loss of generality, we drop the facility index j. First, consider the case with flow payoffs. I index residents by their stayer-type ϕ, flow profit function function $\pi_i(\cdot)$ and arrival time τ_i^A. Therefore, it is helpful to articulate the facility state I_t as $(I_t^{\phi,\pi,\tau})_{\phi,\pi,\tau}$, where $I_t^{\phi,\pi,\tau}$ an indicator for if a resident of stayer-type ϕ with profitability function π was admitted at τ and is still resident at time t.[1] At each time t, the facility chooses an adapted admission plan $\{a_\tau\}_{\tau\geq t}$. The facility believes that the state I_t evolves according to the Poisson arrival and exponential discharge processes described in the main text, where residents of type (ϕ,π,τ) are made offers of admission if and only if the facility has an available bed and $(\phi,\pi,\tau)\in a_\tau^{\phi,\pi}$.[2] The sequence control problem for the facility is:[3]

$$
\begin{aligned}
V(I_t) &= \sup_a \mathbb{E}\left[\int_t^\infty e^{-\rho(\tau-t)}\left(\sum_{\left\{(\phi,\pi,\tau'):I_\tau^{\phi,\pi,\tau'}>0\right\}}\pi(\tau-\tau')+\Psi(N_\tau)\right)d\tau|\mathcal{F}_t^j\right]\\
&= \sum_{\left\{(\phi,\pi,\tau'):I_t^{\phi,\pi,\tau'}>0,\tau'<t\right\}}\mathbb{E}\left[\int_t^\infty e^{-\rho(\tau-t)}I_\tau^{\phi,\pi,\tau'}\pi(\tau-\tau')d\tau\right]\\
&\quad+\sup_a\left\{\mathbb{E}\left[\sum_{\left\{(\phi,\pi,\tau'):\tau'\geq t,I_{\tau'}^{\phi,\pi,\tau'}>0\right\}}\int_{\tau'}^\infty e^{-\rho(\tau-t)}I_\tau^{\phi,\pi,\tau'}\pi(\tau-\tau')d\tau|\mathcal{F}_t^j\right]+\mathbb{E}\left[\int_t^\infty\Psi(N_\tau)d\tau|\mathcal{F}_t^j\right]\right\} \qquad (40)\\
&= \sum_{\left\{(\phi,\pi,\tau'):I_t^{\phi,\pi,\tau'}>0,\tau'<t\right\}}\Pi(\phi,\pi,\tau',t)\\
&\quad+\sup_a\left\{\mathbb{E}\left[\sum_{\left\{(\phi,\pi,\tau'):\tau'\geq t,I_{\tau'}^{\phi,\pi,\tau'}>0\right\}}e^{-\rho(\tau'-t)}\Pi(\phi,\pi)|\mathcal{F}_t^j\right]+\mathbb{E}\left[\int_t^\infty\Psi(N_\tau)d\tau|\mathcal{F}_t^j\right]\right\},
\end{aligned}
$$

$$\Pi(\phi,\pi,\tau',t):=\mathbb{E}\left[\int_t^\infty I_\tau^{\phi,\pi,\tau'}e^{-\rho(\tau-t)}\pi(\tau-\tau')d\tau|\mathcal{F}_t^j\right], \qquad (41)$$

$$\Pi(\phi,\pi):=\Pi(\phi,\pi,\tau',\tau')\text{ where }I_{\tau'}^{\phi,\pi,\tau'}>0. \qquad (42)$$

Since $\Pi(\phi_i,\pi_i)=\Pi_{ij}$ from (8), the maximand above is identical to that in (9), the sequence problem when the facility receives upfront Π_{ij}. Since the maximands are identical in both problems, the optimal admission policies in both problems are the same.

□

[1] I do not account for simultaneous arrivals because the probability of such an event ever occurring is measure zero.

[2] I require that j's beliefs satisfy Assumption 1.

[3] Note that $N_t=(N_{t\phi})_\phi$ is as in the main text, so $N_{t\phi}$ is the number of elements in $\{(\pi,\tau'):I_t^{\phi,\pi,\tau'}>0,\tau'\leq t\}$.

