Asset Prices, Global Portfolios, and the International Financial System

Maxime Sauzet*

October 27, 2022

Link to latest version

I characterize the global solution to the international portfolio problem in a general setup, a long-standing open issue in international finance. The framework replicates a number of stylized facts about the structure and dynamics of the international financial system. In this economy, a Global Financial Cycle in risk premia emerges naturally, and the model can rationalize the Reserve Currency Paradox. The empirical patterns of the wealth share and relative GDP of the United States support the main underlying mechanisms. Empirically, the level and dynamics of portfolios, as well as unconditional and conditional asset pricing tests, are consistent with theoretical predictions.

Keywords: International Portfolio Choice, Asset Pricing, Global Financial Cycle, Reserve Currency Paradox, International Finance and Macroeconomics, International Financial System, Wealth Allocation. ***JEL codes:*** E0, F3, F4, G1.

---

*Boston University, Questrom School of Business. Email: msauzet@bu.edu. For invaluable support and guidance, I thank my advisor Pierre-Olivier Gourinchas, as well as Nicolae Gârleanu, Martin Lettau, Hélène Rey, and Jón Steinsson. For valuable comments and discussions, I thank Harjoat Bhamra, Markus Brunnermeier, Valentina Bruno, Georgy Chabakauri, Max Croce, Nicolas Coeurdacier, Eduardo Dávila, Paul Ehling, Charles Engel, Xiang Fang, Stefano Giglio, Christian Gollier, Goutham Gopalakrishna, Tarek Hassan, Julien Hugonnier, Chen Lian, Matteo Maggiori, Ulrike Malmendier, Sebastian Merkel, Thomas Mertens, Emi Nakamura, Maurice Obstfeld, Anna Pavlova, Nick Sander, Lisa Simon, David Sraer, Richard Stanton, Aleh Tsyvinski, Annette Vissing-Jørgensen, Adrien Verdelhan, Johan Walden, Fernando Zapatero, David Zerbib, and seminar and conference participants at the Bank of Canada, Berkeley (Economics, Haas), Bocconi, Boston University Questrom, Bristol, the CEPR IMF 2022 Meetings, CERGE-EI, EEA, EFA, Toulouse School of Economics, Yale, the University of Washington Foster School of Business, the University of Wisconsin-Madison (Economics, Finance), and the University of Zurich. This paper evolved from the second part of my Job Market Paper, previously circulated as "Asset Prices, Global Portfolios, and the International Financial System" (November 2, 2020). All errors are my own.

1 Introduction

"At the moment, we have no integrative general-equilibrium (monetary) model of international portfolio choice, although we need one."
Obstfeld (2004)

More than a decade later, the international portfolio choice problem remains a long-standing open issue in international finance to which the literature only provides a piecemeal answer. In parallel, a more recent body of work is emerging that studies the structure of the international financial system, with a particular focus on the role of the United States at its center. Namely, the United States acts as a world banker, with the exorbitant privilege of higher returns on its external portfolio and an easier ability to borrow in normal times, but the exorbitant duty of insuring the rest of the world in times of global crises.

This paper makes three main contributions. First, I characterize the global solution to the international portfolio choice problem in a general setup, and in a way that also allows to capture salient features of the international financial system. Second, I show that the framework can jointly replicate a number of stylized facts about the international financial system. Third, I provide empirical evidence on portfolios and asset prices that supports the theoretical predictions of the model.

Theoretically, I adapt recent advances in multi-agent continuous-time asset pricing models to a two-country, two-tree, two-good general equilibrium context in which investors have recursive preferences, a bias in consumption towards their local good, and asymmetric risk tolerance. The main message from the characterization is that two quantities drive portfolios, asset prices, and risk sharing in this economy: the relative supply of goods in different countries, and the allocation of wealth across international investors. The allocation of wealth has received little emphasis thus far in such a setting, but combined with frictions in financial markets, it is crucial in generating equity portfolios and risk premia that are consistent with empirical observations. In addition, the dynamics of those variables are especially important, an aspect that would not be possible to study without the global solution that I propose. This is in sharp contrast to the low-order local approximations that have been used in the literature, and makes it another key innovation of my approach.

The framework can jointly capture a large number of stylized facts about the structure and dynamics of the international financial system, and of asset returns in that context. For instance, it replicates the exorbitant privilege and duty of the United States, and the countercyclicality of risky returns and their correlations.

Most importantly, the framework allows to make sense of at least two key phenomena that have been the focus of an abundant recent literature.

First, introducing the special role of the United States as the world banker in this general equilibrium setting gives rise to Global Financial Cycle-type dynamics in risk premia, as has been discussed empirically (Rey, 2013 ; Miranda-Agrippino and Rey, 2020), but for which a model has proved elusive. Specifically, because U.S. investors are more willing and able to bear risk in international markets, they hold riskier portfolios on average so that they are more negatively impacted in times of global crisis. The resulting transfer of wealth from the United States to the rest of the world leads to a spike in global risk aversion and in risk premia worldwide, and to a decrease in the global safe rate, consistent with empirical observations. Importantly, the mechanism is broadly supported in the data: as I document in novel Fact 1, the share of world wealth held by the United States strongly decreased throughout the global financial crisis.

Second, the model can rationalize the so-called Reserve Currency Paradox (cf. e.g. Maggiori, 2017). The paradox refers to the fact that the wealth transfer from the United States to the rest of the world in times of global crisis should result in a depreciation of the U.S. dollar, at odds with its strong appreciation in practice. Here again, putting the role of the United States in this general setting provides a way out: not only is the allocation of wealth important, but so is the relative supply of goods. A decrease in the relative GDP of the United States in times of crisis can indeed put upward pressure on the price of U.S. goods, and therefore on the price of the U.S. basket of goods. In other words, such an evolution can provide a countervailing force through which the U.S. dollar can appreciate. In addition, this force can potentially dominate given that, the model shows, the exchange rate changes much more strongly with relative supply than with the allocation of wealth. Is this observation relevant in practice? Here again, in novel Fact 1, I suggest that yes: the U.S. share of world GDP has tended to decrease in times of global crisis, so that the appreciation pressure on the U.S. dollar can very well dominate.

For both findings, additional channels have been proposed that are likely to also be important.[1] In short however, even absent those, the framework highlights that there exist deep conceptual and empirically-relevant reasons why we should expect a Global Financial Cycle, and a potential U.S. dollar appreciation in times of crisis, as soon as we introduce the special role of the United States as the world banker in a general enough setting.

Lastly, I provide novel empirical evidence that strongly supports the theoretical predictions of the framework.

Turning first to portfolios, the model is able to match the *average* share of U.S. assets in the equity portfolios of investors in the United States (65-70%) and the rest of the world (8-15%), as well as their equity home biases (U.S.: 55%, rest of the world: 65%). This is thanks to the combination of frictions in financial markets, and the general setup in particular in terms of preferences. Most importantly, the framework also replicates portfolios in their *dynamics*, an entirely novel dimension that the global solution allows us to study. Namely, as predicted by the model, the weight on U.S. equity assets has increased in both portfolios starting in the global financial crisis. Relatedly, the equity home bias has increased in the U.S., and decreased in the rest of the world.[2]

I then study the implication for asset prices by estimating the implied beta-representation on 128 portfolios from four regions (U.S., Europe, Japan, Asia-Pacific). I proxy for U.S. wealth share movements by changes in global risk aversion, and find evidence for a strongly positive price of wealth share risk, consistent with the model in which assets are unattractive (and therefore risky) if they pay in tranquil times when the U.S. wealth share is large and global compensation for risk is low. I proxy for relative supply movements by changes in real exchange rate, and find evidence for a negative price of relative supply risk, consistent with the model in which the concerns of the rest of the world dominate relative supply risk pricing so that an asset

[1]For instance, among others, U.S. monetary policy and intermediaries leverage are important for the Global Financial Cycle (Miranda-Agrippino and Rey, 2020), and spiking demand in safe assets (Kekre and Lenel, 2021) and binding constraints on the part of foreign exchange dealers matter for the Reserve Currency Paradox. Embedding those in the current framework is an important avenue for future research, and could reinforce the mechanisms presented above.

[2]These statements are based on restated portfolio data from Coppola et al. (2021), who use detailed data on investment funds to reallocate the share of assets held in tax havens to their true destination country. Using non-restated data, the picture is similar. For instance, the home bias in the U.S. also increased in 2008, and although it started decreasing slightly back afterwards, consistent with a continuation of the secular downward trend in equity home bias, it has done so at a much slower pace, again broadly consistent in essence with the model.

that pays when the relative supply of the rest of the world is low is attractive (and therefore a good hedge).[3,4] Those prices of risk are strongly significant (Newey-West t-stats around 5 on average), and economically large and meaningful. For instance, the adjusted-R^2s range from 26.5 to 70.8%, and the premia difference between high-β (90th-percentile) and low-β (10th-percentile) assets is around 5 to 8% in annual terms, which is sizable compared to the average (median) returns of 21.55% (15.98%) per year on those portfolios. This is true across an array of specifications, and whether I run unconditional or conditional empirical asset pricing tests.

Taken together, the empirical evidence strongly supports theoretical predictions. It highlights the relevance of the novel elements brought about by the framework, such as the central role of the wealth share of the United States, and the importance of *dynamic* aspects going beyond the average level of risk premia and portfolios.

The model is also a well-suited building block for many potential extensions. For instance, it makes it possible to introduce in this international setting the type of financial intermediaries that have been discussed in the intermediary asset pricing literature e.g. in Daníelsson et al. (2012), He and Krishnamurthy (2013), Adrian and Shin (2014), or Adrian and Boyarchenko (2015). Illustrations are briefly discussed in Section D.3, e.g. with the inclusion of a global asset manager (Sauzet, 2022c). From the perspective of extensions, solving for the *decentralized* general equilibrium of this economy like in this paper will prove valuable: the framework is readily set to tackle a wide range of market structures beyond imperfect risk sharing. In addition, the implementation of those extensions will likely require higher-dimensional resolution approaches such as the "projection methods via neural networks" that I propose in Sauzet (2022b). I leave all these promising avenues for ongoing and future research.

Related literature This paper contributes to two main strands of literature.

First, I contribute to the literature on multi-agent asset pricing models, which has a long tradition since the seminal contributions of Dumas (1989, 1992), Wang (1996), Basak and Cuoco (1998), Chan and Kogan (2002), and more recently Brunnermeier

[3]Choosing empirical proxy factors for wealth share risk, and relative supply risk, is required in practice to have data at a higher frequency and of better quality. I rely on theoretical results for guidance: global risk aversion is the most cleanly (negatively) monotonically-related variable to the U.S. wealth share, while real exchange rate is for the most part (negatively) driven by relative supply.

[4]The price of relative supply risk is more dependent on the state of the economy, here again consistent with theoretical predictions.

and Pedersen (2009), Weinbaum (2009), Gârleanu and Pedersen (2011), Chabakauri (2013), Brunnermeier and Sannikov (2014), Gârleanu and Panageas (2015), Drechsler et al. (2018), Borovička (2020). This literature is also related to recent contributions on heterogeneous agents in closed-economy macroeconomics such as Kaplan et al. (2018). To those, I bring two goods, two assets, two countries, as well as a home bias in consumption and asymmetries in preferences. The home bias in consumption can be seen as a friction in the goods market, which is important because it introduces a fundamental level of heterogeneity across investors even absent asymmetries. It is responsible for many underlying mechanisms in the economy, and also ensures that there is a well-defined (i.e., non-constant) exchange rate. As such, this is among the main differences with the international model of Brunnermeier and Sannikov (2015, 2019). Having two assets also fundamentally relates my paper to contributions with multiple securities but one investor, e.g. Cochrane et al. (2008), Martin (2013).

Related to my contribution are also those of Zapatero (1995), Pavlova and Rigobon (2007, 2008, 2010), and Stathopoulos (2017), who study a pure exchange economy similar to mine, but in which preferences are log and the elasticity of intertemporal substitution across goods is equal to one. The combination of those assumptions leads the allocation of wealth to be constant, equity assets to be perfectly correlated in the absence of demand shocks, or hedging demands to be absent due to myopic portfolios. All three are important dimensions that arise in my framework once I allow for general recursive preferences and an arbitrary elasticity of substitution across goods. My contribution is therefore a natural continuation of this earlier research effort.

However, breaking those limitations requires solving the model using a whole new set of methods compared to those papers. In particular, the resolution of my framework, which draws on Sauzet (2022a), is based on global projection methods, as presented in Judd (1992, 1998), the NBER Summer SI Lecture by Fernández-Villaverde and Christiano (2011), and in contributions by Fernández-Villaverde et al. (2013, 2015, 2018) or Parra-Alvarez (2018), among others. Those are applied to multi-agent models for instance in Drechsler et al. (2018), Fang (2019), or Kargar (2019). The approximation is based on Chebyshev polynomials and orthogonal collocation, although in concurrent work, I am developing a natural extension based on neural networks (Sauzet, 2022b, cf. Section D.3).[5]

[5] I solve for the decentralized economy throughout, but the method of Dumas et al. (2000), based on a planner, could also potentially be used in cases in which risk sharing is perfect.

In addition, I also introduce asymmetries in preferences, labor income in the form of a constant share of output as in Baxter and Jermann (1997), and most importantly, imperfect financial integration. The latter is captured in a parsimonious way as a wedge on foreign dividends by generalizing Bhamra et al. (2014) to a non-log environment that also features home bias, and following the seminal contribution of Basak and Gallmeyer (2003) who study a dynamic asset pricing model with dividend taxation in a one-country one-asset one-good setting. Compared to Bhamra et al. (2014), the introduction of general preferences makes a significant difference: imperfect risk sharing has a large impact *provided that* the elasticity of intertemporal substitution is modest, a novel insight. In addition, I use a global solution instead of relying on local approximations, and am able to study the effect on the exchange rate and of hedging terms, which are central in this economy.[6]

Other related papers include Cass and Pavlova (2004), Brandt et al. (2006), Martin (2011), and Maggiori (2017) that I discuss below, as well as Fang (2019) who focuses on a small open economy in which the rest of the world is taken as exogenous and in which investors do not have symmetric home bias. On the theoretical front, my paper is also related to contributions introducing recursive preferences in continuous-time, e.g. Duffie and Epstein (1992), and contributions focusing on the existence and uniqueness of equilibria in the presence of multiple agents, and possibly multiple goods and incomplete markets, e.g. Polemarchakis (1988), Geanakoplos and Polemarchakis (1986), Geanakoplos and Mas-Colell (1989), Geanakoplos (1990), Duffie et al. (1994), Berrada et al. (2007), Anderson and Raimondo (2008), Hugonnier et al. (2012), Ehling and Heyerdahl-Larsen (2015, 2017).

Second, I contribute to the literature on the international portfolio problem. Specifically, the advances presented above allow me to characterize the global solution to the international portfolio choice problem in a general setup, a long-standing issue in this literature since the seminar contributions of Stulz (1983), Dumas (1989, 1992), Cole and Obstfeld (1991), Uppal (1993), Zapatero (1995), Baxter and Jermann (1997), Baxter et al. (1998), Obstfeld and Rogoff (2001), Obstfeld (2004), among many others. Obstfeld (2007) and Coeurdacier and Rey (2013) provide surveys.

To a large part of the more recent literature on the topic, such as Heathcote and

[6]Theoretically, the use of a wedge to capture a wide range of frictions is related to the work of Gârleanu et al. (2020), who show that models with investment taxes constitute an equivalent, but substantially simpler, way to capture a rich set of impediments to financial trade.

Perri (2002, 2004, 2013), Corsetti et al. (2008), Coeurdacier (2009), Tille and van Wincoop (2010), Devereux and Sutherland (2011), Evans and Hnatkovska (2012), Coeurdacier and Rey (2013), Coeurdacier and Gourinchas (2016), Stewen (2020), I bring (i) a solution that is global and does not rely on approximations. This allows to complete the picture and trace out the evolution of economic outcomes as we move away from the point of approximation (typically the symmetric point), which proves important in this context where variables are strongly state-dependent and potentially non-linear. I also bring (ii) general preferences, which allow to move away from special cases and study all situations under a unified framework (cf. also the discussion above of Pavlova and Rigobon, 2007, 2008, 2010, Stathopoulos, 2017). A limited number of contributions have relied on global methods in similar settings e.g. Kubler and Schmedders (2003) (one country), Stepanchuk and Tsyrennikov (2015) (one good), Rabitsch et al. (2015), and Coeurdacier et al. (2020) (one good). To those, I bring (iii) continuous-time methods, which make it possible to study portfolio drivers, in particular hedging demands, asset prices and their conditional first and second moments, as well as the determinants of wealth and state variable dynamics, in ways that are inaccessible in a discrete-time formulation and therefore make continuous-time the natural tool of choice to study this type of questions. Finally (iv), to all, in addition to labor income as in Baxter and Jermann (1997) and asymmetries in preferences, I bring imperfect financial integration, an important topic in international finance that had not been studied thus far in such a general international portfolio choice context.[7]

The economy is also related to that of Colacito and Croce (2011, 2013), Colacito et al. (2018), and Colacito et al. (2019), who introduce recursive preferences in an international context. Compared to those, output does not feature long-run risk dynamics. Instead, I bring in an arbitrary elasticity of substitution across goods, which renders the two equity assets no longer perfectly correlated so that the portfolio choice is no longer indeterminate in my context. In addition, I solve the decentralized version of the economy, and am therefore able to discuss portfolios and their dynamics, which are one of the main focuses in the current paper. More generally, I bring (i), (ii), (iii) and (iv) above to that economy.

My contribution also relates to that of Dahlquist et al. (2022), and Kim (2022).

[7]More general specifications of labor such as a time-varying share in the spirit of Coeurdacier and Gourinchas (2016) or idiosyncratic labor income risk as in Kaplan et al. (2018) are interesting avenues for further exploration.

The former in particular emphasizes, using a model based on good-specific risk appetite, that movements in exchange rates could have an important impact on the valuation of international portfolios, an aspect that could be interesting to study in this setting.[8] Both papers however have the implication that the U.S. share of world wealth should increase in times of global crisis, in contrast to the strong decrease that I document in the data (cf. two different measures in Fact 1).[9]

Dou and Verdelhan (2015) also solve an international portfolio problem globally, with general preferences and endowments, portfolio constraints, and incomplete markets. Their focus on the volatility of international capital flows is different, however. In addition, partly because their framework is cast in discrete time, they do not focus on describing the underlying determinant of portfolios, such as hedging demands, which are an important part of my contribution.

Finally, the main application in this paper is in the spirit of Gourinchas and Rey (2007a,b), Caballero et al. (2008), Gourinchas et al. (2017), and Maggiori (2017).[10] Compared to those, I introduce general recursive-CES preferences, labor income, asymmetries, and imperfect risk sharing. More generally, my approach allows to bring those contributions to a general international portfolio choice setting, and to jointly study the special role of the U.S. in international markets –as they do–, and the level and dynamics of international portfolios. Lastly, in recent work, Atkeson et al. (2022) suggest that the exorbitant privilege of the United States may have shrunk in recent years, due to a rapid increase in the valuation of U.S. assets. This finding is not problematic for my results, given that I focus on periods of global crises that occurred before this episode. The model however suggests that this could be a different form of privilege: U.S. investors borrow in safe assets from abroad to lever their portfolios, and invest heavily in risky assets worldwide *but also domestically.* As such, they also appear to have benefited strongly from the rise in U.S. valuations, as seen from the large increase in the U.S. wealth share over the same (recent) period documented in Fact 1.

[8] Section D.3 discusses possible extensions. Some could be along those lines, e.g. adding global foreign exchange dealers, downward-sloping demand for exchange rates, or nominal exchange rates.

[9] The U.S. wealth share also appears to decrease at the onset of the global financial crisis when constructed from a third approach, namely using data as in Jiang et al. (2022), based on 33 investor- and 31 issuer-countries.

[10] A related recent contribution, focusing on one good and log preferences, is Huang and Hu (2022). Additional papers that establish stylized facts about the international financial system are discussed in Section 4.1.

Outline The paper is organized as follows. Section 2 describes the setup of the economy, and introduces two main state variables: the wealth share of the domestic (U.S.) investor, and the relative supply of the domestic (U.S.) good, i.e. fundamentals. In Section 3, I characterize the equilibrium theoretically, focusing on asset prices and portfolios. Section 4 presents the main application to modeling the international financial system, and Section 5 describes empirical evidence. Section 6 concludes. Additional material is provided in Appendix.

2 The Economy

This section presents the theoretical setup. I introduce a pure-exchange general equilibrium economy with two countries, domestic and foreign (*), and two trees that produce two differentiated goods. Each country is populated by a representative investor with recursive preferences and whose consumption is biased towards the local good. I show that the equilibrium can be characterized as a function of two state variables: the wealth share of the domestic investor, x_t, and the relative supply of the two goods, y_t. Appendices A and B gather additional results and figures omitted in the main text, and the setup is summarized in Figure B.1. The model builds on Sauzet (2022a) which develops a theoretical characterization in a general context.

Time is continuous and the horizon is infinite, $t \in [0, \infty)$. Uncertainty is represented by a probability space $(\Omega, \mathcal{F}, \mathbb{F}, P)$ supporting a two-dimensional Brownian motion $\vec{Z} \equiv (Z, Z^*)^T \in \mathbb{R}^2$. The filtration $\mathbb{F} = (\mathcal{F}_t)_{t \in [0,\infty)}$ is the usual augmentation of the filtration generated by the Brownian motions, and $\mathcal{F} \equiv \mathcal{F}_\infty$.

2.1 Endowments, prices, assets

Each country hosts a tree, à la Lucas (1978), which produces a differentiated good with its own price. The output of each tree follows a geometric Brownian motion

$$\begin{aligned} \frac{dY_t}{Y_t} &= \mu_Y dt + \sigma_Y^T d\vec{Z}_t \\ \frac{dY_t^*}{Y_t^*} &= \mu_Y^* dt + \sigma_Y^{*T} d\vec{Z}_t \end{aligned} \tag{1}$$

The price of the domestic and foreign goods are p_t, p_t^*. The terms of trade is $q_t \equiv p_t^*/p_t$, and the real exchange rate is $\mathcal{E}_t \equiv P_t^*/P_t$. Both are defined as is standard so that an increase in q_t corresponds to a worsening of the terms of trade, and an increase in $\mathcal{E}_t$ corresponds to a depreciation. P_t, P_t^* are the prices of the domestic and foreign consumption baskets discussed below. All prices are defined with respect to a global numéraire taken to be a CES-basket with weight a on the local good.[11]

Both trees are traded as equity assets, with returns given by

$$\begin{aligned} dR_t &= \frac{dQ_t}{Q_t} + \frac{p_t Y_t}{Q_t} dt = \frac{d\left(p_t Y_t / F_t\right)}{p_t Y_t / F_t} + F_t dt \equiv \mu_{R,t} dt + \sigma_{R,t}^T d\vec{Z}_t \\ dR_t^* &= \frac{dQ_t^*}{Q_t^*} + \frac{p_t^* Y_t^*}{Q_t^*} dt = \frac{d\left(p_t^* Y_t^* / F_t^*\right)}{p_t^* Y_t^* / F_t^*} + F_t^* dt \equiv \mu_{R*,t} dt + \sigma_{R*,t}^T d\vec{Z}_t \end{aligned} \tag{2}$$

where Q_t, Q_t^* are the equity prices, and $F_t \equiv p_t Y_t / Q_t, F_t^* \equiv p_t^* Y_t^* / Q_t^*$ are the dividend yields, for the domestic and foreign assets. Drifts $\mu_{R,t}, \mu_{R*,t}$, which measure conditional expected returns, and diffusion terms $\sigma_{R,t} \equiv (\sigma_{Rz,t}, \sigma_{Rz*,t})', \sigma_{R*,t} \equiv (\sigma_{R*z,t}, \sigma_{R*z*,t})'$, which measure the loadings on the shocks and therefore the conditional volatilities, are obtained from Itô's Lemma and given in Appendix A.2.

The supply of each equity asset is normalized to unity, and there also exists an international bond in net zero supply, which is locally riskless in units of the numéraire. Its price is B_t, and the corresponding instantaneous interest rate is r_t, so that $dB_t/B_t = r_t dt$.

2.2 Preferences

The representative investor of each country has recursive preferences over consumption à la Duffie and Epstein (1992). For the domestic investor for instance, those are given by

$$\begin{aligned} V_t &= \max_{\{C_{h,u}, C_{f,u}, w_{h,u}, w_{f,u}\}_{u=t}^{\infty}} \mathbb{E}_t \left[\int_t^{\infty} f\left(C_u, V_u\right) du \right] \\ f(C,V) &\equiv \left(\frac{1-\gamma}{1-1/\psi}\right) V \left[\left(\frac{C}{\left[(1-\gamma)V\right]^{1/(1-\gamma)}} \right)^{1-1/\psi} - \rho \right] \end{aligned} \tag{3}$$

[11] Specifically, I normalize $\left[a p_t^{1-\theta} + (1-a) p_t^{*1-\theta}\right]^{1/(1-\theta)}$ to unity. I take $a = 1/2$ in the baseline.

where γ is the coefficient of relative risk aversion, $\psi \neq 1/\gamma$ the elasticity of intertemporal substitution, and ρ is the discount rate.

Recursive preferences are in contrast to a large part of the literature that focuses on log or constant relative risk aversion (CRRA) utility. The former has the drawback that investors are myopic so that state variables are not hedged, and have therefore a limited impact on portfolios, asset prices, and other quantities of interest.[12] Contrary to the CRRA case, recursive preferences also allow to disentangle the risk aversion and elasticity of substitution (EIS) of each investor. This is important to get more realistic risk premia, and because the EIS partly controls the quantitative impact of financial market frictions in this setting (Sauzet, 2022a).

The consumption basket of each investor is composed of the two goods, which are combined according to an aggregator with constant elasticity of substitution θ, and is biased towards their own local good

$$C_t = \left[\alpha^{\frac{1}{\theta}} C_{h,t}^{\frac{\theta-1}{\theta}} + (1-\alpha)^{\frac{1}{\theta}} C_{f,t}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{4}$$

Two characteristics of the consumption baskets are noteworthy.

First, the elasticity of substitution across goods, θ, is not equal to unity. Due to its specificity, the case with $\theta = 1$, in which C_t collapses to a Cobb-Douglas aggregator, has received considerable attention in the literature since the seminal contribution of Cole and Obstfeld (1991).[13] In this case and under some conditions, the Pareto optimal equilibrium that would obtain under complete markets can in fact be attained under financial autarky. This is so because when $\theta = 1$, changes in the relative price of the goods exactly compensate changes in their relative supply. As a result, investors are perfectly insured even without trading financial assets. Another consequence is that the payoffs of the two equity assets are perfectly correlated, so that the portfolio choice of international investors is indeterminate.[14] Further, an economy with unit

[12] Hedging terms are absent more generally as long as the risk aversion is equal to one.

