International Portfolio Investments with Trade Networks

Vu Thanh Chau*

July 11, 2022

Link to latest version

Abstract. What determines the composition of international portfolio investments remains an open question in international finance. In this paper, I propose a theory of international portfolio choice where trade networks play a key role. I solve in closed form for the optimal equity and bond portfolio investments in a multi-country model with arbitrary global input - output linkages and taste differences. I show that a measure of international demand exposure, called the "International Domar Weights" (IDWs), is key in determining international equity portfolios, and that a matrix measuring expenditure switching on network determines the bond portfolios. The IDWs extend the closed-economy "Domar weights" to the international setting and capture countries' interdependence through both direct and indirect trade linkages.

Using data from the World Input - Output Database (WIOD) and Coordinated Portfolio Investment Survey (CPIS), I apply the framework to a network of 43 major developed and emerging economies and obtain four main results. First, the theoretical network portfolio is a significant predictor and explains almost half of the variation in international bilateral portfolio investments. The significance of the network portfolio is robust to controlling for gravity factors (market capitalization, distance, EU membership, etc.). Second, including the network-based portfolio in a gravity model for assets resolves the puzzle of why distance matters for asset trade at all. Third, indirect trade linkages matter for portfolio determination, highlighting the need to explicitly account for trade in intermediate inputs. Finally, the model predicts both the levels and the changes in equity home bias that have occurred since 2000. *JEL* codes: F21, F65, G11.

---

*International Monetary Fund. Email: vchau@imf.org. The main portion of this work was completed during my time at the Harvard Economics Department. I am grateful to my advisors - Xavier Gabaix, Kenneth Rogoff, Adi Sunderam, John Campbell, and Robert Barro - for their continuous guidance and support throughout the development of this paper. I want to thank for their comments Emmanuel Farhi, Pol Antras, Jeremy Stein, Elhanan Helpman, Ludwig Straub, Greg Mankiw, Matteo Maggiori, Zhengyang Jiang, Ernest Liu, and Jing Zhang, who collectively made the paper better. I want to thank the seminar participants at the Chicago Fed, Boston Fed, IMF, World Bank, Princeton, Harvard, NYU Abu Dhabi, Oxford SBS, and the Online International Finance and Macro Seminar for their comments. I thank Shushu Liang, Krishna Dasaratha, Masao Fukui, Hoang Nguyen, Namrata Narain, Maria Voronina, Taehoon Kim, and Robert Siliciano for the many insightful conversations from the onset of this paper. The views in this paper are my personal views and do not necessarily represent the views of the IMF, its Executive Board, or IMF Management.

1. INTRODUCTION

International portfolio investments are an important part of the global economy, yet we know little about their determinants. To be specific, despite a large literature focusing on the asset allocation problem between domestic versus foreign assets (the "home bias" literature, starting with French and Poterba (1991)), far fewer papers have attempted to explain asset allocation between multiple foreign destinations, i.e., the *composition* of external portfolios. The shortage of empirical work is partly due to the lack of a theory of portfolio choice in an $N-$country setting with realistic cross-country asymmetries present in the data.

This paper fills that gap by providing the first theory of international portfolio investments where trade networks play a central role in driving portfolio choices. In particular, I solve in closed form for the optimal equity and bonds portfolio in a workhorse real business cycle (RBC) model with an arbitrary number of countries, arbitrary taste differences, and arbitrary international input-output linkages. In this general setting, I show that a measure of international demand exposure, called the "International Domar Weights" (IDWs), is key in determining international equity portfolios. The IDWs extend the closed-economy "Domar weights" to the international setting and capture countries' interdependence through both direct and indirect trade linkages. On the other hand, the bond portfolios depend crucially on a matrix capturing expenditure switching effects as well as the prevailing equity portfolios.

Using data from the World Input-Output Database (WIOD) and Coordinated Portfolio Investment Survey (CPIS), I apply the framework to a network of 43 major developed and emerging economies and obtain four main results.

First, I show that the equity portfolio predicted by the trade networks (henceforth, the *network portfolio*) explains equity holdings data better than other models that assume symmetry and models without input-output linkages. By itself, the network portfolio explains almost half of the variation of all bilateral equity shares. A regression of data shares against theory shares has an adjusted R^2 of 0.48. This model performs better than a simple model where investors hold the world portfolio, which predicts that the portfolio share is proportional to the market capitalization of the destination country (adj. $R^2 = 0.41$). A gravity model with other covariates (distance, contiguity, EU membership, common language, etc.) has more explanatory power (adj. $R^2 = 0.59$), but also has more degrees of freedom. Importantly, controlling for these bilateral gravity variables, the network portfolio is still a significant predictor of the data portfolio, explaining 10% of the variation that bilateral gravity variables cannot explain. Furthermore, key gravity variables such as the market cap of the destination and distance are no longer significant predictors of portfolio shares after controlling for the network portfolio.

The second finding is that the network portfolio helps resolve the “distance puzzle” for assets. In particular, a gravity equation for assets without the network portfolio yields a large negative coefficient on distance (from -0.6 to -1). It is unclear why geographical distance would greatly affect the trade of assets. Portes and Rey (2005) found that including measures of informational frictions can reduce the effect of distance on asset holding, but the effect remains significant. I show that including the network-based portfolio in a gravity regression makes the effect of distance statistically indistinct from zero.

Third, I show that the portfolio which disregards intermediate input linkages is less successful in explaining the data, highlighting the necessity of considering the whole trade network structure for portfolio determination. Finally, turning focus to the diagonal elements of the portfolio matrix, I show that the model explains cross-country differences in equity home bias, both in level and in long-run changes.

In my model, investors choose the optimal holding of international equities and bonds to hedge against international investment and relative price risks. Such risks propagate through the trade networks and impact country outputs (GDPs). Investors can hedge these risks to have a stable national income and consumption by holding the appropriate hedging portfolios. I show that the effect of shocks on a country's GDP can be decomposed into two terms: fluctuations of aggregate expenditure (consumption and investment) and fluctuations in international relative prices. This decomposition makes clear the hedging role of equities and bonds. Equities are best used to hedge against investment risks since their dividend incomes correlate strongly with investments. Meanwhile, bonds have payoffs closely related to relative prices and, therefore, should be used to hedge against relative price risks.

Several papers that jointly study international equities and bonds, such as Coeurdacier et al. (2010) and Coeurdacier and Gourinchas (2016), performed a similar strategy of projecting shocks onto the space spanned by investment and relative prices. I provide a generalized result in this framework with a full trade network structure and derive the loading matrices of country GDPs on international investment and price fluctuations, which are important determinants of the equity and bond portfolios.

The first key theoretical result is that the equity portfolios are determined by a matrix of International Domar Weights (IDWs), which measures the total value-added produced by a sector that is included in the consumption basket of each country. In equilibrium, the IDWs capture the first-order impact of a country's investment spending on another country's output. If production relied only on the domestic value-added (no intermediate goods), this exposure to investment risk would be well-captured simply by the direct trade shares. With intermediate goods used in production, however, a country's can be exposed to another country's investment fluctuations even if it does not export directly to that country. The exposure could instead come from linkages with trade partners along the value chain. In a world with complicated global value chains, considering only direct bilateral trades likely leads to mismeasurements of international income exposure. Johnson and Noguera (2012) was the first paper to derive the expression for the IDWs (which they call "trade in value added") to study the variation of the value added-to-export ratios across countries. In this paper, I incorporate the IDWs in a fully micro-founded structural macroeconomic model and show how they capture the transmission of shocks through the trade networks.

The international Domar weights differ from their domestic counterpart in an important way. In a closed-economy setting, the Domar weight of a sector corresponds to its sale share and fully captures the impact of its TFP shock on aggregate output. This result is the basis for the "network irrelevance" property of the celebrated Hulten (1978)'s theorem: the network structure is not necessary for understanding the impact of sectoral shocks on aggregate output or welfare.[1] In an open economy setting with heterogeneous consumption preference, however, there is no directly measured counterpart of the IDWs, and calculating the IDWs requires knowing the full network structure.

The second result is that the bond portfolios are determined by a matrix M^P that measures expenditure switching effect on network. In particular, the element M^P_{ij} measures the first-order impact on GDP in country i for a change of price of good j. The two-country intuition is that if a good has become relatively more expensive and the elasticity of substitution is bigger than 1, consumers will switch their expenditure away from that good. With the presence of intermediate goods, an increase in the price of good i affects not only the sales of sector i, but also the sales of sectors using good i as an input, for their marginal cost and price have increased as well. These indirect (1-step, 2-step, etc.) network effects are succintly captured by matrix M^P. To the extent that bonds are used to hedge income loss and gain due to fluctuation in relative prices, the bond portfolio is mainly determined by matrix M^P.[2]

The real business cycle setting here is related to the closed-economy model of Long and Plosser (1983), extended to account for the heterogeneous consumption preference across countries. The portfolio determination channels here are similar to those in Coeurdacier et al. (2010) and Coeurdacier and Gourinchas (2016), with a generalization beyond the symmetric 2-country setting. The model features endogenous investment, as in Heathcote and Perri (2013), but enhances the results in Heathcote and Perri (2013) by incorporating bonds in the analysis, which helps keep the equity portfolios robust to functional specifications.

[1] One of the few counterarguments to the "network irrelevance" result is Baqaee and Farhi (2019), who argue that the production network matters through large and nonlinear second-order effects.

[2] In addition to the relative price-hedging bond portfolio, bonds can be additionally used to hedge consumption risk due to fluctuations in the real exchange rate. This channel is operational when the coefficient of relative risk aversion is different from 1. I explain in full detail this exchange rate-hedging portfolio in the main text, but do not emphasize here.

Section 2 presents the model, introduces the IDWs, and shows the optimal portfolios. Section 3 evaluates how well the theory portfolio explains portfolio holdings in the data. Here I also present a comparison with the gravity model and show the role of indirect linkages in portfolio determination. Section 4 shows the extensions to the baseline model. Section 5 concludes.

Related literature. This paper contributes to three strands of literature.

First, it is related to a large literature on international risk-sharing and diversification motive of international portfolio investments. In a seminal paper, Lucas (1982) argues that investors with identical preferences can perfectly diversify consumption risk by having each country holding half of the other country's output stock in a two-country model. French and Poterba (1991) started a large literature on home bias and noted that equity portfolios in reality are strongly biased towards domestic equity, contradicting the Lucas (1982) argument. Some authors view this empirical evidence as a lack of international risk-sharing caused by either trade costs for goods (Obstfeld and Rogoff (2001), Fitzgerald (2012)) or assets (Portes and Rey (2005)). Lane and Milesi-Ferretti (2008) generalizes the Obstfeld and Rogoff (2001) model to an $N-$country setting and is one of very few studies that explains bilateral equity positions. In their model, equity investment is tightly linked to bilateral imports, because both are affected by trade costs in goods. I argue in this paper that not only bilateral trade but also indirect linkages matter.

Heathcote and Perri (2013) studies a setting with only equities and endogenous investment and finds that perfect risk-sharing can be achieved with a home-biased equity portfolio. The equity portfolio in that setting, however, relies crucially on having Cobb-Douglas production. Coeurdacier and Gourinchas (2016) and Engel and Matsumoto (2009) emphasize the role of bonds in hedging against relative price fluctuations and yielding equity portfolios that are robust to deviating from Cobb-Douglas production, an insight which I leverage here.

The second strand of related literature is on the production network in closed economies. Domar (1961) and Hulten (1978) pioneered this literature and focused on analyzing the effect of sectoral shocks on aggregate output. A large literature that follows focuses on explaining the importance of "granular" or "micro" shocks in spite of Hulten's theorem (Gabaix, 2011; Long and Plosser, 1983; Acemoglu et al., 2012; Baqaee and Farhi, 2019). In an international setting, this paper shows the difference between the international Domar weights and those in a closed economy, and studies portfolio holding.

Finally, this literature is related to an emerging literature on networks in finance. In the closed-economy setting, Herskovic (2018) shows that beta-sorted portfolios using network concentration and sparsity factors generates excess returns that cannot be explained by traditional models. Herskovic et al. (2020) shows empirical evidences that production networks also explain the evolution of firm size and volatility distributions in the data. Gofman et al. (2020) analyzes a firm-level supplier-customer dataset and shows that a measure of a firm's distance from consumption-good producers predicts stock returns and exposure to aggregate shocks.

In the open-economy finance literature, di Giovanni and Hale (2021) provides evidence for the importance of trade network in transmitting shocks globally, arguing that 70% of the total impact of US monetary policy on international stock returns is due to the trade network. Chang et al. (2020) provides similar evidences using sovereign CDS prices. Richmond (2019) shows that countries that are more central in the international trade network tend to have lower interest rates and currency risk premia. Jiang and Richmond (2019) study how the trade networks affect international transmission of shocks and international asset prices in a model without investment. This paper contributes by looking at portfolio ("quantity") instead of asset prices. Section 4.3 also shows how asset pricing can be studied in closed form in a model with investment.

2. Model

2.1. **Environment.** The world economy consists of N countries. Each country produces a country-specific intermediate good and a final good. The intermediate good i is produced using domestic labor, capital, and other intermediate inputs according to a technology:

$$Y_{i,t} = Z_{i,t}^{Y}\left(L_{i,t}^{\theta}K_{i,t}^{\alpha}\right)^{1-\gamma}\left[\sum_{j=1}^{N}\omega_{ij}^{1-\frac{1}{\varepsilon}}X_{ij,t}^{\frac{1}{\varepsilon}}\right]^{\frac{\varepsilon\gamma}{\varepsilon-1}}.$$

The final good is produced using a basket of domestic and imported intermediate goods:

$$G_{i,t} = \left[\sum_{j=1}^{N}\xi_{ij}^{\frac{1}{\varepsilon}}G_{ij,t}^{1-\frac{1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Once produced, final goods are used for domestic consumption and investment: $G_{i,t} = C_{i,t} + I_{i,t}$. We use $P_{i,t}$ and $Q_{i,t}$ to denote the price of intermediate and final good i, respectively. For convenience, I use the tilde notation $\widetilde{X} = P_X X$ to denote the nominal value of a physical quantity X.

Trade network. Let us define *the consumption share matrix* Ξ and the *international input-output (IO) matrix* Ω as:

$$\Xi_{ij,t} = \frac{\widetilde{G}_{ij,t}}{\widetilde{G}_{i,t}}, \qquad \Omega_{ij,t} = \frac{\widetilde{X}_{ij,t}}{\widetilde{Y}_{i,t}} \qquad \text{for } i,j \in \{1,2,\dots,N\}. \tag{1}$$

Element (i,j) of matrix Ξ gives the share of country i's total final expenditure spent on good j. Similarly, the element Ω_{ij} gives the share of sector i's total revenue spent on purchasing intermediate input j. The matrices Ω and Ξ describe the network of trade between consumers and production sectors in the world.

Capital accumulation. In period t, sector i purchases $I_{i,t}$ units of the final good i and transforms them into new capital using a linear technology:

$$K_{i,t+1} = (1-\delta)K_{i,t} + Z^K_{i,t} I_{s,t}.$$

The investment-specific technology (IST) shock $Z^K_{i,t}$ governs the efficiency of transforming the consumption good into capital.[3] Capital owners receive rental income net of investment as dividends: $\Pi_{i,t} = r_{i,t}K_{i,t} - Q_{i,t}I_{i,t}$. The total value of capital in country i is given by the Bellman equation:

$$V_t(K_{i,t}) = \max_{I_{i,t}} \left\{ r_{i,t}K_{i,t} - Q_{i,t}I_{i,t} + \mathbb{E}_t\left[\Theta_{i,t+1}V_{t+1}(K_{i,t+1})\right] \right\},$$

where $\Theta_{i,t+1}$ denotes the 1-period-ahead stochastic discount factor (SDF). Optimal capital accumulation satisfies an Euler equation:

$$1 = \mathbb{E}_t\left[\Theta_{i,t+1}\frac{Q_{i,t+1}}{Q_{i,t}}\frac{Z^K_{i,t}}{Z^K_{i,t+1}}\left(1-\delta + r_{i,t+1}\frac{Z^K_{i,t+1}}{Q_{i,t+1}}\right)\right]. \tag{2}$$

Assets and preferences. The representative household in country i has CRRA utility $u(C_{i,t}) = \frac{C_{i,t}^{1-1/\sigma}}{1-1/\sigma}$ over consumption of final good i. They supply $\overline{L}_i$ units of labor inelastically and earn labor income $W_{i,t}\overline{L}_i$.

