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Real Interest Rates, Inflation, and Default[1]

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Abstract

This paper argues that the comovement between inflation and economic activity is an important determinant of real interest rates. Nominal bonds pay out more in bad times when inflation is procyclical. This makes nominal government debt a good hedge against aggregate risk for domestic risk-averse lenders. We show that procyclical inflation leads to lower real rates in the absence of default risk. However, inflation procyclicality implies that the government needs to make larger (real) payments when the economy deteriorates, which could push up default risk and increase real rates. We present empirical findings from advanced economies that are consistent with these patterns of real rates predicted by our simple model. Finally, we turn to a calibrated model that is consistent with our empirical evidence to quantify the welfare consequences of inflation cyclicality and to investigate how real rates respond when inflation uncertainty increases and how this depends on the interaction between inflation cyclicality and default risk.

Keywords: Inflation risk, government debt, nominal bonds, sovereign default
JEL classification codes: E31, F34, G12, H63

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1 Introduction

Inflation and default risk are two major factors determining real returns on nominal bonds issued by governments. We show that these two risks are linked and that one single factor—the comovement between inflation and economic activity—plays an important role, both theoretically and empirically, in shaping how these risks affect the real return on nominal bonds across time and space. We then investigate, quantitatively, the welfare consequences of inflation procyclicality and how the interplay of these two risks changes when inflation becomes more volatile.

In the first part of the paper, we present a simple two-period model to show how inflation and default risk interact in shaping real rates on nominal bonds. The environment features domestic risk-averse borrowers and lenders, both exposed to the same aggregate growth risk, which trade with each other using nominal bonds, subject to inflation risk. These assumptions are motivated by the fact that a large fraction of government debt in advanced economies is issued in nominal bonds that are held domestically.[2] We first consider a change from countercyclical to procyclical inflation, in a setup in which default is not an option. When inflation is procyclical, real returns on domestic nominal bonds are higher when growth is low and the marginal utility of lenders is high. This implies that nominal bonds provide the lenders with a hedge against aggregate risk, which increases the demand for them. Procyclical inflation, however, also implies that borrowers have to make large real repayments in bad times, when their marginal utility is high. This reduces their supply of nominal bonds. Since demand increases and supply falls, the price of bonds unequivocally rises (i.e., real rates fall) as inflation changes from countercyclical to procyclical.

We then repeat the same exercise in an economy in which borrowers have the costly option to default on bonds, and face lower default costs when aggregate growth is low. When inflation is countercyclical, borrowers' real repayment obligations are low when growth is low, and that reduces their incentive to default. With procyclical inflation instead, nominal bonds prescribe larger real payments when growth is low, increasing borrowers' incentives

[2]For example, as of 2015, the domestic share of public debt was 69 percent in the United Kingdom, 78 percent in Canada, and 64 percent in the United States. Moreover, Treasury inflation-protected securities (TIPS) account for less than 10 percent of U.S. public debt (see U.S. Congressional Budget Office 2020).

to default. In other words, when default is an option, countercyclical inflation *substitutes* default, whereas procyclical inflation *complements* it. A higher probability of default will, *ceteris paribus*, reduce the demand for bonds by lenders and increase the supply of bonds by borrowers. These changes will tend to increase equilibrium interest rates.

In the second part of the paper, we show that the qualitative predictions of our simple model are borne out in the data. To this end, we use data from a large sample of advanced economies to document a novel and robust relation between real interest rates, inflation dynamics, and default risk. We show that periods/countries with more procyclical inflation are associated with lower real interest rates, but especially in times when the risk of default on government debt is low. This relation is robust to controlling for a broad array of macroeconomic controls, and its magnitude is economically significant. As an illustration, consider an increase in the covariance between inflation and economic growth equal to two standard deviations of that variable in our sample, for a country that has a AAA rating on its government debt. Our estimated relation suggests that this change is, *ceteris paribus*, associated with a lowering of real rates of almost 100 basis points. We call this reduction in interest rates the *inflation procyclicality discount*. If the same change is experienced in a country with a rating worse than AAA, however, then the reduction in rates associated with more procyclical inflation is much lower and not significantly different from zero. This interaction between sovereign credit risk and the inflation procyclicality discount would be absent in standard consumption-based asset pricing models.

In the third part of the paper, we investigate the welfare effects of inflation cyclicality and counterfactual interest rate dynamics when inflation volatility increases. To do so, we develop a quantitative model of sovereign default on domestic nominal debt that is consistent with our empirical evidence. The backbone of our setup is a standard sovereign debt/default model (as in Arellano 2008), extended along three dimensions. First it assumes that the government borrows using nominal bonds, so that rates reflect both exogenous inflation risk and endogenous default risk. Second, it introduces domestic risk-averse lenders, in contrast to the common assumption of foreign risk-neutral lenders. Finally, it assumes that the government and households trade long-term debt (as in Hatchondo and Martinez 2009 and Chatterjee and Eyigungor 2013). Long-term debt is consistent with the fact that a majority of debt issued by governments in advanced economies has a maturity longer than five years, and it is important to generate a quantitatively sizeable effect of changes in inflation dynamics on real returns. Moreover, since our objective is to understand the pricing of debt assets, we use lender stochastic discount factors that utilize preferences from the finance literature (i.e., Epstein-Zin preferences with high risk aversion).

We calibrate our model so that a benchmark economy with acyclical inflation (which resembles the median covariance between inflation and aggregate growth in our sample) matches some otherwise standard moments such as debt levels and lower default risk in our sample of advanced economies. Before conducting our welfare and counterfactual experiments, we verify that the model generates a reasonable state-dependent inflation procyclicality discount. To do this external validation exercise, we contrast two economies that are identical in every respect but have two different processes for inflation: one in which inflation is countercyclical (having a covariance between inflation and growth equal to minus 1 standard deviation of that variable in our sample) and one in which inflation is procyclical (having a covariance equal to plus 1 standard deviation). The increase in cyclicality leads to a significant reduction in real rates (around 50 basis points, about half of what we document in the data) when default on government debt is not an issue. We also find that when the government is in fiscal trouble and default is a possibility, a more procyclical inflation does not necessarily reduce rates, but it could actually cause them to increase.

We use our model to calculate the welfare gains (or losses) from having a procyclical inflation process relative to a countercyclical one for different states of the world. We find that there is, on average, a small welfare gain associated with procyclical inflation. The welfare is larger when there is no default risk. In contrast, a countercyclical inflation regime is strongly preferred when default risk is significant. In this way, our paper has implications for the debate on the costs and benefits of joining or exiting a monetary union. Suppose that the union goes into a recession where some, but not all, members of the union get into fiscal trouble. Then the countries in fiscal trouble would prefer a more countercyclical monetary policy, while the others would not: the contrast over monetary policy increases in a recession.

Motivated by the recent spike in inflation uncertainty and concerns about the future path of real interest rates, we also use our model to investigate how real interest rates respond to higher inflation volatility. We find that the answer depends on the cyclical properties of inflation and the extent of default risk. In the absence of material default risk, higher inflation volatility reduces spreads in the procyclical economy because of improved hedging properties; the opposite is true in the countercyclical economy. When default risk is significant, however, higher inflation volatility can cause large spikes in borrowing costs, driven by increased default probabilities. This is especially true in the procyclical economy.

Related literature. Our paper is related to several strands of the literature. On the theoretical side, the backbone of our setup is a debt default model with incomplete markets as in Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), or Arellano (2008). Our paper is especially related to Hatchondo et al. (2016) and Lizarazo (2013), who study default in the context of risk-averse international lenders.[3] Our paper is also related to Kursat Onder and Sunel (2016), Nuño and Thomas (2016), and Arellano et al. (2018) who consider the interaction of inflation and default on foreign investors.[4] While these papers focus on foreign debt, Reinhart and Rogoff (2011) suggest that the connection between default, domestic debt, and inflation is an important one. D'Erasmo and Mendoza (2016), Pouzo and Presno (2014), Arellano and Kocherlakota (2014), and Hur et al. (2021) study default on domestic debt but do not include inflation.[5] Araujo et al. (2013), Sunder-Plassmann (2016), Mallucci (2015), and Fried (2017) study how the currency composition of debt interacts with default crises in emerging economies, while Berriel and Bhattarai (2013), Faraglia et al. (2013), and Perez and Ottonello (2016) study nominal debt with inflation in the absence of default. Du et al. (2016) study the effects of inflation-policy credibility on the pricing and the currency denomination of emerging economy debt.

Much of the existing literature on debt and inflation has focused on strategic inflation, even hyperinflation, as a countercyclical policy option that governments with limited commitment can use when faced with a high debt burden in bad times. That focus is certainly legitimate

[3] Aguiar et al. (2016) provide an excellent compendium on modeling risk-averse competitive lenders in the sovereign default literature.

[4] See Bassetto and Galli (forthcoming) for a model with strategic inflation on nominal domestic debt and strategic default on real foreign debt and how they differ through information frictions.

