Dynamic Competition and Expected Returns *

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ABSTRACT

We build a dynamic model of heterogeneous industries to study how firm competition affects systematic risk. Within an industry, firms invest when the underlying demand is high and divest when it is low, resulting in a bell-shaped relation between industry profitability and systematic risk. Fixing profitability, industries with more irreversible investment are less competitive and more risky. Finally, systematic risk is path-dependent because prior demand shocks affect installed industry capital. These results imply that tests that use simple competition measures, such as industry concentration or markups, can produce conflicting results. Our empirical approach exploits changes in oil prices to show the dynamic effect of competition on systematic risk within industry and uses a measure of trade flows between economic sectors to show the cross-industry effect.

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Product market competition affects many corporate decisions and therefore may have important implications for firms' systematic risk.[1] Several recent studies examine the relation between industry competition and expected stock returns, but the sign of the relation tends to depend on the setting.[2] In this paper, we build an analytical framework for studying competitive industry dynamics and show that the competition-risk relation depends on both the time-varying industry conditions and fixed industry characteristics. Our results imply that the standard cross-sectional tests that rely on simple measures of competition, such as industry markups or firm concentration, will almost surely arrive at the opposite conclusions depending on sample composition.

We build a parsimonious continuous-time model of firms operating in multiple economic sectors or industries. Firms within each sector produce a homogenous good and are exposed to common sectorial shocks that can capture, among other things, variation in input prices, technological innovation, or the underlying product demand. In the model, firms have the ability to instantaneously adjust their output in response to demand shocks by running machines at a higher capacity. In addition, firms can make lumpy investment by installing more machines when the marginal value from investment exceeds the purchase price of capital or disinvest by selling machines when the continuation value falls below the sale price of capital. We assume that there is a sector-specific wedge between the purchase and sale price of capital, which implies that firms' investment decisions are partially irreversibile.[3] Because firms within each sector are exposed to common demand shocks, they choose to invest and disinvest at the same time, with the distance between the investment and disinvestment thresholds defining the competitiveness of each industry.

Focusing first on a single industry, we show that firm interaction during sector-wide

[1] For example, product market competition may affect product pricing and quantity produced, entry/exit and liquidation decisions, mergers and acquisitions (M&As) decisions, risk management, capital structure, innovation, and investment. Important contributions include Schumpeter (1912, 1942), Bain (1951), Baumol (1982), Brander and Lewis (1986), Maksimovic (1988), Poitevin (1989), Bolton and Scharfstein (1990), Leahy (1993), Showalter (1995), Caballero and Pindyck (1996), Grenadier (2002), Aguerrevere (2003), Lambrecht and Perraudin (2003), Novy-Marx (2007), and Carlson, Dockner, Fisher, and Giammarino (2014).

[2] For example, Hou and Robinson (2006) and Gu (2016) document a negative relation between industry concentration and returns. Bustamante and Donangelo (2017) and Corhay (2017) find the opposite result, whereas Ali, Klasa, and Yeung (2009) contend that industry concentration is unrelated to stock returns.

[3] Investment irreversibility produces intermittent or lumpy investment in the model, which is consistent with the empirical evidence provided by Doms and Dunne (1998) and Cooper, Haltiwanger, and Power (1999).

episodes of investment and disinvestment decreases the sensitivity of each firm's market value to the underlying demand shocks and hence reduces its systematic risk. Intuitively, in favorable industry conditions, some of the aggregate industry shocks are absorbed by new investment, thereby limiting incumbents' profits, but also reducing their risk exposure. Similarly, when industry profitability is already low, incumbent firms decrease their output and may exit the market altogether, which prevents further decline in industry profitability and limits firms' losses. In this case, it is the put option to reduce capital that makes these firms safer. In sum, the competitive interaction among firms results in the reflecting boundaries on the industry profitability process, which generate the bell-shaped relation between industry profitability and firms' systematic risk.

In addition to dynamic changes within each industry, there are significant differences across industries, with some industries being able to change their capital relatively easily and others facing significant adjustment costs (e.g., because of regulatory constraints or the nature of business). Holding industry profitability constant, competition in the model is inversely related to the distance between industry expansion and contraction thresholds. For example, when investment is perfectly reversible, all fluctuations in the underlying demand are immediately met by firms' collective investment or disinvestment. As a result, firms in the industry earn zero profits and are insulated from any demand shocks. If, on the other hand, it is not possible to adjust capital, then firms' profits are fully exposed to the underlying demand shocks. Therefore, systematic risk is unconditionally decreasing with competition.

Our model shows that a researcher who uses firm concentration as a sole measure of competition will find that a relation between competition and expected returns depends on the state of the industry. Specifically, for industries where the firms are close to the point where they sell capital, the higher concentration increases systematic risk. In contrast, for the industries that are close to the threshold for additional investment concentration decreases systematic risk.

The model allows for a number of extensions. For example, following existing literature, we consider industries with levered firms that can choose to sell assets or optimally default when the profitability is low. In this case, systematic risk spikes at lower profitability because of the usual leverage effect, but this increase in risk is partially compensated by a firm holding put options that allow it to sell capital or default. Because the timing of default is optimal, we observe that the defaulting firm has lower systematic risk than the unlevered firm.

We also examine the effect of a firm's default and subsequent exit from the industry on the systematic risk of other firms. A firm with higher leverage defaults first and creates positive externality for the competitors. It may appear that the defaulting firm "absorbs" the risk and thus makes surviving firms safer, but this is not always the case.[4] Instead, the direction of the effect depends on the profitability. For example, when other firms in the industry are unprofitable, the liquidation of a competitor effectively moves these firms further from the thresholds for exercising their put options, thus increasing the systematic risk.

We empirically test the dynamic profitability-competition relation using a sample of oil producers and other firms with a positive exposure to oil prices. The oil sector provides a useful setting for our purposes because it is economically important, the state of the industry is readily observable, and firms often adjust their production and make significant irreversible investment.[5] Rather than relying on a pre-specified definition of the oil sector, our sample consists of all industries with a significantly positive unconditional oil price exposure. We then analyze how this oil price exposure varies conditionally as a function of the state of the oil sector, summarized by lagged oil prices. Consistent with the predictions of the model, we find a strongly nonlinear relation. Conditional oil betas are low when oil prices are low, high when oil prices are in the intermediate range, and again low when oil prices are high.

To test the cross-sectional prediction of the model, we construct a proxy for the intensity of product market competition based on the number of different inputs supplied from other industries (or outputs supplied to other industries), using the input-output (I-O) accounts published by the Bureau of Economic Analysis (BEA). These data have been used by Ahern (2013) and Ahern and Harford (2014). We argue that firms operating in industries with a more

[4]As opposed to preservation of firm value, there is no requirement that the systematic risk, aggregated over all firms in the industry, is constant or continuous.

[5]MacKay and Moeller (2007) also advocate for studying the oil sector. They show that it is highly competitive and exhibits a significant sensitivity to oil prices. Further, they argue that large swings in oil prices allow for a better fit of the empirical model.

diverse set of inputs have more flexibility in substituting one input of production for another or, alternatively, they have more differentiated products, and therefore, all else equal, they face lower competition from industry rivals. We propose input dependence, the Herfindahl index over trade flows from supplier industries, as an empirical proxy for cross-sectional measure of competition.

This measure has key advantages over the previously proposed proxies. First, it is obtained from I-O accounts that represent the whole economy and are not limited to publicly traded firms, an important consideration highlighted in Ali, Klasa, and Yeung (2009) and Bustamante and Donangelo (2017). Second, it provides a clear interpretation of firms competing for a given input resource. Thus, this measure is similar in spirit to the measure proposed by Hoberg and Phillips (2016) that relies on firms listing their direct product market competitors.

Using input dependence measure, we confirm the predictions of the model. First, market beta decreases in the level of competition, from the average 1.09 for firms in the bottom quartile of input dependence to the average 0.91 for firms in the top quartile of input dependence. The unconditional spread in betas of 0.17 is highly statistically significant. Second, when we interact product market competition with industry conditions as proxied for by industry asset growth, we find that this difference is concentrated in times of high and low industry investment, and drops to approximately 0.06 and 0.10 in intermediate times.

Further, our model shows that by decreasing the sensitivity of the stock price to a factor, competition can also reduce the magnitude of size-related cross-sectional asset pricing anomalies.[6] For example, because our model allows for additive idiosyncratic profit shocks, it naturally produces a negative relation between firm size and stock returns and a positive relation between book-to-market ratio and stock returns.[7] We expect that this relation is less pronounced with competition. More generally, our results imply that the size and value premia are smaller in more competitive industries because competition decreases the systematic risk of an average firm in the industry and thus also reduces the cross-sectional dispersion in

[6]In the cross-section, firms with small market capitalization and a high ratio of fundamentals to price tend to have high stock returns (Banz (1981), Graham and Dodd (1934)). Fama and French (1992) provide a detailed analysis of both value and size premium.

[7]As Babenko, Boguth, and Tserlukevich (2016) show, the idiosyncratic risk, when it does not have a pure multiplicative effect on the firm value function, can explain the key asset pricing anomalies.

risk among firms.

Empirically, we find a strong negative relation between input dependence and the magnitude of observed size and value anomalies. In particular, the inter-quartile difference in returns of size-sorted portfolios is large and significant at 0.73% per month among industries with low competition. In contrast, it completely disappears in high competition industries. The value premium also decreases by more than half in competitive industries.

The paper is organized as follows. The next section offers a brief overview of the related literature. Section II presents a model of perfect competition with partial investment irreversibility and compares results to the model of monopoly. Section III discusses and tests the asset pricing implications of product market competition. The last section concludes. Several extensions of the model, such as imperfect competition, are provided in the Appendix.

I. Literature

Our work contributes to the theoretical and empirical literature that links product market structure and stock returns. On the empirical side, Hou and Robinson (2006) were the first to document a negative relation between industry concentration and stock returns among public firms; they argue that firms in concentrated industries are less risky because they are insulated from undiversifiable distress risk and less innovative, as originally proposed by Schumpeter (1912). Gu (2016) shows that the negative relation between concentration and returns is stronger among the R&D-intensive firms. However, Ali, Klasa, and Yeung (2009) argue that industry concentration measures based on Compustat data are poor proxies for actual industry concentration and may instead capture industry decline. They show that concentration measures calculated by the U.S. Census have only 13% correlation with corresponding Compustat-based measures and are not related to future returns. In subsequent work by Bustamante and Donangelo (2017) and Corhay (2017), the authors find a positive relation between the HHI industry concentration index from the U.S. Census Bureau data for public and private firms and stock returns.[8]

[8]A somewhat different literature addresses the short-term returns caused by innovations to a competitive environment. For example, Grullon, Larkin, and Michaely (2018) find that industries that have experienced

Other important contributions in the area include Hoberg and Phillips (2010), who find that high industry-level stock market valuation and investment are followed by low abnormal stock returns. Although Hoberg and Phillips (2010) attribute their results to market participants suffering from the signal extraction problem and not fully internalizing the negative externality of competition on cash flows, their empirical results are also consistent with our risk-based explanation that in booms firms in competitive industries become safer. In a recent paper, Hoberg and Phillips (2016) use text-based analysis of firm 10-K product descriptions and show that firms typically compete in multiple products rather than fixed industries, as defined by SIC or NAICS codes. They also show that firms may change their products in response to industry shocks, with more similarity of rivals' products following positive shocks and more product differentiation following negative shocks. Based on their work, it is reasonable to argue that conventional measures of product market structure, such as measures of industry concentration using the HHI index, do not adequately capture the complex market relationships and interactions.

On the theoretical side, Aguerrevere (2009) analyzes Cournot competition in industry composed of N identical firms and link it to systematic risk. In his model, firms have operating leverage, optimize production subject to a capacity constraint, and can incrementally and irreversibly increase capacity at a proportional cost. As a result of the interplay of the operating leverage effect and the higher sensitivity of growth options to systematic risk than assets in place (see also Carlson, Fisher, and Giammarino (2004), Carlson, Fisher, and Giammarino (2006)), Aguerrevere (2009) finds that firms in more concentrated industries earn lower expected returns when demand is low and higher expected returns when demand is high. Corhay (2017) uses a production-based asset pricing model to examine the joint dynamics of credit spreads and levered equity. He argues that firms in more competitive industries have thinner profit margins and therefore are more exposed to risk of default on their debt. As a result, such firms optimally choose lower leverage, have higher credit spreads but lower expected returns. Corhay, Kung, and Schmid (2017) use the general equilibrium model to derive

large increases in product market concentration over the last two decades have realized positive abnormal stock returns, and argue that market power is a significant source of value.

the effect of oligopolistic competition on risk premia and term structure of equity returns.

In a closely related paper, Bustamante and Donangelo (2017) predict that unconditional returns are positively related to industry concentration, especially at lower leverage levels, in a sharp contrast to the traditional view that concentrated industry are "insulated" from the negative shocks and less risky. With the improved measure of industry concentration, Bustamante and Donangelo (2017) confirm the prediction empirically. Their rationale for the positive relation is two-fold. As in our model, the procyclical entry of new firms reduces the sensitivity of incumbent firms to the aggregate market risk; while the reversed effect from the risk to competition also makes the industries with greater exposure to systematic risk and higher discount rate less attractive to potential entrants, which allows them to maintain relatively higher markups. We build on the methodology of Bustamante and Donangelo (2017) to show that the conditional prediction for the competition-risk relation are the opposite for profitable and unprofitable industries. Further, we test the predictions for a particular industry that undergoes profitability changes, as well as across the industries that are differently exposed to the competition.

A large body of literature examines the effect of competition on firms' growth options and innovation (see, e.g., Aguerrevere (2009), Novy-Marx (2009)). In these studies, competition affects firm risk mainly by reducing the value of growth opportunities relative to the value of other assets. In turn, lower value of growth options leads to lower betas and expected returns. Gu (2016) demonstrates the counteracting effect; she shows that the threat of abandoning valuable growth options in presence of fierce competition can imply larger systematic risk for R&D-intensive firms. As a result, the positive R&D-return relation is stronger in competitive industries and the positive competition-return relation is stronger among R&D firms. Bena and Garlappi (2019) build the winner-takes-all patent race to shows that a firm that moves first erodes the value of similar innovations and makes its rivals more risky.

Several papers explore the mechanism of strategic interaction and the resulting effect on firm risk. Carlson, Dockner, Fisher, and Giammarino (2014) argue that competitors' options to adjust capacity reduce own-firm risk. Bustamante (2015) analyzes a duopoly setting and shows that *strategic* interaction among firms in a leader-follower equilibrium amplifies return comovement in industries with low value spread and leads to a negative correlation in betas of laggards and leaders in industries with high value spread. The paper by Bena, Garlappi, and Gruning (2016) develops a Schumpeterian model of investment to analyze the returns on the aggregate market.[9]

Our study is also related to the literature where competition affects the value and exercise strategy of real options.[10] The primary interest of our study, however, lies in identifying the asset pricing implications of competition. We also contribute to the general investment literature that explains the cross-sectional asset pricing anomalies.[11] We show that competition reduces systematic risk for all types of assets, not only for investment options, and that the magnitude of size-related anomalies is smaller in competitive industries.[12]

II. Model

The goal of the model is to formalize intuition and to develop empirical predictions for factor betas. We start with the case of perfect competition with partial investment irreversibility, contrast it with the case of monopoly, and then extend the analysis to consider the effects of leverage and the possibility of firm default for factor betas.

[9] Often, this literature considers strategic use of debt by firms with the goal to affect competition or quantity produced by other firms as in e.g., Brander and Lewis (1986), Poitevin (1989), and Showalter (1995).

[10] For example, Grenadier (1996, 2002) argues that competition erodes the value of real options, thereby reducing the advantage of waiting to invest. In contrast, Leahy (1993) and Caballero and Pindyck (1996) show that, despite the fact that the option to wait is less valuable in a competitive environment, irreversible investment is still delayed because the upside profits are lowered by new firm entry. Lambrecht and Perraudin (2003) show that in the presence of preemption threat, the optimal exercise strategy can lie anywhere between the zero-NPV and the monopolist optimal exercise trigger. By considering a nonlinear production technology Novy-Marx (2007) shows that firms in a competitive industry may delay irreversible investment even longer than suggested by a neoclassical framework.

[11] This literature includes Berk, Green, and Naik (1999), Gomes, Kogan, and Zhang (2003), Carlson, Fisher, and Giammarino (2004), Zhang (2005), and Cooper (2006).

