Progressive Pension and Optimal Tax Progressivity*

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Abstract

We examine the implications of progressive pensions for designing an optimal progressive income tax code. In a simple analytical model, we first show that optimal tax progressivity is negatively linked with pension progressivity. This relationship is then analysed further using a stochastic dynamic general equilibrium overlapping generations model calibrated for Australia, where pension payments are universally means-tested to target low income retirees. Importantly, these payments are financed directly by progressive income taxes. We find that flattening the income tax code and tightening means-testing rules for pension payments improve overall welfare. The optimal design consists of a flat tax rate and a strict means-tested pension scheme. Hence, reforms that shift the social insurance/redistribution role embedded in the tax system to the means-tested pension system can yield better welfare outcomes. Our results also highlight that more generally, a redistributive tax and transfer system can be improved by addressing redistribution concerns directly through more progressive transfers while improving efficiency by reducing tax progressivity.

JEL: E62, H24, H31

Keywords: Taxation, age pension, tax progressivity, income dynamics, inequality, Suits index, heterogeneity, dynamic general equilibrium.

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1 Introduction

Progressive income taxes play potentially beneficial roles in enhancing a more equal distribution of income and social insurance against idiosyncratic uncertainty. There is a growing literature studying optimal levels of tax progressivity (e.g., see Conesa and Krueger 2006, Krueger and Ludwig 2016, Heathcote, Storesletten and Violante 2017). These studies abstract from explicitly modeling the social insurance and redistribution role of the transfer system. Heathcote, Storesletten and Violante (2017) in particular combine all public transfer programs into public goods consumption that enters household preferences. The purpose of this paper is to study the implications of explicitly accounting for progressive public pension when designing an optimal progressive income tax code.

Pension progressivity and generosity is of particular importance to the tax system. Highly progressive pensions that are targeted towards the poor (via resource-based means-tests) reduce pension expenditure. This can potentially lower demand for the social insurance/redistribution role of a progressive tax system. Whereas, less progressive pensions with wider coverage can increase expenditure and tax rates. Thus pension design needs to strike a balance between ensuring social insurance for a broader population and reducing pension expenditure. Pension design and tax design are further entwined by their complementary social insurance roles. They can both provide partial insurance against idiosyncratic shocks and unfavourable initial conditions. Progressive tax relieves the poor of higher tax burdens. Means-tested pensions target benefits to those most in need. Particularly, they induce inter-generational redistribution towards the old and needy.

Unlike many other OECD countries, Australia has a very progressive personal income tax system and a highly targeted means-tested pension system that targets low income retirees. Pension payments are not universal because of income and asset tests that restrict payments to only those below a specified threshold. Simplifying only to account for the income test\(^1\), we model pension payments as

\[
p = \max \{0, p^{\text{max}} - \omega y (y^m - \bar{y}_1)\}
\]

where \(p^{\text{max}}\) is the maximum pension benefit, \(y^m\) is assessable income, \(\bar{y}_1\) is the low income threshold and \(\omega y \in [0, 1]\) is a taper rate.

Moreover, means-tested pension is financed directly by general tax revenue rather than a specific social security tax. Since income tax accounts for nearly half of total government revenue, income tax policy in Australia is remarkably intertwined with pension design. Thus, Australian fiscal policy settings provide a real world laboratory for analyzing interactions between progressive public pension and progressive taxes.

To do so, we first document some stylized facts on trends in progressive income tax and pension from 1991 to 2019. We use administrative tax data from the Australian Longitudinal Information Files (ALife) that contains over a million individual tax returns. We use a progressivity index as per Suits (1977) to examine the distribution of tax and pension across the income distribution. We find that both tax and pension have become more progressive in the last decade than they had ever been in the past 29 years. We also make a distinction between distributional progressivity and the progressivity of the actual tax code. We examine trends in progressivity in the tax code by

\(^1\)This simplification is for two reasons. First, our focus in this paper is on progressivity in relation to income rather than wealth. Second, including the asset test does not change our results as in our modelling framework, the income test is the binding test.
estimating a parametric tax function as per Jakobsson (1976) and recently Heathcote, Storesletten and Violante (2017). As such, income tax is modelled as

\[ t(y) = \max \left\{ 0, y - \lambda y^{(1-\tau_y)} \right\} \quad (2) \]

where \( t(y) \) denotes tax liability as a function of pre-tax income \( y \), \( \tau_y \) is the progressivity parameter, and \( \lambda \) is a scaling factor to match the the average level of taxation in the economy. We show that this parsimonious tax function provides a fairly precise estimate of Australia’s complex income tax code. This gives credence to using it to approximate the income tax code in the dynamic general equilibrium model in the second part of our analysis.

Next, we construct a dynamic general equilibrium, small open economy model with overlapping generations of heterogeneous households born with different innate earnings ability (skill types) and facing idiosyncratic shocks to labor productivity. Both progressive income tax, means-tested pension and other public transfers provide social insurance. We calibrate the model to match key macro aggregates and lifecycle patterns in the Australian data. We use the calibrated model to examine the welfare and economic effects of alternative designs of a progressive pension and income tax system. These effects are examined by varying tax progressivity, pension progressivity, and pension generosity. Specifically, we focus on the optimal level of tax and pension progressivity, their interactions, and how they change as the pension system becomes less generous. We reduce tax progressivity by reducing \( \tau_y \), pension progressivity by reducing the taper rate \( \omega_y \), and pension generosity by reducing the level of maximum benefit \( p^{max} \). In our experiments, we balance the budget by adjusting the average level of taxation \( \lambda \) so that higher pension expenditure is financed by higher income tax (on average). Our main findings are summarized as follows.

We find that when the progressive pension system is left unchanged, reducing tax progressivity results in efficiency gains in terms of higher output, aggregate labour supply and savings. Utilitarian social welfare increases in aggregate as well as across all household types. A proportional income tax code yields maximum utilitarian social welfare.

Further, we find that reducing tax progressivity results in efficiency gains over the lifecycle that lead to a lower reliance on the pension system. At this juncture it is pertinent to recall that the income tax code applies throughout the lifecycle while individuals are only eligible for pension in old age. This suggests that it is optimal from both an efficiency and welfare perspective for the income tax system to have next to no social insurance role. Does this preclusion call for a greater social insurance role for the pension system? We test this by varying both tax and pension progressivity and find that at low levels of tax progressivity, it is indeed socially more desirable to increase pension progressivity. With the optimal flat income tax code, we find that the optimal pension system has a strict means-test (most progressive).

Given that the pension system is funded by general tax revenue, making it more targeted via a strict means-test lowers funding costs and reduces tax burdens. In our model economy, this means lower income tax rates. Thus, the welfare improvement from progressive pension is largely due to lower income tax. We examine the link between tax burdens and the pension system by varying pension generosity by varying the maximum pension benefit.

Reducing benefit levels lowers tax burdens and improves welfare. However, as the pension system becomes less generous, the social insurance that it provides becomes less adequate regardless of its progressivity. Consequently, we find that when the pension benefit is lower than the benchmark, the
optimal income tax system is slightly progressive (bestowing a small social insurance role). Overall, we find that compared to the benchmark, the jointly optimal tax and pension systems have a less generous, strictly means-tested pension and a considerably less progressive (but not completely flat) income tax.

The paper is organized as follows. Section 2 includes our empirical analysis examining trends in income tax and pension progressivity. Before setting out on our large scale model, in Section 3 we show the negative relationship between optimal tax progressivity and pension progressivity using a simple two period model. Section 4 describes the features of the benchmark model central to our analysis. Section 5 explains the policy experiments. Section 6 tests whether our results are sensitive to certain model assumptions. We present detailed description of model and calibration, additional analysis and results in the Appendices.

Related literature. This paper links to three branches within the dynamic general equilibrium literature on public finance - (1) optimal income tax, (2) optimal social security systems, and (3) jointly optimal tax and social security. To the best of our knowledge, the link between optimal tax progressivity and optimal pension progressivity is not considered in any paper within this literature. We provide a detailed literature review in Appendix A and summarise the three branches and their respective research gaps below.

Papers in the first branch examine optimal tax progressivity in incomplete market models. Our paper falls within those that search for the optimal level of tax progressivity within a given parametric class of tax scheme as per Ramsey (1927). Notable papers in this branch include Ventura (1999), Benabou (2002), Conesa and Krueger (2006), Krueger and Ludwig (2016), Heathcote and Tsuiyama (2016) and Heathcote, Storesletten and Violante (2017). While they provide important insights on tax progressivity, social security in general and the pension system are often simplified and not fully considered. The literature often employs benchmark models of the U.S. where pension coverage is universal, and effects from the extensive margin are not relevant.

The second branch of literature closely related to this paper is that of general equilibrium life cycle models that examine optimal pension systems. This branch includes papers such as Imrohoroglu, Imrohoroglu and Jones (1995), Sefton and van de Ven (2008) and Kudrna and Woodland (2011) that examine the effects via the intensive margin arising from means-tested versus PAYG pensions. Tran and Woodland (2014) extend these papers by examining the extensive margin effects. Similar to other papers within this branch, their analysis takes the tax system as given.

Closest to our paper in approach are those that examine the interplay between optimal tax progressivity and optimal social security. These papers analyse whether the generosity of a specific social insurance scheme justifies a more or less progressive tax system. McKay and Reis (2016) study the optimal generosity of unemployment benefits and the progressivity of income taxes. Tran and Jung (2018) examine optimal progressivity together with the design of the health insurance system in a model where individuals are exposed both to idiosyncratic labour productivity and health risks over the lifecycle. The central message of these papers is that optimal progressivity depends on the type of risk being mitigated by social insurance, and the adequacy of relevant social insurance mechanisms.

More recently, Ferriere et al. (2022) is quite significant in providing general insights on the interplay between optimal labour income tax progressivity and the optimal design of the transfer
system. Using a canonical heterogeneous agent model with infinitely lived households as per Aiyagari (1994), they demonstrate an optimally negative relationship between transfers and income tax progressivity due to efficiency and redistribution concerns. They show that more generous transfers (that reduce dispersion in consumption) are optimally financed with less progressive labour income taxes (that enhance efficiency).

While our paper affirms these insights, it differs from Ferriere et al. (2022) and complements it in several aspects. The first of these distinctions is in our tax system. In that, while Ferriere et al. (2022) echoes the US case and takes the flat capital income tax rate as given in their analysis, we consider the case where both labour income and capital income are taxed jointly under the same tax code. Thus, changes in the level of progressivity have more direct incentive effects on both labour and savings. Our results indicate that progressive taxation of capital income is highly distortionary, and reducing progressivity results in a substantial increase in savings over the lifecycle. This in turn strengthens the link between pension design and tax progressivity.

The second distinction from Ferriere et al. (2022) is in our focus on the lifecycle and the incorporation of pension (which is age specific) rather than a general transfer received in all periods. On this aspect, our approach and the general implications of our results are closely related to Heathcote, Storesletten and Violante (2020) who explain the importance of lifecycle dynamics in the optimal progressivity of the tax and transfer system. They consider both jointly using a single net tax function (where a negative net tax is a transfer). They analysis highlights the importance of two channels in regards to optimal progressivity - the uninsurable risk channel and the lifecycle channel.

In regards to the first, permanent idiosyncratic productivity shocks that accumulate over the lifecycle result in increasing inequality especially in old-age. To the extent that these shocks are privately uninsurable, the social planner has an incentive to target redistribution by having progressivity increase with age. Results from our framework with “pure” (non-negative) tax and separate transfer systems echo this intuition. Specifically, in the presence of similar uninsurable productivity shocks over lifetime, we find support for increasing social insurance and redistribution in old-age via a highly progressive means-tested pension.

The second is the lifecycle channel - whereby age-increasing earnings profiles induce wages net of the disutility of work to rise during the first decades of working life. Consequently, the planner has the incentive to have progressivity decline with age in order to smooth marginal tax rates over the lifecycle. Thus, the first channel targets redistributive and social insurance concerns, while the second targets efficiency. Similarly, our results indicate that less progressive income taxes induce an increase in labour and savings in the first decades of life.

While Heathcote, Storesletten and Violante (2020) examine a net tax system, we consider an age-invariant “pure” tax system and model transfers separately. Moreover, in our framework, transfers are age dependent while income tax is age invariant. The uninsurable risk channel as per Heathcote, Storesletten and Violante (2020) is moderated in our framework by progressive public transfers in general and specifically by the means-tested age pension.

2 Progressive pension and income tax in Australia

We contextualize our paper by first examining trends in tax and pension progressivity over the last 29 years. The purpose of this section is to highlight rising tax and pension progressivity and
provide motive for examining the interaction of tax and pension progressivity in our main analysis. For conciseness, we defer technical explanations of progressivity measures, descriptive statistics, and other pertinent empirical results such as trends in income, tax and pension shares to Appendix B and the ALife online technical appendix².

2.1 Data

We measure progressivity for tax and for pension using data from the ATO Longitudinal Information Files (ALife) 1991-2019. ALife consists of confidentialised unit records of individual income tax returns from the Australian Tax Office (ATO). The data consists of a 10% random sample of tax filers for each year. On average the sample includes 1-1.5 million individuals per year. We report details of the sample composition and descriptive statistics in the online technical appendix.

For analysing income tax progressivity and pension progressivity, we take market income (the sum of labour and capital incomes) as the base income concept. This is so as to have a uniform base income concept to examine the distribution of tax and pensions. We restrict the sample to individuals above 20 years with non-negative income and tax.


<table>
<thead>
<tr>
<th></th>
<th>Taxable income</th>
<th>Income tax</th>
<th>Pension</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>Mean</td>
<td>SD</td>
</tr>
<tr>
<td>1991</td>
<td>736,584</td>
<td>46,149</td>
<td>42,192</td>
</tr>
<tr>
<td>1995</td>
<td>770,549</td>
<td>46,739</td>
<td>49,432</td>
</tr>
<tr>
<td>2000</td>
<td>838,057</td>
<td>54,228</td>
<td>401,680</td>
</tr>
<tr>
<td>2005</td>
<td>897,518</td>
<td>57,739</td>
<td>97,203</td>
</tr>
<tr>
<td>2010</td>
<td>976,803</td>
<td>61,144</td>
<td>95,606</td>
</tr>
<tr>
<td>2015</td>
<td>1,095,368</td>
<td>63,789</td>
<td>110,854</td>
</tr>
<tr>
<td>2019</td>
<td>1,185,275</td>
<td>65,252</td>
<td>196,345</td>
</tr>
</tbody>
</table>

We take taxable income (which includes pensions and other taxable transfers) as pre-tax income in order to estimate the parametric tax function. Our income tax variable is total tax liability net of offsets and credits. In order to estimate the tax function, we further restrict the sample to include only those observations above the tax-free threshold for each respective year³.