Intuitively, Theorem 3 derives from the fact that once a resident is admitted, their π_{ij} flow payoff is not affected by and does not affect the flow payoffs received from other current or future residents. Therefore, the continued π_{ij} flow payoffs from current residents do not factor into admissions decisions, and hence disregarding these continued payments in the translated game with lump-sum payoffs does not affect the optimal admission policies.[4] An important implication of Theorem 3 is that even if the facility receives flow payoffs $\pi_{ij}(\cdot)$ that vary throughout a resident's stay, one need only characterize Π_{ij} in (8) to characterize optimal admission policies.

F.2 Relaxing Model Assumptions

Corollary 3.1. *The size of the policy-relevant state space, i.e.* $N_j := \{N : \|N\|_1 \leq b_j\}$, *is* $\binom{b_j+|\Phi|}{|\Phi|}$.

It is worth considering the impact that relaxing either Assumption 1 or 2 even slightly would have on the size of facilities' state space. Under Assumptions 1 and 2, a facility with 150 beds (i.e. $b_j = 150$) has 585,276 policy-relevant states. Slightly relaxing Assumption 2 by allowing Ψ to also be a function of just a count of the number of "high needs" patients at the facility increases the number of payoff-relevant states to 18,161,699,556. Similarly, if the facility were to condition its beliefs about the probability that future residents will accept offers of admission on just the current $N_{j't}$ of its closest competitor j' (with $b_{j'} = 150$), then the size of the facility's state space would be 342,547,996,176.[5]

G Additional Details on Methods

H Online Appendix: Machine Learning

The primary machine learning tool I use in this paper is Microsoft's open source gradient boosted decision tree (GBDT) implementation, LightGBM (Ke et al., 2017). Gradient boosting is a procedure to train a predictive function that is an ensemble of weak learners (Friedman, 2001, 2002). As its name suggests, these weak learners are decision trees in the case of GBDT.

By composing many simple functions (decision trees), the predictive function can approximate highly non-linear patterns in the data. However, without restriction on either the trees' or ensemble's complexity, this flexibility will result in over-fitting. To prevent this, I regularize the number of leaves in each tree, the number of trees in the ensemble, and the relative weight that the predictive function places on successive trees (i.e., the "learning rate"). Tuning these parameters is analogous to tuning the penalty parameter in a LASSO or Ridge regression. Parameter values are selected to minimize the model's k-fold cross-validated out of sample loss. When the number of features are too large for parameter tuning to be computationally feasible with the full feature set, the features input to the tuning process are selected by excluding those with very low influence on a model trained with fixed parameter values.

[4]Note that while the optimal policies are shared between these games, it is not true that the value functions are shared. In particular, they differ by the expected remaining flow payoffs from the current census of residents.

[5]A possible alternative would be to allow facilities to condition on coarse information about competitors' censuses in an experience based equilibrium (Fershtman and Pakes, 2012).

I Preferred Parameterizations

I parameterize $\tilde{\pi}_{ij}$ to be only a function of univariate desirability score z_i, stayer-type ϕ_i, and profitability shock ϵ_i^π that is observed by facilities but unobserved to the econometrician:

$$\tilde{\pi}_{ij} = \tilde{\pi}(z_i, \phi_i; \gamma_\pi) - \epsilon_i^\pi, \tag{43}$$

where $\tilde{\pi}(z_i, \phi_i; \gamma_\pi)$ is a second degree polynomial in z_i and $z_i^{\%le}$ with intercepts in ϕ_i.

I parameterize cutoff rules as functions of facility characteristics and current census counts:

$$\underline{\tilde{\pi}}_{j\phi}(N_{jt}) = \underline{\tilde{\pi}}(N_{jt}, X_j, \phi; \gamma_{\underline{\pi}}), \tag{44}$$

Specifically, I parameterize $\underline{\tilde{\pi}}(N_{jt}, X_j, \phi; \gamma_{\underline{\pi}})$ to include intercepts in ϕ and a polynomial in:

- $\mathbb{E}\left[z_i | N_{jt}, a_{ij}\right]$: the expected z_i of residents admitted to facility j given census counts N_{jt}. I use gradient boosted decision trees (GBDT) described in Appendix H to fit $\mathbb{E}\left[z_i | N_{jt}, a_{ij}\right]$.[6] The use of GBDT allows me to fit $\mathbb{E}\left[z_i | N_{jt}, a_{ij}\right]$ with a highly flexible non-linear function that varies both across facilities as well as over N_{jt} within the same facility.