[5] Broner et al. (2010) examine the role of secondary asset markets, which make the distinction between foreign and domestic default less stark.

for emerging economies, but less warranted in the context of advanced economies mainly because of greater monetary policy independence and monetary union constraints.

Our general question is also related to recent work that studies how joining a monetary union can affect the probability of a self-fulfilling crisis in a debt default model (see Aguiar et al. 2015, Corsetti and Dedola 2016, and Bianchi and Mondragon 2018). We complement these papers by highlighting how the cyclicality of inflation affects fundamental-driven default crises, suggesting a promising extension of existing models of self-fulfilling debt crises such as Bocola and Dovis (2016). Our work is also related to the literature on the costs and benefits of monetary unions (Rose and Van Wincoop 2001, Fuchs and Lippi 2006, and Chari et al. 2019). We show the debt pricing and debt crises implications of different inflation cyclicality regimes. Finally, our findings are related to the literature on the non-neutrality of money in incomplete markets pioneered by Magill and Quinzii (1992) and further explored in the context of monetary unions by Neumeyer (1998).

On the empirical side, our findings are related to studies on the importance of the inflation risk premium and its variation, as in, for example, Boudoukh (1993), Piazzesi and Schneider (2006), or Ang et al. (2008). Kang and Pflueger (2015) studies inflation-induced default premium in corporate credit spreads, relative to government yields. In contrast, we focus on the underlying real sovereign yield.

The paper is structured as follows. Section 2 presents the simple model. Section 3 contains the empirical findings. Section 4 discusses the quantitative model and analysis. Section 5 concludes.

2 Simple Model

In this section, we highlight the main economic mechanism of this paper through a stylized two-period model of inflation and default, where equilibrium outcomes can be characterized analytically and using simple diagrams.

2.1 Simple model without default

Consider a two-period, one-good, closed economy with competitive lenders and borrowers.[6] Both borrowers and lenders receive one unit of the good in the first period and an endowment of x in the second period, where x is a random variable with c.d.f. F over X, with finite support $X = [x_{\min}, x_{\max}]$, $\mathbf{E}(x) = \mu > 0$, and $Var(x) = \sigma^2$. The variable x here captures the aggregate risk of the economy, to which both lenders and borrowers are exposed. We assume that the only difference between lenders and borrowers (i.e., the motive to intertemporal trade) lies in their preferences. In particular, we assume that $\beta_\ell > \beta_b$ are the discount factors of lenders and borrowers, respectively. Lenders and borrowers can trade a nominal bond at price q today, which pays a nominal amount of 1 tomorrow. We normalize the current price level to 1 and assume that the future price level is given by $1 + \pi(x; \kappa) \equiv [1 + \kappa(\mu - x)]^{-1}$, where κ is the key parameter, capturing the cyclicality of inflation. If $\kappa > 0$, prices (and inflation) are procyclical, so the bond pays less in good states of the world (when x is high), while the reverse is true if $\kappa < 0$. We define the real interest rate r as $\mathbf{E}[1/(1 + \pi)]/q - 1$, which, given the chosen process for inflation, is equal to $1/q - 1$.

The borrower solves

$$\max_{b_b} u(1 + qb_b) + \beta_b \int_X v\left(x - \frac{b_b}{1 + \pi(x; \kappa)}\right) dF(x), \tag{1}$$

and the lender solves

$$\max_{b_\ell} u(1 - qb_\ell) + \beta_\ell \int_X v\left(x + \frac{b_\ell}{1 + \pi(x; \kappa)}\right) dF(x). \tag{2}$$

Notice that both borrowers and lenders act competitively, taking bond prices as given. An equilibrium is then simply a bond price and a bond quantity such that, given the price, the bond quantity is optimal for each agent.

Theorem 1 shows that, under certain conditions, an inflation cyclicality discount arises from the hedging benefits of inflation procyclicality.

[6] The assumption of competitive borrowers is inconsistent with the fact that borrowing is done by a large player (the government), which internalizes the effect of its borrowing choices on prices. We use this assumption in the simple model for analytical simplicity. In the quantitative model in section 4, we revert to the standard setup in which borrowing is done by a large agent.

Theorem 1. Inflation procyclicality discount

Assume that both borrowers and lenders have quasi-linear utility such that $u(c) = Ac$*, and* $v(c) = Ac - \frac{\phi}{2}c^2$ *with* $A > 0$*,* $\phi > 0$ *and* $\frac{A}{\phi} > \mu$*. Then, the equilibrium real interest rate* $r \equiv \boldsymbol{E}[1/(1+\pi)]/q - 1 = 1/q - 1$ *features an inflation procyclicality discount. That is,*

$$\frac{\partial r}{\partial \kappa} < 0. \tag{3}$$

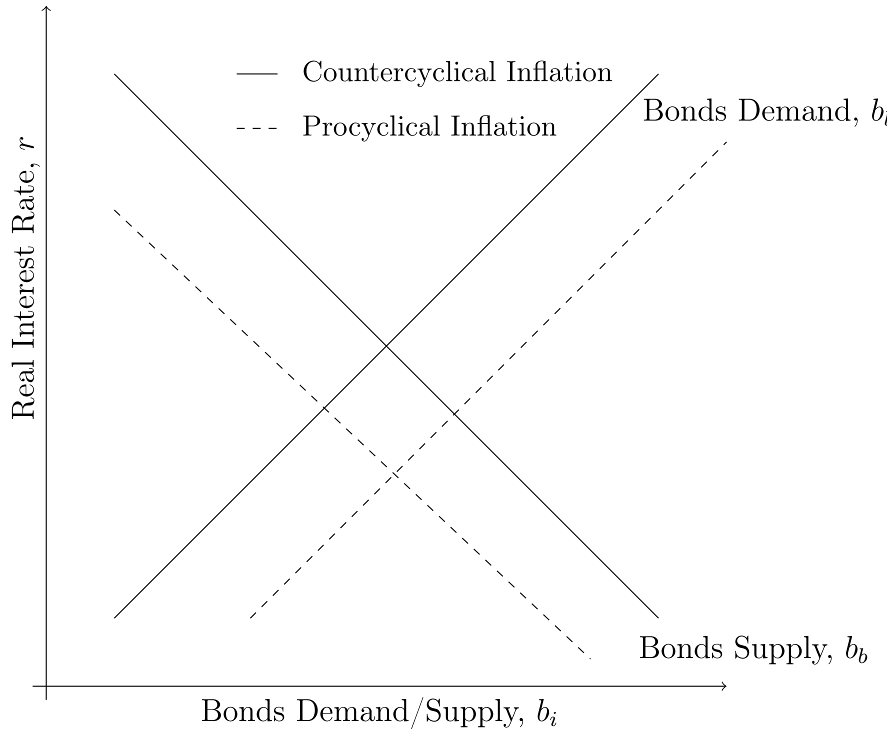
Proof: See Appendix C.1.

Figure 1 provides some visual intuition for this result. The lines in the figure represent the desired demand for bonds by the lender (increasing in the real interest rate) and the desired supply of bonds from the borrowers (decreasing in the real rate). The solid lines are demand and supply with countercyclical inflation, while the dashed lines are demand and supply with procyclical inflation. Note that as inflation goes from countercyclical to procyclical, the demand for bonds increases. Intuitively, with procyclical inflation, for every level of the real rate, risk-averse lenders want to save more. This is because with procyclical inflation, saving in the nominal bond provides insurance to lenders by yielding higher returns in states of the world when income is low. While procyclical inflation makes saving in a nominal bond more attractive for lenders, it makes issuing the nominal bonds less attractive to borrowers, who have to make larger payments when their income is low. This implies that for every real rate, the borrower will borrow less, resulting in an inward shift in their bond supply. Since demand increases and supply falls, the equilibrium interest rate unequivocally falls, while the equilibrium level of debt can move in either direction. This simple model makes it clear why, in the absence of default risk, procyclical inflation results in lower real interest rates.

2.2 Simple model with default

Now consider the possibility that the nominal contract can be defaulted on. In particular, a borrower can default on its bond payments, and if it does so, no payments are made and it incurs a cost $C(x) = \psi(x - x_{\min})^2$. As in Dubey et al. (2005), we maintain the assumption of competitive borrowers, so they do not perceive that their borrowing and default decisions

Figure 1: Interest rates and cyclicality of inflation without default



affect the interest rate they face. In this environment, there will be equilibrium default when default costs are below repayment; hence, the default set $\widehat{X}(\kappa, b_b)$ is given by

$$\widehat{X}(\kappa, b_b) = \left\{ x \in [x_{\min}, x_{\max}] \ : \ C(x) < \frac{b_b}{1 + \pi(x; \kappa)} \right\}, \tag{4}$$

which typically is an interval; that is, default happens when income is low enough and debt is high enough. The key observation is that in a world with default, the cyclicality of inflation can change the default set, thereby altering the hedging properties of bonds. Theorem 2 shows that, under certain regularity conditions, the default set $\widehat{X}$ increases with the level of debt (b_b) and the cyclicality of inflation (κ).