2.2 Trends in income tax progressivity

Progressivity can be measured using different metrics from varying perspectives. We explain these in detail in Appendix B. In this section, we highlight trends in progressivity using the Suits index (Suits (1977)). This index is a Gini type measure that is obtained by plotting the cumulative distribution of a tax or transfer against the cumulative distribution of income. The index ranges from -1 to 1. If the tax system is proportional such that tax shares are equal to income shares, the relative concentration curve lies on the 45° line. In this case, the Suits index equals 0. As tax

²This is a common appendix for all our ALife papers (work in progress).
³Including observations below the threshold leads to over-estimation of taxes for low incomes and under-estimation of high incomes.
becomes more progressive, the relative concentration curve becomes more convex. This increases the index.

**Tax progressivity has been increasing.** Figure 1 plots the trend in the Suits index of tax progressivity from 1991-2019. Throughout the 29 years, income tax in Australia has been highly progressive, with a Suits index in the range of 0.16 (in 1993) to 0.23 (in 2011). Moreover, tax progressivity increased substantially from 2005 to 2013. Since then tax progressivity has been on a slightly downward trend but still high compared to the 1990s.

![Figure 1: Trends in Suits index for income tax](image)

### 2.3 Trends in pension progressivity

We define a progressive pension system as one where benefits are distributed more unequally towards lower incomes. As pension becomes more progressive, the relative curve becomes more concave and the Suits index becomes negative. With pensions, a higher negative value indicates greater progressivity. Suits index equals 0 if pension shares are evenly distributed. For the purpose of illustration, we express the Suits index for pension in absolute terms such that an increase in the Suits index implies an increase in progressivity.
A highly progressive pension system. Figure 2 plots the trend in the Suits index for age-pension. For all years, the value is around 0.9, indicating the targeted, highly progressive nature of the pension system in Australia. In line with the sharp increase in tax progressivity since 2005, we observe a sharp rise in pension progressivity.

2.4 Progressivity of the tax code

According to the Suits index, tax progressivity is measured from a distributional point of view. Alternatively, we can also examine the progressivity that is encoded in the tax system through tax policy (i.e., via legislation).

Measurement. We take from Musgrave and Thirl (1948) and define a progressive tax code as one where tax liability increases when moving up the income scale. Australia’s income tax code consists of multiple income thresholds and statutory marginal tax rates that rise as we progress to higher thresholds. Further, those on lower income thresholds receive various credits and offsets.

We approximate this complex tax code using a parsimonious tax function commonly used in the public finance literature (e.g., see Jakobsson (1976), Persson (1983), Benabou (2002) and more recently Heathcote, Storesletten and Violante (2017)). Specifically, the total tax liability $t(y)$, average tax rate $atr$ and marginal tax rate $mtr$ take the functional form:

\[
\begin{align*}
  t(y) &= y - \lambda y^{(1-\tau y)} \\
  atr &= 1 - \lambda y^{-\tau y} \\
  mtr &= 1 - \lambda (1 - \tau y) y^{-\tau y}
\end{align*}
\]

$y$ is taxable income, $\lambda$ is a scale parameter that controls the level of the average taxation and $\tau y$.

\footnote{See Appendix A for an overview of Australia’s tax code}
is a curvature parameter that controls the curvature of the function\(^5\). When \(\tau^y = 0\), the tax code is proportional with an average tax rate of \(1 - \lambda\). The higher the value of \(\tau^y\), the more progressive is the income tax schedule\(^6\).

**Estimation and empirical fit.** We estimate the tax function using taxable income and tax liability from ALife data via 2 methods - ordinary least squares estimation of the logarithmic transformation of the function, and non-linear least squares. Both methods yield the same estimates and are reported in detail in Appendix B. Table 2 summarizes the OLS estimates of \(\tau^y\), their 95% confidence intervals and the adjusted R-squares of the estimations for some selected years. As evident from the table, we can obtain a very precise estimate of \(\tau^y\). This confirms that the tax function is a fair approximation of the income tax code in Australia.

<table>
<thead>
<tr>
<th>Year</th>
<th>(\tau^y)</th>
<th>95% Confidence interval</th>
<th>Adjusted (R^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991</td>
<td>0.152</td>
<td>(0.151,0.152)</td>
<td>0.97</td>
</tr>
<tr>
<td>2000</td>
<td>0.150</td>
<td>(0.150,0.151)</td>
<td>0.98</td>
</tr>
<tr>
<td>2010</td>
<td>0.129</td>
<td>(0.129,0.129)</td>
<td>0.99</td>
</tr>
<tr>
<td>2019</td>
<td>0.165</td>
<td>(0.165,0.166)</td>
<td>0.99</td>
</tr>
</tbody>
</table>

\(\tau^y\) has risen sharply in years. Figure 3 show the trend in \(\tau^y\) from 1991 to 2019. Throughout the 29 years, Australia’s income tax code has been very progressive. The value has ranged between 0.12 to 0.18. This range is in line with estimates of the parameter by Holter, Krueger and Stepanchuk (2014) for other OECD countries with highly progressive tax codes. In line with the context of this paper, the most relevant point from the figure is that in the last decade, since the sharp rise in 2012-2013, the level of progressivity has been at its highest since 1991.

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\(^5\)On some of the Figures in this paper, the notation for tax progressivity is given as \(\tau\) instead of \(\tau^y\). All such instances without a superscript denote tax progressivity as specified here.

\(^6\)This tax function is fairly general and captures the common cases:

\[
\begin{align*}
(1) \text{ Full redistribution: } t(y) &= y - \lambda y \\ \text{ and } t'(y) &= 1 & \text{ if } \tau^y = 1,
\end{align*}
\]

\[
\begin{align*}
(2) \text{ Progressive: } t'(y) &= 1 - (1 - \tau)\lambda y^{(-\tau y)} \text{ and } t'(y) > \frac{t(y)}{y} & \text{ if } 0 < \tau^y < 1,
\end{align*}
\]

\[
\begin{align*}
(3) \text{ No redistribution (proportional): } t(y) &= y - \lambda y \text{ and } t'(y) = 1 - \lambda & \text{ if } \tau^y = 0,
\end{align*}
\]

\[
\begin{align*}
(4) \text{ Regressive: } t'(y) &= 1 - (1 - \tau)\lambda y^{(-\tau y)} \text{ and } t'(y) < \frac{t(y)}{y} & \text{ if } \tau^y < 0.
\end{align*}
\]

The curvature parameter \(\tau^y\) is a closed-form expression of tax elasticity given by \(\frac{mtr(y) - atr(y)}{1 - atr(y)} = \tau^y\). If the elasticity is larger than unity, \(\varepsilon > 1\), additional tax liability on an additional unit of income (marginal rate) exceeds average tax liability at that income level (average rate), i.e., \(mtr(y) - atr(y) > 0\). This is explained more in Appendix B.
2.5 Pension progressivity and taper rates

Unlike income tax, the pension system cannot be approximated by a parsimonious, two-parameter function. This is because age-pension in Australia depends on whether an individual is single or partnered, and comprises both an income and asset test, with the test that results in a lower pension being used. However, this adds a layer of complexity that is beyond our current scope.

**Income test and taper rate.** We examine pension progressivity by approximating the pension system as follows. Let $p^{\text{max}}$ denote the maximum pension received provided that their assessable income $y^m$ does not exceed the low income threshold $\bar{y}_1$. Above this threshold, pension is reduced at a taper rate $\omega^g \in [0, 1]$ till threshold $\bar{y}_2 = \bar{y}_1 + p^{\text{max}}/\omega^g$. The pension benefit payment is given by

$$
p = \begin{cases} 
p^{\text{max}} & \text{if } y^m \leq \bar{y}_1 \\
p^{\text{max}} - \omega^g (y^m - \bar{y}_1) & \text{if } \bar{y}_1 < y^m < \bar{y}_2 \\
0 & \text{if } y^m \geq \bar{y}_2
\end{cases}$$

The taper rate controls pension progressivity. Given a fixed threshold $\bar{y}_1$, we can control progressivity by changing the taper rate. When $\omega^g = 1$, pension is subject to a strict means-test whereby all below a certain threshold obtain the maximum benefit, and those above receive none. Thus, given a significantly low threshold (as is the case in Australia), the pension system would be highly progressive.

When we decrease $\omega^g$, it leads to an increase in coverage. While those below the income threshold still receive the maximum benefit, those higher up the income scale become eligible for a partial benefit. The amount of benefit received by higher incomes increases as the taper rate decreases, making the pension system less progressive. Thus, decrease in $\omega^g$ implies a reduction in pension progressivity (and vice-versa). An $\omega^g = 0$ indicates a universal pension system.
Thus, the taper rate $\omega_y$ is a valid indicator of pension progressivity giving credence to its use to control pension progressivity in our quantitative experiments.

3 Relationship between optimal tax and pension progressivities

Before setting out on our large scale general equilibrium model, we demonstrate the relationship between optimal tax progressivity and optimal pension progressivity in a highly stylized partial equilibrium model.

3.1 Environment

The model is populated by overlapping generations of individuals of heterogeneous skill types (indexed $i \in I$) who live for two periods indexed $j \in \{1, 2\}$. We assume individuals supply labour inelastically. In the first period, they earn labour income at a skill specific wage rate $w^i$. In the second period, they are retired and live off their savings. In the absence of private insurance, this motivates young individuals to save an amount $s^i$ of their labour income. It also provides motive for age-pension from the government.

The means-tested pension system is given by

\[ p^i = \begin{cases} p^{\text{max}} - \omega r s^i & \text{if } r s^i < \bar{y} \\ 0 & \text{if } r s^i \geq \bar{y} \end{cases} \]  

Pension is funded by imposing labour income tax $\tau^i$.

Simplifying assumptions. Our focus in this section is purely to show the relationship between tax and pension progressivity in terms of the the progression of tax rates $\tau^i$ and the taper rate $\omega$. To do this in an easily tractable manner, we impose some simplifying assumptions on the model. The first, as already evident is inelastic labour supply.

Second, in order to make the pension problem simpler and draw all attention to $\omega$ in terms of pension progressivity, we assume that the income threshold $\bar{y}$ is equal to the maximum income that any household can obtain. Accordingly, the pension system simplifies to $p^i = p^{\text{max}} - \omega r s^i$.

Third, we assume that individual preferences are quadratic in terms of consumption in the two periods such that

\[ U^i = \chi c^1_1 - \frac{(c^1_1)^2}{2} + \beta \left[ \chi c^2_2 - \frac{(c^2_2)^2}{2} \right] \]  

where discount factor $\beta$ is set to 1 and $\chi > 0$ is a parameter determining consumption utility.

Optimal choices. This assumption simplifies the household problem such that individuals would divide their lifetime income equally between the two periods such that optimal consumption and saving is given by

\[ s^i = c^i_1 = c^i_2 = \frac{1}{2} \left[ (1 - \tau^i) w^i + \frac{p^{\text{max}}}{1 + r - \omega r} \right] \]  

This gives the indirect utility function
\[
V^i = \left[ (1 - \tau^i) w^i + \frac{p_{\text{max}}}{1 + r - \omega r} \right] \chi - \left[ (1 - \tau^i) w^i + \frac{p_{\text{max}}}{1 + r - \omega r} \right]^2
\]  

(10)

3.2 Government

For simplicity, we consider two household types: low and high skilled \(i = \{L, H\}\) whereby the former is less productive than the latter and have a lower wage rate, \(w^L < w^H\). This results in an income gap between the two types. The government takes the maximum pension benefit as given and aims to maximize aggregate welfare by redistributing income from high to low skilled households via progressive income tax and/or pension. We assume a utilitarian social welfare criterion given by \(\sum_{i \in I} V^i\). The general problem for the government is

\[
\max_{\tau^H, \tau^L, \omega, p_{\text{max}}} \{ V^L + V^H \text{ s.t. } \tau^L w^L + \tau^H w^H = 2p_{\text{max}} - \omega r (s^L + s^H) \}
\]  

(11)

We further simplify the government problem by assuming that only low skilled households are eligible for age pension. We also assume that the pension parameters are exogenous. The government problem then simplifies to

\[
\max_{\tau^H, \tau^L} \{ V^L + V^H \text{ s.t. } \tau^H w^H + \tau^L w^L = p_{\text{max}} - \omega r s^L \}
\]  

(12)

Given our simple functional form for indirect utility, this yields the equilibrium condition

\[
w^H - \tau^H w^H = w^L - \tau^L w^L + \frac{p_{\text{max}}}{1 + r - \omega r}
\]  

(13)

The optimal tax rates are

\[
\tau^H = \frac{p_{\text{max}} - \omega r}{1 + r - \omega r} \frac{p_{\text{max}}}{1 + r - \omega r} - \omega r \frac{1}{2} w^L + w^H - w^L
\]  

(14)

and

\[
\tau^L = \frac{p_{\text{max}} - \omega r}{1 + r - \omega r} \frac{p_{\text{max}}}{1 + r - \omega r} - \omega r \frac{1}{2} w^L - (w^H - w^L)
\]  

(15)

We can use Equations 14 and 15 to formulate two propositions in regards to the relationship between pension design and optimal tax progressivity.

**Proposition 1.** There is a negative relationship between pension progressivity and optimal tax progressivity.

**Proof.** We use Equations 14 and 15 to obtain a measure of the gap between the tax liability for high skilled and low skilled households.

\[
w^H \tau^H - w^L \tau^L = \frac{p_{\text{max}}}{1 + r - \omega r} + (w^H - w^L)
\]  

(16)

By assumption \(w^H \tau^H > w^L \tau^L\). Yet, this equation can still indicate the change in progressivity as the gap between the two tax liabilities will increase if \(\tau^H\) rises more than \(\tau^L\). Taking the derivative of Equation 16 with respect to \(\omega\) gives

\[

12
\]
\[
\frac{\partial}{\partial \omega} \left( w^H T^H - w^L T^L \right) = \left[ \frac{-r p^{\text{max}}}{(1 + r - \omega r)^2} \right] < 0
\] (17)

This result implies that an increase in the taper rate \( \omega \) would result in a decrease in tax progressivity, and vice versa.