[13] For instance, $\theta = 1$ in Pavlova and Rigobon (2007, 2008, 2010), Colacito and Croce (2013), Maggiori (2017), or Colacito et al. (2018, 2019), among others.

[14] I discuss this case in more detail in Sauzet (2022a). Interestingly, another consequence of the equity assets being perfectly correlated is that markets are dynamically incomplete when the investors can only trade the two equity assets and a bond, as discussed in Ehling and Heyerdahl-Larsen (2015, 2017). Despite this fact, investors are perfectly insured via changes in the relative goods price.

elasticity of substitution across goods satisfy the conditions of the no-trade theorem in Berrada et al. (2007), so that there is no trade in equilibrium, resulting in no realistic international capital flows and no nontrivial portfolio rebalancing. Taken together, those reasons make this case peculiar, and I instead focus on the more general environment in which $\theta \neq 1$, which has received less attention.

Second, the home bias in consumption, captured by parameter $\alpha > \frac{1}{2}$, is well-established in international economics, and turns out to be a core driver of economic outcomes in the model. It can be seen as a simple preference parameter on the part of investors, or as a reduced-form way of capturing various goods market frictions such as trade costs (Coeurdacier, 2009). The home bias in consumption introduces a fundamental degree of heterogeneity across investors, even absent asymmetries, that underpins the determination of risk premia and portfolios in this economy. Note also that without home bias, investors would consume identical baskets so that the real exchange rate –a key quantity in an international context– would be constant, $\mathcal{E}_t = 1$.

The domestic investor allocates a share $w_{h,t}$ of her wealth to the domestic equity asset, earning an expected risk premia $\mu_{R,t} - r_t$, a share $w_{f,t}$ to the foreign equity asset, earning $\mu_{R^*,t} - r_t$, and the rest $(1 - w_{h,t} - w_{f,t})$ to the international bond, earning the riskfree rate r_t. She uses the proceeds to purchase her desired basket of consumption $c_t \equiv C_t/W_t$, at price P_t. In other words, she chooses her consumption and portfolios to maximize (3) subject to the following budget constraint

$$\begin{aligned} \frac{dW_t}{W_t} = {} & \left(r_t + w_{h,t}\left(\mu_{R,t} - r_t\right) + w_{f,t}\left(\mu_{R^*,t} - r_t\right) - P_t c_t\right) dt \\ & + \left(w_{h,t}\sigma_{R,t} + w_{f,t}\sigma_{R^*,t}\right)^T d\vec{Z}_t \end{aligned} \tag{5}$$

The impact on the budget constraint of introducing imperfect financial integration and labor income of the form considered in this paper is discussed in Section 2.4. Finally, to complete the definition of the optimization problem, the investor is subject to a standard transversality condition, and W_0 is given. Note also that $W_t \geqslant 0$.

The problem solved by the foreign investor is similar, and is shown in Appendix A.3. Importantly, all parameters can differ from those of the domestic investor. I make use of this aspect to introduce asymmetries in risk tolerance in order to capture the role of the United States in what follows (Sections 3-5).

2.3 Equilibrium and state variables

The definition of the equilibrium is standard: (1) investors solve their optimization problems by taking aggregate stochastic processes as given, and (2) goods and equity markets clear. It is shown in Appendix A.5. The bond market clears by Walras's law, which gives rise to the following useful relationship: $W_t + W_t^* = Q_t + Q_t^*$. In words, world wealth has to be held in the form of the two equity assets in aggregate.

Stationary recursive Markovian equilibrium Most importantly, the equilibrium can be recast as a stationary recursive Markovian equilibrium in which all variables of interest are expressed as a function of a pair of state variables $X_t \equiv (x_t, y_t)'$, whose dynamics are themselves solely a function of X_t. x_t is the wealth share of the domestic investor, and y_t is the relative supply of the domestic good.[15] Both are defined below.

The characterization of the solution as a system of coupled algebraic and second-order partial differential equations is the focus of Section 3, and additional results and details are provided in Sauzet (2022a). For now, let us discuss the intuition behind both state variables. Note that an additional quantity, which is not a state variable *per se* but is useful throughout, is z_t, the ratio of the home equity price to world wealth. It corresponds to the weight of the domestic asset in the market portfolio, and it can be shown that

$$z_t \equiv \frac{Q_t}{Q_t + Q_t^*} = \left(1 + \left(\frac{F_t}{F_t^*}\right) q_t \left(\frac{1 - y_t}{y_t}\right)\right)^{-1} \tag{6}$$

Wealth share The wealth share of the domestic investor captures the allocation of worldwide wealth, and is therefore a measure of the average investor in the international economy. It is defined as

$$x_t \equiv \frac{W_t}{W_t + W_t^*} \tag{7}$$

Importantly and contrary to a large subset of the literature, the wealth share is

[15] Formally, this is shown using a guess and verify approach like, e.g., in Gârleanu and Panageas (2015). The variables of interest are $\{c_{h,t},\, c_{f,t},\, c^*_{h,t},\, c^*_{f,t},\, w_{h,t},\, w_{f,t},\, w^*_{h,t},\, w^*_{f,t},\, \mu_{R,t},\, \mu_{R*,t},\, r_t,\, F_t,\, F_t^*,\, p_t,\, p_t^*,\, P_t,\, P_t^*,\, q_t,\, \mathcal{E}_t\}$.

not constant, even under perfect risk sharing. This is due to the combination of non-log preferences, and home bias in consumption. Intuitively, the wealth share captures Negishi weights, which are time-varying in this setting.[16] In addition, the wealth share is not purely a function of current fundamentals, again even under perfect risk sharing, so that it is required as an additional state variable. The latter comes from the combination of investor heterogeneity and recursive preferences.

Relative supply The relative supply of the domestic good is the domestic output share. It captures the effect of current fundamentals and is defined as

$$y_t \equiv \frac{Y_t}{Y_t + Y_t^*} \tag{8}$$

This variable has received more attention in the international portfolio choice literature.[17] It is for instance closely related to real exchange rate hedging, as discussed e.g. in Coeurdacier (2009) and Coeurdacier and Rey (2013).[18] The appeal of the framework is to make it possible to study (i) the role of that variable in a more general context with recursive-CES preferences and financial frictions, (ii) how the dynamics of y_t matter beyond the symmetric point that is the typical focus under low-order local approximations in the literature, and (iii) how the relative supply interacts with the allocation of wealth captured by x_t.

Note that because $W_t \geqslant 0$ and $Y_t \geqslant 0$, x_t and y_t are both evolving in the bounded interval $[0, 1]$. This has the advantage that solving for unknown functions on a bounded domain is numerically more stable. Conceptually, as x_t gets closer to either of the boundaries, the economy converges (continuously) to a natural one-investor environment, while as y_t gets closer to either of the boundaries, the economy converges to a one-good one-equity asset economy.

[16]For a discussion, cf. e.g. Dumas et al. (2000), Anderson (2005), or Colacito and Croce (2013).

[17]Note that the ratio involves *quantities* of the two different goods. This poses no particular theoretical issue and is used because it simplifies the characterization of the equilibrium. This definition is a monotonic transformation of Y_t^*/Y_t, $y_t \equiv (1 + Y_t^*/Y_t)^{-1}$, which ensures that the state variable evolves in the bounded interval $[0, 1]$. Y_t^*/Y_t has the clear interpretation of the output of the foreign good per unit of domestic good. An economic intuition is that one compares the economy to the symmetric point in which relative prices are $q_t = \mathcal{E}_t = 1$.

[18]In this economy, the mapping between relative supply, y_t, and real exchange rate, $\mathcal{E}_t$, is not one-for-one because the allocation of wealth, x_t, also impacts $\mathcal{E}_t$. However, the quantitative impact of the relative supply on $\mathcal{E}_t$ still vastly dominates (cf. Figure 8).

Throughout, I focus on the solution to the decentralized, i.e. Radner, equilibrium instead of relying on the social planner's problem. When markets are complete and risk sharing is perfect, both solutions must coincide.[19] However, one of the advantages of the decentralized solution –that is, of characterizing the solution directly as a function of the wealth share– is that the approach remains valid including in situations in which risk sharing is imperfect, and markets are incomplete. In other words, it can be used even when the planner solution can no longer be applied. This is the case when I introduce financial market frictions below. This will also prove useful for further applications and extensions of the framework (Section D.3). An additional benefit is to put the solution closer to observable quantities –e.g. wealth and output shares, or portfolios–, an aspect that I use when comparing the model to empirical evidence in Sections 4 and 5.

The existence and uniqueness of the equilibrium is guaranteed, for instance following the work of Duffie and Epstein (1992), who use partial differential equation techniques to prove them in an infinite-horizon Markov diffusion setting with stochastic differential utility, or Chabakauri (2013) and Bhamra and Uppal (2014), who do so constructively for economies with heterogeneous agents and incomplete/complete markets, respectively. Both are also shown in situations with potentially dynamically complete markets[20] using a planner solution in Anderson and Raimondo (2008), and under complete markets with a full set of Arrow-Debreu securities in Hugonnier et al. (2012). As has been known since the seminal example of Hart (1975), the introduction of multiple goods could complicate the matter, for instance because markets can become dynamically incomplete even if the number of assets should technically be sufficient to span risks. Those multiple-good contexts are discussed e.g. in Berrada et al. (2007), and Ehling and Heyerdahl-Larsen (2015, 2017), again through the lens of the Pareto efficient allocation obtained from a social planner. In the context of this paper, existence and uniqueness obtain provided that $\theta \neq 1$. Further studying the theoretical properties of the equilibrium in this decentralized setting with multiple goods represents an interesting avenue for further research.

[19] In Sauzet (2022a), I show that this is indeed the case for instance under symmetric CRRA preferences. I also check the solution in that case with Monte-Carlo simulations. Solving for the planner solution can potentially be extended to recursive preferences in a Markovian setting, following Dumas et al. (2000).

[20] A securities market is potentially dynamically complete if the number of securities with non-colinear payoffs is equal to one plus the number of risk factors (Brownian motions) to be spanned.

2.4 Additions

Together with the general preferences discussed above, the framework accommodates two important additions: imperfect financial integration, and a stylized form of labor income. I discuss the former below, and the latter, which draws on Baxter and Jermann (1997), in Appendix A.4.

Market structure and imperfect financial integration In the environment described so far, markets are potentially dynamically complete in the sense of Anderson and Raimondo (2008). That is, the number of securities is at least one more than the number of independent sources of uncertainty and they can therefore span all risk. In such a setup, risk sharing is perfect, and the decentralized equilibrium is Pareto efficient and corresponds to the planner's problem.

In practice however, investing internationally comes with a number of frictions –informational, legal, technical, *etc.*–, so that the risk sharing between international investors is likely to be imperfect. This aspect is relevant in this context because as this happens, the impact of the worldwide allocation of wealth, which is one of the novel element and is captured by x_t, is likely to be amplified.

To study this friction, I introduce imperfect financial integration in a parsimonious way as a wedge ("tax") on foreign dividends, adapting Bhamra et al. (2014) to a non-log two-good context with home bias.[21] The assumption allows to study the effect of a range of financial integration degrees without having to take a specific stance on the source of the underlying imperfections. This wedge is meant to encompass the wide array of frictions that prevent investors from freely participating in foreign markets.[22] Note that the spanning condition above remains verified in that the number of securities is still one more than the number of independent sources of uncertainty. As a result, investors still individually face markets that are dynamically complete. However, the opportunity sets that they face are now different due to the wedge that differentially affects the assets for each of them, so that the equilibrium

[21] Cf. also the seminal contribution of Basak and Gallmeyer (2003), who study a dynamic asset pricing model with asymmetric dividend taxation in a one-investor one-tree one-good setting. The friction could also affect the diffusion of asset returns, which could also capture effects about information and uncertainty in the spirit of Gehrig (1993). I leave this exploration for future research.

[22] As shown in Gârleanu et al. (2020), models with investment taxes constitute an equivalent, but substantially simpler, way to capture a rich set of impediments to financial trade.

need not be Pareto efficient and the usual planner solution that has been pervasive in the international finance literature cannot be used.[23] In contrast, this is not an issue in this framework given that, as mentioned previously, I solve for the decentralized version of the equilibrium. Relatedly, the stochastic discount factors of the two international investors are no longer perfectly correlated, and risk sharing is therefore imperfect. This is precisely what the wedge is aiming to capture.[24]

Concretely, each investor pays a tax τ (τ^*) on foreign dividends. For instance, the domestic investor only receives a dividend $(1-\tau)p_t^* Y_t^*$ per share of the foreign equity asset (of which she holds $w_{f,t} W_t / Q_t^*$) because she pays $\tau p_t^* Y_t^*$ as a tax. As a result, the risk premium on the foreign asset faced by the domestic investor and therefore appearing in her budget constraint becomes $\mu_{R^*,t} - r_t - \tau F_t^*$, while the risk premium on the domestic asset faced by the foreign investor and appearing in his budget constraint becomes $\mu_{R,t} - r_t - \tau^* F_t$. This highlights the role of dividend yields in driving the effect of the tax, a point that is important in practice as discussed in Sauzet (2022a). The amount of tax collected from one investor is rebated lump-sum to the other investor, so as not to distort decisions further. The exact details of this rebate do not make a material difference, as discussed in Bhamra et al. (2014). In terms of budget constraints, the domestic investor receives an additional $w_{h,t}^*(1-x_t)\tau^* F_t / x_t$ per unit of wealth each infinitesimal period, while the foreign investor receives $w_{f,t} x_t \tau F_t^* / (1-x_t)$.

2.5 Computation of the equilibrium

All variables of interest can be characterized as a function of the state variables, $X_t = (x_t, y_t)'$, and a set of unknown functions $\mathcal{G} \equiv \{J_t,\ J_t^*,\ F_t,\ F_t^*,\ q_t,\ w_{h,t},\ w_{f,t}\}$.[25] Due to the stationary recursive Markovian structure of the equilibrium, the unknown functions are themselves solely functions of X_t, and are determined by a system of coupled algebraic and second-order partial differential equations. This theoretical

[23] Cf. Basak and Gallmeyer (2003) for details. In simpler cases, e.g. with log preferences, one good and no home bias, one could potentially use a weaker notion of social planner to solve the equilibrium by introducing time-varying Pareto weights, à la Cuoco and He (1994), Basak and Cuoco (1998).

[24] Extensions introducing fully incomplete are promising avenues for future research. They could take the form of time-varying wedges, idiosyncratic labor income as in Kaplan et al. (2018), or capital risk as in Brunnermeier and Sannikov (2014, 2015).

[25] J_t, J_t^* capture (an increasing monotonic transformation of) the marginal values of wealth of each investor. In addition, as a point of notation, for any function g, g_t simply denotes $g(X_t)$, not the time-derivative of g (which is zero because the model is stationary due to the infinite horizon).

characterization is the focus of Section 3, and additional results and details (including on the numerical method) are provided in Sauzet (2022a).

Each of the unknown function $g : [0,1]^2 \rightarrow \mathcal{D}^g \in \mathbb{R}$ in $\mathcal{G}$ is approximated using projection methods based on Chebyshev polynomials and orthogonal collocation. This yields a global solution to the equilibrium, which is one of the key technical innovations in the paper, and on which I rely heavily throughout. Appendix C briefly discusses some of its advantages.

3 Characterization of the Equilibrium

I now characterize the general equilibrium theoretically. I focus on asset prices and portfolios, and show how they depend on the two state variables: the wealth share of the domestic (U.S.) investor, x_t, and the relative supply of the domestic (U.S.) good, y_t. Home bias in equity holdings obtains like in the data for instance due to imperfect financial integration, or when the elasticity of substitution across goods is low. In all cases, the bias in portfolio holdings is amplified by the hedging of wealth share risk. The solution draws on Sauzet (2022a), who provides details and further theoretical results e.g. on marginal values of wealth, consumption and the relative price of goods, and the law of motions of the state variables.[26] The *dynamics* of all variables play a central role, emphasizing the relevance of the global solution, and are also discussed in Sections 4 and 5. Appendices A and B provide additional details.

Calibration Parameters are set according to the calibration of Assumption 1, unless otherwise specified. The elasticity of substitution across goods, θ, partly determines the sign of portfolio biases in the absence of financial market frictions. Our

[26]For instance, I show in Sauzet (2022a) that value functions are given by

$$V^{(*)}(W_t^{(*)}, x_t, y_t) = \left(\frac{W_t^{(*)1-\gamma^{(*)}}}{1-\gamma^{(*)}} \right) J^{(*)}(x_t, y_t)^{\frac{1-\gamma^{(*)}}{1-\psi^{(*)}}} \tag{9}$$

I call J_t, J_t^* the marginal values of wealth of both investors. Those unknown functions solve two Hamilton-Jacobi-Bellman equations, shown in Appendix A.6. They are one of the main drivers of stochastic discount factors, and therefore of asset prices, portfolios, and risk sharing in this economy. Similarly, the characterization of the law of motion of state variables, and consumption and goods prices, are summarized in Appendices A.7 and A.8.

benchmark is a value of $\theta = 2$, consistent with recent estimates e.g. in Imbs and Méjean (2015). The elasticity of intertemporal substitution, ψ, has a limited effect on most variables, but turns out to be a strong determinant of the quantitative impact of imperfect financial integration. Our baseline is $\psi = 1/2$, in part to obtain a plausible riskfree rate.[27] Financial frictions are set to a mild level of $\tau = 15\%$, which are enough to deliver a home bias in equity holdings in this setting. Below, I briefly discuss the impact of a lower $\theta = 1/2$, or of a higher $\psi = 2$, among others, and Sauzet (2022a) provides additional details on the impact of the calibration. Asymmetric risk aversion ($\gamma = 8 < \gamma^* = 15$) allows to capture the special role of the U.S. as the world banker in the international financial system, in the spirit of Gourinchas et al. (2017) and Maggiori (2017). It is the focus of Sections 4 and 5. I discuss the level of risk aversion itself when presenting risk premia below. The home bias in consumption $\alpha = 0.75 > 1/2$ is consistent with the import share of the United States. Other parameters are standard.

Assumption 1 (Baseline calibration)**.** *Unless otherwise specified, theoretical results are obtained under the following calibration*

- *Risk aversion:* $\gamma = 8 < \gamma^* = 15$*,*
- *Elasticity of intertemporal substitution:* $\psi = \psi^* = 1/2$*,*
- *Home bias in consumption:* $\alpha = \alpha^* = 0.75$*, numéraire basket:* $a = 1/2$*,*
- *Elasticity of substitution between goods:* $\theta = \theta^* = 2$*,*
- *Discount rate:* $\rho = \rho^* = 1\%$*,*
- *No labor income:* $\delta = \delta^* = 0$*,*
- *Frictions in financial markets:* $\tau = \tau^* = 15\%$*,*
- *Output:* $\mu_Y = \mu_Y^* = 2\%, \sigma_Y = (4.1\%, 0)^T, \sigma_Y^* = (0, 4.1\%)^T$ *(in annual terms, no fundamental correlation).*

3.1 Asset prices

I first discuss the first and second moments of asset prices in this economy.

[27] The model does not feature long-run risk or disaster, so having $\psi > 1$ is not a requirement, contrary to Bansal and Yaron (2004) and related contributions.

Risk premia Proposition 1 presents the expected risk premia on the domestic and foreign equity assets.

Proposition 1. *The expected risk premia on domestic and foreign equity assets are*

$$\mu_{R,t} - r_t = \gamma_t \sigma_{R,t}^T \sigma_{\widetilde{W},t} + \gamma_t \left(\frac{1 - x_t}{\gamma^*} \right) \tau^* F_t - \gamma_t \sigma_{R,t}^T \sigma_{\widetilde{J},t} \tag{10}$$

$$\mu_{R*,t} - r_t = \gamma_t \sigma_{R*,t}^T \sigma_{\widetilde{W},t} + \gamma_t \left(\frac{x_t}{\gamma} \right) \tau F_t^* - \gamma_t \sigma_{R*,t}^T \sigma_{\widetilde{J},t} \tag{11}$$

where

$$\gamma_t \equiv \left(\frac{x_t}{\gamma} + \frac{1 - x_t}{\gamma^*} \right)^{-1} \tag{12}$$

$$\sigma_{\widetilde{W},t} \equiv z_t \sigma_{R,t} + (1 - z_t) \sigma_{R*,t}$$

$$\sigma_{\widetilde{J},t} \equiv x_t \left(\frac{1}{\gamma} \right) \left(\frac{1 - \gamma}{1 - \psi} \right) \sigma_{J,t} + (1 - x_t) \left(\frac{1}{\gamma^*} \right) \left(\frac{1 - \gamma^*}{1 - \psi^*} \right) \sigma_{J*,t}$$

and γ_t is the wealth-weighted global risk aversion, $\widetilde{W}_t$ is world wealth, $\widetilde{J}_t$ is the world wealth-weighted marginal value of wealth, and $\sigma_{J,t}, \sigma_{J*,t}$ are the geometric diffusion terms of J_t, J_t^* obtained as in Remark A.1 in Appendix.

The intuition for the different components is as follows. It highlights the importance of capturing dynamic aspects to explain prices of risk and risk premia.

The firm term is a market component that is driven by the covariance of equity returns with global wealth (recall that $z_t \equiv Q_t / (Q_t + Q_t^*)$ is the weight on the domestic U.S. asset in the market portfolio). Intuitively, an asset for which this covariance is large is a bad hedge because its returns are large when global wealth is already large, i.e. when it is not valuable for the world economy. Such an asset is therefore risky, and commands a high risk premium in equilibrium.

The second term relates to imperfect integration and frictions in financial markets. Although I do not directly proxy for it in the empirical analysis (Section 5), greater frictions in financial markets lead to a reinforced quantitative impact of the wealth share (cf. Sauzet, 2022a for details). The fact that the proxy for wealth share risk will play an important role empirically is therefore a sign that frictions in financial markets are likely to be relevant in practice.

The third term is a hedging component that is driven by the covariance of equity returns with the world wealth-weighted marginal value of wealth, $\widetilde{J}_t \equiv x_t J_t + (1 - x_t) J_t^*$. Intuitively, an asset for which this covariance is large provides an insurance because its returns are large when $\widetilde{J}_t$ is large, i.e. when the average world investor values it most. Such an asset is therefore a good hedge, and commands a lower risk premium in equilibrium (note the negative sign). This component can in turn be decomposed as follows

$$\begin{aligned}
-\gamma_t \sigma_{R^{(*)},t}^T \sigma_{\widetilde{J},t} = & -\gamma_t \sigma_{R^{(*)},t}^T \sigma_{x,t} x_t \left\{ x_t \left(\frac{1}{\gamma}\right) \left(\frac{1-\gamma}{1-\psi}\right) \frac{J_{x,t}}{J_t} + (1 - x_t) \left(\frac{1}{\gamma^*}\right) \left(\frac{1-\gamma^*}{1-\psi^*}\right) \frac{J_{x,t}^*}{J_t^*} \right\} \\
& - \gamma_t \sigma_{R^{(*)},t}^T \sigma_{y,t} y_t \left\{ x_t \left(\frac{1}{\gamma}\right) \left(\frac{1-\gamma}{1-\psi}\right) \frac{J_{y,t}}{J_t} + (1 - x_t) \left(\frac{1}{\gamma^*}\right) \left(\frac{1-\gamma^*}{1-\psi^*}\right) \frac{J_{y,t}^*}{J_t^*} \right\}
\end{aligned} \tag{13}$$

The first piece of this third term stems from the hedging of wealth share (x_t) risk by both investors, which constitutes the second factor of risk in addition to the market component. In equilibrium, the price of this risk is negative, so that an asset whose returns comove with the domestic U.S. wealth share is a risky asset and commands a higher risk premium. This is consistent with such an asset paying when the U.S. investor, who is more risk tolerant, is larger in world wealth so that less compensation for risk is required and the asset is less valuable a hedge. Empirically, this corresponds to periods of relative calm, as opposed to turbulent global crises in which the U.S. wealth share (x_t) decreases (a point we return to in Section 4, Fact 1). Note that this hedging term is distinct from the effect of x_t through risk aversion γ_t, which impacts the prices of risk on all factors: it corresponds to a *hedging* of those changes in risk compensation. The hedging of x_t also embeds the fact that changes in the share of wealth held by each investor have an impact on the relative prices of goods as well. However, this effect is more muted, and the relative prices of goods play mostly through the second piece below.

The second piece of the third term stems from the hedging of the relative supply of the U.S. good (y_t) by both investors, and constitutes a third factor of risk. Because relative supply strongly (negatively) drives relative prices (cf. Figure 8, and Sauzet, 2022a), this is in the spirit of the real exchange hedging that has been discussed in the literature. The price of that risk depends on which investor dominates the economy. As the U.S. investor gets larger in world wealth (x_t increases), her concern for hedging states of the world in which her preferred U.S. good is rare (low relative supply y_t, high relative price of that good and high U.S. real exchange rate) starts dominating. In that case, the price of y_t risk is positive (i.e., the price of real exchange risk, which is negatively related to relative supply, is negative). In other words, an asset that pays when the relative supply of the U.S. good is low is a good hedge, and commands a lower risk premium on average, while an asset that pays when the relative supply is high is unattractive, risky, and commands a higher risk premium. When the foreign investor starts dominating however, the pattern is reversed and the price of relative supply risk is negative (i.e., the price of real exchange rate risk is positive). Overall, the price of relative supply risk can therefore strongly vary with x_t. In the baseline calibration, it is broadly negative around a U.S. wealth share of 20 to 40% like in the data, although it can turn positive as the relative supply gets relatively small. In other words, the price of real exchange risk is likely to be broadly positive.

In practice, Equations (10)-(11) therefore imply a beta-presentation for risk premia with three factors: a market component, wealth share risk (x_t), and relative supply risk (y_t). It is shown in Proposition 2, together with the predictions for prices of risk discussed above. Equation (14) is the basis for the empirical analysis of Section 5.