Households in all countries can invest frictionlessly into international equities and risk-free perpetual bonds. Each share of equity i costs $P^E_{i,t}$ and pays the capital dividends of intermediate sector i. One unit of country i's bond costs $P^B_{i,t}$ and pays 1 unit of the intermediate good i each period.

Let $\Lambda_{ij,t}$ and $B_{ij,t}$ denote the number of equity shares and bond units of country j held by country i, respectively. Household i maximizes their expected lifetime utility

$$U_i = \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u(C_{i,t})\right] \tag{3}$$

by choosing asset holdings $\left(\Lambda_{ij,t}, B_{ij,t}\right)_{i,j\in\mathcal{N},t\geq 1}$ and consumptions $\left\{C_{i,t}\right\}_{t\geq 0}$ that respect the budget constraint:

$$\widetilde{C}_{i,t} + \sum_{j=1}^{N}\left(P^B_{j,t}B_{ij,t} + P^E_{j,t}\Lambda_{ij,t}\right) = \widetilde{L}_{i,t} + \sum_{j=1}^{N}\left(P^B_{j,t} + P_{j,t}\right)B_{ij,t-1} + \sum_{j=1}^{N}\left(P^E_{j,t} + \Pi_{i,t}\right)\Lambda_{ij,t-1}. \tag{4}$$

[3]IST shocks have been argued to be an important driver of US business cycles (Justiniano et al., 2010) and international trade dynamics (Eaton et al., 2016).

Let us define the *gross national income* (GNI) and *gross domestic product* (GDP):

$$\mathrm{GDP}_{i,t} = (1-\gamma_i)\widetilde{Y}_{i,t}$$
$$\mathrm{GNI}_{i,t} = \underbrace{\widetilde{L}_{i,t}}_{=\theta \mathrm{GDP}_{i,t}} + \underbrace{\sum_{j\in\mathcal{N}} \left(B_{ij,t-1}\mathcal{E}_{i,t} + \Lambda_{ij,t-1}\widetilde{\Pi}_{j,t}\right)}_{\text{Investment income}}.$$

It will be useful later to rewrite the household budget constraint as:

$$\widetilde{C}_{i,t} + \mathrm{PR}_{i,t} = \mathrm{GNI}_{i,t}, \tag{5}$$

where $\mathrm{PR}_{i,t} = \sum_{j=1} P^B_{j,t}\left(B_{ij,t} - B_{ij,t-1}\right) + \sum_{j=1} P^E_{j,t}\left(\Lambda_{ij,t} - \Lambda_{ij,t-1}\right)$ indicates portfolio rebalancing. A positive $\mathrm{PR}_{i,t}$ indicates the extra investment required to increase the number of shares held, while a negative $\mathrm{PR}_{i,t}$ represents capital gain realized when shares are sold.

Market clearing. The market clearing conditions for final and intermediate goods:

$$G_{i,t} = C_{i,t} + I_{i,t}, \quad Y_{i,t} = \sum_{j=1}^{N}\left(G_{ji,t} + X_{ji,t}\right), \quad \forall i,j \in \mathcal{N}. \tag{6}$$

The market clearing conditions for assets are $\sum_{i\in\mathcal{N}} \Lambda_{ij,t} = 1$ and $\sum_{i\in\mathcal{N}} B_{ij,t} = 0$ for all j.

Solution method. I follow Samuelson (1970) and Devereux and Sutherland (2011) and solve for the *zeroth-order portfolio,* obtained by approximating portfolio choice conditions to the second order and the remaining equations to the first order. In particular, if the equilibrium portfolio is $\Lambda(\mathcal{S}_t)$, with $\mathcal{S}$ being the vector of state variables, then we are solving for the zeroth-order term $\overline{\Lambda}$ in the Taylor expansion

$$\Lambda(\mathcal{S}_t) = \overline{\Lambda} + \Lambda_S \cdot \left(\mathcal{S}_t - \overline{\mathcal{S}}\right) + \text{ higher-order terms.}$$

Generally, this method does not deliver a closed-form portfolio. It is possible in this model, however, to obtain a closed-form solution because markets are complete up to the first order of approximation.[4] When markets are complete, the competitive equilibrium allocation replicate that of the Social Planner, and the equilibrium portfolio is one that supports the efficient allocation.[5]

Specifically, if the full set of Arrow-Debreu securities were available, efficient risk-sharing stipulates that consumption levels satisfy the Backus and Smith (1993) condition:

$$\Delta\widetilde{c}_{i,t+1} - \Delta\widetilde{c}_{j,t+1} = (1-\sigma)\Delta e_{ij,t+1}, \quad \forall i,j \in \mathcal{N}. \tag{7}$$

[4]We have complete markets in the linearized model because the number of shocks and assets are exactly equal ($2N$), which makes both income risks and asset returns linear combinations of $2N$ shocks. Thus, there exists a unique portfolio of $2N$ assets that allows households to perfectly hedge against risks. In some special cases (e.g. when $\sigma = \varepsilon = 1$), we have complete markets globally.

[5]This is also the approach in Coeurdacier et al. (2010), Engel and Matsumoto (2009), and Coeurdacier and Gourinchas (2016).

where $e_{ij} \equiv q_i - q_j$ is the real exchange rate. In the special case of log utility ($\sigma = 1$), country consumptions have the same growth rate at all dates and states of the world. From the household budget constraint (5), we can see that relative consumption can be kept stable using a passive portfolio ($\widehat{PR}_{i,t} = 0\ \forall i$) if it generates stable relative national incomes. That passive portfolio is also the zeroth-order portfolio. While the discussion so far has focused on the log utility case, the general logic applies. If households have a higher (lower) risk aversion than implied by log utility, they have an additional motive to shift expenditure into (out of) states of the world where their final good has become relatively more expensive. This real exchange rate (RER) hedging motive requires a modification to the bond portfolio, which I describe and explain in Section 2.3.

I start with characterizing how international shocks propagate through the trade network and affect country GDPs and labor incomes in Section 2.2. Having understood the structure of shock transmission, I give the optimal equity and bond portfolios in Section 2.3.

2.2. **Shock propagation and risks to GDP.** Let us start with an accounting identity that links final expenditure ($\widetilde{G}$) and the income of upstream producer (their GDPs).

Definition 1. Let Ω_t and Ξ_t be the expenditure share matrices defined in eq. (1). Define the International Domar Weight (IDW) matrix as:

$$M_t = (1-\gamma)\left[I - \gamma\Omega_t'\right]^{-1}\Xi_t'. \tag{8}$$

Lemma 1. *Country GDP and expenditure $\widetilde{G}$ satisfy:*

$$GDP_t = M_t\widetilde{G}_t. \tag{9}$$

Proof. See Appendix A.1 □

Lemma 1 shows that, despite a potentially complicated, multi-stage nature of production, a simple matrix M fully captures the relationship between upstream producers and ultimate consumers. In particular, the element M_{ij} gives the value added content of intermediate sector i embedded in the consumption and investment of country j, accounting for both direct and indirect links on the global value chain.[6]

The International Domar weights are closely related to their closed-economy counterpart: both indicate the sales or value added share in consumption.[7] There are two key differences,

[6] For example, a producer of semiconductor chip does not transact directly with car buyers, for they only sell components to car producers. Consequently, direct trade data do not record any transaction between these two parties. On the other hand, the IDWs trace through the input-output structure and measure the fraction of proceeds from car sales that ultimately flow to chip producers.

[7] Seminal works by Hulten (1978) and Long and Plosser (1983) showed that in a closed economy with N sectors, intermediate input trade network Ω, and a representative agent with preference $\ln\mathscr{C} = \sum_{j=1}^{N}\xi_j \ln C_j$ (with $\sum_j \xi_j = 1$), the equilibrium industry sale shares are given by: $m = (1-\gamma)\left[\mathbf{I} - \gamma\Omega'\right]^{-1}\xi$. These sales shares are called the "Domar weights", due to its early usage by Domar (1961) to aggregate sectoral TFP shocks.

Origin	Destination	Intermediate Share	Final Share	Scaled IDWs
Japan	China	1.61%	2.31%	5.57%
Japan	US	0.99%	0.49%	2.64%
Japan	Australia	1.00%	0.10%	1.50%
Japan	Korea	0.73%	0.30%	1.28%
Japan	Russia	0.50%	0.04%	1.11%

TABLE 1. International Domar weights versus Bilateral Trade Shares for Japan

Data: World Input - Output Table. The intermediate import share matrix Ω and final import share matrix Ξ are imputed directly from WIOT. The international Domar weights are calculated as $M = (1-\gamma)\left[\mathbf{I} - \gamma\Omega'\right]^{-1}\Xi'$.

however. First, since there are heterogeneity in preferences across countries, the IDWs are a 2-dimensional object. Secondly, while the closed-economy Domar weights are directly observable (as industry sale shares), the IDWs are not directly observable and must be calculated from the expenditure share matrices. Unlike the closed-economy case, calculating the IDWs requires knowledge of the full network structure. To the best of my knowledge, the IDWs are first derived by Johnson and Noguera (2012) in a study of the cross-country variation in the value added to exports (VAX) ratio.

The IDWs succintly capture the first-order impact of downstream spendings on upstream producers' incomes. This is most obvious in the case of $\varepsilon = 1$, in which all expenditure shares and IDWs are constant. Differentiating equation (9) when M is a constant matrix gives:

$$\frac{\partial \mathrm{GDP}_{i,t}}{\partial \widetilde{G}_{j,t}} = M_{ij}. \tag{10}$$

In other words, any shock that raises country j's final expenditure by \$1 increases country i's GDP by \$$M_{ij}$.

As an illustration, table 1 illustrates the IDWs in the data and shows how measures of direct bilateral trade underestimate actual trade linkage. The first row of Table 1 shows that while Japan accounts for only 1.61% of all Chinese intermediate imports and 2.31% of Chinese imported final goods, the corresponding scaled IDW is 5.57%. Thus, in terms of factor content, Japanese factors account for 5.57% of Chinese final consumption, and the bulk of Japanese factor exports to China is via intermediate goods produced in other countries. In other words, Japan is more exposed to Chinese final expenditure than as indicated by direct trade statistics.

For a general elasticity ε, the expenditure shares will change over time as consumers react to changes in relative prices (expenditure switching). To account for this effect, let us define an expenditure switching matrix. Lemma 2 then generalizes equation (10).

Definition 2. Define the expenditure switching matrix M^P as:

$$M_t^P = (\varepsilon-1)(1-\gamma)\left[I-\gamma\Omega_t'\right]^{-1}\left(D_{\widetilde{Y}} - \Xi_t' D_{\widetilde{G}}\Xi_t - \Omega_t' D_{\widetilde{X}}\Omega_t\right), \tag{11}$$

where $D_X = \text{diag}(X)$.

Lemma 2. *To the first order, the impact of shocks on GDP can be decomposed into two terms: fluctuations in downstream expenditures ($\widehat{\widetilde{G}}$) and in international relative prices ($\widehat{p_t}$). Specifically, we have:*

$$\widehat{GDP}_t = \overline{M}\widehat{\widetilde{G}}_t - \overline{M}^P\widehat{p_t}. \tag{12}$$

where $\overline{M}$ is the IDW matrix, and $\overline{M}^P$ is the expenditure switching matrix in the steady state.

The first term in equation (12) is the same as in the Cobb-Douglas case. The second term is the expenditure switching effect. If $\varepsilon = 1$, there is no expenditure switching, and $\overline{M}^P = 0$. With $\varepsilon > 1$, the good that has become relatively more expensive has lower output and value added (the $D_{\widetilde{Y}}$ term). This effect propagates upstream to intermediate input suppliers of the affected sector, lowering value added in these sectors as well (the Leontief inverse $\left[I-\gamma\Omega_t'\right]^{-1}$). The last two terms in the big bracket of equation (2) represents a *composition* effect: if an intermediate good is already large component of some basket, its price increase relative to the basket is smaller.[8]

The decomposition in Lemma 2 is central to our optimal portfolio analysis. As discussed in Section 2.2, the zeroth-order portfolio implements the Pareto efficient allocations in the linearized model and keeps relative GNI constant at all dates and states. Lemma 2 shows that if we can perfectly hedge income risks caused by any realization of $(\widehat{\widetilde{G}}_t, \widehat{p}_t)$, we can perfectly hedge income against any arbitrary realization of the underlying shocks. I present next the optimal equity and bond portfolios that achieve this goal.

2.3. **Equilibrium portfolio.** In mentioning the bond portfolio below, $\widetilde{B}_{ij}$ denotes country i's holding of bond j evaluated using its steady-state price: $\widetilde{B}_{ij} = B_{ij}\overline{P}_j$.

Proposition 1 (Portfolio with General Elasticities)**.** *Let $\overline{M}$ be the IDW matrix and $\overline{M}^P$ the expenditure switching matrix in the steady state. We have:*

(1) To the first order of approximation, the competitive equilibrium with only bonds and equities is Pareto-efficient.

(2) The steady-state equity portfolio hedges against international investment risks and is given by

$$\Lambda = \theta\overline{M}\left[I-\alpha\overline{M}\right]^{-1}.$$

[8]As an illustration, consider a basket that has two goods: good 1 with weight ω and price p, and good 2 has weight $1-\omega$ and price p^*. The basket price is given by $q = \omega p + (1-\omega)p^*$. The relative price of good 1 (to the basket) is given by $p - q = (1-\omega)(p-p^*)$. Thus, the effect of price change of a good on the relative price decreases with its weight: $d(p-q) = (1-\omega)dp$.

(3) *The steady-state bond portfolio is the sum of two components: $\widetilde{B}^{ER}$, which hedges against exchange rate risk, and $\widetilde{B}^{P}$, which hedges against relative price risk. In particular, $\widetilde{B} = \widetilde{B}^{ER} + \widetilde{B}^{P}$, with:*

$$\widetilde{B}^{ER} = (1-\sigma)[I-\Lambda]D_{\widetilde{C}}\overline{\Xi}, \quad \text{and} \quad \widetilde{B}^{P} = [\theta I + \alpha\Lambda]\overline{M}^{P},$$

where $D_{\widetilde{C}} = diag(\overline{\widetilde{C}})$ is a diagonal matrix of the steady-state consumptions.

Proof. See Appendix A.4 □

I now unpack each component of the optimal portfolio and provide intuitions. Readers who are more interested in the empirical relevance of the theory may wish to skip ahead to Section 3.

2.3.1. *Intuition for optimal equity portfolio.* Recall the decomposition in Lemma 2. Holding relative prices constant ($\widehat{p} = 0$), relative GDPs can fluctuate over time due to differential final expenditures across countries, combined with the fact that the geographical distribution preference for components of the final good differs internationally. Under complete markets, relative consumption expenditures are constant, and the only driver of $\widehat{\widehat{G}}$ is investment. Since capital dividends are tightly linked to investments, equities provide a good hedge against investment risk to cross-country incomes.

