# Share Issues versus Share Repurchases

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**Abstract**

Almost all firms repurchase shares through open market repurchase (OMR) programs. In contrast, issue methods are more diverse: both at-the-market offerings, analogous to OMR programs, and SEOs, analogous to the rarely-used tender-offer repurchases are used by significant fractions of firms. Moreover, average SEOs are larger than at-the-market offerings. We show that this asymmetry in the diversity of transaction methods in issuances and repurchases and the size-method relation in issuances are natural consequences of the single informational friction of a firm having superior information to investors. Finally, repurchasing firms are likely maximizing long-term shareholders' payoffs rather than boosting short-term share prices.

## 1 Introduction

Public firms often tap into the equity market, both issuing new shares to raise funds, and repurchasing existing shares to return cash to investors. In many

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ways, issuing and repurchasing shares are mirror images of each other. Both types of transaction are subject to informational frictions arising from firms' superior knowledge. And for both types of transaction, firms choose transaction size and method. Conceptually, share repurchases are simply negative issuances.

In this paper, we analyze the two transactions side-by-side, under the assumption that firms have superior knowledge about their own prospects, and can choose both transaction size and method. Although many papers analyze security transactions under asymmetric information, the comparison of issues and repurchases is new to the literature, and yields fresh insights. We emphasize three points.

First, and despite the conceptual symmetry between issue and repurchase transactions, their equilibrium *outcomes* are *not* mirror images of each other. Repurchasing firms cannot signal via the efficiency of transaction method—*money burning*—while issuing firms can. Empirically, almost all firms repurchase via open-market transactions; while issuing firms use both seasoned equity offerings (SEOs) and at-the-market offerings (ATMs) with significant frequencies, though the latter has received limited academic attention. Our analysis rationalizes both patterns.

Second, and in contrast, reducing repurchase sizes is a viable signal for repurchasing firms, just as reducing issue size is a viable signal for issuing firms. The predicted patterns of transaction size and market response are consistent with empirical evidence on both issues and repurchases. The contrast between the first and second points highlights that while reducing repurchase size "burns money" by reducing transaction surplus, doing so also has the separate effect of increasing a firm's total value.

Third, and more conceptually, our analysis isolates a precise formal role for total firm value, viz., for any transaction under consideration, a manager should ask, "by what percent will this transaction affect firm value?" The point is starkest for the case of repurchases, in which case transaction size affects transaction surplus and firm value in different directions. But even for issue decisions, a focus on total firm value sheds light on firms' preference for signalling-via-money-burning in preference to signalling-via-issue-size, and operationalizes results Viswanathan's (1995) results on signal-ordering in terms of standard financial quantities.

Our analysis also speaks to the question of whether firms' capital transactions—and repurchases in particular—are excessively driven by a desire to boost short-term share prices at the expense of long-term shareholder payoffs. Our analysis suggests that they are not. Specifically: if one feeds the assumption that firms heavily weight short-term prices into our analysis then it yields counterfactual implications for repurchase behavior.

In more detail, we model issues and repurchases in a unified and symmetric way. A firm privately knows the value of its assets in place (Myers and Majluf (1984)), and has a surplus-creating "project" that can only be implemented through trading equity. If the project requires a positive investment, the firm needs to raise capital by issuing shares. In contrast, if the "investment" is negative, then the firm needs to pay out capital by repurchasing equity; here, the surplus stems from the avoidance of wasteful expenditures that would take place if cash were instead retained. The project is scalable and produces more surplus if more capital is deployed (raised or paid out) up to some maximum. Firms choose both project size—or equivalently, transaction size—and transaction method associated with different levels of efficiency.

The following two points underpin many of our results. First, while issuing firms want to raise investors' perceptions of their value, repurchasing firms instead want to lower perceptions. Second, equity transactions mechanically affect the total firm value even without generating surplus. In the textbook case of public information, only transaction surplus matters for firm decisions (i.e., NPV maximization). In contrast, under asymmetric information the total firm value significantly affects firm decisions too. More specifically, an action needs to simultaneously satisfy two conditions to be a viable signal: it decreases surplus, and its effect on the total firm value is more favourable to the firms who want to distinguish their types relative to those who want to pool with others (the single crossing property).

Repurchasing firms are unable to signal via money-burning because it is worse firms that want to reveal their types, but money-burning is proportionally more costly for such firms. On the other hand, issuing firms are able to signal via money-burning because it is better firms that want to reveal their types, and money-burning is proportionally cheaper for these firms.

In contrast to this asymmetry, both repurchasing and issuing firms can signal via reducing transaction value. A reduction in repurchases increases total firm value, and does so proportionally more for worse firms. A reduction in issuance reduces total firm value, but does so proportionally less for better firms.

Our main implications fit well with empirical findings. We start with the "asymmetry" prediction. In principle, similar transaction methods are available for issuing and repurchasing firms. Specifically, firms can raise equity quickly in an SEO, which typically completes in 2-8 weeks (Gao and Ritter, 2010); or more gradually through at-the-market offerings (ATM) over a couple of years.[1] Likewise, repurchases can be carried out either swiftly in tender offers (henceforth, TOR, often lasting for a month (Masulis, 1980)) or slowly via open market repurchase (OMR) programs. Our asymmetry prediction gives an explanation for the prominent empirical feature that both SEOs and ATMs coexist as frequently observed issue methods, whereas OMR dominates the repurchase market.[2]

Second, the prediction that transaction size reveals firm fundamentals in both issues and repurchases again fits the data well: returns are larger following smaller issues and larger repurchases.[3]

Third, our implication that issuing firms prefer to signal via smaller issues rather than via more inefficient methods implies the following pattern: The worst firms issue the maximum amount using the most efficient method; better firms issue less, still using the most efficient method; and the best firms issue the minimum amount possible to fund the project, but use more inefficient issue methods. As noted, the size-dimension of this pattern is consistent with the empirical observation that

[1] Billett, Floros, and Garfinkel (2019) provides a nice review of this growing popular issue method.

[2] Billett, Floros, and Garfinkel (2019) document that ATMs represent 63% incidence and 26% issue proceeds of those for SEOs. In contrast, in 2004, there are 466 cases of OMR with a total size of 223 billion dollars, and tender offers and dutch auctions only account for 18 and 10 cases, and 1.3 billion dollar and 3.9 billion dollar proceeds respectively (see Banyi, Dyl, and Kahle (2008), and similar patterns have been documented by Grullon and D. L. Ikenberry (2000)).

[3] On the repurchasing front, D. Ikenberry, Lakonishok, and Vermaelen (1995) document respectively 20%, 30%, 31% and 19% of repurchasing firms repurchase 0-2.5%, 2.5-5%, 5-10% and above 10% of their equity. On the issue front, Billett, Floros, and Garfinkel (2019) document standard deviation of issue size (as a fraction of firms' market value) is 20% and 55% for ATMs and SEOs, respectively.

On the size-return relation, Asquith and Mullins Jr (1986) and Masulis and Korwar (1986) document negative relation between issue size and announcement return. In tender offer repurchases, Vermaelen (1981) find abnormal return is positively related to target tender fraction.

large issue sizes are correlated with more efficient methods. We microfound the efficiency associated with the transaction methods (SEO and ATM when firms issue, OMR and TOR when repurchase) in Section 4. There, we argue that one-off SEOs are more efficient than more gradual ATMs, as the former allows the firm to immediately implement the project, whose NPV might diminish over time. Billett, Floros, and Garfinkel (2019) provide evidence that proceeds from an SEO are indeed larger than total proceeds from an average ATM program.[4]

## 1.1 Related Literature

There is a large literature on firms' capital transaction when they have superior information over investors. When selling securities, costly retention of unsold securities or broadly speaking, transaction size, can be informative signals about firms' hidden quality (see Leland and Pyle (1977), Myers and Majluf (1984), Krasker (1986), and DeMarzo and Duffie (1999)). When repurchasing securities, firms can similarly signal by different repurchase amounts (see Vermaelen (1984), M. Brennan and Kraus (1987), Ofer and Thakor (1987), Constantinides and Grundy (1989), Chowdhry and Nanda (1994), Lucas and McDonald (1998), and Bond and Zhong (2016)). In general, higher quality firms buy more or sell less (or even not sell at all). In addition to transaction-size signaling, these papers also show that firms can signal through tax-inefficient dividend payouts, or more generally burning cash (for example advertisement signaling in Milgrom and Roberts 1986), in exchange for a more favorable transaction price. Our analysis contributes to this literature by allowing both size and efficiency signaling simultaneously and compare the two directions of equity transactions (issue and repurchases) side by side. Novel to the literature is the insight that firms can use both transaction size and efficiency as signals when they issue, whereas only size signal is possible when repurchase. We also establish that issuing firms prefer to signal via issuing less rather than via issuing inefficiently.

[4]We calculate from Table 2 of Billett, Floros, and Garfinkel (2019) that the average proceeds per SEO are 256 million dollars, whereas average proceeds per ATM program are 92 million dollars. Even though the ratio of proceeds to market equity is roughly the same between the two methods (18% for SEO and 20% for ATM), it is significantly smaller for ATM than for SEO after controlling for other factors.

Like us, Babenko, Tserlukevich, and Wan (2020) consider issues and repurchases in a unified model, though from a very different perspective. They show that a firm can profitably trade its own equity (market timing), but in doing so harms shareholders who trade against the more informed firm. In contrast, our paper focuses on how these issues and repurchases are carried out, namely the choices of transaction size and methods (efficiency).

Our paper is also related to the literature on firms' choice of equity transaction methods. M. J. Brennan and Thakor (1990) and Oded (2011) study firms' choice between tender offer and open market repurchases. In contrast to our model, which studies firms' choice of transaction methods with privately information, these papers consider the interaction between informed and uninformed shareholders in their tendering strategies, and emphasize the role of shareholders' endogenous decision to acquire information. In contrast, when firms raise equity, Burkart and Zhong (2022) compare public offerings and rights offerings. The key driver in their paper is the wealth transfer between constrained and unconstrained shareholders, and the efficiency choice is left out of the model. Chemmanur and Fulghieri (1994) present a model in which investment banks endogenously acquire information as underwriters, and predict that firms choose underwritten issues over direct issues unless they face little information asymmetry or receive too low an evaluation from the investment bank to procure its services. In contrast, abstracting from the role of underwriters, we establish firms' choice between one-off SEO and gradual ATM emphasizing their difference in efficiency of funding corporate investment. Some of our predictions are not in line with Chemmanur and Fulghieri's but are supported by empirical evidence.[5]

Our paper also speaks to the literature on multi-dimensional signaling/screening. We defer a fuller discussion of this point until page 19 below.

# 2 The model

We model share issues and repurchases in a unified framework. Consider a firm with assets in place $a$ and an opportunity to invest $i$ in a new project. The value

[5]See prediction 3 and 4 in Section 4

of assets in place, $a$, is the firm's private information, whereas others only know that $a$ is distributed according to $F(\cdot)$, which admits a density and has support $[a_{\min}, a_{\max}]$. We refer to $a$ as the firm's type.

The firm chooses investment, i.e., project size, $i$ to lie in the closed interval between $I_L$ and $I_H$, where $I_L$ and $I_H$ are exogenous constants that are common knowledge. Either $I_H > I_L \geq 0$, in which case the project is an investment project; or $I_H < I_L \leq 0$, in which case the project is a divestment project. The case $|I_L| > 0$ corresponds to a minimum project size, which arises for investment projects if the project has a minimum scale, and divestment projects if the firm is compelled to pay out at least a minimum amount of cash (for example, if retaining cash above some level would lead to extremely wasteful spending).[6]

The investment $i$ is associated with equity transactions: Investment projects ($i > 0$) require funding and hence share issues, while divestment projects ($i < 0$) produce cash to be paid out via repurchases. (For reasons outside the model, the firm prefers to raise funding via equity to other securities, and to pay out cash via repurchases rather than dividends.)

In addition to investment $i$, the firm can also choose among equity transaction methods with different levels of efficiencies, captured by the variable $\theta \in [0, 1]$, with efficiency increasing in $\theta$. In empirical applications we typically interpret efficiency in terms of whether a transaction occurs at a single point in time, as

[6] If $I_L > 0$, so that investment projects are being analyzed, then one might also want to allow the possibility of the firm simply doing nothing, i.e., $i \in \{0\} \cup [I_L, I_H]$. We have fully analyzed this case, and it does not yield any additional economic insights relative to $i \in [I_L, I_H]$. Both to avoid distracting complexity in the statements of our results, and also to preserve symmetry across issuance and repurchase analysis, we present our results for the case in which $|I_L| > 0$ indeed precludes the possibility of doing nothing. Effectively, for $I_L > 0$ (the issue setting) we are assuming, in terms of formal objects defined below, that

$$\frac{V(a_{\max}, I_L, 1)}{1 + \frac{I_L}{V(a_{\min}, I_L, 1) - I_L}} > V(a_{\max}, 0, 1), \tag{1}$$

i.e., the best firm prefers issuing $I_L$ at full efficiency but at the most unfavorable price that can be supported in equilibrium over the alternative of doing nothing; along with the analogous assumption for repurchase ($I_L < 0$):

$$\frac{V(a_{\min}, I_L, 1)}{1 + \frac{I_L}{V(a_{\max}, I_L, 1) - I_L}} > V(a_{\min}, 0, 1). \tag{2}$$

in SEOs and tender offer repurchases, or gradually over time, as in at-the-market offerings and open market repurchases. See Section 4 for full details.

Equity transactions are carried out at the competitively determined price $P(i,\theta)$. That is: After a firm announces its investment and transaction efficiency choices $(i,\theta)$, competitive investors update their beliefs about the firm type $a$, and the price $P(i,\theta)$ reflects these updated beliefs.

An equity transaction $i$ carried out with efficiency $\theta$ yields surplus $S(i,\theta)$, increasing in transaction size $|i|$ and efficiency $\theta$. While we can easily accommodate more general function forms, for transparency we parameterize $S$ by

$$S(i,\theta) = i\theta b,$$

where $b$ is a constant that parameterizes surplus created by the investment. For investment projects $(I_H \geq i \geq I_L > 0)$, $b > 0$, and for divestment projects $(I_H \leq i \leq I_L < 0)$, $b < 0$; so in particular, the surplus created by both types of project, $i\theta b$, is positive. For divestments, value creation stems from cash being more valuable in the hands of shareholders than the firm's, either because of internal agency problems in the firm, or because of shareholders' liquidity needs.[7]

A firm's total value $V$ is the combination of its assets in place $a$, the funds raised or disbursed by the equity transaction $i$, and transaction surplus $S$:

$$V(a,i,\theta) \equiv a + i + S(i,\theta) = a + i + i\theta b. \tag{4}$$

For divestment projects we further assume that $b > -1$, which ensures that repurchases indeed reduce a firm's total value. Finally, we naturally assume that the minimum firm value is positive after equity transaction $a_{\min} + I_H > 0$.

[7] Concretely, for the repurchase case, consider a firm with cash $-I_H$ and non-cash assets in place $a_N$. Cash held inside the firm is invested inefficiently, at gross rate of return $1 - |b|$. If all cash is retained in the firm, the firm value is $a = a_N + |I_H|(1-|b|)$, which is the value of its assets in place. Repurchases potentially eliminate inefficient investments. The efficiency choice $\theta$ controls the fraction of the repurchase that is done before the inefficient investments are completed. Hence

$$\begin{aligned} V(a,i,\theta) &= a_N + \theta|i| + [(1-\theta)|i| + (|I_H| - |i|)](1-|b|) - |i| \\ &= a + \theta|i||b| - |i| \end{aligned}, \tag{3}$$

which coincides with (4).