**Proposition 2.** There is a negative relationship between pension generosity and the strength of the link between optimal tax and pension progressivities.

**Proof:** The derivative of Equation 17 with respect to \( p^{\text{max}} \) yields \( \frac{-r}{(1 + r - \omega r)^2} < 0 \). This shows that when maximum pension benefit is less generous, a decline in taper rate will make the tax system more progressive compared to a case where maximum benefit is less generous.

From these two propositions, we can conjecture that the converse in terms of the link between tax progressivity and optimal pension progressivity would also hold. That is, when income tax is more progressive, optimal pension would be less progressive and vice-versa. However, given the non-linearity in terms of solving for the socially optimal level of taper rate, we leave this conjecture to be examined in our large-scale general equilibrium model.

### 4 Quantitative model

We analyse the optimal design of pension and income tax in a rich computational dynamic general equilibrium overlapping generations (OLG) with idiosyncratic labour productivity risk and incomplete markets as per Bewley (1986) and Huggett (1993).

For the sake of conciseness, we defer a detailed description of the model and its calibration to Appendix D. (We encourage any reader who is not familiar with this class of model to read the Appendix thoroughly). In this section, we present the key features and detail parts central to our analysis.

The central feature of our model is the parametric tax function in equation 3 and the means-tested pension function in equation 6. Progressive tax and pension in the model economy provide social insurance to households who face uninsurable idiosyncratic labour productivity risk as per Bewley (1986) and Huggett (1993).

The model economy is on a balanced-growth path in steady-state equilibrium with labour-augmented productivity growth rate \( g \) and population growth rate \( n \). We also assume a stationary demographic structure.

#### 4.1 Features of the model that are central to the analysis

Our model is quite similar to other computational OLG models used to analyse fiscal policy in Australia such as Tran and Woodland (2014) and Kudrna and Tran (2018).

**Household heterogeneity and risk.** Households are heterogeneous with respect to their age \( j = 1, ..., J^P, ..., J \). One model period is equal to 5 years. They are eligible for age-pension at age \( J^P \), which corresponds to real age 65. There is no exogenous retirement age. Retirement from the labour force is through their endogenous labour-leisure choice such that households retire when
they choose 0 hours of work. They face random survival probability $\psi_j$ over their lifecycle, with a maximum age of $j = 14$ (89 years).

Households also face idiosyncratic labour productivity risk. The level of productivity shock at age $j$ is given by $\eta_{z,j} \in \{\eta_{1,j}, \eta_{2,j}, \eta_{3,j}, \eta_{4,j}, \eta_{5,j}\}$. These follow a hump-shaped pattern over the lifecycle. Labour productivity risk follows a Markov switching process with a transition matrix $\pi^\varrho_{z,j} (\eta_{z,j+1} | \eta_{z,j})$. This transition matrix differs by skill type and age, capturing the life cycle shocks faced by those with different levels of education. Productivity shocks, and their transition probabilities are obtained from data from Household Income and Labour Dynamics in Australia (HILDA) data using non-parametric estimation explained in Appendix E.

Skill types are exogenous in the model and are given by $\varrho \in \{low, mid, high\}$. They only affect transition probabilities $\pi^\varrho_{z,j}$. This is so that households with different levels of education could have the same realized productivity shock $\eta_{z,j}$. However, low skilled households have lower probability of attaining higher productivity quintiles compared to higher skills (and vice-versa).

State variables, labour supply and savings. Let the state of the household at age $j$ be

$$\chi_j = (j, \eta_{z,j}, a_j)$$

where $a_j$ is the stock of assets held in period $j$ and $\eta_{z,j}$, labour productivity. The effective wage a household receives is given by $w_{\eta_{z,j}}$. As per Bewley (1986) and Huggett (1993) households partially self-insure against labour productivity risk by accumulating a stock of private assets $a_j$ that earns interest income at a risk-free rate $r$.

Household income and net transfers. Labour and capital income form market income given by

$$y^m_j = w_{\eta_{z,j}} (1 - l_j) + r a_j$$

Before age $J_p$ (65 years), households are eligible for public transfers $st_{j < J_p} (j, \eta_{z,j})$, that depend on their labour productivity shock. In the absence of richer family structure and detailed public transfers, this provides a good approximation of progressive public transfers by age in Australia.

At age $J_p$, households become eligible for means-tested age pension

$$p_{j \geq J_p} = \min \{ \max \left[p_{max} - \omega y \left(y^m_j - \bar{y}_1\right), 0\right] \}$$

We explain the pension function in more detail in equation 6 in Section 2.5.

Households pay income tax on their taxable income $y_j$. Taxable income composes of market income and age pension. We approximate income tax using the tax function

$$t(y_j) = \max \left(y_j - \lambda y^1_j - \tau y^2_j, 0\right)$$

This is explained in more detail in Section 2.4. Given that we model the public transfer system separately, we exclude any negative tax liabilities from the tax function.

Household optimization problem. Given time invariant prices, taxes and transfers, the household problem is written recursively as
\[ V^j(\chi_j) = \max_{c_j, l_j, a_{j+1}} \left\{ u(c_j, l_j) + \beta \psi_{j+1} \sum_{\eta_{z,j+1}} \pi_{\theta,j}(\eta_{z,j+1} | \eta_{z,j}) V^{j+1}(\chi_{j+1}) \right\} \]

subject to:

\[ y_j \text{(taxable income)} = a_{j+1} = y_m \text{(market income)} \]
\[ a_j \geq 0, 0 < l_j \leq 1 \] (22)

where individual quantity variables except for labor hours are normalized by the steady state per capita growth rate \( g \). The household pays consumption tax \( \tau^c \) on their consumption \( c_j \). In our benchmark model, period utility depends on standard Cobb-Douglas preferences

\[ u(c_j, l_j) = \left[ \frac{c_j^{\gamma} (1 - l_j)^{1-\gamma}}{1 - \sigma} \right]^{1-\sigma} \] (23)

**Government policy.** The government finances its fiscal programs by collecting tax revenue via a personal income tax \( t(y_j) \), a tax on consumption at the rate \( \tau^c \in [0, 1] \) and a company income tax at the rate \( \tau^f \in [0, 1] \). Total government revenue is given by

\[ \text{Tax} = \sum_j t(y_j) \mu(\chi_j) + \sum_j t(c_j) \mu(\chi_j) + \tau^f(AK^\alpha H^{1-\alpha} - wH) \] (24)

where \( \mu(\chi_j) \) is the measure of agents in state \( \chi_j \).

In addition to the social welfare system explained above, the government also spends an amount \( G \) on general government purchases. Total government expenditure is financed by tax revenues and the issue of new debt which incurs interest payments \( rD \). In steady state, the level of public debt is constant and the government budget constraint is given by

\[ \text{Tax} = \sum_j p_j(y_m^j) \mu(\chi_j) + \sum_j st_j(\eta_j, j) \mu(\chi_j) + G + rD \] (25)

**Treatment of accidental bequests.** In the absence of annuity markets, deceased households leave behind accidental bequests. We assume that all bequests are taxed away akin to a 100% estate tax and not redistributed to living households. This is so as to preclude any social insurance effect from a strong model assumption such as equal distribution of bequests. As Bagchi (2019) explains, redistribution of bequests can create a insurance mechanism the effects of which could completely undo the effects of social security. Thus we take the position of completely avoiding such effects that arise from making an unrealistic distributive assumption around bequests.
Competitive equilibrium. While we only summarize the salient features central to our results, for completeness, we highlight the competitive equilibrium. Given the government policy settings for the tax system and the pension system, the population growth rate, world interest rate, a steady state competitive equilibrium is such that:

1. a collection of individual household decisions \( \{c_j(\chi_j), l_j(\chi_j), a_{j+1}(\chi_j)\}_{j=1}^J \) solve the household problem given by equation (22);

2. a single representative firm chooses effective labor and capital inputs to solve the profit maximization problem (Appendix D).

3. the government budget constraint defined in equation (25) is satisfied;

4. under small open economy assumptions, the current account is balanced and foreign assets \( A_f \) freely adjust so that domestic interest rates \( r \) equals the world \( r^w \) (not central to our analysis and explained more in Appendix D);

5. the domestic market for capital and labor clear

\[
K = \sum_{j \in J} \mu_j \int a_j(\chi_j) dA_j(\chi_j) + A_f
\]

(26)

\[
H = \sum_{j \in J} \mu_j \int (1 - l_j) \eta_{z,j}(\chi_j) dA_j(\chi_j)
\]

(27)

and factor prices are determined competitively such that \( w = (1 - \alpha) \frac{Y}{M}, q = \alpha \frac{Y}{K} \) and \( r = q - \delta \). \( K \) is the total stock of capital, and \( A_f \) is total amount of foreign capital stock. \( H \) gives aggregate effective labour.

4.2 The benchmark economy

Before proceeding to our policy experiments, we briefly describe our benchmark model economy. Again, we prioritize brevity in the main paper and focus on what is essential to contextualizing our results. We present the model calibration process step by step and provide further benchmark statistics in Appendix E.

Calibration summary. We map the steady state equilibrium to reflect key statistics for the Australian economy for 2012 – 2016. Table 3. presents values for parameters, source data and their respective benchmark targets.
Table 3: Key parameters, targets and data sources

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Demographics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Population growth rate</td>
<td>( n = 1.5% )</td>
<td>WDI</td>
</tr>
<tr>
<td>Survival probabilities</td>
<td>( \psi_j )</td>
<td>Australian Life Tables (ABS)</td>
</tr>
<tr>
<td><strong>Technology and market structure</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital share of output</td>
<td>( \alpha = 0.4 )</td>
<td>Tran and Woodland (2014)</td>
</tr>
<tr>
<td>GDP per capita growth rate</td>
<td>( g = 1.3% )</td>
<td>WDI</td>
</tr>
<tr>
<td>Depreciation</td>
<td>( \delta = 0.055 )</td>
<td>Tran and Woodland (2014)</td>
</tr>
<tr>
<td>Total factor productivity</td>
<td>( A = 1 )</td>
<td>(scaling parameter)</td>
</tr>
<tr>
<td>Interest rates</td>
<td>( r = r^w = 1.01% )</td>
<td>Investment share of GDP</td>
</tr>
<tr>
<td><strong>Preferences</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intertemporal elasticity of consumption</td>
<td>( \sigma = 2 )</td>
<td>Labour supply over the lifecycle</td>
</tr>
<tr>
<td>Share parameter for leisure</td>
<td>( \gamma = 0.36 )</td>
<td>Household savings share of GDP</td>
</tr>
<tr>
<td>Discount factor</td>
<td>( \beta = 0.99 )</td>
<td></td>
</tr>
<tr>
<td><strong>Fiscal policy</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption tax rate</td>
<td>( \tau^c = 10% )</td>
<td>Consumption tax share of GDP</td>
</tr>
<tr>
<td>Income tax</td>
<td>( \lambda = 0.7237 )</td>
<td>Income tax share of GDP,</td>
</tr>
<tr>
<td>Company profits tax rate</td>
<td>( \tau^f = 11% )</td>
<td>Company tax share of GDP and</td>
</tr>
<tr>
<td>Pension income test taper rate</td>
<td>( \omega^y = 0.5 )</td>
<td>Official taper rate</td>
</tr>
<tr>
<td>Maximum pension</td>
<td>( p^{max} )</td>
<td>Pension share of GDP</td>
</tr>
<tr>
<td>Pension thresholds</td>
<td>( \bar{y}_1 )</td>
<td>Pension participation rates</td>
</tr>
<tr>
<td>General government purchases</td>
<td>( G = Y \times 9% )</td>
<td>WDI</td>
</tr>
<tr>
<td>Public debt</td>
<td>( D = Y \times 10% )</td>
<td></td>
</tr>
<tr>
<td>Other public transfers</td>
<td>( ST = Y \times 6.4% )</td>
<td>OECD-SOCX</td>
</tr>
</tbody>
</table>


**Aggregate macro-fiscal variables.** One of our main focuses in the benchmark calibration was to get the key macroeconomic and fiscal aggregates to reflect the Australian economy as closely as possible. As Table 4 shows, we are able to closely match investment and consumption shares of GDP. Most importantly, our model matches the tax revenue shares and the pension and public transfer shares of GDP.

Our model is also able to closely replicate the distribution of income tax liabilities relative to the distribution of taxable income. In this regard, the Gini coefficient of taxable income is at 0.44 and the Suits index \(^7\) is at 0.3. However, net income inequality is slightly lower in the model compared to data.

\(^7\)The Suits index target is based on the distribution of taxable income taken from ALife 2012-16. In that, taxable income is derived from the variable ic_taxable_income_loss.
Table 4: Key variables in the benchmark economy

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model</th>
<th>Targets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment</td>
<td>18.94</td>
<td>26.51</td>
</tr>
<tr>
<td>Consumption</td>
<td>54.87</td>
<td>56.30</td>
</tr>
<tr>
<td>Age-pension</td>
<td>2.62</td>
<td>2.54</td>
</tr>
<tr>
<td>Public transfers other than age-pension</td>
<td>6.49</td>
<td>6.42</td>
</tr>
<tr>
<td>Government debt</td>
<td>11.5</td>
<td>10</td>
</tr>
<tr>
<td>Personal income tax</td>
<td>11.4</td>
<td>11.4</td>
</tr>
<tr>
<td>Consumption tax</td>
<td>5.49</td>
<td>5.55</td>
</tr>
<tr>
<td>Company income tax</td>
<td>4.40</td>
<td>4.25</td>
</tr>
<tr>
<td>Suits index (Income tax distribution)</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Gini coefficient (Taxable income)</td>
<td>0.44</td>
<td>0.45</td>
</tr>
<tr>
<td>Gini coefficient (Net income)</td>
<td>0.28</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Note: All variables are expressed in terms of percentage of GDP. Data are averages of annual variables from 2012-2016 taken from the International Monetary Fund, World Economic Outlook Database, October 2020 and the World Development Indicators.

Lifecycle profiles. For the most part, our model is also able to replicate patterns that are typically observed in lifecycle profiles in OLG models for Australia. Figure 4 displays lifecycle profiles for some key variables by skill type. The average labour hours per week by skill type in Figure 4a indicates that our labour productivity and skill specification generate a labour supply profile that reflects reality.

In this regard, higher skill types on average work more hours, followed by medium skill and low skill types. Consequently they earn higher taxable incomes, pay more income tax on average and consume more over the lifecycle (Figure 4e, Figure 4f and Figure 4c respectively).