[12] Thus, we also contribute to the studies by Ai and Kiku (2013) and Ai, Croce, and Li (2013) who provide a micro-foundation for the value premium using the general equilibrium long-run risk models with intangible capital and growth options and Kung and Schmid (2015), who model innovation to explain asset pricing evidence.

A. Preliminaries

Firms operate in multiple economic sectors indexed by $k = \{1, 2, ..., K\}$ and produce output that is differentiated across sectors, but homogenous within each sector. Firms behave competitively and take output prices as given. A firm's continuous profit flow from an economic sector depends on the sector-wide systematic shock, Z_k, and the output price, p_k. For example, shocks Z_k can capture the variation in the cost of raw materials used in the sector, availability of specialized labor, technological innovation, or changes in consumer preferences and the underlying demand for the final product (we will refer to Z_k as demand shocks). One unit of capital (or one machine) installed in sector k generates the instantaneous profit flow

$$\pi_k = \max_{w_k} \left(p_k Z_k F(w_k) - \psi_k w_k - c_k \right), \tag{1}$$

where the optimization is done with respect to the instantaneous production capacity, w_k, and the firm's production function has a decreasing returns to scale, $F(w_k) = w_k^{\nu}$, where $\nu \in (0, 1)$. To operate one unit of capital in sector k, the firm incurs fixed costs, c_k, and variable costs, $\psi_k w_k$, that increase linearly with the production capacity. We assume that demand shocks Z_k follow the Geometric Brownian motion (GBM)

$$dZ_k / Z_k = m_k dt + s_k dW_k, \tag{2}$$

with constant drift m_k and volatility s_k. Optimization of profit (1) with respect to the production capacity yields the profit per unit of capital

$$\pi_k = (1 - \nu) \left(\left(\frac{\nu}{\psi_k} \right)^{\nu} p_k Z_k \right)^{\frac{1}{1-\nu}} - c_k. \tag{3}$$

The product price, p_k, depends on the aggregate sector output, T_k, and the positive price elasticity of demand, $1/\varphi$. Following Miao (2005), we assume the following iso-elastic functional form

$$p_k = T_k^{-\varphi}. \tag{4}$$

Aggregate sector output is given by the sum of individual firms' outputs

$$T_k = \sum_i q_{ik} Z_k F(w_k^*) = Q_k \left(\left(\frac{p_k \nu}{\psi_k} \right)^{\nu} Z_k \right)^{\frac{1}{1-\nu}}, \tag{5}$$

where q_{ik} is the number of units of capital used by firm i in sector k, and $Q_k = \sum_i q_{ik}$ is the total number of units of capital in the sector. The equilibrium price in equation (4) is derived by setting sector demand equal to supply

$$p_k = \left(Q_k^{1-\nu} Z_k \left(\frac{\nu}{\psi_k}\right)^{\nu}\right)^{\frac{\varphi}{1-\nu+\varphi\nu}}. \tag{6}$$

The price varies with the total number of units of capital used by the sector, Q_k, and demand shocks, Z_k. By substituting the equilibrium price (6) into the profit function (3), we obtain

$$\pi_k = D_k Q_k^{-\varepsilon} Z_k^{\xi} - c_k, \tag{7}$$

where $\varepsilon = \frac{\varphi}{1-\nu+\nu\varphi}$, $\xi = \frac{1-\varphi}{1-\nu+\nu\varphi}$, and $D_k = (1-\nu)(\nu/\psi_k)^{\nu\xi}$ are constants.

The sector profit margin, M_k, can be calculated as a firm's profit generated in sector k divided by revenue

$$M_k = \frac{\pi_k}{p_k Z_k F(w_k^*)} = \frac{(1-\nu)(D_k Q_k^{-\varepsilon} Z_k^{\xi} - c_k)}{D_k Q_k^{-\varepsilon} Z_k^{\xi}}. \tag{8}$$

It follows from (8) that the profit margin decreases with fixed operating cost, c_k, increases with the underlying product demand, Z_k, and decreases with the number of units of capital installed in the sector, Q_k, which also implies that the profit margin changes after sector expansions or contractions.

By summing the firm's profit flows across different sectors and allowing for idiosyncratic shocks, x_i, and financial leverage, C_i, we can write the total profit flow of a firm i as

$$\Pi_i = x_i - C_i + \sum_{k=1}^{K} q_{ik}(D_k Z_k^{\xi} Q_k^{-\varepsilon} - c_k), \tag{9}$$

where the idiosyncratic shock x_i evolves as GBM with constantr drift μ and volatility σ

$$dx_i/x_i = \mu dt + \sigma dB_i. \tag{10}$$

We assume that the increments of standard Wiener processes dB_i and dW_k are uncorrelated, $E[dB_i dW_k] = 0$, and that the idiosyncratic and systematic shocks are uncorrelated across firms and sectors, $E[dB_i dB_j] = 0$ for $i \neq j$, and $E[dW_k dW_l] = 0$ for $k \neq l$.

Intuitively, the term $D_k Z_k^{\xi}$ in the profit function (9) reflects the underlying product demand as well as the endogenous instantaneous capacity choice, whereas $Q_k^{-\varepsilon}$ reflects the effect of the aggregate sector size on the equilibrium output price. Compared to the industry-equilibrium model of Miao (2005), we introduce systematic risk and allow firms to optimize both over capital investment and instantaneous capacity.

We assume that a firm's installed capital q_{ik} decays over time at a constant rate, δ, which can be interpreted as a gradual reduction in the productivity of capital when machines wear out or become obsolete. It follows that the aggregate sector capital decays at the same rate

$$dQ_k = -\delta Q_k dt. \tag{11}$$

We model capital investment by allowing firms to adjust capital, i.e., buy or sell machines, in each sector independently. In particular, firms can continuously add capital in sector k at a cost per machine $\overline{R}_k$ and can also decrease capital by selling machines at a price $\underline{R}_k$. We assume there are real frictions, which manifest themselves in a positive wedge between the purchase and sale cost of capital, i.e., $\overline{R}_k > \underline{R}_k$. Such a wedge between the purchase and sale price results in lumpy investment in the model, consistent with empirical evidence provided in Doms and Dunne (1998), Cooper, Haltiwanger, and Power (1999), and Gourio and Kashyap (2007). Finally, throughout the paper we assume that collusion among competing firms is not possible either because of coordination problems or tight legal constraints.[13]

B. Valuation

To apply the standard valuation tools, it is convenient to define $Y_k = Z_k^{\xi}$, which is also a Geometric Brownian Motion with drift $\mu_k = \xi m_k + \frac{s_k^2}{2}\xi(\xi - 1)$ and volatility $\sigma_k = \xi s_k$. We can then write the sector profitability before fixed costs as

$$y_k = D_k Y_k Q_k^{-\varepsilon}. \tag{12}$$

Without new investment, the aggregate capital deterministically decreases as shown in (11). Using Ito's lemma, we can write the dynamics of sector profitability as

$$\frac{dy_k}{y_k} = \overline{\mu}_k dt + \sigma_k dW_k, \tag{13}$$

[13]See, e.g., Maksimovic (1988), Chen, Dou, Guo, and Ji (2020) for models with industry collusion.

where the adjusted drift is given by

$$\overline{\mu}_k = \mu_k + \varepsilon\delta. \tag{14}$$

Intuitively, the drift in sector profitability is higher because the expected depreciation in the capital installed by competitors implies a higher selling price for the product. Using (12), we can write the firm's profit flow as

$$\Pi_i = x_i - C_i + \sum_{k=1}^{K} q_{ik}(y_k - c_k). \tag{15}$$

The market value of the firm V_i is the value of the claim to all future firm's profits,

$$V_i = E\int \Pi_i e^{-rt} dt, \tag{16}$$

where r is the risk-free rate, and the expectation is taken under the risk-neutral measure. The firm value V_i can be obtained using a standard portfolio replication argument and, assuming the firm follows the optimal investment strategy, it must satisfy the following partial differential equation

$$rV_i = \Pi_i + \sum_{k=1}^{K}\left[\overline{\mu}_k y_k \frac{\partial V_i}{\partial y_k} + \frac{\sigma_k^2 y_k^2}{2}\frac{\partial^2 V_i}{\partial y_k^2} - \delta q_{ik}\frac{\partial V_i}{\partial q_{ik}}\right] + \mu x_i \frac{\partial V_i}{\partial x_i} + \frac{\sigma^2 x_i^2}{2}\frac{\partial^2 V_i}{\partial x_i^2}. \tag{17}$$

Because the profit flow (15) is separable in state variables $\{x_i, y_1, ..., y_K\}$ and because firms are allowed to adjust capital independently in each sector, we conjecture that the firm value can also be written in a separable form, i.e.,

$$V_i = V_i^x(x_i) + \sum_{k=1}^{K} V_i^k(y_k), \tag{18}$$

where $V_i^x(x_i)$ represents the idiosyncratic fraction of firm value related to shocks x_i and leverage C_i, and $V_i^k(y_k)$ is the systematic part of firm value derived from operations in sector k. By substituting conjecture (18) in equation (17), we obtain a system of ordinary differential equations for the idiosyncratic component of firm value

$$rV_i^x(x_i) = x_i - C_i + \mu x_i \frac{\partial V_i^x}{\partial x_i} + \frac{\sigma^2 x_i^2}{2}\frac{\partial^2 V_i^x}{\partial x_i^2}, \tag{19}$$

and for each systematic component linked to sector k

$$rV_i^k(y_k) = q_{ik}(y_k - c_k) + \overline{\mu}_k y_k \frac{\partial V_i^k}{\partial y_k} + \frac{\sigma_k^2 y_k^2}{2} \frac{\partial^2 V_i^k}{\partial y_k^2} - \delta q_{ik} \frac{\partial V_i^k}{\partial q_{ik}}, \tag{20}$$

where we assume the firm follows optimal investment option exercise strategy. We next solve this valuation problem for the case of perfectly competitive sector with partial investment reversibility.

C. Perfect Competition with Partial Investment Reversibility

Without new investment, the aggregate capital decays at rate δ, but firms can choose to increase or decrease their capital in a given sector in response to changes in the underlying demand. Since all firms face an identical decision problem, we conjecture (and later verify) that they buy and sell capital at the same time, i.e., when sector profitability reaches a particular threshold.

As a result of identical decisions of all firms operating in the same sector and the resulting endogenous change in the aggregate capital Q_k, the sector profitability y_k has upper and lower reflecting barriers, $\overline{y}_k$ and $\underline{y}_k$.[14] The upper barrier is the threshold at which firms operating in sector k find it optimal to add capital, which increases firm competition and therefore limits further increase in sector profitability y_k. Similarly, the lower reflecting barrier is the threshold at which firms reduce their capacity by selling unproductive capital, which lessens the effect of the negative systematic shocks and prevents the sector profitability from decreasing further. We first consider exogenous reflecting barriers and solve for the optimal investment thresholds in the next section.

At the reflecting barriers, the change in aggregate capital exactly offsets the change in the shock Y_k to keep the sector profitability y_k constant, i.e., dQ_k, satisfies

$$dy_k = Q_k^{-\varepsilon} dY_k - \varepsilon Y_k Q_k^{-\varepsilon-1} dQ_k = 0. \tag{21}$$

To describe the profitability dynamics, let $\mathbb{1}_B$ be an indicator function equal to one if the process is at one of the reflecting barriers, i.e., $y_k = \overline{y}_k$ and $dY_k > 0$ or $y_k = \underline{y}_k$ and $dY_k < 0$,

[14]As in the investment model of Caballero and Pindyck (1996) and the competitive equilibrium model in Leahy (1993), the reflecting barriers $\overline{y}_k$ and $\underline{y}_k$ are time-invariant in our setting.

and zero otherwise. Then,

$$\frac{dQ_k}{Q_k} = -\delta dt + \mathbb{1}_B \frac{1}{\varepsilon} \frac{dY_k}{Y_k}, \tag{22}$$

and

$$\frac{dy_k}{y_k} = (1 - \mathbb{1}_B) \left(\overline{\mu}_k dt + \sigma_k dW_k \right). \tag{23}$$

For firm i that operates in sector k, the value $V_i^k(y_k)$ derived from this sector is proportional to capital q_{ik}, and hence we can simplify equation (20) to

$$\overline{r} V_i^k(y_k) = q_{ik}(y_k - c_k) + \overline{\mu}_k y_k \frac{\partial V_i^k}{\partial y_k} + \frac{\sigma_k^2 y_k^2}{2} \frac{\partial^2 V_i^k}{\partial y_k^2}, \tag{24}$$

where

$$\overline{r} = r + \delta. \tag{25}$$

The general solution to equation (24) has the following form

$$V_i^k(y_k) = q_{ik} \left(\frac{y_k}{\overline{r} - \overline{\mu}_k} - \frac{c_k}{\overline{r}} + A y_k^{b_1} + B y_k^{b_2} \right), \tag{26}$$

where A and B are constants and exponents $b_1 > 1$ and $b_2 < 0$ are the roots of the standard quadratic equation given in the Appendix. The solution is subject to the boundary conditions. To prevent arbitrage, firm value must be insensitive to changes in y_k at the reflecting barriers,

$$\frac{\partial V_i^k}{\partial y_k}|_{y_k = \overline{y}_k} = 0, \tag{27}$$

$$\frac{\partial V_i^k}{\partial y_k}|_{y_k = \underline{y}_k} = 0. \tag{28}$$

If conditions (27) and (28) were not met, the firm value would change predictably at the reflecting barriers, where the process is almost sure to decrease (at the upper barrier) or increase (at the lower barrier). Hence it would be possible to design a riskless strategy earning a positive profit. Using (27)-(28) to solve for constants A and B, we can write the value of firm's operations in sector k as follows

$$V_i^k(y_k) = q_{ik}\left(V_N(y_k) - V_U(y_k) + V_L(y_k)\right), \tag{29}$$

$$V_N(y_k) = \frac{y_k}{\overline{r} - \overline{\mu}_k} - \frac{c_k}{\overline{r}}, \tag{30}$$

$$V_U(y_k) = \frac{\overline{y}_k \underline{y}_k^{b_2} - \overline{y}_k^{b_2} \underline{y}_k}{\overline{y}_k^{b_1} \underline{y}_k^{b_2} - \overline{y}_k^{b_2} \underline{y}_k^{b_1}} \frac{\frac{y_k^{b_1}}{b_1}}{\overline{r} - \overline{\mu}_k}, \tag{31}$$

$$V_L(y_k) = \frac{\overline{y}_k^{b_1} \underline{y}_k - \overline{y}_k \underline{y}_k^{b_1}}{\overline{y}_k^{b_1} \underline{y}_k^{b_2} - \overline{y}_k^{b_2} \underline{y}_k^{b_1}} \frac{\frac{y_k^{b_2}}{(-b_2)}}{\overline{r} - \overline{\mu}_k}. \tag{32}$$

The first term in (29) represents the value of a claim to an unlimited stream of profits. The second term (proportional to $y_k^{b_1}$) is negative because the profitability of the firm is bounded from above at $\overline{y}_k$, and the third term (proportional to $y_k^{b_2}$) is positive because the profitability is bounded from below at $\underline{y}_k$. The value function is monotonically increasing. Conditions (27) and (28) imply that the function is also convex for low values of y_k and concave for high values. Intuitively, as the sector profitability approaches the lower boundary, a firm is insulated from further negative shocks and benefits from higher volatility. In contrast, the negative externality from competition at the upper boundary makes the firm effectively risk-averse.

D. Optimal Investment

We derive the optimal thresholds for firm expansion and contraction in two steps. First, we conjecture that the thresholds $\overline{y}_k$ and $\underline{y}_k$ are common to all firms operating in the same sector. Next, we verify this conjecture for a firm i, holding the thresholds for all other firms in the sector constant.