The number of shares outstanding before any issue or repurchase is normalized to 1. Given an equity transaction price $p$, the firm needs to issue $\frac{i}{p}$ shares to raise capital $i$, or repurchase $\frac{-i}{p}$ shares for $i < 0$ to disburse $i$. The firm maximizes the payoff of its long-term investors, which is given by

$$\Pi\left(a, i, \theta, p\right) = \frac{V\left(a, i, \theta\right)}{1 + \frac{i}{p}}. \tag{5}$$

Section 5 analyzes the more general case in which firms care about both short- and long-term share prices.

For both intuition and formal analysis, it is frequently convenient to work with the log of the firm's payoff,

$$\ln \Pi\left(a, i, \theta, p\right) = \ln V\left(a, i, \theta\right) - \ln\left(1 + \frac{i}{p}\right). \tag{6}$$

That is, firms trade off percentage changes in total value $V$ with percentage changes in the number of shares outstanding after the equity transaction. The fact that it is percentage changes that is important stems from our focus on equity transactions.

By design, this framework covers both issue and repurchase decisions in a symmetric way. For the remainder of the paper, we refer to the case $I_H \geq i \geq I_L > 0$, $b > 0$ as the *issue game*, and the case $I_H \leq i \leq I_L < 0$, $b < 0$ as the *repurchase game.*

We focus on pure-strategy equilibria, which consist of each firm-type's choices of investment and efficiency, $\left(i\left(a\right), \theta\left(a\right)\right)$; investor beliefs $\mu\left(a|i, \theta\right)$ associated with each choice of $\left(i, \theta\right)$; and competitive investors' pricing function, $P\left(i, \theta\right)$, such that the following three conditions hold:

1. Given $P\left(i, \theta\right)$, firm $a$'s equilibrium strategy $\left(i\left(a\right), \theta\left(a\right)\right)$ maximizes its long-term shareholders' payoff:

$$\left(i\left(a\right), \theta\left(a\right)\right) \in \arg\max_{i,\theta} \Pi\left(a, i, \theta, P\left(i, \theta\right)\right)$$

2. The pricing function $P(i,\theta)$ is such that investors break even, i.e.,

$$P(i,\theta) = E\left[\Pi\left(a,i,\theta,P(i,\theta)\right)|i,\theta\right], \tag{7}$$

where expectations are taken using beliefs $\mu(a|i,\theta)$. Notice $\Pi(a,i,\theta,P(i,\theta))$ is indeed the true value of each share post transaction.

3. Investor beliefs $\mu(a|i,\theta)$ satisfy Bayes' rule for any $(i,\theta)$ such that $(i,\theta) = (i(a),\theta(a))$ for some firm type $a$.

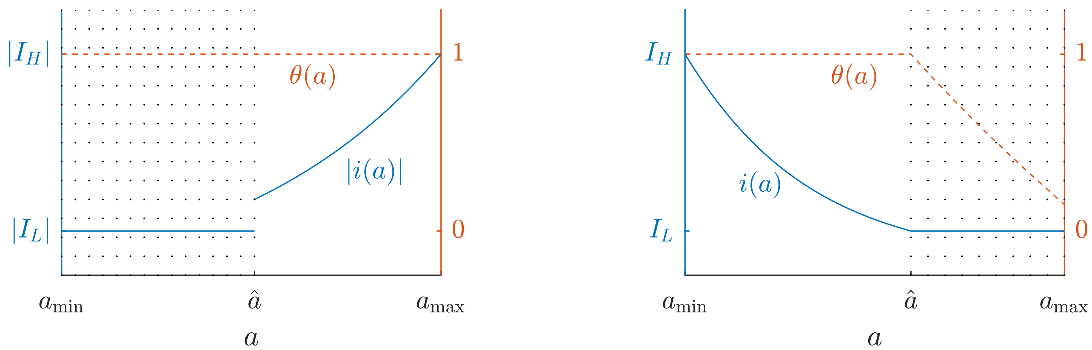
As in many signaling models, there are typically multiple equilibria. We employ the widely accepted D1 criterion (Cho and Kreps, 1987) to eliminate equilibria with "unreasonable" off-equilibrium beliefs. Broadly speaking, D1 requires that the beliefs associated with any off-equilibrium action must place all weight on types most likely to deviate to that action. Formally, let $\Pi^*(a)$ denote the equilibrium payoff of a type $a$ firm. Given an investment and efficiency $(i,\theta)$, define

$$D_a(i,\theta) = \left\{p : \Pi(a,i,\theta,p) > \Pi^*(a)\right\}. \tag{8}$$

An equilibrium satisfies D1 if for any type $a$ for which there exists a second type $a'$ such that $D_a(i,\theta) \subsetneq D_{a'}(i,\theta)$, beliefs satisfy $\mu(a|i,\theta) = 0$. It is worth noting that one of our central results—the impossibility of separation-via-efficiency by repurchasing firms—does not rely on equilibrium refinements.

*Remark:* As noted, we often interpret the efficiency choice $\theta$ in terms of how gradual a transaction. Transactions that are gradual—that is, OMR and ATM programs—also entail optionality, since firms can transact smaller quanitites that than the initial announcement. However, adding optionality to the model does not change the equilibrium outcomes.[8] For brevity, we abstract from this aspect, and assume that firms issue and repurchase the full amount that is announced.

[8]Specifically, we consider the following perturbation of the model: if the firm chooses a method from a subset of $[0,1]$, $\Theta$, then the firm announces a repurchase or issue size $i$, and privately chooses a size $i^A$ with $\left|i^A\right| \in [|I_L|, |i|]$ to repurchase or issue. The market observes $i$ but not $i^A$, and price the shares according to $i$. Under this assumption, the equilibria in Proposition 3 and 5 are still the unique D1 equilibria of the repurchase and issue games. See Appendix B for details.




(a) The equilibrium of the repurchase game. The undotted area represents the range of firm types that separate on different repurchase sizes; The dotted area represents the range of firm types that pool on the minimum repurchase size. All firms pool on the fully efficient repurchase method.

(b) The equilibrium of the issue game. The undotted area represents the range of firm types that separate on different issue sizes and pool on the fully efficient issue method; The dotted area represents the range of firm types that pool on the minimum issue size and separate on different issue methods.

Figure 1: Equilibrium size and efficiency in the repurchase and issue games. Values of $i\left(a\right)$ and $\left|i\left(a\right)\right|$ correspond to the left y-axes, and values of $\theta\left(a\right)$ correspond to the right y-axes.

# 3 Equilibrium Characterization

We fully characterize the equilibria of the repurchase and issue games. Specifically, for repurchases we show that separation-via-efficiency is impossible; while separation-via-size naturally arises, with worse firms repurchasing less, at lower prices. For issues, firms separate by issuing different quantities, with better firms issuing less, at higher prices; and the best firms further separate by issuing inefficiently, at still higher prices. Figure 1 summarizes these results.

## 3.1 Full information benchmark

As a benchmark, consider the case in which a firm's assets $a$ are publicly observed. From (7),

$$P\left(i,\theta\right)=\frac{a+i+S\left(i,\theta\right)}{1+\frac{i}{P\left(i,\theta\right)}},$$

and so

$$P\left(i,\theta\right)=\Pi\left(a,i,\theta,p\right)=a+S\left(i,\theta\right).$$

Hence in this benchmark, and as one would expect, firms choose transaction size $i$ and method $\theta$ to maximize transaction surplus $S(i,\theta)$. Firm value $V(a,i,\theta)$ is irrelevant to the decision.

### 3.2 Repurchases

We first analyze the behavior of firms wishing to pay out funds by repurchasing shares. We start by showing that repurchasing firms are unable to separate from each other by repurchasing with different efficiency levels. As we will see, this impossibility of separation-via-efficiency contrasts sharply with the possibility of such separation by issuing firms that seek to raise funds.

**Proposition 1.** *In the repurchase game, (A) all firms that repurchase the same size $i$ choose the same efficiency $\theta$, and (B) in a D1 equilibrium, all firms that repurchase do so with maximal efficiency $\theta = 1$.*

To understand the economics behind part (A), the impossibility of separation-via-efficiency, start by observing that it is worse firms that wish to reveal their types so as to repurchase shares at lower prices. Focusing on (6), suppose that worse firms attempt to separate by adopting some less efficient method $\theta' \equiv \theta - \Delta\theta < \theta$ in exchange for a lower repurchase price $P(i,\theta')$. On the one hand, the resulting sacrifice in firm value $V$ is $\Delta\theta ib$, which represents a smaller fraction of a better firm. On the other hand, the percentage change in the number of shares is independent of firm type. Consequently, the lower efficiency choice $\theta'$ is more attractive for good firms than bad firms, and so separation of this type is impossible in equilibrium.

Part (B) of Proposition 1 likewise follows from the observation that the worse firms experience the larger (percentage) effects from changing efficiency. Suppose that, contrary to the claimed result, firms repurchase using an inefficient method $\theta < 1$. Then firms would like to deviate and repurchase more efficiently ($\theta = 1$), provided that doing so doesn't significantly increase the repurchase price. The D1 refinement ensures that this condition is met: deviations to $\theta = 1$ tend to induce price decreases, because it is worse firms who experience the larger (percentage) benefit from repurchasing more efficiently, and so the off-equilibrium-path belief is concentrated on worse firms.

In contrast to the impossibility of separation-via-efficiency, repurchasing firms are able to separate by repurchasing less. We first establish this result for the case in which a firm's informational advantage is limited, in the sense that the support of $a$ is sufficiently small. The economic forces are easiest to describe in this case. Subsequently, we characterize the equilibrium when a firm's informational advantage is larger.

**Proposition 2.** *In a D1 equilibrium of the repurchase game, if* $a_{\min}$ *and* $a_{\max}$ *are sufficiently close, then firms separate on transaction size according to strategy* $\hat{i}(\cdot)$*:*

$$\frac{\partial \hat{i}(a)}{\partial a} = -\frac{\hat{i}(a)}{V\left(a, \hat{i}(a), 1\right) b}. \tag{9}$$

*with the boundary condition*

$$\hat{i}(a_{max}) = I_H. \tag{10}$$

Why can repurchasing firms separate using size $i$ even though they cannot separate using efficiency $\theta$ (Proposition 1)? The reason is that reducing transaction size increases total firm value $V$ while reducing efficiency decreases $V$—even though transaction surplus is reduced in both cases. In more detail: In the repurchase setting, it is worse firms that wish to separate themselves from better firms so as to be able to acquire shares at a lower price. Consider a firm that offers a smaller repurchase size $|i'| \equiv |i| - \Delta i$ for some $\Delta i > 0$, in order to obtain a lower price. While this smaller repurchase lowers transaction surplus by $\theta \Delta i |b|$, it increases total firm value by $\Delta i (1 + \theta b)$, because the firm retains more cash. This represents a larger fraction of total value for worse firms. So by (6), a smaller repurchase is more attractive for worse firms, making it a viable signal.

The formal characterization in (9) and (10) of separation via repurchase size is standard (e.g., Mailath (1987)). First, there is no distortion at the "bottom," in this case meaning that the best firm $a_{\max}$ repurchases the maximum amount $I_H$.

Second, worse firms separate by repurchasing less, which has the advantage of reducing the repurchase price. Given separation, repurchases are fairly priced, i.e., $P(i(a),\theta(a)) = a + S(i(a),\theta(a))$. Writing $\hat{\imath}(a)$ for firm $a$'s repurchase strategy, firm $a$'s payoff from mimicking the repurchase strategy of firm $\tilde{a}$ is

$$\frac{V\left(a, \hat{\imath}(\tilde{a}), 1\right)}{1 + \frac{\hat{\imath}(\tilde{a})}{\tilde{a} + S\left(\hat{\imath}(\tilde{a}), 1\right)}}. \tag{11}$$

As standard, the equilibrium condition is that firm $a$ does not gain from mimicking neighboring firms, so that equilibrium strategy $\hat{\imath}(a)$ solves the differential equation

$$\frac{\partial}{\partial \tilde{a}} \left( \frac{V\left(a, \hat{\imath}(\tilde{a}), 1\right)}{1 + \frac{\hat{\imath}(\tilde{a})}{\tilde{a} + S\left(\hat{\imath}(\tilde{a}), 1\right)}} \right)_{\tilde{a}=a} = 0 \tag{12}$$

subject to the boundary condition $\hat{\imath}(a_{\max}) = I_H$. By straightforward manipulation, (12) simplifies to (9).

Note that Proposition 2 builds on Proposition 1's result that firms choose maximal efficiency $\theta = 1$.

Propositions 1 and 2 represent the principle insights of this subsection. First, separation-via-efficiency is impossible for repurchasing firms. It is worse firms that wish to separate to drive down the price, but adopting an inefficient repurchase method is disproportionately costly for such firms. Second, and in contrast, worse firms can separate by scaling down their repurchases. Although scaling down a repurchase reduces the transaction surplus—i.e., "burns money"—just like adopting an inefficient method, doing so *increases* total firm value. The increase in total value is larger (in percentage terms) for worse firms, rendering it an effective way for such firms to signal.

The remainder of the subsection completes the characterization of repurchase equilibria, specifically, by characterizing repurchase outcomes when firm's informational advantage is larger than the case of Proposition 2.

Equation (9) characterizes the form that separation on repurchase size takes. If $a_{\min}$ is sufficiently close to $a_{\max}$ that (9) leads to repurchases above the minimum size $I_L$ ($|i| > |I_L|$, i.e., $i < I_L$) for all firms $a > a_{\min}$, then Proposition 2 is already a complete description of the repurchase equilibrium. For use in Proposition 3 below, define $\hat{a} = a_{\min}$ in this case.

The remaining case in which $\hat{i}(\cdot)$ hits this minimum repurchase level $I_L < 0$ before $a_{\min}$ is reached is more complicated. As a first step, it is instructive to note that it *cannot* be an equilibrium for separation to continue according to (9) all the way until the minimum repurchase size $I_L < 0$ is hit. The reason is that in such a case, there is an interval of firms below the separating firms that pool on the minimum repurchase size $I_L$. But firms marginally better than the pooling firms would gain by deviating marginally and reducing their repurchases to $I_L$, since doing so generates a discrete price reduction.

Instead, the equilibrium consists of a cutoff type $\hat{a}$ that is indifferent between repurchasing $\left|\hat{i}(\hat{a})\right| > |I_L|$ at the separating price $P = \hat{a} + S\left(\hat{i}(\hat{a}), 1\right)$ and repurchasing $|I_L|$ at the pooling price $P = a + E\left[S(I_L, 1) | a \in (a_{\min}, \hat{a})\right]$:

$$\hat{a} + S\left(\hat{i}(\hat{a}), 1\right) = \frac{V(\hat{a}, I_L, 1)}{1 + \frac{I_L}{E[a|a\in[a_{\min},\hat{a})] + S(I_L,1)}}. \tag{13}$$

Firms better than $\hat{a}$ separate according to (9). As discussed immediately above, the separation region ends at $\left|\hat{i}(\hat{a})\right|$ before the minimum repurchase $I_L$ is hit. Firms worse than $\hat{a}$ pool and repurchase the minimum amount, $I_L$; and so in particular, repurchase discretely less than firms better than $\hat{a}$. See Figure 1 for an illustration of this case. If there is no $\hat{a}$ with $\left|\hat{i}(\hat{a})\right| > |I_L|$ that satisfies (13), then the equilibrium is simply that all firms pool and repurchase $|I_L|$, i.e., $\hat{a} = a_{\max}$.