While Figure 4b shows that household assets over the lifecycle match inverted U shaped pattern that is observed in data, all individuals in our model begin their life with no assets, highlighting one of the limitations of our model.

Figure 4d plots pension participation rates by skill type from the age of 60-65 to 85-90. The pattern of higher participation rates for low skill types compared to others confirms the calibration of the means-tested pension system in our model.
Figure 4: Lifecycle profiles in the benchmark economy

(a) Labour hours

(b) Assets

(c) Consumption

(d) Pension participation rates

(e) Taxable income

(f) Income tax liabilities
5 Quantitative analysis

5.1 Tax progressivity with benchmark pension

In our first experiment, we vary $\tau^y$ while maintaining the age pension parameters at their benchmark specification. We also keep other policy variables such as general government expenditure $G$ and other public transfers $ST$ fixed in real terms (while also maintaining its progressivity), and keep consumption and company income tax rates constant. We balance the budget by adjusting $\lambda$, which is the revenue requirement.

How does the tax function change in our experiment? Guner, Lopez-Daneri and Ventura (2016) show that a higher level of revenue requirement implies lower progressivity in terms of $\tau^y$. Similarly, given our budget balancing assumption, when $\tau^y$ increases, $\lambda$ decreases and vice-versa. In turn, this implies that the average level of taxation $(1 - \lambda)$ decreases as progressivity declines (Figure 5).

![Figure 5: Relationship between progressivity $\tau^y$ and the average rate of taxation $1 - \lambda$](image)

The change in both parameters changes the distribution of average and marginal tax rates across the income scale. Figure 6a illustrates the average tax function and Figure 6b illustrates the marginal tax function at different levels of $\tau^y$. We observe that as $\tau^y$ decreases, both tax functions pivot clockwise, resulting in a reduction of average and marginal tax rates for much of the income

![Figure 6: Average and marginal tax functions at different levels of $\tau^y$](image)
tax scale. This reduction is larger at higher income levels. However, this clockwise pivot increases tax rates at the lower end of the income tax scale. It also reduces the tax-free threshold (denoted by $\lambda^{\tau}$).

Figure 7: Average and marginal tax rates by skill type at different levels of $\tau^y$

(a) Average tax rates
(b) Marginal tax rates

The flattening of the tax code depicted in Figures 6a and b imply a reduction in the tax burden for higher income groups and an increase in the burden for lower income groups. In addition, it also implies a positive incentive effect due to declining marginal tax rates for all groups. Figures 7a and 7b illustrate the group averages of average tax rates and marginal tax rates for the three skill types. Figure 7a shows that as $\tau^y$ decreases, average tax rates on average decrease for the high skill types while the other two experience a sharp increase. Figure 7b depicts a sharp decline in marginal tax rates for all skill types. Both figures show a convergence in tax rates among skill types as $\tau^y$ decreases.

5.1.1 Effect on household behaviour

Figure 7 hints at the incentive effects at play for different skill types in our benchmark economy. In this section, we examine how these incentive effects change household behaviour as $\tau^y$ decreases. Figures 8a-d plot the change in savings (a), labour hours (b), consumption (c) and net income (d). All variables are expressed in percentage change relative to benchmark.

Savings. Figure 8a shows reducing progressivity leads to a sharp rise in household savings. This is most pronounced for high skilled households. We know from Figure 7 that they experience a sharp decline in both the marginal and average tax rates with declining progressivity. This strong positive income and substitution effect that encourages saving, resulting in a 105% increase with a proportional tax code.

In the case of medium and low skill types, rising average tax rates (Figure 7a) implies a negative income effect on savings. However, we see that the sharp decline in marginal tax rates (Figure 7b) for these households induces a stronger positive substitution effect. As a result, when the tax code is proportional savings for medium and low skill types increase by 70% and 60% respectively. The increase in savings across skill types results in aggregate savings increasing by 77%. 
Labour supply. Declining $\tau^y$ incentivizes low and medium skilled workers to work more on two fronts - an income effect from increasing average tax rates, and a substitution effect from reducing marginal tax rates. Thus, we observe that these two skill types experience larger increases in labour hours (Figure 8b) compared to high skill types. In the case of the former, although their marginal tax rates decline, declining average tax rates (Figure 7) result in a negative income effect. Hence they experience a smaller increase in hours compared to low and medium skilled workers.

The positive effect on hours from the tax code is dampened to a large extent by a substantial rise in capital income. We observe a smaller increase in labour hours (Figure 8b) compared to the increase in household savings as $\tau^y$ decreases. By increasing capital income through savings, a negative income effect is induced on the labour hours. Nevertheless, the fact that labour hours increase by over 10% shows that the positive incentive effects from a flatter tax code outweighs the negative income effect from increasing savings. We find minimal effects on the extensive margin as depicted by labour force participation rates in Figure 8d.

Consumption. Increase in savings and labour hours increase both capital and labour income. Figure 8d shows that this results in an increase in net income for all household types. As a result, there is a significant increase in consumption with decreasing progressivity (Figure 8c). Under a proportional tax code, aggregate consumption is 23% higher than the benchmark. The increase in consumption is rather uniform across household types. High skilled households experience a 27% rise while medium and low skilled households experience 21% and 20% respectively.
5.1.2 Welfare effects

Figure 9: Aggregate welfare gains and across skill types

Section 5.1.1 shows a significant gain in aggregate efficiency as well as across different types of households. In this section we show that this also translates to welfare improvements as illustrated in Figure 9. Overall, welfare as measured by consumption equivalent variation (CEV) increases with decrease in progressivity. Utilitarian social welfare (aggregate CEV, “All” households) is maximized with a proportional tax code, yielding a 5% gain relative to the benchmark.

However, we find that the gain in welfare is quite uneven between different skill types. In this regard, corresponding with the large decline in average tax rates that the high skill types experience (Figure 7a) and the resultant rise in savings and consumption (Figures 8a and c), we observe a sharp increase in their welfare. They attain a maximum of 8% with a proportional tax code.

In contrast, low and medium skill types experience a smaller welfare gain of 3% and 4% respectively. Further, their welfare gains plateau after $\tau = 0.04$. (This tax function is depicted in Figure 6). Figure 7a shows that the average tax rate sharply rises for the low and medium skill types when $\tau$ decreases below 0.04.

5.1.3 Effect of lower tax progressivity on benchmark pension

Efficiency gains from lower $\tau$ increases retirement savings and income. The substantial increase in savings due to lower progressivity translates into larger asset accumulation and higher market income over the lifecycle (Figures 10a and b). In a flat tax system, by the time they are eligible for age-pension (65 years), households earn a market income that is on average 21% higher than they would under the benchmark progressive tax code. Similarly, on average they hold 87% more assets at age 65 when the tax code is flat.
Figure 10: Average asset holdings and market income (benchmark vs. flat tax)

(a) Asset holdings

(b) Market income

(c) Pension

(d) Pension participation rate

**Less reliance on the pension system.** Hence, efficiency gains over the lifecycle results in fewer households relying on the pension system in old age. Figure 10c shows a significant shift in average pension by age when progressivity reduces from $\tau_y = 0.2$ to a flat tax with $\tau_y = 0$. On average, a flat tax system results in between 19% - 25% benefits for those between 65 and 80 years. Lowering progressivity has a smaller impact on pension in the very last decade of life. We also observe that lower progressivity has a smaller impact on pension participation rates (Figure 10d). There is only a modest downward shift in participation rates in the very early years of old age.

This reduction in pension reliance is not only true for the case of the flat tax. Figure 11a plots the percentage change in total pension benefits, and benefits by skill types against different levels of progressivity $\tau_y$. Reducing $\tau_y$ by a small amount from 0.2 to 0.14 does not have a significant impact on pensions. However, reducing $\tau_y$ further results in a steep decline. Consistent with all other results so far, higher skill types experience the sharpest decline. Yet, we also see a 10% reduction in pension benefits for the low skilled, showing that households across the skill distribution rely less on age-pension as $\tau$ decreases. Pension participation rates also decline by a small amount (Figure 11b) for all skill types.
5.2 Interactions between pension and tax progressivity

In Section 5.1, we focused on the effect of varying income tax progressivity with the age-pension parameters unchanged from their benchmark values. In this section we explore the effect of decreasing \( \tau^y \) on the benchmark pension system. Next we explore whether changing the progressivity of the pension system in terms of changing taper rates affects optimal tax progressivity.

5.2.1 Pension progressivity (taper rate \( \omega^y \)) and tax progressivity \( \tau^y \)

Varying pension taper rate \( \omega^y \) and tax progressivity \( \tau^y \). In our model, at any given level of maximum pension benefit and its respective income threshold, the maximum benefit taper rate \( \omega^y \) controls the progressivity of the pension system. When \( \omega^y = 1 \), pension is subject to a strict means-test whereby all below a certain threshold obtain the maximum benefit, and those above receive none. Thus, given a significantly low threshold (as is the case in our benchmark economy), the pension system would be highly progressive.

When we decrease \( \omega^y \), it leads to an increase in coverage. While those below the income threshold still receive the maximum benefit, those higher up the income scale become eligible for a partial benefit. The amount of benefit received by higher incomes increases as the taper rate decreases, making the pension system less progressive. Thus, decrease in \( \omega^y \) implies a reduction in pension progressivity (and vice-versa). An \( \omega^y = 0 \) indicates a universal pension system.

In Section 5.1, we saw that given the progressive pension system in our benchmark economy, it is socially desirable to reduce tax progressivity. Further, the optimal income tax system is a flat tax with no social insurance role. Given this context, we now examine whether social insurance from progressive pensions becomes more important (from a welfare perspective) as tax progressivity decreases.

To do so, we repeat Experiment 1, but this time varying the taper rate \( \omega^y \) between 0 and 1 while varying tax progressivity \( \tau^y \). At each level of tax progressivity \( \tau^y \), we examine whether welfare increases or decreases as we increase the pension taper rate \( \omega^y \) and make the pension system more progressive. To ease welfare comparisons, for each given level of \( \tau^y \), we compute CEV between economies with means-testing (\( 0 < \omega^y \leq 1 \)) and the economy with universal pension (\( \omega^y = 0 \)).

\[^8\] Same as the first experiment, we balance the budget by adjusting the income tax scale parameter \( \lambda \), and keep all other policy variables fixed at their benchmark rates or levels.
That is, at any given level of tax progressivity, we examine whether households are better off or worse off when the taper rate increases from 0 towards 1. This will make the pension system more progressive. If CEV rises as $\omega^y$ increases, it means households are better off with more progressive pension, and vice-versa.

**Optimal pension progressivity increases with decreasing tax progressivity.** Panels a-c in Figure 12 plot the change in CEV as the taper rate increases from 0 to 1 for four levels of tax progressivity. Tax progressivity increases from Figure 12a to Figure 12d. Figure 12a plots results for economies with a flat tax. Figure 12b shows benchmark $\tau^y = 0.2$. Finally Figure 12c shows a tax progressivity level that is higher than benchmark at $\tau^y = 0.3$. Moving from Figure 12a-c, we observe a clear relationship between tax progressivity and pension progressivity.

Figure 12: Welfare effects of increasing pension progressivity at different levels of tax progressivity

(a) $\tau^y = 0$

(b) $\tau^y = 0.2$

(c) $\tau^y = 0.3$

Starting from Figure 12a, when income tax is flat, welfare improves for all skill types with an increase in the taper rate. Thus, when the social insurance role is completely removed from income tax, social insurance via a more progressive pension system becomes more desirable. The relationship between taper rates and welfare is completely different in Figure 12b. All skill types gain by making the pension less progressive by lowering the from the benchmark ($\omega^y = 0.5$) to 0.2. When income tax is progressive, high skilled households prefer universal pension to means tested pension with any taper rate. When the tax system is even more progressive at $\tau^y = 0.3$ (Figure 12b), we observe an even stronger negative linear trend. Generally, all households experience a

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9 Although low skilled households prefer a slightly progressive taper rate around $\omega^y = 0.2$, this is only preferable...
welfare loss when pension becomes more progressive.

**Optimal tax progressivity and taper rate.** Changing the taper rate does not affect the optimal progressivity level of income tax. Figure 13 plots CEV at different levels of tax and pension progressivity with respect to the benchmark $\tau^y = 0.2$ and $\omega^y = 0.5$. Relative to our benchmark economy, holding the maximum benefit fixed, social welfare is maximized with a flat income tax and a strict means-tested pension system.

It is also important to note the small magnitude of welfare effects at any given level of tax progressivity. This is evident in Figure 12 as well as Figure 13. In this regard, we find that changing the taper rate does not have significant welfare effects compared to changing tax progressivity. This is because the distortionary effects of progressive income tax prevails throughout the lifecycle. In contrast, the pension system affects only in old age. As we saw in Section 5.1.3, efficiency gains from lower tax progressivity translate into less reliance on pension in old age.

Figure 13: Aggregate welfare at different levels of tax and pension progressivity

Reducing tax progressivity mitigates market distortions from progressive pension. We also observe an increase in aggregate efficiency with decreasing tax progressivity at any given level of pension taper rate. Figure 14a and Figure 14b plot the percentage change in aggregate hours and savings relative to benchmark for different $\tau^y$ at $\omega^y = \{0, 0.5, 1\}$. Overall, the effects of reducing tax progressivity on work and savings are the same as Section 5.1.1. The Figure reveals that the effect from tax progressivity dominates the effect from alternative pension taper rates.

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27
Further, reducing tax progressivity mitigates against distortions arising from progressive pensions. At any given level of tax progressivity, means-tested pensions distort labour supply and savings. The taper rate is an implicit tax on labour and capital income. Thus, an increase in the taper rate has a negative incentive effect that discourages savings and work as they face a higher marginal tax rate on their market income at older ages. At the same time, increasing the taper rate may have effects via the extensive margin. On one hand, as the taper rate increases, it lowers the possibility that an individual may be eligible for pension in old age. This possibility could induce some households to save more. On the contrary, it could also induce some to save less to increase their chance of being eligible.

Progressive taxation amplifies the negative incentive effect of increasing taper rates. By increasing the taper rate, the implicit marginal income tax rate becomes even more progressive. Thus, reducing tax progressivity would mitigate against this rise in marginal tax. Further, as explained in Section 5.1.3, efficiency improvements from lower tax progressivity over the lifecycle could lead to less reliance on pensions. The extensive margin effects from rising taper rates would thus be less pronounced.

