Growth through Diversity in Beliefs*

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Abstract

We study a macro-finance model with entrepreneurs who have diverse views about the likelihood that their ideas will lead to successful innovations. These views and the resulting experimentation stimulates economic growth and overcomes market failures that would otherwise occur in an equilibrium without this diversity. The resulting benefits for future generations come at the cost of higher wealth and consumption inequality because a few entrepreneurs will *ex*-post be successful while most entrepreneurs will fail. Hence, our model provides a potential explanation for the "entrepreneurial puzzle" in which entrepreneurs choose to innovate despite taking on substantial idiosyncratic risk accompanied by low expected returns. Taxes and a venture capital fund improve risk sharing among entrepreneurs and thus increase the likelihood of innovation. This reduction in entrepreneurial risk increases growth unless the tax scheme reduces the amount of entrepreneurial capital. Tax redistribution across cohorts lowers inequality and increases interest rates.

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JEL Classification: D51, G10, G11, G18, L26, O30, O40

1 Introduction

Entrepreneurial innovation and experimentation across diverse ideas are key for economic growth. However, innovations are also associated with substantial risk. By their nature, innovations require venturing into the unknown where it is difficult to predict the outcome. There are numerous examples and anecdotes of successful inventions that were discarded as impossible or even ridiculed but turned out to be hugely successful. Of course, there are also numerous examples of innovations, most of them we do not know about today, that turned out to be spectacular failures. Consider the invention of the light bulb by Thomas Edison in 1879. Not everyone at that time was enthusiastic about the prospects of this innovation. Henry Morton, a renowned scientist and president of the Stevens Institute of Technology, stated in the New York Times on December 28, 1879, that "Everyone acquainted with the subject will recognize it as a conspicuous failure." Henry Morton was not the only skeptic. A British parliamentary committee commented on the light bulb: "...good enough for our transatlantic friends...but unworthy of the attention of practical or scientific men." While these statements are ridiculous in retrospect, at the time it was not an uncommon view among intellectuals. Edison was also not the first one to invent an incandescent lamp. Friedel and Israel (2010) discuss more than 20 unsuccessful inventors prior to Edison's version. What made Edison succeed while so many others failed maybe be understood with the benefit of hindsight, but it was clearly important that so many were not discouraged or even prevented from trying because the light bulb is considered as one of the most important innovations in history.

The story about the light bulb is far from unique. Many important inventions have similar stories of uncertainty, doubt, and in particular very diverse views about its prospects. It just very difficult to predict much less agree on the next big idea and the right path forward to successfully implement the idea. Similarly, entrepreneurs often expose themselves to huge risks with seemingly low average returns when judged by an outsider. For instance, Moskowitz and Vissing-Jørgensen (2002) show that the average return to non-publicly traded firms is not higher than the return on publicly traded firms even tough ownership of the non-publicly traded firms is highly concentrated (often more than 70 percent in a single firm). The lack of diversification and the poor risk-return tradeoff from an outside perspective is difficulty to explain and thus the authors refer to it as the entrepreneurial puzzle.

Does diversity in beliefs shed some new light on the entrepreneurial puzzle? More generally, does belief diversity foster economic growth or does it divert resources to wasteful activities? Does diversity lead to more wealth inequality? Does it benefit only the lucky few and who bears the cost? In this paper, we address these questions by studying an equilibrium model with diverse views about the prospects of innovations. Like the light bulb where there were several possible ways of inventing an incandescent lamp and different inventors had different views, we allow entrepreneurs to have different beliefs about the right path forward. If the belief in success is high enough, an agent becomes an entrepreneur. However, only a few are ex post successful. When choosing to become an entrepreneur, the inventor has to bear a cost and cannot easily diversify risk due to a skin in the game constraint.

We show that belief diversity is instrumental to economic growth as entrepreneurs with different ideas decide to innovate and thus the society as a whole is "drawing from the entire distribution of ideas." Hence, diverse beliefs mitigate the skin in the game constraints stemming from moral hazard problems ex-ante and even though most ideas turn out to be failures ex-post, the few successful ones hugely benefit society and generate high economic growth for generations to come. This increase in economic growth comes at the cost of an increase in wealth and consumption inequalities. The cost from an ex-post point of view is mainly borne by the entrepreneurs as a group and the intergenerational growth among non-entrepreneurs is always the same as aggregate growth in the economy. Moreover, we show that these entrepreneurial activities are offering on average very low returns with substantial idiosyncratic risk and hence our model sheds new light on the entrepreneurial puzzle.

We study an overlapping generations model in the spirit of Blanchard (1985). Agents entering the economy can choose to become entrepreneurs to engage in innovative activities. To innovate, an entrepreneur has to put a significant fraction of her endowment at risk; that is, there are limits to risk sharing. There are several possible avenues or ways to innovate, but only one will turn out to be successful ex post. In our model there are as many agent types as ways of innovating. The agents have very diverse views about the right course of action and thus there is a distribution of beliefs about each way to innovate. In equilibrium, only the agents that are sufficiently optimistic choose to become entrepreneurs. However, once the choice to become an entrepreneur is made, the uncertainty is resolved and the agent can trade in complete markets.

The key friction in the model is that agents cannot diversify across the different ways of innovating. We do not micro found this friction, but we think of it as a skin in the game constraint due to moral hazard that works in the background. Importantly, it implies that the entrepreneur has to hold at least some fraction of the company's equity which in our model is the whole equity stake for simplicity. Moreover, it eliminates the possibility of pooling across all projects in equilibrium which would eliminate idiosyncratic risk.

We show that diverse beliefs among innovators overcomes the skin in the game constraint because the resulting optimism about an invention can make an investment that looks poor from an outsiders' perspective, be perceived as a high Sharpe ratio investment, even though everybody is well aware of the fact that they fail on average. This is akin to most people believing they are better drivers than the average. In addition, belief diversity works as a sampling device from the distribution of different ways to implement ideas. If everyone has the same view or an institution would settle on a view; e.g. the consensus view, then only one way of innovating would be implemented. While these views may have a higher probability ex-ante it would only lead to innovation if it is ex-post correct. In our model, there is a continuum of investors with different ideas and therefore the entire distribution of ideas is tested every period, leading to smooth economic growth for society that is much higher than when only one view is implemented. Hence, diverse views and the resulting experimentation is beneficial for society even though it is ex-ante and ex-post not beneficial for the average individual and thus leads to wealth and consumption inequality.

We also study two different mechanisms to share entrepreneurial risk: taxes and a venture capital fund. In the absence of costly effort to innovate taxes and investing in a venture capital fund reduces entrepreneurial risk and thus increases the likelihood of innovation. The reduction in entrepreneurial risk increases economic growth unless the tax scheme reduces the amount of entrepreneurial capital. While tax redistribution always lowers inequality, it only impacts asset price when taxes are redistributed across cohorts.

Our model is stylized, but very tractable. We derive all quantities in closed form and hence the economic forces are transparent. For instance, we do not consider aggregate uncertainty, agents have log utility, uncertainty related to innovations are resolved immediately, and there is a continuum of agents so that the law of large number applies. All of these features clearly contribute to the tractability of the model, but could easily by relaxed at the cost of less transparency of our economic point.

Our paper is related to the literature examining the asset pricing implications of technological innovation, such as Garleanu, Panageas, and Yu (2012), Kogan, Papanikolaou, and Stoffman (2013), Kung (2015), Kung and Schmid (2015), Garleanu, Panageas, Papanikolaou, and Yu (2016), Haddad, Ho, and Loualiche (2018), Lin, Palazzo, and Yang (2017), and Opp (2019). We also connect to papers looking at the impact of taxes on asset prices through the growth margin, such as Croce, Kung, Nguyen, and Schmid (2012) and Croce, Nguyen, and Schmid (2012). We also relate to papers studying wealth inequality and asset prices, such as, Gomez et al. (2016), Pástor and Veronesi (2016), and Pastor and Veronesi (2018).

Our paper also relates to the literature on that studies how heterogeneous beliefs effects asset prices such as Miller (1977), Harrison and Kreps (1978), Detemple and Murthy (1994), Zapatero (1998), Basak (2000), Scheinkman and Xiong (2003), Basak (2005), Berrada (2006), Buraschi and Jiltsov (2006), Jouini and Napp (2007), David (2008), Dumas, Kurshev, and Uppal (2009), Xiong and Yan (2010), Cvitanic and Malamud (2011), Cvitanic, Jouini, Malamud, and Napp (2012), Simsek (2013b), Simsek (2013a), Bhamra and Uppal (2014), Buraschi, Trojani, and Vedolin (2014), Cujean and Hasler (2017), Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2018), Collin-Dufresne, Johannes, and Lochstoer (2017), Ehling, Graniero, and Heyerdahl-Larsen (2018), and Borovička (2019).

Finally our paper also relates to the literature that studies the asset pricing implication of OLG models, such as Constantinides, Donaldson, and Mehra (2002), Gomes and Michaelides (2005), Gârleanu, Kogan, and Panageas (2012), Gârleanu and Panageas (2021) Kogan, Papanikolaou, and Stoffman (2019), Ehling, Graniero, and Heyerdahl-Larsen (2018), and Heyerdahl-Larsen and Illeditsch (2021).