Summarizing:

**Proposition 3.** *The repurchase game has a unique D1 equilibrium, in which firms with $a > \hat{a}$ separate and repurchase according to $\hat{i}(\cdot)$ defined by (9) and (10), and firms $a < \hat{a}$ pool at the minimum repurchase size $I_L$. All repurchases take place at maximal efficiency, $\theta = 1$.*

Finally, we characterize the analytical solution to the ODE given by (9) and (10),

$$\left(-\hat{i}(a)\right)^b \left(a + \hat{i}(a) b\right) = (-I_H)^b (a_{\max} + I_H b). \tag{14}$$

Note that if the minimum repurchase size is $I_L = 0$, the solution $\hat{\imath}$ in (14) never reaches $I_L = 0$. In this case, the cutoff type $\hat{a}$ is $a_{\min}$, and all firms separate on their size choice according to $\hat{\imath}$.

## 3.3 Issues

We now turn to the behavior of firms wishing to raise funds by issuing shares ($I_H > I_L \geq 0$). We establish a stark asymmetry between the two cases, namely that separation-via-efficiency is possible for issuing firms even though it is not for repurchasing firms.

To show that separation-via-efficiency occurs for issuing firms, we fully characterize the unique D1 equilibrium of the issue game. We start by showing that although issuing firms separate via both efficiency and size choices, they prefer to do so via size, and use inefficient methods as a signal only when they have exhausted the use of size as a signal.

**Proposition 4.** *In any D1 equilibrium of the issue game, if a firm issues $i > I_L$ then it uses the most efficient method $\theta = 1$.*

The economic intuition for an issuing firm's preference to separate via size is as follows. Suppose to the contrary that a D1 equilibrium exists in which some firm $a$ issues more than the minimum amount, $i > I_L$, but uses an inefficient method $\theta < 1$. By issuing less but transacting more efficiently, the firm can both increase transaction surplus $S$ and reduce its total value $V$. That is, there exists a deviation to $i' < i$ and $\theta' > \theta$ such that

$$i'\theta' b > i\theta b, \tag{15}$$

$$i' + i'\theta' b < i + i\theta b. \tag{16}$$

The economic principle that makes the combination of (15) and (16) possible is that increasing efficiency ($\theta$) raises transaction surplus and total firm value by the same amount; while issuing less leads to a larger reduction in total firm value than in surplus.

The percentage reduction in total firm value associated with (16) is smaller for better firms. From (6), it follows from D1 that the beliefs associated with this deviation are no worse than $a$. So the deviation $(i', \theta')$ is at least fairly priced for firm $a$, and since it strictly raises transaction surplus, it strictly raises firm $a$'s payoff.

Given Proposition 4, the structure of the equilibrium of the issue game follows naturally. There is an interval of firms that separate by issue size. This interval is potentially followed by an interval of firms that issue the minimum amount $i = I_L$ and separate by inefficient issue methods.

Taken in isolation, the construction of each of the signaling-via-size and signaling-via-efficiency intervals is standard. The new element in our analysis (relative to, for example, retention signaling models of Leland and Pyle, 1977; Myers and Majluf, 1984; DeMarzo and Duffie, 1999) is to analyze both signaling possibilities together. To reiterate, the key tool is Proposition 4.

The specific form of the issuing equilibrium is as follows. First, there is no distortion at the bottom: the worst firm $a_{\min}$ issues the maximum size ($i = I_H$) at maximum efficiency ($\theta = 1$).

Second, an interval of firms better than $a_{\min}$ separate by scaling down the project, while retaining maximal issue efficiency $\theta = 1$. The construction is the same as for the equilibrium of the repurchase game, with the exception that it starts from the worst firm $a_{\min}$ rather than the best firm $a_{\max}$. Writing $\hat{i}(a)$ for firm $a$'s issue strategy, the function $\hat{i}(\cdot)$ must solve the differential equation (12), subject to the boundary condition $\hat{i}(a_{\min}) = I_H$. The economic force behind separation-via-size is similar to in Leland and Pyle (1977), viz., better firms separate by retaining a larger fraction of equity, which is more valuable for them.

Note that although repurchase and issue sizes share the same differential equation (9), the prediction on transaction size is reversed across the two cases, with better firms repurchasing more but issuing less.

Third, separation on issue size according to (9) continues as long as there is room. Specifically, if $\hat{i}(a_{\max}) \geq I_L$, all firms issue, separating on issue size, and the equilibrium characterization is complete; for use in Proposition 5, define $\hat{a} = a_{\max}$.

If instead there is $a$ such that $\hat{i}(a) = I_L$, define $\hat{a}$ as the value of $a$ such that $\hat{i}(a) = I_L$.

Firms better than $\hat{a}$ issue the minimum amount $I_L$, and separate by adopting less efficient methods. Writing $\hat{\theta}(a)$ for firm $a$'s efficiency strategy, for firms $a > \hat{a}$, the equilibrium strategy $\hat{\theta}(a)$ solves the differential equation

$$\frac{\partial}{\partial \tilde{a}} \left( \frac{V\left(a, I_L, \hat{\theta}(\tilde{a})\right)}{1 + \frac{I_L}{\tilde{a} + S\left(I_L, \hat{\theta}(\tilde{a})\right)}} \right)_{\tilde{a}=a} = 0, \tag{17}$$

subject to the boundary condition $\hat{\theta}(\hat{a}) = 1$. Equation (17) simplifies to

$$\frac{\partial \hat{\theta}(a)}{\partial a} = -\frac{1}{V\left(a, I_L, \hat{\theta}(a)\right) b}. \tag{18}$$

Recall that we assume that the best firm prefers issuing $I_L$ with efficiency $\theta = 1$ under the worst belief to doing nothing (see footnote 6). Under this assumption, there is enough room in efficiency choices $\theta$ for all firms better than $\hat{a}$ to fully separate, i.e., $\hat{\theta}(a)$ remains positive for all $a \in [a_{\min}, a_{\max}]$.

Summarizing:

**Proposition 5.** *The issue game has a unique D1 equilibrium, in which firms with $a \in [a_{\min}, \hat{a}]$ issue $\hat{i}(a)$ in the most efficient way ($\theta = 1$), and firms with $a \in (\hat{a}, a_{\max}]$ issue $i = I_L$ at efficiency $\hat{\theta}(a)$, where $\hat{a}$, $\hat{i}(\cdot)$, and $\hat{\theta}(\cdot)$ are as defined above.*

We highlight that D1 rules out pooling on any issue size or efficiency level. Pooling would necessarily entail some firms issuing at prices below their fair prices. Such firms could profitably deviate to an alternative issue size and efficiency that marginally decreases total firm value but discretely improves the issue price. The reason why the issue price improves again follows from (6), i.e., better firms care less in percentage terms about reductions in total firm value, and so D1 beliefs about such deviations heavily weight good firms.

Proposition 4 establishes that a necessary condition for firms to separate using transaction efficiency is that the possibilities from separation on size are exhausted. Proposition 5 shows that, for issuing firms, this condition is also sufficient: once the ability to separate via size is exhausted, firms indeed switch to separating via transaction efficiency.

Proposition 4's ordering of signaling-via-size versus signaling-via-efficiency can be understood by operationalizing Viswanathan (1995)'s "benefit-cost criterion." When multiple signaling devices are available, Viswanathan establishes that the Pareto-optimal separating equilibrium uses the signal with the highest "benefit-cost ratio". In our model, this implies size as a signal is preferred to efficiency, because size has a higher benefit-cost ratio, in the sense that

$$\frac{-\pi_{ai}(a,i,\theta,p)}{\pi_i(a,i,\theta,p)}|_{p=P(i,\theta)} > \frac{-\pi_{a\theta}(a,i,\theta,p)}{\pi_\theta(a,i,\theta,p)}|_{p=P(i,\theta)}, \tag{19}$$

where $\pi = \ln \Pi$. The numerators and denominators are what Viswanathan dubs the benefits and costs of the signals. Condition (19) reduces to

$$\frac{V_i}{S_i} > \frac{V_\theta}{S_\theta}. \tag{20}$$

(20) indeed holds, precisely because transaction size $i$ affects total firm value directly as well as via transaction surplus $S$. Conceptually, (20) formalizes the role of a signal's effects on total firm value and transaction surplus in determining its use.

Viswanathan (1995) characterizes Pareto-optimal separating equilibria. Abstract papers such as Engers (1987), Cho and Sobel (1990), and Ramey (1996) in turn show that the D1 refinement typically select such equilibria. [9]

Finally, we characterize the analytical solutions to the ODEs given by (9) and (18):

$$\hat{i}(a)^b\left(a+\hat{i}(a)\,b\right) = I_H^b\left(a_{\min}+I_H b\right), \tag{21}$$

and

$$e^{b\hat{\theta}(a)}\left(a+I_L\hat{\theta}(a)\,b\right) = e^b\left(\hat{a}+I_L b\right). \tag{22}$$

[9] For other uses of Pareto-optimality to select among signals in corporate finance settings, see John and Williams (1985), Ambarish, John, and Williams (1987), Besanko and Thakor (1987), Ofer and Thakor (1987), and Williams (1988).

Parallel to the repurchase game: When $I_L = 0$, the solution $\hat{\imath}$ in (21) never reaches $I_L = 0$ for any domain $[a_{\min}, a_{\max}]$, and all firms separate on issue size according to $\hat{\imath}$.

# 4 Empirical Implications

In this section, we explore the empirical implications of our model. Broadly speaking, there are two ways to issue seasoned equity in practice. The first method is a one-off SEO, which is typically completed within several weeks.[10] A lesser known but increasingly popular method is an at-the-market offering (henceforth, ATM). Billett, Floros, and Garfinkel (2019) provide a nice review of ATMs. In an ATM, the firm first registers new shares with the SEC, and then anonymously sells these shares in the secondary market. Compared to SEOs, ATMs take much longer to complete, on average 6.2 quarters. Similarly, firms can repurchase equity in a quick one-off fashion through a tender offer repurchase (henceforth TOR) within a month.[11] Alternatively, they can carry out an open-market repurchase program (henceforth, OMR) over a horizon of several years.[12]

The starkest prediction to emerge from our analysis (see Propositions 5 and 3) is:

*Prediction 1:* A greater variety of methods is used in issue transactions than in repurchase transactions.

Consistent with this prediction, significant issuing activities occur via both SEOs and ATMs, while an overwhelming fraction of repurchases are OMRs, with only a very small fraction being TORs.

More specific predictions about transaction methods, and their correlation with transaction size and future outcomes, require us to take a stand of how the efficiency parameter $\theta$ in the model maps to different methods. While different

[10] A non-shelf bookbuilt SEO, which accounts for 91% of all SEOs, often takes 2-8 weeks, while an accelerated bookbuilt SEO often takes 2 days from announcement to completion (Gao and Ritter, 2010; Huang and Zhang, 2011).

[11] It takes 25 days on average from announcement of an TOR to the expiration of the offer (Masulis, 1980).

[12] On average, 46.2%, 66.9%, and 73.9% of the target amount is completed by end of the first, second, and third year, respectively (Stephens and Weisbach, 1998).

assumptions are possible here, we next show that a natural and unified specification based on the availability of cash implies that, for issues, a one-off SEO is more efficient than a more gradual ATM, and that for repurchases, a more gradual OMR is more efficient than a one-off TOR. In a nutshell: The value creation from issues stems from raising capital to deploy in a productive way, without waiting for a firm's retained earnings to accumulate to fund the new investment; and a one-off SEO allows capital to be deployed more quickly than a more gradual ATM. Conversely, the value creation from repurchases stems from paying out a firm's earnings to avoid agency problems inside the firm; and a more gradual OMR allows earnings to be paid out as they arrive, as opposed to a one-off TOR that entails retaining earnings inside the firm until a significant quantity has accumulated.

Formally, first consider a firm that encounters an investment opportunity at time 0, but lacks funds to undertake it. The project exhibits constant returns to scale over a range of investment levels. The firm chooses both an investment amount $i$, i.e., project scale; and a time $t$ to start the project. The project can only start after the firm has raised funds $i$. If implemented at time 0, the project's NPV for each unit of investment is $b$. As time passes, the project becomes more and more obsolete (for example, due to the entry of competitors), and the NPV decreases at the rate of $\alpha$. Defining $\theta(t) = e^{-\alpha t}$, the project NPV is of the form $i\theta(t)\,b$. As such, an immediate SEO corresponds to the highest efficiency level $\theta = 1$, while more gradual ATMs correspond to lower efficiency levels.

Next, consider a firm that generates free cash flows at continuous rate $\lambda$ over a time interval $[0, T]$. If retained inside the firm, these cash flows are deployed in bad projects, generating a negative rate of return $-\beta < 0$. As such, if no payouts are made over some arbitrary time interval $[0, t]$, then these cash flows accumulate to a date-$t$ value of

$$\int_0^t \lambda e^{-\beta(t-s)} ds = \frac{\lambda}{\beta}\left(1 - e^{-\beta t}\right).$$

Suppose that over the interval $[0, T]$, the firm chooses to pay out a total amount of capital $|i| \leq |I_H|$ with $I_H = -\lambda T$ through $n$ share repurchases at dates $\frac{T}{N}, 2\frac{T}{n}, \ldots, T$, then at date $T$, the firm's cash balance becomes

$$\left(\frac{\lambda}{\beta}\left(1 - e^{-\beta\frac{T}{n}}\right) - \frac{|i|}{n}\right)\sum_{m=1}^{n} e^{-\beta\left(T - m\frac{T}{n}\right)} = \frac{\lambda}{\beta}\left(1 - e^{-\beta T}\right) - \frac{|i|}{n}\frac{1 - e^{-\beta T}}{1 - e^{-\beta\frac{T}{n}}}.$$

Suppose a firm has assets in place $a$, which implies net of free cash flows over the interval $(0,T)$, the value of the firm's existing assets is

$$a_N = a - \frac{\lambda}{\beta}\left(1 - e^{-\beta T}\right).$$

Then under the firm's choices of repurchase size $|i|$ and gradualness of repurchase $n$, the firm's value is

$$\begin{aligned} V &= a_N + \frac{\lambda}{\beta}\left(1 - e^{-\beta T}\right) - \frac{|i|}{n}\frac{1 - e^{-\beta T}}{1 - e^{-\beta \frac{T}{n}}} \\ &= a - |i| + |i|\left(1 - \frac{1}{n}\frac{1 - e^{-\beta T}}{1 - e^{-\beta \frac{T}{n}}}\right) \end{aligned}. \tag{23}$$

Define $b = -\left[1 - \frac{1}{\beta T}\left(1 - e^{-\beta T}\right)\right]$ and $\theta(n) = \frac{1 - \frac{1}{n}\frac{1 - e^{-\beta T}}{1 - e^{-\beta \frac{T}{n}}}}{|b|}$. Note that $\theta$ increases monotonically from 0 to 1 as the $n$ increases from 1 (single repurchase at date $T$) to $\infty$ (continuous repurchases over $[0,T]$). Hence, (23) coincides with our general specification of firm value (4), with more gradual repurchases corresponding to higher efficiency levels $\theta$.

This formalization allows us to replace Prediction 1 with the more specific:

*Prediction 1':* Firms issue equity using both SEOs and ATMs, while firms repurchase equity via OMRs.