Proposition 2. *The expected risk premium on any asset j follows a beta-representation*

$$\mu_{R_j,t} - r_t = \alpha_{j,t} + \beta_{j,M,t}\lambda_{M,t} + \beta_{j,x,t}\lambda_{x,t} + \beta_{j,y,t}\lambda_{y,t} + \varepsilon_{j,t} \tag{14}$$

Denote $dR_{M,t} \equiv z_t dR_t + (1 - z_t)dR_t^*$ the return on the market. The quantities of risk are defined as

$$\beta_{j,x,t} \equiv \frac{cov_t\left(dR_{j,t}, dx\right)}{var_t(dx_t)}\ ,\ \beta_{j,y,t} \equiv \frac{cov_t\left(dR_{j,t}, dy_t\right)}{var_t(dy_t)}\ ,\ \beta_{j,M,t} \equiv \frac{cov_t\left(dR_{j,t}, dR_{M,t}\right)}{var_t(dR_{M,t})}$$

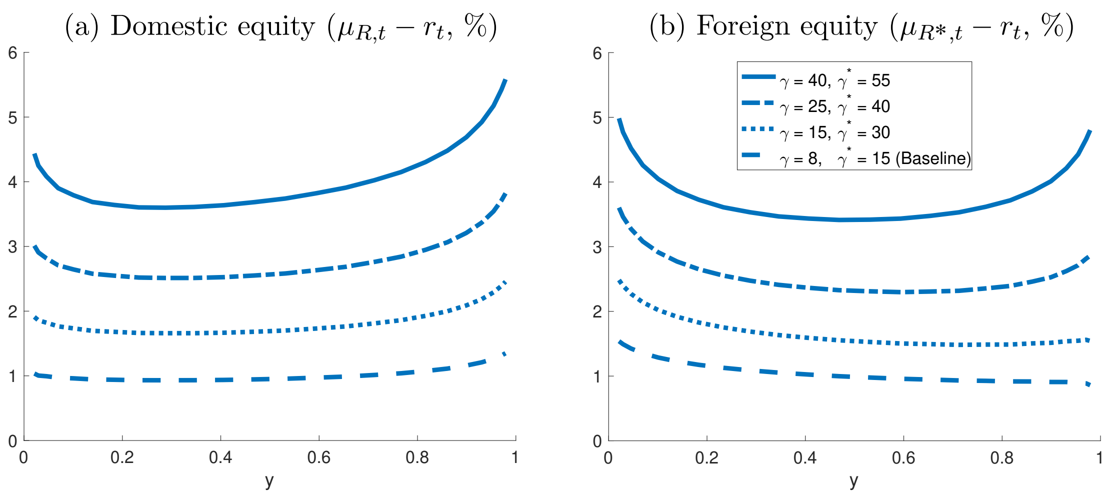
The predicted prices of risk are

- $\lambda_{x,t} > 0$ *throughout the state space, i.e. for any* $X_t = (x_t, y_t)' \in [0, 100\%]^2$,
- $\lambda_{y,t} < 0$ *in the relevant region of the state space, i.e.* $X_t \in [20\%, 40\%]^2$, *but can turn positive in some states.*

Figure 1 shows the results in the baseline calibration, for a U.S. wealth share of $x_t = 30\%$ like in the data (Fact 1). The risk premia on both assets are driven primarily by the common market component. For the domestic asset for instance

(Panel (a)), the risk premium increases as the domestic good becomes dominant (y_t increases), because the domestic asset incidentally starts to dominate global wealth (z_t, its weight in the market portfolio, increases) and is therefore risky. The pattern is broadly symmetric for the foreign asset (Panel (b)), and the risk premia on both assets also increase when the opposite good becomes dominant due to imperfect good substitutability. In practice however, those effects of the relative supply are dwarfed by that of the wealth share x_t: the risk premia on both assets strongly increase as x_t, the share of wealth held by risk-tolerant U.S. investors, decreases. This evolution is at the core of the Global Financial Cycle in risk premia. It is discussed in Section 4.3, and shown in Figure 7.

Figure 1: Risk premia



Notes: Based on the baseline calibration of Assumption 1, except for the specified risk aversion. The figure shows a cut in which the share of worldwide wealth held by the domestic investor is consistent with that of the United States in the data ($x_t = 30\%$, Fact 1). y_t is the relative supply of the domestic good, which captures fundamentals.

In terms of magnitude, risk premia are sizable. Although they are lower than in the data in the baseline calibration, which is expected in a consumption-based asset pricing framework, they can be made close to their historical empirical values by increasing risk aversion γ or by considering that equity assets are leveraged claims on the endowments of both countries. For instance, risk premia reach around 4 to 6% annually for $\gamma = 40, \gamma^* = 55$ at $x_t = 30\%$, and increase further as x_t decreases.[28]

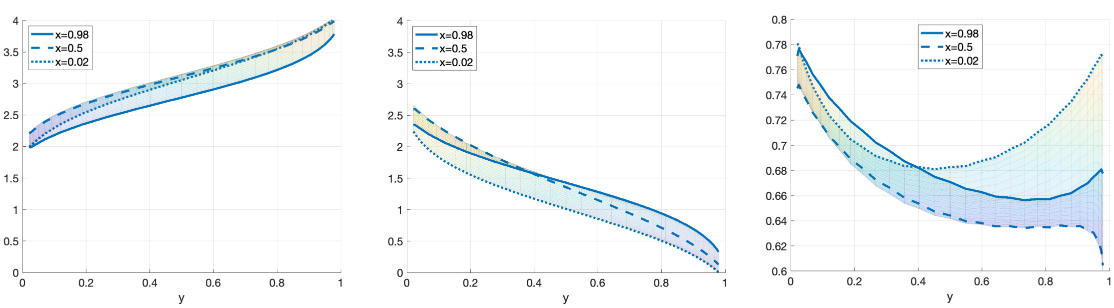
[28]Risk premia can also be made to reach higher values by introducing additional elements, e.g.

The interest rate on the riskfree bond (Figure 7) and Sharpe ratios (Figure B.2 in Appendix) are broadly consistent with their historical averages (e.g. in Jordà et al., 2019a), and the evolution of the latter with the state of the economy closely mimics that of risk premia. Finally, Figure B.4 in Appendix shows the prices of x_t- and y_t-risk, which are consistent with the discussion above.

Second moments In the baseline calibration, the returns on each asset load more on their own fundamental shock. For instance, Panels (a) and (b) of Figure 2 shows that the diffusion of the domestic asset with respect to the domestic shock is larger for most X_t than that with respect to the foreign shock: $\sigma_{Rz,t} > \sigma_{Rz*,t}$. In practice, this is the case as long as movements in the relative price of goods are muted in comparison to movements in the relative supply of goods (Sauzet, 2022a), and the picture is broadly symmetric for the foreign asset. Those patterns imply that the returns on the domestic (foreign) asset tend to comove positively (negatively) with the relative supply of the domestic good, y_t. In other words, the domestic asset has a positive quantity of y_t-risk in the baseline calibration, while it is negative for the foreign asset, as shown in Figure B.5 in Appendix.

Figure 2: Second moments of returns

(a) Loading of domestic asset on domestic shock ($\sigma_{Rz,t}$, %) | (b) Loading of domestic asset on foreign shock ($\sigma_{Rz*,t}$, %) | (c) Conditional returns correlation ($\text{corr}_t(dR_t, dR^*_t)$)



Notes: Based on the baseline calibration of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

consumption habits à la Campbell and Cochrane (1999). However, this comes in part because average risk aversion is much larger in such settings, and is therefore broadly equivalent to increasing γ itself for the purpose of this economy. For instance, an average risk aversion of 40 or 50 is relatively common in the habit literature.

Interestingly, both assets still load on both shocks, despite no fundamental correlation in the output of the underlying trees ($\sigma_{Yz^*} = \sigma_{Y^*z} = 0$). In other words, a large correlation between asset returns of around 0.7 emerges purely endogenously, as shown in Panel (c) of Figure 2. This result stems from the combination of movements in the relative price of goods and in the allocation of wealth, which in turn play a role because of the general recursive-CES preferences of investors. It suggests large endogenous contagion channels taking place through financial markets in this economy.

Correlation and other second moments –e.g. (instantaneous) volatilities shown in Figure B.3 in Appendix– also change with X_t, pointing to the fact that the diversification benefits provided by both assets evolve endogenously. Namely, in the region of the state space relevant empirically, $X_t \in [20\%, 40\%]^2$, they increase as the U.S. wealth share x_t and output share y_t decrease, which is the case in times of crisis (Fact 1). In other words, second moments, together with risk premia and Sharpe ratios, are countercyclical, as observed empirically (Fact 9, Section 4).

3.2 Portfolios

The portfolios of both investors are in the spirit of Merton (1973), and are summarized in Proposition 3. As an example, the weight on the domestic U.S. asset in the domestic U.S. portfolio, $w_{h,t}$, as well as its underlying components, are shown in Figure 3. The patterns for other equity portfolio weights, $w_{f,t}, w^*_{h,t}, w^*_{f,t}$, the weights invested in the riskless bond, b_t, b^*_t, and the market portfolio, z_t, are shown in Figure B.6.

Proposition 3. *The portfolios of the domestic and foreign investors are*

$$\begin{pmatrix} w_{h,t} \\ w_{f,t} \end{pmatrix} = \frac{1}{\gamma}\left(\Sigma_t^T \Sigma_t\right)^{-1} \left\{ \begin{pmatrix} \mu_{R,t} - r_t \\ \mu_{R^*,t} - r_t - \tau F^*_t \end{pmatrix} + \left(\frac{1-\gamma}{1-\psi}\right) \Sigma_t^T \left(\frac{J_{x,t}}{J_t} x_t \sigma_{x,t} + \frac{J_{y,t}}{J_t} y_t \sigma_{y,t} \right) \right\}$$

$$b_t = 1 - w_{h,t} - w_{f,t} \tag{15}$$

$$\begin{pmatrix} w^*_{h,t} \\ w^*_{f,t} \end{pmatrix} = \frac{1}{\gamma^*}\left(\Sigma_t^T \Sigma_t\right)^{-1} \left\{ \begin{pmatrix} \mu_{R,t} - r_t - \tau^* F_t \\ \mu_{R^*,t} - r_t \end{pmatrix} + \left(\frac{1-\gamma^*}{1-\psi^*}\right) \Sigma_t^T \left(\frac{J^*_{x,t}}{J^*_t} x_t \sigma_{x,t} + \frac{J^*_{y,t}}{J^*_t} y_t \sigma_{y,t} \right) \right\}$$

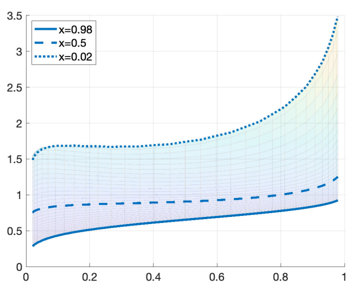
$$b^*_t = 1 - w^*_{h,t} - w^*_{f,t} \tag{16}$$

where $\Sigma_t \equiv \begin{bmatrix} \sigma_{R,t} & \sigma_{R^*,t} \end{bmatrix}$.

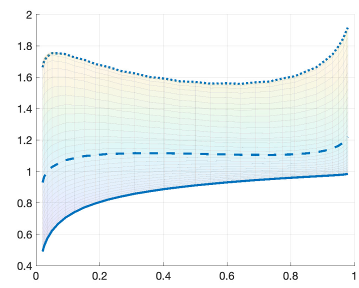
Equity portfolios feature three main elements. The first one is a global common component, which corresponds to the myopic portfolio that would be chosen by a mean-variance or log investor. This piece drives the overall shape of the portfolios of both investors with the state of the economy, X_t. The second element, captured by $\tau F_t^*, \tau^* F_t$, stems from frictions in financial markets. Finally, the third element are hedging terms, which are driven by the comovements of the marginal value of wealth of each investor J_t, J_t^* with the state of the economy X_t. Through those, investors aim to insure against changes in X_t that they particularly dislike, that is, states in which their marginal value of wealth is high. Hedging terms are particularly relevant in this economy with heterogenous investors (both in risk aversion and consumption bias), because they drive the *differential* tilt of the portfolios of both investors.

Figure 3: Components of the domestic portfolio

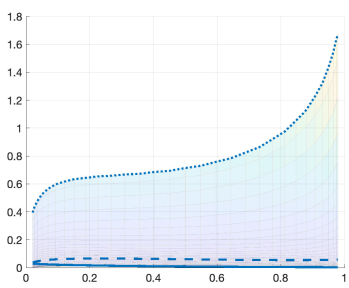
(a) Domestc weight in domestic asset ($w_{h,t}$)



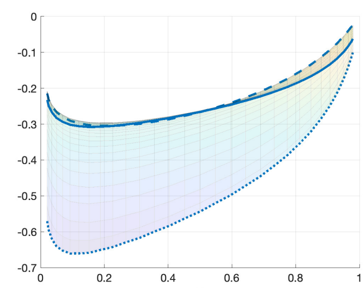
(b) Common component (including wedge)



(c) Hedging of x_t component



(d) Hedging of y_t component



Notes: Based on the baseline calibration of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

The combination of those elements yields average portfolios in the baseline calibration that are in line with empirical observations. Namely, like in the data, the weight on U.S. equity assets in the portfolio of U.S. (rest of the world) investors is around 65-70% (8-15%) in the model, in the region of the state space that is relevant empirically, $X_t = (x_t, y_t)' \in [20\%, 40\%]^2$ (cf. Fact 1). Similarly, like in the data, the home bias in equity holdings for the U.S. (rest of the world) investor when compared to the market portfolio, z_t, is around 55% (65%). This is remarkable given that the model remains stylized, and highlights that allowing for general recursive-CES preferences together with financial frictions goes a long way. In particular, a mild degree of frictions ($\tau = 15\%$) is enough to overturn the foreign bias in equity holdings that could obtain from the hedging of relative supply risk under perfect risk sharing, and this is partly driven by the hedging of wealth share risk x_t.

Additional details on the underlying drivers of portfolios, and the impact of various aspects of the specification, are provided in Sauzet (2022a). For instance, the elasticity of substitution across goods, θ, determines the magnitude of movements in the relative price of goods, q_t, in comparison to movements in the relative supply of goods, y_t. This relative magnitude in turn underpins movements in the relative dividends of the two assets, $p_t Y_t / (p_t^* Y_t^*) = q_t^{-1} y_t / (1 - y_t)$, which are important for the direction of portfolio biases. For example, a home bias can obtain without financial frictions with a low $\theta < 1$, although such values are not consistent with standard estimates (cf. e.g. Imbs and Méjean, 2015) and have counterfactual implications for relative dividends.[29] Second, the impact of financial market frictions, τ, τ^*, depends on the elasticity of intertemporal substition, ψ, via its effect on the magnitude of dividend yields, F_t, F_t^*. Lastly, note that a third possible way to obtain home bias in equity holdings, among others, is to assume that the preferences of investors feature a tilt towards a particular asset that they appreciate, as in Sauzet and Zerbib (2022).

Portfolio weights, and how biased they are with respect to the market portfolio (z_t), are also inherently time-varying, as shown in Figure 3. This aspect can be studied thanks to the global solution proposed in this paper, and the model has clear predictions for the impact of changes in X_t on portfolios. Section 5.1 below discusses those dynamic predictions, and compares them to the evolution of portfolios empirically. It also provides additional details about the underlying data and measures.

[29]Namely, $\theta < 1$, implies immiserizing growth, that is, that dividends at market value, $p_t Y_t / (p_t^* Y_t^*)$, *de*crease when relative output, Y_t / Y_t^*, *in*creases. Coeurdacier (2009) discusses the impact of θ further.

3.3 Imperfect financial integration, investor heterogeneity

The allocation of wealth across investors, x_t, is one of the new important elements put to light by the framework. Together with the relative supply of goods, y_t, it is central to the replication of stylized facts in Section 4. In turn, the quantitative impact of x_t is magnified by (i) the heterogeneity of investors including asymmetries in risk tolerance, and (ii) the imperfect risk sharing that arises due to imperfect financial integration. Both make the identity of who holds most of world wealth matter more. This highlights the value of the framework proposed in this paper because (i) and (ii) are made possible by the introduction of general recursive-CES preferences with potential asymmetries, and studying the dependence of economic variables and their dynamics on X_t is made possible by the global solution. Sauzet (2022a) provides additional details on those aspects. For instance, the paper shows that the hedging of wealth share risk is quantitatively important even under perfect risk sharing and symmetric preferences, provided that the heterogeneity stemming from consumption bias, α, is large enough.

4 The International Financial System

The framework explains several important dimensions of the international financial system. I start by summarizing those empirical findings in Section 4.1. The domestic country is taken to represent the United States, the country at its center, and U.S. investors have higher risk tolerance in international markets ($\gamma = 8 < \gamma^* = 15$). This asymmetry naturally captures the U.S. role as the world banker, and introducing it in this general equilibrium setting replicates stylized facts about the international financial system on average (Section 4.2), and in its dynamics (Sections 4.3).

Most importantly, the model can rationalize a Global Financial Cycle in risk premia and the so-called Reserve Currency Paradox. Those two phenomena have received considerable attention in the literature and have been puzzling from the perspective of existing models. In contrast, they emerge naturally in this economy, and in a way that is consistent with the large decrease in the U.S. wealth share and U.S. output share observed empirically in times of global crisis (novel Fact 1).

In Appendix, I discuss additional dynamic aspects (Section D.1, Facts 8 and 9), as well as longer-term trends (Section D.2, Facts 4, 10 and 11), that the model can shed light on. I also mention possible extensions in Section D.3. The dynamics of portfolio weights is discussed together with empirical evidence in Section 5.1.

4.1 Stylized facts

I start by documenting the novel fact that the share of world wealth held by the United States has tended to strongly decrease throughout the 2007-2008 global financial crisis and subsequent years, the main global crisis in the last two decades (Fact 1). The U.S. share of world GDP also declined over the same period. Those patterns broadly support the mechanisms through which the model can jointly rationalize the several other stylized facts about the international financial system, which I summarize next (Facts 2-11).

Fact 1 (Empirical patterns of the state variables X_t)**.** *Empirically, the share of world wealth held by the United States (x_t) strongly decreased throughout the 2007-2009 global financial crisis and subsequent years, and gradually started increasing back as it got resolved. The U.S. share of world GDP (y_t) also declined over the same period.*

This novel fact is shown in Figure 4, which plots both the U.S. share of world wealth (x_t, Panel (a)), the U.S. share of world GDP (y_t), and the deviations from their trends (Panels (c) and (d)).[30]

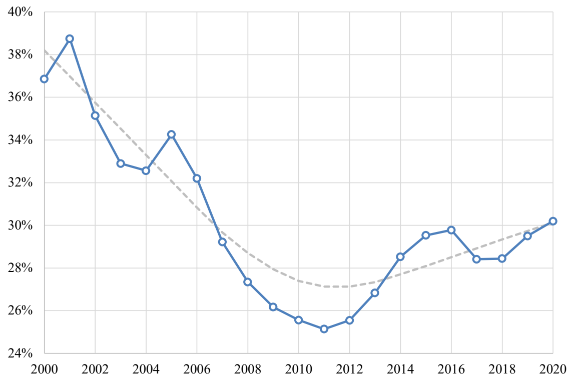
x_t is measured as the share of world wealth held by U.S. households, where wealth is defined as the marketable value of financial assets plus non-financial assets (principally housing and land) less debts. World wealth includes all countries for which appropriate data is available. The data covers 2000-2020, and is from Davies (2008), Davies et al. (2011), as updated in Crédit Suisse (2021). It constitutes one of the main and most comprehensive sources on global wealth allocation available. The measure is particularly adequate as the empirical equivalent of state variable x_t, given that it focuses on the wealth held by households, which are the natural interpretation

[30] The trend is extracted using a STL decomposition, i.e. a Seasonal-Trend decomposition using LOESS (Locally Estimated Scatterplot Smoothing). This approach is standard in the time-series literature, and is used to decompose a series into three components: trend, seasonal, and residual. The deviation from trend is the sum of the residual and the (small) seasonal component. This method has the advantage of being robust, notably to outliers, and of allowing for a flexible trend. The pattern is similar when using simple growth rates ($dx_t/x_t, dy_t/y_t$), as shown in Appendix E.1.

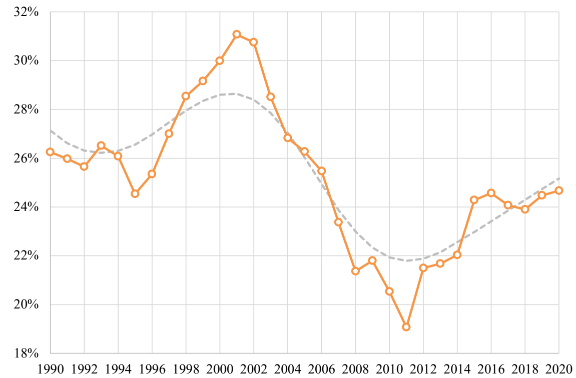
of the representative agent of each country in this consumption-based asset pricing framework.

Figure 4: Empirical patterns of the state variables $X_t \equiv (x_t, y_t)'$

(a) U.S. share of world wealth (x_t)

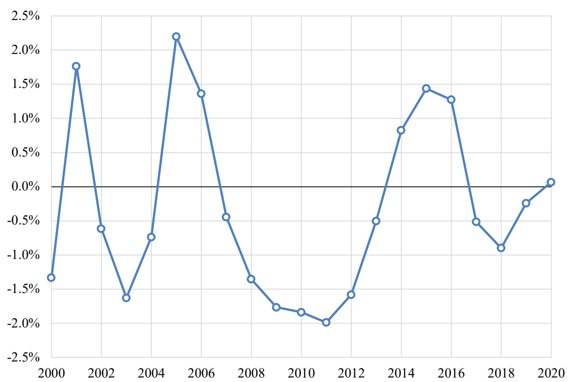


(b) U.S. share of world GDP (y_t)



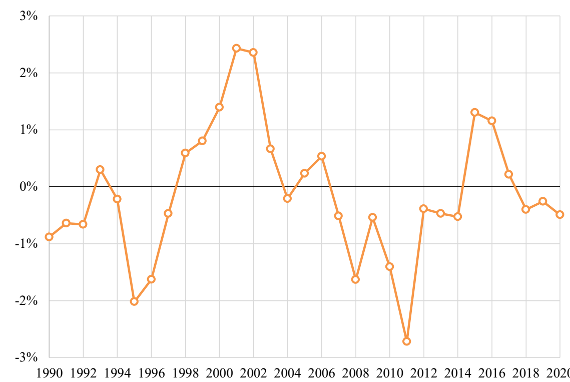
(c) U.S. share of world wealth (x_t)

(deviations from trend)



(d) U.S. share of world GDP (y_t)

(deviations from trend)



Notes: x_t is measured as the share of world wealth held by U.S. households. Wealth is defined as the marketable value of financial assets plus non-financial assets (principally housing and land) less debts. World wealth includes all countries for which appropriate data is available. Data is from Davies (2008), Davies et al. (2011), and as updated in Crédit Suisse (2021). y_t is measured as the U.S. share of world GDP in current U.S. dollars. Data is from the World Bank. Alternative measures shown in Figures E.1 to E.6 in Appendix display similar patterns. The dashed gray curves are trends extracted from standard STL decompositions.

Most importantly, this pattern of x_t broadly supports the mechanism through which the model can jointly rationalize the stylized facts mentioned below. Focusing solely on financial wealth (or non-financial wealth) yields the same pattern, as shown in Figures E.2 and Figures E.3 in Appendix. Alternative measures constructed from the World Inequality Database based on wealth measured as the market-value national wealth (Figure E.4) or market-value net private wealth (Figure E.5) also display the same evolution for x_t in the global financial crisis.[31] In addition, those measures highlight that the pattern for the wealth share also occurred, although in a more muted fashion, following the (less severe) crisis resulting from the 2001 dotcom-bubble burst.

The empirical equivalent of the relative supply (y_t) is computed as the U.S. share of world GDP in current U.S. dollars, using data from the World Bank. Panels (b) and (d) highlight that the U.S. share of world GDP also declined over the 2007-2009 global financial crisis, in particular in 2008 when the U.S. dollar spiked. This constitutes an important mechanism supporting the rationalization of the Reserve Currency Paradox, as discussed in Section 4.3. This mechanism is also consistent with the fact that, in times of crisis, U.S. GDP appears to decline more than does the GDP of the rest of the world, as shown in Panel (d) of Figure E.6.

Fact 2 (U.S. as world banker)**.** *The United States borrows from the rest of the world in safe assets, and uses it to lever up its investment in risky assets worldwide, resulting in a negative net foreign asset position. The country therefore plays the role of the world banker. (Gourinchas and Rey, 2007b; Gourinchas et al., 2017)*

Fact 3 (Exorbitant privilege)**.** *The United States earns excess returns on average on its net foreign asset position, in particular in normal (non-crisis) times. (Gourinchas and Rey, 2007b; Gourinchas et al., 2017)*

Fact 4 (Home bias in equity holdings)**.** *The aggregate portfolio of the United States, as well as that of most countries around the world, exhibit a strong bias towards domestic equity securities. This bias has slowly decreased in recent decades. (French and Poterba, 1991; Coeurdacier and Rey, 2013)*

Fact 5 (Exorbitant duty)**.** *In times of crisis, the United States plays the role of the world insurer, transferring wealth to the rest of the world. This exorbitant duty is the flip of its exorbitant privilege in normal times, and is associated with a strong deterioration in the U.S. net foreign asset position. (Gourinchas et al., 2017)*

[31] The U.S. wealth share also appears to decrease at the onset of the global financial crisis when constructed from a third approach, namely using data as in Jiang et al. (2022), based on 33 investor- and 31 issuer-countries.

Fact 6 (Global Financial Cycle)**.** *Periods of global stress are characterized by risk-off scenarios in which global risk aversion, as well as risk premia worldwide, spike up, and global interest rates decline. (Rey, 2013; Miranda-Agrippino and Rey, 2020)*

Fact 7 (Reserve Currency Paradox)**.** *Periods of global stress are accompanied by a large appreciation of the U.S. dollar, in contrast to the implication of existing models that predict a U.S. dollar depreciation when the wealth share of the U.S. world banker decreases. (Gourinchas et al., 2017; Maggiori, 2017)*

Fact 8 (International Financial Adjustment)**.** *Periods of strong deterioration in the net foreign asset position of the United States predict higher expected risk premia on its external balance sheet in the short to medium term, consistent with valuation effects playing a key role in its external adjustment process. (Gourinchas and Rey, 2007a, Gourinchas et al., 2019)*

Fact 9 (Countercyclicality of asset return dynamics)**.** *The (i) risk premia, (ii) Sharpe ratios, (iii) volatilities, and (iv) correlation of risky returns are countercyclical, i.e. they increase in times of crisis. This is true in particular in times of global stress. Those evolutions are consistent with crises being periods in which not only the quantity of risk rises, but so does the price of risk that is received as a compensation. (Among others, in the context of the United States: for (i), (ii), (iii), cf. Lettau and Ludvigson, 2010; for (i), cf. Fama and French, 1989, Ferson and Harvey, 1991, Harrison and Zhang, 1999, Campbell and Diebold, 2009; for (ii), cf. Harvey, 2001; for (iii), cf. Schwert, 1989, Brandt and Kang, 2004)*

Fact 10 (Global trend in asset comovements)**.** *The comovement of equity prices worldwide has increased over the last 150 years, and particularly rapidly in the past three decades. This sharp increase goes beyond the growing synchronization in real sector variables, and is driven in part by fluctuations in risk premia, which are themselves strongly impacted by fluctuations in global risk appetite. (Jordà et al., 2019b)*

Fact 11 (Global trend in interest rates)**.** *The real rate of interest has trended down globally in recent decades. This evolution can be related to the relative size of the risk-tolerant world banker (the United States) decreasing in the world economy. (Caballero et al., 2008; Hall, 2016; Gourinchas et al., 2017)*

4.2 External portfolio and the exorbitant privilege

I first focus on facts about the international financial system on average.