- $x_j^{\Delta z}$: the difference between the average z_i of j's residents and the average z_i of j's local community.[7] I define the average z_i of j's local community using a weighted average of the z_i of all patients in the data with prior residences near j, including patients admitted to a facility other than j. Because patients are more likely to be admitted at facilities near their prior residence, I use a weight that decreases in a patient's distance of prior residence from the facility. Specifically, I weight local residents' z_i by their predicted admission probability at the facility from a logit regression of admission on a second degree polynomial in the distance between prior residence and the facility. Intuitively this weights residents according to their likelihood of admission at the facility when only conditioning on distance.

I expect both across-facility and within-facility variation in $\mathbb{E}\left[z_i | N_{jt}, a_{ij}\right]$ to be informative about facilities' admission policies. Across-facility variation is likely to be informative because high average z_i scores may indicate that a facility is highly selective.[8] Within-facility variation is likely to be informative since increasing thresholds suggest an increasingly censored distribution of z_i admitted at the facility. Similarly, large values of $x_j^{\Delta z}$ may also correspond to facilities censoring the distribution of z_i that they admit.

It is important to emphasize that $\underline{\tilde{\pi}}(N_{jt}, X_j, \phi; \gamma_{\underline{\pi}})$ is not structural. The objective in parameterizing $\underline{\tilde{\pi}}(N_{jt}, X_j, \phi; \gamma_{\underline{\pi}})$ is to capture the variation in facilities' policies in a parsimonious way.[9] While this allows use of predictive but non-structural terms such as $x_j^{\Delta z}$ and $\mathbb{E}\left[z_i | N_{jt}, a_{ij}\right]$ when parameterizing the policy functions, it also places a severe

[6] I additionally impose monotonicity in N_{jt} since I expect facilities to become more selective in z_i as they become more full.

[7] I weight these averages by residents' lengths of stay.

[8] It may also indicate that the facility's local community of residents has generally high z_i scores. This is also likely to correspond to a high degree of selectivity since the facility likely anticipates an abundance of high-margin patients willing to accept an offer of admission.

[9] Since facility policies are not observed, a more precise statement is that we would like to parameterize $\underline{\tilde{\pi}}$ so that $\tilde{\pi} - \underline{\tilde{\pi}}$ is highly informative about the covariation between facility occupancy and admitted resident composition.

restriction on the counterfactuals that can be simulated using those policy functions. In particular, $\gamma_{\underline{\pi}}$ cannot be used to simulate how facilities adjust their admission policies in counterfactuals.

J Simulating the Applicant Distribution

Observe that:

$$\begin{aligned}
F^{\Pi}_{j\phi}(\Pi) &= \int_{\{c_i:\Pi_{ij}<\Pi\}} \mu_j(c_i) dF\left(c_i | j \in J_i, \phi_i = \phi\right) \\
&= \int_{\{\boldsymbol{X}_i,\epsilon_i^\pi,\epsilon_i^u:\tilde{\pi}(z_i,\phi_i)-\epsilon_i^\pi<(\rho+\lambda_{j\phi_i})\Pi\}} \mathbf{1}\left\{u_{ij} \geq u_{ij'} \forall j' \in \mathcal{O}_i\right\} dP\left(\boldsymbol{X}_i, \epsilon_i^\pi, \epsilon_i^u | j \in J_i, \phi_i = \phi\right) \\
&= \int_{\{\boldsymbol{X}_i,\epsilon_i^\pi:\tilde{\pi}(z_i,\phi_i)-\epsilon_i^\pi<(\rho+\lambda_{j\phi_i})\Pi\}} s_{ij}^{\mathcal{O}_i\cup\{j\}} dP\left(\boldsymbol{X}_i, \epsilon_i^\pi | j \in J_i, \phi_i = \phi\right) \\
&= \int\int_{\{\epsilon_i^\pi:\tilde{\pi}(z_i,\phi_i)-\epsilon_i^\pi<(\rho+\lambda_{j\phi_i})\Pi\}} s_{ij}^{\mathcal{O}_i\cup\{j\}} dF_{\epsilon^\pi}(\epsilon_i^\pi) dP\left(\boldsymbol{X}_i | j \in J_i, \phi_i = \phi\right) \\
&= \int \sum_k s_{ij}^{[k]\cup\{j\}} P\left(\tilde{\pi}(z_i,\phi_i) - \epsilon_i^\pi < (\rho+\lambda_{j\phi_i})\Pi | \boldsymbol{X}_i, \mathcal{O}_i = [k]\right) P\left(\mathcal{O}_i = [k] | \boldsymbol{X}_i\right) dP\left(\boldsymbol{X}_i | j \in J_i, \phi_i = \phi\right),
\end{aligned} \tag{45}$$