Empirically, when firms issue equity, both SEOs and ATMs are widely adopted. Billett, Floros, and Garfinkel (2019) document that ATMs represent 63% incidences and 26% issue proceeds of those of SEOs. In contrast, almost all firms repurchase by conducting an OMR. For example, in 1999, there are 1,212 cases of new OMR programs or program extensions with a total size of \$137 billion. In comparison, tender offers and Dutch auctions account for 21 and 19 cases, and \$1.7 billion and \$3.8 billion proceeds, respectively (see Grullon and D. L. Ikenberry (2000)).

Proposition 5 also delivers the cross-sectional prediction that a firm carries out larger issues using efficient methods, which in our interpretation corresponds to an SEO. In contrast, for smaller issues a firm sometimes uses more inefficient methods, corresponding to ATM issues:

*Prediction 2:* SEOs are larger than ATM programs.

Empirically, Billett, Floros, and Garfinkel (2019) document average SEO proceeds of \$256 million, compared to average ATM program proceeds of \$92 million.[13]

Proposition 5 also implies:

*Prediction 3:* Firms with better unobservable qualities are more likely to use ATM issues.

*Prediction 4:* Firms facing larger informational frictions are more likely to use ATM issues.

In Prediction 4, the size of the informational friction is captured by the dispersion of firm types, $a_{\max} - a_{\min}$. Proposition 5 predicts that separation via transaction efficiency arises only when the dispersion of firm types is large.

Consistent with Prediction 3, Hartzell et al. (2019) shows in a dataset of REITs that the announcement returns of ATMs are less negative than of SEOs.[14]

Billett, Floros, and Garfinkel (2019) use *future* analyst recommendation updates as their proxy for firm quality unobservable to the market at the time of issuance. Consistent with our Prediction 3, their regression result in Table 4 shows that ATM firms receive better future analyst recommendation updates than SEO firms. Consistent with Prediction 4, the same authors show that higher levels of information asymmetry, proxied by unexplained current accruals, are indeed associated with the choice of ATM over SEO.

# 5 Preference for Share Price

If firms solely care about their long-term shareholders' payoff as in our baseline model, then they should prefer a lower price when repurchase. However, it is

[13] See Table 2 in Billett, Floros, and Garfinkel (2019). Table 4 in the same paper reports regression results after controlling for additional observable factors, including size of the issuing firm, and likewise indicates that SEOs are larger.

[14] That both ATMs and SEOs are followed by negative returns can be generated in our model by relaxing the assumption of (1), in which case the best firms do not issue any shares.

a well received idea among financial commentators and politicians that public firms repurchase shares in order to boost their share price.[15] Indeed, it seems plausible that when firms repurchase shares, they may not wish their share price to collapse, even though a lower price enables them to repurchase more shares. In this section, we modify the firm's objective function (5) to incorporate both preferences. Specifically, firms maximize a geometrically weighted average of the short-term share (transaction) price $p$ and the long-term shareholders' payoff (5):

$$\Pi(a, i, \theta, p) = p^{\epsilon} \left( \frac{V(a, i, \theta)}{1 + \frac{i}{p}} \right)^{1-\epsilon}, \tag{24}$$

where the weight $\epsilon \in [0, 1]$ reflects the degree to which firms care about their share prices directly. When $\epsilon = 0$, this objective function reduces to the original model (5). All other ingredients are the same as in the baseline model.

We first consider the repurchase game. The interesting case is when $\epsilon$ is sufficiently large. In this case, the firm may favor a high repurchase price, even though buying more expensive shares hurts the long-term shareholders. As a result of this shift in preference over the transaction price, the equilibrium outcome qualitatively changes compared with the baseline model. All firms pool on the same repurchase size and separate by choosing different methods. In particular, good firms signal their values by using inefficient methods.

**Proposition 6.** *If $\epsilon > \frac{-I_H}{a_{\min}}$, then the repurchase game has a unique D1 equilibrium, in which all types repurchase the maximum amount $I_H$. There is a cutoff type $\hat{a}$ such that firms with $a < \hat{a}$ separate on methods according to $\tilde{\theta}(\cdot)$ defined by*

$$\frac{d\tilde{\theta}(a)}{da} = -\frac{\epsilon V(a, I_H, \theta) + (1-\epsilon) I_H}{V(a, I_H, \theta) I_H b} \tag{25}$$

*and boundary condition $\tilde{\theta}(a_{\min}) = 1$. Firms with $a > \hat{a}$ use the least efficient method $\theta = 0$.*

[15]For instance, Segal (2021) writes in an article on investopedia.com that one of the reasons that a company buy back its own shares is "boosting its financial ratios". In fact this view is also shared by some academics, see Dittmar 2000 for a review of the reasons behind share repurchases. From the political side, congresswoman Alexandria Ocasio-Cortez writes on Twitter that "96% of airline profits over the last decade went to buying up their own stocks to juice the price," among many other similar criticisms when the U.S. government debates the airline bailout package in response to the Covid-19 crisis in early 2020.

In contrast to the baseline model ($\epsilon = 0$) where separation on efficiency $\theta$ is impossible, when firm's overall preference is for a higher repurchase price, separation on size becomes impossible, and the only possible signal is efficiency. Similar to before, this result does not rely on D1 refinement. Intuitively, this contrast is because the single-crossing property remains the same as in the baseline model, whereas the preference over the repurchase price is reversed. Good firms prefer to reveal their superior quality and receive a higher price by sacrificing NPV – either a smaller repurchase size or less efficient method. Suppose good firms attempt to separate on size by repurchasing less and retaining more cash. Since the retained cash is a larger fraction of bad firms, they are more willing to adopt a smaller size and hence will mimic good firms. In contrast, separation on transaction efficiency is possible, since a direct sacrifice in efficiency is relatively more costly for bad firms.

On the other hand, when $\epsilon$ is small in the repurchase game, or in the issue game with any $\epsilon \in [0, 1]$, equilibrium outcomes are qualitatively similar to that in the baseline model.[16] This is hardly surprising because firms' preference in these situations is similar to that in the baseline model. When $\epsilon$ is small in the repurchase game, firms put little weight on a high repurchase price directly, and their incentive to repurchase shares cheaply dominates. In the issue game, firms prefer to issue at a high price even without an explicit preference for price ($\epsilon = 0$). Therefore, introducing a positive $\epsilon$ only strengthens this preference, and the equilibrium outcome does not change qualitatively.

**Proposition 7.** *With $\epsilon \in [0, 1]$, the issue game has a unique D1 equilibrium, in which firms' strategy is the same as in the D1 equilibrium with $\epsilon = 0$ except that ODE (9) is substituted with*

$$\frac{d\hat{i}(a)}{da} = -\frac{\epsilon V(a, \hat{i}, 1) + (1 - \epsilon)\hat{i}}{V(a, \hat{i}, 1)b} \tag{27}$$

[16] Similar to (1), we make a simplifying assumption to ensure all types participate:

$$\left(V(a_{\min}, I_L, 1) - I_L\right)^{\epsilon} \left( \frac{V(a_{\max}, I_L, 1)}{1 + \frac{I_L}{V(a_{\min}, I_L, 1) - I_L}} \right)^{1-\epsilon} > V(a_{\max}, 0, 1), \tag{26}$$

*and ODE (18) is substituted with*

$$\frac{d\hat{\theta}(a)}{da} = -\frac{\epsilon V(a, I_L, \hat{\theta}) + (1-\epsilon) I_L}{V(a, I_L, \hat{\theta}) I_L b}. \tag{28}$$

*With $\epsilon < \frac{-I_H}{a_{\max}+I_H b}$, D1 equilibria of the repurchase game exist. In any D1 equilibrium, firms' strategy has the same format as in the D1 equilibrium with $\epsilon = 0$ except that ODE (9) is substituted with (27). If $\epsilon \leq \frac{-I_L}{E[a]+I_L b}$ is also satisfied, the D1 equilibrium is unique.*

The sharp contrast between the big- and small-$\epsilon$ cases in repurchases sheds light on firms' objective when conducting repurchases. On the one hand, should firms conduct repurchases to boost current share price as many financial commentators and politicians have argued, then $\epsilon$ is large (perhaps even close to 1), and Proposition 6 implies firms tend to repurchase similar size but adopt various methods with different efficiency. On the other hand, should firms care predominantly their long-term shareholders' payoff, Proposition 7 implies the opposite: firms separate on size but not on methods with different efficiency levels. Empirically, repurchase size as a fraction of the firm's market value is rather dispersed: D. Ikenberry, Lakonishok, and Vermaelen (1995) document respectively 20%, 30%, 31% and 19% of repurchases are 0-2.5%, 2.5-5%, 5-10% and above 10% of the firm value. On the other hand, firms tend to pool on the repurchase method as 95% of the repurchases are carried out in the form of an open market repurchase program. This observation suggests that repurchasing firms' main objective is likely to maximize long-term shareholders' value rather than boosting share prices in the short run.

When firms issue equity, the $\epsilon$-preference on share price only has a quantitative impact on the equilibrium outcome. The direct preference on share price (a bigger $\epsilon$) strengthens firms' preference for a higher issue price, resulting in stronger signaling incentives. As a result, the equilibrium features higher costs of signaling, reflected by firms' choices of lower issue size and efficiency. Let $i^*(a;\epsilon)$ and $\theta^*(a;\epsilon)$ denote firms' equilibrium choice of issue size and efficiency given the preference parameter $\epsilon$.

**Corollary 1.** *Firms' equilibrium issue size strategy $i^*(a;\epsilon)$ and efficiency strategy $\theta^*(a;\epsilon)$ are non-increasing in $\epsilon$.*

We conclude our discussion of Propositions 6 and 7 with a technical note on the parameter range of $\epsilon$. We do not fully characterize the equilibrium outcomes of the repurchase game for $\epsilon \in \left[\frac{-I_H}{a_{\max}-I_H b}, \frac{-I_H}{a_{\min}}\right]$. The difficulty is that for the intermediate values of $\epsilon$, the firms' overall preference on issue price is indeterminate and varies with the transaction size and efficiency. In addition, when $\epsilon > \frac{-I_L}{E[a]-I_L b}$, there might be multiple equilibria due to the fact that the boundary indifference condition that pins down $\hat{a}$ may have multiple solutions.[17]

# 6 Private Information on Project Profitability

In our main analysis, a firm's private information is about its assets in place $a$. Here, we summarize the outcomes that arise under the alternative assumption that a firm instead has private information about the profitability of the project to be implemented, i.e., $b$. A firm's financial value continues to be given by (4). For comparability with our main model, we assume that it is common knowledge that the project is profitable, i.e., $b > 0$ for investment projects and $b < 0$ for divestment projects, and that private information is solely about the level of $b$.

The implications of this alternative assumption are easy to summarize (see Proposition 8): No signaling occurs, and all firms undertake the project at its maximal scale, with maximal efficiency $\theta = 1$. We sketch the argument here, while relegating all details to the appendix.

For the issue game, this result is close to analysis in the existing literature, including Myers and Majluf (1984). In brief: It is firms with better projects (higher

[17] The cutoff type $\hat{a}$ satisfies

1. $\hat{a} = a_{\min}$, if $\hat{i}(a) < I_L$ for all $a > a_{\min}$;
2. $f(\hat{a}) = 0$, if such a $\hat{a} \in (a_{\min}, a_{\max})$;
3. $\hat{a} = a_{\max}$, if $f(a_{\max}) > 0$

for

$$f(a') \equiv \Pi\left(a', I_L, 1, V\left(E[a|a \in [a_{\min}, a']], I_L, 1\right) - I_L\right) - \left[V\left(a', \hat{i}(a';\epsilon), 1\right) - \hat{i}(a';\epsilon)\right].$$

When $\epsilon > \frac{-I_L}{E[a]-I_L b}$, the three cases may not be mutually exclusive, and (2) may be satisfied by multiple values of $\hat{a}$.

$b$) that wish to signal their type in order to increase the issue price. But both reducing project size and adopting inefficient methods are more expensive for firms with more profitable projects, and so neither approach is a viable signal.

We now turn to the repurchase game. As a starting point, recall from footnote 7 that $a = a_N + |I_H|(1 - |b|)$, where $a_N$ is non-cash assets-in-place, and post-transaction firm value is

$$V = a_N + (|I_H| - |i|) - |b|(|I_H| - \theta |i|). \tag{29}$$

In words: the value created by repurchases stems from mitigating wasteful spending inside the firm, and so firms with more to gain from repurchases (higher $|b|$) are less valuable (for any given level of repurchase).

Because of this natural property, firms that benefit most from repurchases (high $|b|$) would like to signal their types in order to reduce repurchase prices. But a reduction in the efficiency of repurchases leads to an especially large percentage reduction in the post-transaction value of such firms, and so cannot serve as a signal.

Can a smaller repurchase be used as a signal? After all, in the case in which a firm's private information is about assets in place, repurchase size is a viable signal even though efficiency is not, because of the fact that smaller repurchases increase firm value by a larger percentage for low-valued firms. However, if private information is about the benefit $|b|$ of repurchases, this same force acts against the signaling function of smaller repurchase sizes, because in this case a smaller repurchase disproportionately boosts the firm value of a low-$|b|$ firm. To see this, note that a small reduction in repurchase size affects firm value by[18]

$$-\frac{\partial \ln V}{\partial |i|} = \frac{1 - |b|}{a_N + (1 - |b|)(|I_H| - |i|)}. \tag{30}$$

The denominator embodies the same effect that arises when private information is about assets in place: 1 dollar is a larger proportion of the firm value of a worse firm (here, higher $|b|$). In contrast, the numerator embodies an offsetting

[18]Here, we focus on the case $\theta = 1$, which is the efficiency level chosen by all firms as implied by the D1 refinement.

effect specific to private information about project value $b$: reducing repurchases destroys more project value for worse firms (higher $|b|$), and hence the increase in firm value due to retained cash is smaller for worse firms. The latter effect is the dominant one, so that (30) is decreasing in $|b|$, i.e., it is better firms (lower $|b|$) that experience the greatest proportional increase in firm value from repurchasing less. Consequently, a worse firm (higher $|b|$) is unable to signal its type by reducing repurchases.

Formally, we establish:[19]

**Proposition 8.** *If the firm privately knows $b$, while $a$ is public information in the issue game, and $a_N$ is public information in the repurchase game, then in both the issue game and the repurchase game, the unique D1 equilibrium is that all firms undertake the maximum transaction ($i = I_H$) at maximal efficiency.*

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[19]Proposition 8 is established under the assumption that transaction surplus $S$ is multiplicatively separable in a firm's observable efficiency choice and its privately observed project profitability $b$. This property arises naturally for the issue game, and holds in the repurchase game for the microfoundation of footnote 7. However, this property isn't satisfied by the microfoundation of Section 4. For this latter case, we can still establish a version of Proposition 8 provided that the support of $|b|$ is a sufficiently small neighborhood $\left[0, \bar{b}\right]$. Moreover, since the proof boils down to showing $\frac{\partial^2 S(\beta,i,n)}{\partial n \partial \beta} > 0$ for all $n$ and $\beta$, which is independent of the parameters $I_H, I_L$ and $a_N$, we have numerically verifed that the result extends to the support of $\beta$ being as wide as $[0, 0.49]$. See Appendix C for details.