The domestic country represents the United States, and the U.S. investor has higher risk tolerance in international markets ($\gamma = 8 < \gamma^* = 15$). This assumption is in the spirit of Caballero et al. (2008), Gourinchas et al. (2017), and Maggiori (2017). It captures at a high level the greater development and depth of U.S. financial markets, and the higher participation of U.S. investors in equity markets, which both make the country as a whole better able and willing to carry financial risks in the world economy.

Her higher tolerance for risk leads the U.S. investor to borrow from the rest of the world in the riskfree bond, so as to lever up risky portfolio. Panel (a) of Figure 5 shows that this borrowing is large. For instance, around $x_t = 30\%$, which corresponds to the share of the United States in world wealth on average (Fact 1), the country borrows about 50% ($b_t = -0.5$) of its wealth in international markets, so as to invest 150% of its wealth in risky equity securities. As a result, the net foreign asset position of the United States is strongly negative. The latter is shown in Panel (b) of Figure 5 and averages $NFA_t/W_t = -30\%$ at $x_t = 30\%$. It is computed as the difference between the wealth invested in the foreign equity asset and in the world bond by the domestic investor, and that invested in the domestic asset by the foreign investor. As a fraction of domestic wealth, this yields[32]

$$\frac{NFA_t}{W_t} = w_{f,t} + b_t - w^*_{h,t} \left(\frac{1 - x_t}{x_t} \right) \tag{18}$$

Those results are consistent with the findings of Gourinchas et al. (2017), who document a strongly negative net foreign position in safe securities for the United States, which uses those safe liabilities to finance its investments in risky assets worldwide. In other words, the model naturally replicates Fact 2 about the role of the United States as the world banker, or more accurately, given the amount of leverage, as the

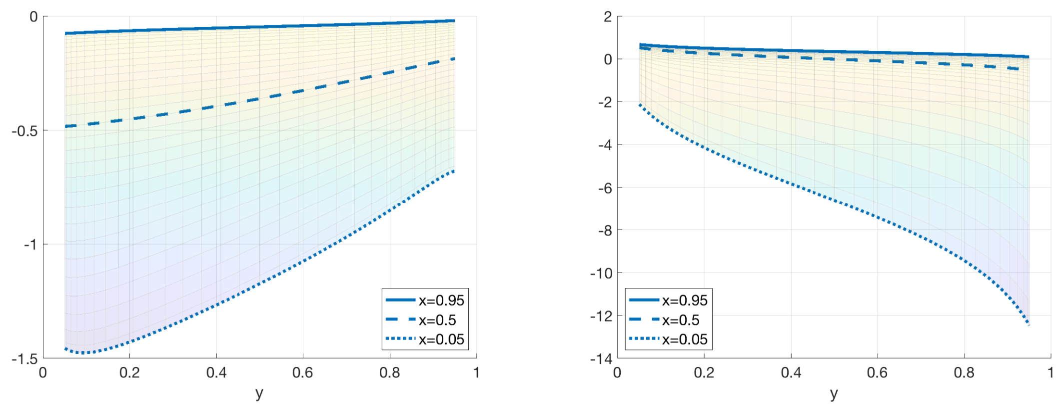
[32]The measure can also be expressed as a fraction of domestic output as follows

$$\frac{NFA_t}{Y_t} = \left(w_{f,t} + b_t - w^*_{h,t} \left(\frac{1 - x_t}{x_t} \right) \right) \left(\frac{x_t}{z_t} \right) \left(\frac{(1-\delta)p_t}{F_t} \right) \tag{17}$$

world venture capitalist, as pointed out in Gourinchas and Rey (2007b). Note also that both the amount of borrowing and the net foreign asset position vary strongly with the state of the economy, an aspect that I discuss below.

Figure 5: External position of the domestic country (U.S.)

(a) Share of bond in U.S. portfolio (b_t) (b) Net foreign asset position (NFA_t/W_t)



Notes: Based on the baseline calibration of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic U.S. investor. y_t is the relative supply of the domestic U.S. good, which captures fundamentals.

Relatedly, the model also reproduces the exorbitant privilege –i.e. the higher returns– that the United States has benefited from on its external portfolio (Fact 3).

This is first visible by comparing the expected returns on the *total* portfolio of the U.S. investor, $\overline{\mu_{R,t}} \equiv w_{h,t}\mu_{R,t} + w_{f,t}\mu_{R*,t} + b_t r_t$, to that of the foreign investor, $\overline{\mu_{R*,t}} \equiv w^*_{h,t}\mu_{R,t}+w^*_{f,t}\mu_{R*,t}+b^*_t r_t$. Panels (a) and (b) of Figure 6 plot both as a function of the wealth share. The former is larger regardless of the state of the economy, with $\overline{\mu_{R,t}} = 4.9\%$ and $\overline{\mu_{R*,t}} = 3.9\%$ on average, reflecting the riskier position taken by the United States that is financed by borrowing internationally in the safe asset and earns higher returns in expectation. Interestingly, this result highlights the fact that the United States borrows to invest not only internationally but in its own domestic assets as well.

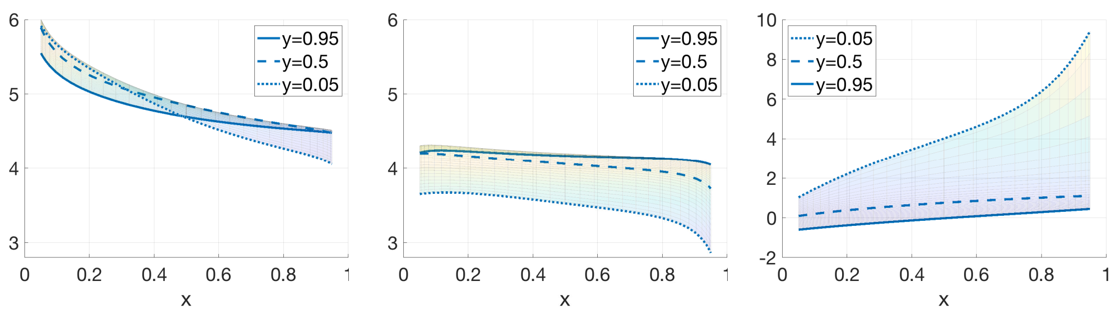
Second, the exorbitant privilege is also apparent in the returns on the *external* portfolio itself. Here, the United States earns $r^a_t = \mu_{R*,t}$ on its external assets, which are comprised of the foreign equity asset, and pays on its external liabilities r^l_t, the weighted average of the returns on the domestic equity and on the riskfree bond. The excess returns on the external position are therefore

$$r_t^a - r_t^l \equiv \mu_{R^*,t} - \left\{ \left(\frac{w_{h,t}^*}{w_{h,t}^* + b_t^*} \right) \mu_{R,t} + \left(\frac{b_t^*}{w_{h,t}^* + b_t^*} \right) r_t \right\} \tag{19}$$

Panel (c) of Figure 6 shows the results, with $r_t^a - r_t^l = 1.4\%$ on average, and about 0.7% when $x_t = 30\%$, consistent with the lower range of estimates in Gourinchas et al. (2017). The excess returns earned on its external portfolio allow the United States to finance its external deficit, and help reduce the burden of its external adjustment process, as discussed in Gourinchas and Rey (2007a) and below.

Figure 6: Average expected returns on countries' portfolios (%)

(a) United States: $\overline{\mu_{R,t}}$ (b) Rest of the world: $\overline{\mu_{R^*,t}}$ (c) United States: $r_t^a - r_t^l$



Notes: Based on the baseline calibration of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic U.S. investor. y_t is the relative supply of the domestic U.S. good, which captures fundamentals.

Finally, the modest degree of imperfect financial integration is able to generate a plausible home bias in equity holdings for both investors (Fact 4) on average. As discussed previously (Section 3.2), like in the data, the weight on U.S. equity assets in the portfolio of U.S. (rest of the world) investors is around 65-70% (8-15%) in the model, in the region of the state space that is relevant empirically $X_t = (x_t, y_t)' \in [20\%, 40\%]^2$ (cf. Fact 1). Similarly, like in the data, the home bias in equity holdings for the U.S. (rest of the world) investor is around 55% (65%). I touch upon the long-run evolution of those portfolio weights in Section D.2, and their dynamics at higher frequency is the main focus of Section 5.1.

In summary, the introduction of asymmetries in risk aversion, together with a modest degree of imperfect financial integration, is able to reproduce a number of stylized facts about the average level of the external portfolio of the United States in the world economy (Facts 2, 3, 4).

4.3 Exorbitant duty, Global Financial Cycle, and Reserve Currency Paradox

The better ability and willingness of the United States, the center country, to bear financial risks also have implications for the *dynamics* of the international financial system and asset returns.

To see this, let us consider a negative shock to the output of either country ($dZ_t < 0$ or $dZ_t^* < 0$). Either shock leads one of the equity assets to do poorly. In turn, because U.S. investors are more invested in equities on average, either shock leads to a decline in x_t, the share of wealth held by the United States. However, the decline in x_t is particularly strong when focusing on U.S. shocks because they affect primarily U.S. assets in which U.S. investors are more heavily invested due to their home bias in equity holdings. A negative shock to U.S. output is also associated with a decline in y_t, the U.S. output share. In short: a negative shock to U.S. output ($dZ_t < 0$) is consistent with the strong decline in x_t and y_t documented empirically in times of global crisis in novel Fact 1. I therefore focus on it for the remainder of this section, and refer to it as the "global shock".

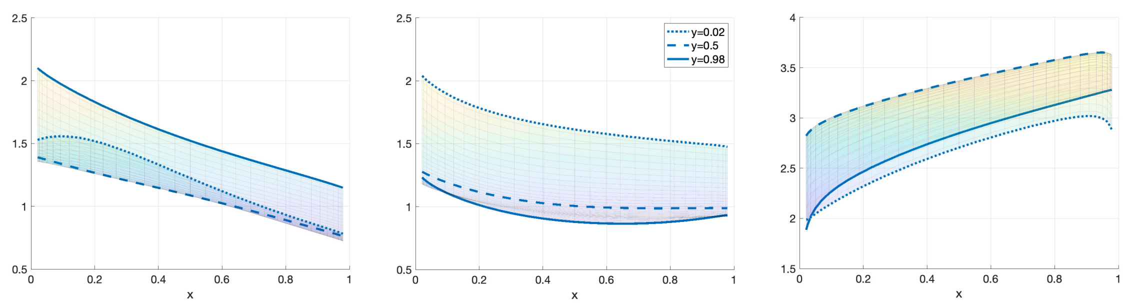
Exorbitant duty The decrease in the share of world wealth held by the center country directly captures the transfer of wealth that occurs from the United States to the rest of the world in times of international crisis. As put forward by Gourinchas et al. (2017) and summarized in Fact 5, this is the flip side of the banker role played by the United States in international markets. In good times, the world banker reaps the exorbitant privilege of its role by earning higher returns and running a large negative net foreign asset position, as we have seen. In bad times however, the country bears the exorbitant duty of insuring the rest of the world by transferring wealth to other countries. Empirically, consistent with my results, the exorbitant duty is large: Gourinchas et al. (2017) estimate that the transfer of wealth amounted to around 19% of U.S. GDP during the 2007-2009 global financial crisis. In the model, the phenomenon can also be observed from the patterns of consumption, with the share of consumption (at market value) enjoyed by the United States declining monotonically with its share of world wealth, consistent with the rest of the world receiving a transfer and therefore consuming relatively more in times of crisis.[33]

Global Financial Cycle The global shock also naturally gives rise to a Global Financial Cycle in asset premia, a phenomenon that has been documented empirically (Fact 6) but for which a theoretical explanation remains elusive.

Figure 7: Asset returns and the U.S. wealth share (x_t): the Global Financial Cycle

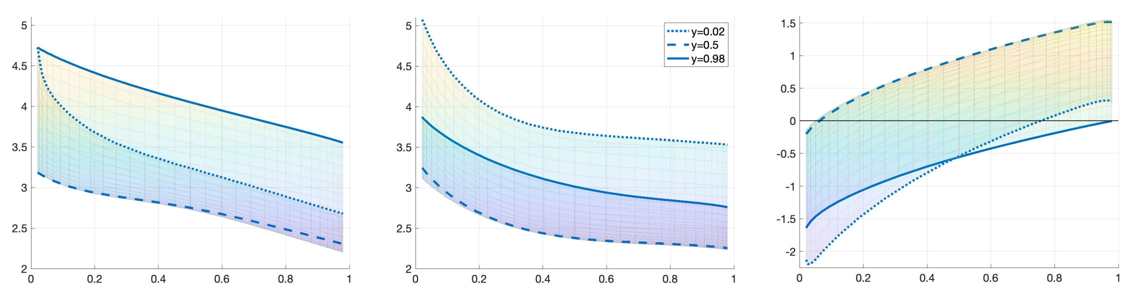
A. Baseline calibration

(a) Domestic (U.S.) risk premia ($\mu_{R,t} - r_t$, %) | (b) Foreign risk premia ($\mu_{R^*,t} - r_t$, %) | (c) Riskfree rate (r_t, %)



B. Baseline calibration, except that $\gamma = 25 < \gamma^* = 40$

(a) Domestic (U.S.) risk premia ($\mu_{R,t} - r_t$, %) | (b) Foreign risk premia ($\mu_{R^*,t} - r_t$, %) | (c) Riskfree rate (r_t, %)



Notes: Based on the baseline calibration of Assumption 1, except for the specified risk aversion. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic U.S. investor. y_t is the relative supply of the domestic U.S. good, which captures fundamentals.

[33] The share of domestic consumption at market value is computed as $P_t C_t / (P_t C_t + P_t^* C_t^*) = c_t x_t / (c_t x_t + \mathcal{E}_t c_t^* (1 - x_t))$.

As the share of wealth held by the United States x_t decreases, global risk aversion spikes up. Indeed, global risk aversion, γ_t, is time-varying in this context, and this reflects a transfer of wealth to foreign investors that are less risk tolerant on average.[34] This pattern captures the emergence of a risk-off scenario worldwide with the compensation for taking risk rising globally as the wealth position of the world banker deteriorates. In turn, this leads risk premia on *all* assets worldwide to increase sharply (Panels (a)-(b) of Figure 7), and riskfree interest rates to decline globally (Panel (c)). Taken together, those are three main markers of the Global Financial Cycle documented in Rey (2013), and Miranda-Agrippino and Rey (2020) (Fact 6).

Reserve Currency Paradox The model can also rationalize the Reserve Currency Paradox that has been a recent focus in the literature (Fact 7).

The paradox refers to the fact that the wealth transfer from the United States to the rest of the world in times of crisis (decrease in x_t) should result in a *depreciation* of the U.S. dollar (increase in $\mathcal{E}_t$ in our price convention), as seen on Panel (a) of Figure 8. Indeed, the decrease in the dominance of the U.S. investor in world wealth puts downward pressure on the price of her preferred good (p_t/p_t^* decreases), and therefore on the price of her consumption basket. This prediction is common to a large class of models, as documented in Maggiori (2017), but is at odds with the strong U.S. dollar *appreciation* that occurs in practice in times of global crisis.

Here again, putting the role of the United States in this general setting provides a way out: not only is the allocation of wealth across international investors (x_t) important, but so is the relative supply of goods (y_t). A decrease in y_t, the relative output of the United States, in times of crisis can indeed put upward pressure on the price of U.S. goods, and therefore on the price of the U.S. basket of goods. In other words, such an evolution can provide a natural countervailing force through which the U.S. dollar can appreciate. This can be seen on Panel (b) of Figure 8 which shows the real exchange rate, $\mathcal{E}_t$, as a function of relative supply, y_t. In addition, this force can potentially dominate quantitatively given that, as shown on the figure, the exchange rate changes much more rapidly with relative supply than with the allocation of wealth.

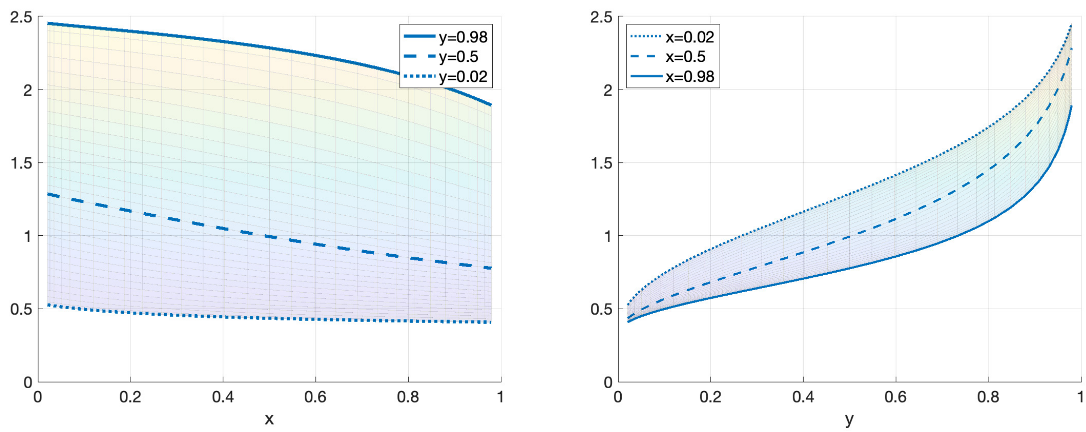
Is this observation relevant in practice? The second part of novel Fact 1 suggests

[34] As a reminder, the global wealth-weighted risk aversion is defined as $\gamma_t \equiv (x_t/\gamma + (1-x_t)/\gamma^*)^{-1}$.

that yes: the U.S. share of world GDP has tended to decrease in times of global crisis, especially so in 2008 when the U.S. dollar spiked up. Combined with the stronger quantitative impact of y_t, the appreciation pressure on the U.S. dollar can therefore very well dominate. In short, the model provides a strong reason why the appreciation in the U.S. dollar in times of crisis does not have to be a paradox with respect to the role of the United States as the world banker.

Figure 8: Reserve Currency Paradox and potential resolution

(a) $\mathcal{E}_t \equiv P_t^*/P_t$ and x_t (b) $\mathcal{E}_t \equiv P_t^*/P_t$ and y_t



Notes: Based on the baseline calibration of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic U.S. investor. y_t is the relative supply of the domestic U.S. good, which captures fundamentals.

To be sure, for both findings, additional channels have been proposed that are likely to also be important. For instance, among others, U.S. monetary policy and intermediaries leverage are important for the Global Financial Cycle (Miranda-Agrippino and Rey, 2020), and spiking demand in safe assets (Kekre and Lenel, 2021) and binding constraints on the part of foreign exchange dealers matter for the Reserve Currency Paradox. Embedding those in the current framework is an important avenue for future research, and could in fact reinforce the mechanisms above by giving them more bites in extreme circumstances such as in 2008. In short however, even absent those, the framework highlights that there exist deep conceptual and empirically-relevant reasons why we should expect a Global Financial Cycle, and a potential U.S. dollar appreciation in times of crisis, as soon as we introduce the special role of the United States as the world banker in a general enough setting.

In summary, in this economy, a global shock consistent with the empirical evolution of X_t (Fact 1) can therefore rationalize the exorbitant duty of the center country in times of global crises (Fact 5), as well as the Global Financial Cycle in risk premia (Fact 6), and the Reserve Currency Paradox (Fact 7).

Overall, the general equilibrium framework proposed in this paper is able to jointly replicate many stylized facts about both the structure and the dynamics of the international financial system. The two state variables highlighted by the model –U.S. wealth share (x_t) and U.S. output share (y_t)–, play a central role, and so do their dynamics. As such, those elements emphasize the importance of studying these questions in a general setup with recursive-CES preferences and financial market frictions, and of using a global solution method. More generally, the framework is a versatile building block for several applications and extensions (Section D.3).

5 Empirical Evidence

This section, together with Appendix E, provide empirical evidence consistent with the implications of the framework for portfolios (Section 5.1), and asset prices (Section E.2). Beyond confirming the main predictions of the model, the results for both emphasize once more two of its key novel elements: (i) the central role played by the two state variables –allocation of wealth and relative supply–, and (ii) the importance of *dynamic* aspects, in particular for portfolios.

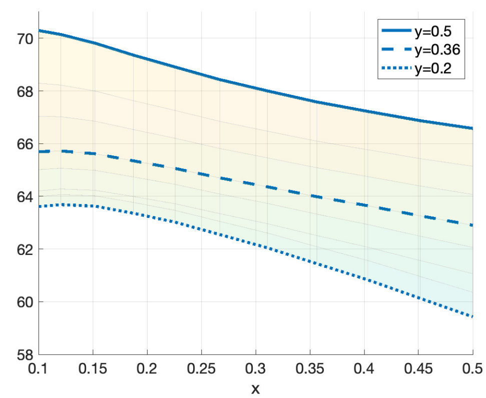
5.1 Portfolios

I first turn to portfolios. In times of global crisis, the wealth share of the U.S. investor strongly decreases, as discussed previously both theoretically and empirically in Fact 1. This is accompanied in the model by an increase in the weights of the U.S. asset in the equity portfolios of both the U.S. and foreign investors, $w_{h,t}/(w_{h,t} + w_{f,t})$, $w^*_{h,t}/(w^*_{h,t} + w^*_{f,t})$. The increase in this portfolio weight is more muted for the domestic U.S. investor, as shown in Panel (a) of Figure 9, and starker for the foreign investor (Panel (b)). The increase arises due to the combination of changes to each of the drivers of portfolios in Proposition 3: common component, financial market frictions, hedging terms. Intuitively, the U.S. asset becomes more attractive in part because its expected returns grow faster than that of the foreign asset.[35]

Figure 9: Share of U.S. asset in equity portfolios (%)

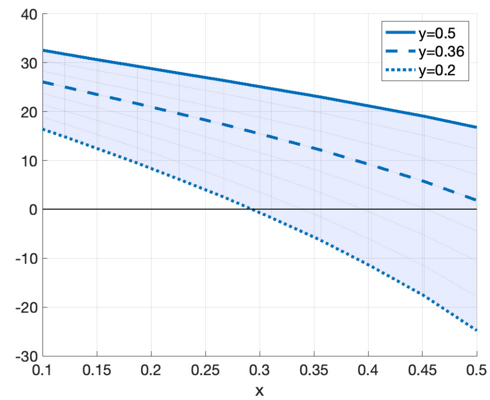
(a) U.S. portfolio (model)

$w_{h,t}/(w_{h,t}+w_{f,t})$



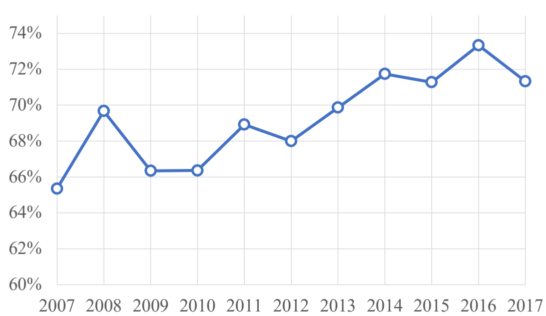
(b) Rest of the world portfolio (model)

$w^*_{h,t}/(w^*_{h,t}+w^*_{f,t})$



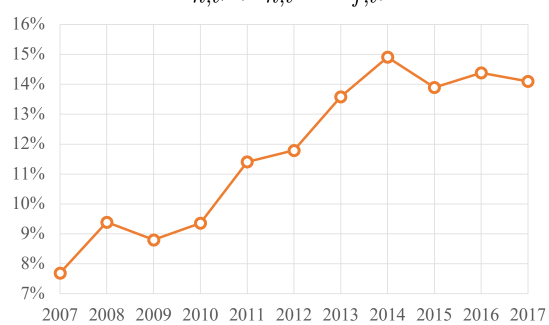
(c) U.S. portfolio (data)

$w_{h,t}/(w_{h,t}+w_{f,t})$



(d) Rest of the world portfolio (data)

$w^*_{h,t}/(w^*_{h,t}+w^*_{f,t})$



Notes: Panels (a) and (b) are based on the model of Sections 2-4 under the baseline calibration of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic U.S. investor. y_t is the relative supply of the U.S. good, which captures fundamentals. Recall that the U.S. wealth share decreases in times of global crisis (Fact 1). Panels (c) and (d) plot the empirical equivalents based on restated portfolio data by Coppola et al. (2021).

Panels (c) and (d) construct the equivalent weights empirically, using the restated global portfolio data of Coppola et al. (2021). This data reallocate the share of investments held in tax havens towards the ultimate country, using detailed data on

[35]Like in the data, the U.S. asset is on average held more by the U.S. investor in the model. When the wealth share of the U.S. investor, x_t, decreases, the larger change in asset ownership for the U.S. asset therefore leads to a larger increase in its expected risk premium compared to the foreign asset.

investment fund holdings, and therefore provide a more accurate description of the true portfolio of U.S. and foreign investors. In short, the weights of U.S. equities in both portfolios (around 65-70% for the U.S. investor, and 8-15% for the rest of the world) are broadly consistent with the model on average, but most importantly, so are the evolutions of those portfolio weights. Namely, they both increase starting around the 2008 global financial crisis, consistent with the *decrease* in the U.S. wealth share (x_t) discussed in Fact 1. The pattern of those weights based on non-reallocated data is broadly consistent, as shown in Figure E.7 in Appendix, which is based on standard data as used for instance in Coeurdacier and Rey (2013) that I extend to the recent period. Namely, the share of the U.S. asset in the rest of the world equity portfolio strongly increases from 2008 onwards. For the U.S. portfolio, that share also increases in 2008, as predicted by the model, and while it starts decreasing back afterwards, it does so at a much slower pace than pre-2008. This is consistent with a continuation of the secular downward trend in domestic portfolio weights (in the model, this can be captured as a decrease in financial market frictions τ), which gets strongly slowed down by the higher frequency pattern shown in Panel (a).