2 Exogenous growth

In this section we introduce the benchmark model with exogenous output growth. There is no heterogeneity within birth cohorts but the exogenous growth will lead to heterogeneity across birth cohorts. In the next section we introduce a decision to become an entrepreneur when entering the economy that leads to successful innovations or failure. Hence, there will be endogenous growth and heterogeneity within and across birth cohorts. Both models are based on a continuous-time overlapping generations setting in the spirit of Blanchard (1985) and, more recently, Gârleanu and Panageas (2015).

Every agent in the economy faces a stochastic time of death τ that is exponentially distributed with hazard rate, $\nu > 0$. A new cohort of mass ν is born every period. Consequently, the population size remains constant, that is,

$$\int_{-\infty}^{t} \nu e^{-\nu(t-s)}\, ds = 1, \tag{1}$$

where $\nu e^{-\nu(t-s)}$ denotes the population density. An agent born at time s is entitled to the endowment stream $Y_{s,t}$ as long as the agent is alive. Enforcing the aggregate resource constraint, that is, integrating over all agents currently alive leads to

$$\int_{-\infty}^{t} \nu e^{-\nu(t-s)} Y_{s,t}\, ds = Y_t, \tag{2}$$

where Y_t denotes total output at time t. There is no aggregate uncertainty in the economy and the output growth rate μ_Y is exogenous. Hence, the dynamics of Y_t are

$$dY_t = \mu_Y Y_t \, dt. \tag{3}$$

Let $Y_{s,t} = \gamma Y_s$ with $\gamma > 0$ and thus there is no endowment growth within a birth cohort. If there is aggregate growth, then new birth cohorts receive more output and γ exceeds one. Specifically, applying Itô's lemma to the aggregate resource constraint leads to

$$\mu_Y = \nu(\gamma - 1). \tag{4}$$

Hence, aggregate output growth depends on the endowment growth of the new cohort, $\gamma - 1$, and the probability of having a new cohort over the next instant, νdt.

Agents can trade two assets: (i) an instantaneously risk-free asset in zero-net-supply and (ii) a life insurance/annuity contract that is offered by a competitive insurance industry as in Blanchard (1985) and Gârleanu and Panageas (2015). The dynamics of the real risk free asset with price B_t are

$$dB_t = r_t B_t \, dt, \tag{5}$$

where the real short rate, r_t, is determined in equilibrium. The life insurance/annuity contract pays the actuarily fair rate ν per unit of wealth in case of an annuity and it charges the rate ν per unit of wealth the case of a life insurance. Hence, the dynamics of the insurance contract from the agent's perspective are

$$d\mathcal{L}_t = \nu W_t^{\mathcal{L}} \, dt, \qquad \mathcal{L}_\tau = -W_\tau^{\mathcal{L}}, \qquad \forall t \leq \tau, \tag{6}$$

where $\mathcal{L}_t$ denotes the value of the insurance contract at time t and $W_t^{\mathcal{L}}$ is the amount of wealth invested in the insurance contract. It is optimal for an investor with positive financial wealth to annuitize all her wealth because she does not have any bequest motive and, hence, does not get any utility from dying with positive financial wealth. Moreover, investors have to buy life insurance for their negative financial wealth to avoid default at death because they are no longer entitled to an income stream after death. The financial wealth of all agents currently alive is always zero and thus the insurance market clears.

Agents have log utility and thus an agent born at time s has life time utility

$$U_s = E_s \left[\int_s^{\tau} e^{-\rho(t-s)} \log C_{s,t} \, dt \right] = E_s \left[\int_s^{\infty} e^{-(\rho+\nu)(t-s)} \log C_{s,t}, \, dt \right], \tag{7}$$

where $C_{s,t}$ denotes time-t consumption of an agent born at time s. Mortality risk, captured by the random time of death τ, increases the effective time discount rate from ρ to $\rho+\nu$. Once an agent is born, she can trade the risk free asset and the insurance contract, and therefore faces a dynamically complete market. Hence, we can solve the model by maximizing Equation (7) subject to the static budget condition (see Cox and Huang (1989) and Karatzas and Shreve (1998))

$$E_s \left[\int_s^{\infty} e^{-\nu(t-s)} \frac{M_t}{M_s} C_{s,t} \, dt \right] = H_s,$$

where H_s is the initial wealth of an agent born at time s and M_t is the stochastic discount factor with dynamics $dM/M = -r \, dt$ since there is no aggregate uncertainty. The first order conditions (FOCs) are

$$e^{-(\rho+\nu)(t-s)} \frac{1}{C_{s,t}} = \kappa_s e^{-\nu(t-s)} \frac{M_t}{M_s}, \tag{8}$$

and solving for $C_{s,t}$ leads to the optimal path for consumption

$$C_{s,t} = \kappa_s^{-1} e^{-\rho(t-s)} \frac{M_s}{M_t} = C_{s,s} e^{-\rho(t-s)} \frac{M_s}{M_t}, \qquad \forall \, s \leq t \leq \tau. \tag{9}$$

Inserting optimal consumption into the static budget condition leads to the optimal time-t total wealth of an agent born at time s. Specifically,

$$W_{s,t} = \frac{1}{\nu + \rho} C_{s,t}, \qquad \forall \, s \leq t \leq \tau. \tag{10}$$

To pin down optimal consumption we still need to solve for initial consumption $C_{s,s}$ or, equivalently, initial consumption per unit of output defined as $\beta_s = C_{s,s}/Y_s$. To determine β_s we value the endowment stream of an investor and then use the fact that investors consume a constant fraction out of their wealth (see equation (10)). Specifically, the time-t value of the endowment stream of an investor born at time s is

$$H_{s,t} = E_s\left[\int_s^{\infty} e^{-\nu(t-s)}\frac{M_t}{M_s}Y_{s,t}\,dt\right] = \gamma Y_s \psi_s, \qquad \forall\, s \le t \le \tau,$$

where ψ_s is the price of a life annuity that continuously pays one unit of the consumption good until time of death τ. Specifically,

$$\psi_s = E_s\left[\int_s^{\tau}\frac{M_t}{M_s}\,dt\right] = E_s\left[\int_s^{\infty} e^{-\nu(t-s)}\frac{M_t}{M_s}\,dt\right]. \tag{11}$$

Investors are born with no financial wealth and thus

$$\beta_s = \frac{C_{s,s}}{Y_s} = (\nu+\rho)\frac{W_{s,s}}{Y_s} = (\nu+\rho)\frac{H_s}{Y_s} = \gamma\psi_s. \tag{12}$$

To determine the risk-free rate we combine the FOC for optimal consumption with the aggregate resource constraint. Specifically,

$$\begin{aligned} Y_t &= \int_{-\infty}^{t} \nu e^{-\nu(t-s)} C_{s,t}\,ds = \int_{-\infty}^{t} \nu e^{-\nu(t-s)}\beta_s Y_s e^{-\rho(t-s)}\frac{M_s}{M_t}\,ds \\ \Rightarrow M_t Y_t &= \int_{-\infty}^{t} \nu e^{-(\nu+\rho)(t-s)}\beta_s Y_s M_s\,ds \end{aligned} \tag{13}$$

Applying Itô's lemma to the previous equation leads to the risk-free rate

$$r_t = \rho + \mu_Y + \nu(1-\beta_t) = \rho, \qquad \forall\, t \tag{14}$$

because $\beta_t = \gamma$ and $\psi_t = 1/(\nu + r)$ for all t.[1] Hence, the equilibrium interest rate is constant and does not depend on output growth. Intuitively, the interest rate is determined by all agents

[1] We formally verify in Proposition 1 that the risk-free rate is constant.

currently alive and even though there is endowment growth from generation to generation when γ exceeds one, nobody experiences any endowment growth after birth and therefore the interest rate does not depend on growth.

To determine the wealth-output ratio we determine total wealth in the economy by integrating over financial wealth of all agents currently alive. Specifically,

$$\begin{aligned} W_t &= \int_{-\infty}^{t} \nu e^{-\nu(t-s)} W_{s,t}\, ds = \frac{1}{\nu+\rho} \int_{-\infty}^{t} \nu e^{-\nu(t-s)} C_{s,t}\, ds \\ &= \frac{1}{\nu+\rho} \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \beta_s Y_s e^{-\rho(t-s)} \frac{M_s}{M_t}\, ds = \frac{1}{\nu+\rho} \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \gamma Y_s e^{(r-\rho)(t-s)}\, ds \\ &= \frac{Y_t}{\nu+\rho} \end{aligned} \tag{15}$$

The market portfolio is a claim on the total wealth of all agents currently alive and thus the price-dividend ratio is $P_t/Y_t = \psi = 1/(\nu+\rho)$. We summarize the results in Proposition 1.

Proposition 1. *In the exogenous growth model with endowment heterogeneity across birth cohorts, there is an equilibrium in which all agents consume their endowment, that is,*

$$C_{s,t} = C_{s,s} = \beta_s Y_s = \gamma Y_s = Y_{s,t}, \qquad \forall\, s \leq t \leq \tau. \tag{16}$$

The risk-free rate is constant and equal to the the subjective time-discount rate, that is, $r = \rho$. Moreover, the price-dividend ratio is equal to $P_t/Y_t = \frac{1}{\nu+\rho}$.