Theorem 2. Inflation procyclicality and default

Assume that $-(\mu - x_{\min})^{-1} < \kappa < (x_{\max} - \mu)^{-1}$*. For* ψ *large enough, there exists a unique threshold,* $\widehat{x}(\kappa, b_b) \in [x_{\min}, \mu]$*, such that default occurs if and only if* $x \in [x_{\min}, \widehat{x}]$*. Fur-*

thermore, the default threshold is increasing in debt (b_b) *and the cyclicality of inflation* (κ)*, ceteris paribus. That is,*

$$\frac{\partial \widehat{x}(\kappa, b_b)}{\partial b_b} > 0 \tag{5}$$
$$\frac{\partial \widehat{x}(\kappa, b_b)}{\partial \kappa} > 0. \tag{6}$$

Proof: See Appendix C.2.

Given this result we can then write the problem of the borrower as

$$\max_{b_b} u\left(1 + q b_b\right) + \beta_b \left(\underbrace{\int_{\widehat{x}(b_b,\kappa)}^{x_{\max}} v\left(x - \frac{b_b}{1+\pi(x)}\right)}_{\text{Repayment}} + \underbrace{\int_{x_{\min}}^{\widehat{x}(b_b,\kappa)} v\left(x - C(x)\right)}_{\text{Default and suffer cost}} \right) dF\left(x\right). \tag{7}$$

The lender, taking as given the default threshold $\widehat{x}$, solves

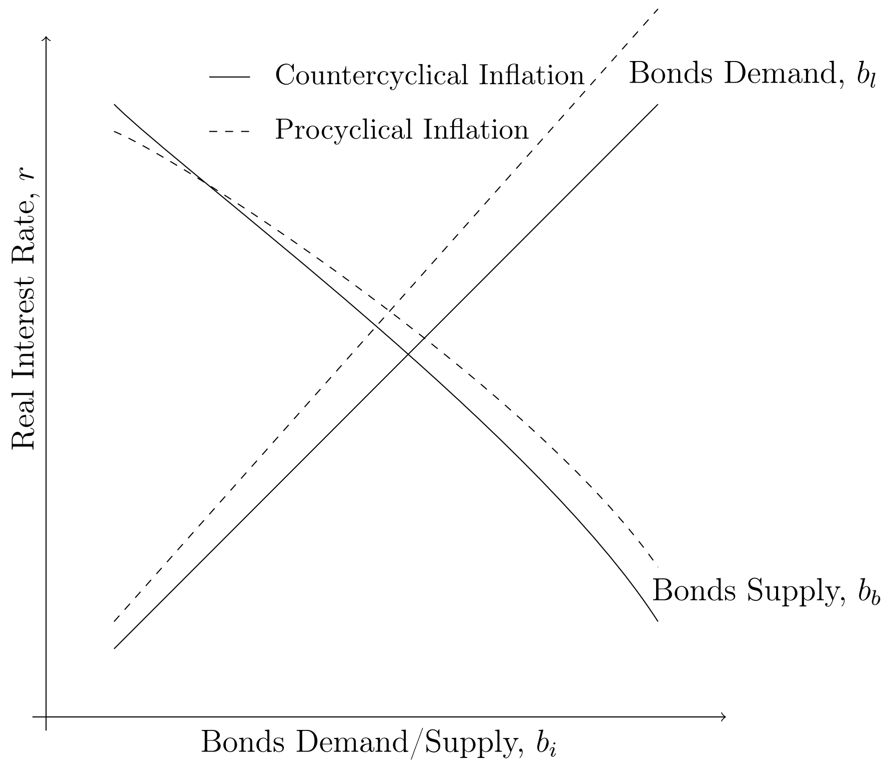
$$\max_{b_\ell} u\left(1 - q b_\ell\right) + \beta_\ell \left(\underbrace{\int_{\widehat{x}}^{x_{\max}} v\left(x + \frac{b_\ell}{1+\pi(x)}\right)}_{\text{Repayment}} + \underbrace{\int_{x_{\min}}^{\widehat{x}} v\left(x\right)}_{\text{Defaulted on}} \right) dF\left(x\right). \tag{8}$$

An equilibrium in this setup is then simply a bond price q, a bond quantity, and a default threshold $\widehat{x}$ such that i) given the bond price and default threshold, the bond quantity is optimal for the lender, and ii) and given the bond price, the bond quantity and the default threshold are optimal for the borrower.

In the model with default, changes in covariance lead to changes not only to quantities but also to the default threshold, complicating the analysis. Thus, to gain further intuition, we use a simple numerical illustration. Figure 2 shows that, unlike the model without default in which higher inflation procyclicality unequivocally reduced interest rates, in the model with default, higher inflation procyclicality can increase real rates.

To understand why, consider first the demand for bonds with and without default. In the absence of default (Figure 1), as inflation goes from countercyclical to procyclical, the demand curve shifts to the right: lenders are willing to accept a lower interest rate because

Figure 2: Interest rates and cyclicality of inflation with default



of the hedging properties of inflation. In Figure 2 instead, the curve shifts to the left because of default risk. This is because countercyclical inflation, which implies low repayments in bad states, *substitutes* default, while procyclical inflation, which implies high repayments in bad states, *complements* default. Thus a move from counter- to procyclical inflation causes an increase in default risk, which, in this example, shifts the demand for bonds to the left. Note that the same increase in default risk that causes the reduction in bond demand also causes an increase in bond supply. Since with default the borrowers will not repay in the bad states, they are now willing to borrow more. So procyclical inflation, by triggering more equilibrium default, can at the same time shift the bond demand in and shift the bond supply out, thereby causing an *increase* in the real interest rate.

This simple model highlights a relation between inflation cyclicality, interest rates, and default. It shows that when default is not a concern, a more procyclical inflation unambiguously results in lower rates. Instead, when default is a possibility, a more procyclical inflation can increase real rates.[7] In the next section, we show that these predictions are consistent with the data.

3 Inflation and Real Interest Rates

In this section, we study the empirical relation between several moments of inflation and real interest rates on government debt. The main novel finding is that stronger comovement of inflation with economic activity is significantly associated with lower real interest rates on government debt. This relation appears to be negative and significant when default risk on government debt is small.

Our data set includes quarterly observations on real consumption growth, inflation, interest rates on government bonds, and government debt-to-GDP ratios for a panel of 19 OECD economies from 1985Q1 to 2015Q4. This is the widest and longest panel of developed countries for which we could get comparable high-quality data for all our variables. The countries in the data set are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Korea, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

Our main data sources are the IMF International Financial Statistics (IFS) and the OECD Quarterly National Accounts (QNA). We compute inflation as the change in the log GDP deflator using data from QNA. We use nominal interest rates on long-term government bonds from the IFS. For government debt, we use quarterly series from Oxford Economics on gross government debt relative to GDP, extended with quarterly OECD data on central government debt relative to GDP. Quarterly real consumption is constructed as the sum of private and public real consumption using the data from QNA.

Using these cross-country quarterly data, we estimate the conditional comovement between inflation and consumption growth, and derive real interest rates by subtracting the expected inflation estimated from nominal yields. To do so, we follow Boudoukh (1993) and formulate the following vector autoregression (VAR) model for inflation and consumption

[7] The simple model also shows that a low interest rate environment, driven for instance by a more procyclical inflation, might make public debt more risky. This case illustrates the risk associated with public debt accumulation in low rate environments discussed by Blanchard (2019).

growth:

$$\begin{bmatrix} \pi_{it} \\ g_{it} \end{bmatrix} = \mathbf{A_i} \begin{bmatrix} \pi_{it-1} \\ g_{it-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{\pi it} \\ \varepsilon_{git} \end{bmatrix} \tag{9}$$

where π_{it} is inflation, g_{it} is the change in log consumption in country i in period t, $\mathbf{A_i}$ is a country-specific 2-by-2 matrix, and $\varepsilon_{\pi it}$ and ε_{git} are innovations in the two time series. We then estimate the VAR using standard OLS and construct the time series for residuals $\varepsilon_{\pi it}$ and ε_{git} for each country. In Appendix B.2, we show that our results are robust to estimating a rolling VAR à la Lunsford and West (2019) and a time-varying parameter VAR with stochastic volatility à la Primiceri (2005) on a longer annual dataset.[8]

We measure the expected inflation as the forward-looking predicted inflation from the VAR, that is, $\mathbf{E}[\pi_{i,t+1}]$. We then derive real rates on government debt as nominal rates less expected inflation. Finally, we measure the conditional comovement between inflation and consumption growth as the covariance/correlation between the two innovations, $\varepsilon_{\pi it}$ and ε_{git}, in overlapping 40-quarter country-windows.

With this data set, we estimate how the conditional covariance of inflation and consumption growth relates to interest rates faced by governments. In all the regressions that follow, each variable is computed on the same 10-year overlapping windows used to compute the conditional covariance. All specifications include a full set of country and time fixed effects.