Relatedly, the empirical level and pattern of the equity *home bias*, which compares the weights in individual portfolios to that in the *market* portfolio, are also strongly consistent with theoretical predictions. In Panels (a) and (b) of Figure E.9 in Appendix, I show that a decrease in U.S. wealth share (x_t) in times of crisis leads to a strong increase in equity home bias for the U.S. investor, and a strong decrease of the equity home bias for the foreign investor. Here, to be consistent with empirical measures described e.g. in Coeurdacier and Rey (2013), I define home bias as[36]

$$HB_t \equiv 1 - \frac{\text{Share of foreign equities in country } i \text{ equity holdings}}{\text{Share of foreign equities in the world market portfolio}} \tag{20}$$

Empirically, again, the levels and patterns for the equity home bias of both investors are strongly consistent with the model, as shown in Panels (c) and (d) based on the data in Coppola et al. (2021). Using non-reallocated data as in Coeurdacier and Rey (2013), the patterns are similar (Figure E.8 in Appendix): (i) the equity home bias strongly decreases in the portfolio of the rest of the world investor, and (ii) the equity home bias increases in 2008 for the U.S. investor, and while it starts de-

[36] In the notation of this paper, the home bias for the U.S. and foreign investors are, respectively, $HB_t \equiv 1 - [w_{f,t}/(w_{h,t} + w_{f,t})]/(1 - z_t)$, $HB_t^* \equiv 1 - [w_{h,t}^*/(w_{h,t}^* + w_{f,t}^*)]/z_t$.

creasing again afterwards, it does so at a much slower pace. Like for portfolio weights, this is consistent with a continuation of the secular downward trend in equity home bias (decreasing τ), which gets slowed down by those higher frequency patterns.

Taken together, these findings are strongly consistent with the novel theoretical predictions of the model for portfolios and their dynamics, and confirm the central role of the wealth share held by U.S. investors in driving those patterns. Note that as the wealth share starts slowly increasing back around 2012, the patterns of portfolio weights and especially the home bias, start tapering off, although they do not fully reverse. This suggests that the sudden change in portfolios from 2008 onwards predicted by the model, might have started to alter the deeper motives of portfolio construction by international investors in ways that could persist for the longer run. Exploring these further patterns, and how they interact with recent evolutions in the valuation of U.S. equity assets (e.g. the large increase documented in Atkeson et al., 2022), is left for future research. Those recent evolutions are not problematic for my results, given that I focus on periods of global crises that occurred before this episode. The model however suggests that this could be a different form of privilege: U.S. investors borrow in safe assets from abroad to lever their portfolios, and invest heavily in risky assets worldwide *but also domestically*. As such, they also appear to have benefited strongly from the rise in U.S. valuations, as seen from the large increase in the U.S. wealth share over the same (recent) period (Fact 1).

In summary, empirical evidence on portfolios (Section 5.1), as well as unconditional and conditional asset pricing tests (Section E.2), strongly support the theoretical predictions of the model. They also emphasize the novel elements brought about by the framework, e.g. the central role of U.S. wealth and output shares, and the importance of *dynamic* aspects going beyond *average* portfolios and risk premia.

6 Conclusion

This paper makes three main contributions.

First, I characterize the global solution to the international portfolio choice problem in a general setup, a long-standing open issue in international finance.

The main message from the characterization is that two quantities are the core drivers of all economic outcomes: the relative supply of goods in different countries, and the allocation of wealth across investors. The importance of the latter had received little emphasis thus far in a general equilibrium international portfolio choice setting, but resonates with an emerging theme in the broader economic literature that has recently highlighted the role of the wealth distribution in various contexts.[37]

Studying those questions in a general setup makes it possible to highlight the importance of several elements –e.g. financial market frictions, the elasticity of intertemporal substitution, or the elasticity of substitution across goods– for asset prices, portfolios, and risk sharing.

Similarly, relying on a global solution method makes it possible to trace out the evolution of economic outcomes with the state of the economy, and to uncover the dynamic relationships between them. Those dynamics, in turn, are central to the ability of the model to replicate empirical observations, and underlie for instance the hedging components that are key drivers of portfolios and risk premia.

Second, when introducing the special role of the United States in the form of asymmetries in risk tolerance, I show that the framework is able to replicate a number of stylized facts about both the structure and dynamics of the international financial system. In particular, a Global Financial Cycle in risk premia emerges in this economy, a fact that has been documented empirically (Rey, 2013; Miranda-Agrippino and Rey, 2020) but for which a theoretical explanation remains elusive, and the model can make sense of the so-called Reserve Currency Paradox (Maggiori, 2017) via the role of the relative supply of goods on exchange rates. Both phenomena have received considerable attention in the recent literature, and were puzzling from the perspective of existing models, while they arise naturally in this framework. Importantly, the empirical patterns of the wealth share and relative GDP of the United States, which I document in a novel fact, support the main underlying mechanisms.

Third, I provide novel empirical evidence on portfolio levels, portfolio dynamics, and asset prices, that strongly supports the theoretical predictions of the framework.

The framework is a well-suited building block for several applications and extensions (Section D.3) that are promising avenues for future research.

[37] The impact of the distribution of wealth has been emphasized in macroeconomics (e.g. Brunnermeier and Sannikov, 2014, Kaplan et al., 2018), finance (e.g. Gomez, 2017, Lettau et al., 2019, Greenwald et al., 2020), and economics more generally (e.g. Piketty and Zucman, 2014).

References

Adrian, Tobias, and Nina Boyarchenko, 2015, Intermediary Balance Sheets, 2015 Meeting Papers 239, Society for Economic Dynamics.

Adrian, Tobias, and Hyun Song Shin, 2014, Procyclical leverage and value-at-risk, *Review of Financial Studies* 27, 373–403.

Anderson, Evan W., 2005, The dynamics of risk-sensitive allocations, *Journal of Economic Theory* 125, 93 – 150.

Anderson, Robert M., and Roberto C. Raimondo, 2008, Equilibrium in continuous-time financial markets: Endogenously dynamically complete markets, *Econometrica* 76, 841–907.

Atkeson, Andrew, Jonathan Heathcote, and Fabrizio Perri, 2022, The end of privilege: A reexamination of the net foreign asset position of the united states, Working Paper 29771, National Bureau of Economic Research.

Backus, David K., and Gregor W. Smith, 1993, Consumption and real exchange rates in dynamic economies with non-traded goods, *Journal of International Economics* 35, 297 – 316.

Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *The Journal of Finance* 59, 1481–1509.

Basak, Suleyman, and Domenico Cuoco, 1998, An Equilibrium Model with Restricted Stock Market Participation, *The Review of Financial Studies* 11, 309–341.

Basak, Suleyman, and Michael Gallmeyer, 2003, Capital Market Equilibrium with Differential Taxation, *Review of Finance* 7, 121–159.

Baxter, Marianne, and Urban Jermann, 1997, The international diversification puzzle is worse than you think, *American Economic Review* 87, 170–80.

Baxter, Marianne, Urban J Jermann, and Robert G King, 1998, Nontraded goods, nontraded factors, and international non-diversification, *Journal of International Economics* 44, 211 – 229.

Berrada, Tony, Julien Hugonnier, and Marcel Rindisbacher, 2007, Heterogeneous preferences and equilibrium trading volume, *Journal of Financial Economics* 83, 719 – 750.

Bhamra, Harjoat S., Nicolas Coeurdacier, and Stéphane Guibaud, 2014, A dynamic equilibrium model of imperfectly integrated financial markets, *Journal of Economic Theory* 154, 490 – 542.

Bhamra, Harjoat S., and Raman Uppal, 2014, Asset Prices with Heterogeneity in Preferences and Beliefs, *The Review of Financial Studies* 27, 519–580.

Borovička, Jaroslav, 2020, Survival and long-run dynamics with heterogeneous beliefs under recursive preferences, *Journal of Political Economy* 128, 206–251.

Boudoukh, Jacob, Michael Katz, Matthew P. Richardson, and Ashwin Thapar, 2015, Risk without reward: The case for strategic fx hedging, Aqr white paper.

Brandt, Michael W., John H. Cochrane, and Pedro Santa-Clara, 2006, International risk sharing is better than you think, or exchange rates are too smooth, *Journal of Monetary Economics* 53, 671 – 698.

Brandt, Michael W., and Qiang Kang, 2004, On the relationship between the conditional mean and volatility of stock returns: A latent var approach, *Journal of Financial Economics* 72, 217–257.

Brunnermeier, Markus, and Yuliy Sannikov, 2014, A macroeconomic model with a financial sector, *American Economic Review* 104, 379–421.

Brunnermeier, Markus K., and Lasse Heje Pedersen, 2009, Market liquidity and funding liquidity, *The Review of Financial Studies* 22, 2201–2238.

Brunnermeier, Markus K., and Yuliy Sannikov, 2015, International credit flows and pecuniary externalities, *American Economic Journal: Macroeconomics* 7, 297–338.

Brunnermeier, Markus K., and Yuliy Sannikov, 2019, International monetary theory: A risk perspective, *Working Paper* .

Caballero, Ricardo J., Emmanuel Farhi, and Pierre-Olivier Gourinchas, 2008, An equilibrium model of "global imbalances" and low interest rates, *American Economic Review* 98, 358–93.

Campbell, John Y., and John H. Cochrane, 1999, By force of habit: A consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy* 107, 205–251.

Campbell, Sean D., and Francis X. Diebold, 2009, Stock returns and expected business conditions: Half a century of direct evidence, *Journal of Business & Economic Statistics* 27, 266–278.

Cass, David, and Anna Pavlova, 2004, On trees and logs, *Journal of Economic Theory* 116, 41 – 83.

Chabakauri, Georgy, 2013, Dynamic Equilibrium with Two Stocks, Heterogeneous Investors, and Portfolio Constraints, *The Review of Financial Studies* 26, 3104–3141.

Chan, Yeung Lewis, and Leonid Kogan, 2002, Catching up with the joneses: Heterogeneous preferences and the dynamics of asset prices, *Journal of Political Economy* 110, 1255–1285.

Cochrane, John, 2005, Asset pricing: Revised edition.

Cochrane, John H., Francis A. Longstaff, and Pedro Santa-Clara, 2008, Two Trees, *The Review of Financial Studies* 21, 347–385.

Coeurdacier, Nicolas, 2009, Do trade costs in goods market lead to home bias in equities?, *Journal of International Economics* 77, 86 – 100.

Coeurdacier, Nicolas, and Pierre-Olivier Gourinchas, 2016, When bonds matter: Home bias in goods and assets, *Journal of Monetary Economics* 82, 119 – 137.

Coeurdacier, Nicolas, and Hélène Rey, 2013, Home bias in open economy financial macroeconomics, *Journal of Economic Literature* 51, 63–115.

Coeurdacier, Nicolas, Hélène Rey, and Pablo Winant, 2020, Financial integration and growth in a risky world, *Journal of Monetary Economics* 112, 1 – 21.

Colacito, R., M. M. Croce, and Zhao Liu, 2019, Recursive allocations and wealth distribution with multiple goods: Existence, survivorship, and dynamics, *Quantitative Economics* 10, 311–351.

Colacito, Ric, Max Croce, Steven Ho, and Philip Howard, 2018, Bkk the ez way: International long-run growth news and capital flows, *American Economic Review* 108, 3416–49.

Colacito, Riccardo, and Mariano M. Croce, 2011, Risks for the long run and the real exchange rate, *Journal of Political Economy* 119, 153–181.

Colacito, Riccardo, and Mariano M. Croce, 2013, International asset pricing with recursive preferences, *Journal of Finance* 68, 2651–2686.

Cole, Harold L., and Maurice Obstfeld, 1991, Commodity trade and international risk sharing: How much do financial markets matter?, *Journal of Monetary Economics* 28, 3 – 24.

Coppola, Antonio, Matteo Maggiori, Brent Neiman, and Jesse Schreger, 2021, Redrawing the Map of Global Capital Flows: The Role of Cross-Border Financing and Tax Havens, *The Quarterly Journal of Economics* 136, 1499–1556.

Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc, 2008, International Risk Sharing and the Transmission of Productivity Shocks, *The Review of Economic Studies* 75, 443–473.

Crédit Suisse, 2021, Global wealth report 2021, Credit suisse research institute.

Cuoco, Domenico, and Hua He, 1994, Dynamic equilibrium in infinite-dimensional economies with incomplete financial markets, Working paper, UPenn.

Dahlquist, Magnus, Christian Heyerdahl-Larsen, Anna Pavlova, and Julien Pénasse, 2022, International Capital Markets and Wealth Transfers, CEPR Discussion Papers 17334, C.E.P.R. Discussion Papers.

Daníelsson, Jón, Hyun Song Shin, and Jean-Pierre Zigrand, 2012, Procyclical leverage and endogenous risk, Mimeo, LSE, http://www.riskresearch.org.

Davies, James, ed., 2008, *Personal Wealth from a Global Perspective* (Oxford University Press).

Davies, James B., Susanna Sandström, Anthony Shorrocks, and Edward N. Wolff, 2011, The level and distribution of global household wealth*, *The Economic Journal* 121, 223–254.

Devereux, Michael B., and Alan Sutherland, 2011, Country portfolios in open economy macro-models, *Journal of the European Economic Association* 9, 337–369.

Dou, Winston Wei, and Adrien Verdelhan, 2015, The volatility of international capital flows and foreign assets, *Working Paper* .

Drechsler, Itamar, Alexi Savov, and Philipp Schnabl, 2018, A model of monetary policy and risk premia, *The Journal of Finance* 73, 317–373.

Duffie, D., J. Geanakoplos, A. Mas-Colell, and A. McLennan, 1994, Stationary markov equilibria, *Econometrica* 62, 745–781.

Duffie, Darrell, and Larry Epstein, 1992, Stochastic differential utility, *Econometrica* 60, 353–94.

Dumas, Bernard, 1989, Two-Person Dynamic Equilibrium in the Capital Market, *The Review of Financial Studies* 2, 157–188.

Dumas, Bernard, 1992, Dynamic Equilibrium and the Real Exchange Rate in a Spatially Separated World, *The Review of Financial Studies* 5, 153–180.

Dumas, Bernard, Raman Uppal, and Tan Wang, 2000, Efficient intertemporal allocations with recursive utility, *Journal of Economic Theory* 93, 240–259.

Ehling, Paul, and Christian Heyerdahl-Larsen, 2015, Complete and incomplete financial markets in multi-good economies, *Journal of Economic Theory* 160, 438 – 462.

Ehling, Paul, and Christian Heyerdahl-Larsen, 2017, Correlations, *Management Science* 63, 1919–1937.

Evans, Martin D.D., and Viktoria Hnatkovska, 2012, A method for solving general equilibrium models with incomplete markets and many financial assets, *Journal of Economic Dynamics and Control* 36, 1909 – 1930.

Fama, Eugene F., and Kenneth R. French, 1989, Business conditions and expected returns on stocks and bonds, *Journal of Financial Economics* 25, 23 – 49.

Fama, Eugene F., and Kenneth R. French, 2012, Size, value, and momentum in international stock returns, *Journal of Financial Economics* 105, 457–472.

Fang, Xiang, 2019, Intermediary leverage and currency risk premium, Working paper.

Farhi, Emmanuel, and Matteo Maggiori, 2018, A Model of the International Monetary System*, *The Quarterly Journal of Economics* 133, 295–355.

Fernández-Villaverde, Jesús, and Lawrence Christiano, 2011, Nber summer institute, computational tools & macroeconomic applications, Methods lectures.

Fernández-Villaverde, Jesús, Grey Gordon, Pablo Guerrón-Quintana, and Juan F. Rubio-Ramírez, 2015, Nonlinear adventures at the zero lower bound, *Journal of Economic Dynamics and Control* 57, 182–204.

Fernández-Villaverde, Jesús, and Oren Levintal, 2018, Solution methods for models with rare disasters, *Quantitative Economics* 9, 903–944.

Fernández-Villaverde, Jesús, Juan F. Rubio-Ramírez, and Olaf Posch, 2013, Solving the new keynesian model in continuous time, Mimeo.

Ferson, Wayne E., and Campbell R. Harvey, 1991, The variation of economic risk premiums, *Journal of Political Economy* 99, 385–415.

French, Kenneth, and James Poterba, 1991, Investor diversification and international equity markets, *American Economic Review* 81, 222–26.

Gabaix, Xavier, and Matteo Maggiori, 2015, International liquidity and exchange rate dynamics*, *The Quarterly Journal of Economics* 130, 1369–1420.

Gârleanu, Nicolae, and Stavros Panageas, 2015, Young, old, conservative, and bold: The implications of heterogeneity and finite lives for asset pricing, *Journal of Political Economy* 123, 670 – 685.

Gârleanu, Nicolae, Stavros Panageas, and Jianfeng Yu, 2020, Impediments to Financial Trade: Theory and Applications, *The Review of Financial Studies* 33, 2697–2727.

Gârleanu, Nicolae, and Lasse Heje Pedersen, 2011, Margin-based asset pricing and deviations from the law of one price, *The Review of Financial Studies* 24, 1980–2022.

Geanakoplos, John, 1990, An introduction to general equilibrium with incomplete asset markets, *Journal of Mathematical Economics* 19, 1 – 38.

Geanakoplos, John, and Andreu Mas-Colell, 1989, Real indeterminacy with financial assets, *Journal of Economic Theory* 47, 22 – 38.

Geanakoplos, John D, and Heraklis M Polemarchakis, 1986, Existence, regularity, and constrained suboptimality of competitive allocations, *Essays in Honor of Kenneth J. Arrow: Volume 3, Uncertainty, Information, and Communication* 3, 65.

Gehrig, Thomas, 1993, An information based explanation of the domestic bias in international equity investment, *The Scandinavian Journal of Economics* 95, 97–109.

Gomez, Matthieu, 2017, Asset prices and wealth inequality, 2017 Meeting Papers 1155, Society for Economic Dynamics.

Gourinchas, Pierre-Olivier, Walker D Ray, and Dimitri Vayanos, 2022, A preferred-habitat model of term premia, exchange rates, and monetary policy spillovers, Working Paper 29875, National Bureau of Economic Research.

Gourinchas, Pierre-Olivier, and Hélène Rey, 2007a, International financial adjustment, *Journal of Political Economy* 115, 665–703.

Gourinchas, Pierre-Olivier, and Hélène Rey, 2007b, From World Banker to World Venture Capitalist: U.S. External Adjustment and the Exorbitant Privilege, in *G7 Current Account Imbalances: Sustainability and Adjustment*, NBER Chapters, 11–66 (National Bureau of Economic Research, Inc).

Gourinchas, Pierre-Olivier, Hélène Rey, and Nicolas Govillot, 2017, Exorbitant Privilege and Exorbitant Duty, Working paper, University of California, Berkeley, LBS.

Gourinchas, Pierre-Olivier, Hélène Rey, and Maxime Sauzet, 2019, The international monetary and financial system, *Annual Review of Economics* 11, 859–893.

Greenwald, Daniel L, Martin Lettau, and Sydney C Ludvigson, 2020, How the wealth was won: Factors shares as market fundamentals, Working Paper 25769, NBER.

Hall, Robert E, 2016, The role of the growth of risk-averse wealth in the decline of the safe real interest rate, Working Paper 22196, NBER.

Hansen, Lars Peter, Paymon Khorrami, and Fabrice Tourre, 2018, Comparative valuation dynamics in models with financing restrictions .

Harrison, Paul, and Harold H. Zhang, 1999, An investigation of the risk and return relation at long horizons, *The Review of Economics and Statistics* 81, 399–408.

Hart, Oliver D, 1975, On the optimality of equilibrium when the market structure is incomplete, *Journal of Economic Theory* 11, 418 – 443.

Harvey, Campbell R., 2001, The specification of conditional expectations, *Journal of Empirical Finance* 8, 573 – 637, On the Predictability of Asset Returns.

Hau, Harald, and Hélène Rey, 2006, Exchange Rates, Equity Prices, and Capital Flows, *The Review of Financial Studies* 19, 273–317.

He, Zhiguo, and Arvind Krishnamurthy, 2013, Intermediary Asset Pricing, *American Economic Review* 103, 732–770.

Heathcote, Jonathan, and Fabrizio Perri, 2002, Financial autarky and international business cycles, *Journal of Monetary Economics* 49, 601–627.

Heathcote, Jonathan, and Fabrizio Perri, 2004, Financial globalization and real regionalization, *Journal of Economic Theory* 119, 207–243, Macroeconomics of Global Capital Market Imperfections.

Heathcote, Jonathan, and Fabrizio Perri, 2013, The international diversification puzzle is not as bad as you think, *Journal of Political Economy* 121, 1108 – 1159.

Huang, Ji, and Hao Hu, 2022, Endogenous global risk, Technical report.

Hugonnier, J., S. Malamud, and E. Trubowitz, 2012, Endogenous completeness of diffusion driven equilibrium markets, *Econometrica* 80, 1249–1270.

Imbs, Jean, and Isabelle Méjean, 2015, Elasticity optimism, *American Economic Journal: Macroeconomics* 7, 43–83.

Jiang, Zhengyang, Arvind Krishnamurthy, and Hanno Lustig, 2020, Dollar safety and the global financial cycle, Working Paper 27682, NBER.

Jiang, Zhengyang, Robert J Richmond, and Tony Zhang, 2022, A portfolio approach to global imbalances, Working Paper 30253, NBER.

Jordà, Òscar, Katharina Knoll, Dmitry Kuvshinov, Moritz Schularick, and Alan M Taylor, 2019a, The Rate of Return on Everything, 1870–2015*, *The Quarterly Journal of Economics* 134, 1225–1298.

Jordà, Òscar, Moritz Schularick, Alan Taylor, and Felix Ward, 2019b, Global financial cycles and risk premiums, *IMF Economic Review* 67, 109–150.

Judd, Kenneth, 1992, Projection methods for solving aggregate growth models, *Journal of Economic Theory* 58, 410–452.

Judd, Kenneth, 1998, *Numerical Methods in Economics*, volume 1, first edition (The MIT Press).

Kaplan, Greg, Benjamin Moll, and Giovanni L. Violante, 2018, Monetary policy according to hank, *American Economic Review* 108, 697–743.

Kargar, Mahyar, 2019, Heterogeneous intermediary asset pricing, *Working Paper* .

Kekre, Rohan, and Moritz Lenel, 2021, The flight to safety and international risk sharing, Working paper.

Kim, Sun Yong, 2022, The dollar, global growth risks and the us safety puzzle, *Global Growth Risks and the US Safety Puzzle (August 30, 2022)* .

Kollmann, Robert, 1995, Consumption, real exchange rates and the structure of international asset markets, *Journal of International Money and Finance* 14, 191–211.

Kubler, Felix, and Karl Schmedders, 2003, Stationary Equilibria in Asset-Pricing Models with Incomplete Markets and Collateral, *Econometrica* 71, 1767–1793.

Lettau, Martin, and Sydney Ludvigson, 2001, Resurrecting the (c)capm: A cross-sectional test when risk premia are time-varying, *Journal of Political Economy* 109, 1238–1287.

Lettau, Martin, and Sydney C. Ludvigson, 2010, Chapter 11 - measuring and modeling variation in the risk-return trade-off, in Yacine Aït-Sahalia, and Lars Peter Hansen, eds., *Handbook of Financial Econometrics: Tools and Techniques*, volume 1 of *Handbooks in Finance*, 617 – 690 (North-Holland, San Diego).

Lettau, Martin, Sydney C. Ludvigson, and Sai Ma, 2019, Capital share risk in u.s. asset pricing, *The Journal of Finance* 74, 1753–1792.

Lettau, Martin, Matteo Maggiori, and Michael Weber, 2014, Conditional risk premia in currency markets and other asset classes, *Journal of Financial Economics* 114, 197 – 225.

Lucas, Robert, 1978, Asset prices in an exchange economy, *Econometrica* 46, 1429–45.

Maggiori, Matteo, 2017, Financial intermediation, international risk sharing, and reserve currencies, *American Economic Review* 107, 3038–71.

Martin, Ian, 2011, The forward premium puzzle in a two-country world, Working Paper 17564, National Bureau of Economic Research.

Martin, Ian, 2013, The lucas orchard, *Econometrica* 81, 55–111.

Mertens, Thomas M., and Kenneth L. Judd, 2018, Solving an incomplete markets model with a large cross-section of agents, *Journal of Economic Dynamics and Control* 91, 349 – 368, Special Issue in Honour of Prof. Carl Chiarella.

Merton, Robert, 1973, An intertemporal capital asset pricing model, *Econometrica* 41, 867–87.

Miranda-Agrippino, Silvia, and Hélène Rey, 2020, U.S. Monetary Policy and the Global Financial Cycle, *The Review of Economic Studies* rdaa019.

Nurkse, Ragnar, 1944, *International Currency Experience: Lessons of the Inter-war Period*, Series of League of Nations Publications. 2. Economic and financial. 1944. 2 (UN).

Obstfeld, Maurice, 2004, External adjustment, *Review of World Economics (Weltwirtschaftliches Archiv)* 140, 541–568.

Obstfeld, Maurice, 2007, International risk sharing and the costs of trade, *The Ohlin Lectures* Stockholm School of Economics.

Obstfeld, Maurice, and Kenneth Rogoff, 2001, The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?, in *NBER Macroeconomics Annual 2000, Volume 15*, NBER Chapters, 339–412 (NBER, Inc).

Parra-Alvarez, Juan, 2018, A comparison of numerical methods for the solution of continuous-time dsge models, *Macroeconomic Dynamics* 22, 1555–1583.

Pavlova, Anna, and Roberto Rigobon, 2007, Asset Prices and Exchange Rates, *The Review of Financial Studies* 20, 1139–1180.

Pavlova, Anna, and Roberto Rigobon, 2008, The Role of Portfolio Constraints in the International Propagation of Shocks, *The Review of Economic Studies* 75, 1215–1256.

Pavlova, Anna, and Roberto Rigobon, 2010, An asset-pricing view of external adjustment, *Journal of International Economics* 80, 144 – 156, Special Issue: JIE Special Issue on International Macro-Finance.

Piketty, Thomas, and Gabriel Zucman, 2014, Capital is Back: Wealth-Income Ratios in Rich Countries 1700–2010*, *The Quarterly Journal of Economics* 129, 1255–1310.

Polemarchakis, Heraklis M, 1988, Portfolio choice, exchange rates, and indeterminacy, *Journal of Economic Theory* 46, 414–421.

Rabitsch, Katrin, Serhiy Stepanchuk, and Viktor Tsyrennikov, 2015, International portfolios: A comparison of solution methods, *Journal of International Economics* 97, 404 – 422.