where facilities are are numbered in increasing order of $\underline{\tilde{\pi}}_k(N_{kt})$, as in (22). This suggests a number of simulators for $F^{\Pi}_{j\phi}$, the most direct of which is:

$$\hat{F}^{\Pi}_{j\phi}(\Pi) = \frac{\sum_{\{i:j\in J_i,\phi_i=\phi\}} \sum_k \hat{s}_{ij}^{[k]\cup\{j\}} P\left(\tilde{\pi}(z_i,\phi_i;\hat{\gamma}_\pi) - \epsilon_i^\pi < (\rho+\lambda_{j\phi_i})\Pi | \hat{\theta}_1, \boldsymbol{X}_i, \mathcal{O}_i = [k]\right) P\left(\mathcal{O}_i = [k] | \hat{\theta}_1, \boldsymbol{X}_i\right)}{|\{i : j \in J_i, \phi_i = \phi\}|}, \tag{46}$$

where $P(\mathcal{O}_i = [k]|\hat{\theta}_1, \boldsymbol{X}_i)$ is from equation (25), and $\hat{s}_{ij}^{[k]\cup\{j\}}$ is computed according to (17).[10] The remaining term, $P\left(\tilde{\pi}(z_i,\phi_i;\hat{\gamma}_\pi) - \epsilon_i^\pi < (\rho+\lambda_{j\phi_i})\Pi|\hat{\theta}_1, \boldsymbol{X}_i, \mathcal{O}_i = [k]\right)$, can be computed analytically using F_{ϵ^π}.

K Computing Expected Out-of-Pocket Expenditure

In this section I describe my model of resident beliefs about their expected admission-discounted daily out-of-pocket expenditure, $\mathbb{E}[OoP_{ij}]$. I model residents as transitioning from Medicare to private pay to Medicaid based on the resident's Medicare eligibility and wealth. First, I assume a cap of 100 days on Medicare coverage ($\bar{L}_i^{mcare} = 100$) if i's stay qualifies for Medicare coverage and 0 days ($\bar{L}_i^{mcare} = 0$) if i's stay does not. Second, I infer i's wealth from the number of days i was private pay at her admitting facility until she transitioned to Medicaid—i.e. $W_i = P_j^{priv} L_{ij}^{priv}$, where j was i's admitting facility. In cases where i was never observed to transition to Medicaid, I assume that i did not expect to ever transition to Medicaid at any facility ($W_i = \infty$). Inferred wealth is then used to forecast the number of days that i could have afforded as a private payer at other $j' \in J_i$ (i.e., $\bar{L}_{ij'}^{priv} = \frac{W_i}{P_{j'}^{priv}}$).

[10] I use first stage ($\hat{\theta}_1$) preference estimates in computing $\hat{s}_{ij}^{[k]\cup\{j\}}$.

Since Medicare out-of-pocket rates do not vary with facilities, I only model the out-of-pocket expenditures during private pay.[11] Then the resident's expected out-of-pocket contribution can be computed analytically:

$$
\begin{aligned}
\mathbb{E}\left[OoP_{ij}\right] = P_j^{priv}\left(\rho + \lambda_{j\phi_i}^D\right) & \left(\int_{\overline{L}_i^{mcare}}^{\overline{L}_i^{mcare}+\overline{L}_{ij}^{priv}} \int_{\overline{L}_i^{mcare}}^{L_{ij}} \exp(-\rho l) dl dF(L_{ij})\right. \\
& \left. + \int_{\overline{L}_i^{mcare}+\overline{L}_{ij}^{priv}}^{\infty} \int_{\overline{L}_i^{mcare}}^{\overline{L}_i^{mcare}+\overline{L}_{ij}^{priv}} \exp(-\rho l) dl dF(L_{ij})\right) \\
& = \left(\exp\left(-(\lambda_{j\phi_i}^D + \rho)\overline{L}_i^{mcare}\right) - \exp\left(-(\lambda_{j\phi_i}^D + \rho)(\overline{L}_i^{mcare} + \overline{L}_i^{priv})\right)\right) P_j^{priv},
\end{aligned}
\tag{47}
$$