Let $\widehat{\widehat{I}}$ be a realization of sectoral investments in some state of the world. From Lemma 2, the impact on producer income is captured by the IDWs: $\widehat{\text{GDP}} = \overline{M}\widehat{\widehat{I}}$. Intuitively, sectors that supply a large value added content for a country's investment good benefit more when that country experiences an investment boom. This extra income is split into labor and capital income by their respective shares:

$$\widehat{\widehat{L}} = \theta\widehat{\text{GDP}}_t = \theta\overline{M}\widehat{\widehat{I}}, \quad \widehat{\widehat{K_t}} = \alpha\widehat{\text{GDP}}_t = \alpha\overline{M}\widehat{\widehat{I}}.$$

Since making investment lowers dividends (capital income minus investment) one-for-one, the net impact of investment on dividends is given by:

$$\widehat{\widehat{\Pi}}_t = \left(\alpha\overline{M} - I\right)\widehat{\widehat{I}}_t.$$

The optimal equity portfolio Λ is the unique portfolio that makes the sum of labor and financial income relatively constant across country:

$$\widehat{\widehat{L}} + \Lambda\widehat{\widehat{\Pi}} = \theta\overline{M}\widehat{\widehat{I}} + \Lambda\left(\alpha\overline{M} - I\right)\widehat{\widehat{I}}_t = \left[\theta\overline{M} + \alpha\Lambda\overline{M} - \Lambda\right]\widehat{\widehat{I}} = 0$$

for all realization of $\widehat{\widehat{I}}$. Therefore, we must have $\theta\overline{M} + \alpha\Lambda\overline{M} - \Lambda = 0$, which delivers a unique solution for Λ:

$$\Lambda = \theta\overline{M}\left[I - \alpha\overline{M}\right]^{-1}.$$

This is the equity portfolio presented in Proposition 1.

We can expand the equity portfolio into an infinite sum:

$$\Lambda = \theta\overline{M} + \theta\alpha\overline{M}^2 + \theta\alpha^2\overline{M}^3 + \dots$$

The first-order term of Λ is simply $\theta\overline{M}$ – the IDW matrix. In words, the most important determinant of international equity holding is given by the IDWs, which measure downstream expenditure risk via the trade network. Country i holds a higher share of country j's equity if country i has a large value added content embedded in country j's investment.

This result is driven by the negative correlation between labor income and dividends in the model when driven by investment, *ceteris paribus*. Suppose country j experiences an investment boom, which could be due to a positive TFP or investment shock. This leads to increased labor incomes for all suppliers of the investment good j (including its components). Simultaneously, the dividends of country j, $\widetilde{\Pi}_j = \alpha\widetilde{K}_j - \widetilde{I}_j$, is significantly lower due to investment financing. Thus, a supplier country i can hedge against the higher labor income by holding country j's equity shares. The amount of shares required for hedging is approximately proportional to how much country i's income depends on investment spending in country j, measured by the IDW M_{ij}.

Several earlier papers have explored the role of $cor(\widetilde{L},\widetilde{\Pi}) < 0$ in determining the hedging equity portfolio in two-country models (Heathcote and Perri, 2013; Coeurdacier et al., 2010). My contribution here is to show that, in a multi-country model with arbitrary preferences and input-output linkages, the IDWs are the correct measure of income exposure to investment fluctuations, and, therefore, the determinant of international equity holdings.

We can also understand the optimal equity portfolio from a network perspective. In general, consider a directional, weighted network described by an adjacency matrix A. Let b be a vector of nodes' *intrinsic importance*, i.e. its importance without considering network linkages. Let $\mathscr{B}_i$ be a measure of node i's importance, either intrinsic (b_i) or via links with other important nodes:

$$\mathscr{B}_i = \phi \sum_j A_{ij}\mathscr{B}_j + b_i. \tag{13}$$

The parameter ϕ describes the strength of network effects compared to intrinsic importance. Equation (13) can be written and solved in matrix form:

$$\mathscr{B}(A,\phi,b) = \left[I - \phi A\right]^{-1} b.$$

This measures of importance, or centrality, is also called the *Bonacich centrality measure*.

Mapping to the equity portfolio in this paper, we can write the equity portfolio as:

$$\Lambda' = \left[\mathbf{I} - \alpha M'\right]^{-1}(\theta M').$$

Thus, the equity portfolio of country i is precisely a vector of Bonacich centrality measure with the adjacency matrix M', network strength α, and intrinsic importance equals $\theta M_{\text{row } i}$.

$$\Lambda_{\text{row}i} = \mathscr{B}(M', \alpha, \theta M_{\text{row } i}).$$

Intuitively, country i should hold higher equity shares in countries that are more central (in the Bonacich sense) to country i. The relevant network for portfolio consideration, however, is not the trade network but the network described by the IDWs because the latter more correctly captures income risks in the presence of intermediate input trades.

2.3.2. *Intuition for the relative price-hedging bond portfolio* $\widetilde{B}^P$. The bond portfolio $\widetilde{B}^P$ hedges against the income risk coming from relative price fluctuations, i.e. the second term in Lemma 2. Let us first consider how changes in relative prices of intermediate goods affect relative GNI:

$$\frac{\partial \text{GNI}_t}{\partial \widehat{p}_t} = \underbrace{[\theta I + \alpha \Lambda]}_{\frac{\partial \text{GNI}_t}{\partial \text{GDP}_t}} \underbrace{(-\overline{M}^P)}_{\frac{\partial \text{GDP}_t}{\partial \widehat{\text{p}}_t}}. \tag{14}$$

The second term, $\overline{M}^P$, captures how relative price changes affects country GDPs via the expenditure switching effect. The first term describes how GDP changes affects GNI: a fraction θ of GDP is labor income that enters GNI directly, while a fraction α of GDP is the capital income that gets paid out to international investors via the equity holding structure Λ.

The relative price-hedging bond portfolio then must have payoffs that exactly compensate the income risk for all realizations of $\widehat{p}$:

$$\widetilde{B}^P \widehat{p}_t = [\theta I + \alpha \Lambda] \overline{M}_P \widehat{p}_t. \tag{15}$$

which is the portfolio given in Proposition 1.

In general, when the elasticity of substitution is greater than 1, a domestic price increase causes consumers to substitute away from the domestic good, causing an income loss. This is reflected through positive diagonal elements of $\overline{M}_P$. When equities exhibit full home bias ($\Lambda = I$), the relative price-hedging bond portfolio is exactly $\overline{M}_P$ and also has large positive diagonal elements, implying an optimal long position on the domestic bond. Intuitively, this happens because the bond returns is high (high $P_{i,t}$) precisely when there is income loss due to substitution.

Interestingly, as the level of equity home bias is reduced, the long domestic bond position is also reduced. In particular, the domestic bond position is:

$$\widetilde{B}^P_{ii} = \sum_{j=1}^{N} (\theta + \alpha \Lambda_{ij}) \overline{M}^P_{ji}.$$

All else equal, an ε reduction in domestic asset holding also reduces domestic bond holding by $\overline{M}^P_{ii}$. This happens a reduction in domestic equity holding breaks the link between GDP and GNI: a relative price increase can lower domestic output and value added while not affecting domestic income as much if domestic residents own less of the domestic production..

2.3.3. *Intuition for the exchange rate-hedging bond portfolio.* The discussion thus far focuses on the case of log utility. In this case, it is optimal to hold relative GNI constant. If investors have a higher level of risk-aversion ($\sigma < 1$), they additionally want to shift expenditures into states where their domestic final good becomes expensive, and vice versa for the case of $\sigma > 1$. They can use bond holding for this purpose, as described below.

Suppose some combination of shocks have made certain final goods more expensive than others, captured by a realized vector $\widehat{q}$ of final good prices. Complete markets imply that country i wants to increase its consumption expenditure by an amount $\widehat{\widetilde{c}}_{i,t} = (1-\sigma)\widehat{q}_{i,t}$. Rewriting in matrix form, we have:

$$\widehat{\widetilde{c}}_t = (1-\sigma)\widehat{q}_t.$$

It is then straightforward to calculate the extra income needed to hedge against the exchange rate risk:

$$\frac{\partial \mathrm{GNI}_t}{\partial \widehat{q}_t} = \underbrace{[\theta I + \alpha\Lambda]}_{\frac{\partial \mathrm{GNI}}{\partial \mathrm{GDP}}}\underbrace{\overline{M}D_{\widetilde{C}}}_{\frac{\partial \mathrm{GDP}}{\partial \widehat{\widetilde{c}}_t}} \cdot \underbrace{(1-\sigma)}_{\frac{\partial \widehat{\widetilde{c}}_t}{\widehat{q}_t}}.$$

The optimal exchange rate-hedging bond portfolio has payoffs that exactly compensate for the budget surpluses/shortages induced by changes in the desired consumption levels:

$$\widetilde{B}^{ER}\widehat{p} = (1-\sigma)\left[I - (\theta I + \alpha\Lambda)\overline{M}\right]D_{\widetilde{C}}\widehat{q}.$$

Using $\widehat{q} = \overline{\Xi}\widehat{p}$ and $(\theta I + \alpha\Lambda)\overline{M} = \Lambda$ gives the exchange rate-hedging bond portfolio:

$$\widetilde{B}^{ER} = (1-\sigma)[I-\Lambda]D_{\widetilde{C}}\overline{\Xi},$$

where $D_{\widetilde{C}} = \mathrm{diag}(\overline{\widetilde{C}})$.

To get at the core intuition, let us focus on the case where agents only consume their domestically produced good ($\overline{\Xi} = I$). In this case, the exchange rate-hedging portfolio can be written as:

$$\frac{\widetilde{B}^{ER}_{ij}}{\widetilde{C}_j} = (1-\sigma)(1_{i=j} - \Lambda_{ij}).$$

Thus, country i holds a position of bond j that is proportional to the its net equity position of country j. If investors are more risk averse than log utility ($\sigma < 1$), then the theory predicts that (1) agents go long on domestic bond and short foreign bonds, and (2) the long domestic position is higher if a higher fraction of domestic stocks is held by foreigners.

Prediction (1) yields from the fact that the domestic bond pays higher returns in states of higher consumption (both positively correlated with domestic price $\widehat{p}$). Prediction (2) is due to a "leakage" phenomenon: if foreigners own a large part of domestic production, a higher fraction of domestic expenditure "leaks" to foreign investors instead of becoming domestic income. This implies that hedging against exchange rate risk requires a larger expenditure transfers between states, done via a higher domestic bond position.

3. Explaining International Portfolio Investment in the data

This section evaluates the empirical usefulness of the theory in explaining actual portfolio equity investments that we observe in the data. The model has predictions for both the country equity and bond portfolios. Equity in the model corresponds to claims on capital and maps naturally to equity investment in the data. Bonds, on the other hand, are modeled as claims on the country-specific good and do not have a natural counterpart. An empirical analysis of bonds requires data on bonds used for hedging motives (for example, currency forward contracts). Without that data available, I focus on the equity portfolio here.

I perform the analysis in two steps: I first focus on the composition of the external portfolio in section 3.1, before turning to explaining asset home bias in section 3.2.

3.1. Explaining the composition of the external portfolio.

This section compares the optimal portfolio presented in Proposition 1 (henceforth, *network portfolio*) with the bilateral portfolio equity investment recorded in the Coordinated Portfolio Investment Surveys (CPIS) dataset. I calculate the network portfolio using trade shares matrices (Ω, Ξ) taken from the World Input-Output Database (WIOD):

$$\Omega_{ij} = \frac{\text{Export from } i \text{ for use as intermediate input in } j}{\sum_k \text{Export from } k \text{ for use as intermediate input in } j}.$$

$$\Xi_{ij} = \frac{\text{Export from } i \text{ for use as final consumption in } j}{\sum_k \text{Export from } k \text{ for use as final consumption in } j}.$$

The WIOD dataset records annual total export flows from a country-sector pair to another country-sector pair, decomposed into usage for intermediate versus final consumption. The dataset covers 43 countries, including 28 EU countries, 14 other major advanced and emerging economies, and a Rest of World (ROW) aggregation for the remaining countries. Importantly, WIOD contains the decomposition of export flows for intermediate uses (56 sectors) and for final uses (5 sectors). The data is available for 1997, and every year from 2000 to 2014. Sectoral data is aggregated the national level to map directly to the baseline model, but a more granular analysis at the sector level can be done using the result in section 4.1. For the purpose of analyzing the country portfolio, granular sectoral analysis does not change the main results presented here.

Having calculated the trade share matrices (Ω, Ξ), the network portfolio rule is given by Proposition 1:

$$\Lambda = \theta M [\mathbf{I} - \alpha M]^{-1}, \qquad \text{with } M \equiv \left[\mathbf{I} - \gamma \Omega'\right]^{-1} \Xi'.$$

Note that Λ_{ij} is the portfolio fraction of j's equity market cap owned by i. The portfolio share of country j in the external equity portfolio of country i is given by:

$$\text{Theory equity share}_{ij} = \frac{\Lambda_{ij} P_j^S}{\sum_{k \neq i} \Lambda_{ik} P_k^S}, \tag{16}$$

where P_j^S is the stock market cap of country j in the steady state.[9] In calculating the theory portfolio, the intermediate input shares γ are taken from the data. I calibrate the capital α and labor share θ of value added to be $\alpha = 1 - \theta = 1/3$, besides which the calculation of equity portfolios is done with zero free parameters.

The portfolio shares (16) is then compared to its data counterpart, which is calculated from the IMF's Coordinated Portfolio Investment Surveys (CPIS) data set. The CPIS data contains bilateral equity and bond investment data for more than 200 countries over the period 2001-2018. Since the WIOD country sample is more restrictive, I aggregate the non-WIOD countries into a Rest of World (ROW) block. Being a data set of external investments, CPIS does not contain investments into own country. The data equity share between origin i and destination j for a given year is given by:

$$\text{CPIS equity share}_{ij} = \frac{\text{Portfolio equity investment from } i \text{ to } j}{\text{Total portfolio equity investment from } i \text{ to all countries } k}. \tag{17}$$

In the analysis, I drop the country pairs for which the CPIS equity share is extremely small (less than 0.001%).

The CPIS data is subject to the "residency" vs. "nationality" problem (Coppola et al., 2020). In particular, since global firms sometimes finance themselves via shell companies in tax havens or financial centers, CPIS data often obscures the true nationality of the issuers or investors. For example, Coppola et al. (2020) shows that US investment into China is understated while investments into tax havens are overstated. Indeed, the model struggles to explain the large equity inflows into small tax havens that are otherwise insignificant in the global trade network, and this reflects a data problem rather than an error of the model. To resolve this problem, I use the equity portfolio reallocation matrices provided by Coppola et al. (2020) to "restate CPIS," i.e. restating flows into tax havens to the correct ultimate issuing countries. Coppola et al. (2020) does not provide reallocation matrices for all investor countries, and I retain the original CPIS flows in those cases.

[9]In the steady state, the stock market cap of each country can be shown to be the eigenvector of the optimal portfolio rule matrix Λ corresponding to an eigenvalue of 1.

3.1.1. *Fit of network portfolio.* To evaluate the performance of the network portfolio, I run a simple baseline regression:

$$\log\left(\text{CPIS portfolio share}_{ij}\right) = \alpha + \beta \log\left(\text{Network portfolio share}_{ij}\right) + \varepsilon_{ij}. \tag{18}$$

Here, the indices (i, j) denotes a distinct country-pair ($i \neq j$). If the theory is a "perfect" explanation of the data, one expects $\alpha = 0$ and $\beta = 1$ and an R^2 of 1. Data for the year 2005 is used for the empirical analysis, but similar results are obtained for other years and reported in the online appendix.