Rey, Hélène, 2013, Dilemma not trilemma: the global cycle and monetary policy independence, *Proceedings - Economic Policy Symposium - Jackson Hole* 1–2.

Sauzet, Maxime, 2022a, Two investors, two trees, two goods, Working paper.

Sauzet, Maxime, 2022b, Projection methods via neural networks for continuous-time models, Working paper.

Sauzet, Maxime, 2022c, Global asset manager and the global financial cycle, Work in progress.

Sauzet, Maxime, and Olivier David Zerbib, 2022, When green investors are green consumers, Working paper.

Schwert, G. William, 1989, Why does stock market volatility change over time?, *The Journal of Finance* 44, 1115–1153.

Stathopoulos, Andreas, 2017, Asset Prices and Risk Sharing in Open Economies, *The Review of Financial Studies* 30, 363–415.

Stepanchuk, Serhiy, and Viktor Tsyrennikov, 2015, Portfolio and welfare consequences of debt market dominance, *Journal of Monetary Economics* 74, 89 – 101.

Stewen, Iryna, 2020, Is Real Exchange Rate Hedging Motive Still Important in Determining Equity Home Bias?, Working paper.

Stulz, René M., 1983, The demand for foreign bonds, *Journal of International Economics* 15, 225 – 238.

Tille, Cédric, and Eric van Wincoop, 2010, International capital flows, *Journal of International Economics* 80, 157–175.

Uppal, Raman, 1993, A general equilibrium model of international portfolio choice, *The Journal of Finance* 48, 529–553.

Wang, Jiang, 1996, The term structure of interest rates in a pure exchange economy with heterogeneous investors, *Journal of Financial Economics* 41, 75 – 110.

Weinbaum, David, 2009, Investor heterogeneity, asset pricing and volatility dynamics, *Journal of Economic Dynamics and Control* 33, 1379 – 1397.

Zapatero, Fernando, 1995, Equilibrium asset prices and exchange rates, *Journal of Economic Dynamics and Control* 19, 787–811.

Online Appendix

A Additional Equations and Results

The model draws on Sauzet (2022a), who provides additional details and further theoretical results.

A.1 Drift and diffusion terms for any variable

Remark A.1. *By Itô's Lemma, the geometric drift and diffusion term for any function $g_t = g(X_t)$ are given by:*

$$\frac{dg_t}{g_t} = \frac{dg(X_t)}{g(X_t)} \equiv \mu_{g,t}dt + \sigma_{g,t}^T d\vec{Z}_t \tag{A.1}$$

where:

$$\mu_{g,t} = \frac{g_{x,t}}{g_t}x_t\mu_{x,t} + \frac{g_{y,t}}{g_t}y_t\mu_{y,t} + \frac{1}{2}\frac{g_{xx,t}}{g_t}x_t^2\sigma_{x,t}^T\sigma_{x,t} + \frac{1}{2}\frac{g_{yy,t}}{g_t}y_t^2\sigma_{y,t}^T\sigma_{y,t} + \frac{g_{xy,t}}{g_t}x_ty_t\sigma_{x,t}^T\sigma_{y,t} \tag{A.2}$$

$$\sigma_{g,t} = \frac{g_{x,t}}{g_t}x_t\sigma_{x,t} + \frac{g_{y,t}}{g_t}y_t\sigma_{y,t} \tag{A.3}$$

This result is used repeatedly throughout the paper.

As a point of notation, recall that for any function g, g_t simply denotes $g(X_t)$, not the time-derivative of g (which is zero because the model is stationary due to infinite horizon). $g_{x,t}, g_{y,t}, g_{xx,t}, g_{yy,t}, g_{xy,t}$ denote the partial derivatives of $g(X_t)$.

A.2 Returns, and risk premia

The (geometric) drifts and diffusion terms for asset returns are obtained from Itô's Lemma and are as follows

$$
\begin{aligned}
dR_t &= \mu_{R,t}dt + \sigma_{R,t}^T d\vec{Z}_t \qquad\qquad (A.4)\\
&\equiv \left(F_t + \mu_{p,t} + \mu_Y + \sigma_{p,t}^T\sigma_Y - \mu_{F,t} + \sigma_{F,t}^T\sigma_{F,t} - (\sigma_{p,t}+\sigma_Y)^T\sigma_{F,t}\right)dt\\
&+ (\sigma_{p,t}+\sigma_Y-\sigma_{F,t})^T d\vec{Z}_t\\
dR_t^* &= \mu_{R*,t}dt + \sigma_{R*,t}^T d\vec{Z}_t \qquad\qquad (A.5)\\
&\equiv \left(F_t^* + \mu_{p*,t} + \mu_Y^* + \sigma_{p*,t}^T\sigma_Y^* - \mu_{F*,t} + \sigma_{F*,t}^T\sigma_{F*,t} - (\sigma_{p*,t}+\sigma_Y^*)^T\sigma_{F*,t}\right)dt\\
&+ (\sigma_{p*,t}+\sigma_Y^*-\sigma_{F*,t})^T d\vec{Z}_t
\end{aligned}
$$

where $\mu_{p,t}$, $\mu_{p*,t}$, $\mu_{F,t}$, $\mu_{F*,t}$, $\sigma_{p,t}$, $\sigma_{p*,t}$, $\sigma_{F,t}$, $\sigma_{F*,t}$ are obtained using Remark A.1 above.

Proposition A.1. *The expected risk premia on the equity assets are given by*

$$
\begin{aligned}
\mu_{R,t} - r_t &= \gamma_t\sigma_{R,t}^T\{z_t\sigma_{R,t} + (1-z_t)\sigma_{R*,t}\} \qquad\qquad (A.6)\\
&- \gamma_t\sigma_{R,t}^T\sigma_{x,t}x_t\left\{x_t\left(\frac{1}{\gamma}\right)\left(\frac{1-\gamma}{1-\psi}\right)\frac{J_{x,t}}{J_t} + (1-x_t)\left(\frac{1}{\gamma^*}\right)\left(\frac{1-\gamma^*}{1-\psi^*}\right)\frac{J_{x,t}^*}{J_t^*}\right\}\\
&- \gamma_t\sigma_{R,t}^T\sigma_{y,t}y_t\left\{x_t\left(\frac{1}{\gamma}\right)\left(\frac{1-\gamma}{1-\psi}\right)\frac{J_{y,t}}{J_t} + (1-x_t)\left(\frac{1}{\gamma^*}\right)\left(\frac{1-\gamma^*}{1-\psi^*}\right)\frac{J_{y,t}^*}{J_t^*}\right\}\\
&+ \gamma_t\left(\frac{1-x_t}{\gamma^*}\right)\tau^* F_t\\
\mu_{R*,t} - r_t &= \gamma_t\sigma_{R*,t}^T\{z_t\sigma_{R,t} + (1-z_t)\sigma_{R*,t}\} \qquad\qquad (A.7)\\
&- \gamma_t\sigma_{R*,t}^T\sigma_{x,t}x_t\left\{x_t\left(\frac{1}{\gamma}\right)\left(\frac{1-\gamma}{1-\psi}\right)\frac{J_{x,t}}{J_t} + (1-x_t)\left(\frac{1}{\gamma^*}\right)\left(\frac{1-\gamma^*}{1-\psi^*}\right)\frac{J_{x,t}^*}{J_t^*}\right\}\\
&- \gamma_t\sigma_{R*,t}^T\sigma_{y,t}y_t\left\{x_t\left(\frac{1}{\gamma}\right)\left(\frac{1-\gamma}{1-\psi}\right)\frac{J_{y,t}}{J_t} + (1-x_t)\left(\frac{1}{\gamma^*}\right)\left(\frac{1-\gamma^*}{1-\psi^*}\right)\frac{J_{y,t}^*}{J_t^*}\right\}\\
&+ \gamma_t\left(\frac{x_t}{\gamma}\right)\tau F_t^*
\end{aligned}
$$

where $\gamma_t \equiv \left(\frac{x_t}{\gamma} + \frac{1-x_t}{\gamma^*}\right)^{-1}$ *is the wealth-weighted global risk aversion.*

A.3 Foreign investor problem

The representative consumer of the foreign country solves:

$$V_t^* = \max_{\{C_{h,u}^*, C_{f,u}^*, w_{h,u}^*, w_{f,u}^*\}_{u=t}^{\infty}} \mathbb{E}_t \left[\int_t^{\infty} f\left(C_u^*, V_u^*\right) du \right] \tag{A.8}$$

$$f(C^*, V^*) \equiv \left(\frac{1-\gamma^*}{1-1/\psi^*} \right) V^* \left[\left(\frac{C^*}{\left[(1-\gamma^*)V^*\right]^{1/(1-\gamma^*)}} \right)^{1-1/\psi^*} - \rho^* \right] \tag{A.9}$$

subject to:

$$\frac{dW_t^*}{W_t^*} = \left(r_t + w_{h,t}^* \left(\mu_{R,t} - r_t\right) + w_{f,t}^* \left(\mu_{R^*,t} - r_t\right) - P_t^* c_t^* \right) dt$$
$$+ \left(w_{h,t}^* \sigma_{R,t} + w_{f,t}^* \sigma_{R^*,t} \right)^T d\vec{z}_t \tag{A.10}$$

$$C_t^* = \left[(1-\alpha)^{\frac{1}{\theta}} C_{h,t}^{*\frac{\theta-1}{\theta}} + \alpha^{\frac{1}{\theta}} C_{f,t}^{*\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{A.11}$$

All parameters can differ from those of the domestic investor. Cf. the main text for a discussion. To complete the definition of the optimization problem, the investor is subject to a standard transversality condition, and W_0^* is given. Note also that $W_t^* \geqslant 0$.

A.4 Labor income

Another aspect that the literature emphasized can have a large impact on portfolios is labor income. I leave it aside in this paper, but it can also be accommodated in the framework, e.g. as a constant share of the output of each tree in the spirit of Baxter and Jermann (1997). Specifically, a share δ (δ^*) of the output of each tree is paid as labor income, while the remainder $1-\delta$ ($1-\delta^*$) is paid as dividends. In turn, this means that the dividend yields of the equity assets become $F_t \equiv (1-\delta)p_t Y_t / Q_t$ and $F_t^* \equiv (1-\delta^*)p_t^* Y_t^* / Q_t^*$, while the budget constraints have an additional term, $\delta F_t z_t / ((1-\delta)x_t)$ and $\delta^* F_t^* (1-z_t)/((1-\delta^*)(1-x_t))$, for the domestic and foreign investor respectively.[38] Sauzet (2022a) provide additional details.

[38] z_t, the ratio of the home equity price to world wealth, is updated accordingly: $z_t \equiv Q_t/(Q_t + Q_t^*) = \left(1 + ((1-\delta^*)/(1-\delta)) \left(F_t/F_t^*\right) q_t \left(1-y_t\right)/y_t\right)^{-1}$.

Even though such a form of labor income has a large impact on portfolios, additional elements would be required for it to impact other economic outcomes such as asset prices. This could take the form of portfolio constraints (e.g. Chabakauri, 2013), time-varying output shares δ_t (e.g. Coeurdacier and Gourinchas, 2016), or idiosyncratic labor income risk (e.g. Kaplan et al., 2018). Again, those could naturally give rise to incomplete markets, and to additional hedging motives. I leave those avenues for future research.

A.5 Equilibrium

The definition of the equilibrium is standard.

Definition 1. *A competitive equilibrium is a set of aggregate stochastic processes adapted to the filtration generated by $\vec{Z}$: the price of the equity asset (Q_t, Q_t^*), and the interest rate (r_t), together with a set of individual stochastic processes for each investor: consumption of each good $(C_{h,t}, C_{f,t}, C_{h,t}^*, C_{f,t}^*)$, wealth (W_t, W_t^*), and portfolio shares $(w_{h,t}, w_{f,t}, w_{h,t}^*, w_{f,t}^*)$, such that, given the output of the two endowment trees (Y_t, Y_t^*):*

1. *Given the aggregate stochastic processes, individual choices solve the investor optimization problem given above.*
2. *Markets clear.*
   (a) *Good markets:*

$$\begin{aligned} C_{h,t} + C_{h,t} &= Y_t \\ C_{f,t} + C_{f,t}^* &= Y_t^* \end{aligned} \tag{A.12}$$

   (b) *Equity markets:*

$$\begin{aligned} w_{h,t}W_t + w_{h,t}^*W_t^* &= Q_t \\ w_{f,t}W_t + w_{f,t}^*W_t^* &= Q_t^* \end{aligned} \tag{A.13}$$

Most importantly, as shown in Section 2.3 of the main text, the equilibrium can be recast as a stationary recursive Markovian equilibrium in which all variables of interest are expressed as a function of a pair of state variables $X_t \equiv (x_t, y_t)'$, whose dynamics are also solely a function of X_t. x_t is the wealth share of the domestic investor, and y_t is the relative supply of the domestic good.

A.6 Marginal values of wealth

Proposition A.2. *J_t satisfies the Hamilton-Jacobi-Bellman equation:*

$$0 = \left(\frac{1}{\psi - 1}\right) P_t^{1-\psi} J_t - \left(\frac{1}{1 - 1/\psi}\right) \rho + r_t + \frac{\gamma}{2} \left(w_{h,t}\sigma_{R,t} + w_{f,t}\sigma_{R^*,t}\right) \tag{A.14}$$

$$+ \left(\frac{1}{1-\psi}\right) \mu_{J,t} + \frac{1}{2}\left(\frac{1}{1-\psi}\right)\left(\frac{\psi - \gamma}{1-\psi}\right) \sigma_{J,t}^T \sigma_{J,t}$$

$$+ \left(\frac{\delta}{1-\delta}\right) F_t \left(\frac{z_t}{x_t}\right) + w_{h,t}^* \left(\frac{1 - x_t}{x_t}\right) \tau^* F_t \tag{A.15}$$

where $\mu_{J,t}, \sigma_{J,t}$ are the geometric drift and diffusion terms of J_t obtained as in Remark A.1:

$$\frac{dJ_t}{J_t} \equiv \mu_{J,t} dt + \sigma_{J,t}^T d\vec{Z}_t \tag{A.16}$$

J_t^* satisfies a similar Hamilton-Jacobi-Bellman equation.

A.7 Evolutions of the state variables

Due to the Markovian nature of the equilibrium, the laws of motion of the state variables underlie the dynamics of the economy. They are summarized in Proposition A.3.

Proposition A.3. *Laws of motion for the wealth share x_t, and relative supply y_t:*

$$\frac{dx_t}{x_t} \equiv \mu_{x,t}dt + \sigma_{x,t}^T d\vec{Z}_t \tag{A.17}$$
$$\frac{dy_t}{y_t} \equiv \mu_{y,t}dt + \sigma_{y,t}^T d\vec{Z}_t$$

where:

$$\begin{aligned}
\mu_{x,t} &= (w_{h,t} - z_t)(\mu_{R,t} - r_t) + (w_{f,t} - (1 - z_t))(\mu_{R*,t} - r_t) \\
&\quad + (F_t z_t + (1 - z_t)F_t^*) - P_t c_t + \left(\frac{\delta}{1-\delta}\right) F_t \left(\frac{z_t}{x_t}\right) + \tau^* F_t \left(\frac{z_t}{x_t} - w_{h,t}\right) - \tau F_t^* w_{f,t} \\
&\quad - ((w_{h,t} - z_t)\sigma_{R,t} + (w_{f,t} - (1 - z_t))\sigma_{R*,t})^T (z_t\sigma_{R,t} + (1 - z_t)\sigma_{R*,t}) \\
\sigma_{x,t} &= ((w_{h,t} - z_t)\sigma_{R,t} + (w_{f,t} - (1 - z_t))\sigma_{R*,t}) \\
\mu_{y,t} &= (1 - y_t)(\mu_Y - \mu_{Y*}) - (1 - y_t)(\sigma_Y - \sigma_{Y*})^T (y_t\sigma_Y + (1 - y_t)\sigma_{Y*}) \\
\sigma_{y,t} &= (1 - y_t)(\sigma_Y - \sigma_{Y*})
\end{aligned}$$

A.8 Consumptions, goods prices

Proposition A.4. *The consumption of the home investor is given by:*

$$c_t \equiv \frac{C_t}{W_t} = P_t^{-\psi} J_t \tag{A.18}$$
$$c_{h,t} = \alpha \left(\frac{p_t}{P_t}\right)^{-\theta} c_t$$
$$c_{f,t} = (1-\alpha) \left(\frac{p_t^*}{P_t}\right)^{-\theta} c_t$$
$$P_t = \left[\alpha p_t^{1-\theta} + (1-\alpha) p_t^{*1-\theta}\right]^{1/(1-\theta)}$$

The consumption of the foreign investor is given by:

$$c_t^* \equiv \frac{C_t^*}{W_t^*} = P_t^{*-\psi^*} J_t^* \tag{A.19}$$

$$c_{h,t}^* = (1-\alpha)\left(\frac{p_t}{P_t^*}\right)^{-\theta} c_t^* \tag{A.20}$$

$$c_{f,t}^* = \alpha\left(\frac{p_t^*}{P_t^*}\right)^{-\theta} c_t^* \tag{A.21}$$

$$P_t^* = \left[(1-\alpha)p_t^{1-\theta} + \alpha p_t^{*1-\theta}\right]^{1/(1-\theta)} \tag{A.22}$$

Proposition A.5. *The terms of trade, $q_t = q(X_t)$, solves the following non-linear equation:*

$$q_t = S_t^{1/\theta}\left(\frac{y_t}{1-y_t}\right)^{1/\theta} \tag{A.23}$$

where:

$$S_t = \frac{(1-\alpha)P_t^{\theta-\psi} J_t x_t + \alpha P_t^{*\theta-\psi^*} J_t^*(1-x_t)}{\alpha J_t x_t P_t^{\theta-\psi} + (1-\alpha)P_t^{*\theta-\psi^*} J_t^*(1-x_t)} \tag{A.24}$$

Using the definition of the numéraire, prices follow:

$$p_t = \left(a + (1-a)q_t^{1-\theta}\right)^{1/(\theta-1)} \tag{A.25}$$

$$p_t^* = p_t q_t = \left(a q_t^{\theta-1} + (1-a)\right)^{1/(\theta-1)} \tag{A.26}$$

$$P_t = \left[\alpha p_t^{1-\theta} + (1-\alpha)p_t^{*1-\theta}\right]^{1/(1-\theta)} \tag{A.27}$$

$$P_t^* = \left[(1-\alpha)p_t^{1-\theta} + \alpha p_t^{*1-\theta}\right]^{1/(1-\theta)} \tag{A.28}$$

$$\mathcal{E}_t = P_t^*/P_t \tag{A.29}$$

Under the assumption that there are no wedges on foreign dividends so that risk sharing is perfect, the stochastic discount factors of both investors are perfectly correlated and we can derive in this environment a generalized version of the Backus-Smith condition (Backus and Smith, 1993; Kollmann, 1995), provided that preferences are symmetric. This condition, shown in Proposition A.6, emphasizes that the real exchange rate is not only determined by relative consumption, as in the usual CRRA case, but also depends on relative wealth and the marginal values of wealth of international investors. However, one deviates from this condition as soon as one introduces asymmetries and frictions in financial markets, as is partly the goal in this paper.

Proposition A.6 (Generalized Backus-Smith condition)**.** *Under symmetric recursive preferences and perfect risk sharing*

$$\mathcal{E}_t = \phi^{\frac{1}{\gamma\psi}} \exp\left\{\int_0^t \frac{1}{\gamma\psi}\left(\Theta_1(P_u^{1-\psi}J_u - P_u^{*1-\psi}J_u^*)du\right)\right\}\left(\frac{C_t^*}{C_t}\right)^{-1/\psi}\left(\frac{J_t^*}{J_t}\right)^{-\frac{1-1/(\gamma\psi)}{1-\psi}} \tag{A.30}$$

where $\Theta_1 \equiv -\left(\frac{\gamma-1/\psi}{1-1/\psi}\right)$*, and* ϕ *is the relative Pareto weight of the two investors.*

B Additional Figures

B.1 Economic setup

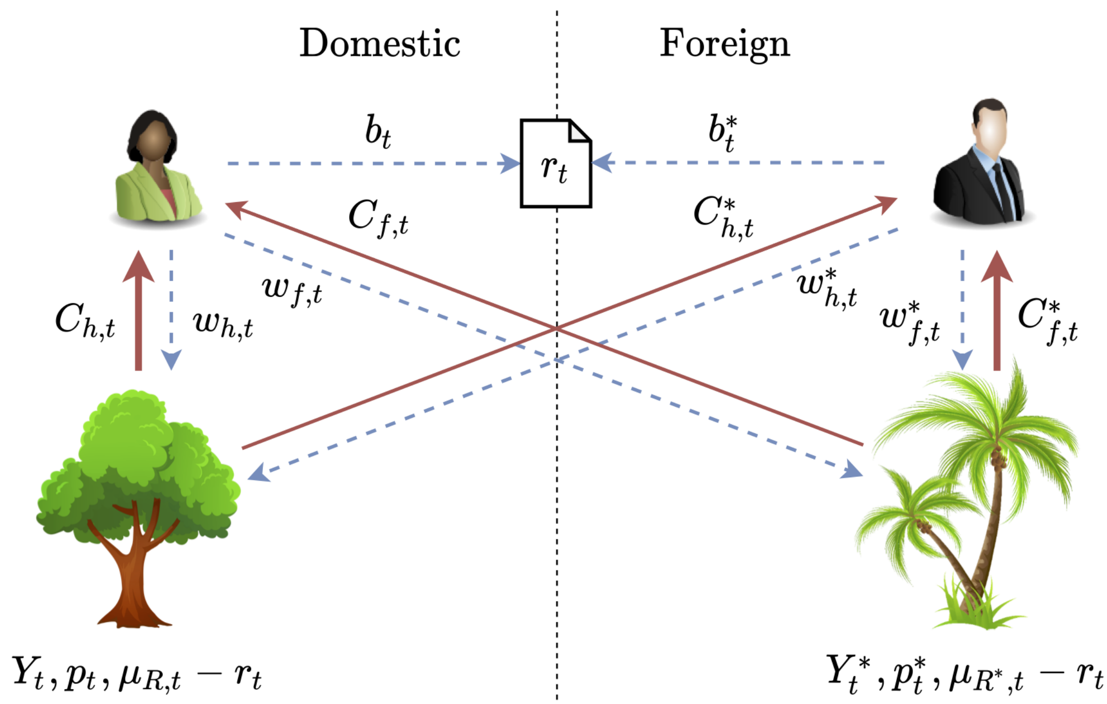


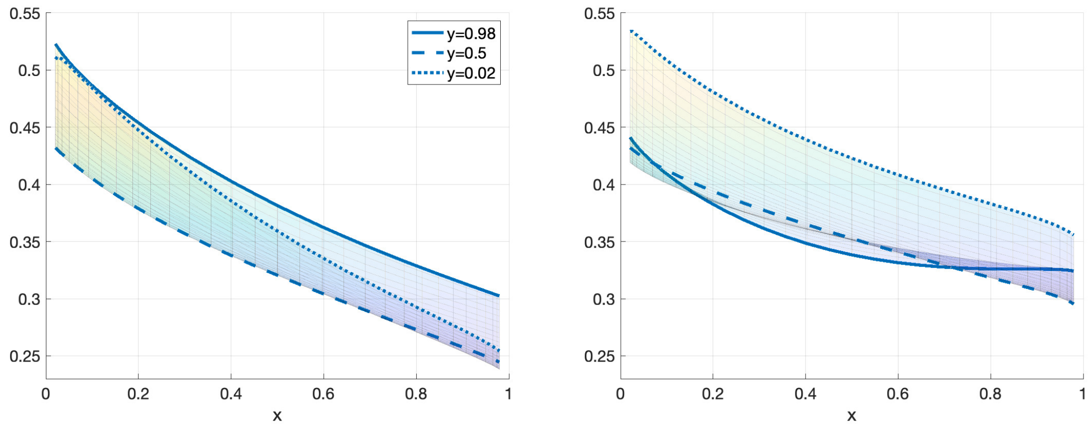
Figure B.1: Baseline international economy

Notes: Back to main text: Section 2.