There is no consumption or wealth inequality within a birth cohort and the annualized log consumption growth rate across birth cohorts is equal to aggregate output growth. Specifically,

$$\frac{1}{\Delta} \log\left(\frac{C_{s+\Delta,t}}{C_{s,t}}\right) = \mu_Y, \qquad \forall\, \Delta > 0 \qquad \textit{and} \qquad s \leq t \leq \tau. \tag{17}$$

The risk-free rate does not depend on output growth and thus there is no growth in individual consumption which usually occurs when the risk-free rate exceeds the subjective time discount factor. However, the price-dividend ratio does not depend on growth which is surprising since the economy is growing and the risk-fee rate does not depend on growth. The reason is that we define the market portfolio as a claim on total wealth of all agents currently alive which does not include wealth that is generated by future generations.

We conclude this section by comparing the results to an exogenous growth model without output heterogeneity summarized in Corollary 1.

Corollary 1. *In the exogenous growth model without endowment heterogeneity, that is,* $Y_{s,t} = Y_t$ *there is an equilibrium in which all agents consume their endowment, that is,*

$$C_{s,t} = Y_s e^{(r-\rho)(t-s)} = Y_t, \qquad \forall\, s \leq t \leq \tau. \tag{18}$$

The risk-free rate is constant and equal to $r = \rho + \mu_Y$ *and the price-dividend ratio is* $P_t/Y_t = \frac{1}{\nu+\rho} = \psi$.

3 Endogenous growth

In this section we introduce the choice to become an entrepreneur and this choice will feed back into aggregate economic growth. Specifically, we consider the same demographic structure as in Section 2 in which each agent receives the endowment stream $Y_{s,t} = \gamma Y_s$. The choice to become an entrepreneur or innovator and the ex-post separation of agents in successful, unsuccessful, and non innovators determines γ and hence leads to endogenous growth and inequality within and across birth cohorts.

3.1 Innovation

Agents can choose to become entrepreneurs and engage in an activity that may lead to an innovation. Only entrepreneurs can innovate. Specifically, an agent entering the economy at time s has an endowment stream Y_s. They can only make the decision to become an entrepreneur at the time of entry. If the agent chooses not to innovate, then $Y_{s,t} = Y_s$ for all $s \leq t \leq \tau$. Hence, any agent who chooses not to innovate will not experience any growth in her endowment stream. An agent that chooses to become an entrepreneur must pay the cost $(1-\delta)Y_s$ with $0<\delta<1$. This cost can be thought of as an irreversible investment into the entrepreneurial activity. The entrepreneur must bear the entire investment risk due to an unmodeled skin-in-the-game constraint.[2] Importantly, the choice to become an entrepreneur is made just prior to entering the economy, and all uncertainty about the success of the innovation is resolved immediately after the decision to innovate has been made.

If the entrepreneur is successful, then her endowment is $Y_{s,t} = A\delta Y_s$ for all $s \leq t \leq \tau$. Let $A\delta > 1$ because otherwise a successful innovation would not raise output even if successful. If the entrepreneur is unsuccessful, then her endowment is $Y_{s,t} = \delta Y_s$ for all $s \leq t \leq \tau$. Agent's endowment stream is passed on to new generation when the agent dies. Hence, neither the knowledge created by a successful innovation nor the cost of an unsuccessful innovation vanishes. Equation (19) summarizes the endowment in the three different scenarios:

$$Y_{s,t} = Y_s \cdot \begin{cases} A\delta > 1, & \text{if} \quad \text{successful innovation} \\ \delta < 1, & \text{if} \quad \text{unsuccessful innovation} \\ 1, & \text{if} \quad \text{no innovation.} \end{cases} \tag{19}$$

We focus on a winner-takes-all-innovation economy in which many entrepreneurs try but only a handful capture a large share of the reward. Hence, every period there are $H \gg 1$ possible ways to innovate and only one leads to success with a large reward $A\delta \gg 1$. The expected per period endowment for an entrepreneur who thinks that each of the H possibilities are equally likely is

$$E^0\left[Y_{s,t}\right] = \left(A\frac{1}{H} + \left(1 - \frac{1}{H}\right)\right)\delta Y_s = \left(1 + \frac{1}{H}(A-1)\right)\delta Y_s. \tag{20}$$

Hence, a risk neutral agent would become an entrepreneur as long as this expectation exceeds the endowment of agents who do not become entrepreneurs (Y_s), that is, she becomes an entrepreneur if $\delta > \delta^* \equiv \frac{H}{H+A-1}$. However, a risk averse agent might require a large entrepreneurial risk premium due to the skin-in-the-game constraint that prevents her from diversifying and

[2] Relaxing this assumption by allowing the entrepreneur to sell part of the firm to an outside investor would lead to qualitatively similar results.

thus she has to bear the idiosyncratic entrepreneurial risk. Specifically, an agent with log-utility would innovate if

$$E^0 \left[\log Y_{s,t}\right] = \log\left(A\delta Y_s\right)\frac{1}{H} + \log\left(\delta Y_s\right)\left(1 - \frac{1}{H}\right) > \log(Y_s) \tag{21}$$

and thus she innovates if

$$\delta > \delta^{**} \equiv A^{-\frac{1}{H}} > \delta^*. \tag{22}$$

To stress the winner-takes-all innovation economy we consider the following parametric example for the remainder of this paper.

Example 1. *Suppose there are $H = 1000$ different ways to innovate and the reward of a successful innovation is very large, that is, $A = 1501$. Hence, $\delta^* = 40\%$ and $\delta^{**} = 99.27\%$.*

Hence, a risk-neutral entrepreneur or an entrepreneur who could diversify by investing in all H projects and thus eliminating the idiosyncratic risk would do so as long as she does not lose more than 60% of her investment if or when the project fails. Moderate risk aversion increases that threshold from 40% to 99.27% and thus the inability to diversify entrepreneurial risk due to the skin-in-the-game constraint leads to no innovation except for the rare situations in which to cost to innovate is less than 0.73%.

When agents are ex-ante indifferent between the H different ways of innovating, then the ex-post fraction of successful innovators is indeterminate since we do not know the fraction of entrepreneurs that choose the right way to innovate, that is, we do not know the fraction of entrepreneurs that choose the one successful innovations out of the H possible ones. We show in the next section that diverse views about the right way to innovate leads to more agents deciding ex-ante to become entrepreneurs and it uniquely determines the ex-post fraction of successful innovators, which is increasing in the degree of diversity.

3.2 Diverse Views and Experimentation

We define in this section the beliefs of entrepreneurs. Let p_h denote the subjective probability of an entrepreneur that path h is the right way to innovate with $p_h > \frac{1}{H}$ and p_j denote the subjective probability that path j is the right way to innovate with $p_j \equiv \frac{1-p_h}{H-1} < \frac{1}{H}$ for all $j \in \{1, \ldots H\} \backslash \{h\}$. Hence, all paths other than h are percevied to be equally likey by this entrepreneur. Hence, an entrepreneur who prefers path h to innovate behaves as if this path is the one out of the H possibilities that most likely leads to a successful innovation. Since only one out of the H possible ways to innovate leads to success we have that the probability of success and failure is p_h and $1 - p_h$, respectively.

Entrepreneurs not only have different views about the right way to innovate but they also differ in their degree of optimism and confidence about their chosen path. Hence, we assume that there is a unit interval of agents who differ across two dimensions: (i) their preferred way of innovating indexed by h and (ii) their perceived probability of success p_h. For each of the H different ways of innovating there is an equal mass of agents with different beliefs about the probability of a successful innovation, that is, $p_h = \frac{\Delta_h}{H}$ for all $h \in \{1, \ldots, H\}$. The parameter Δ_h which captures the degree of optimism and confidence across entrepreneurs who chose path h to innovate is uniformly distributed on the interval $\left[1, \bar{\Delta}\right]$ with $1 \leq \bar{\Delta} < H$. The lower bound of 1 for Δ_h guarantees that for each path h to innovate the mass $\frac{1}{H}$ of entrepreneurs think that the path h most likely leads to success and the upper bound of $\bar{\Delta} < H$ guarantees that no entrepreneur thinks that success is certain. Moreover, an entrepreneur who is optimistic about a specific path is pessimistic about all other paths and by symmetry we have that the consensus view of all entrepreneurs about the probability that a specific path leads to success is $1/H$. However, the consensus probability of success for entrepreneurs who prefer path h is

$$\bar{p}_{success} \equiv \int_1^{\bar{\Delta}} \frac{\Delta_h}{H} \frac{1}{\bar{\Delta} - 1} d\Delta_h = \frac{1 + \bar{\Delta}}{2H}. \tag{23}$$

The consensus view of all entrepreneurs about the success probability is also equal to $\bar{p}_{success}$ because the distribution of beliefs is the same for all paths. Similarly, the belief dispersion across all entrepreneurs defined as the cross-sectional standard deviation of beliefs is $\bar{\Delta}_{success} = \frac{\bar{\Delta}-1}{H\sqrt{12}}$ since

$$\bar{\Delta}_{success}^2 \equiv \int_1^{\bar{\Delta}} \left(\frac{\Delta_h}{H} - \bar{p}_{success} \right)^2 \frac{1}{\bar{\Delta}-1} d\Delta_h = \frac{1}{12} \left(\frac{\bar{\Delta}-1}{H} \right)^2. \tag{24}$$

We measure diversity across investors with the diversity index $\mathcal{D} = \bar{\Delta} - 1$ which monotonically increases the consensus success probability and belief dispersion but holds the consensus probability that a specific path leads to success equal to $1/H$.