This regression is reported in column (1) of Table 2. The network portfolio share is a statistically significant predictor of the actual portfolio share in the data with a estimated coefficient $\widehat{\beta} = 1.19$. That $\widehat{\beta} > 1$ suggests that the model slightly under-predicts the data. The network portfolio alone explains half of the variation in bilateral equity portfolio holding ($R^2 = 0.48$). Figure 1 plots the network portfolio shares on the x-axis and the CPIS portfolio shares on the y-axis. It is easy to see that the network portfolio fits the data well, reflecting the high R^2 of the regression. There are a few outliers on the upper-left corner, for example Russia (RUS)'s holding of Cyprus (CYP) and Czech (CZE)'s holding of Malta (MLT) equities, which again reflects the "nationality vs. residency" symptom that plagues the CPIS data.

I benchmark the network portfolio's performance against two other parsimonious models: "size" and distance. The "size model" is the world CAPM model: if all countries share the same preferences (which make their portfolio optimization problems identical), they would hold the same portfolio in equilibrium. In this case, the portfolio share of country j in country i's external portfolio is proportional to country j's share of the world market capitalization excluding i. To test this simple theory of bilateral holding, I regress the log CPIS portfolio share against the log market caps of the origin and destination and report results in column (2) of Table 2. The theory would predict a coefficient of 1 for the destination market cap, and close to zero for the origin. Column (2) shows that the coefficient for the destination country's market cap is 0.77 (0.04), while that for the origin is statistically indistinguishable from zero. This model performs slightly worse than the network portfolio ($R^2 = 0.41$), and has a large negative intercept of -7. This means that the data shares is typically smaller than that predicted by the size model by a factor of 10^{-7}, a problem that does not exist for the network portfolio in column (1). The tight confidence interval also allows us to reject the null hypothesis of everyone holding a world CAPM portfolio (of which, the coefficient in front of the destination's market cap should be 1).

Another alternative theory is that portfolio shares are influenced by informational frictions, which I proxy using the geographical distance between the origin and destination country. To test this, I augment the "size" model in column (2) by including a measure of geographic distance. Column (3) reports the result of this specification: the model, with 3

	Dependent variable: CPIS portfolio share		
	(1)	(2)	(3)
Network portfolio share	1.19***		
	(0.05)		
Market cap, origin		0.06	0.19***
		(0.04)	(0.03)
Market cap, destination		0.77***	0.89***
		(0.04)	(0.04)
Distance			-0.76***
			(0.06)
Constant	-0.05	-7.04***	-5.72***
	(0.09)	(0.34)	(0.35)
Adj. R-squared	0.48	0.41	0.51
Number of observations	684	684	684

TABLE 2. Regression results for three models: network, size, and distance

Column (1) reports the regression result of data portfolio share (CPIS) on network equity portfolio share as predicted by the theory (construction detailed in section 3.1). The model is compared against two alternative simple models: "size" and "distance", reported in corresponding columns (2) and (3). Numerical variables are in log form, so coefficients can be interpreted as elasticities. Heteroskedasticity-consistent standard errors are given in parentheses. Data: CPIS, WIOD in 2013.

explanatory variables, does only slightly better than the network model ($R^2 = 0.51$). The coefficient for the market cap of the destination is slightly higher, but we can still reject the null that it is 1. This model still suffers from having a large and negative intercept (-5.72). Furthermore, the effect of distance is also large, with a coefficient of -0.76 (SE = 0.06). We will return to discussing the effect of distance in the next section.

Network portfolio in a gravity model for assets. Next, I use a larger set of variables often included in gravity equations in the trade literature to proxy for familiarity and/or frictions, and see whether the network portfolio retains its explanatory power after controlling for these factors. Several authors have suggested estimating a gravity equation for assets that is similar to the popular gravity model in the international trade literature.[10] Portes and Rey (2005) empirically estimated a gravity model for equity trade flows[11] of the type:

$$\log\left(\text{Equity flow}_{ij}\right) = \beta_1 \log(\text{mkcap}_i) + \beta_2 \log(\text{mkcap}_j) + \delta' \tau_{ij} + u_{ij}$$

[10]For a review of the gravity model in trade, see Anderson (2011).

[11]As opposed to explaining asset holdings, a stock concept.

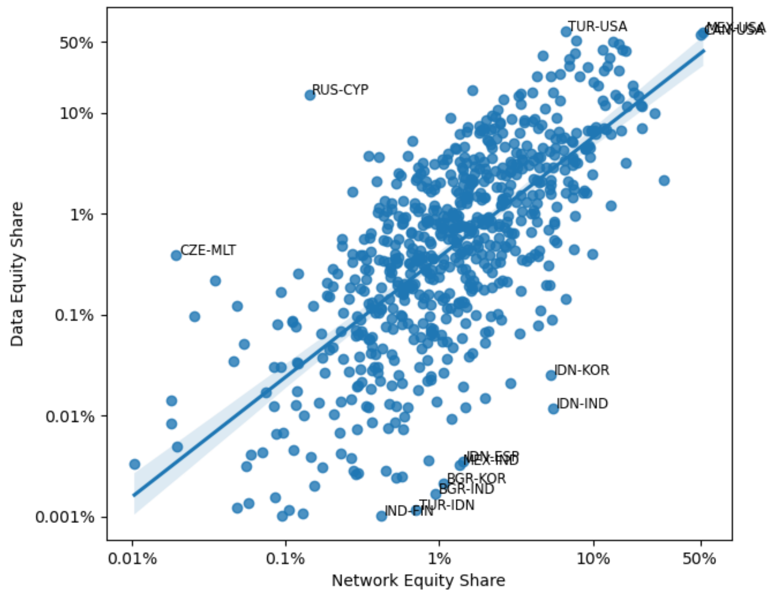

FIGURE 1. Bilateral portfolio equity shares: network portfolio versus CPIS data
This figure plots the CPIS portfolio equity shares against the theoretical network shares predicted by the model. Theoretical shares were calculated using WIOD data on international input-output linkages, as detailed in section 3.1. Equity share here is the total equity investment in a particular foreign destination divided by total foreign equity investment by a given origin country. CPIS data has been "restated" to account for indirect investments through tax havens and financial centers using the procedure by Coppola et al. (2020). Some points in the scatter plot have been labeled in the format "origin-destination." The adjusted R-squared of the corresponding regression including the constant is 0.49, and the regression slope is 1.17 (standard error = 0.04). Data: CPIS, WIOD in 2013.

where τ_{ij} is a vector of control variables that represent transaction costs in the asset market.[12] In Table 3, I compare the network portfolio presented in this paper with the gravity model. Column (1) repeats the baseline regression result in Table 2. Column (2) reports a gravity model that includes market caps, distance, and proxies of trade frictions such as contiguity, common language, historical dependence (colony). I also include two dummies for the US as an origin or destination due to its special role in international finance. Unsurprisingly, the gravity model (column 2, $R^2 = 0.57$), with 16 variables (and 17 free parameters

[12]While popular, the theoretical underpinning of a gravity model for asset less understood, and there are few papers on this topic. An exception is Chaney (2018), who provides the first theory of the role of distance in gravity equation for trade using a network model of contact acquisition. In terms of gravity models for assets, Martin and Rey (2004) provides a model in which Arrow-Debreu assets are endogenously created and traded internationally, and gives rise to a gravity equation. The Arrow-Debreu assets traded in this framework, however, do not resemble equities in the data (Okawa and van Wincoop, 2012).

including the constant term), explains the data better than the network model (column 1, $R^2 = 0.48$), with 2 free parameters including a constant term.

Many variables are significant predictors in the gravity model. Most notably, the coefficient of the destination's market cap is now 0.97 (SE = 0.07) is now in line with a CAPM model. There is a large and negative effect of distance, as in a simpler regression before, of -0.72 (SE = 0.08). Including the network portfolio into the gravity regression, however, makes the effect of key gravity variables disappear. In particular, destination market cap and distance are no longer significant predictor of equity shares. Meanwhile, controlling for frictions makes the coefficient in front of network portfolio become 1.00 (SE=0.13). Figure 2 plots the data versus network portfolio shares again, now having residualized out all that can be explained by gravity variables. The partial R^2 of the network portfolio is 0.08.

Other robust variables include outflow capital control at the origin country (negative effect) and a US destination indicator (positive). The effect of capital control is expected: if investors want to make outward investment, but legally barred by their country from doing so, will not hold as high a share of foreign equities as the optimal theory predicts they would. The positive coefficient of US as destination points to the special status of the US in the global trade and financial system. Finally, whenever the network portfolio is included, the significantly negative constant term disappears.

It is also worth noting that the model taken literally implies that once the estimated network portfolio share is held fixed, the other explanatory variables should have coefficients of zero. This hypothesis is rejected for some variables individually (particularly for outflow capital control), and for all non-network portfolio variables jointly (p-value = 0.000). The significance of gravity variables after controlling for the network portfolio share can be due to possible measurement errors for the network portfolio shares, which can come from the measurement errors in the input - output data. Another reason is that the gravity variables proxy for frictions that are not present in the model.

Effect of distance on asset holdings. Distance is a significant predictor of bilateral asset holdings, as shown in column (2) of Table 3. The coefficient (elasticity) of log distance is -0.72, which is slightly smaller than previous estimates.[13] However, presuming other covariates have proxied well for informational cost, it remains puzzling why physical distance matters at all for trade in assets. Adding the network portfolio to the gravity regression resolves the distance puzzle. Column (3) of Table 3 shows that the effect of distance becomes statistically insignificant when network portfolio is added. In summary, the empirical result in Table 3 shows that the effect of distance on international trade in assets is purely through its effect

[13] Portes and Rey (2005) estimates a coefficient in the range of $(-0.9, -0.5)$ for distance, depending on specifications.

	Dependent variable: CPIS portfolio share		
	(1)	(2)	(3)
Network portfolio share	1.19***		1.00**
	(0.05)		(0.13)
Market cap, origin		0.13**	0.15**
		(0.06)	(0.06)
Market cap, destination		0.97***	0.07
		(0.07)	(0.14)
Distance		-0.72***	-0.10
		(0.08)	(0.10)
Outflow capital control of origin		-0.34***	-0.36***
		(0.08)	(0.07)
Inflow capital control of destination		-0.14	-0.13
		(0.10)	(0.10)
Destination = US		0.58***	0.70***
		(0.11)	(0.11)
Origin = US		0.32***	0.16
		(0.10)	(0.10)
Destination = US		0.2684	0.4775
		(0.104)	(0.101)
Common official language		0.22***	0.19***
		(0.07)	(0.07)
Market cap/GDP, origin		-0.10	-0.12
		(0.14)	(0.13)
Market cap/GDP, destination		-0.41***	0.48***
		(0.14)	(0.18)
Constant	-0.05	-5.95***	6.175
	(0.09)	(0.78)	(0.844)
Adj. R-squared	0.48	0.56	0.59
Number of observations	684	684	684

TABLE 3. Regression result of network model versus gravity model

Column (1) reports again the baseline regression result of data portfolio share (CPIS) on network equity portfolio share (detailed in section 3.1). Column (2) regresses CPIS equity share against bilateral covariates often included in a gravity equation. Column (3) adds the network portfolio to the gravity model to show that network portfolio has explanatory power beyond the gravity factors. Numerical variables are included in log form, so coefficients can beinterpreted as elasticities. Heteroskedasticity-consistent standard errors are given in parentheses. Other gravity variables included in the regression but are statistically insignificant and are not reported here in the interest of space: contiguity, EU membership indicators for origin and destination, WTO member indicators. Data: CPIS, WIOD, CEPII gravity database. * p < 0.1, ** p < 0.5, *** p < 0.01.

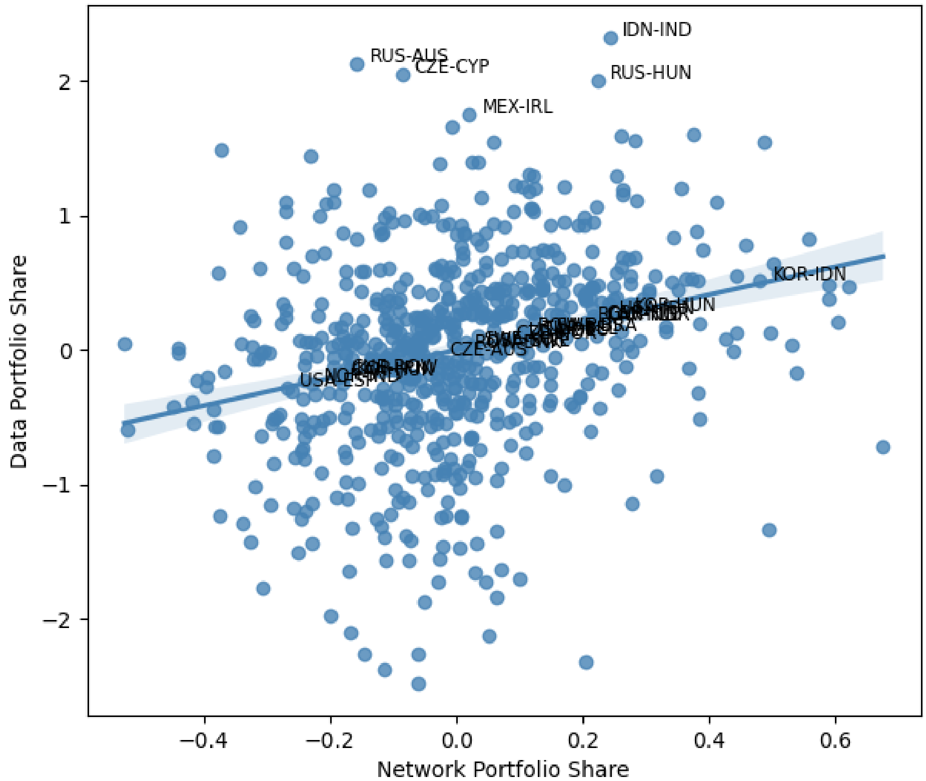

FIGURE 2. Partial regression plot: network portfolio versus CPIS data

This figure is the partial regression plot of CPIS portfolio equity shares against the theoretical network shares predicted by the model. Both variables were residualized by gravity variables (market caps, common language, contiguity, etc.). The partial R-squared associated with this graph is 0.08.

on trade in goods. Holding constant the trade structure, distance by itself does not have an effect on asset holdings.

Role of indirect linkages. In this section, I test for the role of indirect trade linkages in determining optimal equity portfolios by controlling for the "direct-trade-only portfolio." The optimal equity holding in the model is a nonlinear function of the trade matrices (Ω, Ξ). In particular, performing a Taylor expansion on the optimal equity holding matrix Λ gives:

$$\Lambda = \theta M [\mathbf{I} - \alpha M]^{-1} = \theta M + \theta \alpha M^2 + \text{higher order terms}$$

The matrix of international Domar weights is in turn given by:

$$M = (1-\gamma) [\mathbf{I} - \gamma \Omega']^{-1} \Xi' = (1-\gamma) \Xi' + (1-\gamma) \gamma \Omega' \Xi' + \text{higher order terms}$$

So, to the first-order, optimal equity holding Λ is simply proportional to the transpose of the consumption matrix Ξ.[14] This portfolio has the portfolio decision rule $\Lambda_{ij} = \Xi'_{ij} = \Xi_{ji}$, so that the portfolio share of j in i's portfolio is given by:

$$\text{Direct-trade-only portfolio}_{ij} = \frac{\Xi_{ji} P_j^S}{\sum_{k \neq i} \Xi_{ki} P_k^S},$$

where again P^S denotes the steady state stock market cap.