B.2 Asset returns

Figure B.2: Sharpe ratios on equity assets

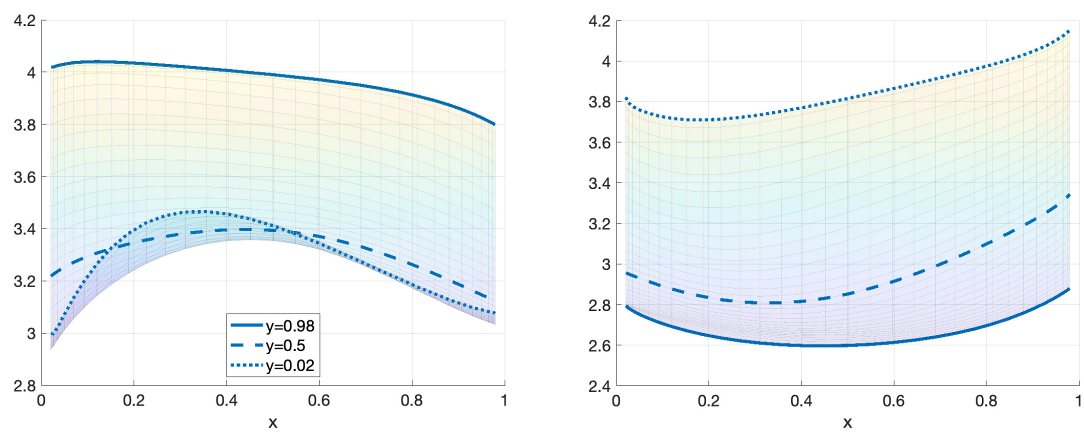
(a) Sharpe ratio on domestic asset (SR_t) (b) Sharpe ratio on foreign asset (SR_t^*)



Notes: Based on the baseline calibration of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

Figure B.3: Instantaneous volatilities of equity asset returns

(a) Domestic returns volatility (%) $\text{vol}_t(dR_t) = \left(\sigma_{R,t}^T \sigma_{R,t}\right)^{1/2}$ (a) Foreign returns volatility (%) $\text{vol}_t(dR_t^*) = \left(\sigma_{R*,t}^T \sigma_{R*,t}\right)^{1/2}$



Notes: Based on the baseline calibration of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

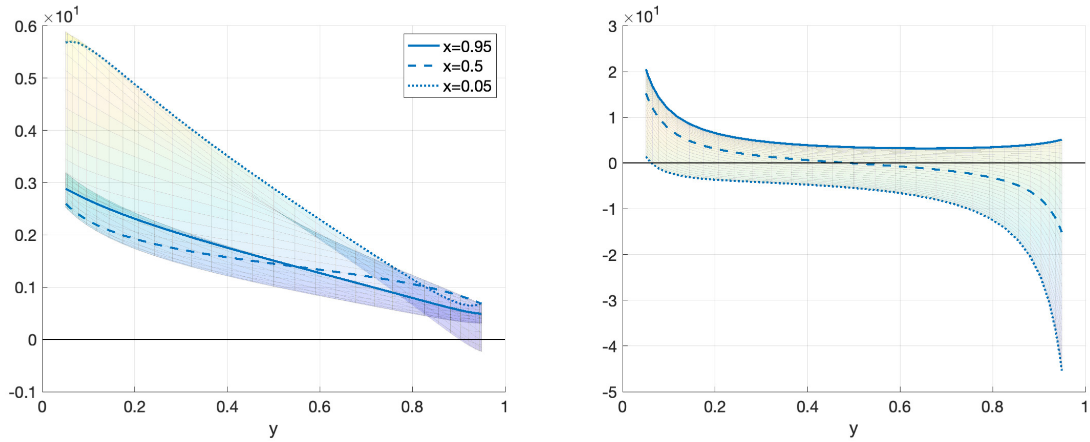
Figure B.4: Prices of risk

(a) Price of x_t risk

$x_t \left(\frac{1}{\gamma}\right)\left(\frac{1-\gamma}{1-\psi}\right)\frac{J_{x,t}}{J_t} + (1-x_t)\left(\frac{1}{\gamma^*}\right)\left(\frac{1-\gamma^*}{1-\psi^*}\right)\frac{J^*_{x,t}}{J^*_t}$

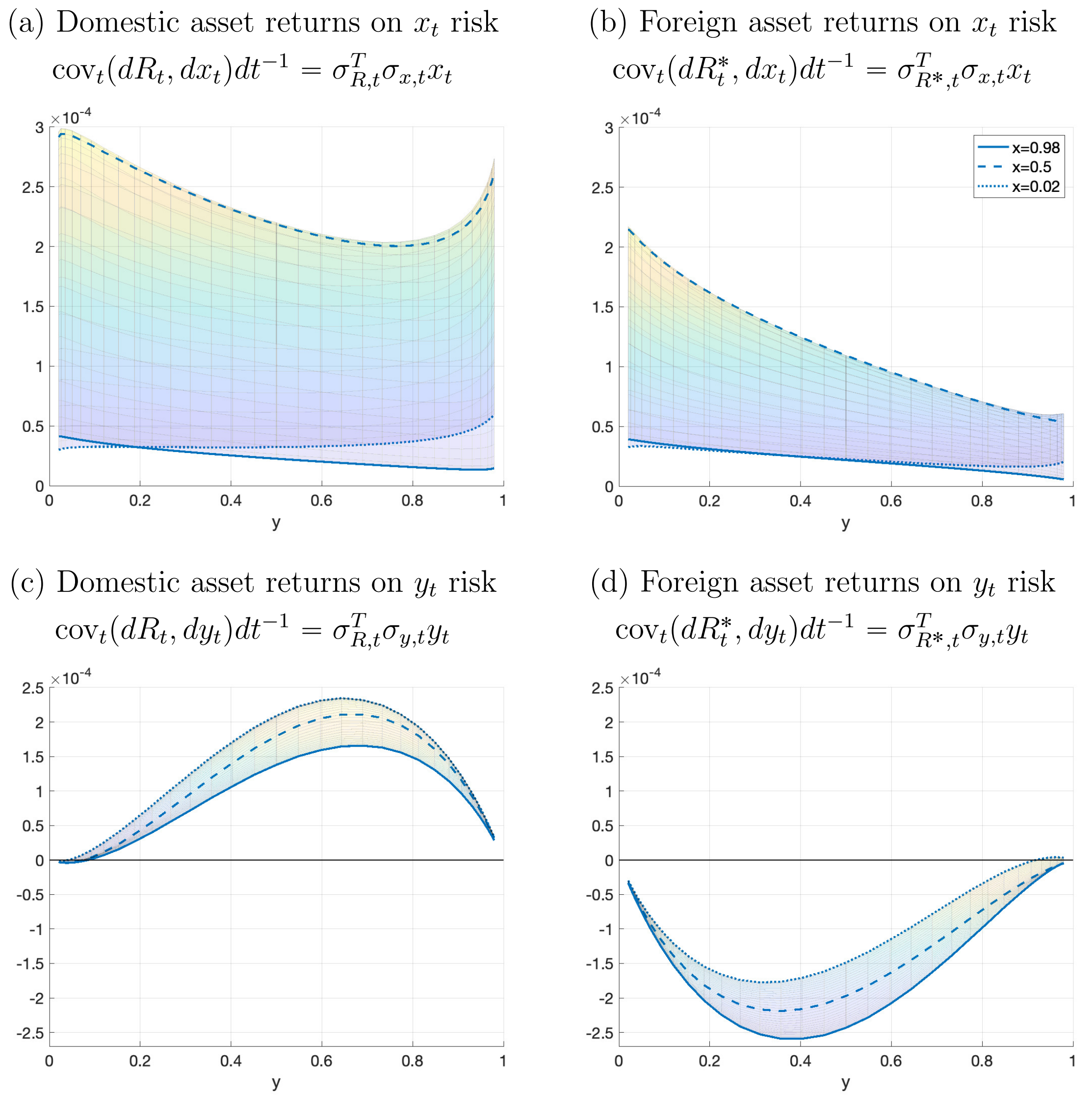
(b) Price of y_t risk

$x_t \left(\frac{1}{\gamma}\right)\left(\frac{1-\gamma}{1-\psi}\right)\frac{J_{y,t}}{J_t} + (1-x_t)\left(\frac{1}{\gamma^*}\right)\left(\frac{1-\gamma^*}{1-\psi^*}\right)\frac{J^*_{y,t}}{J^*_t}$



Notes: Based on the baseline calibration of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

Figure B.5: Quantities of risk

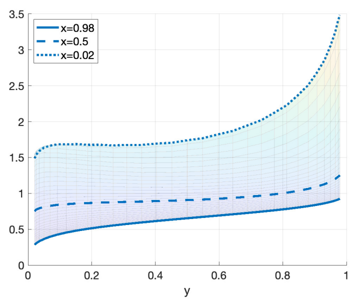


Notes: Based on the baseline calibration of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

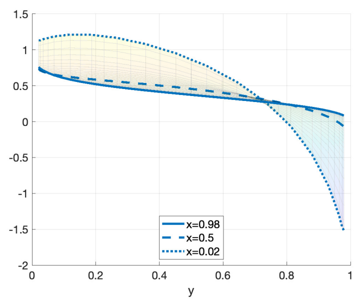
B.3 Portfolios

Figure B.6: Portfolio weights

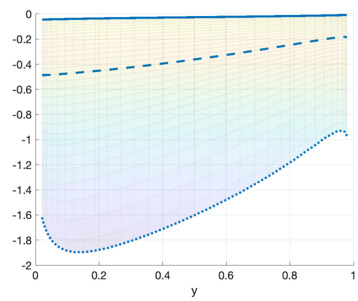
(a) Domestic weight in domestic asset ($w_{h,t}$)



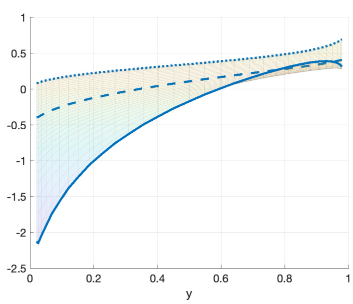
(b) Domestic weight in foreign asset ($w_{f,t}$)



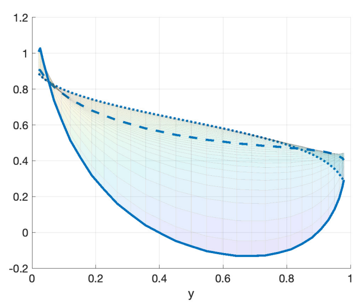
(c) Domestic weight in bond ($b_t = 1 - w_{h,t} - w_{f,t}$)



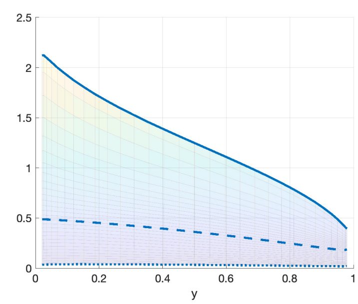
(d) Foreign weight in domestic asset ($w^*_{h,t}$)



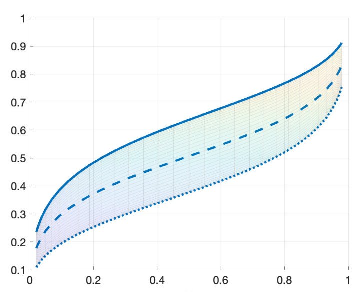
(e) Foreign weight in foreign asset ($w^*_{f,t}$)



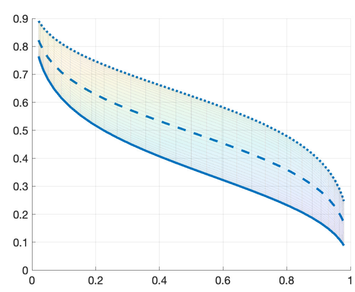
(f) Foreign weight in bond ($b^*_t = 1 - w^*_{h,t} - w^*_{f,t}$)



(f) Market weight in domestic asset (z_t)



(g) Market weight in foreign asset ($1 - z_t$)



Notes: Based on the baseline calibration of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals. The market portfolio weight is $z_t \equiv Q_t/(Q_t + Q^*_t)$, where Q_t, Q^*_t are the equity prices of the domestic and foreign equity assets, respectively.

C Computation of the Equilibrium

All variables of interest can be characterized as a function of the state variables, $X_t = (x_t, y_t)'$, and a set of unknown functions $\mathcal{G} \equiv \{J_t,\ J_t^*,\ F_t,\ F_t^*,\ q_t,\ w_{h,t},\ w_{f,t}\}$.[39] Due to the stationary recursive Markovian structure of the equilibrium, the unknown functions are themselves solely functions of X_t, and are determined by a system of coupled algebraic and second-order partial differential equations. This theoretical characterization is the focus of Section 3, and additional results and details (including on the numerical method) are provided in Sauzet (2022a).

Each of the unknown function $g : [0,1]^2 \to \mathcal{D}^g \in \mathbb{R}$ in $\mathcal{G}$ is approximated using projection methods based on Chebyshev polynomials and orthogonal collocation. This approach has a number of advantages.

First, projection methods are a global solution approach. This is in sharp contrast to most of the international portfolio choice literature that focuses on local approximations in neighborhoods of specific points.[40] The use of a global solution approach instead makes it possible to trace out economic outcomes throughout the state space, which is particularly relevant in settings in which variables are very state-dependent. This is the case in this economy that features strong heterogeneity, and the global solution method makes it possible to study the dynamics of risk premia, portfolios, and hedging terms. Those dynamics play a central role in the application of Sections 4 and 5.

Second, projection methods are well-suited to contexts with multiple state variables in which other approaches like finite differences become rapidly computationally too costly.[41]

[39] J_t, J_t^* capture (an increasing monotonic transformation of) the marginal values of wealth of each investor. In addition, as a point of notation, for any function g, g_t simply denotes $g(X_t)$, not the time-derivative of g (which is zero because the model is stationary due to the infinite horizon).

[40] A typical point around which the local approximation is performed is the deterministic or risky steady state if it is well-defined, or the symmetric point in the middle of the state space ($X_t = (1/2, 1/2)$ in this context). A notable exception is Rabitsch et al. (2015), who use a global method. However, their framework is cast in discrete time so that the authors do not discuss the underlying drivers of portfolios, in particular hedging demands, the conditional (time-varying) moments of asset returns, as well as the conditional (time-varying) state variable dynamics. Under a set of assumptions, local methods could be used to study an economy further in the state space, cf. for instance Mertens and Judd (2018). However, such methods remain difficult to use in an international portfolio context due to the portfolio indeterminacy that arises in the corresponding deterministic economy.

[41] The method currently developed in Hansen et al. (2018) could help from that perspective.

More generally, a global solution will prove crucial for many applications and extensions (Section D.3) because those are likely to give rise to strong non-linearities. For settings with additional state variables such as those however, higher-dimensional methods could become necessary. One such method consists in extending the concept of projection approaches, but to replace the Chebyshev polynomials in the approximation by neural networks, which are designed specifically to handle high-dimensional contexts. Those "projection methods via neural networks" for continuous-time models are developed in Sauzet (2022b), and have the additional benefit that they can naturally handle more extreme cases of non-linearities such as kinks.

D The International Financial System: Additional Results

D.1 Additional dynamic aspects

The framework also makes it possible to make sense of additional aspects about the process of international financial adjustment (Fact 8), and the countercyclicality of asset return dynamics (Fact 9). The dynamics of portfolio weights is discussed together with empirical evidence in Section 5.1.

International Financial Adjustment As the global shock hits, not only does the U.S. wealth share decreases as discussed previously (Fact 1), but so does the net foreign asset position of the United States. This is shown in Panel (b) of Figure 5, and consistent with findings in Gourinchas et al. (2017). In this context, this arises because the U.S. investor borrows an increasing share of her wealth (Panel (a)), consistent with the decline in the U.S. net foreign position in safe asset observed empirically. The sharp increase in risk premia discussed above is also in line with the large role for valuation effects in the external adjustment process discussed in Gourinchas and Rey (2007a). These higher returns in expectation ease part of the burden of adjustment for the center country. Taken together, those patterns can replicate the predictability relationship between net foreign asset position and expected returns documented in Gourinchas and Rey (2007a) and Gourinchas et al. (2019): a deterioration in

NFA_t is associated with higher expected returns in the short to medium-run, but the relationship reverses at longer horizons as the net foreign asset position improves while expected returns get back down to their average level.

Countercyclicality of asset return dynamics The sharp increase in expected excess returns in times of crisis (Figure 7) is not only consistent with the Global Financial Cycle (Fact 6) but is also in line with the countercyclical risk premia that have been documented empirically in a wide variety of asset pricing contexts, such as e.g. Lettau and Ludvigson (2010), Fama and French (1989), Ferson and Harvey (1991), Harrison and Zhang (1999), and Campbell and Diebold (2009)). The same is true for Sharpe ratios, which naturally increase in this setting, like in Lettau and Ludvigson (2010), and Harvey (2001).

The countercyclicality is also apparent for second moments. For the type of shocks that we have focused on in this section, the volatility on both assets are mildly countercyclical, as shown in Figure B.3 in Appendix, consistent with empirical observations in a domestic context e.g. in Schwert (1989), Brandt and Kang (2004), and Lettau and Ludvigson (2010). For the foreign asset, this occurs regardless of the U.S. wealth share (x_t) because its volatility naturally increases following the decrease in the U.S. output share (y_t) that also ensues. For the U.S. asset, this occurs provided that x_t is not too small, and y_t is not too large, which are both likely to occur given the empirical values of X_t documented in Fact 1. In addition, the conditional returns correlation across asset is itself countercyclical, as shown in Figure 2, and again consistent with empirical observations.

Those patterns emerge from movements in the wealth share across investors, and relative supply of goods, in times of crisis that are consistent with the empirical patterns of Fact 1, and are due to the combination of general recursive-CES preferences with heterogeneity, asymmetries, and frictions in financial markets.

D.2 Counterfactuals and long-term trends

The economic setup in this paper can also be used to study questions about longer-term trends. For instance, the secular improvement in financial market integration worldwide can be captured as a progressive decrease in financial frictions τ. In the model, such a decrease in τ gives rise to a slow-moving reduction in the home bias in equity holdings, consistent with empirical observations over the last 30 years in Coeurdacier and Rey (2013) (Fact 4), and to a gradual (although mild) decrease in average returns correlation, consistent with long-term evidence in Jordà et al. (2019b) (Fact 10). Asymmetric improvement in financial market integration, i.e. different evolutions in τ and τ^*, could also be explored, and dividend wedges could also be interpreted more literally to analyze the impact of macroprudential policy on international risk premia, portfolios, and risk sharing.

The framework can also be interpreted at a lower frequency to analyze phenomena such as the secular decline in interest rates (Fact 11), and the long-term evolution of risk premia worldwide. For instance, even though there is no measure of U.S. wealth share (x_t) going back prior to 2000, this share is likely to have decreased over time and to continue to do so, as large emerging economies such as China take center stage. The model predicts that this should come with an overall decline in the global riskfree rate, consistent with empirical observations over recent decades. This decline arises as a larger share of world wealth is held by more risk averse investors who prefer safer assets, and is in the spirit of Caballero et al. (2008) and Hall (2016). Integrating additional heterogeneity in EIS ψ, as the framework permits, could further allow to capture differences in propensity to save, and is likely to reinforce this mechanism.

Studying those and additional longer-term phenomena, and performing counterfactual exercises, could benefit from additional extensions of the framework, some of which I briefly discuss now.

D.3 Extensions

The model is also a well-suited building block for many potential applications and extensions.

For instance, the framework can be generalized to $N > 2$ investors, $M > 2$ trees, and $L > 2$ goods. Such an extension can allow to study situations with more than two countries or regions, and could shed light on the transition from one center country to another (Nurkse (1944), Farhi and Maggiori (2018)) or on the interaction between the U.S., China, Europe, and other regions, that occur during various global crises.

Heterogeneity within countries can also be included to tackle topics related to wealth inequality. Those could be coupled with refined labor income processes (e.g. Gârleanu and Panageas (2015)), idiosyncratic labor-income risk (e.g. Kaplan et al. (2018)) or capital risk (e.g. Brunnermeier and Sannikov (2014, 2015)), portfolio and borrowing constraints (e.g. Chabakauri (2013)), and more general market structures (e.g. with stochastic wedges or additional sources of risk).

Among those, particularly promising extensions relate to the introduction in an international setting of the type of financial intermediaries that have been discussed in the recent intermediary asset pricing literature e.g. in Daníelsson et al. (2012), He and Krishnamurthy (2013), Adrian and Shin (2014), or Adrian and Boyarchenko (2015). Those global intermediaries, which are relevant in practice, can be involved in the dealing of foreign currencies, in the spirit of Hau and Rey (2006), Gabaix and Maggiori (2015), and Gourinchas et al. (2022), or can play the role of bankers as in Maggiori (2017) and Jiang et al. (2020). As an illustration, in ongoing work (Sauzet, 2022c), I explore a third possibility: the introduction of a global asset manager.

From the perspective of extensions, solving for the *decentralized* general equilibrium of this economy, like in this paper, will prove valuable: the framework is readily set to tackle a wide range of market structures beyond imperfect risk sharing. In addition, the implementation of those extensions will likely require higher-dimensional (as well as more non-linear) resolution methods as the number of state variable increases. I propose such an approach for continuous-time models in "Projection Methods via Neural Networks" (Sauzet, 2022b).

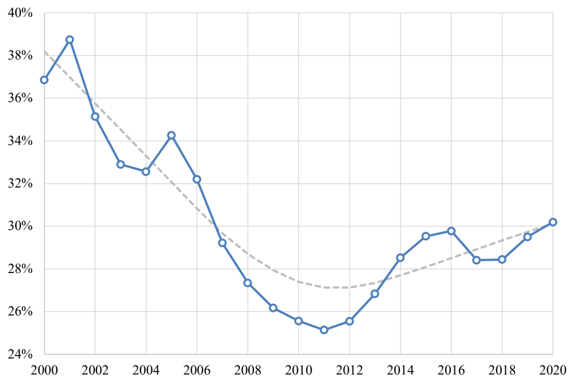
I leave all these promising avenues for ongoing and future research.

E Additional Empirical Results

E.1 Empirical equivalent of X_t

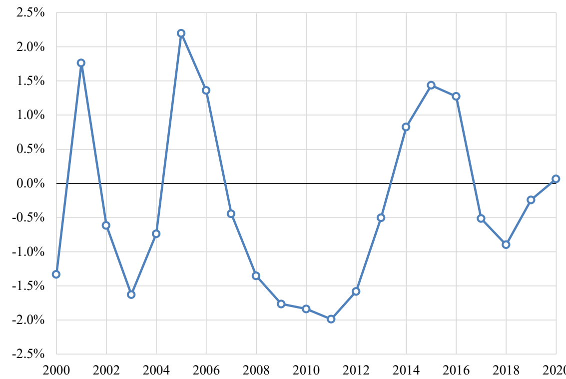
Figure E.1: Empirical pattern of the allocation of wealth

(a) U.S. share of world wealth (x_t)

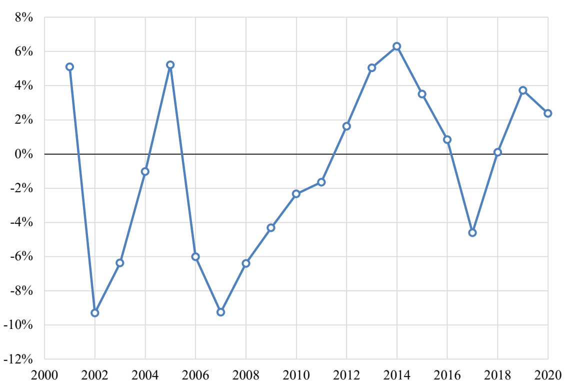


(b) U.S. share of world wealth (x_t)

(deviations from trend)



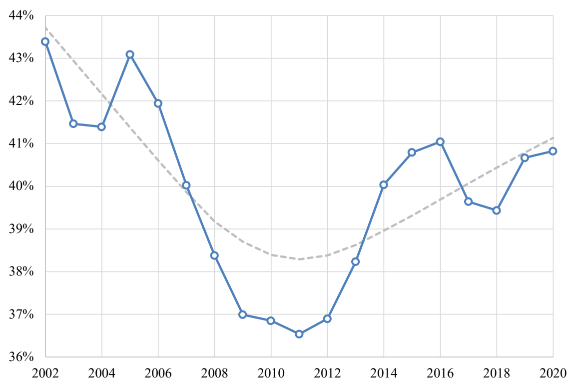
(c) Change in U.S. share of world wealth (dx_t/x_t)



Notes: x_t is measured as the share of world wealth held by U.S. households. Wealth is defined as the marketable value of financial assets plus non-financial assets (principally housing and land) less debts. World wealth includes all countries for which appropriate data is available. Data is from Davies (2008), Davies et al. (2011), and as updated in Crédit Suisse (2021). The dashed gray curve is the trend extracted from a standard STL decomposition. This figure is identical to Figure 4 in the main text.

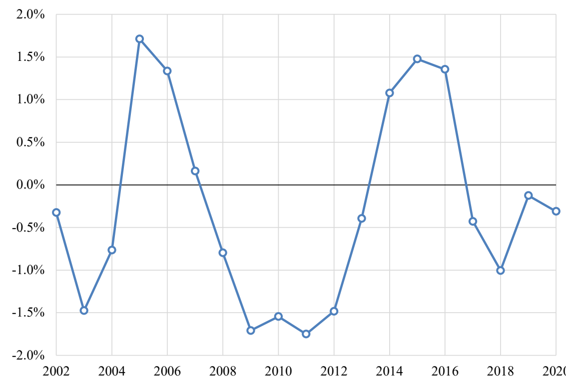
Figure E.2: Empirical pattern of the allocation of wealth (financial wealth only)

(a) U.S. share of world wealth (x_t)

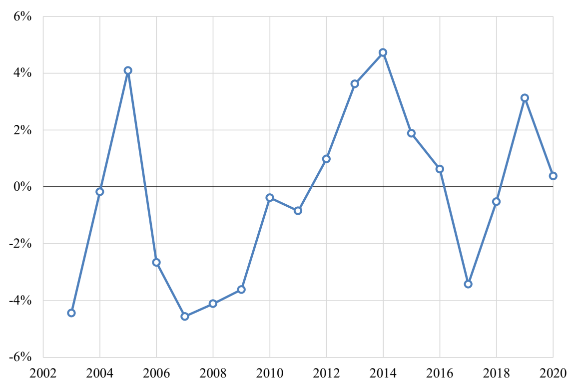


(b) U.S. share of world wealth (x_t)

(deviations from trend)



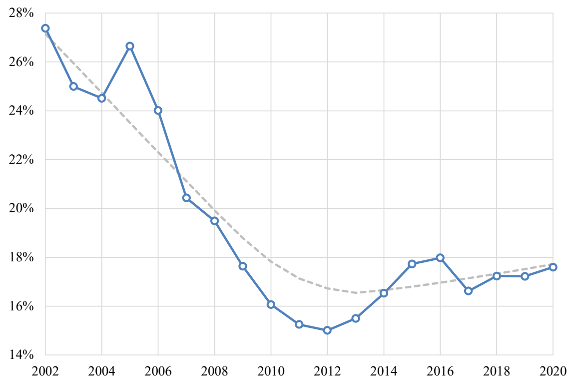
(c) Change in U.S. share of world wealth (dx_t/x_t)



Notes: x_t is measured as the share of world wealth held by U.S. households. Wealth is defined as the marketable value of financial assets less debts. World wealth includes all countries for which appropriate data is available. Data is from Davies (2008), Davies et al. (2011), and as updated in Crédit Suisse (2021). The dashed gray curve is the trend extracted from a standard STL decomposition.

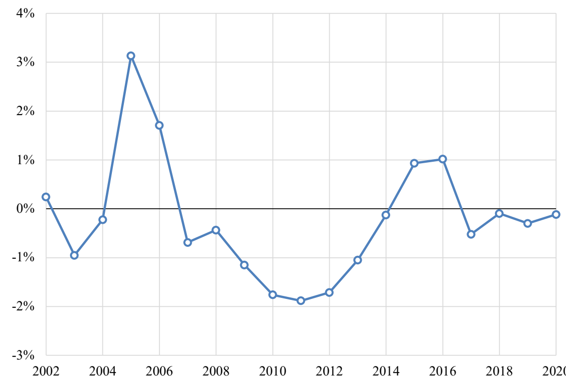
Figure E.3: Empirical pattern of the allocation of wealth (non-financial wealth only)

(a) U.S. share of world wealth (x_t)

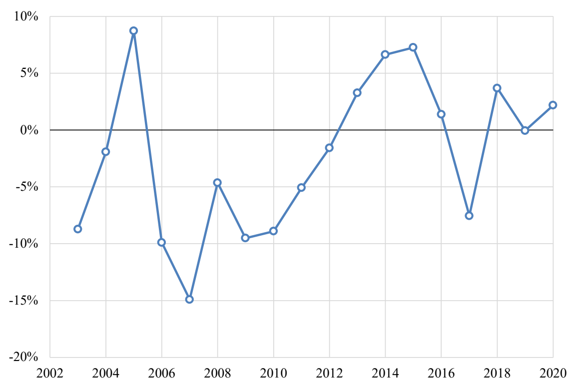


(b) U.S. share of world wealth (x_t)

(deviations from trend)



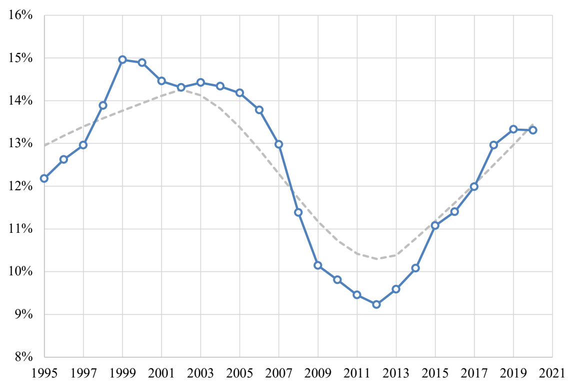
(c) Change in U.S. share of world wealth (dx_t/x_t)



Notes: x_t is measured as the share of world wealth held by U.S. households. Wealth is defined as the marketable value of non-financial assets (principally housing and land) less debts. World wealth includes all countries for which appropriate data is available. Data is from Davies (2008), Davies et al. (2011), and as updated in Crédit Suisse (2021). The dashed gray curve is the trend extracted from a standard STL decomposition.