Table 1 reports the results from regressing the real interest rate on the conditional comovement between inflation and consumption growth. The main result from the table is that the coefficients in the first row of the table are always negative and significantly different from 0. This means that in periods with higher comovement between inflation and consumption growth (measured using either covariance in columns 1–3 or correlation in column 4), governments face lower real interest rates. This finding is robust to the inclusion of the lagged government debt-to-GDP ratio and average residual inflation and consumption growth in the period (columns 2, 3, and 4).[9] This association is also robust to the inclusion of the

[8] We prefer the time-invariant VAR as the benchmark specification for the quarterly data as it requires neither a training sample nor many initial lag years. Credit ratings, our preferred measure of default risk, is also unavailable for many countries prior to 1985.

[9] The coefficients on debt are estimated significantly positive; that is, governments with higher debt-to-GDP ratios tend to pay higher real rates.

variances of residual inflation and consumption growth as additional regressors (columns 3 and 4).

Table 1: Inflation consumption growth comovement and real interest rates

	Real yield on government debt			
	covariance			correlation
	(1)	(2)	(3)	(4)
Inflation consumption comovement	–1.89***	–1.64***	–1.80**	–1.06**
	(0.60)	(0.38)	(0.64)	(0.43)
Lagged government debt to GDP		0.02***	0.02***	0.02***
		(0.00)	(0.00)	(0.00)
Average inflation residual		2.41**	2.14*	1.91*
		(0.99)	(1.02)	(0.93)
Average cons. growth residual		–1.75	–1.65	–1.52
		(1.07)	(1.04)	(1.08)
Variance of inflation residual			0.30	0.26
			(0.29)	(0.31)
Variance of cons. growth residual			–0.06	0.23*
			(0.18)	(0.12)
standard deviation of comovement	0.17	0.17	0.17	0.21
adj. R^2	0.87	0.90	0.90	0.90
N	1764	1726	1726	1726

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parentheses. Standard errors are clustered by country. All regressions include country and time fixed effects. All variables are computed over 10-year overlapping windows.

Overall, these results show that stronger comovement of inflation and consumption growth is associated with lower real interest rates on government bonds; that is, it induces an *inflation procyclicality discount*. Our second main finding is that this procyclicality discount is only significant in times when default on government debt is not an issue.

Columns (2) and (3) of Table 2 report the results from a regression similar to the one from Table 1, with the difference that now the inflation-consumption covariance is interacted with a dummy for no default risk and with a dummy for its complement, positive default risk.

Table 2: Inflation procyclicality discount with and without default risk

	Real yield on government debt		
	(1)	(2)	(3)
		Credit rating	Cons. growth
		as *default risk* measure	
Inflation consumption covariance	–1.80**		
	(0.64)		
Interaction term (No default risk)		–2.70***	–2.99***
		(0.91)	(0.70)
Interaction term (Positive default risk)		–1.31	–1.16
		(0.79)	(0.68)
Additional controls	Yes	Yes	Yes
adj. R^2	0.90	0.92	0.91
N	1726	1438	1726

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parentheses. Standard errors are clustered by country. Additional controls include country and time fixed effects, lagged government debt-to-GDP, the averages and variances of residual inflation and consumption growth, and, in columns (2)-(3), dummies for no default risk. All variables are computed over a 10-year window.

In column (2), we define a window with no default risk for a country as a 10-year window in which the average credit rating for government bonds of that country is AAA. In column (3), we experiment with an alternative measure of no default risk; that is, a 10-year window in which the average residual aggregate consumption growth for that country is positive. The second measure is based on the observation that default on domestic debt appears only to "occur under situations of greater duress than for pure external defaults" (Reinhart and Rogoff 2011, p. 320).

Both columns show that the interaction term between the inflation-consumption growth covariance and the no-default risk dummy is negative, statistically significant, and larger than the discount estimated on the full sample. The interaction of the same covariance with the indicator for times with positive default risk, however, is smaller and not statistically significant. Moreover, the estimated coefficients on the interaction terms with no default risk and positive default risk in column (3) are statistically different at the one percent level. These results suggest that procyclical inflation is associated with lower real rates only at times when domestic default on government debt is very unlikely.

The magnitude of the procyclicality discount in times of no-default risk is economically significant. As an illustration of its magnitude, consider an increase in the inflation-consumption growth covariance equal to 0.34, which is equal to two times the standard deviation of that covariance in our sample. Using the coefficients estimated in columns (2) and (3) of Table 2, we can see that such an increase in cyclicality in no-default times is associated with a lowering of real rates of between 92 and 102 basis points.

These empirical results are robust to alternative measures of our variables and to alternative estimation techniques. In Table 9 of the appendix, we show that our baseline findings are robust to different window lengths, a quantile regression approach, or realized inflation measures for the construction of real returns. We also use an annual panel data from 1950 to 2016 based on the Jordà et al. (2017) Macrohistory database to document similar facts. First, using rolling VARs à la Lunsford and West (2019), we estimate time-varying inflation cyclicality and document similar facts in Tables 10 and 11. We also estimate time-varying parameter VARs (TVP-VAR) à la Primiceri (2005) to extract time-varying inflation cyclicality measures. Our results using these TVP-VAR estimates are reported in Tables 12 and 13. See Appendix B for detailed description of our robustness exercises.

The standard consumption-based asset pricing model suggests that the hedging benefits (for the lender) of procyclical inflation rationalize an inflation procyclicality discount. However, in periods in which default risk is material, the procyclicality discount appears to be much attenuated. We conjecture that this is because, from the government's perspective, inflation procyclicality implies that it has to make larger real payments when aggregate growth is low and this, *ceteris paribus*, reduces the government's willingness to pay in those states. When default risk is material, inflation procyclicality is going to increase this risk, thereby attenuating the hedging property of procyclical inflation.

This section presented some novel but suggestive evidence that support the predictions of the main economic mechanisms highlighted in the simple model. However, the simple model cannot be used to quantitatively assess how large of an interest rate differential can be explained by the different inflation process we see in the data, nor to assess how much a given change in inflation cyclicality can affect default risk. For these questions, we now turn to a standard quantitative model of default, augmented with nominal long-term debt and risk-averse domestic lenders.

4 Quantitative Analysis

In this section, we extend the standard sovereign default model of Eaton and Gersovitz (1981) and Arellano (2008) along three dimensions: exogenous *inflation*, domestic *risk-averse lenders*, and long-term debt. Note that risk-averse lenders are important to capture the impact of inflation cyclicality on the pricing of nominal bonds, while long-term debt is important to generate a quantitatively relevant impact of inflation cyclicality on returns to nominal debt. The model abstracts from the exact drivers of the changes in inflation cyclicality and focuses on their implications for debt pricing and default decisions. Such changes in inflation cyclicality might arise because of changes in the mix of macroeconomic shocks, changes in monetary policy stance, changes in the independence of the monetary authority, or some combination of these factors.[10]

4.1 Environment

We consider a closed economy inhabited by a continuum of (relatively patient) risk-averse lenders and a (relatively impatient) government. Both government and lenders are exposed to the same aggregate risk and, in equilibrium, the difference in patience results in the government borrowing from lenders. Importantly, the government has the option of defaulting on debt obligations to lenders, and if it does so, suffers a temporary utility loss. Time is discrete and indexed by $t = 0, 1, 2, ...$, and we let s_t denote the state of the world in period t. In each period, the economy receives a stochastic endowment $y(s_t)$. The government receives a fraction τ of the endowment, and lenders receive the remaining fraction $1 - \tau$.

[10] See, for example, Bianchi (2012), Campbell et al. (2014), and Song (2017) for studies that estimate changes in macroeconomic shocks and monetary policy regime switches using New Keynesian models. The exogenous inflation-output process considered in our model can be rationalized as the process implied by such exogenous macroeconomic shocks in the absence of default risk. See also Albanesi et al. (2003) and Bianchi and Melosi (2018), among others, for studies that focus on the interaction between monetary and fiscal policy for determining inflation dynamics.

Preferences The government uses its fraction of output plus proceeds from borrowing to finance public spending $g(s_t)$, which is valued according to[11]

$$E_0 \sum_{t=0}^{\infty} \beta_g^t \frac{\left(g(s_t) - \phi^d(s_t)\right)^{1-\gamma_g}}{1-\gamma_g}, \tag{10}$$

where $0 < \beta_g < 1$ is the government's discount factor, γ_g is the risk aversion of the government, and $\phi^d(s_t)$ is the utility loss suffered if the government defaults.

Lenders evaluate payments in two states of the world s_t and s_{t+1} using a stochastic discount factor $m(s_t, s_{t+1})$, and thus value a sequence of payments $\{x(s_t)\}_{t=0}^{\infty}$ as

$$E_0 \sum_{t=0}^{\infty} m(s_0, s_t) x_t, \tag{11}$$

where $m(s_0, s_t) = \Pi_{j=0}^{t-1} m(s_j, s_{j+1})$. We later specify the stochastic discount factor $m(s_t, s_{t+1})$ such that it is negatively correlated with aggregate output growth. That is, low economic activity is associated with high marginal utility.