For completeness, I also consider several candidate explanations for portfolio shares. The first candidate is the destination's share in the origin's total imports, inspired by the consumption-based mechanism presented in Obstfeld and Rogoff (2001). The second candidate is "trade intensity," defined as the ratio of the bilateral trade volume to the sum of the origin and destination's GDPs. This is a model-free variable that is popular in the literature. Finally, I also test for "familiarity" – the hypothesis that investors tend to be more familiar with their large trade partners. The familiarity variable is then constructed as the destination's share of the origin's total trade (import plus export).

The result is presented in Table 4. Column (1) shows again the result of the baseline regression (18), while columns (2) - (6) report the performance of alternative variables: direct links only, import shares (Obstfeld and Rogoff (2001)), trade intensity, and familiarity. The network portfolio explains the data the best ($R^2 = 0.49$), and significantly better than the model where only direct trade links are considered ($R^2 = 0.02$) and the model-free variable trade intensity ($R^2 = 0.25$). Import shares (OR 2001) and familiarity have relatively high explanatory power, with R^2 equal 0.41 and 0.45 respectively. However, in a horse race with the network portfolio, these two variables become insignificant.

In sum, this section shows that indirect trade linkages, not just direct trades, matter in explaining international equity portfolios. This result highlights the need to account for intermediate inputs and their trade networks in theoretical and empirical works.

3.2. **Explaining Home bias.** I now turn to evaluate the model's predictive power on equity home bias.

I measure equity home bias (EHB) in the data as the share of country equity portfolio made up by domestic assets:

$$\text{Data EHB}_i = \frac{\text{Domestic equity investment}_i}{\text{domestic equity investment}_i + \text{external equity investment}_i}$$

The external equity component of a country's portfolio is calculated simply by summing its portfolio investments across destinations using CPIS data. However, CPIS does not include the value of domestic equities held by domestic investors. I calculate this object as the total

[14]This first-order term is the exact result, not approximation, in a model without intermediate inputs ($\gamma = 0$).

	Dependent variable: CPIS portfolio share					
	(1)	(2)	(3)	(4)	(5)	(6)
Network portfolio share	1.19***					0.96***
	(0.05)					(0.16)
Direct links only		0.26***				-0.13*
		(0.08)				(0.07)
Import shares (OR 2001)			0.84***			0.00
			(0.05)			(0.14)
Trade intensity				0.81***		0.15
				(0.05)		(0.11)
Familiarity					0.98***	0.14
					(0.04)	(0.22)
Constant	-0.05	-1.76***	-0.61***	-0.18	-0.35***	-0.10
	(0.09)	(0.17)	(0.09)	(0.14)	(0.08)	(0.18)
Adj. R-squared	0.48	0.02	0.41	0.25	0.45	0.49
Number of observations	684	684	684	684	684	684

TABLE 4. Explanatory power of network portfolio after controlling for direct trade measures

Column "Network" reports the regression result of data portfolio share (CPIS) on network equity portfolio share as predicted by the theory. This portfolio is constructed in section 3.1 using WIOD data. All numerical variables are in log form. The variable "direct-trade-only portfolio" is the portfolio constructed by ignoring intermediate input-output linkages and looking at direct imports only. "Import share" is the destination's share of the origin's total imports. "Trade intensity" equals total trade volume divided by the sum of the GDPs of the origin and destination. "Familiarity" refers to the destination share of the origin's total trade (import plus export).

equity market cap less equity held by foreigners:

$$\text{Domestic equity investment}_i = \text{Market cap}_i - \text{Equities held by foreigners}_i.$$

I smooth out the time series of equity home bias by calculating a 5-year moving-average for each year in the sample. As the theory put forth in this paper focuses more on longer-run changes, smoothing out the time series helps focus more on the trend of the data and less of transitory movements. Figure 3 plots the equity home bias in the data for five major economies: US, Japan, France, Germany, and Brazil. We can see that there is a large heterogeneity in levels of equity home bias. France and Germany tend to be more diversified than larger economies like US or Japan. We can also see that the level of asset home bias has decreased across the board (with the exception of Brazil). In fact, in my sample, the average home equity bias has declined from 0.88 in 1997 to 0.77 in 2013.

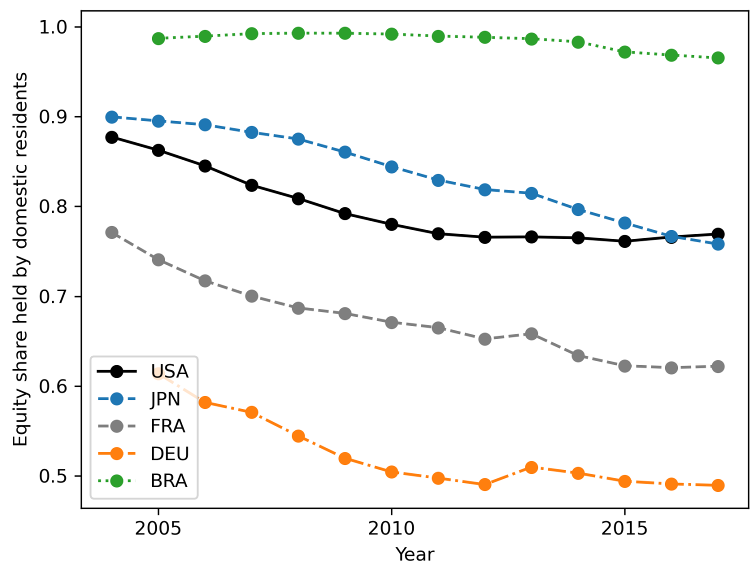

FIGURE 3. Declining asset home bias in the data

Next, I compare how well the theory can explain the equity home bias observed in the data. Again, let Λ be the optimal equity portfolio given in Proposition 1 and P_i^E the stead-state stock price (market cap). The theoretical equity home bias is given by:

$$\text{Theory equity home bias} = \frac{\Lambda_{ii} P_i^E}{\sum_j \Lambda_{ij} P_j^E},$$

I then regress:

$$\text{Data equity home bias}_i = \alpha + \beta \cdot \text{Theory equity home bias}_i + \text{controls} + \varepsilon_i, \tag{19}$$

where i denotes country. In addition to the theory portfolio, and informed by the gravity regression, I include measures of capital controls that could make the equity home bias higher in the data than predicted by the model.

Table 5 reports the results of regressing data home bias on the theory's prediction. The first column of Table 5 reports the cross-country regression in *level* using data of the year 2010. The theory equity home bias level is a significant predictor of the data, with a coefficient equal 0.98 (0.25). We can also see that countries with higher capital controls on outward investments see a higher equity home bias. The variable market cap, included to test against the null of the simple world CAPM model, is insignificant. The R^2 of the regression is 0.73. Figure 4 visualizes the correlation between network versus data home bias, having residualized out the other covariates. The partial R^2 of the network portfolio is 0.32.

	Dependent variable: Equity home bias, data Level, 2010
Equity home bias, theory	0.98***
	(0.25)
Market cap	-0.10
	(0.03)
Capital control	0.46***
	(0.09)
Constant	0.66***
	(0.13)
Adj. R-squared	0.73
Number of observations	20

TABLE 5. Regression results for equity home bias, theory vs. data

This table reports the regression result of data equity home bias versus that predicted by the network model. Data from CPIS and WIOD, with author's calculations. Column "Level, 2010" refers to the regression in level (2010 is the year with median R-squared), and column "Changes, 2015-2014" reports the corresponding regression in log changes.

The model also does quite well in explaining changes in the trend of equity home bias in the cross section. Figure 5 plots the model's predicted changes in country equity home bias (which comes from the increase in intermediate good trade in the data) versus the actual equity home bias in the data. While the fit is less good than the level regression ($R^2 = 0.14$), the theory is a significant predictor of the data, with a slope of 1.15.

4. EXTENSIONS

4.1. **Model with multiple intermediate sectors.** So far, I have assumed that each country has a unique country-specific intermediate good. I now show that the analysis can be more "granular" and extend the model to allow for many intermediate industries within a country.

Let $\mathscr{S}_i$ be the set of intermediate industries in country i, and $\mathscr{S} = \cup_{i=1}^{N} \mathscr{S}_i$ is the set of intermediate industries of the world. Each industry is now given as a nation-industry pair, e.g. "USA, Agriculture" versus "Japan, Agriculture." The production function of intermediate sector s is now given by:

$$Y_{s,t} = Z_{s,t}^{Y} \left((K_t^s)^{\alpha} (L_t^s)^{\theta} \right)^{1-\gamma} \left(\sum_{k \in \mathscr{S}} \omega_{sk}^{1/\varepsilon} X_{sk,t}^{1-1/\varepsilon} \right)^{\frac{\gamma\varepsilon}{\varepsilon-1}}, \quad \forall k \in \mathscr{K}.$$

The final good i is now produced with technology:

$$G_t^i = \left(\sum_{k \in \mathscr{S}} \xi_{ik}^{1/\varepsilon} G_{ik,t}^{1-1/\varepsilon} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad \forall i \in \mathscr{N}.$$

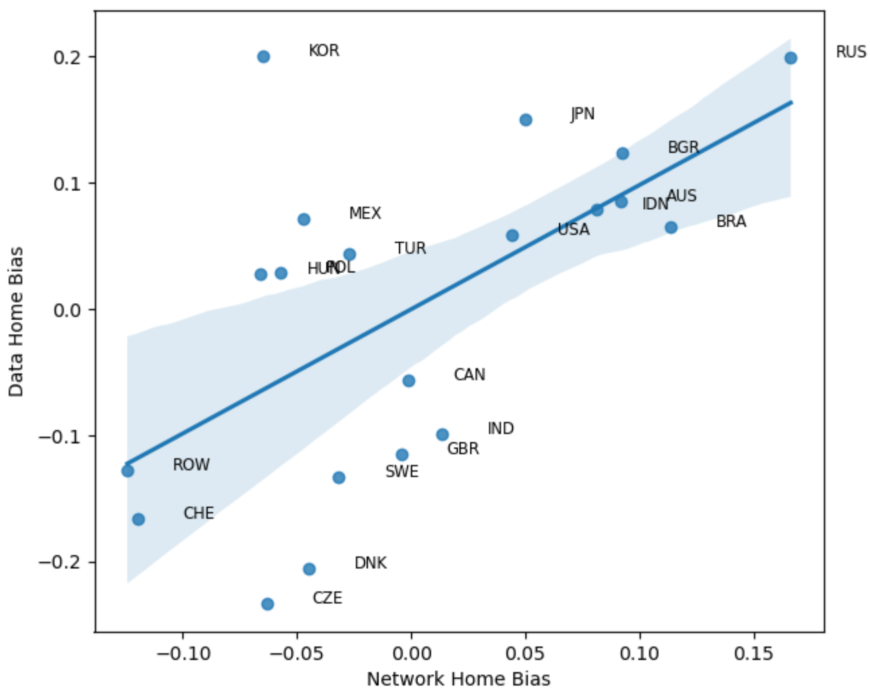

FIGURE 4. Model fit of asset home bias in level.
This scatterplot visualizes the fit of the *level* of equity home bias predicted by the network model versus that observed in the data.Data: CPIS, WIOD, and author's calculations.

Let Ω and Ξ be the matrix of trade shares as before. Ω now has dimension $S \times S$, with $S = |\mathscr{S}|$ is the total number of intermediate sectors in the world; Ξ has dimension $N \times S$. Note that each intermediate sector has its own idiosyncratic TFP shock $Z^Y_{s,t}$. Investors can buy claims on capital for each industry, so the number of assets still matches the number of shocks. Let Λ_{is} denote the fraction of shares of industry s owned by investors of country i, and $\Lambda_{N \times S}$ be the portfolio matrix.

Define the following "modified identity matrix":[15]

$$E = \begin{bmatrix} 1_{1\times S_1} & 0 & \dots & 0 \\ 0 & 1_{1\times S_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1_{1\times S_N} \end{bmatrix} \tag{20}$$

[15] E serves the role of an aggregating matrix. Suppose $\widetilde{Y}^*$ is a $|\mathscr{K}| \times 1$ vector of output for each industry, then $\widetilde{Y} = E\widetilde{Y}^*$ is an $N \times 1$ vector of output for each country, with $\widetilde{Y}_i = \sum_{k\in\mathscr{K}_i} \widetilde{Y}_k$.

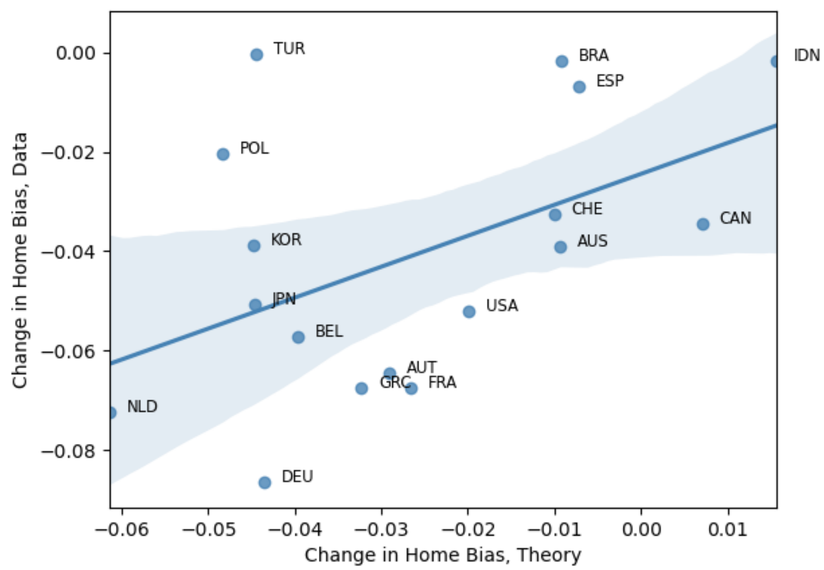

FIGURE 5. Model fit of asset home bias in changes, 2005-2014.
This scatterplot visualizes the fit of the changes in equity home bias as predicted by the theory versus that observed in the data. Data: CPIS, WIOD, and author's calculations. $R^2 = 0.14$, and slope of best fit line equals 1.15.

The matrix of International Domar Weights (IDWs) is still exactly as before:

$$\overline{M} = (1-\gamma)\left[\mathbf{I}-\gamma\overline{\Omega}'\right]^{-1}\overline{\Xi}'.$$

The optimal equity portfolio is now given by a modified version of the portfolio in Proposition 1.

Proposition 2. *The optimal portfolio when there are multiple industries within each country is given by:*

$$\Lambda = \theta E\overline{M}E\left[\mathbf{I}-\alpha\overline{M}E\right]^{-1}.$$

where E is the modified identity matrix (defined in (20)) and $\overline{M} \equiv (1-\gamma)\left[\mathbf{I}-\gamma\overline{\Omega}'\right]^{-1}\overline{\Xi}'$ is the modified IDWs.

Proof. See Appendix A.4 □

When $S_i = 1$ for all i, E is the identity matrix, and we recover the optimal portfolio in Proposition 1.

4.2. **Model with nontraded sector.** The model so far has abstracted from a nontraded sector. Adding a nontraded sector may be desirable in quantitative models that attempt to match low international consumption correlations or deviations from the Purchasing Power

Parity (PPP). I now show that the baseline model in this paper can incorporate in nontraded sectors without complicating the portfolio analysis.

Suppose that a share ι of final consumption and investment now comes from the nontraded sector:

$$G_{it} = \left[\left(\sum_j \xi_{ij}^{1/\varepsilon}\left(G_{jt}^i\right)^{1-\frac{1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right]^{1-\iota} D_{it}^{\iota}$$

For simplicity, suppose that the nontraded good in country i is produced using labor only: $\overline{D}_t^i = z_{it}^{NT} L_{it}^{NT}$. In equilibrium, since all nontraded goods must be consumed domestically, we have $D_t^i = \overline{D}_t^i$. On the asset side, investors in all countries can buy claims on the nontraded sector in other countries.