where L_{ij} signifies resident i's length of stay at j, and the coefficient on P_j^{priv} is the admission-discounted share of a resident's stay that is anticipated to be private pay.

An important feature of this procedure to compute $\mathbb{E}[OoP_{ij}]$ is that it takes into account the fact that many residents who end up on Medicaid would likely have exhausted their private resources at any facility and should therefore have been largely indifferent to variation in facility prices.[12]

L Changing Reimbursement Rates

Because $\tilde{\pi}_{ij}$ is a discounted average anticipated flow profit over a resident's stay, it does not distinguish the precise flow profit generated on different days under different reimbursements. In order to translate raising the Medicaid reimbursement rate into a change in $\tilde{\pi}_{ij}$, I assume the following functional form for resident-specific flow profitability:

$$
\pi_{ij}(t - \tau_i^A) = \begin{cases} \pi_{ij}^{mcare} & \text{if } t - \tau_i^A \leq \overline{L}_i^{mcare} \\ \pi_{ij}^{priv} & \text{if } \overline{L}_i^{mcare} < t - \tau_i^A \leq \overline{L}_i^{mcare} + \overline{L}_{ij}^{priv} \\ \pi_{ij}^{mcaid} & \text{if } t - \tau_i^A > \overline{L}_i^{mcare} + \overline{L}_{ij}^{priv}. \end{cases}
\tag{48}
$$

Under this assumption, the average present discounted value of resident-specific profit that j expects to receive from admitting i can be computed as:

$$
\tilde{\pi}_{ij} = w_{ij}^{mcare}\pi_{ij}^{mcare} + w_{ij}^{priv}\pi_{ij}^{priv} + w_{ij}^{mcaid}\pi_{ij}^{mcaid},
\tag{49}
$$

$$
w_{ij}^{mcare} = 1 - \exp\left(-(\lambda_{j\phi_i}^D + \rho)\overline{L}_i^{mcare}\right),
\tag{50}
$$

$$
w_{ij}^{priv} = \exp\left(-(\lambda_{j\phi_i}^D + \rho)\overline{L}_i^{mcare}\right) - \exp\left(-(\lambda_{j\phi_i}^D + \rho)(\overline{L}_i^{mcare} + \overline{L}_i^{priv})\right),
\tag{51}
$$

$$
w_{ij}^{mcaid} = \exp\left(-(\lambda_{j\phi_i}^D + \rho)(\overline{L}_i^{mcare} + \overline{L}_i^{priv})\right).
\tag{52}
$$

[11] Note that this is assumption excludes the possibility that residents adjust their beliefs about out-of-pocket expenditures made while on Medicare based on across-facility variation in discharge rates. This assumption primarily affects short-stay residents likely to discharge between the start of Medicare coinsurance (20 days) and the end of Medicare coverage (100 days).

[12] While such residents are largely indifferent to private rates, they are not entirely so. For example, there is nonzero probability that a resident could be discharged after a time that would have resulted in exhaustion of private wealth at an expensive facility but not at a less expensive facility.

The coefficients $\{w_{ij}^{mcare}, w_{ij}^{priv}, w_{ij}^{mcaid}\}$ sum to 1 and represent the present discounted share of time that the resident is expected to spend as each payer type. The derivation of these coefficients is shown in Appendix L.1. I additionally place the following functional form restrictions on the flow profitabilities:

$$\pi_{ij}^{mcare} := \pi^{mcare}(h_i) + \epsilon_{ij} \tag{53}$$

$$\pi_{ij}^{priv} := \bar{P}^{priv} - mc(h_i) + \epsilon_{ij}, \tag{54}$$

$$\pi_{ij}^{mcaid} := \bar{P}^{mcaid} - mc(h_i) + \epsilon_{ij}. \tag{55}$$

This functional form allows that Medicare mark-ups may vary flexibly with the health characteristics of i. It also allows that the residents' marginal costs may vary with health characteristics when on private pay or Medicaid. These marginal costs may differ from the marginal costs while the resident is on Medicare since the type of care that qualifies for Medicare coverage is likely to differ from the typical care required when on private pay or Medicaid.[13] The parameterization of $\tilde{\pi}_{ij}$ in my estimation procedure implies that $\tilde{\pi}_{ij}$ is common to all facilities $j \in J_i$ under current reimbursements. In the same vein, I impose the average state-wide private rate and Medicaid reimbursement rates on all facilities in the parameterizations of π_{ij}^{priv} and π_{ij}^{mcaid}.[14]

This parameterization suggests regressing estimated individual specific profitabilities $\tilde{\pi}(z_i, \phi_i; \hat{\gamma}_\pi)$ on $\{w_{ij}^{mcare}, w_{ij}^{priv}, w_{ij}^{mcaid}\}$, health controls, and health controls interacted with w_{ij}^{mcare}.[15] Since marginal cost is assumed to be the same whether a patient is on Medicaid or private pay, the difference in the coefficients on w_{ij}^{priv} and w_{ij}^{mcaid} identifies the relationship between $\tilde{\pi}_{ij}$ and $\bar{P}^{priv} - \bar{P}^{mcaid}$. Table L.1 gives the results of such regressions. Column (3) is the preferred specification that allows marginal costs to differentially vary with health characteristics depending on whether the patient is currently receiving care reimbursed by Medicare. The estimates suggest that after controlling for health characteristics, the difference in desirability between patients on private pay and Medicaid corresponds to a .765 difference in $\tilde{\pi}_{ij}$. Attributing this disparity entirely to the difference in reimbursement rates suggests modeling raising the Medicaid reimbursement rate to match the private rate by increasing $\tilde{\pi}_{ij}$ by $.765 w_{ij}^{mcaid}$. Since private-pay rates are on average \$37.26 higher than Medicaid reimbursement rates, this also implies that the scale of $\tilde{\pi}_{ij}$—which is usually in terms of the standard deviation of the normally distributed error representing the unobserved component of profitability (equation 21)—is approximately \$48.71.

[13] The marginal cost when the resident is on Medicare is in $\pi^{mcare}(h_i)$.

[14] Note that I must allow the error term ϵ_{ij} to vary across j in order to match the modeling assumption in my estimation procedure that $\tilde{\pi}_{ij}$ is common across all $j \in J_i$.

[15] I include only realized admissions in these regressions.

Table L.1: Payer Source Regression

	(1) Model 1	(2) Model 2	(3) Model 3
Medicare Weight	5.506	4.992	4.989
	(0.00170)	(0.0209)	(0.0388)
Private Weight	4.780	4.340	4.220
	(0.00275)	(0.0210)	(0.0245)
Medicaid Weight	3.923	3.532	3.455
	(0.00415)	(0.0210)	(0.0244)
Stayer-Type Controls	Y	Y	Y
Health Controls	N	Y	Y
Health Controls X Rehab Weight	N	N	Y
R^2	0.525	0.544	0.549
N	555,170	555,170	555,170

Notes. Standard errors in parenthesis.

L.1 Derivation of Payer Source Coefficients

Observe that:

$$
\begin{aligned}
\Pi_{ij} &= \int_0^\infty \int_0^{L_i} \exp\left(-\rho l\right)\pi_{ij}(t) dl dF(L_i) \\
&= \int_0^{\overline{L}_i^{mcare}} \pi_{ij}^{mcare} \int_0^{L_i} \exp\left(-\rho l\right) dl dF(L_i) \\
&+ \int_{\overline{L}_i^{mcare}}^{\overline{L}_i^{mcare}+\overline{L}_{ij}^{priv}} \left(\pi_{ij}^{mcare}\int_0^{\overline{L}_i^{mcare}} \exp\left(-\rho l\right) dl + \pi_{ij}^{priv}\int_{\overline{L}_i^{mcare}}^{L_i} \exp\left(-\rho l\right) dl\right) dF(L_i) \\
&+ \int_{\overline{L}_i^{mcare}+\overline{L}_{ij}^{priv}}^{\infty} \left(\pi_{ij}^{mcare}\int_0^{\overline{L}_i^{mcare}} \exp\left(-\rho l\right) dl + \pi_{ij}^{priv}\int_{\overline{L}_i^{mcare}}^{\overline{L}_i^{mcare}+\overline{L}_{ij}^{priv}} \exp\left(-\rho l\right) + \pi_{ij}^{mcaid}\int_{\overline{L}_i^{mcare}+\overline{L}_{ij}^{priv}}^{L_i} \exp\left(-\rho l\right) d\tau\right) dF(L_i) \\
&= \pi_{ij}^{mcare} \underbrace{\left(\int_0^{\overline{L}_i^{mcare}} \int_0^{L_i} \exp\left(-\rho l\right) dl dF(L_i) + \int_{\overline{L}_i^{mcare}}^{\infty} \int_0^{\overline{L}_i^{mcare}} \exp\left(-\rho l\right) d\tau dF(L_i)\right)}_{\frac{w_{ij}^{mcare}}{\rho+\lambda_{j\phi_i}^D}} \\
&+ \pi_{ij}^{priv} \underbrace{\left(\int_{\overline{L}_i^{mcare}}^{\overline{L}_i^{mcare}+\overline{L}_{ij}^{priv}} \int_{\overline{L}_i^{mcare}}^{L_i} \exp\left(-\rho l\right) dl dF(L_i) + \int_{\overline{L}_i^{mcare}+\overline{L}_{ij}^{priv}}^{\infty} \int_{\overline{L}_i^{mcare}}^{\overline{L}_i^{mcare}+\overline{L}_{ij}^{priv}} \exp\left(-\rho l\right) d\tau dF(L_i)\right)}_{\frac{w_{ij}^{priv}}{\rho+\lambda_{j\phi_i}^D}} \\
&+ \pi_{ij}^{mcaid} \underbrace{\int_{\overline{L}_i^{mcare}+\overline{L}_{ij}^{priv}}^{\infty} \int_{\overline{L}_i^{mcare}+\overline{L}_{ij}^{priv}}^{L_i} \exp\left(-\rho l\right) d\tau dF(L_i)}_{\frac{w_{ij}^{mcaid}}{\rho+\lambda_{j\phi_i}^D}}.
\end{aligned}
\tag{56}
$$

The result follows from integrating where F is an exponential distribution with rate parameter $\lambda_{j\phi_i}^D$.

M Algorithm for Allocating Additional Beds

Determining the true optimal allocation of additional beds to benefit Medicaid residents is a complex problem that requires forecasting dynamic equilibrium changes in facility policies and realized resident admissions. Therefore, I implement a simple algorithm that a policymaker might plausibly use to target capacity expansions. Before enumerating the algorithm, it is helpful to define R_j to be the set of simulated residents for whom j was their first choice but did not receive an offer of admission from j. The algorithm aims to allocate additional beds to facilities where R_j includes a large share of Medicaid-enrollees that would gain substantial additional utility from being admitted at j.

The specific algorithm I implement to allocate a fixed number of additional beds is:

1. For each facility, compute:
$$\frac{\sum_{i\in R_j}(u_{ij}-u_{ij'})w_{ij}^{mcaid}L_{ij}}{\sum_{i\in R_j}L_{ij}}, \tag{57}$$
where j' is the facility that i was admitted to. This metric is intended to approximate the Medicaid-enrollee welfare increase that would occur if an extra bed were allocated to an arbitrary rejected resident for whom j was their first choice.

2. Iterate cyclically over facilities in decreasing order of (57) and allocate up to half of the beds required to satisfy the Medicaid-weighted bed-days in R_j:[16]
$$\frac{1}{2T}\sum_{i\in R_j} w_{ij}^{mcaid}L_{ij}, \tag{58}$$
where T is the length of the simulation. Repeat until all additional beds are allocated.

In my counterfactuals, I allocate 200, 400, 600, and 800 additional beds by iteratively applying the algorithm above in batches of 200 beds. In addition to the algorithm above, I have also implemented other allocation algorithms, such as allocating in proportion to the probability that facilities are full under FCFS, and the counterfactual results are qualitatively similar.

[16] I also prohibit expanding facilities above twice their original capacity.