To obtain the thresholds, we use the *dynamic zero excess profit condition*, i.e., we require that the additional firm value from investment at the upper threshold is exactly equal to the purchase price of capital, and that the firm value associated with one unit of capital at the lower threshold is equal to the sale price of capital

$$V_i^k(\overline{y}_k)/q_{ik} = \overline{R}_k, \tag{33}$$

$$V_i^k\left(\underline{y}_k\right)/q_{ik} = \underline{R}_k. \tag{34}$$

These equations define the reflecting barriers $\overline{y}_k$ and $\underline{y}_k$ for firms that competitively buy and sell capital. Substituting value function (29) into equations (33) and (34) yields a system of equations

$$\underline{y}_k \overline{y}_k^{b_1} - \overline{y}_k \underline{y}_k^{b_1} = \frac{b_2 (\overline{r} - \overline{\mu}_k)}{1 - b_2} \left(\left(\overline{R}_k + \frac{c_k}{\overline{r}} \right) \underline{y}_k^{b_1} - \left(\underline{R}_k + \frac{c_k}{\overline{r}} \right) \overline{y}_k^{b_1} \right), \tag{35}$$

$$\underline{y}_k \overline{y}_k^{b_2} - \overline{y}_k \underline{y}_k^{b_2} = \frac{b_1 (\overline{r} - \overline{\mu}_k)}{1 - b_1} \left(\left(\overline{R}_k + \frac{c_k}{\overline{r}} \right) \underline{y}_k^{b_2} - \left(\underline{R}_k + \frac{c_k}{\overline{r}} \right) \overline{y}_k^{b_2} \right). \tag{36}$$

Using the standard approach, we verify that the optimal threshold selected by a firm is equal to the one conjectured for other competitors.[15] In Appendix B.1, we solve for the value and the optimal exercise policy for a stand-alone option of increasing or decreasing capital by one unit. We derive the system of two equations for an individual firm's thresholds and show that this system of equations is identical to (35)-(36). Thus, we have defined the market values and the equilibrium optimal strategies of firms competing in sector k.

E. Pricing and Factor Betas

We assume an exogenous linear pricing kernel

$$\frac{d\Lambda}{\Lambda} = -rdt + \sum_{k=1}^{K} \theta_k \left(\frac{dY_k}{Y_k} - \mu_k dt \right), \tag{37}$$

where θ_k is the price of risk per unit of exposure to the systematic shock in sector k. For a linear pricing kernel, the k-th factor beta is the elasticity of the market value of the firm with respect to Y_k.[16]

$$\beta_i^k = \frac{\partial V_i}{\partial Y_k} \frac{Y_k}{V_i}. \tag{38}$$

Because Y_k and y_k have the same exposure to the underlying risk process between the reflecting barriers, the factor betas with respect to these variables are identical.[17] This allows us to compute factor betas using expressions (29) to (32).

[15] See Section B.1 of the appendix for details.

[16] To obtain this result, substitute $R_i = dV_i/V_i$ in the definition of factor beta

$$\beta_i^k = Cov\left(\frac{dV_i}{V_i}, \frac{dY_k}{Y_k} \right) / Var\left(\frac{dY_k}{Y_k} \right),$$

and note that all systematic and idiosyncratic shocks are uncorrelated.

[17] Alternatively, the priced risk can be expressed in terms of the underlying shock Z_k. Betas with respect to Z_k and Y_k are proportional, $\beta_Z = \xi \beta_Y$.

Proposition 1. *The factor beta of the firm operating in sector k is*

$$\beta_i^k = 1 - q_{ik}\left(b_1 - 1\right)\frac{V_U}{V_i} - q_{ik}\left(1 - b_2\right)\frac{V_L}{V_i} + \frac{q_{ik}}{V_i}\frac{c_k}{\overline{r}} - \frac{V_i - V_i^k}{V_i}, \tag{39}$$

where functions V_i^k, V_U, and V_L are given in (29), (31), and (32), and constants $b_1 > 1$, $b_2 < 0$, and firm value V_i are given in the Appendix.

Proof. All proofs are in the Appendix. □

In equation (39), the first term is a normalization. The second and third terms appear because of the limiting effect of competition on the systematic risk at the upper and lower reflecting barriers on sector profitability. The fourth term reflects higher risk due to operating costs. The last term captures the effect of firm's operations outside of sector k, including idiosyncratic profit and financial leverage. For example, if operations in sector k make up only a small part of the total firm, exposure to factor k will be low.[18]

Next, we investigate how factor betas depend on sector profitability. The following corollary shows that factor betas have a bell-shaped relation with respect to sector profitability y_k, and, in particular, they decrease to zero in the vicinity of the sector contraction and expansion thresholds.

Corollary 1. *Factor beta decreases to zero at the contraction and expansion thresholds within each sector, i.e.,*

$$\beta_i^k|_{y_k \to \overline{y}_k} \to 0,\ \beta_i^k|_{y_k \to \underline{y}_k} \to 0. \tag{40}$$

Using Proposition 1, we can show that for any given level of sector profitability y_k, factor beta decreases with competition. Effectively, competition among firms in the sector decreases beta through two terms in (39), which are proportional to V_U and V_L, but it also affects the total firm value V_i in the denominator. We show in the corollary below that the first effect always dominates.

[18]Note that if we have a closed economy with all firms representing the market portfolio, it is also possible to derive the market betas. Appendix B.3 discusses the relation between factor betas with respect to the systematic shock and equity betas with respect to the aggregate market. Such relation is only simple under strong assumption, and therefore we work with the factor betas in the model.

Corollary 2. *For any profitability y_k, firms operating in sectors with greater investment irreversibility are more risky, i.e.,*

$$\frac{\partial \beta_i^k}{\partial \overline{R}_k} > 0, \; \frac{\partial \beta_i^k}{\partial \underline{R}_k} < 0. \tag{41}$$

The intuition is that as the purchase price of capital $\overline{R}_k$ increases, firms choose to expand at a higher profitability threshold $\overline{y}_k$, which in turn decreases the influence of the limiting effect of competition on firm value and factor beta through the value term V_U. Similarly, as the sale price of capital $\underline{R}_k$ increases, firms choose to divest sooner (at a higher threshold $\underline{y}_k$), which increases value term V_L and hence decreases factor beta in (39).

Corollary 3. *Define y_{max} as $\partial \beta_i^k(y_{max})/\partial y_k = 0$. Then, for $y_k < y_{max}$, β_i^k monotonically increases in concentration $1/Q_k$, while for $y_k > y_{max}$, β_i^k monotonically decreases in concentration $1/Q_k$.*

This result follows directly from (39) and the definition of y_k in (12). Because beta has the largest value at y_{max}, it increases in concentration for the lower profitability, but decreases in concentration for the higher profitability. The corollary is important for understanding the cross-sectional regressions that use concentration as measure of competition.

Finally, we examine how factor beta depends on the idiosyncratic shocks which affect firm size in the model and therefore are important for understanding size-related cross-sectional anomalies.

Corollary 4. *Factor beta decreases with the idiosyncratic shock x_i, but this relation is attenuated by competition.*

The first part of the corollary follows directly from (39). Intuitively, the larger idiosyncratic component of firm value reduces the influence of any systematic shocks and hence lowers the overall beta. Competition further reduces betas at low and high industry profitability. As a result, beta is negatively affected by the idiosyncratic shock and this relation is less pronounced with competition.

F. Illustrations

Figure 1 illustrates the dynamic features of the model. In Panel A, we display a sample path of the systematic shock Y_k, plotted over time. The next panel gives the corresponding dynamics of sector concentration, where concentration is defined as an inverse of aggregate number of units of capital in the sector, i.e., concentration is $\sum_{i=1}^{N}\left(\frac{q_{ik}}{Q_k}\right) = \frac{1}{Q_k}$. In general, sector concentration can change for three reasons. First, it increases over time because of the natural decay of installed capital at the constant rate δ. Second, concentration decreases when the systematic shock Y_k reaches the expansion threshold and firms install additional capital. Third, it increases when the systematic shock falls to the contraction threshold. Panel C displays the dynamics of the sector profitability, $y_k = D_k Y_k Q_k^{-\varepsilon}$, which increases with the underlying demand Y_k, but is limited by the high and low reflecting barriers. Finally, Panel D shows the dynamics of factor beta corresponding to the path of Y_k in Panel A.

We next use the analytic solution of the model to plot the firm value and factor betas as a function of sector profitability and perform comparative statics. Unless stated otherwise in the captions, we use the base case parameters in all subsequent figures: $\delta = 0.01$, $\varepsilon = 0.5$, $\mu_k = 0.03$, $\mu = 0.03$, $r = 0.05$, $\sigma_k = 0.25$, $\underline{R}_k = 100$, $\overline{R}_k = 200$, $x_i = 0$, $c_i = 0$, $N = 100$, and $q_{ik} = 1$.[19] The solution for the high and low reflecting barriers, $\overline{y}_k$ and $\underline{y}_k$, is obtained by solving numerically the system of non-linear equations (35)-(36).

In Figure 2, we vary the sale and purchase price of capital, $\underline{R}_k$ and $\overline{R}_k$. As the sale price of capital $\underline{R}_k$ increases, firms start exercising their contraction options earlier, which leads to a faster decrease in the systematic risk in downturns. Similarly, as the purchase price of capital $\overline{R}_k$ increases, firms wait longer to install additional capital, which causes a slower decrease in betas in booms. It can be seen from the figure that the higher purchase price of capital for competitors translates into higher firm values. The reason behind this result is that a higher purchase price of capital is associated with greater irreversibility of investment in the model and thus helps to limit the negative influence of firm's rivals on its profit.

[19] Our goal here is to illustrate the main mechanism, rather than to build a realistic simulation. For this reason, parameter values are adopted from the previous studies or simply normalized. Specifically, we have fixed the risk-free interest rate at $r = 0.05$; therefore the drift parameters, which may not exceed the discount and must be positive, were set at 0.03. Parameters $\underline{R}_k$ and $\overline{R}_k$ are normalized to 100 and 200, respectively.

In Figure 3, we show the effect of the idiosyncratic shocks on firm value and beta. The figure shows that the range of changes of beta is smaller when idiosyncratic risk is larger, with the larger effect at smaller values of profitability y_k. We therefore expect that the effect of competition on betas is attenuated by the idiosyncratic risk. It follows from (18) that a higher value of the idiosyncratic profit shock x_i implies a larger market value of the firm and a smaller book-to-market ratio. At the same time, it follows from (39) that shock x_i decreases beta. Intuitively, a larger value of x_i makes a larger proportion of the profit come from the idiosyncratic source. Therefore our model naturally produces a negative relation between firm value and stock returns and a positive relation between book-to-market ratio and stock returns.

We next show in Figure 4 the comparative statics with respect to the volatility of the systematic shock and the depreciation rate of capital. Interestingly, high-systematic-volatility sectors earn have a lower valuation and earn higher expected returns than low-volatility sectors when the underlying demand is high. In contrast, this relation tends to reverse in downturns. This result is driven by the concavity of the value function at higher profitability. The option to add capacity is optimally delayed, the industry expands at a higher threshold when volatility is high. Finally, we observe that the firm value decreases and both thresholds increase with parameter δ leading to the higher systematic risk at high profitability and the lower risk at low profitability.

As we argued above, the model gives rise to the size and value anomalies through the idiosyncratic component of firm profit. We next provide intuition for why the size-related anomalies are stronger in less competitive environments. Figure 5 demonstrates that more intensive competition reduces the difference in betas between the firms with different idiosyncratic shocks.

G. Model Extension: Effect of Firm Default

In this section, we consider a modification of the base model and examine the effect of levered firm default on its systematic risk.[20] Suppose the firm has a choice to sell capital or default. At some high level of leverage, firms finds it optimal to default *before* the threshold for the unproductive capital sale is triggered. In this case, firm systematic risk is affected and the firm's exit from the industry can have a feedback effect on the optimal strategies and firm values of the competitors. We show below that the solution with the possibility of default is very similar to our base case.

To consider the implications of default for firm risk, we take the amount of debt as given and leave the capital structure considerations (e.g., the optimal mix of debt and equity in firm financing) out of the picture.[21] We consider a firm i that operates in a single sector k and we assume for simplicity that it is not subject to the idiosyncratic shocks. The solution to the levered equity value is

$$E_i^k\left(y_k\right) = q_{ik}\left(\frac{y_k}{\overline{r}-\overline{\mu}_k} - \frac{c_k}{\overline{r}} + A_l y_k^{b_1} + B_l y_k^{b_2}\right) - \frac{C_i}{r}, \tag{42}$$

where A_l and B_l are some constants, and C_i is the coupon payment to debtholders. Default is modeled as optimal, which means that the value-matching and smooth-pasting conditions must be satisfied at the default threshold $y_d > 0$, i.e.,

$$\frac{\partial E_i^k}{\partial y_k}|_{y_k=y_d} = 0, \tag{43}$$

$$E_i(y_d) = 0. \tag{44}$$

It is important to understand that although the equity smooth-pasting condition (43) is similar to the previously used condition (28) for firm value, the meaning is different. The smooth-pasting condition ensures that the default timing is optimal and is not a consequence of competition.

[20] The base model already allows for leverage, as a negative term in the idiosyncratic part of firm value V_i^x, but we assumed that the leverage is small enough so that a firm never finds it optimal to default and instead sells the unproductive capital before hitting the default threshold. Therefore, the only effect of leverage in the base model was that, by removing the safe part of cash flows, leverage increased factor betas.

[21] Note that in the absence of any benefits of debt, such as tax benefits or mitigation of overinvestment incentives, leverage represents a pure disadvantage to a firm in our model because, by creating a possibility of a future default, it simply reduces the expected useful life of capital.

Since the threshold for optimal default by the levered firm satisfies $y_d > \underline{y}_k$, it is also true that the levered firm would generally find it optimal to invest at a higher threshold than its unlevered competitors operating in the same sector. This implies that the levered firm will be preempted and will actually never invest, and the upper reflecting barrier on sector profitability $\overline{y}_k$ will be the same as before (since it is set by unlevered firm competitors). Therefore, our solution for firm beta in (39) still applies, with the exception that in functions V_U, V_L, and V_i the threshold $\underline{y}_k$ is replaced by the firm's default threshold y_d from (44) and it simplifies to

$$\beta_i^k = 1 - q_{ik}\left(b_1 - 1\right)\frac{V_U}{V_i} - q_{ik}\left(1 - b_2\right)\frac{V_L}{V_i} + \frac{q_{ik}}{V_i}\frac{c_k}{\overline{r}} + \frac{1}{V_i}\frac{C_i}{r}, \tag{45}$$

Figure 6 illustrates the effect of default in our model. The two panels show the equity value and beta for a firm that optimally chooses to default prior to reaching the threshold for the sale of capital $\underline{y}_k$. Note that the equity value pastes smoothly to zero at the default threshold y_d and is equal to zero at this threshold, indicating that the timing of the default is equity's control.

We model the effect of a levered firm default on other firms in the industry by assuming that the levered firm is operating a proportion α of the sector's capacity. Following the default, the sector capacity decreases to $(1-\alpha)Q$ and therefore sector profitability y_k increases by a factor $(1-\alpha)^{-\varepsilon}$

$$y_k^{post} = y_k(1-\alpha)^{-\varepsilon}. \tag{46}$$

For example, if half of the competitors leave the industry, their departure increases profitability for the remaining competitors by a factor $2^{\varepsilon} > 1$. Therefore, the value and beta of a surviving firm (in the "post-game" following the competitor's default) is described by the solution to our main model, but with y_k^{post}.

Finally, the solution for the firm value at any instant anticipates the default of a levered competitor with the following boundary condition.

$$E_i^{pre}\left(y_d^*\right) = E_i^{post}\left(y_d^*\right). \tag{47}$$

Finally, Figure 7 demonstrates the effect of the levered firm's default on other firms operating in the same sector. We assume, for the purpose of illustration, that the default of a levered firm induces its exit from the sector and cuts the sector capacity in half, to $Q/2$, and plot the value of the equity and the beta against the profitability shock Y_k (note that because y_k is scaled by Q it cannot be used to show the relation between the size of the sector and systematic risk). We observe from the figure that the competitor's default creates a positive exernality on the remaining firms and increases the value of their equity. Moreover, because the competitors anticipate a positive externality at the time when the levered firm defaults, their systematic risk is smaller. Effectively, the default of one of the firms in the sector serves as a put option for all other firms in the sector.

H. Contrasting the Model's Results with the Cases of Monopoly and N-firm Symmetric Oligopoly

To better understand the effect of firms' interaction on asset prices, we contrast the expression for beta in our model with that in a model of monopoly and the symmetric case with N-firm competition. Appendix C provides the comprehensive list of assumptions and details the full solution. In short, we consider a firm that has monopoly power in sector M, i.e., it owns all capacity in this sector and faces no immediate threat of new entry (e.g., because of special technology, government regulation, trade secrets, or patents). To ensure that the problem has an interior solution, we assume that the monopolist can optimize over the one-time decisions when to expand and scale down capital as well as over the size of expansion or contraction. Similarly, in Appendix D, we provide the solution for a symmetric Cournot-Nash equilibrium with N competing firms that equally share capital and can increase or decrease capacities by a fixed amount. This case may be viewed as intermediate case between the environment with a monopolist and our base case.[22] In the following proposition, we derive the expression for the firm value and systematic risk.

Proposition 2. *The solution to firm value for the monopolist is provided in the Appendix*

[22]Note that because we model expansion by a fixed amount, this case is not fully isomorphic to the case of perfect competition when the number of competitors N is very large.