We now discuss the ex-ante decision to become an entrepreneur. This decision is the same for each path and thus we drop the subscript h and use the parameter Δ when referring to an agent's perceived success probability. The expected per period endowment growth from innovating as perceived by an agent with belief Δ is

$$E^{\Delta} \left[\frac{Y_{s,t}}{Y_s} \right] = \left(A \frac{\Delta}{H} + \left(1 - \frac{\Delta}{H} \right) \right) \delta = \left(1 + \frac{\Delta}{H} (A-1) \right) \delta. \tag{25}$$

This expectation is strictly increasing in the degree of optimism Δ and attains its minimum if $\Delta = 1$. However, the risk is also changing with Δ because the variance of the per period endowment growth from innovating is

$$Var^{\Delta} \left(\frac{Y_{s,t}}{Y_s} \right) = (A(A-1)+1) \frac{\Delta}{H} \left(1 - \frac{\Delta}{H} \right) \delta^2. \tag{26}$$

The risk is a quadratic function of Δ that is maximized when $\Delta = \frac{H}{2}$.

To illustrate the entrepreneurial choice model we consider the following numerical example.

Example 2. *Suppose there are $H = 1000$ different ways to innovate and the reward of a successful innovation is very large, that is, $A = 1501$. Hence, $\delta^* = 40\%$ and $\delta^{**} = 99.27\%$. Let, $\bar{\Delta} = 199$ and thus the consensus success probability and belief dispersion is $\bar{p}_{success} = \frac{1+\bar{\Delta}}{2H} = 10\%$ and $\bar{\Delta}_{success} = \frac{\bar{\Delta}-1}{H\sqrt{12}} = 5.72\%$, respectively. Moreover, the maximum success probability is less than 20%.*

We now discuss the ex-post outcome. The belief distribution of entrepreneurs is the same for each path to innovate and thus if a fraction α decides to choose path h to innovate given their beliefs, then the same fraction chooses path j to innovate for all $j \in \{1, \dots H\} \backslash \{h\}$. Hence, by the law of large numbers we have with probability one that $\frac{\alpha}{H}$ entrepreneurs will be successful, $\alpha\left(1-\frac{1}{H}\right)$ will be unsuccessful, and $(1-\alpha)$ will not become entrepreneurs.

3.3 Decision to innovate

All agents have log utility and time discount rate ρ but differ w.r.t. they way to innovate indexed by h and their perceived probability of success index by Δ. Let i denote the index of an agent where each i refers to a pair (h, Δ). Agent i maximizes lifetime expected utility

$$U_s^i = E_s\left[\int_s^{\tau} e^{-\rho(t-s)} \log C_{s,t}^i \, dt\right] = E_s\left[\int_s^{\infty} e^{-(\rho+\nu)(t-s)} \log C_{s,t}^i \, dt\right]. \tag{27}$$

Agents know if the innovation was successful or not when entering the economy and they face complete markets once the innovation uncertainty has been resolved. Hence, the wealth of agent i born at time s immediately after the innovation uncertainty is resolved equals

$$W_{s,s}^i = E_s\left[\int_s^{\tau} \frac{M_t}{M_s} Y_{s,t}^i \, dt\right] = \omega_i Y_s \psi_s, \tag{28}$$

where ω_i denotes the fraction of total wealth of agent i given by

$$\omega_i = \begin{cases} A\delta > 1, & \text{if } i \text{ is} \quad \text{successful} \\ \delta < 1, & \text{if } i \text{ is} \quad \text{unsuccessful} \\ 1, & \text{if } i \text{ is} \quad \text{not innovating.} \end{cases} \tag{29}$$

and the valuation ratio ψ_s is given in Equation (11). All newborn agents are the same with the exception of their initial wealth immediately after the innovation uncertainty is resolved and thus we can solve the dynamic consumption-saving problem by using the static martingale approach as in Section 2. Specifically,

$$C^i_{s,t} = C^i_{s,s} e^{-\rho(t-s)} \frac{M_s}{M_t} = (\rho + \nu) W^i_{s,t} \qquad \forall\, s \leq t \leq \tau. \tag{30}$$

Plugging Equation (30) into lifetime expected utility given in equation (27) leads to

$$U^i_s = \frac{\log\left(W^i_{s,s}\right)}{\rho+\nu} + \frac{log\,(\rho+\nu)}{\rho+\nu} + \mathrm{E}\left[\int_s^\infty e^{-(\rho+\nu)(t-s)} \log\left(\frac{M_s}{M_t}\right)\, dt\right]. \tag{31}$$

Plugging in for wealth of agent i born at time s given in Equation (28) leads to

$$U^i_s = \frac{\log\left(\omega^i\right)}{\rho+\nu} + \underbrace{\frac{\log\,(\rho+\nu) + \log(\psi_s Y_s)}{\rho+\nu} + \mathrm{E}\left[\int_s^\infty e^{-(\rho+\nu)(t-s)} \log\left(\frac{M_s}{M_t}\right)\, dt\right]}_{=\bar{U}_s}, \tag{32}$$

where the second term of lifetime utility is independent of the agent's type. Hence, an agent born at time s will choose to become an entrepreneur if the expected lifetime utility from innovating is higher than the lifetime expected utility from not innovating. Specifically,

$$\mathrm{E}^i\left[U^{i,e}_s\right] \equiv \frac{\Delta}{H}\frac{\log\,(A\delta)}{\rho+\nu} + \left(1 - \frac{\Delta}{H}\right)\frac{\log\,(\delta)}{\rho+\nu} + \bar{U}_s \geq \frac{\log\,(1)}{\rho+\nu} + \bar{U}_s \equiv \mathrm{E}^i\left[U^{i,ne}_s\right]. \tag{33}$$

Hence, agents become entrepreneurs if their success probability exceeds the threshold Δ^*/H with

$$\Delta^* = -H\frac{\log\ \delta}{\log A} \geq 0.$$

The decision to innovate does neither depends on the valuation ratio ϕ_t nor the stochastic discount factor M_t and hence the threshold Δ^* is constant. To determine the equilibrium fraction of entrepreneurs for each cohort we determine the probability that belief type Δ exceeds threshold Δ^* using the cross-sectional distribution of belief types Δ, that is, Δ is uniformly distributed on the interval $[1, \bar{\Delta}]$. The threshold is constant and the cross-sectional beliefs distribution does not vary over time and thus the equilibrium fraction of entrepreneurs is the same for each birth cohort s. We summarize the results in the next proposition.

Proposition 2. *In equilibrium an agent with type* $i = (h, \Delta)$ *decides to innovate by choosing path* $h \in \{1, \ldots, H\}$ *if her success probability* Δ/H *exceeds the threshold* Δ^*/H *with*

$$\Delta^* = -H \frac{\log\ \delta}{\log A}. \tag{34}$$

The equilibrium fraction of innovators for every birth cohort is

$$\alpha = \begin{cases} 1 & \text{if} \quad \Delta^* \leq 1 \\ \frac{\bar{\Delta} + H \frac{log(\delta)}{log(A)}}{\bar{\Delta} - 1} & \text{if} \quad \Delta^* \in \left(1, \bar{\Delta}\right) \\ 0 & \text{if} \quad \Delta^* \geq \bar{\Delta}. \end{cases} \tag{35}$$

The fraction of entrepreneurs is weakly increasing in diversity in beliefs $\mathcal{D}$ *because it is strictly increasing in* $\mathcal{D}$ *for all* $\Delta^* \in \left(1, \bar{\Delta}\right)$*, that is*

$$\frac{\partial \alpha}{\partial \mathcal{D}} = \frac{1}{\mathcal{D}} \left(1 - \alpha\right) > 0 \text{ for } \Delta^* \in \left(1, \bar{\Delta}\right). \tag{36}$$

Ex-post only one of the H *innovation paths leads to success and thus for birth cohort* s *the fraction* α/H *is successful and has total wealth* $A\delta\psi_s Y_s$*, the fraction* $\alpha(1-1/H)$ *is unsuccessful and has total wealth* $\delta\psi_s Y_s$*, and the fraction* $(1-\alpha)$ *does not innovate and has total wealth* $\psi_s Y_s$*.*

The left plot of Figure 1 shows the minimum perceived success probability for an agent that results in innovation and the right plot shows the equilibrium fraction of entrepreneurs as a function of the parameter δ. The figure shows that no one innovates when the cost parameter δ is less than $A^{-\frac{\bar{\Delta}}{H}} = 23\%$ because in thus case the perceived success probability in order to have innovation would have to exceed the maximum perceived success probability of 20%. However, once the cost of innovating decreases such that δ exceeds 23%, the cutoff probability drops below 20% and the most optimistic agents start to innovate. If δ is between 23% and 40% (red line in both plots) it would not be optimal for a central planner who thinks each path to success is equally likely and can diversify away all the idiosyncratic risk to innovate. We will see in the next section that in this range innovation is detrimental for society because it leads to negative growth. If δ exceeds 40% (blue line), then there is innovation that benefits society because it leads to positive growth. We conclude this subsection with a numerical example