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# A Proofs for Section 3

**Lemma 1.** *In both issue and repurchase games, suppose $a' > a$ and $V(a, i_1, \theta_1) < V(a, i_2, \theta_2)$, then $\Pi(a, i_1, \theta_1, p_1) \geq \Pi(a, i_2, \theta_2, p_2)$ implies $\Pi(a', i_1, \theta_1, p_1) > \Pi(a', i_2, \theta_2, p_2)$.*

*If instead $V(a, i_1, \theta_1) = V(a, i_2, \theta_2)$, then $\Pi(a, i_1, \theta_1, p_1) \geq \Pi(a, i_2, \theta_2, p_2)$ if and only if $\Pi(a', i_1, \theta_1, p_1) \geq \Pi(a', i_2, \theta_2, p_2)$.*

*Proof.* (6) implies

$$\ln \Pi(a, i_1, \theta_1, p_1) - \ln \Pi(a, i_2, \theta_2, p_2) = \ln \frac{V(a, i_1, \theta_1)}{V(a, i_2, \theta_2)} - \ln \frac{1 + \frac{i_1}{p_1}}{1 + \frac{i_2}{p_2}} \geq 0. \tag{31}$$

If $\frac{V(a,i_1,\theta_1)}{V(a,i_2,\theta_2)} \in (0, 1)$, then this ratio increases in $a$. Therefore, for $a' > a$, $\ln \Pi(a, i_1, \theta_1, p_1) - \ln \Pi(a, i_2, \theta_2, p_2) > 0$.

If $V(a, i_1, \theta_1) = V(a, i_2, \theta_2)$, then $V(a', i_1, \theta_1) = V(a', i_2, \theta_2)$ holds for any $a'$ due to linearity, and

$$\ln \Pi(a', i_1, \theta_1, p_1) - \ln \Pi(a', i_2, \theta_2, p_2) = -\ln \frac{1 + \frac{i_1}{p_1}}{1 + \frac{i_2}{p_2}} = \ln \frac{\Pi(a, i_1, \theta_1, p_1)}{\Pi(a, i_2, \theta_2, p_2)} \geq 0,$$

completing the proof. $\square$

**Lemma 2.** *In a D1 equilibrium, if a type-a firm chooses* $(i^*(a), \theta^*(a))$*, then for any alternative strategy* $(i, \theta)$ *such that* $V(a, i, \theta) < V(a, i^*(a), \theta^*(a))$*, the associated price must satisfy*

$$P^*(i, \theta) \geq V(a, i, \theta) - i;$$

*For* $(i, \theta)$ *such that* $V(a, i, \theta) > V(a, i^*(a), \theta^*(a))$*,*

$$P^*(i, \theta) \leq V(a, i, \theta) - i.$$

*Proof.* We first consider the deviation to a lower $V(a, i, \theta) < V(a, i^*, \theta^*)$ and show that $(i, \theta)$ cannot be associated with any type $a' < a$ under the D1 refinement. Let $p$ be a price associated with $(i, \theta)$ such that type $a' < a$ weakly prefers to deviate:

$$\Pi(a', i, \theta, p) \geq \Pi^*(a').$$

In equilibrium, type $a'$ cannot benefit from mimicking type $a$, that is

$$\Pi^*(a') \geq \Pi\left(a', i^*(a), \theta^*(a), P^*\left(i^*(a), \theta^*(a)\right)\right),$$

which implies

$$\Pi(a', i, \theta, p) \geq \Pi\left(a', i^*(a), \theta^*(a), P^*\left(i^*(a), \theta^*(a)\right)\right).$$

Then Lemma 1 implies type $a$ strictly prefers to deviate to $(i, \theta)$ at price $p$, that is

$$\Pi(a, i, \theta, p) > \Pi(a, i^*(a), \theta^*(a), P^*\left(i^*(a), \theta^*(a)\right)) = \Pi^*(a).$$

This implies $D_{a'}(i, \theta) \subsetneq D_a(i, \theta)$, that is $a$ is more like to deviate. D1 therefore implies that the deviation $(i, \theta)$ cannot be associated with type $a$. Hence, $P^*(i, \theta) \geq V(a, i, \theta) - i$.

The other case $V(a, i, \theta) > V(a, i^*, \theta^*)$ is similar. The same logic yields that a worse firm is more likely to deviate, that is $D_{a'}(i,\theta) \subsetneq D_a(i,\theta)$ for all $a' > a$. D1 refinement implies $(i,\theta)$ cannot be associated with any type $a' > a$, and $P^*(i,\theta) \leq V(a,i,\theta) - i$. $\square$

**Proof of Proposition 1:**

We first show all types that repurchase the same size choose the same efficiency. Suppose otherwise that $(i,\theta)$ and $(i,\theta')$ are equilibrium strategies adopted by two nonempty sets of firms $A$ and $A'$, respectively. The transaction prices are therefore $p = S(E[a|a \in A], i, \theta)$ and $p' = S(E[a|a \in A'], i, \theta')$. Without loss of generality, we assume $\theta' < \theta$. Since $A'$ firms prefer $(i,\theta')$ over $(i,\theta)$, Lemma 1 implies that all $a > \inf A'$ must share the same preference. Hence, $\sup A \leq \inf A'$, and consequently $E[a|a \in A] < E[a|a \in A']$. However, this implies type $E[a|a \in A']$ strictly prefers $(i,\theta)$ to $(i,\theta')$:

$$\begin{aligned}
\Pi(E[a|a \in A'], i, \theta, p) &> \Pi(E[a|a \in A'], i, \theta, S(E[a|a \in A'], i, \theta)) \\
&= S(E[a|a \in A'], i, \theta) \\
&> S(E[a|a \in A'], i, \theta') \\
&= \Pi(E[a|a \in A'], i, \theta', p').
\end{aligned}$$

where the first inequality uses the fact that $\Pi(a,i,\theta,p)$ is decreasing in $p$ in the repurchase game. Lemma 4 further implies that there is a type in $A'$ that strictly prefers $(i,\theta)$ to $(i,\theta')$. Contradiction! Hence, there is no separation-via-efficiency given repurchase size.

We next show that in a D1 equilibrium, all repurchasing firms choose $\theta = 1$. Suppose in contrast, $(i,\theta)$ with $\theta < 1$ is an equilibrium strategy associated with the set of firms $A$. Consider any firm type $a \in A$ such that $a \leq E(A)$ and the potential deviation to a more efficient strategy $(i,1)$. Lemma 2 implies

$$P^*(i,1) \leq S(a,i,1).$$

However, under such belief, type-$a$ firm strictly prefers $(i, 1)$ to $(i, \theta)$:

$$\begin{aligned}\Pi\left(a, i, 1, P^{*}(i, 1)\right) &> \Pi\left(a, i, 1, S\left(a, i, 1\right)\right) \\ &= S\left(a, i, 1\right) \\ &> S\left(a, i, \theta\right) \\ &= \Pi\left(a, i, \theta, S\left(a, i, \theta\right)\right) \\ &> \Pi\left(a, i, \theta, P^{*}\left(i, \theta\right)\right).\end{aligned}$$

This contradiction completes the proof.

**Lemma 3.** *In an equilibrium of the issue or repurchase game, if there is an open interval $A$ such that size and method choices $i^{*}(a)$ and $\theta^{*}(a)$ are continuous, and their actions fully reveal their types, then*

$$\frac{d\left(i^{*}(a)\theta^{*}(a)b\right)}{da} = -\frac{i^{*}(a)}{V\left(a, i^{*}(a), \theta^{*}(a)\right)}. \tag{32}$$

*Conversely, if in an interval $A$ such that (32) holds, $\frac{di^{*}(a)}{da} \cdot \frac{d\theta^{*}(a)}{da} = 0$ in the issue game, $\frac{d\theta^{*}(a)}{da} = 0$ in the repurchase game, and price is fully revealing $P^{*}\left(i^{*}(a), \theta^{*}(a)\right) = S\left(a, i^{*}(a), \theta^{*}(a)\right)$, then no firm in $A$ has incentive to deviate to mimic another firm in $A$.*

*Proof.* The full revelation of types implies that for all $a \in A$,

$$\Pi^{*}(a) = S\left(a, i^{*}(a), \theta^{*}(a)\right).$$

Consider any two firm types $a_1, a_2 \in A$ such that $[a_1, a_2] \subset A$. Equilibrium conditions imply

$$\Pi\left(a_1, i^{*}(a_2), \theta^{*}(a_2), P^{*}\left(i^{*}(a_2), \theta^{*}(a_2)\right)\right) \leq \Pi^{*}(a_1), \tag{33}$$

$$\Pi\left(a_2, i^{*}(a_1), \theta^{*}(a_1), P^{*}\left(i^{*}(a_1), \theta^{*}(a_1)\right)\right) \leq \Pi^{*}(a_2). \tag{34}$$

Using the functional form of $\Pi$ and $V$, condition (33) can be explicitly written as

$$
\begin{aligned}
& \tfrac{\frac{a_1+i_2+i_2\theta_2 b}{\frac{a_2+i_2+i_2\theta_2 b}{a_2+i_2\theta_2 b}}} \leq a_1 + i_1\theta_1 b \\
\Leftrightarrow\quad & \left(V(a_2,i_2,\theta_2) - a_2 + a_1\right)\left(a_2 + i_2\theta_2 b\right) \leq \left(a_1 + i_1\theta_1 b\right) V(a_2,i_2,\theta_2) \\
\Leftrightarrow\quad & V(a_2,i_2,\theta_2)\left(a_2 + i_2\theta_2 b - a_1 - i_1\theta_1 b\right) \leq \left(a_2 - a_1\right)\left(a_2 + i_2\theta_2 b\right) \\
\Leftrightarrow\quad & V(a_2,i_2,\theta_2) + \tfrac{V(a_2,i_2,\theta_2)(i_2\theta_2 b - i_1\theta_1 b)}{a_2-a_1} \leq a_2 + i_2\theta_2 b \\
\Leftrightarrow\quad & \tfrac{V(a_2,i_2,\theta_2)(i_2\theta_2 b - i_1\theta_1 b)}{a_2-a_1} \leq -i_2 \\
\Leftrightarrow\quad & \tfrac{i_2\theta_2 - i_1\theta_1}{a_2-a_1} b \leq -\tfrac{i_2}{V(a_2,i_2,\theta_2)}.
\end{aligned}
$$

Similarly, condition (34) yields $\frac{i_1\theta_1 - i_2\theta_2}{a_1-a_2} b \geq -\frac{i_1}{V(a_1,i_1,\theta_1)}$, and hence

$$
-\frac{i^*(a_1)}{V\left(a_1, i^*(a_1), \theta^*(a_1)\right)} \leq \frac{i^*(a_2)\theta^*(a_2) - i^*(a_1)\theta^*(a_1)}{a_2 - a_1} b \leq -\frac{i^*(a_2)}{V\left(a_2, i^*(a_2), \theta^*(a_2)\right)}.
$$

Since $i^*(a)$ and $\theta^*(a)$ and hence $V\left(a, i^*(a), \theta^*(a)\right)$ are continuous on $A$, the squeeze theorem implies (32).

To see that (32) deters deviation, notice the partial derivatives of $\Pi\left(a,i,\theta,p\right)$ satisfy

$$
\frac{\Pi_2\left(a,i,\theta,p\right)}{\Pi\left(a,i,\theta,p\right)} = \frac{1+\theta b}{V\left(a,i,\theta\right)} - \frac{1}{p+i},
$$

$$
\frac{\Pi_3\left(a,i,\theta,p\right)}{\Pi\left(a,i,\theta,p\right)} = \frac{ib}{V\left(a,i,\theta\right)},
$$

and

$$
\frac{\Pi_4\left(a,i,\theta,p\right)}{\Pi\left(a,i,\theta,p\right)} = \frac{i}{p\left(p+i\right)}.
$$

Consider the payoff of a type-$a$ firm mimicking $a'$:

$$
\begin{aligned}
& \frac{d}{da'}\Pi\left(a, i^*(a'), \theta^*(a'), S\left(a', i^*(a'), \theta^*(a')\right)\right) \\
& = \Pi_2 \frac{di^*(a')}{da'} + \Pi_3 \frac{d\theta^*(a')}{da'} + \Pi_4 \frac{dS\left(a', i^*(a'), \theta^*(a')\right)}{da'}.
\end{aligned}
$$

Condition (32) implies that at $a' = a$,

$$
\begin{aligned}
&\tfrac{d}{da'}\Pi\left(a, i^*(a'), \theta^*(a'), S\left(a', i^*(a'), \theta^*(a')\right)\right)|_{a'=a} \\
&= \Pi \cdot \left[\tfrac{\theta b}{V(a,i^*(a),i^*(a))}\tfrac{di^*(a')}{da'} + \tfrac{ib}{V(a,i^*(a),i^*(a))}\tfrac{d\theta^*(a')}{da'} + \tfrac{i}{S(a,i^*(a),i^*(a))V(a,i^*(a),i^*(a))}\left(1 + \tfrac{d(i^*(a)\theta^*(a)b)}{da}\right)\right] \\
&= 0.
\end{aligned}
\tag{35}
$$

Furthermore, note that in the decomposition (35), $\frac{\Pi_2(a,i,\theta,p)}{\Pi(a,i,\theta,p)}$ is strictly decreasing in $a$, and $\frac{\Pi_3(a,i,\theta,p)}{\Pi(a,i,\theta,p)}$ is strictly decreasing in $a$; According to (32), in the issue game, that $\frac{di^*(a)}{da} \cdot \frac{d\theta^*(a)}{da} = 0$ implies $\frac{di^*(a)}{da} < 0$ and $\frac{d\theta^*(a)}{da} = 0$ or $\frac{di^*(a)}{da} = 0$ and $\frac{d\theta^*(a)}{da} < 0$; in the repurchase game, that $\frac{d\theta^*(a)}{da} = 0$ implies $\frac{di^*(a)}{da} < 0$. Therefore, when $a' \gtrless a$, we have

$$\frac{d}{da'}\Pi\left(a, i^*(a'), \theta^*(a'), S\left(a', i^*(a'), \theta^*(a')\right)\right) \lesseqgtr 0. \tag{36}$$

Hence, type-$a$ firm has no incentives to mimic any other firm in $A$, completing the proof. □

**Proof of Proposition 2:**

*Proof.* Proposition 1 implies that all repurchasing firms use method $\theta = 1$ in D1 equilibria. Furthermore, Lemma 1 implies that better firms repurchase more, that is $|i^*(a)|$ is weakly increasing (or $i^*(a)$ is weakly decreasing). By assumption (2), all types repurchase. Therefore, there is a lower interval of firms $[a_{\min}, \hat{a})$ that repurchase the minimum size ($i = I_L$) and an upper interval of firms $(\hat{a}, a_{\max}]$ that repurchase more ($i < I_L$). One of the two intervals may be empty.

Step 1. Firms with $a > \hat{a}$ separate on different $i$.

Suppose otherwise, there is an equilibrium repurchase strategy $(i, 1)$ for some $i < I_L$ adopted by a non-singleton set of firms $A$. Consider type $a' < E[a|a \in A]$ in $A$. Since $P^*(i, 1) = S(E[a|a \in A], i, 1) > S(a', i, 1)$, the equilibrium payoff of type-$a'$ firm satisfies

$$\Pi\left(a', i, 1, P^*(i, 1)\right) < S\left(a', i, 1\right).$$

Since $\lim_{i' \downarrow i} S(a', i', 1) = S(a', i, 1)$, it is possible to choose an $i' > i$ (i.e., repurchase less $|i'| < |i|$), such that

$$S\left(a', i', 1\right) \in \left(\Pi\left(a', i, 1, P^*(i, 1)\right), S\left(a', i, 1\right)\right)$$

Lemma 2 implies that under D1, the price associated with the potential deviation $(i', 1)$ satisfies

$$P^*\left(i', 1\right) \leq S\left(\inf A, i', 1\right).$$

Type-$a'$ firm therefore strictly prefers $(i', 1)$ to $(i, 1)$:

$$\Pi\left(a', i', 1, P^*(i', 1)\right) \geq S\left(a', i', 1\right) > \Pi\left(a', i, 1, P^*(i, 1)\right),$$

contradiction.