(76). The monopolist's factor beta is

$$\beta^M(z_M) = 1 - \frac{(1-c_2)\,B_M z_M^{c_2}}{V} + \frac{(c_1-1)\,A_M z_M^{c_1}}{V} - \frac{V-V_M}{V}, \tag{48}$$

and beta for symmetric oligopoly is given by

$$\beta^K(z_K) \;=\; 1 + \frac{(c_1-1)\,C_M z_K^{c_1}}{V} - \frac{(1-c_2)\,D_M z_K^{c_2}}{V} - \frac{V-V_K}{V}, \tag{49}$$

where the constants, $c_1 > 1$, $c_2 < 0$, z_K and z_M are analogous to y_K in the perfect competition case, and constants A_M, B_M, C_M, and D_M are given in the Appendix.

Compare the expressions for beta of a monopoly to expression (39) for the case of perfectly competitive sector. In both cases, the disinvestment in the low-profitability states has a negative effect on the firm's exposure to systematic risk (note the second term in (48)), albeit for different reasons. The monopolist decreases the total sector capacity following a series of negative profitability shocks and thereby reduces its sensitivity to the adverse shock. In case of the perfect competition, the reduced risk exposure in the low-profitability states comes mainly from the reduced production by competitors and their aggregate effect on the price of output.

Figure 8 illustrates the solution for firm value and beta of the monopolist and highlights differences with the base case. In contrast to the perfect competition case described earlier, the beta of monopolist is higher near the expansion threshold (the third term in (48)). This is because a monopolist has an exclusive right to exercise its investment option, and this option increases the sensitivity of the firm value to the systematic shock. Therefore, the factor beta of a monopolist increases near the expansion threshold until the firm exercises the option; immediately following the expansion, beta falls. Additionally, Figure 8 shows that the monopolistic beta is gradually decreasing as the firm is approaching the disinvestment threshold. When the firm decreases its capacity, beta increases again. Overall, the changes in the firm value sensitivity for the monopolist are driven by the value of the investment options. This is in contrast to the perfect competition case, where changes in systematic risk exposure are dictated by the actions of firm competitors. Figure 8 contrasts the dynamic behavior of beta in the case of monopoly with the case of duopoly. In the oligopolistic setting, firms choose to expand more aggressively, which has a feedback effect on their optimal exercise timing. The investment options are less valuable and the firm systematic risk increases prior to exercise for small N (similar to monopolist case). As the number of competitiors increases, the option effect is dominated by the negative externality from the competitors exercising their own options; therefore, beta decreases for large N (similar to the perfect competition case). Finally, Figure 8 shows the case with idiosyncractic shock. Idiosyncratic risk, modeled as additive component in our framework, makes beta insignificant when systematic shock decreases.

III. Empirical Analysis

We now empirically test the main predictions of the model, which are the following: (1) risk factor exposure decreases in very good and bad industry conditions; (2) betas are lower with competition, particularly when industry conditions are either very good or very bad; and (3) size-related asset pricing anomalies are less pronounced with competition.

A. Data Sources and Sample Construction

Our firm and stock return data come from the merged CRSP-Compustat file. We define industries by the four-digit Standard Industrial Classification (SIC) code. Following Hoberg and Phillips (2010), we obtain the initial SIC codes from Compustat because they are generally deemed more accurate than those from CRSP. Since SIC codes ending in zero or nine mask the broader three-digit classification, we follow Bustamante and Donangelo (2017) and replace those by the corresponding SIC code of the firm's segment with the highest sales from the Compustat segment database. Firm-years with SIC codes still ending in zero or nine after the adjustment are deleted from the sample. We further exclude firms that operate in more than three segments. When Compustat SIC codes are missing, we rely on the SIC codes from CRSP. Finally, we exclude financial industries (SIC codes 6000 to 6999) and regulated industries (4900 to 4999).

We follow Fama and French (1993) and match stock return data between July of year t and June of year $t+1$ to the annual report information from the last fiscal year end in year

$t-1$ to ensure that all used accounting data are publicly available. We restrict our universe to common stocks (*shrcd* 10 or 11) that trade on NYSE, AMEX, or NASDAQ (*exchcd* 1, 2, or 3). Monthly returns are adjusted for possible delistings, as proposed by Shumway (1997).

To construct a proxy for the intensity of product market competition, we use industry input or output dependence. To measure the dependence on specific input factors, we use the benchmark input-output (I-O) accounts that have been published by the Bureau of Economic Analysis (BEA) every five years since 1947. These accounts show how industries provide input to, and use output from, each other to produce gross domestic product. The information is provided at two levels of aggregation that differ by the coarseness of industry definitions. We use the summary-level data, which contain, depending on the year of the report, between 77 and 126 industries. Starting in 1972, I-O data are reported in MAKE and USE tables, which show, respectively, the output and input commodities for each industry. We therefore use the I-O reports from years 1972, 1977, 1982, 1987, 1992, 1997, 2002, and the last available report from 2007.

Following Ahern and Harford (2014) and Ahern (2013), we create a matrix of industry-to-industry dollar flows. We first normalize the MAKE table so that total production of each commodity sums to one. An entry in the table then represents the percentage of a commodity produced by a given industry. Multiplying this normalized MAKE table by the USE table produces a matrix of dollar flows from the customer industries (columns) to the supplier industries (rows). For each customer industry j, we measure input dependence by the Herfindahl index

$$HHI_i = \sum_i \left(\frac{f_{ij}}{\sum_i f_{ij}} \right)^2, \tag{50}$$

where f_{ij} are the dollar flows between the supplier industry i and the customer industry j.

For illustration purposes, in Table I we display the industries with lowest and highest input dependence based on the 2002 BEA report. The industries that are highly dependent on a particular factor are dominated by commodity producers such as forestry and logging, natural gas distribution, and petroleum and coal products manufacturing. In contrast, service industries tend to have low input dependence.

For completeness, in Tables II and III we also display the industries with the lowest and highest output dependance and input-output dependance, which is the average of input and output dependence. Overall, there is some overlap in the top and bottom lists of industries by input and output dependence, which is not surprising given the positive correlation of 0.20 between the two measures.

We next merge the input and output dependence data in year t to firm-level accounting data from year t to $t+4$, and stock return data from July in year $t+1$ to June of year $t+6$ using four-digit SIC codes (1972–1996) and six-digit NAICS codes from COMPUSTAT (1997–2011). Since firms with identical NAICS in Compustat can have different SIC codes, we assign each NAICS code to the most frequently appearing SIC industry in the respective sample year.[23]

B. Oil Price Exposure

We start our empirical analysis by testing whether there is an inverse U-shaped relation between factor risk and industry conditions. For this exercise, we require a well-defined factor that is a major driver of prices for a subset of assets. Further, the economic state of the factor needs to be easily observable. For this analysis, we use oil prices and the subset of stocks in industries that have a positive exposure to oil price risk.

We obtain monthly oil prices and the consumer price index (CPI) from Federal Reserve Economic Data (FRED), and deflate oil prices by the CPI. The return on oil is defined as the relative change in prices of this deflated series from one month to the next. Panel A of Figure 9 shows the time-series. The 1973 oil crisis occurs at the very beginning of our sample, where adjusted prices jump from about 20 to nearly 60. Importantly, over our entire sample, the deflated oil price appears to be stationary with only two big, decade long swings.

Rather than selecting industries that might have a large oil price exposure based on our

[23]In general, there is no one-to-one match between I-O industry definitions and SIC or NAICS codes. Typically, an I-O industry corresponds to several SIC/NAICS codes, and we assign the same input dependence to all firms with the corresponding SIC/NAICS codes. In the few cases where an SIC or NAICS code corresponds to multiple I-O industries, we average the input dependence. We also merge I-O industries 11 and 12 (years 1963, 1967, 1972, and 1977) and 2301, 2302, and 2303 (years 1997, 2002, and 2007) before computing input dependence because their matched SIC/NAICS codes make them indistinguishable from each other.

priors, we regress industry excess returns onto market excess returns and oil returns. After we require industries to have at least 120 monthly return observations in our data set, 52 industries have a significantly positive oil exposure. While not all of these 52 industries are actually oil producers, and there is room for statistical errors, the five industries with the highest t-statistics are Crude Petroleum and Natural Gas (SIC code 1311), Drilling Oil and Gas Wells (1381), Oil and Gas Field Machinery and Equipment (3533), Oil and Gas Field Exploration Services (1382), and Petroleum Refining (2911).

Our test asset is the value-weighted average of all firms in the 52 industries. By construction, this test asset will have a large exposure to oil prices. However, we are interested in how this exposure changes with factor realizations. Our model predicts that factor exposure declines in bad times, when oil producers scale down production, and in good times, when the sector as a whole expands. To test for this possible non-linearity, we model the conditional oil beta of our test asset as a quadratic function

$$\beta_t^{OIL} = \gamma_0 + \gamma_1 OIL_{t-1} + \gamma_2 OIL_{t-1}^2, \tag{51}$$

where OIL_{t-1} is the average oil price over the preceding 12-month period, scaled to be between zero and one. The γ coefficients are estimated from a regression of excess returns of the test portfolio onto excess market returns (R_t^M), oil returns (R_t^{OIL}), as well as the two interaction terms between lagged oil prices and oil returns:

$$R_t = \alpha + \beta^M R_t^M + \gamma_0 R_t^{OIL} + \gamma_1 OIL_{t-1} R_t^{OIL} + \gamma_2 OIL_{t-1}^2 R_t^{OIL} + \varepsilon_t. \tag{52}$$

In this regression, the coefficients β^M (0.97, t = 22.25), γ_1 (0.85, t = 2.56), and γ_2 (-0.86, t = -3.04) are all significantly different from zero. The coefficient γ_0, which is large and significant when the interaction terms are omitted (0.09, t = 2.31), is insignificant (-0.01, t = -0.09).

The resulting oil beta is plotted in Panel B of Figure 9. The beta is somewhat smoother than the oil price in Panel A since the conditioning variable is smoothed over 12 months. While the conditional beta is clearly correlated with oil prices, the most striking feature of the data is that it drops to zero three times in the sample: When oil prices are really low

(1973, 1998) or high (2007). This is highlighted in Figure 10, which plots the conditional oil beta against the lagged oil price. Not only does the negative estimate γ_2 cause concavity in this relation, but oil betas decrease nearly exactly to zero when oil prices are either very low or very high. Just as our model predicts, oil producing firms therefore are most exposed to the oil risk at the intermediate levels of the factor realization, but competition erodes this exposure when the sector contracts or expands.

C. Input and Output Dependence and Betas

We want to extend the intuition from the oil sector studied above to other industries that might be exposed to different factors. As such, our measure of competition cannot be based on the number or concentration of firms in the industry alone, especially since Hoberg and Phillips (2016) show that firms compete in multiple products and may consider only a few firms as relevant rivals in their industry. Rather than relying on competition in the product market, we propose a measure of competition in input or output factors. To construct input dependence measure, we use input-output (I-O) accounts and analyze from which industries a firm obtains its production input factors. Our reasoning is that firms operating in industries that use a diverse set of inputs are either heterogeneous (e.g., because they produce differentiated products), or have the ability to substitute between different input factors. Therefore, we expect that industries with higher input dependence (relying on one factor) are more competitive than industries with lower input dependence (relying on a diverse set of factors). Similarly, it can be argued that a firm that supplies its products to a diverse set of industries can substitute between these industries and therefore faces lower competition in the product market. As such, we expect betas to be lower among firms with lower input or output dependence.

We are interested in systematic factor risk, but since we cannot identify the risk factors for the universe of stocks as cleanly as in the oil sector, we perform our test on market beta as a stand-in for the total of systematic risks. Market betas are estimated at the firm level using 12 monthly observations between July of year t and June of year $t + 1$, account for non-synchronous trading with one Dimson (1979) lag, and are value weighted.

Panel A of Table IV shows market betas for four groups with increasing input dependence. Market betas decline with input dependence, from 1.09 in the low competition group, to 0.91 in the high competition group. The difference of -0.17 is statistically significant. Consistent with our model's prediction, market betas decrease with competition.

Panel B shows that systematic risk not only decreases with competition, but also that it decreases by more in bad or good economic states. Specifically, stocks are sorted first into four groups by input dependence, and then are sorted into four groups by the last year's industry asset growth. Looking at the last column suggests that competition decreases market betas by 0.24 in times of low investment (contraction) and 0.16 in times of high investment (expansion). In intermediate times, the effect of competition on beta is much more modest (0.06 and 0.10). We find similar patterns when instead of the input dependence we use output dependence or input-output dependence measures of competition. We report the results in Table V and Table VI.

D. Competition and Size Related Anomalies

The last prediction of our model concerns the magnitude of size related anomalies. As competition lowers all systematic risk, it also affects the dispersion of systematic risk, which drives anomalies in our model. We therefore expect anomalies to be less pronounced among firms with higher input dependence.

We empirically test this prediction in Table VII. We first sort all firms in the sample into four groups by input dependence and then, within each input dependence quartile, sort firms into four groups by their market capitalizations (left side) and book-to-market ratios (right side). We report in the tables the time-series averages of value-weighted monthly returns (Panel A) and four-factor alphas (Panel B) for each of the 16 portfolios, as well as for portfolios long in the top quartile and short in the bottom quartile.

The table shows that average portfolio returns generally decrease with firm size within each first three input dependence groups. The magnitude of the size effect, however, declines sharply with input dependence. For example, among firms with low input dependence, i.e., with low degree of product market competition, small stocks outperform large stocks by

0.73% per month, while the effect is only 0.42% and 0.37% per month in the second and third quartile. The size premium disappears completely in the high input dependence quartile. The difference between these size premiums is both economically large and statistically significant with a t-statistic of 3.56.

The evidence on the value premium on the right side mirrors the evidence on the size premium. In particular, value firms outperform growth firms for each input dependence group. However, the value premium is largest for firms with low input dependence. Specifically, the value premium for stocks with low input dependence is 0.88%, and is significantly smaller at 0.38% for stocks with high input dependence.

Large differences in returns are not surprising if they can be explained by exposure to risk factors. Therefore, Panel B reports alphas from the Fama and French (1993) and Carhart (1997) four-factor model. Adjustment for these factors has little effect on the impact of competition. The alphas of the size premium still decrease by 0.70% per month when moving from low to high competition, and the value premium decreases by 0.41%.

Panel C shows that the differences in size and value premiums cannot be attributed to differences in the levels of average market capitalization or book-to-market ratios across input dependence quartiles. Overall, as predicted by the theory, we find that both size and value anomalies are weaker in industries with high input dependence, or more intense competition.

IV. Conclusion

In this paper, we study how strategic interaction among firms affects their systematic risk. Competition effectively bounds profitability within each economic sector because competing firms simultaneously contract or expand production in response to systematic demand shocks. We build a model of competition in different economic sectors and show that no arbitrage implies that sensitivity of firm value to systematic factors must decrease to zero near the contraction and expansion thresholds. Therefore, for a given level of competition, the model predicts a bell-shaped relation between systematic risk and the underlying industry demand. Guided by theory, we test this prediction using a set of firms from the oil sector and find supporting evidence.

Our model also predicts that more competitive industries have lower systematic risk and that competition attenuates size-related cross-sectional anomalies. Using trade flows between economic sectors from the input-output accounts, we construct a new measure of competition based on each sector's dependence on input factors. We confirm that market betas decline with competition. Size and value anomalies are strong in non-competitive industries. In competitive industries, the value anomaly is reduced by more than half, and the size anomaly disappears.