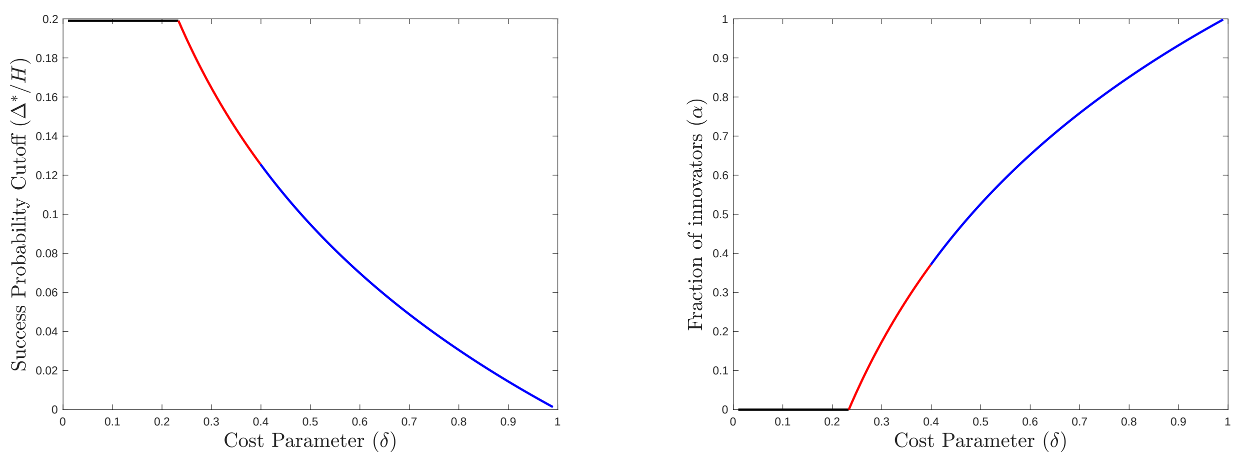

Figure 1: *Cutoff success probability and equilibrium fraction of innovators.* The left plot shows the cutoff probability for an agent to become an entrepreneur (Δ^*/H) and the right plot shows the equilibrium fraction of innovators (α) as a function of the cost parameter (δ). The black line segment denotes the case when the fraction of innovators is zero, the red segment denotes the segment when the fraction of innovators is positive, but as a group they should not innovate as they destroy value. The blue segment denotes the part when the fraction of innovators is positive and they create value as a group. In the figure we set $H = 1000$, $A = 1501$ and $\bar{\Delta} = 199$.

and further discuss the equilibrium fraction of entrepreneurs in the next section in which we derive aggregate output growth.

Example 3. *Suppose there are $H = 1000$ different ways to innovate and the reward of a successful innovation is very large, that is, $A = 1501$. Suppose the cost of innovating is 40%, that is, $\delta = 60\%$. In this case the perceived success probability has to exceed $\Delta^*/H = 6.98\%$ in order for an agent to become an entrepreneur, the equilibrium fraction of entrepreneurs is $\alpha = 65.23\%$, and the ex-post fraction of successful and unsuccessful entrepreneurs is 0.065% and 65.17%, respectively.*

3.4 Aggregate output growth

Total output in the economy is equal to the sum of all agents' endowments that are currently alive. Specifically,

$$\int_{-\infty}^{t} \nu e^{-\nu(t-s)} Y_{s,t} \, ds = Y_t, \qquad \text{where} \qquad Y_{s,t} = \gamma Y_s. \tag{37}$$

We know from Proposition 2 that for every birth cohort there are three types of agents after the innovation uncertainty is resolved. Specifically, (i) the fraction α/H of successful innovators with endowment stream $A\delta Y_s$, (ii) the fraction $\alpha(1-1/H)$ of unsuccessful innovators with endowment stream δY_s, and the fraction $(1-\alpha)$ of non-innovators with endowment stream Y_s. Hence,

$$\gamma = \alpha\delta \left(\frac{A}{H} + \left(1 - \frac{1}{H} \right) \right) + (1-\alpha) \tag{38}$$

and the decision to innovate leads to constant endowment growth $\gamma - 1$ of each new cohort. The endogenous output growth rate is presented in Proposition 3.

Proposition 3 (Endogenous Output Growth). *Aggregate output is given by*

$$Y_t = Y_0 e^{\mu_Y t}, \qquad \mu_Y = \nu(\gamma - 1) = \nu\alpha NCF \tag{39}$$

NCF denotes the increase in net cash flows from innovating and is given by

$$NCF = A\frac{\delta}{H} + \left(1 - \frac{1}{H} \right) \delta - 1. \tag{40}$$

The endogenous growth rate μ_Y can be decomposed into three components. First, only newborns can innovate, and therefore only the measure ν of endowments can potentially be used to innovate. Second, among the newborns only the fraction α of newborns innovates in equilibrium. Third, only one out of H innovation paths leads to success and entrepreneurial output after investment of $A\delta$ per unit of aggregate output whereas the other paths fail with remaining output after investment of δ per unit of aggregate output. Putting it all together we have that

$$\mu_Y = \underbrace{\nu}_{\text{Mass of newborns}} \overbrace{\alpha}^{\text{Fraction of innovators}} \underbrace{NCF}_{\text{Increase in endowment from innovation}} \tag{41}$$

Importantly, the growth in aggregate output is proportional to the difference in the net present value of a portfolio consisting of all possible innovations and the no-innovation value of a birth cohort. Specifically, if one could perfectly diversify over all H possible projects, then for every unit of endowment invested the difference in the net present value (dNPV) would be

$$dNPV_t = \overbrace{\frac{1}{H}A\delta Y_t\psi_t + \left(1 - \frac{1}{H}\right)\delta Y_t\psi_t}^{\text{Value of Innovation Portfolio}} - \underbrace{Y_t\psi_t}_{\text{No innovation value}} = Y_t\psi_t NCF \tag{42}$$

Hence, we have that the sign for $dNPV_t$ is fully determined by the sign of NCF. The left plot of Figure 2 illustrates how the difference in the NPVs (dNPV) changes with the cost parameter δ. Specifically, the difference in the NPV between the innovation portfolio and the no-innovation portfolio is negative if the cost to innovate exceeds 60%, that is, $\delta < 40\%$. We know that in this case investing in the all the projects and thus eliminating the idiosyncratic risk would not be optimal. However, with belief diversity some agents decide to become entrepreneurs which leads to negative output growth. If δ exceeds 40%, then $dNPV$ is positive and the endowment will grow as long as agents are innovating. This is the case with sufficient diversity in beliefs but does not happen without belief diversity, that is, if $\bar{\Delta} = 1$ unless the cost is less than 0.73%, that is, $\delta \geq A^{-\frac{1}{H}} = 99.27\%$. Hence, there is innovation only for a very low cost to innovating. Yet, the NPV of innovating would be positive for all $\delta > 40\%$, hence the skin-in-the-game constraint prevents pooling of risk and positive NPV projects to be undertaken. Importantly, even for sufficiently high δ so that innovation happens even with homogeneous beliefs, one would need that entrepreneurs independently randomize over the H possible ways of innovating for innovations to always be value creating. However, if people ended up coordinating by chance or truly had the same beliefs, then only one out of the H possibilities would be chosen and therefore there would be $1 - \frac{1}{H}$ chance that a non-productive innovation would be chosen and hence we would have negative growth. This illustrates the important of belief diversity in "drawing from the tails" of the distribution.

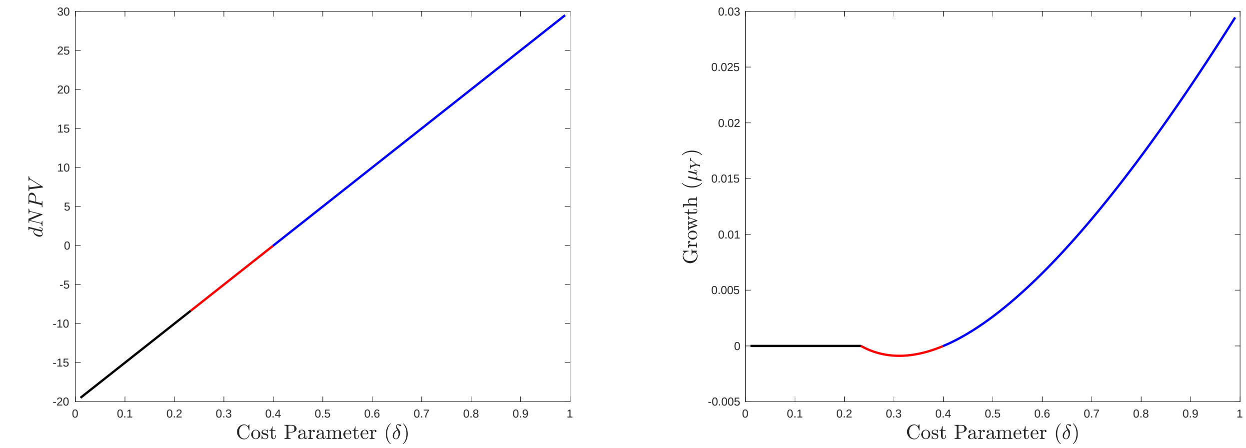

Figure 2: **Endogenous growth.** The left plot shows the NPV from innovating and the right plot shows the endogenous output growth rate as a function of the cost parameter δ. The black line segment denotes the case when the fraction of innovators is zero and thus there is no growth. Clearly, the NPV from innovating would be zero for such large cost. The red segment shows a range of relatively high cost of innovation for which the NPV from innovating is negative but there is still a fraction of very optimistic entrepreneurs who innovate and cause negative growth. The blue segment shows a wide range of cost parameters for which the NPV from innovating is positive and there is a lot of innovation that results in positive output growth. In the figure we set $H = 1000$, $A = 1501$, $\bar{\Delta} = 199$.