Market structure The government issues nominal long-term non-contingent bonds to the domestic lenders. Payouts of the bonds are nominal, so they are subject to inflation risk. In particular, a nominal payout in state s_t, $x(s_t)$, is worth $\frac{x(s_t)}{1+\pi(s_t)}$, where $\pi(s_t)$ follows an exogenous Markov process, possibly correlated with the process for $y(s_t)$. Bonds have a fixed coupon payment of r and mature in each period with probability δ, as in Arellano and Ramanarayanan (2012), Hatchondo and Martinez (2009), and Chatterjee and Eyigungor (2013). Setting $\delta = 1$ corresponds to the model with one-period debt and $\delta = 0$ corresponds to the model with consols.

Default choices The government enters the period with outstanding assets B and, upon realization of the state of the world, it decides whether to default on its obligations. We

[11] An alternative interpretation is that the government uses its revenues to finance and smooth the consumption of "median" agents who have lower income and no access to financial markets. This interpretation is similar to the baseline setting in Bhandari et al. (2017) where the planner sets full weight on lower income agents.

define the value of the government at this point as $V^o(B,s)$, which satisfies

$$V^o(B,s) = \max_{d} \ \left\{(1-d)V^c(B,s) + dV^d(B,s)\right\}, \tag{12}$$

where V^c is the value of not defaulting, V^d is the value of default, and $d \in \{0,1\}$ is a binary variable capturing the default choice.

When the government defaults, it suspends payments on all existing debt, in which case the government is excluded from debt markets for a stochastic number of periods, and during those periods, suffers a utility loss. Upon reentry after k periods, the government's debt obligation is $-\lambda^k B$, where $1-\lambda$ is the rate at which the government's debt obligation decays each period. This tractable way of modeling partial default is also consistent with the fact that longer default episodes are associated with lower recovery rates, as documented by Benjamin and Wright (2009). Setting $\lambda = 0$ corresponds to the case with full default and $\lambda = 1$ to the case of no debt forgiveness upon reentry into credit markets.

The government's value of default is then given by

$$\begin{aligned} V^d(B,s) = {} & \frac{\left(\tau y(s) - \phi^d(s)\right)^{1-\sigma_g}}{1-\sigma_g} \\ & + \beta_g \mathbf{E}_{s'|s}\left[\theta V^o\left(\frac{\lambda B}{1+\pi(s')}, s'\right) + (1-\theta)V^d\left(\frac{\lambda B}{1+\pi(s')}, s'\right)\right], \end{aligned} \tag{13}$$

where $0 < \theta < 1$ is the probability that the government will regain access to credit markets, and $\phi^d(s)$ is the state-contingent utility loss during default. In particular, we assume a quadratic function,

$$\phi^d(s) = d_1 \max\left\{0, \frac{1}{d_0}y(s) + \left(1 - \frac{1}{d_0}\right)y(s)^2\right\}, \tag{14}$$

similar to Chatterjee and Eyigungor (2013), except that the expression has been written such that d_1 is the default cost at mean output ($y=1$) and d_0 determines the output threshold above which the default costs are positive.

In this setup, there are two possible exogenous shocks that increase the likelihood of default. The first (present in most standard models) is a low realization of the endowment

$y(s)$, which raises the marginal value of current resources and makes repayment more costly. The second, and specific to our setup, is a low realization of inflation $\pi(s)$, which increases the real value of the government's repayment and remaining debt obligations, and thus makes default a more attractive option. It turns out that both of these forces play an important role in our quantitative results.

The value of not defaulting is given by

$$V^c(B,s) = \max_{B'\leq 0} \begin{Bmatrix} \frac{1}{1-\sigma_g}\left(\tau y - q(s,B')\left(B'-(1-\delta)B\right)+B(r+\delta)\right)^{1-\sigma_g} \\ +\beta_g \mathbf{E}_{s'|s}\left[V^o\left(\frac{B'}{1+\pi(s')},s'\right)\right] \end{Bmatrix}, \tag{15}$$

where $B(r+\delta)$ represents the payment the government needs to make to lenders (maturing bonds plus coupon), and $q(s,B')$ is the price schedule that the government faces on its new issuance, $(B'-(1-\delta)B)$. Note that the real return on government debt is stochastic, even in the absence of default, because of inflation risk.

In this environment, the bond price schedule satisfies

$$\begin{aligned} q(s,B') \quad = \quad & \mathbf{E}_{s'|s}\left[\frac{1-d'}{1+\pi(s')}\left(r+\delta+(1-\delta)q\left(s',B''\right)\right)m(s,s')\right] \\ & +\mathbf{E}_{s'|s}\left[\frac{d'}{1+\pi(s')}q^{def}\left(\frac{B'}{1+\pi(s')},s'\right)m(s,s')\right], \end{aligned} \tag{16}$$

where d' and B'' are the optimal default and debt decisions given the state $(\frac{B'}{1+\pi(s')},s')$, and q^{def} is the value of a bond in default and is given by

$$\begin{aligned} q^{def}(B,s) \quad = \quad & \lambda\mathbf{E}_{s'|s}\left[\frac{\theta(1-d')}{1+\pi(s')}\left(r+\delta+(1-\delta)q\left(s',B''\right)\right)m(s,s')\right] \\ & +\lambda\mathbf{E}_{s'|s}\left[\frac{1-\theta+\theta d'}{1+\pi(s')}q^{def}\left(\frac{\lambda B}{1+\pi(s')},s'\right)m(s,s')\right], \end{aligned} \tag{17}$$

where d' and B'' are the optimal default and debt decisions given the state $(\frac{\lambda B}{1+\pi(s')},s')$. The first line of equation (17) represents the value in the case in which the government regains access to financial markets and does not immediately default on its debt. The second line represents the value when the government is either still excluded from markets or it regains access and immediately defaults. Notice that in both cases the value of debt decays by $1-\lambda$ each period.

Recursive equilibrium A Markov-perfect equilibrium for this economy is defined as value functions for the government $\left\{V^o, V^c, V^d\right\}$, the associated policy functions $\{B', d\}$, and bond pricing functions $\left\{q, q^{def}\right\}$ such that: (a) given $\left\{q, q^{def}\right\}$, $\left\{V^o, V^c, V^d, B', d\right\}$ solve the government's recursive problem in (12), (13), and (15); and (b) given the government policy functions $\{B', d\}$, the bond pricing functions $\left\{q, q^{def}\right\}$ satisfy (16) and (17).

Real bond price and spread It is convenient to define the real bond price as

$$\begin{aligned}\widehat{q}(s, B') \;&=\; \mathbf{E}_{s'|s}\left[(1-d')\frac{1+\bar{\pi}(s)}{1+\pi'}\left(r+\delta+(1-\delta)\widehat{q}\left(s', B''\right)\right)m(s,s')\right] \\ &\quad+\mathbf{E}_{s'|s}\left[d'\frac{1+\bar{\pi}(s)}{1+\pi(s')}\widehat{q}^{def}\left(\frac{B'}{1+\pi(s')}, s'\right)m(s,s')\right],\end{aligned} \tag{18}$$

where lenders adjust for expected inflation, defined as $1+\bar{\pi}(s) \equiv 1/\mathbf{E}_{s'|s}\left[1/\left(1+\pi(s')\right)\right]$. As before, d' and B'' are the optimal default and debt decisions given the state $(B'/(1+\pi(s')), s')$, and the real price of a bond in default $\widehat{q}^{def}$ is similarly defined as

$$\begin{aligned}\widehat{q}^{def}(B, s) \;&=\; \lambda\mathbf{E}_{s'|s}\left[\theta(1-d')\frac{1+\bar{\pi}(s)}{1+\pi(s')}\left(r+\delta+(1-\delta)\widehat{q}\left(s', B''\right)\right)m(s,s')\right] \\ &\quad+\lambda\mathbf{E}_{s'|s}\left[(1-\theta+\theta d')\frac{1+\bar{\pi}(s)}{1+\pi(s')}\widehat{q}^{def}\left(\frac{\lambda B}{1+\pi(s')}, s'\right)m(s,s')\right],\end{aligned} \tag{19}$$

where d' and B'' are the optimal default and debt decisions given the state $(\lambda B/(1+\pi(s')), s')$. We can now define our main object of interest, the equilibrium spread, $spr(B, s)$ as

$$spr(B,s) \equiv \frac{q^{RF}(s)-\widehat{q}(B,s)}{q_t^{RF}(s)}, \tag{20}$$

where $q^{RF}(s) \equiv \mathbf{E}_{s'|s}\left[\left(\delta+r+(1-\delta)q^{RF}(s')\right)m(s,s')\right]$ is the risk-free price, that is, the price of a non-defaultable real bond with the same maturity structure. Note that $q^{RF}(s)$ is not affected by default risk nor by the inflation process. Thus, the spread is the component of the real interest rate that is affected by the inflation process and default risk. To make this more transparent, in the special case in which $\lambda = 0$ and $\delta = 1$, we can express the equilibrium spread as