Figure 15a plots the percentage change in aggregate hours worked for three different levels of tax progressivity as the pension taper rate increases. As the taper rate increases, initially there is a sharp reduction in hours worked at all levels of tax progressivity. The effect is more pronounced and prevails till higher taper rates (around $\omega = 0.6$) in the two progressive tax regimes where the increase in the marginal tax rate would be higher. In contrast, in the flat tax regime, we observe the positive incentive effect via the extensive margin slightly dominating as the taper rate increases above 0.4.
Figure 15: Savings and labour supply effects of increasing pension progressivity at different levels of tax progressivity

Figure 15b plots the change in aggregate savings as the taper rate increases. At all levels of tax progressivity, when we initially introduce taper rates, the positive extensive margin effect dominates. However, in the two progressive tax regimes, the adverse effects due to increasing implicit tax rates dominate as the taper rate increases beyond 0.4 (in the benchmark economy) and 0.6 (in the economy with higher tax progressivity). Importantly, when the tax code is flat, the effect of taper rate on savings plateaus at the very low taper rate of 0.2 due to a reduction in the reliance on pensions.

5.2.2 Pension generosity and tax progressivity

In addition to its progressivity, the desirability of any social insurance program depends on its generosity. For instance, consider the extreme case where the maximum benefit is a measly $100 dollars per annum. Regardless of whether that benefit is targeted only towards the very poor (a strict means test) or whether it is tapered so that richer households also receive a fraction of that $100, it may not provide adequate social insurance compared to a case where the maximum benefit is $50,000.

In this section, we test whether a less generous pension benefit warrants for social insurance via progressive income tax (and vice-versa). To do so, we vary the maximum pension benefit $p^{max}$ along with tax progressivity. We index the maximum benefit in an alternative economy to that in the benchmark as $p^{max}(\varphi^p)$ where $\varphi^p \in \{0, 0.5, 1, 1.5\}$. An increase in $\varphi^p$ increases the maximum benefit. When $\varphi^p = 0$, there is no pension, when $\varphi = 1$ it is equal to the benchmark.

**Optimal tax progressivity at different levels of maximum pension.** Figure 16a tracks the change in welfare relative to benchmark across the range of tax progressivity $\tau^y$ at the different levels of maximum pension. Decreasing the generosity of the maximum benefit improves welfare. Such a reduction results in lower pension expenditure. This in turn lowers the average rate of taxation $(1 - \lambda)$ at any level of tax progressivity. Thus, households experience welfare gains as their tax burdens decrease. Yet, this does not warrant a complete shut down of the pension system. As the figure reveals, low pension benefits ($\varphi^p = 0.5$) results in slightly higher welfare compared to when the pension system is shut down ($\varphi^p = 0$).

For each of the maximum benefit levels, reducing tax progressivity from the benchmark value of $\tau^y = 0.2$ improves aggregate welfare. However, welfare changes that households experience at the
very low levels (near optimal) tax progressivity are significantly different between economies with different levels of pension generosity. Figures 16b-d zoom into welfare changes from $\tau^{y} = 0.06$ to $\tau^{y} = 0$. In all 3 figures, there is a steep increase in CEV $\tau^{y}$ reduces from 0.06 to 0.04. In fact, when the pension system is shut down, welfare is maximized with a progressive tax system at $\tau^{y} = 0.04$ rather than a flat tax system. Comparing 16c and Figure 16d, we observe that welfare gains from moving from $\tau^{y} = 0.04$ to a flat tax is larger when the pension system is more generous.

Figure 16: Aggregate welfare at different levels of tax progressivity and maximum pension

5.2.3 Summing it altogether: the optimal tax and pension system

We extend our analysis in Section 5.2.2 further by considering the pension taper rate $\omega^{y}$ at different levels of tax progressivity and pension benefit. To do so, we examine the CEV relative to benchmark for different combinations of $\tau^{y} \in [0,1]$, $\omega^{y} \in [0,1]$ and $\varphi^{p} = \{0,0.5,1,1.5\}$. For the sake of conciseness, we shall focus on the “optimal” combinations of tax and pension progressivity that maximizes social welfare at each level of maximum benefit $\varphi^{p}$.

Table 5 details the change in welfare and macroeconomic aggregates at the optimal tax and pension taper rate at each level of pension benefit. Except for welfare and tax rates, all other variables are expressed in terms of percentage change relative to the benchmark economy. Average and marginal tax rates are averaged by household and across skill types.
Table 5 shows that in our dynamic general equilibrium economy, the optimal policy mix involves a slightly less progressive income tax system with $\tau^y = 0.02$, a highly progressive pension system with the taper rate $\omega^y = 0.9$ that has a lower pension benefit at $\phi^P = 0.5$.

At all levels of pension benefit and pension taper rate, welfare improves by reducing tax progressivity from our benchmark level of $\tau^y = 0.2$. Further, this is not only in aggregate, but across all skill types including the lowest. This suggests that the income tax system in Australia is presently more progressive than is socially optimal. This is however conditional on the generosity of the pension system. A reduction in pension benefit results in a social insurance role via a slightly progressive income tax.

In our general equilibrium economy, any discrepancy between government expenditures and revenues is financed by raising or lowering the average rate of taxation $1 - \lambda$. Hence, reducing the pension benefit results in lower average and marginal tax rates and consequently, increases welfare. Unsurprisingly, this is not uniform across skill types. Medium and high skill types gain by reducing
pension benefits as their tax burden falls significantly (from around 11% to 5-7%). However, low and medium skill types are better off in the low pension ($\varphi^p = 0.5$) economy compared to the no pension economy.

Reduction in pension benefit and tax progressivity results in substantial efficiency improvements. Lower pensions in old age encourage households to work and save more during over the lifecycle. In addition, reducing pension benefits considerably lowers the marginal tax rates at the top. Observe in Table 5 that even when the tax rate is slightly progressive, the top marginal tax rate does not exceed 11%. As a result, aggregate savings is 126% higher than benchmark when $\varphi^p = 0$ compared 78% when $\varphi^P = 1$. The effect of the pension system on labour hours is less pronounced than savings. Reducing pension benefits increase hours worked only slightly, implying that efficiency gains in terms of labour is mainly due to lower tax progressivity.

6 Extensions and sensitivity analysis

6.1 Progressive public transfers for workers

Our main focus is the interaction between age-pension and income tax. However, it is imperative that we briefly examine the importance of other public transfers to our results. In our model economy, public transfers before 65 years are a non-parametric approximation by age and labour income distribution. This provides a suitable approximation of the overall progressive public transfer system that provides social insurance for the majority of the lifecycle.

In this section, we examine whether optimal tax system is flat or almost flat because of progressive transfers. We do this by shutting down the public transfer system before 65 years and repeating our experiments. In order to maintain the income tax code in our benchmark economy unchanged, we increase general government expenditures $G$ to compensate for zero public transfers.

Figure 17 plots the CEV against different progressivity levels in this alternative economy. Aggregate welfare is maximized at $\tau^y = 0.04$. Without public transfers, low skilled households maximize welfare at even higher level of tax progressivity at $\tau^y = 0.06$. This suggests that public transfers play an important social insurance role in the economy. Moreover, it also gives weight to our previous argument for a flatter income tax given Australia’s highly progressive transfer system.
Figure 17: Aggregate welfare gains in economy with no public transfers < 65 years

Figures 18a-d compare the welfare gains from increasing taper rates at different tax progressivity levels between the benchmark model and the model without public transfers. While the general results that we obtained in Section 5.2.1 are robust to excluding transfers, the welfare gains from progressive pensions are higher. In the absence of other public transfers, increasing pension progressivity (higher taper rate) becomes even more important at lower levels of tax progressivity.

Figure 18: Welfare effects of increasing pension progressivity at different levels of tax progressivity (Benchmark vs. No transfer economies)
Similar to the pension system, it is reasonable to believe that the structure and progressivity of other public transfers also matter in determining the progressivity of the income tax code. However, a detailed examination of these aspects requires more details in terms of the actual transfers received by households. An important extension to our analysis of optimal progressivity is to model the transfer system in greater detail. Since public transfers are dependent on demographic characteristics such as age and family structure, such an analysis requires a model with greater household heterogeneity. We deem this beyond the scope of this paper and leave it for future research.

6.2 Labour supply elasticity

In our benchmark economy, household preferences are specified in terms of \( u(c, l) = \left[ \frac{c^{1-\gamma}l^{1-\gamma}}{1-\sigma} \right]^{1-\sigma} \). Under this specification, the Frisch elasticity is given by \( \frac{l}{1-l} \frac{1-\gamma(1-\sigma)}{\sigma} \) which varies over the lifecycle relative to labor supply. Under our benchmark specification, \( \sigma = 2 \), the Frisch elasticity of labour for the average household in our benchmark economy is 2.3.

We consider a higher Frisch elasticity by changing the risk aversion parameter to \( \sigma = 1.5 \) (Frisch elasticity of 2.7) and a lower elasticity of 1.8 by changing \( \sigma \) to 4. In each case, we ensure that the benchmark economy meets its calibration targets. Figure 19 plots aggregate welfare gains from reducing progressivity in the economies with these alternative parameterizations. The general trajectory of the results are robust to alternative labour supply elasticity assumptions.

That is, at both lower and higher Frisch elasticities, we find welfare and efficiency gains by lowering tax progressivity, lowering tax burden by lowering benefit levels and complementing that with higher pension progressivity. However, we find that when the magnitude of welfare gains is highly sensitive to labour supply elasticity. Further, when the elasticity is lower, the gains from reducing tax progressivity plateau sooner at \( \tau^y = 0.04 \).

Figure 19: Aggregate welfare gains with alternative values risk aversion and Frisch elasticity

It is reasonable to expect greater responses to changes in progressivity if labor supply is more elastic - that is, households increase their hours worked by a larger amount. Similarly, a lower (higher) \( \sigma \) also implies that household savings would be more (less) responsive to changes in tax progressivity. This is evident in Figure 20a and b where we plot the increase in aggregate hours worked and savings against decreasing tax progressivity. As the figure shows, the responsiveness
household savings to changes in progressivity is more sensitive to alternative values of $\sigma$ than hours worked.

Figure 20: Changes in aggregate hours worked and household savings with alternative values of risk aversion and Frisch elasticity

We also check the robustness of changing the maximum pension and taper rate with these alternative parameterizations. Table 6 summarizes key statistics at the optimal combination of tax progressivity and pension parameters in the three alternatives. The column in boldface lists the values from our benchmark model. We find that the optimal pension system is robust to the alternative assumptions on labour supply elasticity. In contrast, when labour supply elasticity is higher ($\sigma = 1.5$), even at the optimal strict means-tested pension system with lower benefit levels, the optimal tax system is a flat tax. When labour elasticity is lower ($\sigma = 4$), we find that it is socially desirable to have a tax system that is more progressive than our benchmark results.

Table 6: Optimal progressivity and taper rate under alternative labour supply elasticities

<table>
<thead>
<tr>
<th></th>
<th>$\sigma = 1.5$</th>
<th>$\sigma = 2$</th>
<th>$\sigma = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax progressivity $\tau^y$</td>
<td>0</td>
<td>0.02</td>
<td>0.06</td>
</tr>
<tr>
<td>Pension taper rate $\omega^y$</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Pension level $\varphi^p$</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Welfare (%CEV)</td>
<td>6.32</td>
<td>5.56</td>
<td>3.22</td>
</tr>
<tr>
<td>Savings (%CEV)</td>
<td>129</td>
<td>106</td>
<td>71</td>
</tr>
<tr>
<td>Hours (%$\Delta$)</td>
<td>21</td>
<td>20</td>
<td>16</td>
</tr>
</tbody>
</table>

7 Conclusion

In this paper, we examine the interaction between income tax progressivity and pension progressivity. We use Australia as a case study due to its unique combination of a highly progressive means-tested pension system and progressive income tax.

We motivate our paper by first quantifying tax and pension progressivity in Australia. We used data from ALife to quantify the level of tax and pension progressivity from 1991-2019 by calculating the Suits Index. We also estimated the progressivity of the income tax code. We found that both income tax and pensions in Australia have been very progressive over the last 29 years. Moreover it has sharply risen in the last decade.
Given this context, we next examined the optimal level of tax and pension progressivity using a dynamic general equilibrium OLG model calibrated to match key macroeconomic, fiscal and distributional features of the Australian economy. Our results show that, given its highly progressive pension system, the model economy gains welfare and efficiency improvements from changing to a flat income tax (no social insurance role).

As tax progressivity decreases, social insurance via progressive pensions becomes more critical. With a flat tax, the optimal pension system has a strict means-test (the most progressive). The economy experiences further welfare improvements with a lower maximum pension. But when pension is less generous, a slightly progressive income tax is socially desirable relative to a completely flat tax.

This paper is a first attempt to examine the Australian progressive income tax system's social insurance in a dynamic general equilibrium framework. Our paper affirms a general conclusion that the optimal tax system is contingent on a public transfer system's design. The Australian public transfer system is relatively generous and progressive compared to similar OECD countries.

As such, the adverse incentive effects on labor supply and savings from progressive taxation can be reduced by moving towards a less progressive income tax system. Public transfers in general and the means-tested age-pension, in particular, complement a less progressive tax system by providing social insurance for the poorer segment of the population.

This conclusion carries important policy implications. Governments interested in flattening the income tax code should give careful consideration to the design and generosity of the public transfer system to mitigate any reduction in the social insurance role of the income tax system.
References


URL: http://www.sciencedirect.com/science/article/pii/S0304393206000638


URL: https://www.sciencedirect.com/science/article/pii/S0047272719301355


Appendix: Further details to the main paper

A Related literature

A.1 Literature on optimal income tax and social transfers

Our paper links to three main branches within the tradition of dynamic general equilibrium literature on public finance - (1) the literature on optimal income tax progressivity, (2) optimal pension systems, and (3) optimal progressivity and optimal social security.

In regards to optimal progressivity, we follow the approach of searching for optimal progressivity within a given parametric class of tax scheme. This implies the use of a parametric tax function that closely approximates a given income tax code (where the slope of the function determines progressivity); and searching for the value of the progressivity parameter that maximizes the utilitarian social welfare function. This approach has a long standing tradition in the public finance literature going back to Ramsey (1927), Ventura (1999), Benabou (2002), Conesa and Krueger (2006), Krueger and Ludwig (2016), Heathcote and Tsujiyama (2016) and Heathcote, Storesletten and Violante (2017). This is in contrast to the the Mirrlees (1971) approach to optimal taxation that imposes no constraints on the form of the tax schedule. Heathcote and Tsujiyama (2016) shows that for a wide range of welfare functions, the best policy derived from utilizing a parametric class of tax function delivers the vast majority of potential gains from the fully optimal non-parametric Mirrlees tax schedule.