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Appendix

A. Notational Key

q_{ik}	Firm i size
Q_k	Aggregate capital in sector k
δ	Depreciation rate of capital Q_k
$1/\varepsilon$	Price elasticity of demand
Y_k	Systematic shock in sector k
y_k	Adjusted profitability shock in sector k, $Y_k Q_k^{-\varepsilon}$
x_i	Idiosyncratic shock of firm i
c_i	Operating leverage or financial leverage, of firm i
$\overline{R}_k$	Purchase price of a unit of capital in sector k
$\underline{R}_k$	Sale price of a unit of capital in sector k
$\overline{y}_k$	Optimal expansion threshold in sector k
$\underline{y}_k$	Optimal contraction threshold in sector k
V_i	Market value of firm i
V_i^k	Component of firm i's market value derived from sector k
V_i^x	Market value of assets associated with idiosyncratic profitability
V_L	A component of firm value associated with the effect of the contraction
V_U	A component of firm value associated with the effect of the expansion
V_N	Value of a perpetuity of cash flows y_k
Λ	Pricing kernel
θ_k	price of risk of the systematic shock Y_k

B. Firm Value and Beta

B.1. Derivation of Contraction and Expanasion Thresholds

We need to show that for any firm i the individually optimal expansion and contraction thresholds $\overline{y}_k^*$ and $\underline{y}_k^*$ are equal to the rationally anticipated thresholds for all other firms, i.e., $\overline{y}_k = \overline{y}_k^*$, $\underline{y}_k = \underline{y}_k^*$. The fixed point of the process determines the equilibrium within each sector. Just as the firm value itself, the solution for the option value has to satisfy equation (20), but without the dividend term because the option pays nothing until it is exercised. Hence, the solution V_{opt} is

$$V_{opt}(y_k) = A_0 y_k^{b_1} + B_0 y_k^{b_2}, \tag{53}$$

where the first component is the activation value at an upper barrier and the second component is due to the lower reflecting barrier. The value is subject to the following boundary conditions:

$$V_{opt}(\overline{y}_k) = V_i^k(\overline{y}_k) - q_{ik}\overline{R}_k, \tag{54}$$

$$V'_{opt}(\overline{y}_k) = V_i^{k\prime}(\overline{y}_k), \tag{55}$$

where $V_i^k(.)$ is the value of the active asset in sector k, which we derived in (29). The first condition says that the value of the option at exercise is equal to the present value of the incremental profit, minus the cost of the exercise. Similarly, at the low barrier, we have two conditions

$$V'_{opt}\left(\underline{y}_k\right) = 0, \tag{56}$$

$$V_{opt}\left(\underline{y}_k\right) = V_i^k\left(\underline{y}_k\right) - q_{ik}\underline{R}_k. \tag{57}$$

These four conditions pin down two constants for the inactive firm value, A_0 and B_0, and also the thresholds $\overline{y}_k$ and $\underline{y}_k$.This system of equations is identical to the system of equation for the optimal barriers in (35)-(36).

B.2. Derivation of Beta

Proof of Proposition 1. To obtain the market value of firm i, use (18), (19), and (29)

$$V_i(x_i, y_1, ..., y_K) = V_i^x(x_i) + \sum_{k=1}^{K} q_{ik}\left(V_N(y_k) - V_U(y_k) + V_L(y_k)\right), \tag{58}$$

where the idiosyncratic component of firm value is given by

$$V_i^x(x_i) = \frac{x_i}{\overline{r} - \mu} - \frac{C_i}{r}, \tag{59}$$

functions $V_N(y_k)$, $V_U(y_k)$, and $V_L(y_k)$ are given by (30)-(32) and exponents $b_1 > 1$ and $b_2 < 0$ solve

$$b^2\sigma_k^2 + b\left(2\overline{\mu}_k - \sigma_k^2\right) - 2\overline{r} = 0. \tag{60}$$

Using definition (38), the factor beta can be obtained by differentiation

$$\beta_i^k = 1 - q_{ik}(b_1 - 1)\frac{V_U}{V_i} - q_{ik}(1 - b_2)\frac{V_L}{V_i} - \frac{V_i - V_i^k}{V_i}, \tag{61}$$

where V_i^k is given by (29). □

B.3. Relation between Factor and Market Betas

The analysis in the text is based on factor betas,

$$\beta_i^k = \frac{\partial V_i / V_i}{\partial Y_k / Y_k}, \tag{62}$$

where k indexes sectors and i firms in each sector. The relation between the factor and "market beta" can be recovered from the condition that the sum of values of all firms forms the market portfolio. Starting from a linear projection of stock returns on the priced factor,

$$R_i = \alpha_i + \sum_{k=1}^{K} \beta_i^k Y_k + \varepsilon_i, \tag{63}$$

it is straightforward to show that

$$\beta_i^M = \sum_{k=1}^{K} \beta_i^k \beta_k^M, \tag{64}$$

where β_k^M measures the correlation between the factor and market,

$$\beta_k^M = \frac{\partial Y_k / Y_k}{\partial M / M}. \tag{65}$$

Aggregating the values of N stocks, we obtain

$$M = V^x + \sum_{i=1}^{N} \sum_{k=1}^{K} q_{ik}\left(V_N^i(Y_k) - V_U^i(Y_k) + V_L^i(Y_k)\right), \tag{66}$$

and thus

$$\beta_k^M = \frac{M}{\sum_{i=1}^N q_{ik} \left(V_N\left(Y_k\right) - b_1 V_U\left(Y_k\right) + b_2 V_L\left(Y_k\right)\right)} \tag{67}$$

Using the definition of β_i^k, we can write

$$\beta_i^k \beta_k^M = \frac{q_{ik}}{V_i} \frac{\sum_{i=1}^N V_i}{\sum_{i=1}^N q_{ik}}. \tag{68}$$

Then the market beta is

$$\beta_i^M = \sum_{k=1}^K \beta_i^k \beta_k^M = \sum_{k=1}^K \left(\frac{\Sigma_i V_i}{V_i} \frac{q_{ik}}{\Sigma_i q_{ik}} \right), \tag{69}$$

By construction, when there is only one common shock, the weighted sum of market betas is equal to one, $\Sigma_i V_i \beta_i^M / \Sigma_i V_i = 1$.

Proof of Corollary 1. The claim follows directly by evaluating β_i^k in Proposition 1 at the high and low reflecting barriers. □

Proof of Corollary 2. The value function $V_i^k\left(y_k\right)$ in (29) is monotonically increasing in sector profitability y_k. Therefore, it follows from condition (33) that the upper threshold barrier increases with the purchase price of capital, i.e., $\partial \overline{y}_k / \partial \overline{R}_k > 0$. As the upper threshold barrier increases, the term in firm value responsible for it, V_U, decreases. Hence, it follows from (39) that

$$\frac{\partial \beta_i^k}{\partial \overline{R}_k} > 0. \tag{70}$$

A mirror argument with respect to the sale price of capital $\underline{R}_k$ and condition (34) shows that $\partial \underline{y}_k / \partial \underline{R}_k > 0$. As the lower threshold barrier increases with the higher sale price of capital, the term in firm value responsible for it, V_L, increases. Hence, it follows from (39) that

$$\frac{\partial \beta_i^k}{\partial \underline{R}_k} < 0. \tag{71}$$

□

Proof of Corollary 4. To establish the first part of the claim, examine expression (39) and note that the idiosyncratic shock x_i enters only through the value of the firm in the denominator. Therefore, whenever β_i^k is positive, it must decrease with idiosyncratic risk. To establish the second part of the claim, explicitly differentiate β_i^k with respect to the idiosyncratic shock x_i using (39)

$$\frac{\partial \beta_i^k}{\partial x_i} = -\frac{q_{ik}\left(V_N - b_1 V_U + b_2 V_L\right)}{V_i^2\left(x_i, y_1, ..., y_K\right)\left(\overline{r} - \mu\right)}. \tag{72}$$

Rewriting this expression, we obtain

$$\frac{\partial \beta_i^k}{\partial x_i} = \left(-\frac{V_i^k}{V_i^2} + \frac{q_{ik}\left(b_1 - 1\right)V_U + q_{ik}\left(1 - b_2\right)V_L}{V_i^2}\right)\frac{1}{\overline{r} - \mu}, \tag{73}$$

where the second term in parentheses is due to competition. This term is positive and hence it reduces the negative sensitivity of factor beta to the idiosyncratic shock. □

C. Case of Monopolist

We assume that a firm is a monopolist in sector M

$$\Pi = x - c + Y_M Q_M^{1-\varepsilon} + \sum_{k \neq M} q_k Y_k Q_k^{-\varepsilon}. \tag{74}$$

Because the solution is additive, we can focus on obtaining the solution only for the value derived from operations in sector M. By denoting monopolist's profit

$$z_M = Y_M Q_M^{1-\varepsilon}, \tag{75}$$

and applying Ito's lemma, we obtain the ODE for the component of firm value derived from sector M

$$rV_M\left(z_M\right) = z_M + \left(\overline{\mu}_M - \delta\right) z_M \frac{\partial V_M\left(z_M\right)}{\partial z_M} + \frac{\sigma_M^2 z_M^2}{2}\frac{\partial^2 V_M\left(z_M\right)}{\partial z_M^2}. \tag{76}$$

The candidate solution for this firm value is

$$V_M\left(z_M\right) = \frac{z_M}{\overline{r} - \overline{\mu}_M} + A_M z_M^{c_1} + B_M z_M^{c_2}, \tag{77}$$

where $c_1 > 1$, $c_2 < 0$ are the roots of the standard characteristic equation

$$c^2\sigma_M^2 + c\left(2\left(\overline{\mu}_M - \delta\right) - \sigma_M^2\right) - 2r = 0. \tag{78}$$

We assume that the option may be exercised only once, and that upon exercise of the expansion option at z^*, the firm increases its capital by $\theta^* Q_M$ at a proportional cost $\overline{R}_M$. The respective sector's cash flow increases from z_M to $z_M(1+\theta^*)^{1-\varepsilon}$. Similarly, upon exercise of the contraction option at some low value z_*, the firm sells a portion of its capital at a proportional cost $\underline{R}_M$ and thus decreases the cash flow from z_M to $z_M(1+\theta_*)^{1-\varepsilon}$, where θ_* is negative.

Proof of Proposition 2. The solution always exists and is unique given fixed θ_* and θ^*. For generality, we give the full solution with optimal θ_* and θ^*. Immediately following the exercise, the continuation value of cash flows, $V_R(z_M)$, is

$$V_R(z_M) = \frac{z_M(1+\theta^*)^{1-\varepsilon}}{\overline{r}-\overline{\mu}_M}. \tag{79}$$

To find the optimal expansion scale θ^*, we maximize the incremental value from expansion net of the costs, at the threshold z^*

$$\max_{\theta^*}\left(\frac{z^*(1+\theta^*)^{1-\varepsilon}}{\overline{r}-\overline{\mu}_M} - Q_M\overline{R}_M\theta^*\right), \tag{80}$$

which yields

$$1+\theta^* = \left(\frac{z^*(1-\varepsilon)}{Q_M\overline{R}_M(\overline{r}-\overline{\mu}_M)}\right)^{\frac{1}{\varepsilon}}. \tag{81}$$

Similarly, after the exercise of the option to decrease capital, the continuation value is

$$V_L(z_M) = \frac{z_M(1+\theta_*)^{1-\varepsilon}}{\overline{r}-\overline{\mu}_M}. \tag{82}$$

The maximization with respect to θ_* at the contraction threshold z_* yields the optimal contraction scale $\theta_* < 0$

$$1+\theta_* = \left(\frac{z_*(1-\varepsilon)}{Q_M\underline{R}_M(\overline{r}-\overline{\mu}_M)}\right)^{\frac{1}{\varepsilon}}. \tag{83}$$

Then the value and slope matching conditions pin down the optimal expansion and contraction thresholds and free constants

$$V_M(z^*) = V_R(z^*) - Q_M\overline{R}_M\theta^*, \tag{84}$$

$$V_M'(z^*) = V_R'(z^*), \tag{85}$$

$$V_M(z_*) = V_L(z_*) - Q_M\underline{R}_M\theta_*, \tag{86}$$

$$V_M'(z_*) = V_L'(z_*). \tag{87}$$

Solving these equations for constants A_M and B_M, we obtain

$$A_M = \frac{\frac{z^{*c_2}(1+\varepsilon\theta_*)Q_M\underline{R}_M}{1-\varepsilon} - \frac{z_*^{c_2}(1+\varepsilon\theta^*)Q_M\overline{R}_M}{1-\varepsilon} + \frac{z_*^{c_2}z^* - z^{*c_2}z_*}{\overline{r}-\overline{\mu}_M}}{z_*^{c_1}z^{*c_2} - z^{*c_1}z_*^{c_2}}, \tag{88}$$

$$B_M = \frac{\frac{z_*^{c_1}(1+\varepsilon\theta^*)Q_M\overline{R}_M}{1-\varepsilon} - \frac{z^{*c_1}(1+\varepsilon\theta_*)Q_M\underline{R}_M}{1-\varepsilon} + \frac{z^{*c_1}z_* - z_*^{c_1}z^*}{\overline{r}-\overline{\mu}_M}}{z_*^{c_1}z^{*c_2} - z^{*c_1}z_*^{c_2}}, \tag{89}$$

where the optimal expansion and contraction parameters, θ^* and θ_*, are given in (81) and (83). The optimal exercise thresholds z_* and z^* can be obtained as a solution of the system of two non-linear equations

$$\frac{z^{*c_2}\underline{R}_M - z_*^{c_2}\overline{R}_M}{1-c_1\varepsilon} = \frac{z^{*c_2}\underline{R}_M\theta_* - z_*^{c_2}\overline{R}_M\theta^*}{c_1-1} - \frac{\left(z_*^{c_2}z^* - z^{*c_2}z_*\right)(1-\varepsilon)}{(1-c_1\varepsilon)\,Q_M\left(\overline{r}-\overline{\mu}_M\right)}, \tag{90}$$

$$\frac{z^{*c_1}\underline{R}_M - z_*^{c_1}\overline{R}_M}{1-c_2\varepsilon} = \frac{z_*^{c_1}\overline{R}_M\theta^* - z^{*c_1}\underline{R}_M\theta_*}{1-c_2} + \frac{\left(z^{*c_1}z_* - z_*^{c_1}z^*\right)(1-\varepsilon)}{(1-c_2\varepsilon)\,Q_M\left(\overline{r}-\overline{\mu}_M\right)}. \tag{91}$$

The factor beta of the monopolist firm operating in sector K is obtained by differentiation of firm value (77)

$$\beta_M^K = \frac{\partial V}{dY_K}\frac{Y_K}{V} = \frac{\frac{z_K}{\overline{r}-\overline{\mu}_K} + c_1A_Mz_K^{c_1} + c_2B_Mz_K^{c_2}}{V} \tag{92}$$

$$= 1 + \frac{(c_1-1)\,A_Mz_K^{c_1}}{V} - \frac{(1-c_2)\,B_Mz_K^{c_2}}{V} - \frac{V-V_M}{V}. \tag{93}$$

Similarly, in the left and right regions, where the option to increase or decrease production scale has already been exercised, the factor beta is obtained by differentiation of expressions (79) and (82), respectively

$$\beta_R^K = \frac{\frac{z_K(1+\theta^*)^{1-\varepsilon}}{\overline{r}-\overline{\mu}_K}}{V} = 1 - \frac{V-V_R}{V}, \tag{94}$$

$$\beta_L^K = \frac{\frac{z_K(1-\theta_*)^{1-\varepsilon}}{\overline{r}-\overline{\mu}_K}}{V} = 1 - \frac{V-V_L}{V}. \tag{95}$$

□

D. Symmetric Cournot-Nash Equilibrium with N Firms

Here we provide the treatment for the N-firm symmetric oligopoly, with each firm sharing $1/N$ of the total sector capital

$$q_{iK} = \frac{Q_K}{N}, \tag{96}$$

As in the main specification, we will construct the the profit flow that consists of the firm-specific component, operating costs or leverage, the profits generated from operations in "other" competitive sectors, and finally the profit from the share in the oligopolistic sector K

$$\Pi_i = x_i - c_i + \sum_{k=1}^{K-1} q_{ik} Y_k Q_k^{-\varepsilon} + \frac{Q_k}{N} Y_K Q_K^{-\varepsilon}. \tag{97}$$

By denoting the per-unit-of-capital value in sector K as

$$z_K = \frac{Y_K Q_K^{1-\varepsilon}}{N}, \tag{98}$$

we can write the valuation ODE for the component of firm value derived from sector K,

$$rV^K(z_K) = z_K + (\overline{\mu}_K - \delta) z_K \frac{\partial V^K}{\partial z_K} + \frac{\sigma_K^2 z_K^2}{2} \frac{\partial^2 V^K}{\partial z_K^2}. \tag{99}$$

Similar to the monopolist case, we assume that each of the competitors can increase capital from $\frac{Q_K}{N}$ to $(1+\theta_i^*)\frac{Q_K}{N}$ at a cost $\frac{Q_K \overline{R}_K \theta_i^*}{N}$ at some threshold z^*. In the middle region, where no options are yet exercised, the candidate solution for the firm value is

$$V_M^K(z_K) = \frac{z_K}{\overline{r} - \overline{\mu}_K} + C_M z_K^{c_1} + D_M z_K^{c_2}, \tag{100}$$

where the exponents c_1 and c_2 are the same as provided for the monopolist's firm value solution.