$$
\begin{aligned}
spr(B, s) = \; & \underbrace{\Pr\left[d' = 1\right]}_{\text{Default probability}} + \underbrace{\mathbf{cov}_t\left[\frac{m(s, s')}{\bar{m}(s)}, d'\right]}_{\text{Default risk premium}} \\
& - \underbrace{\Pr\left[d' = 0\right] \mathbf{cov}_t\left[\frac{m(s, s')}{\bar{m}(s)}, \frac{1 + \bar{\pi}(s)}{1 + \pi(s')}\right]}_{\text{Inflation procyclicality discount}} \\
& + \underbrace{\mathbf{cov}_t\left[\frac{1 + \bar{\pi}(s)}{1 + \pi(s')}, d'\right]}_{\text{Inflation/default interaction}},
\end{aligned} \tag{21}
$$

where $\bar{m}(s) \equiv \mathbf{E}_{s'|s}\left[m(s, s')\right]$. Recall that the lender's stochastic discount factor, $m(s, s')$ is negatively correlated with output growth.

The first two terms add to the spread and reflect the probability of default and the compensation for countercyclical default risk—effects that are standard but are now endogenous to the cyclicality of inflation. The term in the second line reflects the inflation procyclicality discount in the absence of default risk; it depends on the conditional comovement between surprise inflation and surprise output growth, and is positive in the procyclical inflation regime. The third term captures how the interaction between inflation and default affects bond returns. To see how this interaction works, consider the case of procyclical inflation and countercyclical default, in which case, the last term is positive. When inflation is procyclical, nominal bonds pay the most in the worst (low income) states of the world. Default, which happens in exactly those states, cuts these returns to 0 (when $\lambda = 0$) and thus makes the nominal bond less attractive.

Overall, equation (21) elicits the intuition from the simple model: the cyclicality of inflation in a model with domestic default entails various endogenous channels including, but not limited to, an endogenous default risk and the standard hedging argument. The interplay between these channels also varies over the cycle: inflation procyclicality is likely to be associated with a discount when default risk is low, but not in bad times as default motives increase with inflation procyclicality. Next we turn to a quantitative analysis of these forces.

4.2 Functional forms and calibration

We first calibrate the model with zero covariance between output and inflation, and then compare and contrast the models with procyclical and countercyclical inflation to assess the differential impact of inflation cyclicality on interest rates, debt dynamics, and default crises. Table 3 reports the value of the parameters of the model.

Income and inflation processes Endowments y and inflation π follow a joint process:

$$\begin{bmatrix} \log y' \\ \pi' \end{bmatrix} = \begin{bmatrix} \rho_{y,y} & \rho_{\pi,y} \\ \rho_{y,\pi} & \rho_{\pi,\pi} \end{bmatrix} \begin{bmatrix} \log y \\ \pi \end{bmatrix} + \begin{bmatrix} \epsilon_y \\ \epsilon_\pi \end{bmatrix} \tag{22}$$

where

$$\begin{bmatrix} \epsilon_y \\ \epsilon_\pi \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_y^2 & \sigma_{\pi,y} \\ \sigma_{\pi,y} & \sigma_\pi^2 \end{bmatrix} \right).$$

We set the persistence of output $\rho_{y,y}$ to 0.8, the persistence of inflation $\rho_{\pi,\pi}$ to 0.8, the spillover terms $\rho_{y,\pi}$ and $\rho_{\pi,y}$ to zero, and both variance terms σ_y and σ_π to 0.01 based on the parameters estimated for the cross section of OECD economies in our data set. Table 8 in Appendix A contains the detailed estimates by country. We consider two values for the covariance of inflation and output $\sigma_{\pi,y}$: $+0.17e^{-4}$ and $-0.17e^{-4}$, which correspond to positive and negative one standard deviation of the covariance of inflation and consumption growth estimated in Section 3.

Preferences and lender's stochastic discount factor Following the recent work that focuses on long-term interest rates with default risk (see, for example, Bocola and Dovis 2016 and Hatchondo et al. 2016) we assume that the lender's stochastic discount factor $m(s_t, s_{t+1})$ is a stochastic random variable and takes the form,

$$m(s_t, s_{t+1}) = \beta_\ell \left(\frac{y(s_{t+1})}{y(s_t)} \right)^{-1} \left(\frac{W(s_{t+1})^{1-\gamma_\ell}}{\mathbf{E}_t \left[W(s_{t+1})^{1-\gamma_\ell} \right]} \right), \tag{23}$$

where β_ℓ and γ_ℓ can be interpreted as the lender's discount factor and risk aversion, respectively, and $W(s_t)$ is defined recursively as

$$\log W(s_t) = (1-\beta_\ell)\log y(s_t) + \frac{\beta_\ell}{1-\gamma_\ell}\log\left(E_t\left[W(s_{t+1})^{1-\gamma_\ell}\right]\right). \tag{24}$$

Thus, the lender's stochastic discount factor is derived from recursive preferences as in Epstein and Zin (1989) and Weil (1989) where the intertemporal elasticity of substitution has been set to 1. Note that the lender's stochastic discount factor depends on total endowment $y(s_t)$, which is equal to the lender's consumption if we assume that the government's public expenditures are lumpsum rebated to the lender.

We set the discount factor β_ℓ of the lender to be 0.99 to match an annual risk-free rate of 4 percent. We set the lender's risk aversion γ_ℓ to be 59, following Hatchondo et al. (2016) and Piazzesi and Schneider (2006). This higher level of risk aversion of the lender is also common in the finance and equity premium puzzle literature (for example, see Bansal and Yaron 2004 and Mehra and Prescott 1985). We set the government's risk aversion γ_g to be 2, as is standard in the macro and sovereign debt literature.[12]

Jointly calibrated parameters We jointly choose the mean income loss parameter $d_1 = 0.20$ and the government's discount factor $\beta_g = 0.9875$ to match the cyclical properties of default risk. Specifically, we choose these parameters so that the acyclical economy has (i) an unconditional default probability of 0.2 percent and (ii) a conditional default probability of 0.0 percent when output is above average.

The unconditional default probability of 0.2 percent implies that defaults, on average, occur once every 500 years, which is the average frequency at which the countries in our data set have defaulted between 1900 and 2015, excluding the two world wars, according to the default and debt rescheduling episodes reported by Reinhart and Rogoff (2009). Since all four of these default and debt rescheduling episodes occurred during the midst of the Great Depression, we set the probability of default in tranquil times (above mean output) to 0.0 percent. Note that our unconditional default probability of 0.2 percent is an order of magnitude lower than those typically used in the literature for emerging economies, which

[12]We show in Appendix D that the results are robust to alternative lender or government preferences.

Table 3: Calibration – Baseline economy with acyclical inflation

Parameters	Values	Targets / Source
Gov't discount factor β_g	0.988	Unconditional default probability: 0.2 percent
Default cost at mean d_1	0.200	Default probability in good times: 0.0 percent
Lender discount factor β_ℓ	0.990	Risk-free rate: 4 percent
Lender risk aversion γ_ℓ	59	Hatchondo et al. (2016)
Gov't risk aversion γ_g	2	Hatchondo et al. (2016)
Default cost threshold d_0	-0.028	Sensitivity analysis in Appendix D
Probability of re-entry θ	0.100	Average exclusion: 10 quarters†
Recovery parameter λ	0.960	Average recovery rate: 50 percent‡
Tax rate τ	0.193	Government consumption (percent GDP)
Debt maturity δ	0.054	OECD average maturity: 4.6 years
Persistence $\rho_{y,y} = \rho_{\pi,\pi}$	0.800	VAR estimates (OECD cross section)
Spillovers $\rho_{\pi,y} = \rho_{y,\pi}$	0.000	VAR estimates
Volatility $\sigma_y = \sigma_\pi$	0.010	VAR estimates
Covariance of innovations $\sigma_{\pi,y}$	0.000	Acyclical baseline ± 1 s.d. $= \pm 0.17e^{-4}$

Note: † : See Richmond and Dias (2008). ‡ : See Benjamin and Wright (2009).

is around 2 percent.[13] We discuss the sensitivity of our main findings in section 4.3.

Other externally calibrated parameters We set the default cost parameter $d_0 = -0.0275$, which implies that additional default costs (over and above exclusion) are positive when output is more than 1.5 standard deviations above its mean. We show in Table 18 of Appendix D that the main results are robust to alternative values.

We set δ to be 0.054 to match the average domestic debt maturity of 4.6 years in our sample (1999–2010). We set the tax rate τ to be 19 percent to match the government consumption share of GDP in OECD economies between 1985 and 2015.