A standard prediction from this branch of literature that employs dynamic general equilibrium models with idiosyncratic risk and incomplete markets is that, reductions in progressivity have positive effects on welfare and aggregate activity and adverse impact on distribution. Also, common among these models is that, social security in general and the pension system are often simplified and not fully considered. The literature often employs benchmark models of the U.S. where coverage is universal, and effects from the extensive margin are not relevant.

The second branch of literature closely related to this paper is that of general equilibrium life cycle models that examine optimal pension systems. This branch includes papers such as Imrohoroglu, Imrohoroglu and Jones (1995), Sefton and van de Ven (2008) and Kudrna and Woodland (2011) that examine the effects via the intensive margin arising from means-tested versus PAYG pensions. A majority of these models show positive welfare outcomes in means-tested systems compared to PAYG systems. Tran and Woodland (2014) extend these papers by examining the extensive margin effects. They show that the interactions between taper rates and the maximum pension benefit via the extensive margin results in opposing effects on incentives and welfare effects to changes in taper rates vary significantly over the levels of maximum pension benefits. Similar to other papers within this branch, their analysis takes the tax system as given.

Closest to our paper in approach are those that examine the interplay between optimal tax progressivity and optimal social security. They analyse whether the generosity of specific social insurance schemes justify a more or less progressive tax system. McKay and Reis (2016) study the optimal generosity of unemployment benefits and progressivity of income taxes in a model with macroeconomic aggregate shocks and individual unemployment risk. They solve for the ex-ante socially optimal replacement rate of unemployment benefits and progressivity of personal income taxes in the presence of uninsured income risks, precautionary savings motives, labor market frictions
and nominal rigidities. Their results imply that, more generous unemployment benefits justify a more progressive income tax system. Tran and Jung (2018) examine optimal progressivity together with the design of the health insurance system in a model where individuals are exposed both to idiosyncratic and health risks over the lifecycle. They find that the design of the health insurance system strongly affects optimal progressivity, whereby in the presence of health risk the optimal tax system is more progressive compared to those that abstract from health risk such as Conesa and Krueger (2006) and Heathcote, Storesletten and Violante (2017). When health risk is reduced or removed, the optimal tax system becomes less progressive and closer to the optimal tax system reported in previous literature. The central message of these papers is that optimal progressivity depends on the type of risk being mitigated social insurance, and the adequacy of relevant social insurance mechanisms.

This paper contributes to the literature studying tax progressivity in Australia. Tran and Zakariyya (2021) document stylized facts and estimate the progressivity trends of the Australian personal income tax system. This paper builds a dynamic general equilibrium model that can match these facts and studies the optimal design of a progressive income tax system in conjunction with the means-tested pension system. This paper also contributes to the growing body of research on the impacts of fiscal policy reforms in Australia analysed using general equilibrium OLG models that incorporate the behaviour of households and firms (e.g., see Kudrna and Woodland (2011), Tran and Woodland (2014) and Kudrna and Tran (2018)). Different from these previous studies, we focus on the personal income tax system.

Recent work by Ferriere et al. (2022) examines the optimal design of means-tested transfers and progressive income tax. Using a canonical heterogeneous agent model with infinitely lived households as per Aiyagari (1994), they demonstrate an optimally negative relation between transfers and income tax progressivity due to efficiency and redistribution concerns. They approximate public transfers in the US using a flexible transfer function with two parameters: a level and phase out (taper) rate. Capital income is subject to a flat tax rate while labour income tax is approximated by a non-linear tax function.

A.2 Related studies on income tax progressivity in Australia

One of the earliest papers that examined tax progressivity in Australia is by Kakwani (1977), in which the author examined income tax statistics for Australia (1962 – 1972), Canada (1966 – 1972), Britain (1959 – 1967) and the United States (1958 – 1970). Kakwani found that there were relatively small differences in the degrees of income inequality before and after tax, except for the US. He also found that during the period, Australia had the highest degree of tax progressivity compared to the other advanced economies. Hodgson (2014) explores the relationship between personal income tax rates and means tested transfer payments in Australia from 1970 to 2014. She documents the major reforms in taxes and transfers during that period. She argues that the Australian tax and transfer system shifted from one with highly progressive tax rates coupled with universal benefits to flatter tax rates coupled with more targeted and means tested benefits.

Smith (2001) applies the tax distribution approach and provides a comprehensive study on tax progressivity in Australia. She estimates the degree of income tax progressivity from 1917 to 1997 from Australian official income taxation statistics, using 3 indices of tax progressivity - the Kakwani (1977) index, Suits (1977) index and Musgrave and Thin (1948) index. She finds a peak
in tax progressivity in the early 1950s on the Kakwani and Suits indices and a strong decline till the late 1970s followed by a relatively steady trend until 1997. She also finds that only a slight temporary increase in progressivity was associated with tax reforms in the 1970s and 1980s. The results with Musgrave and Thin index were ambiguous in direction with occasional peaks. Smith (2001) only uses taxation statistics and does not extend beyond 1997. Herault and Azpitarte (2015) use the Australian Survey of Income and Housing Costs (SIHC) from 1994 and 2009. They find the Kakwani index declined from a peak value of 0.27 in 1997 to 0.23 in 2005, and increased in 2007 and 2009. We extend the tax distribution approach to a more recent and important period since the introduction of New Tax System Act 1999. We employ two new datasets: survey data (HILDA) and administrative data (ATO sample of tax records). We show that the levels of tax progressivity in Australia have been deteriorated sharply after 2010.

Our paper is related to a number of empirical studies on the redistributive effects of the Australian tax and transfer system. Whiteford (2010; 2014), Wilkins (2014) and Herault and Azpitarte (2015) are notable studies that examine trends in the redistribution and progressivity of both taxes and transfers in Australia. Whiteford (2010) provides a detailed examination of the progressivity of the Australian transfer system together with taxes by examining the ratio of transfers paid to the poorest quintile to those paid to the richest quintile between the mid 1990s to 2005 and the concentration coefficient for transfers from 1980 to 2000. He concludes that Australia has one of the most progressive systems of direct taxes of any OECD country. Wilkins (2014) studies income inequality between 2001 and 2010, using the Survey of Income and Housing (SIH) and the Household Income and Labour Dynamics in Australia (HILDA) survey. He shows that the effect of taxes on reducing income inequality declined in all income series used in the analysis. Wilkins (2014) and Whiteford (2010; 2014) are descriptive in essence and focus more on summary statistics of redistribution at various income levels rather than on examining measures of progressivity.

Our study overlaps with Herault and Azpitarte (2015) that examines trends in the redistributive impact of the tax and transfer system between 1994 and 2009 using the Australian Survey of Income and Housing Costs (SIHC). They measure the redistributive effect as per Reynolds and Smolensky (1977). They also compare the Gini index of pre-fiscal income (before tax and transfers) to post-fiscal income (after tax and transfers). They find that after reaching a peak value in the late 1990s, the redistributive effect of the tax and transfer system declined sharply. Differently, we HILDA and ATO data and find a similar declining trend in the redistributive effect from 2001 to 2009. However, when we go beyond 2009 we find a reversed tax progressivity trend.

There is a large literature on inequality in Australia. For example, Leigh (2005) deriv ed long-run inequality series from tax data. Wilkins (2015) documents trends in income inequality in Australia using household survey data and find a slight increase in income inequality over recent years. Chatterjee, Singh and Stone (2016) examine the rise in labour income inequality over the past decade using HILDA. Kaplan, Cava and Stone (2018) document the facts on consumption and income inequality among households in Australia, emphasizing the role of the rents imputed to home owners for conclusions about inequality. Differently, we document the joint distribution of income and tax liability using ATO data and also HILDA data. Our focus is different as we aim to estimate the progressivity level of the Australian personal income tax system.
B Measuring tax progressivity

In this section we provide more a detailed description of the analytical framework that we rely on to measure tax progressivity. In general, there is no consensus on how to measure the progressivity of an income tax system. The variety of measures can be classified into two main approaches: one based tax liability progression and one based on tax liability distribution. The former measures tax progressivity in terms of tax elasticity as income progresses, namely tax progression metric or tax progression-based measure. Meanwhile, the latter measures tax progressivity in terms of tax liability shares relative to income shares across income distribution, namely tax distribution metric or tax distribution-based measures.

B.1 Tax progression metric

In a progressive tax system, tax liability rises with income. The progressive level of a tax system can be measured in terms of tax progression at a given income level, which has a long standing in public finance going back to Pigou (1929) and Slitor (1948). Musgrave and Thin (1948) summarise three common measures of the tax progression approach in Table 7.

<table>
<thead>
<tr>
<th>Definition</th>
<th>Formula</th>
<th>Progressive</th>
<th>Regressive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average rate progression</td>
<td>$\frac{dT}{dy}$</td>
<td>$&gt; 0$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>Liability progression</td>
<td>$\frac{dT}{dy} \cdot \frac{y}{T}$</td>
<td>$&gt; 1$</td>
<td>$&lt; 1$</td>
</tr>
<tr>
<td>Residual income progression</td>
<td>$\frac{n(y-T)}{dy} \cdot \frac{y}{(y-T)}$</td>
<td>$&lt; 1$</td>
<td>$&gt; 1$</td>
</tr>
</tbody>
</table>

Note: $T$ denotes the total tax liability and $y$ is pre-government income.

The income tax schedule is progressive if the elasticity of tax liability is greater than unity, $\varepsilon > 1$. Let $m(y) = \frac{dT}{dy}$ and $t(y) = \frac{T}{y}$ denote marginal tax rate and average tax rate, respectively. The elasticity of tax liability can be expressed in terms of a ratio of marginal tax rate to average tax rate as $\varepsilon = \frac{m(y)}{t(y)}$.

This ratio implies an interpretation of tax progressivity. That is, the income tax schedule is progressive if the additional tax burden on an additional unit of income exceeds the average tax burden at that income level

$$\frac{m(y)}{t(y)} > 1 \text{ or } m(y) - t(y) > 0$$ (29)

Intuitively, an income tax system is progressive if the marginal tax rate is higher than the average
tax rate and becomes more progressive when the gap between marginal and average tax rates, \( m(y) - t(y) \), is relatively larger.

**A parametric tax function.** The elasticity of tax liability can be calculated by assuming a parametric tax function summarizing the complicated structure of taxes in easy-to-interpret and an easy-to-use parametric form. We consider a parametric tax function that maps pre-government income to post-tax income as

\[
\tilde{y} = \lambda y^{(1-\tau y)}, \quad \lambda > 0, \quad 0 \leq (1 - \tau y) \leq 1
\]

where \( \tilde{y} \) is post-tax income, \( y \) is pre-government income, \( \lambda \) is a scale parameter that controls the level of the tax rate and \( \tau y \) is a curvature parameter that controls the slope of the function. This function is commonly used in the public finance literature (e.g., Jakobsson (1976), Persson (1983) and more recently, Heathcote, Storesletten and Violante (2017)).

Using this function, we can work out the total tax payment \( T \) and the average tax rate \( t(y) \) as a function of pre-government income \( y \) as

\[
T = y - \lambda y^{(1-\tau y)} \quad \text{and} \quad t(y) = 1 - \lambda y^{-\tau y}. 
\]

The elasticity of tax liability can be expressed in term of the adjusted gap between marginal and average tax rates as

\[
\frac{m(y) - t(y)}{1 - t(y)} = \tau y
\]

According to the interpretation of tax liability progression in Musgrave and Thin (1948), \( \tau y \) is a measure of the progressivity level in the tax schedule. When marginal tax rate is identical to average tax rate, \( \tau y = 0 \), it implies a proportional income tax system. When marginal tax rate is higher than average tax rate, \( \tau y > 0 \), the elasticity of tax liability is greater than unity and the income tax schedule is progressive.

Alternatively, the elasticity of residual income with respect to pre-government income is given by

\[
\frac{1 - m(y)}{1 - t(y)} = 1 - \tau y.
\]

According to the interpretation of residual income progression in Musgrave and Thin (1948), \((1 - \tau y)\) is the measure of residual income progression (see the third row of Table 7). An increase in the elasticity implies a reduction in progressivity and vice-versa. A tax system with a lower \((1 - \tau y)\) is more progressive than one with a higher \((1 - \tau y)\).

Thus, the curvature parameter \( \tau y \) can be used to a measure of how progressive a income tax system is. Note that, the elasticity approach to measuring tax progressivity can only give an indication of progressivity at a given point on income distribution. This can be viewed as a local measure of tax progressivity that is dependent on the income level.

---

\footnote{The parametric function approach also provides valuable inputs for quantitative studies of fiscal policy in models with heterogeneous agents. Krueger, K and Perri (2016) provide a review of this literature.}
B.2 Tax distribution metric

The tax distribution approach account for changes in income distribution over time that potentially affects tax progressivity. The tax distribution approach measures tax progressivity in terms of the tax liability distribution relative to the income distribution. This approach accounts for both the income tax schedule and income distribution in one measure.

We specifically consider a more general index that takes into account both the income tax schedule and the underlying distribution of income (e.g. see Pfahler (1987)). There are two common global measures that take this perspective: Kakwani index (Kakwani (1977)) and Suits index (Suits (1977)). Both indices examine the extent to which the tax system deviates from proportionality by comparing the distribution of pre-government income with the distribution of tax liabilities ordered by pre-government income. Intuitively, these two indices measure how tax liabilities are distributed across the income distribution. A more progressive tax system is simply one where the tax liabilities are distributed more unequally toward the higher end of the income distribution.

To formally define these two indices, we first define the cumulative distribution function and the associated concentration curves. Let $Y$ represent pre-government income and $T$ represent tax liabilities where both are non-negative and continuous random variables where $T = f(y)$. Let $\mu_Y$ and $\mu_T$ be the means of the pre-government income and tax liabilities respectively. The cumulative distribution function (c.d.f.) is $F_Y(y), 0 \leq p \leq 1$. Thus, the Lorenz curve of pre-government income is defined as $L_Y(p) = \mu_Y^{-1} \int_0^p y(x) dx$ where $y(p)$ is the $p$th quantile of the pre-government income distribution. The tax concentration curve is defined as $L_T(p) = \mu_T^{-1} \int_0^p t(x) dx$ where $t(p) = f[y(p)]$. Figure 21(a) illustrates the Lorenz curve and the tax concentration curves.