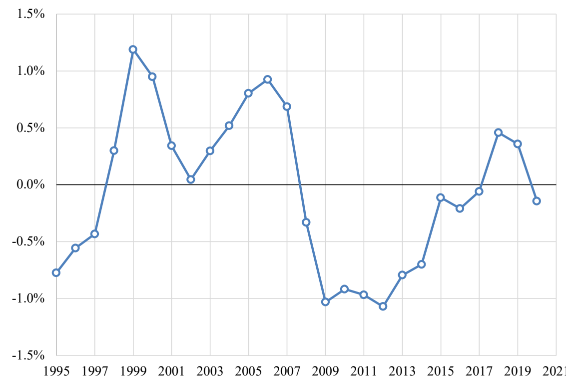
Figure E.4: Empirical pattern of the allocation of wealth (alternative measure)

(a) U.S. share of world wealth (x_t)

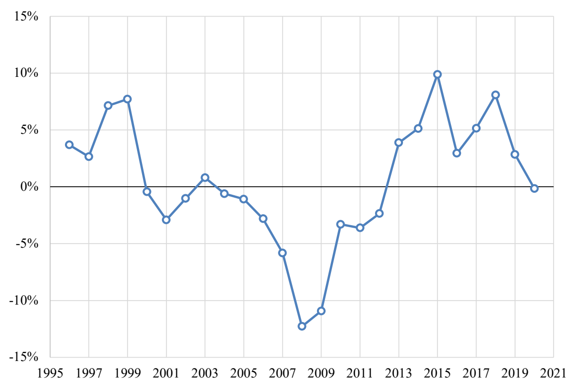


(a) U.S. share of world wealth (x_t)

(deviations from trend)



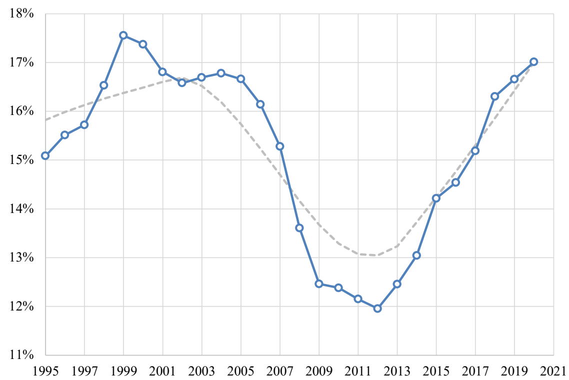
(c) Change in U.S. share of world wealth (dx_t/x_t)



Notes: x_t is measured as the share of world wealth held by the U.S. Wealth is defined as the market value of national wealth. World wealth includes all countries for which appropriate data is available. Data is from the World Inequality Database. The dashed gray curve is the trend extracted from a standard STL decomposition.

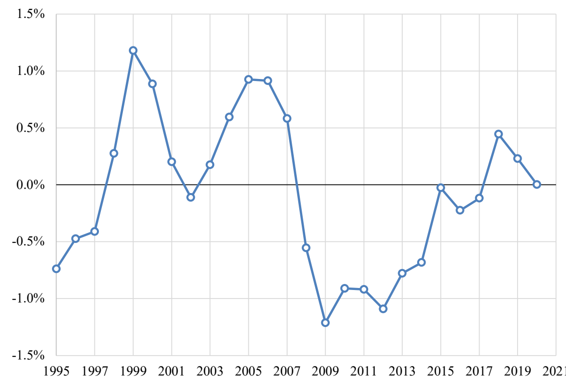
Figure E.5: Empirical pattern of the allocation of wealth (alternative measure, private wealth only)

(a) U.S. share of world wealth (x_t)

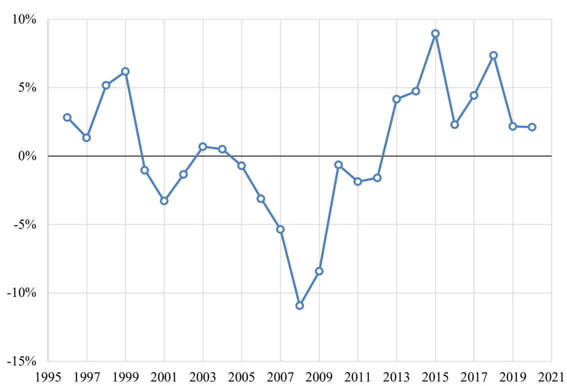


(a) U.S. share of world wealth (x_t)

(deviations from trend)



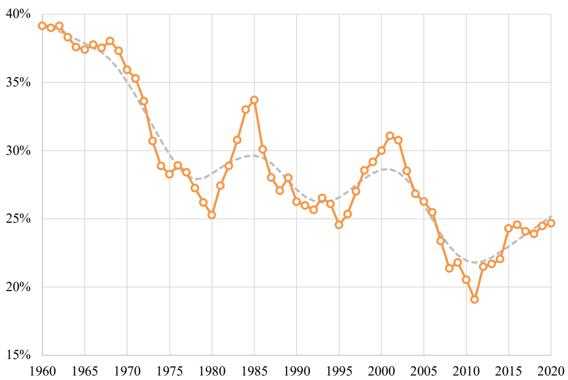
(c) Change in U.S. share of world wealth (dx_t/x_t)



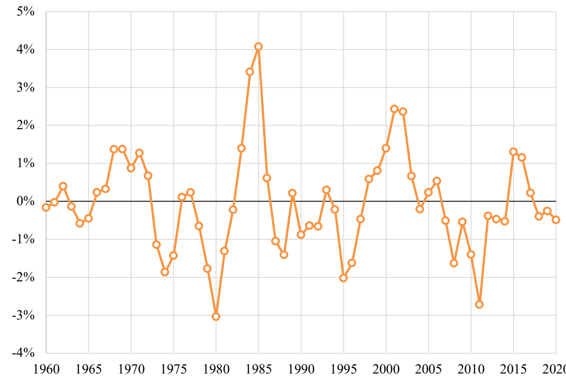
Notes: x_t is measured as the share of world wealth held by U.S. households. Wealth is defined as the market value net private wealth. World wealth includes all countries for which appropriate data is available. Data is from the World Inequality Database. The dashed gray curve is the trend extracted from a standard STL decomposition.

Figure E.6: Empirical pattern of the relative supply

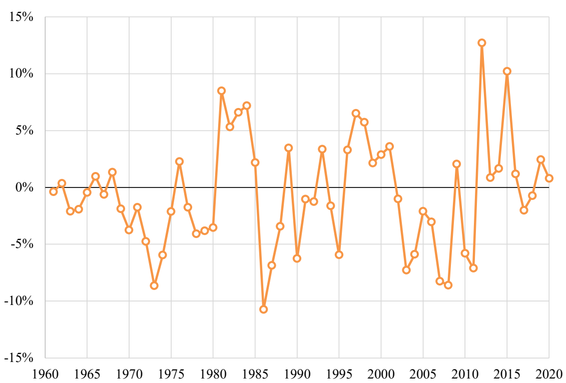
(a) U.S. share of world GDP (y_t)



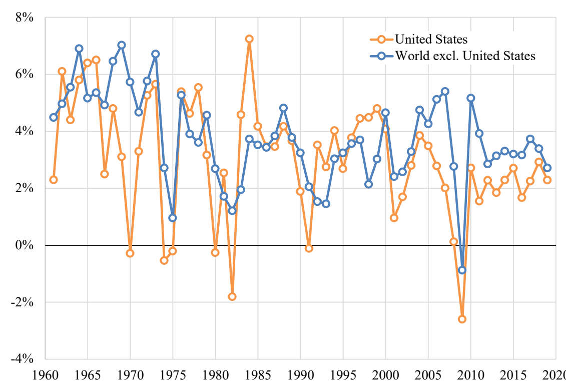
(b) U.S. share of world GDP (y_t)
(deviations from trend)



(c) Change in U.S. share of world GDP (dy_t/y_t)



(d) Real GDP growth



Notes: y_t is measured as the U.S. share of world GDP in current U.S. dollars. Data is from the World Bank. The dashed gray curve is the trend extracted from a standard STL decomposition.

E.2 Asset prices

I turn to providing suggestive evidence for asset prices by estimating an empirical equivalent of the theoretical risk premia expressions in Equations (10) and (11).

Testing those asset prices implications empirically requires proxies for the two state variables of interest.

As is common in the international portfolio choice literature, I proxy movements in relative supply y_t by movements in the real exchange rate $\mathcal{E}_t$.[42] This is consistent with the fact that y_t is by far the main driver of relative prices, so that there exists a strong (negative) monotonic relationship between y_t and $\mathcal{E}_t$ (cf. Figure 8). The main advantage is that prices data are available at a significantly higher frequency, and are overall of better quality. In practice, because I focus on assets from the U.S. and the rest of the world, I use the real broad dollar index from the Federal Reserve Board for $\mathcal{E}_t$. Because the price of relative supply risk is likely to be negative in the relevant region of the state space, the model predicts that the price of $\mathcal{E}_t$ risk should be positive, although it could turn negative in particular as the relative supply gets low ($\mathcal{E}_t$ high).

The ideal proxy for x_t would be the U.S. wealth share shown in Fact 1 and discussed throughout. However, it is available for too short a sample, and too low a frequency to be usable in practice. Instead, I follow the insight of the model and note that the wealth-weighted world risk aversion γ_t is the variable that as the strongest and cleanest direct (negative) relationship with the U.S. wealth share x_t. I therefore proxy for x_t with the global risk aversion ($GlobalRA_t$) extracted from worldwide data in Miranda-Agrippino and Rey (2020). The fact that γ_t is also part of the price of risk for other factors is not of particular concern. Indeed, although γ_t could impact those other prices of risk –an aspect that the conditional version of the specification below partly controls for–, the empirical test is about whether changes to x_t or γ_t risks are priced as a *distinct factor*. Finally, recall that the price of x_t risk predicted by the model is positive throughout, so that we expect the price of $GlobalRA_t$ risk

[42] I do not clean the real exchange rate from changes in nominal exchange rate because a majority of equity investors do not appear to be hedging currency risk in practice. For instance, Boudoukh et al. (2015) find that out of 428 international equity funds available in the U.S., totaling \$1,610 billion in assets, only 20.9% include some form of currency hedging via options *etc.*, with data up to March 2015. The relevance of real exchange rate movements for international asset prices is also consistent with recent evidence for several industrial and emerging countries (e.g. Stewen, 2020).

to be negative.

Lastly, as is common in the asset pricing literature, I proxy for the market component by using various factors. Specifically, I use the global excess returns on the market ($Mkt - RF_t$), global value factor (HML_t), global size factor (SMB_t), and global momentum factor (MOM_t), from Fama and French (2012). The table below shows that the results are robust across various combinations of those factors.

In practice, combining those proxies with Equation (14) imply the following beta-representation for the risk premia on both equity assets j

$$\mu_{j,t} - r_t = \alpha_{j,t} + \lambda_{M,t}\beta_{j,M,t} + \lambda_{GlobalRA,t}\beta_{j,GlobalRA,t} + \lambda_{\mathcal{E},t}\beta_{j,\mathcal{E},t} + \varepsilon_{j,t} \tag{E.1}$$

$\boldsymbol{\lambda}_t$ are the prices of risk and can also be derived theoretically from Equations (10) and (11). Denote the return on the market as $dR_{M,t} \equiv z_t dR_t + (1 - z_t)dR_t^*$. The quantities of risk for any U.S. or foreign assets are

$$\beta_{j,GlobalRA,t} \equiv \frac{\text{cov}_t\left(dR_{j,t}, dGlobalRA_t\right)}{\text{var}_t(dGlobalRA_t)}\ ,\ \beta_{j,\mathcal{E},t} \equiv \frac{\text{cov}_t\left(dR_{j,t}, d\mathcal{E}_t\right)}{\text{var}_t(d\mathcal{E}_t)}$$

$$\beta_{j,M,t} \equiv \frac{\text{cov}_t\left(dR_{j,t}, dR_{M,t}\right)}{\text{var}_t(dR_{M,t})}$$

Test assets are taken to be a set of portfolios, again, as is standard in the empirical asset pricing literature. Specifically, I use 128 portfolios sorted on a mix of factors (Size, Book-to-Market, and Operating Profitability), 32 in each of four regions – the United States, Europe, Japan, and Asia Pacific– so as to cover a wide range of countries. The data is from Fama and French (2012). The main reason portfolios are used is to smooth out idiosyncratic movements in returns stemming from individual stocks, which allows for a more precise and stable estimation of prices and quantities of risk. I base the analysis on quarterly returns on each of those 128 portfolios expressed in U.S. dollars, as is standard in the literature. The final sample starts in 1990Q4, and ends in 2010Q4 (because of the availability of the real broad dollar index).

Because the beta representation of risk premia in Equation (E.1) is conditional, and because dynamic aspects are an important novel element of the framework and could be relevant (like for portfolios), I test the model both unconditionally and conditionally. For the latter, I proceed by using a conditioning variable Z_{t-1} in the spirit of Cochrane (2005), and as used e.g. in Lettau and Ludvigson (2001) to test the consumption-CAPM conditionally in a domestic context. In practice, the conditional aspect comes from interacting the various factors in Equation (E.1) with Z_{t-1} so that Z_{t-1}, $Z_{t-1} \times dR_{M,t}$, $Z_{t-1} \times dGlobalRA_t$, and $Z_{t-1} \times d\mathcal{E}_t$ become additional factors. The conditioning variable must be chosen so as to predict returns well in the sample. For instance, Lettau and Ludvigson (2001) use cay_{t-1}, which they have shown to predict returns in the U.S. context. In the same spirit, I choose $Z_{t-1} \equiv nxa_{t-1}$, where nxa_{t-1} is the measure of cyclical external imbalances that Gourinchas and Rey (2007a) and Gourinchas et al. (2019) show is a good predictor of returns in an international context.[43]

Table E.1 shows the results. In short, regardless of the specifications, the empirical evidence appears to strongly support the predictions of the model. This is true both unconditionally (columns (1)-(4)) and conditionally ((5)-(8)).

The price of $GlobalRA_t$ risk, our (inverse) proxy of wealth share risk, is strongly negative across specifications, consistent with the theoretical predictions discussed above. It is both strongly statistical significant (t-stats between -3.281 and -6.243), and economically large. For instance, the risk premia difference in annual terms between assets with $\beta_{j,GlobalRA,t}$ in the top 90% and the bottom 10% percentile ranges from -5.334% to -8.773%, and is even more negative when including the conditional components $dGlobalRA_t \times Z_{t-1}$. Those magnitudes are large compared to the average of portfolios returns (21.553%), or their median (15.981%). This is true regardless of what proxies are used for the market component (labeled as *Controls* in Table E.1).

The price of real exchange risk $d\mathcal{E}_t$, our (inverse) proxy of relative supply risk, is strongly positive in most specifications, although its conditional component sometimes turns negative. Both are consistent with theoretical predictions discussed above that suggest that this risk price should be mostly positive, although its dynamic component can make it turn negative depending on the state of the economy. The price of $d\mathcal{E}_t$ risk is strongly statistical significant (t-stats between 2.391 and 5.706), and economically large. The risk premia difference in annual terms between assets with $\beta_{j,\mathcal{E},t}$ in the top 90% and the bottom 10% percentile ranges from 2.476% to 7.075% across

[43] Other conditioning variables, such as the level of risk aversion, $GlobalRA_{t-1}$, or market excess returns in the previous period, $Mkt\text{-}RF_{t-1}$, could be used. An alternative method to capture time-variations in expected returns could also be to condition on periods of market turbulences, as in the downward-risk-CAPM of Lettau et al. (2014).

specifications, and is often more positive when including the conditional components $dGlobalRA_t \times Z_{t-1}$. Again, those magnitudes are large compared to the average of portfolios returns (21.553%), or their median (15.981%).

Lastly, the adjusted-R^2 for those empirical tests are noticeably large, ranging from 26.5% to 70.8%.

Table E.1: Estimation of the implied beta-representation of Equation (E.1)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Risk Prices (monthly, %)								
$dGlobalRA_t$	-4.147	-6.817	-6.254	-6.281	-3.822	-5.730	-6.198	-5.014
	(-3.281)	(-5.455)	(-5.550)	(-5.050)	(-4.248)	(-4.954)	(-6.243)	(-4.683)
$dGlobalRA_t \times Z_{t-1}$					-1.398	-1.174	-1.086	-1.185
					(-6.950)	(-5.614)	(-5.016)	(-4.875)
$d\mathcal{E}_t$	0.725	0.826	0.636	0.637	0.464	0.628	0.485	0.399
	(5.168)	(5.706)	(5.325)	(5.339)	(2.783)	(4.286)	(3.125)	(2.391)
$d\mathcal{E}_t \times Z_{t-1}$					0.156	0.177	-0.031	-0.067
					(3.437)	(4.484)	(-0.574)	(-1.045)
Z_{t-1}					-3.552	-3.114	-0.628	-2.015
					(-3.706)	(-2.610)	(-0.345)	(-1.342)
Panel B: Premia difference (annual, %)								
$dGlobalRA_t$	-5.865	-8.524	-8.643	-8.773	-5.334	-6.973	-8.431	-6.785
$dGlobalRA_t \times Z_{t-1}$					-7.927	-8.871	-6.831	-8.669
$d\mathcal{E}_t$	5.402	7.075	5.307	5.281	2.733	3.639	2.971	2.476
$d\mathcal{E}_t \times Z_{t-1}$					4.043	4.047	-0.655	-1.391
Z_{t-1}					-3.354	-3.653	-0.441	-1.224
Controls		CAPM	FF3	FF3MOM		CAPM	FF3	FF3MOM
Conditioning var. (Z_{t-1})					nxa_{t-1}	nxa_{t-1}	nxa_{t-1}	nxa_{t-1}
Adjusted-R^2 (%)	26.5	38.7	57.4	57.1	55.0	57.7	68.4	70.8

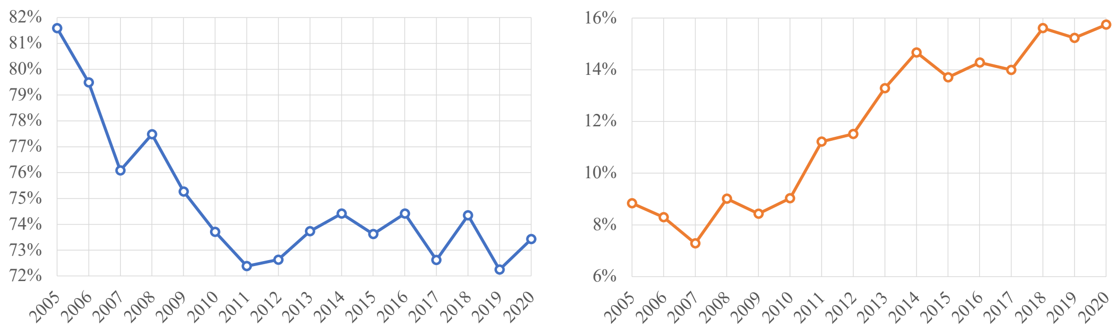
Notes: Test assets are 128 portfolios sorted on Size, Book-to-Market, and Operating Profitability, for four regions: the United States, Europe, Japan, and Asia Pacific (32 portfolios per region). Sample: 1990Q4-2010Q4. The premia difference is computed as the risk premia difference in annual terms between assets in the top 90% and bottom 10%-beta percentile for each factor. t-stats from Newey-West standard errors in parenthesis. Cf. Section E.2 for additional details.

E.3 Portfolios

Figure E.7: Share of U.S. asset in equity portfolios (%)

(c) U.S. portfolio (data) $w_{h,t}/(w_{h,t}+w_{f,t})$

(d) Rest of the world portfolio (data): $w^*_{h,t}/(w^*_{h,t}+w^*_{f,t})$

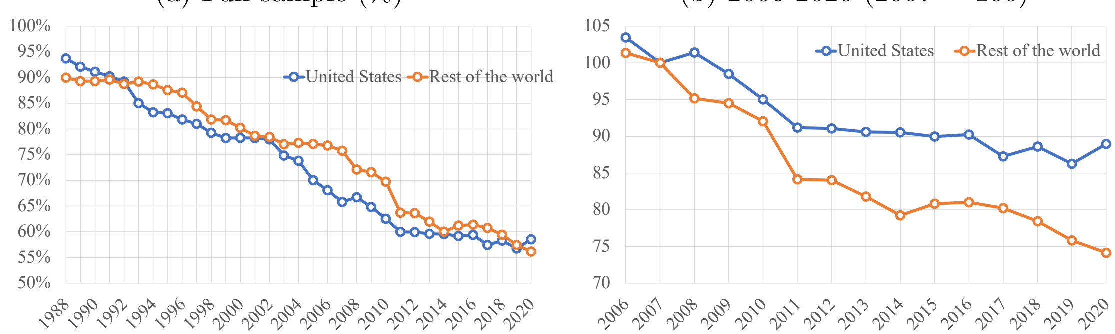


Notes: Based on non-restated data, following Coeurdacier and Rey (2013) that I extend to the recent period. Cf. Section 5.1.

Figure E.8: Equity home portfolio (data)

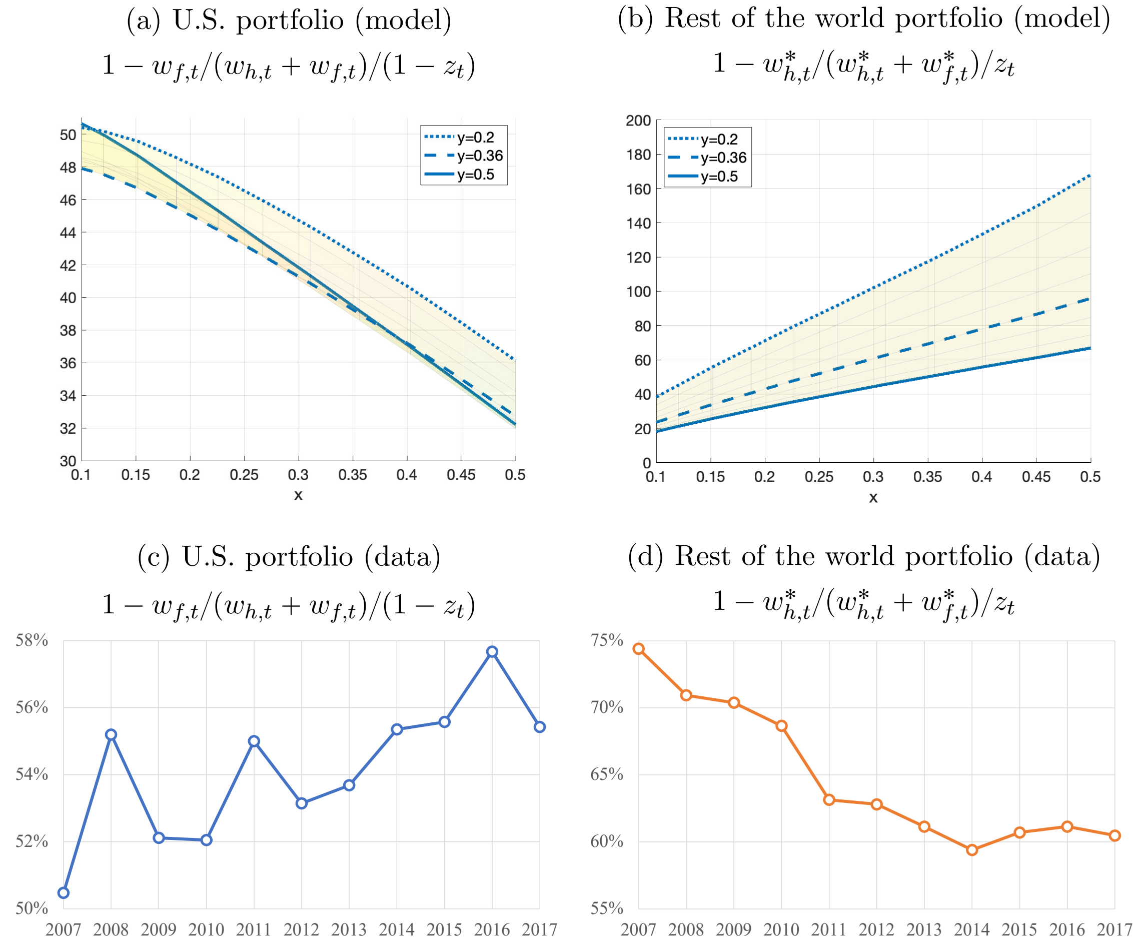
(a) Full sample (%)

(b) 2006-2020 (2007 = 100)



Notes: Based on non-restated data, following Coeurdacier and Rey (2013) that I extend to the recent period. Cf. Section 5.1. The home bias is defined as $HB_t \equiv 1 - w_{f,t}/(w_{h,t}+w_{f,t})/(1-z_t)$ for the United States, and $HB^*_t \equiv 1 - w^*_{h,t}/(w^*_{h,t}+w^*_{f,t})/z_t$ for the rest of the world.

Figure E.9: Home bias in equity portfolio (%)



Notes: Panels (a) and (b) are based on the model of Sections 2-4 under the baseline calibration of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic U.S. investor. y_t is the relative supply of the U.S. good, which captures fundamentals. Recall that the U.S. wealth share decreases in times of global crisis (Fact 1). Panels (c) and (d) plot the empirical equivalents based on restated portfolio data by Coppola et al. (2021). Cf. Section 5.1.