The probability of reentry $\theta = 0.1$ is set to match the average exclusion of 10 quarters as documented by Richmond and Dias (2008), and the recovery parameter $\lambda = 0.96$ is set to be consistent with the average recovery rate of 50 percent reported by Benjamin and Wright (2009). To compute the average recovery rate, we consider a default to be over when the government regains access to credit, and we discount the payment back to the period of default at an annualized interest rate of 10 percent, as in Benjamin and Wright (2009).

[13] See, for example, Aguiar et al. (2016) for a benchmark calibration for emerging economies.

Table 4: The unconditional procyclicality discount

	Negative comovement (–1 s.d.)	Positive comovement (+1 s.d.)	Difference
Spreads (percent)	1.57	1.31	–0.26
Default probability (percent)	0.16	0.21	+0.05
Public debt (percent of tax receipts)	70.9	66.7	–4.24

4.3 Results

Using the calibrated model, we contrast the two inflation regimes: countercyclical and procyclical. The goal of this exercise is to quantitatively assess how different inflation regimes affect interest rates in periods with and without default risk.[14]

The unconditional inflation procyclicality discount First, we present unconditional results from our calibrated benchmark model. In Table 4, we show the average equilibrium interest rates, debt, and default risk across inflation regimes.

We find that, relative to its countercyclical counterpart, the economy with procyclical inflation faces spreads that are 26 basis points lower. To compare this magnitude with our empirical findings, we use the regression coefficients estimated in the first row of Table 1 to show that a change in covariance like the one we feed into the model is associated in our data-set with a reduction in spreads of 61 basis points. This suggests that the mechanism highlighted in the model can account for a little less than half of the unconditional inflation procyclicality discount documented in the data. Table 4 shows that despite the discount, the procyclical economy is marginally more prone to debt crises and sustains lower debt burdens compared with the countercyclical economy.

These results are also qualitatively consistent with the intuition given in the spread decomposition equation (21) and the simple model in section 2: spreads feature an inflation procyclicality hedging discount in addition to an inflation procyclicality default premium.

[14]See the computational appendix for a description of our solution algorithm and the model simulation.

Table 5: The procyclicality discount with and without default risk

	Negative comovement (–1 s.d.)	Positive comovement (+1 s.d.)	Difference
Spreads (percent)			
No default risk (Low prob.)	1.08	0.67	–0.42
No default risk (High y)	1.31	0.73	–0.58
Positive default risk (High prob.)	5.17	5.62	+0.45
Positive default risk (Low y)	1.82	1.86	+0.04
Default prob. (percent)			
No default risk (Low prob.)	0.00	0.00	+0.00
No default risk (High y)	0.00	0.00	+0.00
Positive default risk (High prob.)	0.47	0.52	+0.05
Positive default risk (Low y)	0.31	0.39	+0.09

The conditional procyclicality discount Moreover, the procyclicality discount is state-contingent, as in the data. To show this, we report spreads (and default probabilities), conditional on periods with no default risk and with positive default risk. As we did in the data section, we experiment with two ways of selecting periods with and without default risk. The first (labeled Low/High prob. in the table) is based on actual default probabilities, which in the model we can measure exactly. The second (labeled High/Low y) is based on periods with output realizations above or below the mean. Table 5 reports the results.

In times with no default risk, default probabilities are near zero in both inflation regimes and under both definitions. During those times, the conditional inflation procyclicality discount is between 42 and 58 basis points. The coefficients estimated in the second row of Table 2 imply that a change in covariance like the one we feed into the model is associated in our data set with a reduction in spreads, during periods of low default risk, between 92 and 102 basis points. This suggests that the mechanism highlighted in the model can account for about half of the conditional inflation procyclicality discount documented in the data.

Table 5 also shows that in periods with positive default risk, moving from countercyclical to procyclical inflation increases default risk (by 5 or 9 basis points). During those times, the increase in default risk offsets the reduction in rates coming from the hedging effect, and overall more procyclical inflation causes an increase in rates of 4 or 45 basis points depending on the definition.

Summary Section 3 shows that an increase in the covariance between inflation and aggregate consumption of 0.34 is associated, in times without default risk, with a reduction of real rates of about 100 basis points. The model's results suggest that about half of this reduction can be explained by the economic mechanism highlighted here: When default is not an issue, more procyclical inflation implies that nominal bonds are less risky and thus pay lower rates. When default risk is present, however, the association between lower rates and procyclical inflation disappears in the data. In the model, this is also the case, as in simulated periods when default risk is positive, more procyclical inflation is associated with slightly higher rates. This is because in those periods, a more procyclical inflation, by generating large real debt repayments in bad times, increases the default incentives of the government. These findings suggest that the contingent nature of the inflation procyclicality discount observed in the data is explained by the interaction between inflation cyclicality and default highlighted by the model.

Robustness Our findings about the impact of inflation cyclicality on interest rates are qualitatively robust to alternative preferences, to different debt maturities, and to higher or lower default costs. However, all these factors matter quantitatively. In Tables 14 through 19 in Appendix D, we report the detailed results of several experiments. Table 14 shows that, not surprisingly, the procyclicality discount is increasing in the lender risk aversion. When risk aversion of the lender is sufficiently low ($\gamma_l = 8$), the unconditional procyclicality discount vanishes, as the default risk due to more procyclical inflation now offsets the lower procyclical hedging discount. Yet, the model still features a conditional procyclicality discount, that is in times without default risk the procyclical economy has lower interest rates. Table 15 reports the results of the economies with shorter (4 years) and longer (6 years) debt maturities. The table shows that increasing the maturity increases the procyclicality discount conditional on no default risk, but not the unconditional one. In the absence of default risk, the prices of longer maturity bonds are more sensitive to inflation surprises, and thus with procyclical inflation they provide a better hedge against aggregate risk. However, with default risk, the prices of longer maturity bonds are also more sensitive to the increase in default risk caused by more procyclicality. For our benchmark parameters, the second effect dominates, and the unconditional procyclicality discount falls (from 26 to 19 points) with longer maturity. In Table 16, we experiment with constant relative risk aversion (CRRA) utility for the lender, with two different values for the risk aversion ($\gamma_l = 8$ and $\gamma_l = 4$). As in the benchmark economy, these economies feature an unconditional and a conditional procyclicality discount. One issue with those preferences is that, as highlighted by many papers in the finance literature, they feature too much volatility of the risk-free rate. In Table 17, we experiment with higher and lower government risk aversion. With lower risk aversion, the results are mostly unchanged. When government risk aversion is sufficiently high ($\gamma_g = 3$ in the table), the government never finds it optimal to default and the economy becomes akin to an economy without default risk. Table 18 analyzes the impact of changes in the default costs (as captured by the threshold parameter d_0) and shows that procyclicality discounts and default probabilities are not significantly affected.

Finally, in Table 19, we report the results of the economy with higher and lower government discount factors. Note that when the government has a lower discount factor (relative to the benchmark) default probabilities are much higher than in the benchmark, and the economy features a conditional procyclicality discount but not an unconditional one. In other words, the unconditional inflation procyclicality discount does not materialize when default probabilities are on the order of magnitude of those observed in emerging economies.

4.4 When is procyclicality preferred?

The paper so far has shown that changes in inflation cyclicality can have sizable effects on real interest rates and default risk. In this section, with the aim of providing some guidance for policy, we discuss if and when the government prefers a procyclical inflation regime. Table 6 reports across different states the welfare gain, measured in consumption equivalents, that a government experiences with a change from counter- to procyclical inflation.

Table 6 reveals that the government typically prefers the procyclical regime, especially when default risk is low. Without default risk, the government can borrow at lower real interest rates, and since the borrower risk aversion is lower relative to the one of the lender, the benefits of paying lower interest rates outweigh the cost of making higher payments in bad

Table 6: Government preferences for procyclical inflation regime

	Consumption equivalent (percent)
Overall	+0.03
No default risk (Low prob.)	+0.04
No default risk (High y)	+0.08
Positive default risk (High prob.)	–0.06
Positive default risk (Low y)	–0.02
High default risk (Prob. $>$ 2 percent)	–0.15

times. However, during periods with positive default risk (measured either by low output or by high default probability), the government has a preference for countercyclicality. In very bad states, when the annualized probability of default exceeds 2 percent, the government has a strong preference for countercyclicality. This finding is consistent with the endogenous state- and regime-dependent default premium present in this model and the implied debt pricing.

As discussed above, when default is possible, a procyclical inflation regime is more likely to default, thus leading to higher, instead of lower, interest rates for the borrowers. These higher rates eliminate the source of welfare gain for the government and explain why in those states procyclicality is not preferred. Note that the welfare cost of higher rates is partially offset by the fact that in default states, the borrower repays less. However, the lender is more risk averse than the borrower, and that implies that the higher interest rate cost is larger than the reduction in payments during default.