Step 2. For $a > \hat{a}$, $i^*(a)$ satisfies (9).

Since $i^*(a)$ is monotonic, it is sufficient to show $i^*(a)$ has no jump on $(\hat{a}, a_{\max}]$, which in turn implies continuity, and Lemma 3 establishes (9).

Suppose in contrast, there exists $a^* > \hat{a}$ such that

$$\bar{i} \equiv \lim_{a \uparrow a^*} i^*(a) > \lim_{a \downarrow a^*} i^*(a) \equiv \underline{i}.$$

For any $i \in [\underline{i}, \bar{i}]$, $V(a, i, 1) > V(a, \underline{i}, 1) > V(a, i^*(a), 1)$ for any $a > a^*$ and $V(a, i, 1) < V(a, \bar{i}, 1) < V(a, i^*(a), 1)$ for any $a < a^*$. Lemma 2 implies

$$\sup_{a < a^*} S(a, i, 1) \leq P^*(i, 1) \leq \inf_{a > a^*} S(a, i, 1)$$

and in particular for $i = \underline{i}$,

$$P^*(\underline{i}, 1) = S(a^*, \underline{i}, 1).$$

Combined with $|\underline{i}| > |\bar{i}|$, this implies

$$\Pi\left(a^*, \underline{i}, 1, P^*(\underline{i}, 1)\right) = S(a^*, \underline{i}, 1) > S(a^*, \bar{i}, 1). \tag{37}$$

Let $\epsilon < \frac{S(a^*, \underline{i}, 1) - S(a^*, \bar{i}, 1)}{2}$ be a small constant. By continuity, there exists a type

$a \in (\hat{a}, a^*)$ sufficiently close to $a^*$ such that

$$\Pi\left(a, \underline{i}, 1, P^*(\underline{i}, 1)\right) > S(a^*, \underline{i}, 1) - \epsilon,$$

and type-$a$'s equilibrium payoff

$$\Pi^*(a) = S(a, i^*(a), 1) < S(a^*, \bar{i}, 1) + \epsilon.$$

The choice of $\epsilon$ implies that type $a$ benefits from deviating to $(\underline{i}, 1)$:

$$\Pi\left(a, \underline{i}, 1, P^*(\underline{i}, 1)\right) > \Pi^*(a),$$

a contradiction.

Step 3. If type $a_{\max}$ chooses $i < I_L$, it chooses $i = I_H$.

Suppose in contrast, type $a_{\max}$ chooses repurchase size $i \in (I_H, I_L)$. Step 1 implies type $a_{\max}$ is fairly priced and has payoff

$$\Pi^*(a_{\max}) = S(a_{\max}, i, 1).$$

Lemma 2 implies the price associated with the deviation $(I_H, 1)$

$$P^*(I_H, 1) = S(a_{\max}, I_H, 1),$$

and hence

$$\begin{aligned}\Pi\left(a_{\max}, I_H, 1, P^*(I_H, 1)\right) &= S\left(a_{\max}, I_H, 1\right) \\ &> S\left(a_{\max}, i, 1\right) \quad , \\ &= \Pi^*(a_{\max})\end{aligned}$$

that is type $a_{\max}$ benefits from deviating to $(I_H, 1)$, contradiction!

Step 4. If $a_{\min}$ and $a_{\max}$ are close enough, such that $\hat{i}(a) < I_L$ for all $a$, where $\hat{i}$ is the unique solution to the ODE (9) and boundary condition (10), then all types repurchase strictly more than $|I_L|$, i.e., $\hat{a} = a_{\min}$.

Suppose in contrast, the solution to the ODE $\hat{i}(a) < I_L$ for all $a$, yet $\hat{a} > a_{\min}$.

Lemma 3 implies

$$\Pi\left(\hat{a},\hat{i}(\hat{a}),1,S\left(\hat{a},\hat{i}(\hat{a}),1\right)\right)>\Pi\left(\hat{a},\hat{i}(a_{\min}),1,S\left(a_{\min},\hat{i}(a_{\min}),1\right)\right),$$

that is type $\hat{a}$ prefers $\left(\hat{i}(\hat{a}),1\right)$ at the equilibrium price over $\left(\hat{i}(a_{\min}),1\right)$ at price $S\left(a_{\min},\hat{i}(a_{\min}),1\right)$.

On the other hand, type $a_{\min}$ prefers $\left(\hat{i}(a_{\min}),1\right)$ at price $S\left(a_{\min},\hat{i}(a_{\min}),1\right)$ over $(I_L,1)$ at the equilibrium price $S\left(E\left[a|a\in[a_{\min},\hat{a})\right],I_L,1\right)$, since the latter leads to lower surplus and more unfavorable (meaning higher) market belief:

$$\begin{aligned}\Pi\left(a_{\min},\hat{i}(a_{\min}),1,S\left(a_{\min},\hat{i}(a_{\min}),1\right)\right)&=S\left(a_{\min},\hat{i}(a_{\min}),1\right)\\&>S\left(a_{\min},I_L,1\right)\\&=\Pi\left(a_{\min},I_L,1,S\left(a_{\min},I_L,1\right)\right)\\&>\Pi\left(a_{\min},I_L,1,S\left(E\left[a|a\in[a_{\min},\hat{a})\right],I_L,1\right)\right)\end{aligned}.$$

Since ,$\hat{i}(a_{\min})<I_L$, Lemma 1 implies type $\hat{a}$ has the same preference. Therefore, type $\hat{a}$ strictly prefers $\left(\hat{i}(\hat{a}),1\right)$ over $(I_L,1)$. By continuity, some type $a\in(a_{\min},\hat{a})$ has the same preference, and hence deviates to $\left(\hat{i}(\hat{a}),1\right)$, leading to a contradiction. □

**Proof of Proposition 3:**

*Proof.* Following Propositions 1 and 2, we only need to show that the previously constructed cutoff $\hat{a}$ uniquely exists, and the conjectured equilibrium indeed satisfies D1 refinement.

We first show the uniqueness of $\hat{a}$. Proposition 2 establishes the unique $\hat{a}=a_{\min}$ when the range of $[a_{\min},a_{\max}]$ is small enough such that $\hat{i}(a)<I_L$ for all $a$. Now, suppose that the range of $[a_{\min},a_{\max}]$ is big enough such that there exists an $a_0>a_{\min}$ such that $\hat{i}(a_0)=I_L$. In this case, it must be that $\hat{a}>a_{\min}$.

The equilibrium payoff must be a continuous function with respect to firm types because it is the higher of the full separation payoff $S\left(\hat{a},\hat{i}\left(\hat{a}\right),1\right)$ implied by the ODE (9) and the pooling payoff at $(I_L,1)$ — both are continuous. Therefore, if

$\hat{a} < a_{\max}$, type-$\hat{a}$ firm must be indifferent between $\left(\hat{i}(\hat{a}), 1\right)$ and $(I_L, 1)$, which implies that $\hat{a}$ solves (13). To show the uniqueness of $\hat{a}$, it is sufficient to show for $\hat{a}' > \hat{a}$, (13) holds with inequality $>$.

Suppose $\hat{a}$ satisfies (13). Lemma 3 implies type $\hat{a}' > \hat{a}$ strictly prefers $\left(\hat{i}(\hat{a}'), 1\right)$ at price $S\left(\hat{a}', \hat{i}(\hat{a}'), 1\right)$ to $\left(\hat{i}(\hat{a}), 1\right)$ at price $S\left(\hat{a}, \hat{i}(\hat{a}), 1\right)$. Lemma 1 implies type $\hat{a}'$ strictly prefers $\left(\hat{i}(\hat{a}), 1\right)$ at price $S\left(\hat{a}, \hat{i}(\hat{a}), 1\right)$ to $(I_L, 1)$ at price $E\left[S\left(a, I_L, 1\right) | a \in [a_{\min}, \hat{a})\right]$. That firm payoff decreases with repurchase price implies type $\hat{a}'$ strictly prefers $(I_L, 1)$ at price $E\left[S\left(a, I_L, 1\right) | a \in [a_{\min}, \hat{a})\right]$ to $(I_L, 1)$ at price $E\left[S\left(a, I_L, 1\right) | a \in [a_{\min}, \hat{a}')\right]$. The above chain implies type $\hat{a}'$ strictly prefers $\left(\hat{i}(\hat{a}'), 1\right)$ at price $S\left(\hat{a}', \hat{i}(\hat{a}'), 1\right)$ to $(I_L, 1)$ at price $E\left[S\left(a, I_L, 1\right) | a \in [a_{\min}, \hat{a}')\right]$, which implies (13) holds for $\hat{a}'$ with inequality $>$.

If $\hat{a} = a_{\max}$, then all firms with $a < a_{\max}$ prefer $(I_L, 1)$ over $\left(\hat{i}(a_{\max}), 1\right)$, which implies (13) holds with inequality $\leq$ at $a_{\max}$. Suppose in this case (13) has a solution $\hat{a}'$, then the proof above concludes for $a_{\max} > \hat{a}'$, (13) holds with inequality $>$, contradiction. Hence, condition (13) has no solution.

The remainder of the proof shows that the conjectured equilibrium indeed satisfies D1 refinement.

Step 1. No firm mimics another type.

It follows Lemma 3 that types with $a > \hat{a}$ do not mimic each other.

If $\hat{a} > a_{\min}$, (13) holds with $\leq$, which implies type $\hat{a}$ weakly prefers $(I_L, 1)$ over $\left(\hat{i}\left(\hat{a}\right), 1\right)$, and hence over $\left(\hat{i}\left(a\right), 1\right)$ for any $a \geq \hat{a}$. Lemma 1 hence implies types with $a < \hat{a}$ do not mimic $a \geq \hat{a}$.

If $\hat{a} < a_{\max}$, (13) holds with $\geq$, which implies type $\hat{a}$ weakly prefers $\left(\hat{i}\left(\hat{a}\right), 1\right)$ over $(I_L, 1)$. Lemma 3 implies types with $a > \hat{a}$ has the same preference, and therefore do not mimic $a < \hat{a}$ and deviate to $(I_L, 1)$.

Step 2. No firm deviates to an off-equilibrium action $(i, \theta)$ with $i\left(1 + \theta b\right) < i^*\left(a_{\max}\right)\left(1 + b\right)$.

Lemma 2 implies $P^*\left(i, \theta\right) = S\left(a_{\max}, i, \theta\right)$. Meanwhile, $P^*\left(I_H, 1\right) = S\left(a_{\max}, I_H, 1\right)$. Therefore, type $a_{\max}$ is fairly priced under both $(i, \theta)$ and $(I_H, 1)$. It prefers $(I_H, 1)$

since it leads to higher surplus:

$$\begin{aligned}\Pi\left(a_{\max}, I_H, 1, P^*\left(I_H, 1\right)\right) &= S\left(a_{\max}, I_H, 1\right) \\ &> S\left(a_{\max}, i, \theta\right) \\ &= \Pi\left(a_{\max}, i, \theta, P^*\left(i, \theta\right)\right).\end{aligned}$$

If $\hat{a} < a_{\max}$, $i^*\left(a_{\max}\right) = I_H$. If $\hat{a} = a_{\max}$, (13) holds with $\leq$, implying $a_{\max}$ weakly prefers $\left(i^*\left(a_{\max}\right), 1\right)$ to $\left(I_H, 1\right)$. In both cases, type $a_{\max}$ prefers $\left(i^*\left(a_{\max}\right), 1\right)$ to $(i, \theta)$. Lemma 1 then implies all types have the same preference. Since no type mimics type $a_{\max}$ according to Step 1, no type deviates to $(i, \theta)$.

Step 3. No firm deviates to an off-equilibrium action $(i, \theta)$ with

$$i\left(1+\theta b\right) \in \left[i^*\left(a_{\max}\right)\left(1+b\right), \hat{i}\left(\hat{a}\right)\left(1+b\right)\right]. \tag{38}$$

Condition (38) and the continuity of $\hat{i}(\cdot)$ imply that there is $\bar{a} \in \left[\hat{a}, a_{\max}\right]$ such that

$$i\left(1+\theta b\right) = \hat{i}\left(\bar{a}\right)\left(1+b\right). \tag{39}$$

Since $\theta \leq 1$, condition (39) implies that $|i| < |\hat{i}\left(\bar{a}\right)|$, which in turn implies that $i\theta b < \hat{i}\left(\bar{a}\right) b$. Lemma 2 implies $P^*\left(i, \theta\right) = S\left(\bar{a}, i, \theta\right)$. Similar to Step 2, type $\bar{a}$ is fairly priced under both $(i, \theta)$ and $\left(\hat{i}\left(\bar{a}\right), 1\right)$, and therefore it weakly prefers $\left(\hat{i}\left(\bar{a}\right), 1\right)$ — the one with weakly higher fundamental value $\bar{a} + \hat{i}\left(\bar{a}\right) b$. Lemma 1 implies all types weakly prefer $\left(\hat{i}\left(\bar{a}\right), 1\right)$ over $(i, \theta)$. Since no type deviates to $\left(\hat{i}\left(\bar{a}\right), 1\right)$ according to Step 1, no type deviates to $(i, \theta)$.

Step 4. No firm deviates to an off-equilibrium action $(i, \theta)$ with $i\left(1+\theta b\right) \in \left(\hat{i}\left(\hat{a}\right)\left(1+b\right), I_L(1+b)\right]$.

Repeating the analysis in Step 3, we have $|i| < |\hat{i}\left(\bar{a}\right)|$ and $i\theta b < \hat{i}\left(\hat{a}\right) b$. Lemma 2 implies $P^*\left(i, \theta\right) = S\left(\hat{a}, i, \theta\right)$. Hence, type $\hat{a}$ is fairly priced under both choices $(i, \theta)$ and $\left(\hat{i}\left(\hat{a}\right), 1\right)$, with the latter generating higher surplus. Therefore, type $\hat{a}$ prefers $\left(\hat{i}\left(\hat{a}\right), 1\right)$ to $(i, \theta)$. Lemma 1 implies all types $a > \hat{a}$ have the same preference, and hence do not deviate to $(i, \theta)$. If $\hat{a} > a_{\min}$, then (13) holds with $\leq$.