![Figure 21: Tax concentration curves, Kakwani index and Suits index](image)

The areas under the curves give the concentration index for each respective curve. As such, the concentration index for pre-government income is
\[ G_Y = 1 - 2\mu_Y^{-1} \int_0^1 \int_0^p y(x) \, dx \]  

(33)

and the concentration index for tax liabilities is

\[ G_T = 1 - 2\mu_T^{-1} \int_0^1 \int_0^p t(x) \, dx \]  

(34)

**Kakwani index** measures the deviation from proportionality by measuring the difference between the two concentration indices.

\[ K = G_T - G_Y \]  

(35)

If each individual’s income share is equal to her tax share, the two concentration curves will be equal such that \( G_T = G_Y \rightarrow K = 0 \) and the tax system is proportional. If tax shares exceed income shares, the concentration curve for tax will be more convex compared to the concentration curve for income such that \( K > 0 \) indicating a progressive tax system. Similarly if \( K < 0 \), the tax system is regressive such that the tax share for each respective individual is lower than the income share.

**Suits index** takes a different approach but uses the same concept of tax shares relative to income shares. Instead of relying on two concentration curves, the index relies on the relative concentration curve of taxes. The curve plots the cumulative proportion of tax liabilities ordered by pre-government income against the cumulative proportion of pre-government income. The 45 degree line indicates proportionality where tax shares equal income shares. A curve below the line indicates a progressive system where tax shares increase with rising income shares and vice-versa. The Suits index is the area between the 45-degree line and the relative concentration curve. The index ranges from -1 for the most regressive tax possible to +1 for the most progressive tax possible, and takes the value zero for a proportional tax. This is expressed as

\[ S = 2 \int_0^1 [q - L_T(q)] \, dq \]  

(36)

where \( L_T(q) \) is the relative concentration curve for tax liabilities where \( q \equiv L_Y(p), 0 \leq q \leq 1 \) is the value of the Lorenz curve for pre-government income associated with the population rank \( p \).

**C Detailed empirical results using ALife data**

**C.1 Data and sample composition**

We use data from ATO Longitudinal Information Files (ALife) 1991-2019. ALife consists of confidentialised unit records of individual income tax returns from the Australian Tax Office (ATO). It is based on a random sample of the total population of individuals on the ATO’s 2016 client register as described in Abhayaratna, Carter and Johnson (2021). Each subsequent year, a 10% sample of all new individuals who are added to the client register is randomly selected.

Our unit of measurement is an adult individual who legally pays taxes in Australia. The individual tax filer is tracked over time by their unique client identification. Individual information available in Tax Return forms, Super Member Contribution Statements (MCS) forms and the Self
Managed Superannuation Fund (SMSF) annual returns are included in ALife, including age, gender, geographic location and occupation. In the current standard release of ALife, there is no partner identifier.

Tax return forms consists of annual financial-year’s incomes, deductions, tax rebates and offsets, medicare levy and surcharge and other tax information from the individual tax returns. In years where a tax return was not lodged, the individual’s information for that year is missing in ALife.

Our analysis uses cross-sectional data from 1991-2019. In each year, we exclude those who earned negative pre-government income from our sample. Table 8 provides the number of individuals in our sample for 1991, 2010 and 2019.

<table>
<thead>
<tr>
<th>Year</th>
<th>Data</th>
<th>Sample</th>
<th>% Included</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991</td>
<td>983,476</td>
<td>736,584</td>
<td>75</td>
</tr>
<tr>
<td>1995</td>
<td>1,012,619</td>
<td>770,549</td>
<td>76</td>
</tr>
<tr>
<td>2000</td>
<td>1,076,254</td>
<td>838,057</td>
<td>78</td>
</tr>
<tr>
<td>2005</td>
<td>1,203,103</td>
<td>897,518</td>
<td>75</td>
</tr>
<tr>
<td>2010</td>
<td>1,338,919</td>
<td>976,803</td>
<td>73</td>
</tr>
<tr>
<td>2019</td>
<td>1,530,918</td>
<td>1,185,275</td>
<td>77</td>
</tr>
</tbody>
</table>

The sample for each year is quite balanced between males and females, with males composing of 50%-55% of individuals. The proportion of females in the sample steadily increases from 45% in 1991 to 49% in 2019 (Table 9). The age distribution is fairly constant across all years and genders with the mean age for both males and females are around 40 (SD = 15).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Male (%)</td>
<td>55</td>
<td>55</td>
<td>54</td>
<td>54</td>
<td>51</td>
</tr>
<tr>
<td>Female (%)</td>
<td>45</td>
<td>45</td>
<td>46</td>
<td>46</td>
<td>49</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Male</td>
<td>41</td>
<td>41</td>
<td>42</td>
<td>41</td>
<td>42</td>
</tr>
<tr>
<td>SD Male</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>Median Male</td>
<td>38</td>
<td>39</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Mean Female</td>
<td>40</td>
<td>41</td>
<td>42</td>
<td>41</td>
<td>42</td>
</tr>
<tr>
<td>SD Female</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>Median Female</td>
<td>38</td>
<td>38</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
</tbody>
</table>

C.2 Income, tax and pension concepts

We rely on 2 main income concepts. The first is market income which is the sum of total labour and total capital income. We use market income as the base income concept for our distributional analysis. That is, income shares, pension shares, tax shares and their related concentration coefficients are calculated by ordering individuals by market income.

The second income concept is taxable income. In Australia some public transfers including age-pension is taxable. We use taxable income as the base income concept when estimating the parametric tax function. In doing so, we make a further restriction on the sample to exclude
all income below the statutory tax-free threshold for a given year. We impose this restriction as including incomes below the threshold results in over-estimating tax rates at the bottom and under-estimating tax rates at the top.

We take income tax liability directly from the net tax ("tc_net_tax") variable included in the data. This measures income tax liability of an individual after deducting all eligible deduction, tax-offsets and credits. Pension is calculated using government pensions and allowances in the data. For each year, we use the pension eligibility age to infer whether that payment is age-pension or whether it is another type of government transfer. All income, tax and pension variables are expressed in real 2019 Australian dollars by adjusting for inflation using the consumer price index.

C.3 Tax and pension statistics 1991-2019

In this section we summarize tax and pension statistics from 1991-2019. Table 10 compares average market income, income tax and pension by quantile of market income between 1991, 2010 and 2019.

Table 10: Average income, tax and pension by quantile of market income 1991, 2005, 2019

<table>
<thead>
<tr>
<th>Market income</th>
<th>Income tax</th>
<th>Pension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quintile 1</td>
<td>5,998 9,295 7,901</td>
<td>379 716 262</td>
</tr>
<tr>
<td>Quintile 2</td>
<td>24,941 30,423 31,371</td>
<td>3,024 3,994 2,495</td>
</tr>
<tr>
<td>Quintile 3</td>
<td>41,726 47,811 51,418</td>
<td>6,709 8,848 7,542</td>
</tr>
<tr>
<td>Quintile 4</td>
<td>56,643 67,394 74,679</td>
<td>11,759 14,527 15,259</td>
</tr>
<tr>
<td>Quintile 5</td>
<td>94,916 133,844 155,449</td>
<td>25,442 38,499 45,578</td>
</tr>
<tr>
<td>Top 10%</td>
<td>116,558 177,189 206,979</td>
<td>33,052 55,116 65,567</td>
</tr>
<tr>
<td>Top 1%</td>
<td>250,535 514,100 601,027</td>
<td>70,861 181,830 226,107</td>
</tr>
<tr>
<td>Top 0.1%</td>
<td>608,582 1,671,701 2,116,809</td>
<td>143,576 598,681 818,022</td>
</tr>
</tbody>
</table>

Table 11 compares shares of market income, income tax and pension by quantile of market income between 1991, 2010 and 2019.

Table 11: Shares of income, tax and pension by quantile of market income 1991, 2005, 2019

<table>
<thead>
<tr>
<th>Market income</th>
<th>Income tax</th>
<th>Pension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quintile 1</td>
<td>3 3 2</td>
<td>1 1 0</td>
</tr>
<tr>
<td>Quintile 2</td>
<td>11 11 10</td>
<td>6 6 4</td>
</tr>
<tr>
<td>Quintile 3</td>
<td>19 17 16</td>
<td>14 13 11</td>
</tr>
<tr>
<td>Quintile 4</td>
<td>25 23 23</td>
<td>25 22 21</td>
</tr>
<tr>
<td>Quintile 5</td>
<td>42 46 48</td>
<td>54 58 64</td>
</tr>
<tr>
<td>Top 10%</td>
<td>26 31 32</td>
<td>35 41 46</td>
</tr>
<tr>
<td>Top 1%</td>
<td>6 9 9</td>
<td>7 14 16</td>
</tr>
<tr>
<td>Top 0.1%</td>
<td>1 3 3</td>
<td>2 4 6</td>
</tr>
</tbody>
</table>
D Quantitative model

We construct a large-scale overlapping generations model in the spirit of Auerbach and Kotlikoff (1987) that includes households who are ex-ante heterogeneous with respect to education level and ex-post heterogeneous due to uninsurable idiosyncratic labor productivity risk. Our model is an extended version of the general equilibrium OLG model developed for the Australian economy by Tran and Woodland (2014). Similar to other OLG models of the Australian economy, our benchmark is modeled under small open economy assumptions.

D.1 Demographics

The model economy is populated by 14 overlapping generations aged 20 – 89 years. One model period corresponds to 5 years. In each period, a new generation aged 20 enters the model and faces random survival probability $\psi_j$ with a maximum age of 89 years. We assume a stationary demographic structure such that the fraction of population of age $j$ at any point in time is given by

$$\mu_j = \mu_{j-1} \psi_j (1+n),$$

where $n$ is the constant rate of population growth.

Each cohort consists of 3 exogenous skill types that are based on education level $\varrho \in \{\text{low, medium, high}\}$. Those whose highest education attained is high school or below are classified as low skilled, those with a further tertiary training but without a graduate level qualification are classified as medium skilled, and graduates and higher are high skilled. These classifications capture differences in life cycle earnings profiles in Australia.

D.2 Endowments and preferences

Endowments. In each period, households are endowed with 1 unit of labor time with labor productivity $\eta_{z,j} \in \{\eta_1,j, \eta_2,j, \eta_3,j, \eta_4,j, \eta_5,j\}$ which follows a Markov switching process with a transition matrix $\pi_{\varrho,j}(\eta_{z,j+1}|\eta_{z,j})$. This transition matrix differ by skill type, capturing the life cycle shocks faced by those with different levels of education. It also provides for even low skill types to attain higher wage quantiles (albeit with a low probability).

The wage a household faces in the market is given by $w \cdot \eta_{z,j}$. Thus, households face two types of risk - idiosyncratic wage risk and mortality risk. As per Bewley (1986) and Huggett (1993) we assume that these cannot be explicitly insured. Rather, households self-insure against them by accumulating a stock of private assets $a_j$ that earns interest income at a risk-free rate $r$.

Household income and net transfers. The household’s market income thus includes labor income and capital income given by

$$y^n_j = w \cdot \eta_{z,j} \cdot (1-l_j) + r a_j$$

In addition to market income, households obtain public transfers $st_j(\eta_{z,j},j)$ that are dependent on the level of stochastic shock $\eta_j$ and age $j$. In this absence of a richer family structure, this provides an fairly approximate mapping from public transfers by age to market incomes.

Upon reaching the pension eligibility age $J^p$, households are also entitled to a public pension $p\left(y^n_j\right)$ that is dependent on their market income. These transfers from the government are explained in detail in section D.4. Households also pay consumption tax at the rate $\tau^c$ on their consumption $c_j$ and income tax $t_j$ on their taxable income $y_j$, which is the sum of their market income and age-pension.
Preferences. Households have preferences over stochastic streams of consumption \(c_j\) and leisure \(l_j\).

Let the state of the household at age \(j\) be \(\chi_j = (j, \eta_{z,j}, a_j)\). Given time invariant prices, taxes and transfers, the household problem is written recursively as

\[
V^j(\chi_j) = \max_{c_j, l_j, a_{j+1}} \left\{ u(c_j, l_j) + \beta \psi_{j+1} \sum_{\eta_{z,j+1}} \pi_{\eta,j} (\eta_{z,j+1} | \eta_{z,j}) V^{j+1}(\chi_{j+1}) \right\}
\]

subject to:

\[
a_{j+1} = \frac{1}{1+g} [a_j + \eta_{z,j} (1 - l_j) w + ra_j + b_j + st_j + p_j - t_j - (1 + \tau^c) c_j]
\]

\[
a_j \geq 0, \quad 0 < l_j \leq 1
\]

where individual quantity variables except for labor hours are normalized by the steady state per capita growth rate \(g\).

D.3 Technology

We assume a representative, competitive firm that hires capital \(K\) and effective labor services \(H\) (human capital) to operate the constant returns to scale technology \(Y = AK^\alpha H^{1-\alpha}\), where \(A \geq 0\) parameterizes the total factor productivity which grows at a constant rate \(g\) and \(\alpha\) is the capital share of output. Capital depreciates at a rate \(\delta\) in every period. The firm chooses capital and labor inputs to maximize its profit given rental rate \(q\) and the market wage rate \(w\) according to

\[
\max_{K,H} \left\{ (1 - \tau_f) (AK^\alpha H^{1-\alpha} - wH) - qK \right\}
\]

where \(\tau_f \in [0, 1]\) is the company income tax rate. The firm pays tax on a portion of its income denoted by its revenue minus wages.

D.4 Fiscal policy

Government revenues. The government finances its fiscal programs by collecting tax revenue via a personal income tax \(t(y_j)\), a tax on consumption at the rate \(\tau^c \in [0, 1]\) and a company income tax at the rate \(\tau_f \in [0, 1]\) (explained in the previous sections).

The government levies a progressive income tax on taxable income \(y_j\) that includes both labor income, capital income and pension. We approximate the Australian personal income tax code using the following parametric tax function explained earlier in Section X.

\[
t_j = \max \left(0, y_j - \lambda y_j^{1-\tau} \right)
\]

Total government revenue is given by

\[
Tax = \sum_j t(y_j) \mu (\chi_j) + \sum_j t(c_j) \mu (\chi_j) + \tau_f (AK^\alpha H^{1-\alpha} - wH)
\]
where $\mu(\chi_j)$ is the measure of agents in state $\chi_j$.