Proposition 3. *The optimal equity portfolio when nontraded goods are present is given by:*

$$\Lambda = \left[\iota\mathbf{I} + (1-\iota)\theta\overline{M}\right]\left[\mathbf{I} - (1-\iota)\alpha\overline{M}\right]^{-1},$$

where ι is the nontraded good expenditure share, and $\overline{M}$ is the steady-state IDWs.

Incorporating nontraded goods into the model tends to increase equity home bias. When $\iota = 1$ (only nontraded good), then we have perfect equity home bias $\Lambda = \mathbf{I}$. When $\iota = 0$ (no nontraded good), we get back to the original optimal equity portfolio in Proposition 1. The portfolio here is a weighted average of the two cases.

The increase in equity home bias is because domestic investors will end up holding all of the equities of nontraded sectors (Obstfeld and Rogoff, 2001). Because (i) returns have to be paid in tradable goods and (ii) nontraded goods enter separably in utility, no further risk-sharing can be achieved using the equities of nontraded sectors.

4.3. **Network Implication for International Asset Prices .** Up until now, the focus has been on portfolio quantity. I now show how the model can be used to analyze the network implications for international asset prices.

In general, it is hard to describe the behavior of asset prices in closed form in models with endogenous investment and general elasticities. Recent works by Richmond (2019) and Jiang and Richmond (2019) study the network implications for international asset prices in a model without investment and Cobb-Douglas production function. Here, I show that it is possible to study asset prices in an environment with networks and investment together, in closed form, if we assume (i) log utility, (ii) Cobb-Douglas production, and (iii) full depreciation. This specification follows after Brock and Mirman (1972), who made the same assumptions to attain a constant saving rate in a closed-economy model.

In the following, I use $\otimes$ to denote the Kronecker product, and $vec(\cdot)$ is the vectorization operator (stacking a matrix into a column).

Proposition 4. *Suppose that $\delta = \varepsilon = \sigma = 1$, and that TFP follows a random walk: $\Delta z_t = \varepsilon_t \sim \mathcal{N}(0, \Sigma)$. Then:*

(1) The covariance matrix for equity returns, $\Sigma_r = cov(r_S)$, is given by:

$$vec(\Sigma_r) = \frac{1}{(1-\gamma)^2}\left[\mathbf{I} - \alpha^2 M' \otimes M'\right]^{-1} M' \otimes M' vec(\Sigma)$$

(2) The covariance matrix of exchange rate is given by:

$$vec(\Sigma_e) = (A \otimes A) vec(\Sigma_r)$$

with

$$A = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 1 & -1 & 0 & \dots & 0 \\ 1 & 0 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & -1 \end{bmatrix}.$$

Proof. See Appendix A.7. □

This model nests the network model with consumption only by Jiang and Richmond (2019) by setting $\alpha = 0$ (no capital). In that case, the returns on equity are given by:

$$r_t^{JR} = \rho + \frac{M'}{1-\gamma}\Delta z_t.$$

Equity in Jiang and Richmond (2019) is modeled as claims on consumption stream. Innovations to productivity increase output and lower consumption price, raising stock returns. The multiplication by the transpose of IDWs represents *supply effect*: how productivity shocks upstream affecting output downstream.

In this paper, capital is present in the model. This adds another intertemporal transmission channel of productivity shocks. Since capital is built one period in advance, a productivity shock that stimulates investment increases the stock of capital, lowering the cost of capital next period, increasing output next period. Equity returns in this model are given by:

$$r_t = \rho + \frac{M'}{1-\gamma}\Delta z_t - \alpha M' \Delta \mu_{t-1}. \tag{21}$$

The last term in equation (21) represents the intertemporal channel as described: a lowering in the price of investment yesterday boosts equity returns today. The higher the capital share, the more pronounced is the effect of this channel.

Note that the network matters for the intertemporal channel as well: an investment booms yesterday in a country affects not only the return of that country, but that of its key trade partners as well. This network investment effect has implications for the correlation of returns that is captured by the terms $\left[\mathbf{I}-\alpha^2 M' \otimes M'\right]^{-1}$ in Proposition 4.

In this section, I have described how the model with investment can be used to study network implication for asset prices analytically. Evaluating quantitatively whether the model prediction changes significantly when we depart from the Brock and Mirman (1972) specification, as well as quantifying the strength of the intertemporal network channel are interesting problems for future research.

5. Conclusion

This paper was motivated by the lack of a theory that can explains the rich heterogeneity of country equity and bond portfolios that we observe in the data. I proposed the first theory of international portfolios that accounts for international production linkages and taste differences, all in a standard real business cycle setting. The generality of the framework allows us to study not just the allocation problem between Home versus Foreign (home bias), but also the composition of external portfolio as well. Empirically, I showed that the network-theory portfolio explains well the data on portfolio holding, and its significance is robust to controlling for a large variety of bilateral gravity factors.

I anticipate three directions for future research that can greatly improve our understanding of the role of networks for international portfolios. First, while I focused on the portfolio problem in a real setting without nominal frictions or nominal shocks, the paper is not meant to evaluate or understate the importance of such factors. Studying how networks interact with pricing regimes and nominal frictions will be interesting for future research. Second, this paper studies portfolios in an environment with complete markets in order to obtain insights from analytical solutions. It is important to study (perhaps quantitatively) how the optimal portfolios change when we depart from complete markets. Finally, the focus on tractability given a complex production structure has come at the cost of model ingredients that are useful in matching well moments of asset prices. Incorporating those ingredients and checking to what extent the optimal portfolios change would be a very useful exercise.

Appendix A. Proofs of Theorems and Lemmas

A.1. **Proof of Lemma 1.** The market clearing condition for intermediate good j is:

$$Y_{j,t} = \sum_{i \in \mathcal{N}} G_{ij,t} + \sum_{i \in \mathcal{N}} X_{ij,t}.$$

Multiply both sides by $P_{j,t}$ and use the tilde notation, we have:

$$\widetilde{Y}_{j,t} = \sum_{i \in \mathcal{N}} \widetilde{G}_{ij,t} + \sum_{i \in \mathcal{N}} \widetilde{X}_{is,t},$$

Use the definition of the expenditure shares matrices Ω and Ξ, we have:

$$\widetilde{Y}_{j,t} = \sum_{i\in\mathcal{N}} \Xi_{ij,t}\widetilde{G}_{i,t} + \sum_{i\in\mathcal{N}} \Omega_{ij,t}\gamma_j\widetilde{Y}_{j,t}.$$

Put in matrix form:

$$\widetilde{Y}_t = \Xi_t'\widetilde{G}_t + (\Gamma\Omega_t)'\widetilde{Y}_t$$
$$\widetilde{Y}_t = \left[I - (\Gamma\Omega_t)'\right]^{-1}\Xi_t'\widetilde{G}_t.$$

Value added is given by:

$$GDP_t = (I-\Gamma)\widetilde{Y}_t = M_t\widetilde{G}_t$$

with $M_t \equiv (I-\Gamma)\left[I-(\Gamma\Omega_t)'\right]^{-1}\Xi_t'$. □

A.2. **Proof of Lemma 2 .** Consider the market clearing condition for sector j, using tilde notation:

$$\widetilde{Y}_{j,t} = \sum_{i=1}^{N}\left(\widetilde{G}_{ij,t} + \widetilde{X}_{ij,t}\right) = \sum_{i=1}^{N}\left(\Xi_{ij,t}\widetilde{G}_{i,t} + \Omega_{ij,t}\widetilde{X}_{i,t}\right).$$

Linearize this equation around the deterministic steady state and use the hat notation to denote deviations from the steady state:

$$\widehat{\widetilde{Y}}_{j,t} = \sum_{i=1}^{N}\left(\overline{\widetilde{G}}_i\widehat{\Xi}_{ij,t} + \overline{\Xi}_{ij}\widehat{\widetilde{G}}_{i,t}\right) + \sum_{i=1}^{N}\left(\overline{\widetilde{X}}_j\widehat{\Omega}_{ij,t} + \overline{\Omega}_{ij}\widehat{\widetilde{X}}_{j,t}\right) \tag{22}$$

$$= \sum_{i=1}^{N}\left(\overline{\widetilde{G}}_i\overline{\Xi}_{ij}\widehat{\ln\Xi}_{ij,t} + \overline{\Xi}_{ij}\widehat{\widetilde{G}}_{i,t}\right) + \sum_{i=1}^{N}\left(\overline{\widetilde{X}}_j\overline{\Omega}_{ij}\widehat{\ln\Omega}_{ij,t} + \overline{\Omega}_{ij}\widehat{\widetilde{X}}_{j,t}\right) \tag{23}$$

Equation (22) decomposes changes in output of a sector into two sets of terms: (a) an increase in aggregate spending by customers ($\widehat{G},\widehat{X}$) and (b) a reallocation effect ($\widehat{\Xi}$, $\widehat{\Omega}$). The reallocation effect can be written in terms of changes in input prices (relative to the basket):

$$\widehat{\ln\Xi}_{ij,t} = (1-\varepsilon)\left(\widehat{p}_{j,t} - \widehat{q}_{i,t}\right)$$
$$\widehat{\ln\Omega}_{ij,t} = (1-\varepsilon)\left(\widehat{p}_{j,t} - \widehat{p}^X_{i,t}\right).$$

Substitute into equation (22) and collect terms:

$$\widehat{\widetilde{Y}}_{j,t} = \sum_{i=1}^{N}\left(\overline{\Xi}_{ij}\widehat{\widetilde{G}}_{i,t} + \overline{\Omega}_{ij}\widehat{\widetilde{X}}_{i,t}\right) + (1-\varepsilon)\sum_{i=1}^{N}\overline{\widetilde{G}}_i\overline{\Xi}_{ij}\left(\widehat{p}_{j,t} - \widehat{q}_{i,t}\right) + (1-\varepsilon)\sum_{i=1}^{N}\overline{\widetilde{X}}_i\overline{\Omega}_{ij}\left(\widehat{p}_{j,t} - \widehat{p}^X_{i,t}\right)$$
$$= \sum_{i=1}^{N}\left(\overline{\Xi}_{ij}\widehat{\widetilde{G}}_{i,t} + \overline{\Omega}_{ij}\widehat{\widetilde{X}}_{i,t}\right) - (\varepsilon-1)\left(\overline{\widetilde{Y}}_j\widehat{p}_{j,t} - \sum_{i=1}^{N}\overline{\widetilde{G}}_i\overline{\Xi}_{ij}\widehat{q}_{i,t} - \sum_{i=1}^{N}\overline{\widetilde{X}}_i\overline{\Omega}_{ij}\widehat{p}^X_{i,t}\right).$$

Put in matrix form:

$$\widehat{\widetilde{Y}} = \overline{\Xi}'\widehat{\widetilde{G}} + \overline{\Omega}'\widehat{\widetilde{X}} - (\varepsilon-1)\left[\Phi_Y\widehat{p}_t - \overline{\Xi}'D_{\widetilde{G}}\widehat{q}_t - \overline{\Omega}'D_{\widetilde{X}}\widehat{p}^X_t\right], \tag{24}$$

where $D_A \equiv diag(\overline{A})$ is a diagonal matrix of steady-state values for variable A in the steady state.

The CES price index of the final good i is given by $Q_{i,t} = \left[\sum_j \xi_{ij}\left(P_{j,t}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$, which is linearized into $\widehat{q}_{i,t} = \sum_j \overline{\Xi}_{ij}\widehat{p}_{j,t}$. Similarly, the price index for the intermediate good bundle of country i is given by: $\widehat{p}^X_{i,t} = \sum_j \overline{\Omega}_{ij}\widehat{p}_{j,t}$. Put in matrix form:

$$\widehat{q}_t = \overline{\Xi}\widehat{p}_t, \qquad \widehat{p}^X_t = \overline{\Omega}\widehat{p}_t. \tag{25}$$

Substitute the price indices into equation (24):

$$\widehat{\widetilde{Y}}_t = \overline{\Xi}'\widehat{\widetilde{G}}_t + \overline{\Omega}'\widehat{\widetilde{X}}_t - (\varepsilon-1)\left[D_{\widetilde{Y}} - \overline{\Xi}'D_{\widetilde{G}}\overline{\Xi} - \overline{\Omega}'D_{\widetilde{X}}\overline{\Omega}\right]\widehat{p}_t.$$

Note that $X_j = \gamma_j Y_j$, so $\widehat{\widetilde{X}} = \Gamma\widehat{\widetilde{Y}}$. We arrive at:

$$\widehat{\widetilde{Y}}_t = \overline{\Xi}'\widehat{\widetilde{G}}_t + \left(\Gamma\overline{\Omega}\right)'\widehat{\widetilde{Y}}_t - (\varepsilon-1)\left[D_{\widetilde{Y}} - \overline{\Xi}'D_{\widetilde{G}}\overline{\Xi} - \overline{\Omega}'D_{\widetilde{X}}\overline{\Omega}\right]\widehat{p}_t. \tag{26}$$

Define the International Domar Weights matrix M as in Definition 1, and the Expenditure Switching matrix M^P as in Definition 2. Re-arrange (26) concludes our proof:

$$\widehat{GDP}_t = (I-\Gamma)\widehat{\widetilde{Y}}_t = \overline{M}\widehat{\widetilde{G}}_t - \overline{M}^P\widehat{p}_t.$$

□

A.3. **The Social Planner problem.** Given a vector of Pareto weights $\chi = (\chi_1, \chi_2, ..., \chi_N)$, with $\sum_{i=1}^N \chi_i = 1$, the Social Planner maximizes

$$\sum_{i=1}^N \chi_i \left(\mathbb{E}_0 \sum_{t=0}^\infty \beta^t u\left(C_{i,t}\right)\right)$$

by choosing for each country i and time t their consumption $C_{i,t}$, investments $I_{i,t}$ (equivalently, next period capital $K_{i,t+1}$), and intermediate inputs $\{G_{ij,t}, X_{ij,t}\}_{i,j\in\mathcal{N}}$.

The Social Planner faces the resource constraints:

$$C_{i,t} + \frac{1}{Z^K_{i,t}}\left(K_{i,t+1} - (1-\delta)K_{i,t}\right) = \left[\sum_{j=1}^N \xi_{ij}^{\frac{1}{\varepsilon}}\left(G_{ij,t}\right)^{1-\frac{1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \equiv G_{i,t} \qquad \forall i \in \mathcal{N}, \tag{27}$$

$$\sum_{i\in\mathcal{N}} G_{ij,t} + \sum_{i\in\mathcal{N}} X_{ij,t} = Z^Y_{j,t}\left(K^\alpha_{j,t}L^\theta_{j,t}\right)^{1-\gamma} X^\gamma_{j,t} \equiv Y_{i,t} \tag{28}$$

$$X_{i,t} = \left[\sum_{j=1}^N \omega_{ij}^{\frac{1}{\varepsilon}} X_{ij,t}^{1-\frac{1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{29}$$

Let $\left(Q^*_{i,t}, P^*_{i,t}, P^{X*}_{i,t}\right)$ be the Lagrange multipliers corresponding to the resource constraints for final goods (27), intermediate goods (28), and the intermediate bundle (29) respectively.