The right plot of Figure 2 shows the growth rate, μ_Y, as a function of the cost parameter δ. As one can see, the growth is first zero for very low values of δ, then declines to become negative as entrepreneurs take negative NPV projects, but then eventually increases. The next proposition shows how the output growth depends on belief diversity.

Proposition 4. *Aggregate output growth, μ_Y, increases with belief diversity, $\mathcal{D}$, for positive NPV innovations and decreases with $\mathcal{D}$ for negative NPV innovations. Specifically,*

$$\frac{\partial \mu_Y}{\partial \mathcal{D}} = \begin{cases} \frac{1}{\mathcal{D}}\left(1-\alpha\right) NCF \geq 0 & \textit{if} \quad NCF > 0 \\ \frac{1}{\mathcal{D}}\left(1-\alpha\right) NCF \leq 0 & \textit{if} \quad NCF < 0. \end{cases} \tag{43}$$

There is no growth if $NCF = 0$ in which case μ_Y does not depend on belief diversity.

3.5 Asset Pricing

The derivation of the risk-free rate and price-dividend ratio are similar to derivation in the exogenous growth economy discussed in Section 2. Specifically, the choice to become an entrepreneur is made just prior to entering the economy and thus everyone trading in the economy is facing complete markets. Agents within a birth cohort just differ w.r.t. the wealth when entering the economy because they are either successful, unsuccessful, or not even entrepreneurs. Therefore the FOCs from the exogenous growth model still holds, that is, $C^i_{s,t} = C^i_{s,s} e^{-\rho(t-s)} \frac{M_s}{M_t}$. Plugging the FOC into the aggregate resource constraint leads to

$$Y_t = \int_{-\infty}^{t} \nu e^{-(\rho+\nu)(t-s)} \frac{M_s}{M_t} \int_{i \in \mathcal{I}} \lambda^i_s C^i_{s,s} \, di \, ds, \tag{44}$$

where we define λ^i_t to be the measure of agents of type i with index $\mathcal{I} = \{1, \ldots, H\} \times \left[1, \bar{\Delta}\right]$ and $\int_{i \in \mathcal{I}} \lambda^i_s \, di = 1$. The ratio of total consumption of newborns to output is

$$\beta_s = \frac{\int_i \lambda^i_s C^i_{s,s} di}{Y_t}. \tag{45}$$

It follows from Equations (44) and (45) that

$$M_t Y_t = \int_{-\infty}^{t} \nu e^{-(\rho+\nu)(t-s)} \beta_s M_s Y_s \, ds. \tag{46}$$

The dynamics of the SDF are $dM/M = -r \, dt$ and applying Itô's Lemma to Equation (46) leads to the following expression for the interest rate

$$r_t = \rho + \mu_Y + \nu \left(1 - \beta_t\right) \tag{47}$$

We define the market portfolio with price P_t and dividend $D_t = Y_t$ as a claim on total wealth of all agents currently alive. The asset pricing results are given in the next proposition.

Proposition 5. *The equilibrium SDF is $M_t = e^{-rt}$ with risk-free interest rate $r = \rho$. The* *price-dividend ratio is* $\phi = \frac{1}{\rho+\nu}$*, the price of the life annuity is* $\psi = \frac{1}{\rho+\nu}$*, and* $\beta = 1 + \alpha NCF$*.*

Newborn agents innovate and cause aggregate output growth but they do not determine the interest rate when entering the economy. The interest rate is determined by all agents currently alive and they experience no growth in their endowments. Hence, the interest rate is the same as in an economy without growth. Consumption of newborns relative to output is $\beta = 1 + \alpha NCF = \frac{1}{\nu}(1 + \mu_Y)$ and the displacement effect in the interest rate due to the overlapping generations is $\nu(1-\beta) = -\mu_Y$ implying that $r = \rho + \mu_Y - \mu_Y = \rho$. The market portfolio is a claim on total wealth of all agents currently alive which does not include wealth that is generated by future generations. Hence, the price-dividend ratio does not depend on growth even though there is growth in the economy and the discount rate does not depend on growth.

3.6 Wealth and consumption inequality

In this section we examine the cross-sectional distribution of consumption and wealth in equilibrium. Consider the consumption of an agent of type i born at time s relative to the average consumption given by

$$\beta^i_{s,t} = \frac{C^i_{s,t}}{Y_t}. \tag{48}$$

The next proposition measures the consumption/wealth inequality as the cross-sectional variance of the $\beta^i_{s,t}$. There is no difference between consumption and wealth inequality because everybody has log utility.

Proposition 6. *Let* $NCF > 0$*, then*

$$\mathcal{V} = Var\left(\beta^i_{s,t}\right) = \frac{\alpha}{1 + 2\alpha NCF}\left(\left(\frac{A^2}{H} + \left(1 - \frac{1}{H}\right)\right)\delta^2 - 2NCF\right). \tag{49}$$

Moreover, the consumption inequality as measured by $\mathcal{V}$ *is weakly increasing in disagreement, that is* $\frac{\partial \mathcal{V}}{\partial \mathcal{D}} \geq 0$*.*

Not surprisingly, belief diversity increase inequality but this implies that inequality is increasing in economic growth that is valuable for society as the next proposition shows.

Proposition 7. *Assume that $NCF > 0$, i.e., the innovators as a group provide valuable innovations, then consumption inequality, $\mathcal{V}$ is weakly increasing with the economic growth μ_Y.*

Although agents that are currently alive do not benefit form innovations by newborns, they have benefited from past innovations as long as $NCF > 0$ and consequently $\mu_Y > 0$. The next proposition formalizes this.

Proposition 8. *Consider the population of agents that do not innovate. The inter-generational growth in the consumption of this group is μ_Y. That is the consumption of the cohort born at time u relative to those born at time s for $u < s$ is $e^{\mu_Y(s-u)}$.*

As Proposition 8 illustrates there is a positive spillover of innovations to agents that do not innovate. However, this spillover is intergenerational. Moreover, as long as $NCF > 0$ younger generations of non-entrepreneurs are better off than older generations. The reverse is true if $NCF < 0$. It is clear that under the subjective belief, the group of innovators are better off in expected utility terms when they can choose to become entrepreneurs as this is an optimal decision. However, it is also clear that not all innovators can be correct since only one out of the H possible ways of innovating is successful. Hence, one might argue that a social planner should consider the ex post utility based on a success probability of $\frac{1}{H}$. The next proposition derives the ex post utility gain/loss from innovating based on a representative cohort.

Proposition 9. *Define the ex-post utility difference as $dU = U^{yes} - U^{no}$ where U^{yes} is the lifetime utility if becoming an entrepreneur and calculated based on the success probability of $\frac{1}{H}$ and U^{no} is the lifetime utility when choosing not to become an entrepreneur. We have the following*

$$dU = \frac{1}{\rho + \nu} \left(\frac{1}{H} log\left(A\right) + log\left(\delta\right) \right). \tag{50}$$

Moreover, $dU < 0$ for $A^{-\frac{1}{M}} > \delta$.

Proposition 9 illustrates that based on the ex post utility, the average entrepreneur is worse off unless the cost of innovating is sufficiently low. If this is the case, then everyone would innovate. It follows immediately that for a given cohort, the group of innovators are worse of in ex-post utility terms when increasing disagreement when the cost is of innovating is sufficiently high such that $A^{-\frac{1}{H}} > \delta$. This follows from the fact that the fraction of innovators is increasing in the disagreement as illustrated in Proposition 2.

4 Entrepreneurial Risk Sharing

In this section we study two different mechanisms to share entrepreneurial risk: taxes and a venture capital fund. There is no costly effort to innovate and thus in both case the likelihood of innovation increases because the entrepreneurial risk decreases.

4.1 Government

In this section we study the effects of taxes on the likelihood of innovation α, economic growth μ_Y, and asset prices. We consider two cases: (i) a lump-sum tax with redistribution within birth cohorts and (ii) a flat tax with redistribution within and across birth cohorts. Taxes provide a risk-sharing mechanism for entrepreneurs and thus increase the likelihood of innovation in both cases. However, taxes reduce the amount of entrepreneurial capital in the first case and thus lower economic growth. Moreover, the tax redistribution within birth cohorts reduces inequality and does not affect asset prices. In the second case, taxes levied by the government are used to innovate and they are equally redistributed over time and thus there is a transfer from younger generations to older generations when there is growth in the economy. This affects the interest rate and valuation-ratios.

4.1.1 Lump-Sum Tax with Redistribution within Birth Cohort

Consider birth cohort s with endowment Y_s that pays the lump-sum tax $T_s = \tau Y_s$ over their lifetime. We refer to the tax amount as lump sum because it does not depend on the endowment post innovation. The remaining endowment stream $(1-\tau)Y_s$ can be used to innovate and the taxes levied by the government are redistributed within the birth cohort s. Hence, the endowment stream and its value after the innovation decision is made for all $s \leq t \leq \tau$ are

$$Y^i_{s,t} = \omega_i(1-\tau)Y_s + \tau Y_s \tag{51}$$

$$W^i_{s,t} = \omega_i(1-\tau)\psi Y_s + \tau\psi Y_s, \tag{52}$$

where ψ is the constant price of a life annuity given in Equation (11) and ω_i captures the different wealth of successful, unsuccessful, and non innovators given in Equation (29). Taxes collected within a birth cohort are redistributed within a birth cohort and thus the budget constraint of the government is satisfied. Effectively, the tax imposed forces agents to take less entrepreneurial risk.