The individual investment/disinvestment strategy must be an accord with the strategies of other firms. We therefore build the Cournot-Nash equilibrium as follows. We first conjecture that all other $N-1$ firms in sector K expand at the same threshold z^* by a factor $(1+\theta_{-i}^*)$. Then, upon the exercise, the cash flow of firm i changes from z_K to

$$z_K(1+\theta_i^*)\left(\frac{\theta_i^* - \theta_{-i}^*}{N} + 1 + \theta_{-i}^*\right)^{-\varepsilon}. \tag{101}$$

Further, upon exercise of the contraction option at z_* (at a cost $\frac{Q_K \underline{R}_K \theta_{i*}}{N}$) the capacity is decreased from Q_k to $(1+\theta_{i*})Q_K$ so that the cash flow changes from z_K to

$$z_K(1+\theta_{i*})\left(\frac{\theta_{i*} - \theta_{-i*}}{N} + 1 + \theta_{-i*}\right)^{-\varepsilon}. \tag{102}$$

We next solve for the optimal thresholds. Upon the irreversible expansion, the firm continuation value is

$$V_R^K(z_K) = \frac{z_K(1+\theta_{i*})}{\overline{r}-\overline{\mu}_K}\left(\frac{\theta_{i*}-\theta_{-i*}}{N}+1+\theta_{-i*}\right)^{-\varepsilon}. \tag{103}$$

To find the optimal expansion scale θ_i^*, we maximize the value from expansion net of costs at the threshold z^*

$$\theta_i^* = \max_{\theta_i^*}\left(\frac{z^*(1+\theta_i^*)\left(\frac{\theta_i^*-\theta_{-i}^*}{N}+1+\theta_{-i}^*\right)^{-\varepsilon}}{\overline{r}-\overline{\mu}_K} - \frac{Q_K\overline{R}_K\theta_i^*}{N}\right), \tag{104}$$

which gives the first order condition for the optimal expansion scale

$$1-\frac{\frac{\varepsilon}{N}(1+\theta_i^*)}{\frac{\theta_i^*-\theta_{-i}^*}{N}+1+\theta_{-i}^*} = \frac{Q_K\overline{R}_K(\overline{r}-\overline{\mu}_K)}{z^*N}\left(\frac{\theta_i^*-\theta_{-i}^*}{N}+1+\theta_{-i}^*\right)^{\varepsilon}. \tag{105}$$

Since all firms are identical, it must be that in equilibrium $\theta_{-i}^* = \theta_i^*$. Substituting the last expression in the first order condition and solving for the optimal expansion constant yields

$$1+\theta_i^* = \left(\frac{(N-\varepsilon)z^*}{Q_K\overline{R}_K(\overline{r}-\overline{\mu}_K)}\right)^{\frac{1}{\varepsilon}}. \tag{106}$$

By comparing the expression for the optimal expansion of a firm operating in the oligopolistic sector with the one for the monopolist, it is easy to see that as number of firms N increases, firms expand by a larger amount as $\frac{\partial\theta_i^*}{\partial N} > 0$. Similarly, after the option to decrease capacity, the continuation value is

$$V_L^K(z_K) = \frac{z_K(1+\theta_{i*})\left(\frac{\theta_{i*}-\theta_{-i*}}{N}+1+\theta_{-i*}\right)^{-\varepsilon}}{\overline{r}-\overline{\mu}_K}. \tag{107}$$

Maximization with respect to θ_{i*} at the disinvestment threshold z_* and use of the equilibrium condition $\theta_{-i_*} = \theta_{i*}$ yield the optimal contraction scale θ_{i*}

$$1+\theta_{i*} = \left(\frac{(N-\varepsilon)z_*}{Q_K\underline{R}_K(\overline{r}-\overline{\mu}_K)}\right)^{\frac{1}{\varepsilon}}. \tag{108}$$

The value and slope matching conditions pin down the optimal expansion and contraction thresholds and free constants

$$
\begin{aligned}
V_M\left(z^*\right) &= V_R\left(z^*\right) - \frac{Q_K \overline{R}_K \theta_i^*}{N}, && (109)\\
V_M'\left(z^*\right) &= V_R'\left(z^*\right), && (110)\\
V_M\left(z_*\right) &= V_L\left(z_*\right) - \frac{Q_K \underline{R}_K \theta_{i*}}{N}, && (111)\\
V_M'\left(z_*\right) &= V_L'\left(z_*\right). && (112)
\end{aligned}
$$

Solving these equations for constants C_M and D_M, we obtain

$$
\begin{aligned}
C_M &= \frac{\frac{z^{*c_2} z_* \left((1+\theta_{i*})^{1-\varepsilon}-1\right)}{\overline{r}-\overline{\mu}_K} - \frac{z_*^{c_2} z^* \left((1+\theta_i^*)^{1-\varepsilon}-1\right)}{\overline{r}-\overline{\mu}_K} + \frac{Q_K \overline{R}_K z_*^{c_2} \theta_i^*}{N} - \frac{Q_K \underline{R}_K z^{*c_2} \theta_{i*}}{N}}{z^{*c_2} z_*^{c_1} - z_*^{c_2} z^{*c_1}}, && (113)\\
D_M &= \frac{\frac{z_*^{c_1} z^* \left((1+\theta_i^*)^{1-\varepsilon}-1\right)}{\overline{r}-\overline{\mu}_K} - \frac{z^{*c_1} z_* \left((1+\theta_{i*})^{1-\varepsilon}-1\right)}{\overline{r}-\overline{\mu}_K} + \frac{Q_K \underline{R}_K z^{*c_1} \theta_{i*}}{N} - \frac{Q_k \overline{R}_K z_*^{c_1} \theta_i^*}{N}}{z_*^{c_1} z^{*c_2} - z^{*c_1} z_*^{c_2}}. && (114)
\end{aligned}
$$

where optimal θ_i^* and θ_{i*} can be substituted from (106) and (108). The optimal exercise thresholds z_* and z^* solve the system of two non-linear equations

$$
\begin{aligned}
\frac{c_2}{1-c_2}\frac{Q_K}{N} &= \frac{z_*^{c_1} z^* \left((1+\theta_i^*)^{1-\varepsilon}-1\right) - z^{*c_1} z_* \left((1+\theta_{i*})^{1-\varepsilon}-1\right)}{\left(z^{*c_1} \underline{R}_K \theta_{i*} - z_*^{c_1} \overline{R}_K \theta_i^*\right)\left(\overline{r}-\overline{\mu}_K\right)}, && (115)\\
\frac{c_1}{1-c_1}\frac{Q_K}{N} &= \frac{z^{*c_2} z_* \left((1+\theta_{i*})^{1-\varepsilon}-1\right) - z_*^{c_2} z^* \left((1+\theta_i^*)^{1-\varepsilon}-1\right)}{\left(z_*^{c_2} \overline{R}_K \theta_i^* - z^{*c_2} \underline{R}_K \theta_{i*}\right)\left(\overline{r}-\overline{\mu}_K\right)}. && (116)
\end{aligned}
$$

This system can be solved numerically for the thresholds z_* and z^*.

Finally, using (97) and (99), we show that firm beta is given by

$$
\beta^K(z_K) = 1 + \frac{(c_1-1)\, C_M z_K^{c_1}}{V} - \frac{(1-c_2)\, D_M z_K^{c_2}}{V} - \frac{V - V_K}{V}. \qquad (117)
$$

Table I
Industries with Lowest and Highest Input Dependence in 2002

This table illustrates the top and bottom ten industries by input dependence based on the Bureau of Economic Analysis report for 2002. Input dependence is measured as a Herfindahl index of dollar flows to supplier industries.

Rank	Industry	Input Dependence
1	Electronic, commercial, and household goods repair	0.030
2	Scientific research and development services	0.034
3	Travel arrangement and reservation services	0.036
4	Other professional, scientific, and technical services	0.037
5	All other administrative and support services	0.038
6	Commercial and service industry machinery manufacturing	0.038
7	Internet service providers, web search portals, and data processing	0.039
8	Other miscellaneous manufacturing	0.040
9	Management of companies and enterprises	0.040
10	Personal and laundry services	0.040
116	Pharmaceutical and medicine manufacturing	0.181
117	Textile product mills	0.182
118	Converted paper product manufacturing	0.231
119	Nonferrous metal production and processing	0.234
120	Motor vehicle manufacturing	0.349
121	Forestry and logging	0.351
122	Insurance carriers and related activities	0.362
123	Natural gas distribution	0.418
124	Petroleum and coal products manufacturing	0.468
125	Funds, trusts, and other financial vehicles	0.565

Table II
Industries with Lowest and Highest Output Dependence in 2002

This table illustrates the top and bottom ten industries by output dependence based on the Bureau of Economic Analysis report for 2002. Output dependence is measured as a Herfindahl index of dollar flows from customer industries.

Rank	Industry	Output Dependence
1	Air transportation	0.022
2	Wholesale trade	0.025
3	Automotive equipment rental and leasing	0.026
4	Management of companies and enterprises	0.026
5	Electronic, commercial, and household goods repair	0.028
6	Accounting, tax preparation, bookkeeping, and payroll services	0.028
7	Accommodation	0.029
8	Transit and ground passenger transportation	0.030
9	All other administrative and support services	0.030
10	Other professional, scientific, and technical services	0.030
116	Ambulatory health care services	0.425
117	Other information services	0.430
118	Funds, trusts, and other financial vehicles	0.461
119	Religious, grantmaking, giving, and social advocacy organizations	0.493
120	Oil and gas extraction	0.496
121	Support activities for agriculture and forestry	0.533
122	Coal mining	0.570
123	Animal production	0.632
124	Aerospace product and parts manufacturing	0.741
125	Tobacco manufacturing	0.788

Table III
Industries with Lowest and Highest Input-Output Factor Dependence in 2002

This table illustrates the top and bottom ten industries by input-output factor dependence based on the Bureau of Economic Analysis report for 2002. Input-output factor dependence is measured as average of the Herfindahl indices of dollar flows to supplier industries and dollar flows from customer insdustries.

Rank	Industry	Input-Output Dependence
1	Electronic, commercial, and household goods repair	0.029
2	Management of companies and enterprises	0.033
3	Other professional, scientific, and technical services	0.033
4	All other administrative and support services	0.034
5	Wholesale trade	0.035
6	Accommodation	0.036
7	Scientific research and development services	0.037
8	Specialized design services	0.037
9	Accounting, tax preparation, bookkeeping, and payroll services	0.038
10	Internet service providers, web search portals, and data processing	0.038
116	Religious, grantmaking, giving, and social advocacy organizations	0.306
117	Oil and gas extraction	0.308
118	Coal mining	0.311
119	Support activities for agriculture and forestry	0.337
120	Motor vehicle manufacturing	0.343
121	Forestry and logging	0.357
122	Animal production	0.395
123	Tobacco manufacturing	0.446
124	Aerospace product and parts manufacturing	0.447
125	Funds, trusts, and other financial vehicles	0.513

Table IV
Market Betas and Input Dependence

This table reports in Panel A market betas of portfolios sorted by input dependence, as well as the difference in betas between high and low input dependence quartiles. In Panel B, stocks are sorted first by input dependence, then by industry asset growth. Betas are computed for each portfolio from regressions of excess stock returns on excess market returns using all monthly observations. Input dependence is computed at the industry level as the Hirshman-Herfindahl index of the dollar flows from all supplier industries. Details are provided in the main text. Industries are defined at the four-digit SIC level. t-statistics are based on Newey and West (1987) adjusted standard errors using 12 lags. The sample period is July 1973 to June 2013.

	Input Dependence				
	Low	2	3	High	H–L
A. Univariate Sorts					
	1.09	1.04	1.12	0.91	-0.17
	[31.45]	[24.32]	[39.16]	[36.48]	[-3.80]
B. Split by Industry Asset Growth					
Low	1.09	0.94	1.04	0.85	-0.24
	[20.69]	[16.80]	[17.63]	[17.10]	[-4.20]
2	0.97	0.94	0.96	0.92	-0.06
	[20.40]	[18.97]	[15.74]	[21.52]	[-0.80]
3	1.06	1.09	1.17	0.95	-0.10
	[24.54]	[28.27]	[25.66]	[16.21]	[-1.88]
High	1.34	1.25	1.24	1.18	-0.16
	[17.70]	[17.58]	[16.51]	[20.90]	[-2.23]
H–L	0.25	0.31	0.20	0.33	0.08
	[2.19]	[3.05]	[1.64]	[3.85]	[0.79]
(H+L) - (2+3)					-0.12
					[-1.86]

Table V
Market Betas and Output Dependence

This table reports in Panel A market betas of portfolios sorted by output dependence, as well as the difference in betas between high and low output dependence quartiles. In Panel B, stocks are sorted first by input dependence, then by industry asset growth. Betas are computed for each portfolio from regressions of excess stock returns on excess market returns using all monthly observations. Output dependence is computed at the industry level as the Hirshman-Herfindahl index of the dollar flows from all customer industries. Details are provided in the main text. Industries are defined at the four-digit SIC level. t-statistics are based on Newey and West (1987) adjusted standard errors using 12 lags. The sample period is July 1973 to June 2013.

	Output Dependence				
	Low	2	3	High	H–L
A. Univariate Sorts					
	1.07	0.98	1.10	0.97	-0.10
	[37.27]	[23.77]	[39.24]	[32.87]	[-2.56]
B. Split by Industry Asset Growth					
Low	1.04	0.88	1.01	0.88	-0.15
	[18.32]	[19.46]	[18.22]	[15.70]	[-2.91]
2	0.90	0.94	1.06	0.91	0.01
	[19.91]	[18.99]	[23.49]	[24.19]	[0.13]
3	1.11	1.09	1.06	1.02	-0.09
	[28.63]	[18.42]	[21.37]	[15.97]	[-1.12]
High	1.38	1.24	1.33	1.12	-0.26
	[18.07]	[21.13]	[19.10]	[14.19]	[-3.46]
H–L	0.34	0.36	0.32	0.23	-0.11
	[3.05]	[5.02]	[2.80]	[2.04]	[-1.39]
(H+L) - (2+3)					-0.16
					[-2.66]

Table VI
Market Betas and Input-Output Factor Dependence

This table reports in Panel A market betas of portfolios sorted by input-output factor dependence, as well as the difference in betas between high and low input-output factor dependence quartiles. In Panel B, stocks are sorted first by input-output factor dependence, then by industry asset growth. Betas are computed for each portfolio from regressions of excess stock returns on excess market returns using all monthly observations. Input-output Factor dependence is computed at the industry level as average of the Hirshman-Herfindahl indices of the dollar flows from all supplier industries and the dollar flows from all customer industries. Details are provided in the main text. Industries are defined at the four-digit SIC level. t-statistics are based on Newey and West (1987) adjusted standard errors using 12 lags. The sample period is July 1973 to June 2013.

	Input-Output Factor Dependence				
	Low	2	3	High	H–L
A. Univariate Sorts					
	1.09	1.08	1.10	0.90	-0.19
	[40.01]	[19.32]	[39.89]	[31.38]	[-4.55]
B. Split by Industry Asset Growth					
Low	1.06	0.97	1.07	0.87	-0.19
	[21.50]	[17.09]	[19.92]	[17.79]	[-4.07]
2	0.95	0.96	1.11	0.86	-0.09
	[23.04]	[16.16]	[26.77]	[22.61]	[-1.64]
3	1.04	1.23	1.02	0.95	-0.09
	[35.59]	[23.85]	[28.41]	[15.30]	[-1.33]
High	1.36	1.35	1.31	1.08	-0.27
	[17.94]	[18.38]	[16.18]	[17.44]	[-3.39]
H–L	0.30	0.38	0.24	0.22	-0.08
	[3.11]	[4.28]	[2.03]	[2.39]	[-0.90]
(H+L) - (2+3)					-0.14
					[-2.19]

Table VII
Input Dependence and the Size and Value Anomalies

This table reports average monthly returns (Panel A) and four-factor alphas (Panel B) of value-weighted portfolios sorted first by input dependence and then by market capitalization (left) or book-to-market ratio (right), as well as the difference between high and low input dependence and small and big market capitalization (high and low book-to-market ratio). Input dependence is computed at the industry level as Herfindahl index of the dollar flows from all supplier industries. Details are provided in the main text. We also provide average log market capitalization (book-to-market ratio) in Panel C. t-statistics are based on Newey and West (1987) adjusted standard errors using 12 lags. The sample period is July 1973 to June 2013.