The Social Planner solution is characterized by the resource constraints and the following first-order conditions:

(1) Optimal input choices:

$$g_{ij,t} - g_{i,t} = \ln \xi_{ij} - \varepsilon (p^*_{j,t} - q^*_{i,t}) \tag{30}$$

$$x_{ij,t} - x_{i,t} = \ln \omega_{ij} - \varepsilon \left(p^*_{j,t} - p^{X*}_{i,t} \right), \quad \text{where } p^{X*}_{i,t} = \ln \gamma_i + p^*_{i,t} + y_{i,t} - x_{i,t}, \tag{31}$$

(2) Optimal consumption choices:[16]

$$\ln \chi_i - \frac{1}{\sigma} c_{i,t} = q^*_{i,t}. \tag{32}$$

(3) Optimal investments:

$$\mathbb{E}_t \left[\Theta^*_{i,t+1} \left((1-\delta) \frac{Z^K_{i,t}}{Z^K_{i,t+1}} + \frac{P^*_{i,t+1}}{Q^*_{i,t+1}} \frac{\partial Y_{i,t+1}}{\partial K_{i,t+1}} Z^K_{i,t} \right) \right] = 1 \tag{33}$$

where $\Theta^*_{i,t+1} \equiv \beta u'(C_{i,t+1}) / u'(C_{i,t})$.

A.4. **Proof of Proposition 1.** The competitive equilibrium is characterized by the following equations.

Production. Sector i's profit maximization problem:

$$\max_{L_{i,t}, K_{i,t}, \{X_{ij,t}\}_{j=1}^N} P_{i,t} Y_{i,t} - W_{i,t} L_{i,t} - r_{i,t} K_{i,t} - \sum_j P_{j,t} X_{ij,t}$$

Optimal labor, capital, and intermediate input choices of sector i satisfy:

$$w_{i,t} + l_{i,t} = \ln \theta (1-\gamma) + p_{i,t} + y_{i,t} \tag{34}$$

$$p^X_{i,t} + x_{i,t} = \ln \gamma + p_{i,t} + y_{i,t} \tag{35}$$

$$r_{i,t} + k_{i,t} = \ln \alpha (1-\gamma) + p_{i,t} + y_{i,t} \tag{36}$$

Optimal intermediate input choices for the CES baskets require:

$$g_{ij,t} - g_{i,t} = \ln \xi_{ij} - \varepsilon \left(p_{j,t} - q_{i,t} \right) \tag{37}$$

$$x_{ij,t} - x_{i,t} = \ln \omega_{ij} - \varepsilon \left(p_{j,t} - p^X_{i,t} \right) \tag{38}$$

[16] Condition (32) is the multi-country version of the well-known Backus-Smith condition. When the utility function takes the log form ($\sigma = 1$), (32) reduces to $\mu_{i,t} + c_{i,t} = \chi_i$, which means that the relative *nominal* consumption remains the same across all states and times. With a more general CRRA utility, take the difference of eq. (32) between country i and j, we get:

$$\left(\mu_{i,t} + c_{i,t}\right) - \left(\mu_{j,t} + c_{j,t}\right) = (1-\sigma)\left(\mu_{i,t} - \mu_{j,t}\right) + \sigma \left(\ln \chi_i - \ln \chi_j\right).$$

This is the usual 2-country Backus Smith condition. Intuitively, country i increases its consumption expenditure in states where its consumption has become relatively more expensive (μ_i increases relative to μ_j) if and only if it is more risk averse than a log-utility agent ($\sigma < 1$).

Capital accumulation. Optimal capital accumulation in sector i:

$$\mathbb{E}_t \left[\Theta_{i,t+1} \frac{Q_{i,t+1}}{Q_{i,t}} \frac{Z^K_{i,t}}{Z^K_{i,t+1}} \left(1 - \delta + \frac{Z^K_{i,t+1}}{Q_{i,t+1}} r_{i,t+1} \right) \right] = 1, \tag{39}$$

where $r_{i,t+1} = P_{i,t+1} \frac{\partial Y_{i,t+1}}{\partial K_{i,t+1}}$.

Consumer - investor's problem. The consumer in country i solves:

$$\max_{\left(C^i_t, \Lambda^i_{j,t+1}, B^i_{j,t+1}\right)^\infty_{t=0}} \quad \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u_{i,t}(C_{i,t}) \right]$$

subject to the budget constraint:

$$Q_{i,t} C_{i,t} + \sum_{j=1}^{N} \left(P^B_{j,t} B_{ij,t} + P^E_{j,t} \Lambda_{ij,t} \right) = W_{i,t} L_{i,t} + \sum_{j=1}^{N} \left(B_{ij,t-1} \left(P_{j,t} + P^B_{j,t} \right) + \Lambda_{ij,t-1} \left(\Pi_{j,t} + P^E_{j,t} \right) \right). \tag{40}$$

Let $\kappa_{i,t}$ be the Lagrange multipliers on the budget constraints. The optimality conditions are:

$$u'_{i,t}(C_{i,t}) = Q_{i,t} \kappa_{i,t} \tag{41}$$

$$1 = \mathbb{E}_t \left[\Theta_{i,t+1} \frac{P^E_{j,t+1} + \Pi_{j,t+1}}{P^E_{j,t}} \right], \quad j = 1, 2, ..., N \tag{42}$$

$$1 = \mathbb{E}_t \left[\Theta_{i,t+1} \frac{P^B_{j,t+1} + P_{j,t+1}}{P^B_{j,t}} \right], \quad j = 1, 2, ..., N. \tag{43}$$

with $\Theta_{i,t+1} = \beta \kappa_{i,t+1} / \kappa_{i,t} = \beta \frac{u'_{i,t+1}(C_{i,t+1})/Q_{i,t+1}}{u'_{i,t+1}(C_{i,t})/Q_{i,t}}$ is the SDF.

These optimal conditions, together with the budget constraint (5), market clearing conditions fully characterize the competitive equilibrium.

The equivalence of the SP solution and the competitive equilibrium. By choosing $q_{i,t} = q^*_{i,t}$ and $\kappa_{i,t} = 1/\chi_i$, the decentralized equation (41) becomes equivalent to equation (32) from the SP problem. Setting $p_{i,t} = p^*_{i,t}$ and $p^X_{i,t} = p^{X*}_{i,t}$ makes input choice equations (30) and (31) of the SP solution coincides with the decentralized choices (35), (37), and (38). Equations (34) and (36) give the decentralized wage and rental price of capital, respectively. The decentralized optimal investment condition (39) is equivalent to the SP condition (33), since $Q_{i,t} = Q^*_{i,t}$ and $\Theta_{i,t+1} Q_{i,t+1} / Q_{i,t} = \beta u'(C_{i,t+1}) / u'(C_{i,t})$.

The return to investment by the capital sector is equal to the return to investment in domestic equity by households. To see this, setting $P^E_{i,t} = Q_{i,t} K_{i,t+1} / Z^K_{i,t}$ makes the household's Euler equation (42) equivalent to optimal investment condition (39) (which, in turn, is equivalent to the SP version (33)):

$$\frac{P^E_{i,t+1}+\Pi_{i,t+1}}{P^E_{i,t}} = \frac{Q_{i,t+1}K_{i,t+2}/Z^K_{i,t+1}+r_{i,t+1}K_{i,t+1}-Q_{i,t+1}\frac{K_{i,t+2}-(1-\delta)K_{i,t+1}}{Z^K_{i,t+1}}}{Q_{i,t}K_{i,t+1}/Z^K_{i,t}} = \frac{Q_{i,t+1}}{Q_{i,t}}\left[(1-\delta)\frac{Z^K_{i,t}}{Z^K_{i,t+1}}+Z^K_{i,t}\frac{r_{i,t+1}}{Q_{i,t+1}}\right].$$

It remains to check that under these prices, there exists a portfolio (Λ, B) and a particular Social Planner allocation (corresponding to specific Pareto weights) that satisfy the Competitive Equilibrium budget constraint. We guess-and-verify that we can decentralize using a constant portfolio. The budget constraint (5) reduces to the following static constraint:

$$\widetilde{C}_{i,t} = \widetilde{L}_{i,t} + \sum_{j\in\mathcal{N}}\left(B_{ij}P_{j,t}+\Lambda_{ij}\Pi_{j,t}\right)$$

Put in matrix form:

$$\widetilde{C}_t = \widetilde{L}_t + \Lambda\widetilde{\Pi}_t + BP_t$$

Since in equilibrium, $\widetilde{L}_t = \theta GDP_t$, and $\widetilde{\Pi}_t = \alpha GDP_t - \widetilde{I}_t$, we have:

$$\begin{aligned}\widetilde{C}_t &= \theta GDP_t + \Lambda\left(\alpha GDP_t - \widetilde{I}_t\right) + BP_t \\ &= (\theta I + \alpha\Lambda)GDP_t - \Lambda\widetilde{I}_t + BP_t\end{aligned}$$

Rewrite in terms of deviations from the steady state, and denote by $\widetilde{B} \equiv B\overline{P}$:

$$\widehat{\widetilde{C}}_t = [\theta I + \alpha\Lambda]\widehat{GDP}_t - \Lambda\widehat{\widetilde{I}}_t + \widetilde{B}\widehat{p}_t. \tag{44}$$

Using the decomposition of $\widetilde{VA}$ provided in Lemma 2, we have:

$$\widehat{\widetilde{C}}_t = [\theta I + \alpha\Lambda]\left(\overline{M}\widehat{\widetilde{G}}_t - \overline{M}^P\widehat{p}^{\$}_t\right) - \Lambda\widehat{\widetilde{I}}_t + \widetilde{B}\widehat{p}_t.$$

Use the identity $\widehat{\widetilde{G}}_t = \widehat{\widetilde{C}}_t + \widehat{\widetilde{I}}_t$ and re-arrange:

$$\left[I-(\theta I+\alpha\Lambda)\overline{M}\right]\widehat{\widetilde{C}}_t = \left[(\theta I+\alpha\Lambda)\overline{M}-\Lambda\right]\widehat{\widetilde{I}}_t - (\theta I+\alpha\Lambda)\overline{M}^P\widehat{p}_t + \widetilde{B}\widehat{p}_t. \tag{45}$$

The equity portfolio $\Lambda = \theta\overline{M}\left[I-\alpha\overline{M}\right]^{-1}$ ensures that the first term on the RHS of (45) is zero for all realization of $\widehat{\widetilde{I}}$. In other words, the equity portfolio hedges completely against demand risk arising from fluctuating investment. The second term on the RHS of (45) can be hedged using a corresponding bond portfolio that satisfies:

$$\begin{aligned}\widetilde{B}^P\widehat{p}_t - (\theta I+\alpha\Lambda)\overline{M}^P\widehat{p}^{\$}_t = 0 \\ \widetilde{B}^P = (\theta I+\alpha\Lambda)\overline{M}^P\end{aligned}$$

Finally, we are left with choosing the bond portfolio $\widetilde{B}^{ER}$ that hedges consumption risk:

$$\left[I-(\theta I+\alpha\Lambda)\overline{M}\right]\widehat{\widetilde{C}}_t = (I-\Lambda)\widehat{\widetilde{C}}_t = \widetilde{B}^{ER}\widehat{p}_t.$$

We have:

$$(I-\Lambda)\widehat{\widetilde{C}}_t = (I-\Lambda)D_{\widetilde{C}}\widehat{\widetilde{c}}_t = (I-\Lambda)D_{\widetilde{C}}(1-\sigma)\widehat{q}^*_t$$
$$= (I-\Lambda)D_{\widetilde{C}}(1-\sigma)\widehat{q}_t$$
$$= (I-\Lambda)D_{\widetilde{C}}(1-\sigma)\overline{\Xi}\widehat{p}_t$$

Therefore, the bond portfolio that fully hedges consumption risk is

$$\widetilde{B}^{ER} = (1-\sigma)(I-\Lambda)D_{\widetilde{C}}\overline{\Xi}.$$

A.5. **Proof of Proposition 2.** To focus on the equity portfolio, let us assume $\sigma = \varepsilon = 1$, but the proof follows more generally.

The equity portfolio is chosen to satisfy the household budget constraint

$$\widetilde{C}_t = \widetilde{L}_t + \Lambda\widetilde{\Pi}_t$$

where Λ has dimension $N \times S$ ($S = |\mathcal{K}|$ is the number of world industries), $\widetilde{\Pi} = \left(1-\gamma\right)\alpha\widetilde{Y} - \widetilde{I}$ has dimension $S \times 1$. The labor income is found as the fraction $(1-\gamma)\theta$ of domestic industries' sales:

$$\widetilde{L}_t = \left(1-\gamma\right)\theta E\widetilde{Y}_t$$

where $E_{N\times S}$ denotes the modified identity matrix (20). Expanding, we have:

$$\widetilde{C}_t = \left(1-\gamma\right)\theta E\widetilde{Y}_t + \Lambda\left(1-\gamma\right)\alpha\widetilde{Y}_t - \Lambda\widetilde{I}_t$$
$$= [\theta E + \alpha\Lambda](1-\gamma)\widetilde{Y}_t - \Lambda\widetilde{I}_t$$

We have:

$$\widetilde{Y}^i_t = \sum_{j=1}^{S}\gamma\Omega_{ji}\widetilde{Y}^j_t + \sum_{j=1}^{N}\Xi_{ji}\widetilde{G}^j_t$$
$$\widetilde{Y}_t = \left[\mathbf{I} - \gamma\Omega'\right]^{-1}\Xi'\widetilde{G}_t$$
$$\left(1-\gamma\right)\widetilde{Y}_t = M\widetilde{G}_t,$$

with $M = \left(1-\gamma\right)\left[\mathbf{I} - \gamma\Omega'\right]^{-1}\Xi'$ is the IDWs matrix. M has dimension $S \times N$. Substitute into the budget constraint:

$$\widetilde{C}_t = [\theta E + \alpha\Lambda]M\left(\widetilde{C}_t + E\widetilde{I}_t\right) - \Lambda\widetilde{I}_t$$
$$[\mathbf{I} - [\theta E + \alpha\Lambda]M]\widetilde{C}_t = [[\theta E + \alpha\Lambda]ME - \Lambda]\widetilde{I}_t$$

The optimal equity portfolio satisfies:

$$\Lambda = \theta EME + \alpha\Lambda ME \Rightarrow \Lambda = \theta EME\left[\mathbf{I} - \alpha ME\right]^{-1}.$$

A.6. **Proof of Proposition 3.** To focus on the equity portfolio, let us assume $\sigma = \varepsilon = 1$, but the proof follows more generally.

Labor income now comes from two sources, traded and nontraded sector:

$$\widetilde{L} = (1-\gamma)\theta\widetilde{Y} + \iota\widetilde{G}.$$

(Since labor is the only factor of production in the nontraded sector, they get the entire revenue of that sector, which is $\iota\widetilde{G}$.)

Consider the static budget constraint:

$$\begin{aligned}
\widetilde{C} &= \widetilde{L} + \Lambda\widetilde{I} \\
&= (1-\gamma)\theta\widetilde{Y} + \iota\widetilde{G} + \Lambda\left[(1-\gamma)\alpha\widetilde{Y} - \widetilde{I}\right] \\
&= [\theta\mathbf{I} + \alpha\Lambda](1-\gamma)\widetilde{Y} - \Lambda\widetilde{I} + \iota\widetilde{I} + \iota\widetilde{C} \\
(1-\iota)\widetilde{C} &= [\theta\mathbf{I} + \alpha\Lambda](1-\gamma)\widetilde{Y} - \Lambda\widetilde{I} + \iota\widetilde{I}
\end{aligned}$$

The market clearing condition with nontraded sector is:

$$(1-\gamma)\widetilde{Y} = (1-\iota)M\widetilde{G}$$

since the traded sector only gets fraction $1-\iota$ of final expenditure. Substitute into the budget constraint, we have:

$$\begin{aligned}
(1-\iota)\widetilde{C} &= (1-\iota)[\theta\mathbf{I} + \alpha\Lambda]M\widetilde{G} - \Lambda\widetilde{I} + \iota\widetilde{I} \\
(1-\iota)[\mathbf{I} - [\theta\mathbf{I} + \alpha\Lambda]M]\widetilde{C} &= [(1-\iota)[\theta\mathbf{I} + \alpha\Lambda]M - \Lambda + \iota\mathbf{I}]\widetilde{I}
\end{aligned}$$

The optimal equity portfolio solves:

$$\begin{aligned}
\Lambda &= \iota\mathbf{I} + (1-\iota)[\theta\mathbf{I} + \alpha\Lambda]M \\
\Lambda &= [\iota\mathbf{I} + (1-\iota)\theta M][\mathbf{I} - (1-\iota)\alpha M]^{-1}.
\end{aligned}$$

A.7. **Proof of Proposition 4.** Given complete markets, I work with the Social Planner problem (detailed in Appendix A.3). Let μ_t and ν_t be the (log) Lagrange multiplier on the resource constraint for final and intermediate good, respectively.