**Public pension.**

The amount of pension benefit $p_j$ is given by

$$
p_j(y_j^m) = \begin{cases} 
p_{\text{max}} & \text{if } y_j^m \leq \bar{y}_1 \\
p_{\text{max}} - \omega(y_j^m - \bar{y}_1) & \text{if } \bar{y}_1 < y_j^m < \bar{y}_2 \\
0 & \text{if } y_j^m \geq \bar{y}_2
\end{cases} \quad (42)
$$

where $\bar{y}_1$ and $\bar{y}_2 = \bar{y}_1 + p_{\text{max}}/\omega y$ are the income thresholds and $\omega$ is the income taper rate.

**Other public transfers.** In addition to the pension system, we approximate all other public transfers to households in order to closely reflect the breadth of the social welfare system in Australia. Other public transfers are given by $st_j(\eta_j, j)$ such that they are dependent on the level of stochastic shock $\eta_j$ and evolve over age $j$. This closely approximates the progressive nature of the transfer system, as well as changes in the level of transfers by the age of households.

Most importantly, in contrast to previous OLG models of Australia with similar public transfer assumptions, it does not restrict public transfers to low skill types alone. That is, even a high skilled individual facing an adverse labour productivity shock is eligible for the same public transfer as a low skilled individual of the same age facing the same adverse shock.

**Government budget constraint.** In addition to the social welfare system explained above, the government also spends an amount $G$ on general government purchases. Total government expenditure is financed by tax revenues and the issue of new debt which incurs interest payments $rD$. In steady state, the level of public debt is constant and the government budget constraint is given by

$$
Tax = \sum_j p_j(y_j^m) \mu(\chi_j) + \sum_j st_j(\eta_j, j) \mu(\chi_j) + G + rD \quad (43)
$$

The model allows for the government to have an additional role in distributing bequests (both accidental and intentional) from dead agents to those alive. Bequests are distributed equally across all surviving households. However, in our baseline experiments we assume that all accidental bequests are taxed away akin to a 100% estate tax.

**D.5 Market structure**

The Australian economy fits the description of a small open economy better than a closed economy. Thus, we assume that the domestic capital market is fully integrated with the world capital market. Hence, under free inflows and outflows of capital, the domestic interest rate $r$ is exogenously set by the world interest rate $r^w$. Labor is internationally immobile so that there is no migration. The wage rate $w$ adjusts to clear the labor market in equilibrium.

Markets are incomplete such that households cannot insure against idiosyncratic wage risk and mortality risk by trading state contingent assets. In addition, they are not allowed to borrow against future income, such that asset holdings are non-negative.
D.6 Equilibrium

Given the government policy settings for the tax system and the pension system, the population growth rate, world interest rate, a steady state competitive equilibrium is such that:

(i) a collection of individual household decisions \( \{c_j(\chi_j), l_j(\chi_j), a_{j+1}(\chi_j)\}_{j=1}^{J} \) solve the household problem given by equation (38);

(ii) the firm chooses effective labor and capital inputs to solve the profit maximization problem in equation (39);

(iii) the total lump-sum bequest transfer is equal to the total amount of assets left by all the deceased agents

\[
B = \sum_{j \in J} \frac{\mu_j - 1}{(1 + n)} \int a_j(\chi_j) \, d\Lambda_j(\chi_j) \quad \text{(44)}
\]

(iv) the current account is balanced and foreign assets \( A_f \) freely adjust so that \( r = r^w \), where \( r^w \) is the world interest rate;

(v) the domestic market for capital and labor clear

\[
K = \sum_{j \in J} \mu_j \int a_j(\chi_j) \, d\Lambda_j(\chi_j) + B + A_f \quad \text{(45)}
\]

\[
H = \sum_{j \in J} \mu_j \int (1 - l_j) e_j(\chi_j) \, d\Lambda_j(\chi_j) \quad \text{(46)}
\]

and factor prices are determined competitively such that \( w = (1 - \alpha) \frac{Y}{H} \), \( q = \alpha \frac{Y}{K} \) and \( r = q - \delta \);

(vi) the government budget constraint defined in equation (43) is satisfied.

E Mapping the model to data

We map the steady state equilibrium to reflect key statistics for the Australian economy for 2012 – 2016. We chose 2012 to begin our analysis to eliminate any temporary shocks to economic activity and resultant fiscal shocks due to the Global Financial Crisis. 2016 is the last year for which complete data on all key statistics were available. We present values for parameters that were determined by standard and their respective sources or benchmark targets in Table 12.
### Table 12: Key parameters, targets and data sources

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demographics</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Population growth rate</td>
<td>( n = 1.5% )</td>
<td>WDI</td>
</tr>
<tr>
<td>Survival probabilities</td>
<td>( \psi_j )</td>
<td>Australian Life Tables (ABS)</td>
</tr>
<tr>
<td>Technology and market structure</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital share of output</td>
<td>( \alpha = 0.4 )</td>
<td>Tran and Woodland (2014)</td>
</tr>
<tr>
<td>GDP per capita growth rate</td>
<td>( g = 1.3% )</td>
<td>WDI</td>
</tr>
<tr>
<td>Depreciation</td>
<td>( \delta = 0.055 )</td>
<td>Tran and Woodland (2014)</td>
</tr>
<tr>
<td>Total factor productivity</td>
<td>( A = 1 )</td>
<td>(scaling parameter)</td>
</tr>
<tr>
<td>Interest rates</td>
<td>( r = r^w = 1.01% )</td>
<td>Investment share of GDP</td>
</tr>
<tr>
<td>Preferences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intertemporal elasticity of consumption</td>
<td>( \sigma = 2 )</td>
<td>Labour supply over the lifecycle</td>
</tr>
<tr>
<td>Share parameter for leisure</td>
<td>( \gamma = 0.36 )</td>
<td>Household savings share of GDP</td>
</tr>
<tr>
<td>Discount factor</td>
<td>( \beta = 0.99 )</td>
<td>Households become economically active at age 20, ( j = 1 ). They are eligible for age-pension at age 65, ( j = 10 ). Household survival probability becomes zero (die with certainty) at age 90. We set the population growth rate to ( n = 1.5% ). We use Life Tables for the period from the Australian Bureau of Statistics to determine survival probabilities ( \psi_j ).</td>
</tr>
<tr>
<td>Fiscal policy</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption tax rate</td>
<td>( \tau^c = 10% )</td>
<td>Consumption tax share of GDP</td>
</tr>
<tr>
<td>Income tax</td>
<td>( \lambda = 0.7237 )</td>
<td>Income tax share of GDP, Suits index and Tax distribution</td>
</tr>
<tr>
<td>Company profits tax rate</td>
<td>( \tau^f = 11% )</td>
<td>Company tax share of GDP and investment/GDP ratio.</td>
</tr>
<tr>
<td>Pension income test taper rate</td>
<td>( \omega^y = 0.5 )</td>
<td>Official taper rate</td>
</tr>
<tr>
<td>Maximum pension</td>
<td>( \rho^{\text{max}} )</td>
<td>Pension share of GDP</td>
</tr>
<tr>
<td>Pension thresholds</td>
<td>( y_1 )</td>
<td>Pension participation rates</td>
</tr>
<tr>
<td>General government purchases</td>
<td>( G = Y \times 9% )</td>
<td>WDI</td>
</tr>
<tr>
<td>Public debt</td>
<td>( D = Y \times 10% )</td>
<td>WDI</td>
</tr>
<tr>
<td>Other public transfers</td>
<td>( ST = Y \times 6.4% )</td>
<td>OECD-SOCX</td>
</tr>
</tbody>
</table>


### E.1 Demographics

One model period lasts 5 years. Households become economically active at age 20, \( j = 1 \). They are eligible for age-pension at age 65, \( j = 10 \). Household survival probability becomes zero (die with certainty) at age 90. We set the population growth rate to \( n = 1.5\% \). We use Life Tables for the period from the Australian Bureau of Statistics to determine survival probabilities \( \psi_j \).

### E.2 Technology and market structure

Production in the economy is characterized by the Cobb-Douglas function \( AK^\alpha H^{1-\alpha} \). We follow Tran and Woodland (2014) and set the capital share of output \( \alpha = 0.4 \), the parameter \( A = 1 \) and the depreciation rate of physical capital \( \delta = 0.055 \). GDP per capita growth rate \( g \) is set at \( 1.3\% \) which is the average rate for Australia during the period, taken from the World Development Indicators database of the World Bank.

Under our small open economy assumption, we take the world interest rate on bonds as given...
and assume the world (and domestic) interest rate is \( r = 4\% \).

### E.3 Labor productivity

We estimate labor productivity using data drawn from the Household, Income and Labour Dynamics in Australia (HILDA) longitudinal survey for the years 2001-2018. We follow Nishiyama and Smetters (2007) to approximate the dynamics of labour productivity over the life-cycle. We define working ability/labour productivity as the hourly average wage rate, defined as gross labour income divided by total hours worked. We first group individuals aged between 20 and 64 into cohorts of 5 year age groups. We then classify individuals in each of these age groups in 5 quintiles of hourly wage rate. We assume that labour productivity declines linearly for those age 65 and above, reaching 0 at age 80.

The mobility of individuals from quintile to quintile over the lifecycle is governed by Markov transition matrices that are skill and age dependent. The following steps outline the estimation procedure for these matrices.

1. For each wave of the HILDA survey, we group individuals by skill type, age and quintile. Let \( N_{j,s}^{i=v} \) be the total number of individuals of skill type \( s \) and age \( j \) in quintile \( i = v \in [1, 2, 3, 4, 5] \).

2. Next, we track the movement of individuals in each group from age \( j \) to \( j + 1 \). That is, we see whether they have stayed in one quintile or moved to another, and if so, which quintile they moved to. Let \( n_{j+1,s}^{i=k} \) be the total number of individuals in the pool \( N_{j,s}^{i=v} \) in age \( j \) that moved to quintile \( i = k \in [1, 2, 3, 4, 5] \) at age \( j + 1 \).

3. The transition probability from quintile \( v \) at age \( j \) to quintile \( k \) at age \( j + 1 \) is then calculated as

\[
\pi_{j,j+1}^{i=k|v} = \frac{n_{j+1,s}^{i=k}}{N_{j,s}^{i=v}} \quad (47)
\]

To make the transition matrix more persistent, we use the average of estimates between 2001 and 2018. We assume that labor productivity declines at a constant rate, reaching zero at 80 years.

The difference between skill types in our model is thus not directly dependent on a skill specific labour productivity profile over the lifecycle. Rather, it depends on the transition probabilities that are different between skill types. For example, at the age of 40-45, both a high skilled individual and a medium skilled individual could be at the top quintile. However, a high skilled individual could be more likely to persist at the top, while a low skilled individual is more likely to descend to a lower quintile. We present more details of the estimation procedure and the Markov transition probability matrices by skill and age in the Online Appendix.

The main reason for choosing this method to estimate labour productivity is that we approximate public transfers below the age of 65 by wage quintile rather than by skill type as explained in Section D.4. This is a better approximation of reality as public transfers do not distinguish between skill type, but is highly correlated on labour income regardless of your educational background.

### E.4 Preferences

We assume that the period utility function is given by
\[ U(c,l) = \left( \frac{c^\gamma l^{1-\gamma}}{1-\sigma} \right) ^{1-\sigma} \]

where \(\gamma\) is the weight on utility from consumption relative to that of leisure, \(\sigma\) is coefficient of relative risk aversion. The subjective discount factor \(\beta\) is calibrated to match gross household savings to GDP ratio which has averaged around 0.2 according to ABS data.

E.5 Fiscal policy

We base our policy settings and their parameter values for the period between 2012-2016 to calibrate the fiscal policy in the benchmark model.

**Income tax.** As explained in Section D.4, we approximate the Australian income tax code using a parametric tax function. We calibrate the parameters of the function to approximate the tax-free threshold and average tax rates by income level during the period. We set the tax level parameter \(\lambda = 0.7237\) and the curvature parameter \(\tau_y = 0.2\) so as to match the income tax share of GDP, and the distribution of tax liabilities and the Suits (1977) index which calculates the concentration of tax liability relative to the distribution of taxable income.

**Consumption tax and company income tax.** The consumption and company income tax rates during the period were 10% and 30% respectively. However, we adjust these statutory rates in our benchmark model to match the actual tax revenue to GDP ratios. In the case of company income tax, we also target the net investment to GDP ratio. This results in a consumption tax rate of 10% and a significantly lower company income tax rate of 11%. In the case of the company income tax, it is important to note that our model only includes a single representative firm. The lower tax rate on companies reflect the significant number of small and medium enterprises in the economy who would be tax exempt. Thus, the company tax revenue share of GDP rather than the statutory rate is a better target for our model.

**Means-tested age pension.** The income test taper rate is set at \(\omega_y = 0.5\) which reflect the reduction in pension by 50 cents for every $1 above the low income threshold \(\bar{y}_1\). In order to test whether the asset test binds in our model, we also calibrate a version with the asset test where the asset test taper rate is \(\omega_a = 0.0015\) for every $1,000 above the low asset threshold \(\bar{a}_1\). Below these thresholds, households obtain the maximum pension denoted by \(p_{\text{max}}\). We calibrate \(p_{\text{max}}\) and the thresholds \(\bar{y}_1\)and \(\bar{a}_1\)to match pension participation rates over the life cycle and the public pension to GDP ratio.

In our benchmark model economy, the income test binds. Since it is the focus of the paper, we do not run any further experiments with the asset test taper rate. We leave that exploration for future work.

**Other public transfers.** We lump all public transfers other than pension such as family benefits, disability support pension and unemployment benefits in to \(st(\eta_j, j)\). We estimate the share of other public transfers by wage quintile \(\eta_j\) and age \(j\) using HILDA data and set the total amount of public transfers to match its share of GDP.
**General government expenditure and debt.** We define government expenditure other than public transfers and age-pension as general government expenditure $G$. We target government expenditure between 10-20% of GDP to reflect the average of these expenses over the benchmark period. Similarly, public debt is set at 10-20% of GDP which reflects the average net public debt share of GDP during the period. Both these aggregates are increased or decreased within this range during our calibration in order to adjust for tax revenue shares of GDP.