	Size Anomaly						Value Anomaly				
	Input Dependence						Input Dependence				
	Low	2	3	High	H–L		Low	2	3	High	H–L
					Panel A: Returns						
Small	1.57	1.52	1.41	1.03	-0.54	Low	0.61	1.08	0.81	0.86	0.25
	(4.64)	(5.17)	(4.12)	(2.71)	(-3.05)		(2.27)	(4.14)	(3.18)	(3.07)	(1.43)
2	1.29	1.29	1.27	1.22	-0.07	2	0.95	1.23	1.16	0.98	0.03
	(4.43)	(4.80)	(4.62)	(3.35)	(-0.32)		(4.68)	(6.93)	(5.93)	(4.74)	(0.21)
3	1.22	1.23	1.24	1.09	-0.13	3	1.25	1.21	1.38	1.18	-0.06
	(5.21)	(5.17)	(5.60)	(3.62)	(-0.83)		(5.59)	(6.42)	(8.56)	(5.33)	(-0.34)
Big	0.84	1.09	1.04	1.03	0.18	High	1.50	1.40	1.57	1.24	-0.25
	(3.65)	(5.30)	(4.93)	(4.64)	(1.79)		(6.00)	(6.64)	(7.63)	(4.65)	(-1.42)
S–B	0.73	0.42	0.37	0.00	-0.73	H–L	0.88	0.32	0.76	0.38	-0.50
	(2.36)	(1.43)	(1.06)	(0.01)	(-3.56)		(3.21)	(1.12)	(2.92)	(1.32)	(-2.04)

Table VII continued.

	Size Anomaly						Value Anomaly				
	Input Dependence						Input Dependence				
	Low	2	3	High	H–L		Low	2	3	High	H–L
Panel B: Four-Factor Alphas											
Small	0.19	0.20	0.03	-0.32	-0.51	Low	-0.20	0.26	0.02	0.11	0.31
	(1.09)	(1.32)	(0.18)	(-1.29)	(-2.75)		(-2.08)	(2.77)	(0.12)	(0.80)	(1.91)
2	-0.09	-0.10	-0.07	-0.21	-0.12	2	-0.10	0.23	0.07	0.05	0.15
	(-0.74)	(-0.97)	(-0.44)	(-0.93)	(-0.62)		(-0.96)	(2.10)	(0.68)	(0.51)	(1.32)
3	-0.12	-0.12	-0.03	-0.21	-0.09	3	0.03	0.06	0.15	0.13	0.09
	(-1.46)	(-1.39)	(-0.26)	(-1.50)	(-0.58)		(0.33)	(0.49)	(1.26)	(1.21)	(0.69)
Big	-0.06	0.22	0.10	0.13	0.20	High	0.08	0.06	0.24	-0.02	-0.10
	(-0.88)	(3.22)	(1.14)	(1.94)	(1.87)		(0.95)	(0.38)	(1.65)	(-0.13)	(-0.69)
S–B	0.25	-0.02	-0.07	-0.45	-0.70	H–L	0.28	-0.20	0.22	-0.13	-0.41
	(1.34)	(-0.10)	(-0.31)	(-1.76)	(-3.44)		(2.20)	(-1.01)	(1.42)	(-0.69)	(-2.10)
Panel C: Characteristics											
	Average Market Capitalization						Average Book-to-Market Ratios				
Small	8.96	9.02	9.19	9.13	0.17	Low	-1.66	-1.62	-1.57	-1.42	0.24
2	10.54	10.62	10.92	10.82	0.28	2	-0.68	-0.67	-0.61	-0.45	0.24
3	11.81	11.89	12.35	12.30	0.49	3	-0.16	-0.16	-0.11	-0.02	0.14
Big	13.79	13.95	14.46	14.50	0.71	High	0.54	0.54	0.58	0.62	0.08
S–B	-4.83	-4.93	-5.27	-5.37	-0.54	H–L	2.20	2.16	2.14	2.03	-0.16

Figure 1. Simulated Sample Economy

This figure shows one sample path of the systematic sector-wide shock Y_k (Panel A), the respective dynamics of the concentration of firms in this sector over time (Panel B), the dynamics of the sector profitability $y_k = Y_k Q_k^{-\varepsilon}$ (Panel C), and factor beta β_k of a representative firm (Panel D). The horizontal dashed lines in panel C indicate the high and low reflecting barriers due to effect of competition. The parameters are described in the text.

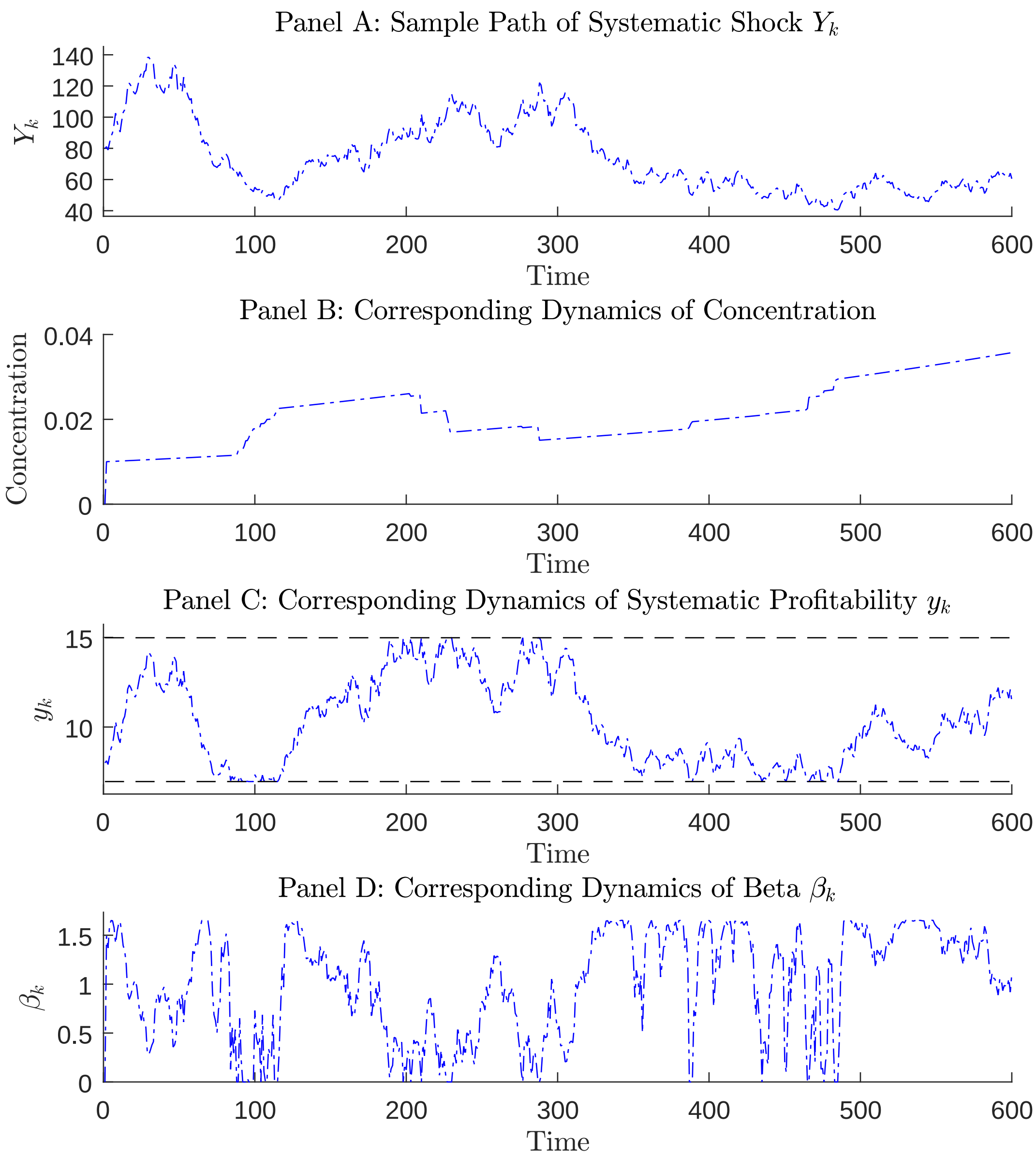
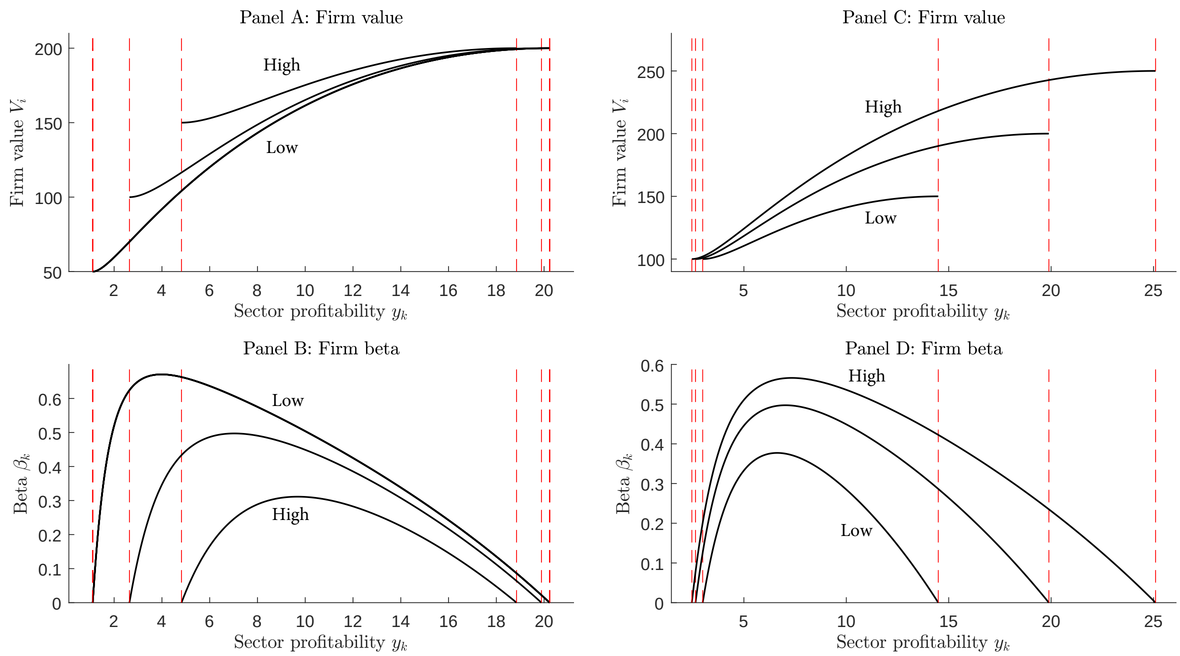


Figure 2. Firm Value and Beta for Different Values of the Purchase and Sale Price of Capital

The figure shows the solution for perfect competition case for firm value and equity beta using different values of the sale and purchase price of capital, $\underline{R}_k$ and $\overline{R}_k$. We use the base set of parameters. In Panels A and B, we set $\overline{R}_k = 200$ and plot values for $\underline{R}_k = 25$, $\underline{R}_k = 50$, and $\underline{R}_k = 100$. In Panels C and D, we set $\underline{R}_k = 50$ and plot values for $\overline{R}_k = 150$, $\overline{R}_k = 200$, and $\overline{R}_k = 250$.



High, Medium, and Low Sale Price of Capital, $\underline{R}_k$

High, Medium, and Low Purchase Price of Capital, $\overline{R}_k$

Figure 3. Firm Value and Beta for Low, Medium, and High Values of Idiosyncratic Shock
The figure shows the solution for firm value and equity beta, as a function of profitability y_k. We use the constant values of the idiosyncratic shock, $x_i = 0$, $x_i = 1$, and $x_i = 2$. We use analytic solution for firm beta. The parameters are as follows: $\delta = 0.01$, $\varepsilon = 0.5$, $\mu_k = 0.03$, $\mu = 0.03$, $r = 0.05$, $\sigma_k = 0.25$, $\underline{R}_k = 100$, $\overline{R}_k = 200$, $c_i = 0$, and $q_{ik} = 1$. Red vertical lines denote the reflecting barriers $\overline{y}_k$ and $\underline{y}_k$.

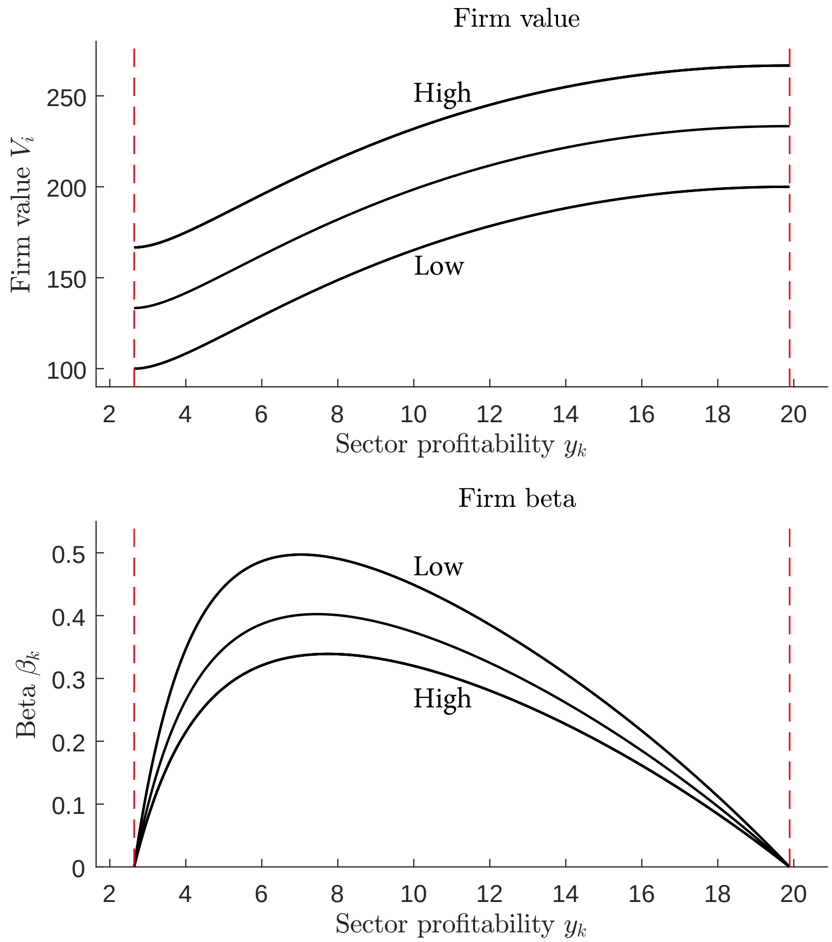
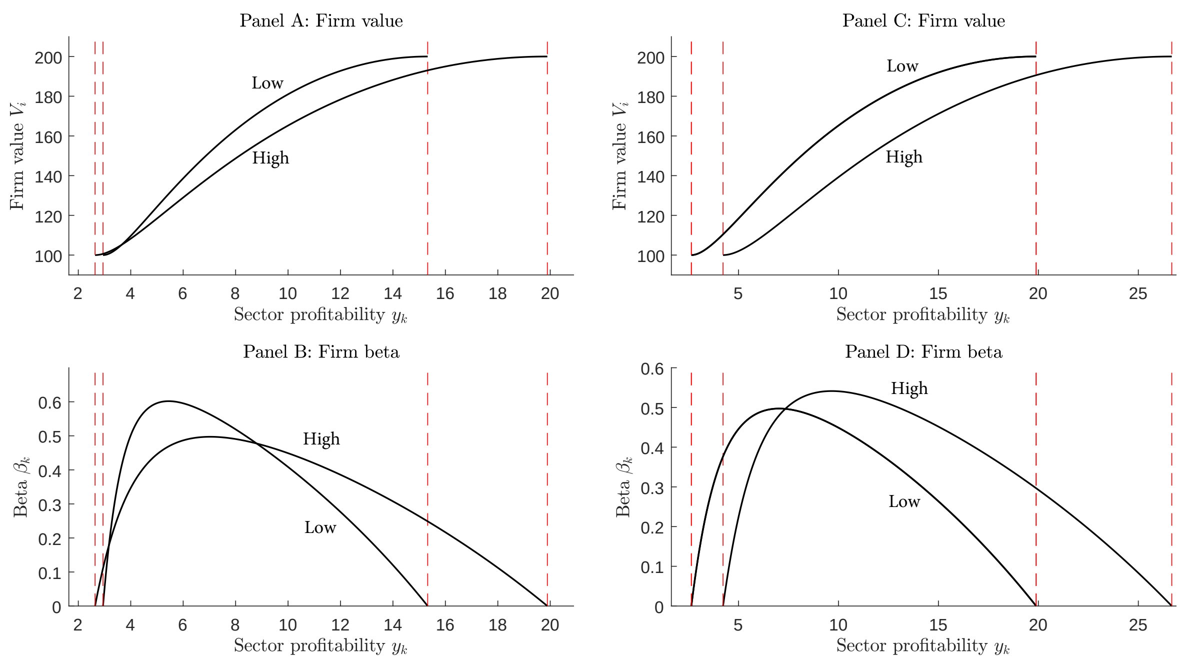


Figure 4. Effect of Volatility and Capital Depreciation Rate

The figure shows the solution for perfect competition case for firm value and equity beta. Panel A and Panel B show the solutions for a high and low volatility of the systematic shock, σ_k. Panel C and Panel D show the solutions for a high and low capital depreciation rate, δ.