An agent born at time s will choose to become an entrepreneur if the expected lifetime utility from innovating is higher than the lifetime expected utility from not innovating. Hence, agents become entrepreneurs if their success probability exceeds the threshold

$$\frac{\Delta^*_\tau}{H} = \frac{\log\left(W^0_{s,s}/W^L_{s,s}\right)}{\log\left(W^H_{s,s}/W^L_{s,s}\right)} = \frac{-\log\left(\tau+\delta(1-\tau)\right)}{\log\left((\tau+A\delta(1-\tau))/(\tau+\delta(1-\tau))\right)}. \tag{53}$$

The likelihood of innovation is

$$\alpha_\tau = \mathbb{P}\left(\Delta > \Delta^*_\tau\right) = \frac{\bar{\Delta}-\Delta^*_\tau}{\bar{\Delta}-1}. \tag{54}$$

The left plot of Figure 3 shows the equilibrium fraction of innovators α_τ as a function of the tax rate τ for different cost parameters δ. The equilibrium fraction of innovators is strictly increasing in taxes for all cost parameters that lead to innovation without taxes, that is, $\delta > 23.33\% = A^{-\bar{\Delta}/H}$. For higher cost it is weakly increasing (e.g. $\delta = 0.1$). Strikingly, a moderate tax rate of 30% leads to an innovation likelihood of 12% in equilibrium despite a high cost of 90%. To determine economic growth we plug Equation (51) into the aggregate

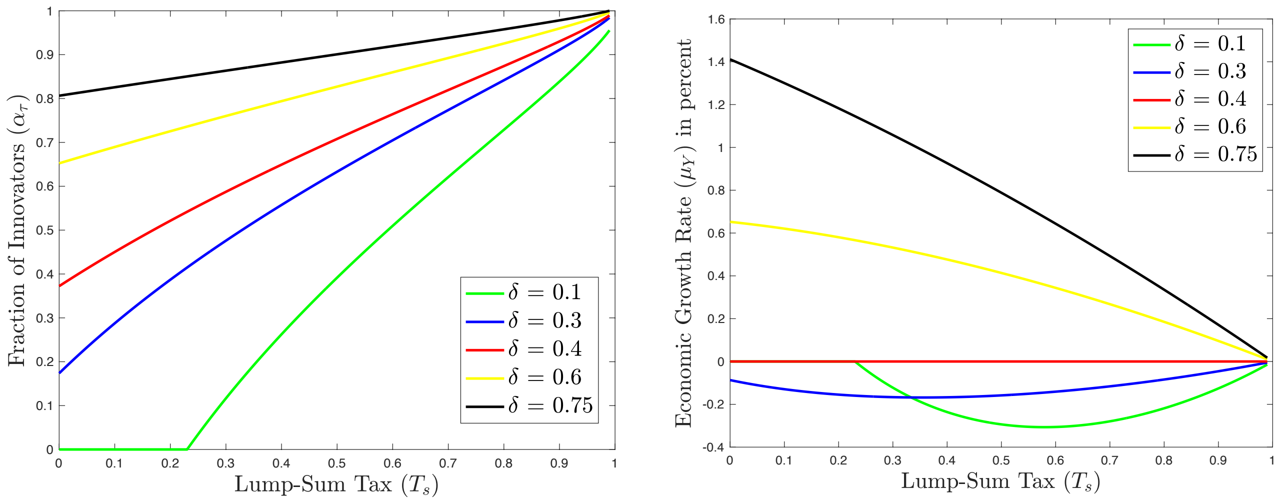

Figure 3: **Entrepreneurial Risk Sharing with Lump Sum Taxes.** The left plot shows the equilibrium fraction of innovators α_τ and the right plot shows growth in equilibrium as a function of the lump-sum taxes (T_s). The likelihood of innovation increases in taxes but the economic growth decreases in taxes if innovation is socially optimal and is hump shaped otherwise. In the figure we set $\nu = 0.02$, $H = 1000$, $A = 1501$, $Y_s = 1$ and $\bar{\Delta} = 199$.

resource constraint. Specifically,

$$Y_t = \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \int_{i \in \mathcal{I}} \lambda_s^i Y_{s,t}^i \, di \, ds = \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \gamma_\tau Y_s \, ds, \tag{55}$$

where λ_t^i is the measure of agents of type i as defined in the previous section and

$$\gamma_\tau = \tau + (1-\tau)\left(1 + \alpha_\tau NCF\right). \tag{56}$$

Increase in net cash flows from innovating, NCF, is given in equation (40) and does not depend on taxes. Hence, economic growth is

$$\mu_Y^\tau = \nu(\gamma_\tau - 1) = \nu(1-\tau)\alpha_\tau NCF. \tag{57}$$

The right plot of Figure 3 shows economic growth as a function of the tax rate τ for different cost parameters δ. Growth is strictly decreasing in the tax rate τ when innovation is social valuable without taxes, that is, if $\delta > H/(H + A - 1) = 40\%$. In this case the increase in the likelihood of innovation due to improved entrepreneurial risk sharing does not outweigh the reduction in entrepreneurial capital because the lump sum taxes levied by the government are not deployed for innovation. For innovations that are not socially optimal, $\delta < 40\%$ growth is hump-shaped.

4.2 Flat tax with redistribution across cohorts

In the case without taxes, the lifetime endowment (after the choice of innovating) of an agent born at time t of type i is $\omega_i Y_s$, where ω_i captures the different wealth of successful, unsuccessful, and non innovators given in Equation (29). In this subsection, we consider the case of a flat tax rate and tax redistribution within and across cohorts.

The total tax revenues, T_t, at time t is

$$T_t = \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \int_{i \in \mathcal{I}} \lambda_s^i \tau w_i Y_s \, di \, ds = \tau Y_t, \tag{58}$$

where λ_t^i is the measure of agents of type i as defined in the previous section. The tax revenues are equally spread among all agents, hence the total (post) transfers endowment, $y_{s,t}^i$, is

$$y_{s,t}^i = \omega_i (1-\tau) Y_s + \tau Y_t. \tag{59}$$

In contrast to the previous tax example taxes are equally distributed among all cohorts. Hence, there is a wealth transfer between generations that the previous examples did not consider. If there is growth, then this implies a net transfer from younger generations to older generations. This transfer impacts the risk-free rate and valuation because tax transfers depend on total output and thus individual endowments that include the tax transfer grow. This is fundamentally different from the previous sections. The total wealth, $W_{s,s}$, of an agent born at time s of type i is

$$W_{s,s}^i = \omega_i (1-\tau) \psi Y_s + \tau \phi Y_s \tag{60}$$

where the valuation ratios ψ and ϕ are constants. The reasons for the different valuation ratios is that the tax transfers are a claim to total output whereas individual endowments before transfers do not grow. Consequently, the two claims have different durations. As before, we can solve for the innovation threshold, Δ_τ^*, and the fraction of newborns that chose to innovate, α_τ. Specifically,

$$\frac{\Delta_\tau^*}{H} = \frac{\log\left(W_{s,s}^0/W_{s,s}^L\right)}{\log\left(W_{s,s}^H/W_{s,s}^L\right)} = \frac{\log\left(\frac{(1-\tau)\psi+\tau\phi}{(1-\tau)\delta\psi+\tau\phi}\right)}{\log\left(\frac{(1-\tau)A\delta\psi+\tau\phi}{(1-\tau)\delta\psi+\tau\phi}\right)} \tag{61}$$

and it follows that the likelihood of innovation is

$$\alpha_\tau = \mathbb{P}\left(\Delta > \Delta^*\right) = \frac{\bar{\Delta} - \Delta^*}{\bar{\Delta} - 1}. \tag{62}$$

The innovation likelihood depends on valuation ratios which depend on the risk-free rate and economic growth which depends on the innovation likelihood. Hence, we have a fixed point problem. The next proposition presents the equation that the interest rate and valuation ratios have to satisfy.

Proposition 10. *The average consumption of newborns relative to output is*

$$\beta_\tau = (\rho+\nu)\left(\left(1+\alpha_\tau NCF\right)(1-\tau)\psi + \tau\phi\right) \tag{63}$$

with

$$\psi = \frac{1}{r+\nu} \qquad \text{and} \qquad \phi = \frac{1}{r+\nu-\mu_Y}. \tag{64}$$

In equilibrium, the interest rate satisfies

$$r = \rho + \mu_Y^\tau + \nu\left(1 - (\rho+\nu)\left(\left(\alpha_\tau NCV + 1\right)\frac{1-\tau}{r^\tau+\nu} + \frac{\tau}{r^\tau+\nu-\mu_Y^\tau}\right)\right). \tag{65}$$

One solution is such that $r = \rho$ for $\tau = 0$ and $r = \rho + \mu_Y$ when τ approaches one.

The likelihood of innovation is increasing in the tax rate because the redistribution lowers entrepreneurial risk as discussed in the previous section. The left plot of Figure 4 shows that the interest rate, r, is strictly increasing in the flat tax rate τ when there is positive growth.