Guess, and later verify, that the investment-GDP ratio is constant:

$$\frac{\widetilde{I}_t}{\widetilde{Y}_t} = \frac{\widetilde{K}_{t+1}}{\widetilde{Y}_t} = \zeta,$$

for some ζ to be determined. Recall that $\widetilde{I}_t = e^{\mu_t}I_t$, $\widetilde{K}_{t+1} = e^{\mu_t}K_{t+1}$, and $\widetilde{Y}_t = e^{\nu_t}Y_t$ are nominal amounts. The market clearing conditions are given by:

$$\widetilde{Y}_t = M\left(\widetilde{C}_t + \widetilde{I}_t\right),$$

where $M=\left(1-\gamma\right)\left[I-\gamma\Omega'\right]^{-1}\Xi'$ is the matrix of IDWs. When utility is log and production function is Cobb-Douglas, $\widetilde{C}_t=\chi$, a vector of constant (χ is the Pareto weights, normalized). Since $\widetilde{I}_t=\zeta\widetilde{Y}_t$, this implies that these two variables are constants as well:

$$\widetilde{Y}_t=M\chi+\zeta M\widetilde{Y}_t\Rightarrow\widetilde{Y}_t=[\mathbf{I}-\zeta M]^{-1}M\chi.$$

The Euler equation when $\delta=1$ is:

$$\mathbb{E}_t\left[\exp\left(-\rho+\Delta\mu^i_{t+1}+\ln r^i_{t+1}\right)\right]=1$$

where $\ln r^i_{t+1}\equiv\ln\left[(1-\gamma)\alpha\right]+\widetilde{y}^i_{t+1}-k^i_{t+1}-\mu^i_{t+1}$ and $\rho=-\ln\beta$. The constant ζ can be solved out from the Euler equation in the steady state:

$$-\rho+\ln\left[(1-\gamma)\alpha\right]-\ln\zeta=0\Rightarrow\zeta=\beta\left(1-\gamma\right)\alpha.$$

The return on capital is then given by:

$$\ln r^i_{t+1}=\ln\left[(1-\gamma)\alpha\right]-\ln\zeta-\Delta\mu^i_{t+1}=\rho-\Delta\ln\mu^i_{t+1}.$$

Thus, the Euler equation is satisfied with a constant investment-GDP ratio.

Finally, cost minimization in the intermediate sector implies:

$$v_t=-z_t+\left(1-\gamma\right)\alpha\left(\mu_t+\ln r_t\right)+\gamma\Omega v_t+h$$

with $h=\left(1-\gamma\right)\alpha\ln\left[(1-\gamma)\alpha\right]+\left(1-\gamma\right)\theta\widetilde{y}+\sum_j\gamma\omega_{ij}\ln\left[\gamma\omega_{ij}\right]$. Re-arranging, we have:

$$v_t=\left[\mathbf{I}-\gamma\Omega\right]^{-1}\left[-z_t+\left(1-\gamma\right)\alpha\mu_{t-1}+h+\left(1-\gamma\right)\alpha\rho\right]$$

Multiply both sides by Ξ, we have:

$$\mu_t=\frac{M'}{1-\gamma}\left[-z_t+\left(1-\gamma\right)\alpha\mu_{t-1}+h_2\right]$$

with $h_2=h+(1-\gamma)\alpha\rho$. Let $\overline{\mu}$ be the solution to $\overline{\mu}=-\left[\mathbf{I}-\alpha M'\right]^{-1}h_2$, we have:

$$\mu_t-\overline{\mu}=-\frac{M'}{1-\gamma}z_t+\alpha M'\left(\mu_{t-1}-\overline{\mu}\right)$$

Take the first difference, we have:

$$\Delta\mu_t=-\frac{M'}{1-\gamma}\Delta z_t+\alpha M'\Delta\mu_{t-1}.$$

When TFP follows a random walk, $\Delta z=\varepsilon\sim\mathcal{N}(0,\Sigma)$ is independent of the second term $\alpha M'\Delta\mu_{t-1}$. We can calculate $\operatorname{cov}\left(\Delta\mu\right)$ as:

$$\operatorname{cov}\left(\Delta\mu_t\right)=\frac{M'}{1-\gamma}\Sigma\frac{M}{1-\gamma}+\alpha^2M'\operatorname{cov}\left(\Delta\mu_{t-1}\right)M.$$

Let $\Sigma_\mu = \text{cov}\left(\Delta\mu_t\right)$. Let vec denote the vectorization operator (stacking matrix into a column) and $\otimes$denote the Kronecker product. We have:

$$vec\left(\Sigma_\mu\right) = \frac{1}{(1-\gamma)^2} M' \otimes M' vec(\Sigma) + \alpha^2 M' \otimes M' vec\left(\Sigma_\mu\right)$$

$$vec\left(\Sigma_\mu\right) = \frac{1}{(1-\gamma)^2}\left[I - \alpha^2 M' \otimes M'\right]^{-1} M' \otimes M' vec(\Sigma).$$

Given that $c+\mu$ is a constant in this setting, $vec(\Sigma_c) = vec\left(\Sigma_\mu\right)$ as well. Since $r_{t+1} = \rho - \Delta\mu_{t+1}$, the return to capital also has the same covariance matrix as μ.

Finally, the vector of exchange rate in the decentralized economy is given by:

$$e = A\mu,$$

with

$$A = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 1 & -1 & 0 & \dots & 0 \\ 1 & 0 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & -1 \end{bmatrix}.$$

Thus:

$$\Sigma_e = A\Sigma_\mu A'$$

$$vec(\Sigma_e) = (A \otimes A) vec\left(\Sigma_\mu\right).$$

References

Daron Acemoglu, Vasco M. Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. The network origins of aggregate fluctuations. *Econometrica*, 80(5):1977–2016, 2012. doi: 10.3982/ECTA9623. URL https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA9623. 1

James E. Anderson. The gravity model. *Annual Review of Economics*, 3(1):133–160, 2011. doi: 10.1146/annurev-economics-111809-125114. URL https://doi.org/10.1146/annurev-economics-111809-125114. 10

David K. Backus and Gregor W. Smith. Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics*, 35(3):297–316, 1993. ISSN 0022-1996. doi: https://doi.org/10.1016/0022-1996(93)90021-O. URL https://www.sciencedirect.com/science/article/pii/002219969390021O. 2.1

David Rezza Baqaee and Emmanuel Farhi. The macroeconomic impact of microeconomic shocks: Beyond hulten's theorem. *Econometrica*, 87(4):1155–1203, 2019. doi: 10.3982/

ECTA15202. URL https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA15202. 1, 1

William A Brock and Leonard J Mirman. Optimal economic growth and uncertainty: The discounted case. *Journal of Economic Theory*, 4(3):479 – 513, 1972. ISSN 0022-0531. doi: https://doi.org/10.1016/0022-0531(72)90135-4. URL http://www.sciencedirect.com/science/article/pii/0022053172901354. 4.3, 4.3

Thomas Chaney. The gravity equation in international trade: An explanation. *Journal of Political Economy*, 126(1):150–177, 2018. doi: 10.1086/694292. URL https://doi.org/10.1086/694292. 12

Jeffery Jinfan Chang, Huancheng Du, Dong Lou, and Christopher Polk. Ripples into waves: Trade networks, economic activity, and asset prices. Technical report, 2020. 1

Nicolas Coeurdacier and Pierre-Olivier Gourinchas. When bonds matter: Home bias in goods and assets. *Journal of Monetary Economics*, 82:119 – 137, 2016. ISSN 0304-3932. doi: https://doi.org/10.1016/j.jmoneco.2016.07.005. URL http://www.sciencedirect.com/science/article/pii/S0304393216300423. 1, 1, 5

Nicolas Coeurdacier, Robert Kollmann, and Philippe Martin. International portfolios, capital accumulation and foreign assets dynamics. *Journal of International Economics*, 80(1): 100 – 112, 2010. ISSN 0022-1996. doi: https://doi.org/10.1016/j.jinteco.2009.05.006. URL http://www.sciencedirect.com/science/article/pii/S0022199609000713. Special Issue: JIE Special Issue on International Macro-Finance. 1, 5, 2.3.1

Antonio Coppola, Matteo Maggiori, Brent Neiman, and Jesse Schreger. Redrawing the map of global capital flows: The role of cross-border financing and tax havens. Working Paper 26855, National Bureau of Economic Research, March 2020. URL http://www.nber.org/papers/w26855. 3.1, 1

Michael B. Devereux and Alan Sutherland. Country portfolios in open economy macro-models. *Journal of the European Economic Association*, 9(2):337–369, 2011. ISSN 15424766, 15424774. URL http://www.jstor.org/stable/25836069. 2.1

Julian di Giovanni and Galina Hale. Stock market spillovers via the global production network: Transmission of u.s. monetary policy. Working Paper 28827, National Bureau of Economic Research, May 2021. URL http://www.nber.org/papers/w28827. 1

Evsey D. Domar. On the measurement of technological change. *The Economic Journal*, 71 (284):709–729, 1961. ISSN 00130133, 14680297. URL http://www.jstor.org/stable/2228246. 1, 7

Jonathan Eaton, Samuel Kortum, Brent Neiman, and John Romalis. Trade and the global recession. *American Economic Review*, 106(11):3401–38, November 2016. doi: 10.1257/aer.20101557. URL https://www.aeaweb.org/articles?id=10.1257/aer.20101557. 3

Charles Engel and Akito Matsumoto. The international diversification puzzle when goods prices are sticky: It's really about exchange-rate hedging, not equity portfolios. *American Economic Journal: Macroeconomics*, 1(2):155–88, July 2009. doi: 10.1257/mac.1.2.155. URL http://www.aeaweb.org/articles?id=10.1257/mac.1.2.155. 1, 5

Doireann Fitzgerald. Trade costs, asset market frictions, and risk sharing. *American Economic Review*, 102(6):2700–2733, May 2012. doi: 10.1257/aer.102.6.2700. URL https://www.aeaweb.org/articles?id=10.1257/aer.102.6.2700. 1

Kenneth R. French and James M. Poterba. Investor diversification and international equity markets. *The American Economic Review*, 81(2):222–226, 1991. ISSN 00028282. URL http://www.jstor.org/stable/2006858. 1, 1

Xavier Gabaix. The granular origins of aggregate fluctuations. *Econometrica*, 79(3):733–772, 2011. doi: 10.3982/ECTA8769. URL https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA8769. 1

Michael Gofman, Gill Segal, and Youchang Wu. Production Networks and Stock Returns: The Role of Vertical Creative Destruction. *The Review of Financial Studies*, 33(12):5856–5905, 03 2020. ISSN 0893-9454. doi: 10.1093/rfs/hhaa034. URL https://doi.org/10.1093/rfs/hhaa034. 1

Jonathan Heathcote and Fabrizio Perri. The international diversification puzzle is not as bad as you think. *Journal of Political Economy*, 121(6):1108–1159, 2013. doi: 10.1086/674143. URL https://ideas.repec.org/a/ucp/jpolec/doi10.1086-674143.html. 1, 1, 2.3.1

Bernard Herskovic. Networks in production: Asset pricing implications. *The Journal of Finance*, 73(4):1785–1818, 2018. doi: 10.1111/jofi.12684. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/jofi.12684. 1

Bernard Herskovic, Bryan Kelly, Hanno Lustig, and Stijn Van Nieuwerburgh. Firm volatility in granular networks. *Journal of Political Economy*, 128(11):4097–4162, 2020. doi: 10.1086/710345. URL https://doi.org/10.1086/710345. 1

Charles R. Hulten. Growth Accounting with Intermediate Inputs. *The Review of Economic Studies*, 45(3):511–518, 10 1978. ISSN 0034-6527. doi: 10.2307/2297252. URL https://doi.org/10.2307/2297252. 1, 1, 7

Zhengyang Jiang and Robert Richmond. Origins of international factor structures. June 2019. 1, 4.3, 4.3

Robert C. Johnson and Guillermo Noguera. Accounting for intermediates: Production sharing and trade in value added. *Journal of International Economics*, 86(2):224 – 236, 2012. ISSN 0022-1996. doi: https://doi.org/10.1016/j.jinteco.2011.10.003. URL http://www.sciencedirect.com/science/article/pii/S002219961100122X. 1, 2.2

Alejandro Justiniano, Giorgio E. Primiceri, and Andrea Tambalotti. Investment shocks and business cycles. *Journal of Monetary Economics*, 57(2):132 – 145, 2010. ISSN 0304-3932. doi: https://doi.org/10.1016/j.jmoneco.2009.12.008. URL http://www.sciencedirect.com/science/article/pii/S0304393210000048. 3

Philip Lane and Gian Maria Milesi-Ferretti. International investment patterns. *The Review of Economics and Statistics*, 90(3):538–549, 2008. URL https://EconPapers.repec.org/RePEc:tpr:restat:v:90:y:2008:i:3:p:538-549. 1

John B. Long and Charles I. Plosser. Real business cycles. *Journal of Political Economy*, 91(1):39–69, 1983. ISSN 00223808, 1537534X. URL http://www.jstor.org/stable/1840430. 1, 1, 7

Robert Lucas. Interest rates and currency prices in a two-country world. *Journal of Monetary Economics*, 10(3):335–359, 1982. URL https://EconPapers.repec.org/RePEc:eee:moneco:v:10:y:1982:i:3:p:335-359. 1

Philippe Martin and Helene Rey. Financial super-markets: size matters for asset trade. *Journal of International Economics*, 64(2):335–361, 2004. URL https://EconPapers.repec.org/RePEc:eee:inecon:v:64:y:2004:i:2:p:335-361. 12

Maurice Obstfeld and Kenneth Rogoff. *The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?*, pages 339–412. MIT Press, January 2001. URL http://www.nber.org/chapters/c11059. 1, 3.1.1, 4.2

Yohei Okawa and Eric van Wincoop. Gravity in international finance. *Journal of International Economics*, 87(2):205 – 215, 2012. ISSN 0022-1996. doi: https://doi.org/10.1016/j.jinteco.2012.01.006. URL http://www.sciencedirect.com/science/article/pii/S0022199612000074. 12

Richard Portes and Helene Rey. The determinants of cross-border equity flows. *Journal of International Economics*, 65(2):269–296, March 2005. URL https://ideas.repec.org/a/eee/inecon/v65y2005i2p269-296.html. 1, 1, 3.1.1, 13

Robert J. Richmond. Trade network centrality and currency risk premia. *The Journal of Finance*, 74(3):1315–1361, 2019. doi: 10.1111/jofi.12755. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/jofi.12755. 1, 4.3

Paul A. Samuelson. The fundamental approximation theorem of portfolio analysis in terms of means, variances and higher moments. *The Review of Economic Studies*, 37(4):537–542, 1970. ISSN 00346527, 1467937X. URL http://www.jstor.org/stable/2296483. 2.1