High and Low Volatility of Systematic Shock, σ_k High and Low Capital Depreciation Rate, δ

Figure 5. Competition and Cross-Sectional Anomalies

The figure plots equity beta, as a function of profitability y_k. On the graph, the dash dotted lines represent the betas for two firms operating in low competition environment ($\overline{R}_k = 400$, $\underline{R}_k = 50$) that differ only in their idiosyncratic shocks, we use $x_i = 0$ and $x_i = 2$. The solid black lines on the left side of the graph are betas of the same two firms that operate in a more competitive environment ($\overline{R}_k = 200$, $\underline{R}_k = 100$).

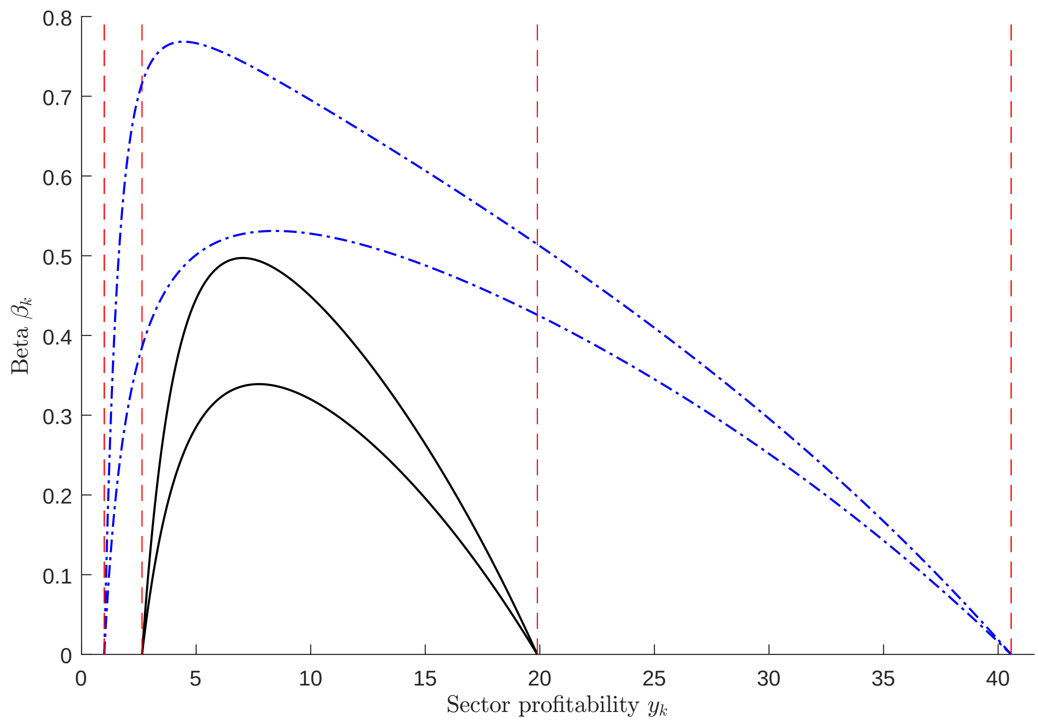


Figure 6. Effect of Firm Default on Systematic Risk

This figure plots the value and equity beta for the firm with high leverage which has an option to default. A firm is exposed to only one industry. Parameters are set so that the firm defaults optimally (at $\overline{y}_d = 1.33$) before reaching the disinvestment threshold. The parameters are as follows: $\delta = 0.01$, $\varepsilon = 0.5$, $\mu_k = 0.03$, $\mu = 0.03$, $r = 0.05$, $\sigma_k = 0.25$, $\underline{R}_k = 100$, $\overline{R}_k = 200$, and $q_{ik} = 1$.

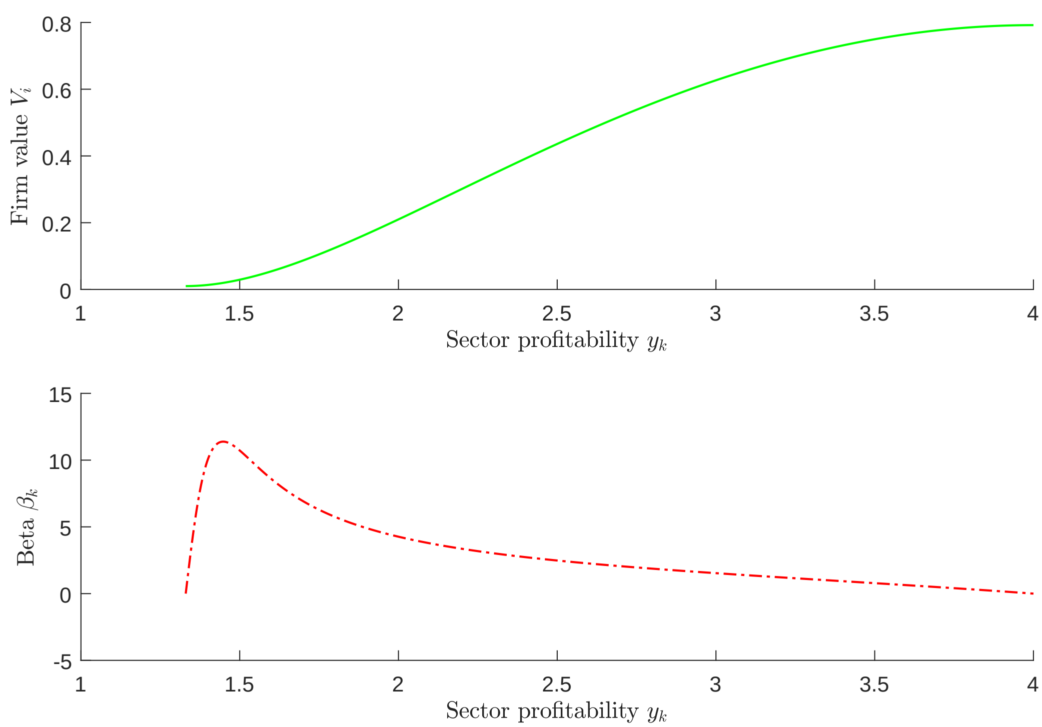


Figure 7. Effect of Firm Leverage and Default on Systematic Risk

This figure illustrates the effect of a levered competitor's exit due to default on other firms operating in the same sector. We assume that a levered competitor's default reduces the sector's capacity in half and plot values against the underlying shock Y_k.

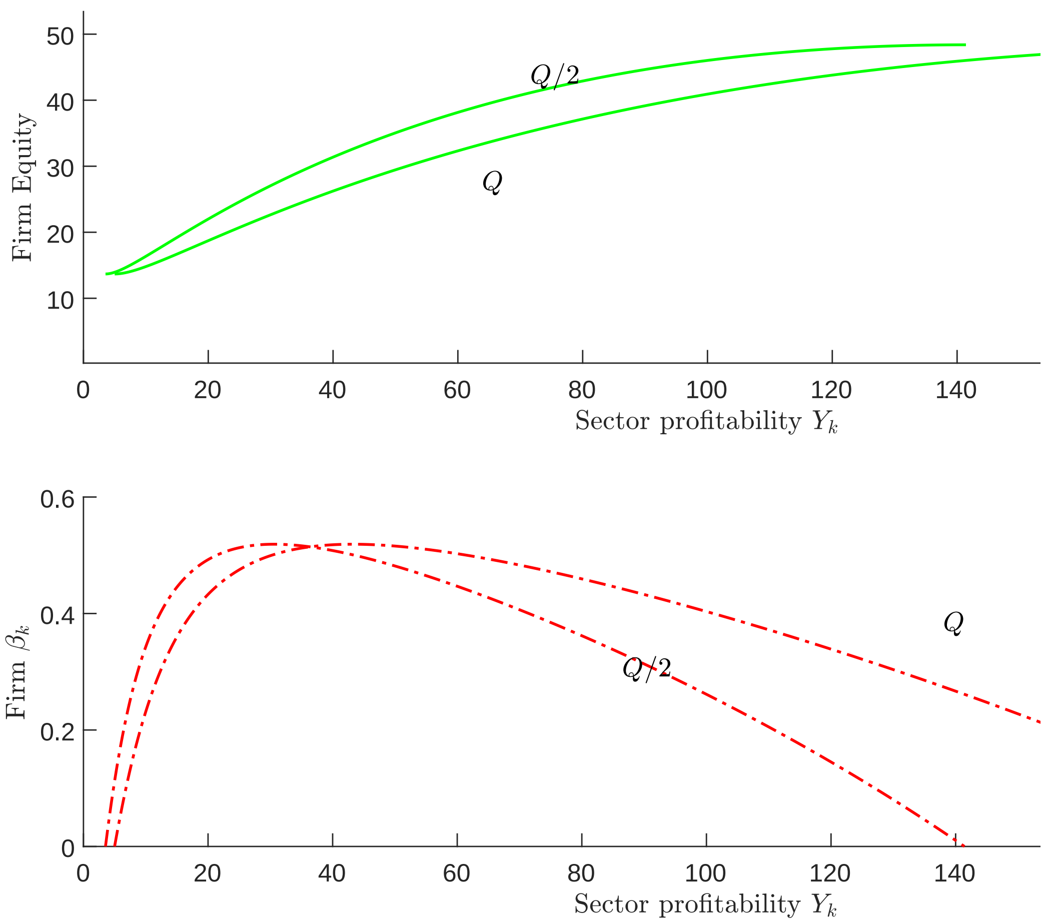


Figure 8. Firm Value and Beta in Case of Imperfect Competition

This figure illustrates the solution to the cases of monopoly and duopoly. In Panel A, we plot firm values for the monopoly and duopoly without idiosyncratic shocks. In Panel B, we plot beta for the monopoly without idiosyncratic shocks. In Panel C, we plot beta for the monopoly with idiosyncratic shock, $x_i = 2$.

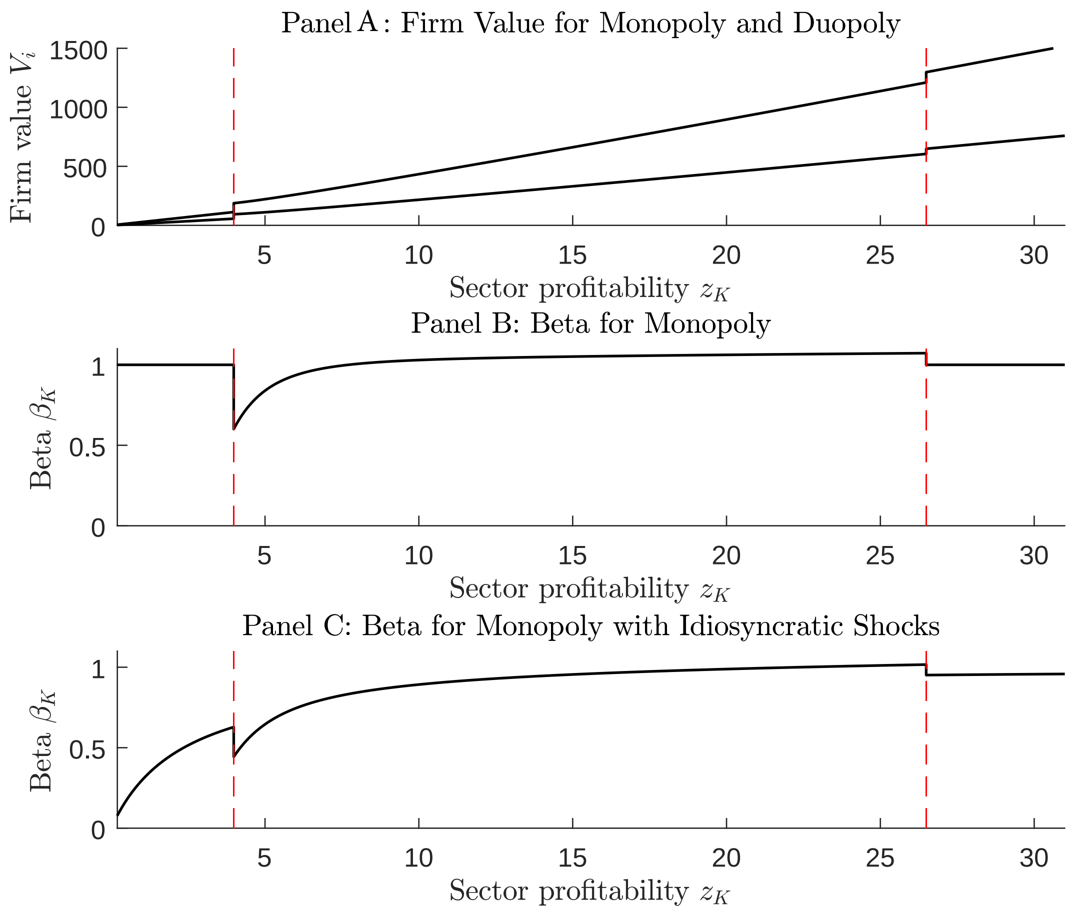


Figure 9. Oil Beta

This figure plots inflation-adjusted oil prices (Panel A), the time-series of the conditional oil beta (Panel B). The asset under consideration is the value weighted portfolio of all stocks in SIC industries whose excess returns have a significantly positive exposure to oil price changes after controlling for excess market returns. The conditional oil beta is modeled as

$\beta_t^{OIL} = \gamma_0 + \gamma_1 OIL_{t-1} + \gamma_2 OIL_{t-1}^2$,

where OIL_{t-1} is the average oil price over the preceding 12-month period, appropriately scaled to always be between zero and one. The γ coefficients are estimated from a regression that also controls for the market return. The sample period is July 1973 to December 2014.

Panel A: Oil Price

Panel B: Oil Beta

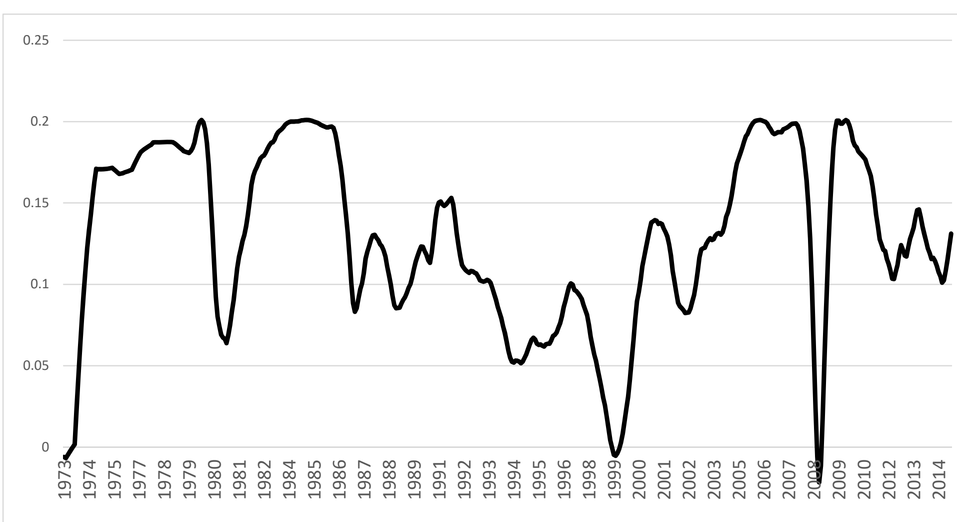


Figure 10. Conditional oil beta and lagged oil price

This figure plots the relation between the conditional oil beta and lagged oil price. The asset under consideration is the value weighted portfolio of all stocks in SIC industries whose excess returns have a significantly positive exposure to oil price changes after controlling for excess market returns.

The conditional oil beta is modeled as $\beta_t^{OIL} = \gamma_0 + \gamma_1 OIL_{t-1} + \gamma_2 OIL_{t-1}^2$, where OIL_{t-1} is the average oil price over the preceding 12-month period, appropriately scaled to always be between zero and one. The γ coefficients are estimated from a regression that also controls for the market return. The sample period is July 1973 to December 2014.

Oil Beta vs. Oil Price – Regression Fit