When the flat tax rate approaches one, then we have perfect sharing of entrepreneurial risk and the same interest rate as in the exogenous growth case without any heterogeneity, that is, $r = \rho + \mu_Y$.

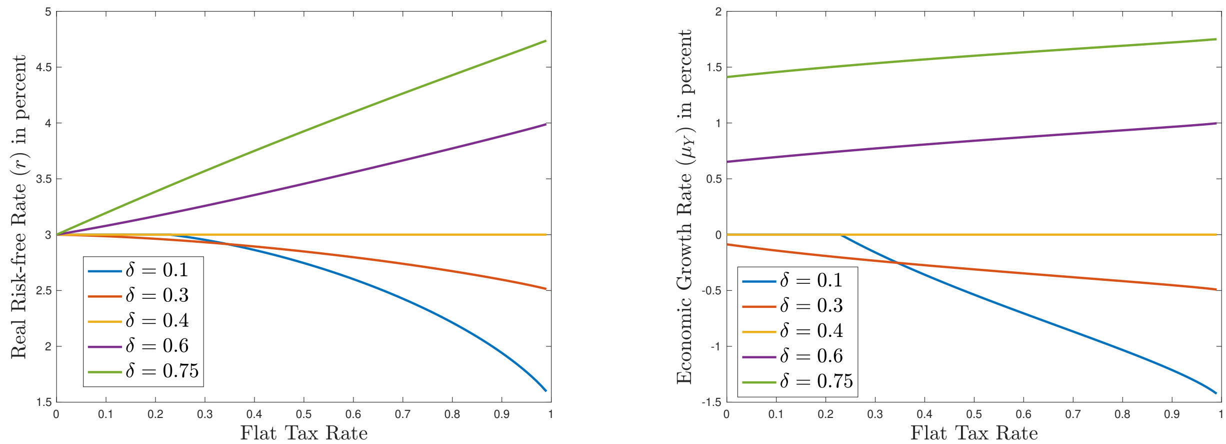

Figure 4: **Entrepreneurial Risk Sharing with Flat Taxes and Redistribution across Cohorts.** The left plot shows the equilibrium interest rate and the right plot shows growth in equilibrium as a function of the flat tax rate (τ). In the figure we set $\nu = 0.02$, $\rho = 0.03$, $H = 1000$, $A = 1501$, and $\bar{\Delta} = 199$.

4.3 Venture Capital

Consider birth cohort s with endowment Y_s that can invest the fraction θ of its endowment into a fund that invest in all H projects. The idiosyncratic entrepreneurial risk is completely diversifiable and thus the fund pays the certain amount

$$\left(A \frac{1}{H} + \left(1 - \frac{1}{H} \right) \right) \delta Y_s = (1 + NCF) Y_s. \tag{66}$$

Suppose $\delta > 40\%$ and thus the return of the fund, NCF, is always positive. Every agent invests θY_s in this fund and decides whether to use the remaining fraction $1 - \theta$ to become an entrepreneur.[3] Hence, the endowment stream and its value after the innovation decision is

[3] The fraction θ is exogenously given but can be endogenized by introducing costly effort and moral hazard.

made for all $s \leq t \leq \tau$ are

$$Y^i_{s,t} = \omega_i(1-\theta)Y_s + \theta(1+NCF)Y_s \qquad \text{and} \qquad W^i_{s,t} = Y^i_{s,t}\psi, \tag{67}$$

where ψ is the constant price of a life annuity given in Equation (11) and ω_i captures the different wealth of successful, unsuccessful, and non innovators given in Equation (29). An agent born at time s will choose to become an entrepreneur if the expected lifetime utility from innovating is higher than the lifetime expected utility from not innovating. Hence, agents become entrepreneurs if their success probability exceeds the threshold

$$\frac{\Delta^*_\theta}{H} = \frac{\log\left(W^0_{s,s}/W^L_{s,s}\right)}{\log\left(W^H_{s,s}/W^L_{s,s}\right)} = \frac{\log\left((1+\theta NCF)\,/\,(\theta(1+NCF)+(1-\theta)\delta)\right)}{\log\left((\theta(1+NCF)+(1-\theta)A\delta)\,/\,(\theta(1+NCF)+(1-\theta)\delta)\right)} \tag{68}$$

The likelihood of innovation in equilibrium is

$$\alpha_\theta = \mathbb{P}\left(\Delta > \Delta^*_\theta\right) = \frac{\bar{\Delta} - \Delta^*_\theta}{\bar{\Delta} - 1}. \tag{69}$$

The left plot of Figure 5 shows the equilibrium fraction of innovators α_θ as a function of the VC investment θ for different cost parameters δ. The equilibrium fraction of innovators is strictly increasing in the VC investment for all cost parameters $\delta > 40\%$ so that NCF is strictly positive because the VC investment lowers entrepreneurial risk. To determine economic growth we plug Equation (67) into the aggregate resource constraint. Specifically,

$$Y_t = \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \int_{i\in\mathcal{I}} \lambda^i_s Y^i_{s,t}\, di\, ds = \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \gamma_\theta Y_s\, ds, \tag{70}$$

where λ^i_t is the measure of agents of type i as defined in the previous section and

$$\gamma_\theta = 1 + \left(1 - (1-\theta)\left(1-\alpha_\theta\right)\right) NCF \tag{71}$$

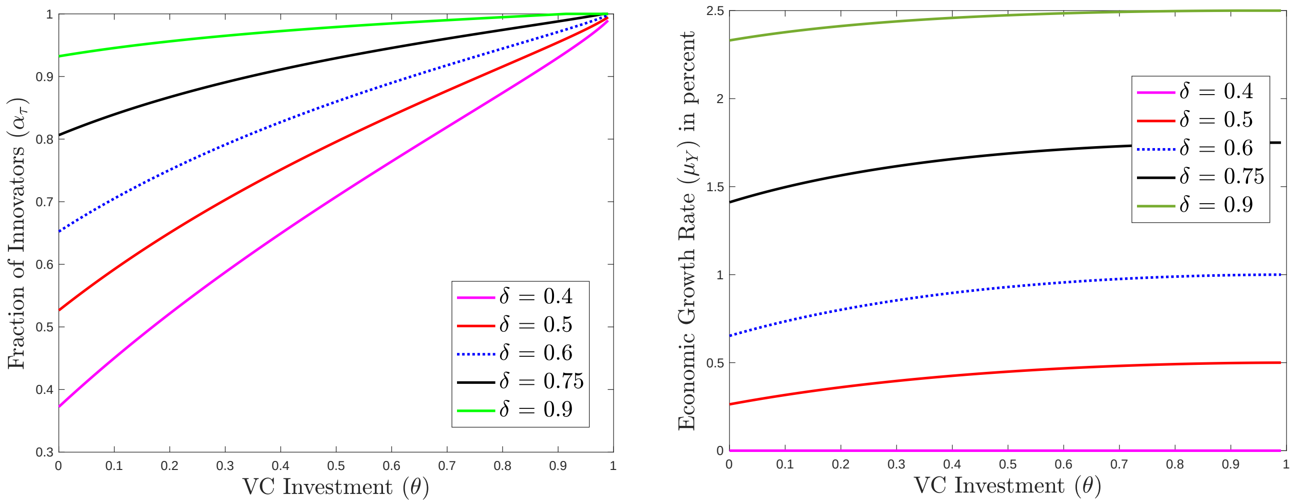

Figure 5: **Entrepreneurial Risk Sharing with a Venture Capital Fund.** The left plot shows the equilibrium fraction of innovators α_θ and the right plot shows growth in equilibrium as a function of the VC investment (θ). If $\delta > 40\%$ and thus NCF is strictly positive, then an increase in VC investment lowers the entrepreneurial risk and thus increases the likelihood of innovation and economic growth. In the figure we set $\nu = 0.02$, $H = 1000$, $A = 1501$ and $\bar{\Delta} = 199$.

Hence, economic growth is

$$\mu_Y^\theta = \nu(\gamma_\theta - 1) = \nu\left(1 - (1-\theta)(1-\alpha_\theta)\right) NCF. \tag{72}$$

The right plot of Figure 5 shows economic growth as a function of the VC investment θ for different cost parameters δ. If $\delta > 40\%$ and thus NCF is strictly positive, then an increase in VC investment lowers the entrepreneurial risk and thus increases economic growth.

5 Conclusion

We study an equilibrium model with diverse views about the likelihood of successful innovations. We show that diversity stimulates aggregate economic growth and overcomes market failures that would otherwise occur in an equilibrium without belief diversity. The higher growth with diversity comes at the cost of a higher wealth and consumption inequality, as a few entrepreneurs will ex post be successful while most entrepreneurs will fail. Hence, belief diversity provides a potential explanation for the "entrepreneurial puzzle" in which entrepreneurs choose to innovate despite taking on substantial idiosyncratic risk accompanied by low expected returns. Taxes and a venture capital fund improve risk sharing among entrepreneurs and thus increase the likelihood of innovation. This reduction in entrepreneurial risk increases growth unless the tax scheme reduces the amount of entrepreneurial capital. Tax redistribution across cohorts lowers inequality and increases interest rates. Introducing costly effort and moral hazard would endogenize the fraction invested in a venture capital fund and it would reduce the incentive to innovate when the tax liability for entrepreneur increases. Hence, the improvement in risk sharing would not always lead to an increase in the likelihood of innovation.

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