Type $\hat{a}$ weakly prefers $(I_L, 1)$ to $\left(\hat{i}\left(\hat{a}\right), 1\right)$, and hence to $(i, \theta)$. Lemma 1 implies all types $a < \hat{a}$ have the same preference, and hence do not deviate to $(i, \theta)$ $\square$

**Proof of Proposition 4:**

*Proof.* Suppose in contrast, in a D1 equilibrium of the issue game, firm types in set $A$ choose $(i, \theta)$ such that $i > I_L$ and $\theta < 1$. We can choose $i' < i$ and a corresponding $\theta' \in \left(\frac{i\theta}{i'}, \left(\frac{i(1+\theta b)}{i'} - 1\right) b^{-1}\right)$, which imply that $i'(1+\theta' b) < i(1+\theta b)$ and $i'\theta' b > i\theta b$. Lemma 2 then implies

$$P^*\left(i', \theta'\right) \geq S\left(E\left[a|a \in A\right], i', \theta'\right) > S\left(E\left[a|a \in A\right], i, \theta\right).$$

This implies $(i', \theta')$ leads to higher issue surplus and better issue price than $(i, \theta)$. Then firm $a' \in A$ with $a' \geq E\left[a|a \in A\right]$ strictly prefers $(i', \theta')$ to $(i, \theta)$:

$$\begin{aligned}
\Pi\left(a', i', \theta', P^*(i', \theta')\right) &\geq \frac{a' + i' + i'\theta' b}{1 + \frac{i'}{S(E[a|a \in A], i', \theta')}} \\
&> \frac{a' + i + i'\theta' b}{1 + \frac{i}{S(E[a|a \in A], i', \theta')}} \\
&\geq \frac{a' + i + i\theta b}{1 + \frac{i}{S(E[a|a \in A], i, \theta)}} \\
&= \Pi\left(E\left[a|a \in A\right], i, \theta, P^*(i, \theta)\right)
\end{aligned},$$

where the second inequality is because $i' < i$ and $a' + i'\theta' b > S\left(E\left[a|a \in A\right], i', \theta'\right)$. This contradiction completes the proof. $\square$

**Lemma 4.** *For any set of firm types $A$, if type $E[a|a \in A]$ strictly prefers $(i', \theta')$ to $(i, \theta)$ given their prices, then there is some type from $A$ that has the same preference.*

*Proof.* Suppose type $E[a|a \in A]$ strictly prefers $(i', \theta')$ to $(i, \theta)$.

Suppose $V(a, i', \theta') < V(a, i, \theta)$. Lemma 1 implies type $a' \in A$ with $a' \geq E[a|a \in A]$ also strictly prefers $(i', \theta')$ to $(i, \theta)$.

Suppose $V(a, i', \theta') > V(a, i, \theta)$. Lemma 1 implies type $a' \in A$ with $a' \leq E[a|a \in A]$ also strictly prefers $(i', \theta')$ to $(i, \theta)$.

Suppose $V(a, i', \theta') = V(a, i, \theta)$. Lemma 1 implies all types strictly prefers $(i', \theta')$ to $(i, \theta)$. $\square$

**Proof of Proposition 5:**

*Proof.* We first show the uniqueness of the D1 equilibrium.

Proposition 4 guarantees a firm either chooses $(i, 1)$ for some $i$ or $(I_L, \theta)$ for some $\theta$. Lemma 1 implies firms' choice of $i$, denoted by $i^*(a)$ for type $a$, and choice of $\theta$, denoted by $\theta^*(a)$ for type $a$, are weakly decreasing with $a$. Therefore, there is a cutoff $\hat{a}$ such that types with $a > \hat{a}$ choose $(i, 1)$ for some $i$ and types with $a < \hat{a}$ choose $(I_L, \theta)$ for some $\theta$.

Step 1. No type chooses $(I_L, 0)$ in equilibrium.

Assumption (1) implies type $a_{\max}$ strictly prefers $(I_L, 1)$ to $(0, 1)$ (doing nothing) under any prices. On the other hand, type $a_{\max}$ prefers $(0, 1)$ over $(I_L, 0)$ under any prices, the latter of which implies zero surplus but weak under-pricing:

$$\begin{aligned}\Pi\left(a_{\max}, 0, 1, P^*\left(0, 1\right)\right) &= a_{\max} \\ &= S\left(a_{\max}, I_L, 0\right) \\ &= \Pi\left(a_{\max}, I_L, 0, S\left(a_{\max}, I_L, 0\right)\right) \\ &\geq \Pi\left(a_{\max}, I_L, 0, P^*\left(I_L, 0\right)\right)\end{aligned}.$$

Therefore, type $a_{\max}$ strictly prefers $(I_L, 1)$ to $(I_L, 0)$. According to Lemma 1, all types have the same preference. Therefore, no type chooses $(I_L, 0)$ in equilibrium.

Step 2. All types separate on different pairs of $(i, \theta)$.

Suppose in contrast, types in set $A$ pool on $(i, \theta)$ in equilibrium.

$$P^*\left(i, \theta\right) = S\left(E\left[a | a \in A\right], i, \theta\right).$$

According to step 1, $(i, \theta) \neq (I_L, 0)$. There is $(i', \theta')$ such that $i'\left(1 + \theta' b\right) < i\left(1 + \theta b\right)$. Lemma 2 implies

$$P^*\left(i', \theta'\right) \geq S\left(\sup A, i', \theta'\right).$$

Types in $A$ benefit from deviating to $(i', \theta')$ that are close enough to $(i, \theta)$, because it leads to marginal changes in $i$ and $\theta$ but a discrete improvement in market belief. This leads to a contradiction.

Step 3. $i^*(a)$ and $\theta^*(a)$ are continuous.

Given $i^*(a)$ and $\theta^*(a)$ are decreasing, it is sufficient to show there is no jump. Suppose in contrast, there is $a^*$ such that

$$\lim_{a \uparrow a^*} i^*(a) > \lim_{a \downarrow a^*} i^*(a)$$

or

$$\lim_{a \uparrow a^*} \theta^*(a) > \lim_{a \downarrow a^*} \theta^*(a).$$

Let $\bar{i} \equiv \lim_{a \uparrow a^*} i^*(a)$, $\underline{i} \equiv \lim_{a \downarrow a^*} i^*(a)$, $\bar{\theta} \equiv \lim_{a \uparrow a^*} \theta^*(a)$, $\underline{\theta} \equiv \lim_{a \downarrow a^*} \theta^*(a)$. Since all types fully separate, as $a$ approaches $a^*$ from above, their equilibrium payoff, $S(a, i^*(a), \theta^*(a))$, approaches $S(a^*, \underline{i}, \underline{\theta})$:

$$\lim_{a \downarrow a^*} \Pi^*(a) = \lim_{a \downarrow a^*} S(a, i^*(a), \theta^*(a)) = S(a^*, \underline{i}, \underline{\theta}).$$

Lemma 2 implies

$$P^*\left(\bar{i}, \bar{\theta}\right) = S\left(a^*, \bar{i}, \bar{\theta}\right).$$

As $a$ approaches $a^*$ from above, their payoff from deviating to $\left(\bar{i}, \bar{\theta}\right)$ approaches $S\left(a^*, \bar{i}, \bar{\theta}\right)$:

$$\lim_{a \downarrow a^*} \Pi\left(a, \bar{i}, \bar{\theta}, P^*\left(\bar{i}, \bar{\theta}\right)\right) = \Pi\left(a^*, \bar{i}, \bar{\theta}, P^*\left(\bar{i}, \bar{\theta}\right)\right) = S\left(a^*, \bar{i}, \bar{\theta}\right).$$

Since $S(a^*, \underline{i}, \underline{\theta}) < S\left(a^*, \bar{i}, \bar{\theta}\right)$, there $a > a^*$ such that

$$\Pi^*(a) < \Pi\left(a, \bar{i}, \bar{\theta}, P^*\left(\bar{i}, \bar{\theta}\right)\right),$$

that is type $a$ deviates to $\left(\bar{i}, \bar{\theta}\right)$. This leads to a contradiction.

Step 4. Lemma 3 implies for $a > \hat{a}$, $i^*(a)$ is differentiable and satisfies ODE (9); for $a < \hat{a}$, $\theta^*(a)$ is differentiable and satisfies ODE (18).

Step 5. Type $a_{\min}$ chooses $(I, 1)$.

Suppose in contrast, $a_{\min}$ chooses $(i, \theta) \neq (I, 1)$. This implies $i\theta < I$. Type $a_{\min}$'s equilibrium payoff is $S(a_{\min}, i, \theta)$. If type $a_{\min}$ deviates to $(I, 1)$, it receives higher surplus and no worse market belief:

$$\begin{aligned}
\Pi\left(a_{\min}, I, 1, P^*(I, 1)\right) &\geq \Pi\left(a_{\min}, I, 1, S\left(a_{\min}, I, 1\right)\right) \\
&= S\left(a_{\min}, I, 1\right) \\
&> S\left(a_{\min}, i, \theta\right).
\end{aligned}$$

Therefore, type $a_{\min}$ deviates to $(I, 1)$, leading to a contradiction.

Next, we show the existence of the D1 equilibrium by showing the strategy is indeed incentive compatible under a D1 belief.

Step 1. There is enough space for all types to separate according to $\hat{i}(\cdot)$ and $\hat{\theta}(\cdot)$.

Suppose in contrast, there is $a > \hat{a}$ such that $\hat{\theta}(a) = 0$. Lemma 3 implies type $a$ strictly prefers $(I_L, 0)$ under belief $a$ over $(I_L, 1)$ under belief $\hat{a}$.

On the other hand, under Assumption (1), type $a_{\max}$ prefers $(I_L, 1)$ under the worst belief, $a_{\min}$, to doing nothing, the latter of which leads to payoff $a_{\max}$, and hence is equivalent to choosing $(I_L, 0)$ under the correct belief $a_{\max}$. This implies type $a_{\max}$also prefers $(I_L, 1)$ under belief $\hat{a}$ to $(I_L, 0)$ under belief $a_{\max}$. Lemma 1 implies all types have the same preference. Therefore, type $a$ prefers $(I_L, 1)$ under belief $\hat{a}$ to $(I_L, 0)$ under belief $a$. This leads to a contradiction.

Step 2. Lemma 3 implies no type benefits from mimicking another type.

Step 3. Under a D1 belief, no type benefits from deviating to an off-equilibrium action $(i, \theta)$ that satisfies

$$i(1 + \theta b) > \hat{i}(a_{\max}) \left(1 + \hat{\theta}(a_{\max}) b\right). \tag{40}$$

(40) implies there is $a^*$ such that

$$i(1 + \theta b) = \hat{i}(a^*) \left(1 + \hat{\theta}(a^*) b\right),$$

that is $(i, \theta)$ leads to the same financial value as type $a^*$'s equilibrium choice. Lemma 2 implies $P^*(i, \theta) = S(a^*, i, \theta)$. Since $(i, \theta)$ is off-equilibrium, $i > \hat{i}(a^*)$, which implies $i\theta b < \hat{i}(a^*)\,\hat{\theta}(a^*)\,b$. Since type $a^*$ is fairly priced under both $(i,\theta)$ and its equilibrium choice $\left(\hat{i}(a^*),\hat{\theta}(a^*)\right)$, it prefers $\left(\hat{i}(a^*),\hat{\theta}(a^*)\right)$, which leads to higher surplus:

$$\begin{aligned}
\Pi\left(a^*,\hat{i}(a^*),\hat{\theta}(a^*),P^*\left(\hat{i}(a^*),\hat{\theta}(a^*)\right)\right) &= S\left(a^*,\hat{i}(a^*),\hat{\theta}(a^*)\right) \\
&> S(a^*,i,\theta) \\
&= \Pi(a^*,i,\theta,P^*(i,\theta)).
\end{aligned}$$

Lemma 1 implies all types have the same preference. By Lemma 3, no type benefits from deviating to $(i,\theta)$.

Step 4. No type benefits from deviating to an off-equilibrium action $(i,\theta)$ that satisfies

$$i(1+\theta b) \leq \hat{i}(a_{\max})\left(1+\hat{\theta}(a_{\max})\,b\right). \tag{41}$$

Lemma 2 implies

$$P^*(i,\theta) = S(a_{\max},i,\theta).$$

(41) implies $i\theta < \hat{i}(a_{\max})\,\hat{\theta}(a_{\max})$. Therefore, type $a_{\max}$ prefers $\left(\hat{i}(a_{\max}),\hat{\theta}(a_{\max})\right)$ over $(i,\theta)$ since the latter leads to lower surplus:

$$\begin{aligned}
\Pi(a_{\max},i,\theta,P^*(i,\theta)) &= S(,i,\theta) \\
&< S\left(a_{\max},\hat{i}(a_{\max}),\hat{\theta}(a_{\max})\right) \\
&= \Pi\left(a_{\max},\hat{i}(a_{\max}),\hat{\theta}(a_{\max}),P^*\left(\hat{i}(a_{\max}),\hat{\theta}(a_{\max})\right)\right)
\end{aligned}.$$

Lemma 1 implies all types have the same preference. Lemma 3 implies no type benefits from deviating to $(i,\theta)$. This completes the proof. □

# B Optionality in Gradual Methods

We show that the equilibrium outcomes remain unchanged in the modification of the model outlined in Footnote 8, which incorporates firms' option to privately choose the actual issue or repurchase size $i^A$ with the announced size $i$ as an upper bound. In particular, firms choose to issue or repurchase the full size that is announced, $i^A = i$, and the unique D1 equilibria of the repurchase game and the issue game are still those described in Proposition 3 and 5.

We redefine D1 belief by substituting (8) with

$$D_a(i, \theta) = \left\{p : \Pi\left(a, i^A, \theta, p\right) > \Pi^*(a) \quad \exists \left|i^A\right| \in \left[\left|I_L\right|, |i|\right]\right\}. \tag{42}$$

D1 hence requires the belief associated with an announced size $i$ and efficiency $\theta$ should only be supported on types $a$ that can benefit from these choices combined with any private choice of $\left|i^A\right| \in \left[\left|I_L\right|, |i|\right]$ under the largest range of prices.

That Proposition 3 and 5 continue to hold follow the following property:

**Lemma 5.** *If*

$$\Pi\left(a, i^A, \theta, p\right) = \frac{a + i^A(1 + \theta b)}{1 + \frac{i^A}{p}} > a, \tag{43}$$

*then* $\Pi\left(a, i^A, \theta, p\right)$ *strictly increases with* $\left|i^A\right|$.

*Proof.* In the issue game, (43) is equivalent to

$$a - (1 + \theta b)\, p < 0,$$

which implies

$$\Pi\left(a, i^A, \theta, p\right) = (1 + \theta b)\, p + \frac{a - (1 + \theta b)\, p}{1 + \frac{i^A}{p}} \tag{44}$$

increases with $i^A$. In the repurchase game, (43) is equivalent to

$$a - (1 + \theta b)\, p > 0.$$

Since $i^A < 0$, this implies (44) decreases with $i^A$, that is, increases with $\left|i^A\right|$. □

By Lemma 5, firms equilibrium choices should satisfy $i^A = i$. Moreover, Lemma 5 implies in any equilibrium or conjectured equilibrium, $D_a(i, \theta)$ is the same as that defined in the original game (8), and hence the set of beliefs that satisfy D1 is the same as in the original game.

Conjecture an equilibrium with an arbitrary equilibrium strategy that satisfies $i = i^A$ and with a D1 belief. Type $a$ benefits from deviating to a public choice $(i', \theta')$

with any private choice $\left|i^A\right| \in [|I_L|, |i|]$ if and only if it benefits from choosing $(i', \theta')$ with $i^A = i$. This is equivalent to that in the conjectured equilibrium of the original game with the same strategy of $(i, \theta)$ choices and the same belief, type $a$ benefits from deviating to $(i', \theta')$. Therefore,

**Proposition 9.** *A strategy $\left(i(a), \theta(a), i^A(a)\right)$ supports a D1 equilibrium of the modified issue or repurchase game that incorporates optionality if and only if $i^A(a) = i(a)$ and $(i(a), \theta(a))$ supports a D1 equilibrium of the original game.*

# C Supplements to Section 6

We first provide the proof for Proposition 8 under the functional form (29) of $V$. To be more specific, we outline the following

The project profitability $b$ is distributed between $b_L$ and $b_H$. In the issue game, $b_H > b_L > 0$. In the repurchase game, $-1 < b_H < b_L < 0$. The firm maximizes existing shareholders' value, which is written as a function of the firm's type $b$, the firm's choice $(i, \theta)$ and market belief $b^E$:

$$\ln \Pi\left(b, i, \theta, b^E\right) = \ln V(b, i, \theta) - \ln\left[1 + \frac{i}{V(b^E, i, \theta) - i}\right],$$

where

$$V(b, i, \theta) \equiv a(b) + i + i\theta b$$

is the total firm value. In the issue game, $a(b) = a$ is the firm's assets in place which is public information. In the repurchase game, as assumed in the micro-foundation (29),

$$a(b) = a_N + |I_H|(1 - |b|)$$

decreases with $|b|$, and hence

$$V(b, i, \theta) = a_N + (|I_H| - |i|) - (|I_H| - \theta|i|)|b|$$

coincides with (29).

**Proof of Proposition 8 for the issue game:**

We first show that in a D1 equilibrium, all types choose size $I_H$ and efficiency 1.

We show this by contradiction. Conjecture an equilibrium in which types in $B$ choose $(i, \theta)$ with $|i\theta| < |I|$. Since

$$\frac{\partial \ln \Pi\left(b, i, \theta, b^E\right)}{\partial b^E} > 0,$$

for each type $b$, there is a $\bar{b}^E(b)$ such that type $b$ strictly prefers deviating to $(I_H, 1)$ over its equilibrium payoff when the average belief about $(I_H, 1)$ is strictly above $b^E(b)$. Consider $b^E(\sup B)$. Since

$$\Pi(\sup B, I_H, 1, \sup B) > \Pi(\sup B, i, \theta, \sup B) > \Pi^*(\sup B), \tag{45}$$

and the equilibrium choice of types in $B$ implies the equilibrium belief about $(I_H, 1)$ is such that

$$\Pi\left(\sup B, I_H, 1, b^E\right) \leq \Pi^*(\sup B),$$

$\bar{b}^E(\sup B) \in [b_L, b_H)$ and

$$\Pi\left(\sup B, I_H, 1, \bar{b}^E(\sup B)\right) = \Pi(\sup B, i, \theta, E[b \in B]).$$

Since $\ln V(b, I_H, 1) - \ln V(b, i, \theta)$ increases with $b$, this implies for types $b < B$,

$$\ln \Pi\left(b, I_H, 1, \bar{b}^E(\sup B)\right) < \ln\Pi(b, i, \theta, E[b \in B]) \leq \ln \Pi^*(b),$$

which implies $\bar{b}^E(b) > \bar{b}^E(\sup B)$. Therefore, D1 implies the choice $(I_H, 1)$ is only associated with types no smaller than $\sup B$:

$$E[b|(I_H, 1)] \geq \sup B.$$

However, this implies type $\sup B$ strictly prefers $(I_H, 1)$ over $(i, \theta)$ by (45). This implies some types in $B$ has the same preference. It contradicts that these types choose $(i, \theta)$ in equilibrium.

We next show that all types choose $(I_H, 1)$ is a D1 equilibrium. In such an equilibrium,

$$E[b|(I_H, 1)] = E[b].$$

Consider a deviation to $(i, \theta) \neq (I_H, 1)$. Again, since

$$\frac{\partial \ln \Pi \left(b, i, \theta, b^E\right)}{\partial b^E} > 0,$$

for each type $b$, there is a $\bar{b^E}(b)$ such that type $b$ prefers deviating to $(i, \theta)$ over its equilibrium payoff when the average belief about $(i, \theta)$ is above $\bar{b^E}(b)$.

If $\bar{b^E}(b) = b_H$ for all $b$, this implies no type has an incentive to deviate to $(i, \theta)$ under any belief.

If $\bar{b^E}(b^*) < b_H$ for some $b^*$, then

$$\Pi \left(b^*, i, \theta, \bar{b^E}(b^*)\right) = \Pi^*(b^*) = \Pi(b^*, I_H, 1, E[b]).$$

Since $\ln V(b, i, \theta) - \ln V(b, I_H, 1)$ decreases with $b$, this implies for $b' \gtrless b^*$,

$$\ln \Pi \left(b', i, \theta, \bar{b^E}(b^*)\right) \lesseqgtr \ln \Pi(b', I_H, 1, E[b]) = \ln \Pi^*(b'),$$

implying $\bar{b^E}(b') \gtreqless \bar{b^E}(b^*)$. D1 therefore requires $(i, \theta)$ be associated with the lowest type $b_L$. Under this belief, no type deviates:

$$\Pi(b, i, \theta, b_L) < \Pi(b, I_H, 1, E[b]),$$

since $(I_H, 1)$ leads to higher surplus and higher market belief.

**Proof of Proposition 8 for the repurchase game:**

The keys to the proof are the signs of the following items:

$$\frac{\partial \ln \Pi \left(b, i, \theta, b^E\right)}{\partial b^E} < 0, \tag{46}$$

implying firms want to signal low $b$, that is, high $|b|$ (to repurchase at a cheap price);

$$\frac{\partial S(b, i, \theta)}{\partial \theta} > 0, \tag{47}$$

$$\frac{\partial S(b, i, \theta)}{\partial i} < 0; \tag{48}$$

where $S(b, i, \theta) = i\theta b$ is the transaction surplus, implying a signal must reduce transaction surplus by decreasing efficiency $\theta$ or decreasing repurchase size $|i|$; and

$$\frac{\partial^2 \ln V(b,i,\theta)}{\partial\theta\partial b} < 0, \tag{49}$$

$$\frac{\partial^2 \ln V(b,i,1)}{\partial i\partial b} > 0, \tag{50}$$

implying a firm with lower $b$ loses more from decreasing $\theta$; Given $\theta = 1$, a firm with lower $b$ loses more from decreasing $|i|$.

We first show that in a D1 equilibrium, no type chooses $\theta < 1$. We then show in a D1 equilibrium, no type chooses $i > I_H$. We finally show that all types choose $(I_H, 1)$ is indeed a D1 equilibrium.

1. In a D1 equilibrium, no type chooses $\theta < 1$. Conjecture an equilibrium in which types in $B$ choose $(i,\theta)$ with $\theta < 1$. Then consider the market belief associated with $(i, 1)$. (46) implies for each type $b$, there is $b^{\bar{E}}(b)$ such that type $b$ strictly prefers to deviate to $(i, 1)$ when the market belief of $(i, 1)$ is strictly below $b^{\bar{E}}(b)$, that is the believed $|b|$ is strictly above $\left|b^{\bar{E}}(b)\right|$.

   Consider $b^{\bar{E}}(\inf B)$. If $(i, 1)$ is associated with average belief $E[b \in B]$, then type $\inf B$ strictly prefers $(i, 1)$ to $(i,\theta)$ since

   $$\ln V(b,i,1) > \ln V(b,i,\theta).$$

   On the other hand, that types in $B$ choose $(i,\theta)$ in equilibrium implies type $\inf B$ weakly prefers $(i,\theta)$ to $(i, 1)$ under the equilibrium belief. Therefore, $b^{\bar{E}}(\inf B) \in (b_H, b_L]$ and

   $$\Pi\left(\inf B, i, 1, b^{\bar{E}}(\inf B)\right) = \Pi(\inf B, i, \theta, E[b \in B]) = \Pi^*(\inf B).$$

   Since (49) implies $\ln V(b,i,1) - \ln V(b,i,\theta)$ decreases with $b$, for types $b > \inf B$,

   $$\Pi\left(b, i, 1, b^{\bar{E}}(\inf B)\right) < \Pi(b, i, \theta, E[b \in B]) = \Pi^*(b).$$

   Therefore, $b^{\bar{E}}(b) < b^{\bar{E}}(\inf B)$. Therefore, $(i, 1)$ cannot be associated with types higher than $\inf B$. However, under this belief, type $\inf B$ strictly prefers $(i, 1)$ to $(i,\theta)$ since it leads to higher surplus and better market belief (meaning market belief about higher agency problem $|b|$, which leads to lower repurchase price). By continuity, this implies some types in $B$ have the same preference, leading to a contradiction.

2. In a D1 equilibrium, no type chooses $(i, 1)$ with $i > I_H$. Repeat the reasoning in step 1, substituting $(i, \theta)$ by $(i, 1)$ and $(i, 1)$ by $(I_H, 1)$. The same process leads to a contradiction. Instead of (47) and (49), we are now utilizing (48) and (50).

3. Every type of the firm pooling on $(I_H, 1)$ is a D1 equilibrium. In such an equilibrium,

$$E\left[b\right| (I_H, 1)] = E\left[b\right].$$

Consider a choice $(i, \theta) \neq (I_H, 1)$. (46) implies for each type $b$, there is a $\bar{b^E}(b)$ such that type $b$ strictly prefers deviating to $(i, \theta)$ over its equilibrium payoff when the average belief about $(i, \theta)$ is strictly below $\bar{b^E}(b)$.

If $\bar{b^E}(b) = b_L$ for all $b$, this implies no type has an incentive to deviate to $(i, \theta)$ under any belief.

If $\bar{b^E}(b^*) > b_L$ for some $b^*$, then

$$\Pi\left(b^*, i, \theta, \bar{b^E}(b^*)\right) = \Pi\left(b^*, I_H, 1, E\left[b\right]\right) = \Pi^*\left(b^*\right).$$

(49) and (50) imply $\ln V(b, i, \theta) - \ln V(b, I_H, 1)$ increases with $b$. This implies for $b \gtreqless b^*$,

$$\Pi\left(b, i, \theta, \bar{b^E}(b^*)\right) \gtreqless \Pi\left(b, I_H, 1, E\left[b\right]\right) = \Pi^*\left(b\right),$$

implying $\bar{b^E}(b) \gtreqless \bar{b^E}(b^*)$. D1 therefore requires $(i, \theta)$ be associated with the highest type $b_L$ (i.e., the type with the lowest agency problem $|b_L|$). This is the worst belief since it leads to the highest repurchase price. Under this belief, no type deviates to $(i, \theta)$, since $(I_H, 1)$ leads to higher surplus and better market belief.

**Proof of Proposition 8 for the repurchase game under the function form in Section 4:**

We have above proved Proposition 8 based on the function form $V$, (29), in which the transaction surplus is multiplicatively separable between $b$ and $\theta$. However, in the function form $V$ of the repurchase game as specified in the microfoundation provided in Section 4, the project profitability $b$, which is now the firm's private information, and its public choice $\theta$ are not multiplicatively separable. Here we show that Proposition 8 still holds under this form of $V$ for $[|b_L|, |b_L|] \in [0, 0.49]$.

In the microfoundation in Section 4, agency cost $b$ is determined by the rate of cash burning, $\beta$:

$$b(\beta) = -\left[1 - \frac{1}{\beta T}\left(1 - e^{-\beta T}\right)\right].$$

For this exercise about asymmetric information on project profitability, we assume the firm privately observes its rate of cash burning $\beta \in \left[0, \bar{\beta}\right]$.

The firm publicly chooses the frequency of repurchases $n = 1, \ldots, \infty$, which affects $\theta$, the efficiency gain from each unit of repurchase normalized by $b$. Since the efficiency gain

$$S(\beta, i, n) = |i|\left(1 - \frac{1}{n}\frac{1 - e^{-\beta T}}{1 - e^{-\beta \frac{T}{n}}}\right)$$

is not multiplicatively separable in $\beta$ and $n$, $\theta$ is determined jointly by $\beta$ and $n$:

$$\theta(n, \beta) = \frac{1 - \frac{1}{n}\frac{1 - e^{-\beta T}}{1 - e^{-\beta \frac{T}{n}}}}{-b(\beta)}.$$

Then the total firm value (23) can be expressed as

$$\begin{aligned} V(\beta, i, n) &= \underbrace{a_N + \frac{\lambda}{\beta}\left(1 - e^{-\beta T}\right)}_{a} + i + i\theta(n, \beta)\, b(\beta) \\ &= a_N + (|I_H| - |i|) - |b(\beta)|\left(|I_H| - \theta(n, \beta)|i|\right). \end{aligned}$$

This is consistent with (29) except that $\theta(n, \beta)$ is not solely determined by the firm's choice variable $n$, but is also affected by the hidden type $\beta$. We next prove that this difference does not change the equilibrium outcome.

Similar to the above proof for the repurchase game under the function form of $V$ (29), the key to this proof is to find those conditions that are parallel to (46)-(50).

We will verify the following conditions:

$$\frac{\partial \ln \Pi \left(\beta, i, n, \beta^E\right)}{\partial \beta^E} > 0, \tag{51}$$

implying firms want to signal high $\beta$ to repurchase at a cheap price;

$$\frac{\partial S \left(\beta, i, n\right)}{\partial n} > 0, \tag{52}$$

$$\frac{\partial S \left(\beta, i, n\right)}{\partial i} < 0, \tag{53}$$

implying a signal must reduce surplus by decreasing frequency $n$ or decreasing repurchase size $|i|$; and

$$\frac{\partial^2 \ln V \left(\beta, i, n\right)}{\partial n \partial \beta} > 0, \tag{54}$$

$$\frac{\partial^2 \ln V \left(\beta, i, \infty\right)}{\partial i \partial \beta} < 0, \tag{55}$$

implying a firm with higher $\beta$ loses more from decreasing $n$; Given the maximum frequency $n = \infty$, a firm with higher $\beta$ loses more from decreasing $|i|$. Given (51)-(55) hold for all $\beta \in \left[0, \bar{\beta}\right]$, one can follow similar steps in the above proof under function form (29) to show that Proposition 8 holds. We omit these steps for brevity.

(51)-(53) can be verified by easy algebra. That (55) holds is because

$$V \left(\beta, i, \infty\right) = a_N + \left(|I_H| - |i|\right) \left(1 - |b \left(\beta\right)|\right)$$

implies

$$\frac{\partial \ln V \left(\beta, i, \infty\right)}{\partial i} = \frac{-1}{\frac{a_N}{1 - |b(\beta)|} + \left(|I_H| - |i|\right)},$$

and $|b \left(\beta\right)|$ increases with $\beta$. In the remainder, we show that (54) holds for $\beta \in \left[0, \frac{1.56}{T}\right]$, which corresponds to $|b| \in [0, 0.49]$. Since

$$\frac{\partial^2 \ln V \left(\beta, i, n\right)}{\partial n \partial \beta} = \frac{\frac{\partial^2 V}{\partial n \partial \beta}}{V} - \frac{\frac{\partial V}{\partial n} \cdot \frac{\partial V}{\partial \beta}}{V^2},$$

and $\frac{\partial V}{\partial n} > 0$ and $\frac{\partial V}{\partial \beta} < 0$,it is sufficient to show

$$\frac{\partial^2 V(\beta, i, n)}{\partial n \partial \beta} = \frac{\partial^2 S(\beta, i, n)}{\partial n \partial \beta} > 0. \tag{56}$$

1. As $\beta$ approaches 0 from the right, (56) holds:

$$\lim_{\beta \searrow 0} \frac{\partial^2 S(\beta, i, n)}{\partial n \partial \beta} = \frac{3n-1}{6n^3} T |i| > 0.$$

2. Since the sign of $\frac{\partial^2 S(\beta,i,n)}{\partial n \partial \beta}$ is independent of the parameters $I_H, I_L$ and $a_N$, we have verified numerically that for all $n$ with $\frac{1}{n} \in (0, 1]$ and $\beta$ with $\beta \in \left(0, \frac{1.56}{T}\right]$, (54) holds.