These findings are relevant for the debate on the costs and benefits of joining or exiting a monetary union, and on the need for fiscal constraints in a monetary union (see Chari and Kehoe 2007). Consider countries within a union that enter a recession with different fiscal deficits (and hence default risk). The findings suggest that those in fiscal trouble would prefer a countercyclical monetary policy, while the others would not: the contrast over monetary policy increases in a recession. The specter of sovereign default in advanced economies or parts of a monetary union also raises financial stability concerns for the monetary authorities, in particular the optimal provision of safe assets and monetary backstops (see, for a discussion of these interactions, Gourinchas and Jeanne 2012).

4.5 Increased inflation uncertainty

Our theoretical, empirical, and quantitative findings suggest that the cyclicality of inflation and sovereign credit risk interact in the determination of the real interest rate. Motivated by the surge in inflation uncertainty during the COVID-19 pandemic recession and recovery, we use the model to ask how different real interest rates would be in an economic environment featuring more inflation variance than in our baseline calibration. To highlight the role of default risk, we contrast these changes across higher/lower government discount factors that make the government less/more prone to default.

Specifically, we consider a hypothetical change in inflation volatility from $\sigma_\pi = 0.01$ to $\sigma_\pi = 0.02$, while keeping unchanged both the volatility of output ($\sigma_y = 0.01$) and the output-inflation correlation ($\rho_{\pi,y} = \pm 0.17$). We emphasize the role of endogenous default risk by considering different government discount factor values ($\beta_g = 0.9888$ and $\beta_g = 0.9850$), in addition to the calibrated baseline value ($\beta_g = 0.9875$).

We report average spreads and annual default probabilities across these counterfactual experiments in Table 7. We focus on the difference in spreads in the high inflation volatility economy relative to the lower volatility regime.

Panel A of Table 7 shows that, when inflation volatility increases, the procyclical economy experiences only a modest increase in spreads (+5 bp.). This modest increase is the result of two offsetting effects. On the one hand, higher inflation volatility implies better hedging from procyclical inflation and reduces spreads. On the other hand, higher inflation volatility also increases the probability of default when inflation is procyclical (+10 pp.) and pushes spreads higher. The latter effect dominates in our baseline calibration. When inflation is countercyclical, higher inflation volatility leads to higher spreads (+16 bp.) since the hedging property becomes worse for the lender (i.e. larger losses from inflation become more likely in bad times).

Panels B and C of Table 7 clearly illustrate this key interaction of default, inflation cyclicality, and interest rates. When the government discount factor is lower (Panel B), the default risk channel dominates and the procyclical economy actually experiences a *larger increase* in spreads (+73 pp.) than the countercyclical economy (+71 pp.). In contrast, when the government is more patient and default motives are more muted (Panel C), the

Table 7: Inflation cyclicality and changes in spreads under increased inflation risk

		Baseline inflation variance ($\sigma_\pi = 0.01$)	Higher inflation variance ($\sigma_\pi = 0.02$)	Difference
Panel A: Baseline government discount factor ($\beta_g = 0.9875$)				
Procyclical inflation	Spreads	1.31	1.35	+0.05
	Default prob.	0.21	0.31	+0.10
Countercyclical inflation	Spreads	1.57	1.73	+0.16
	Default prob.	0.16	0.17	+0.01
Panel B: Lower government discount factor ($\beta_g = 0.9850$)				
Procyclical inflation	Spreads	3.88	4.60	+0.73
	Default prob.	0.60	0.85	+0.24
Countercyclical inflation	Spreads	3.63	4.34	+0.71
	Default prob.	0.49	0.63	+0.15
Panel C: Higher government discount factor ($\beta_g = 0.9888$)				
Procyclical inflation	Spreads	0.38	0.11	–0.27
	Default prob.	0.09	0.12	+0.03
Countercyclical inflation	Spreads	0.75	0.88	+0.13
	Default prob.	0.06	0.03	–0.03

Units: percent.

procyclical economy experiences a *decline* in spreads (-27 pp.) while the countercyclical economy experiences rising spreads (+13 pp.).[15] These counterfactual exercises show how the future path of interest rates will likely be influenced by the evolution of inflation cyclicality and default risk.

5 Conclusion

This paper has shown that inflation cyclicality is an important determinant of real returns on nominal bonds issued by governments across countries and over time.

Theoretically, we have developed a model of sovereign debt with inflation risk and domestic risk-averse lenders. The model shows how inflation cyclicality affects interest rates and the dynamics of default. A more procyclical inflation implies that nominal bonds pay out more in bad times; this makes these bonds desirable for lenders and tends to yield

[15] We also vary the default cost threshold parameter d_0 to explore the role of default risk. The findings are reported in Table 20 of Appendix E. Overall, the findings are similar to the ones we are reporting in the main text using changes in the government discount factor.

lower equilibrium real rates. However, bad times for the lenders are also bad times for the borrower (the government), and these larger payouts in bad times imply higher default incentives. When default risk is material, higher default incentives can result in higher, instead of lower, equilibrium rates. Empirically, consistent with our theory, we find that increased comovement of inflation and aggregate consumption growth is associated with lower real interest rates, but only in times when default on government debt is not an issue. We call this pattern a "conditional inflation procyclicality discount."

Quantitatively, we use a calibrated model that is consistent with our empirical evidence to investigate the welfare consequences of inflation procyclicality and the counterfactual change in real interest rates when inflation becomes more volatile. First, procyclical inflation is strongly preferred to countercyclical inflation only when there is no default risk. Second, when default risk is very low, higher inflation volatility reduces borrowing costs in the procyclical economy because of improved hedging properties; the opposite is true in the countercyclical economy. When default risk is significant, however, higher inflation volatility substantially increases borrowing costs in both the procyclical-inflation and the countercyclical-inflation economies.

Overall, our findings can help us understand the secular decline in real rates observed in recent years in many countries. We believe they also shed light on why some developed countries recently have observed substantial swings in their sovereign default risk. These findings can also help inform to the costs and benefits of public debt in a low interest environment, especially the drivers of credit risk emphasized by Blanchard (2019).

Throughout the paper, we have modeled inflation as an exogenous process and focused on the pricing of debt and on endogenous default decisions. In reality, many studies—starting with Sargent and Wallace (1981)—showed that the process for inflation and its comovement with output are the result of explicit monetary policy choices, and of the interaction between monetary policy and the fiscal authority, all in response to different types of shocks. We think that including the link between inflation cyclicality, debt pricing, and default highlighted by this paper in a study of optimal monetary and fiscal responses to shocks is an interesting and policy-relevant direction for future research.

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Appendix

A Additional Tables and Figures

Table 8: VAR results

country	$\rho_{\pi\pi}$	$\rho_{c\pi}$	$\rho_{\pi c}$	ρ_{cc}	σ_c	σ_π	$\sigma_{\pi,c}$
USA	0.93	0.06	-0.10	0.86	0.17	0.34	0.00
AUS	0.82	0.10	-0.02	0.67	0.67	0.54	0.07
AUT	0.82	0.04	-0.10	0.65	0.27	0.43	0.00
BEL	0.85	0.02	-0.04	0.77	0.33	0.33	0.00
CAN	0.75	0.18	-0.02	0.72	0.63	0.42	0.06
CHE	0.90	0.09	-0.02	0.83	0.27	0.29	0.01
DEU	0.85	0.10	-0.15	0.49	0.32	0.53	0.02
DNK	0.56	-0.05	-0.25	0.71	0.56	0.66	0.02
ESP	0.87	0.01	-0.04	0.91	0.34	0.59	0.01
FIN	0.67	0.12	-0.01	0.87	0.65	0.73	0.05
FRA	0.89	0.10	-0.18	0.67	0.22	0.32	-0.01
GBR	0.83	0.09	-0.11	0.83	0.56	0.51	-0.06
ITA	0.67	-0.03	-0.01	0.88	0.61	0.44	-0.01
JPN	0.92	0.10	-0.26	0.48	0.37	0.70	-0.11
KOR	0.69	0.10	-0.30	0.81	0.97	1.24	-0.32
NLD	0.67	0.04	-0.05	0.85	0.53	0.44	0.00
NOR	0.81	0.14	-0.02	0.68	1.79	0.80	-0.02
PRT	0.88	-0.04	0.02	0.89	0.68	0.71	-0.02
SWE	0.75	-0.12	-0.02	0.75	0.72	0.52	0.09
average	0.80	0.06	-0.09	0.75	0.56	0.56	-0.01
median	0.82	0.09	-0.04	0.77	0.52	0.56	0.00
min	0.56	-0.12	-0.30	0.48	0.29	0.17	-0.32
max	0.93	0.18	0.02	0.92	1.24	1.79	0.09

The data are a quarterly panel from 1985Q1 to 2015Q4.

B Additional empirical analysis

B.1 Robustness of empirical findings

Table 9 documents the robustness of the two main empirical findings from section 3. The top panel documents the robustness of the finding that more procyclical inflation is (unconditionally) associated with lower real rates. The middle and bottom panels of the table show the robustness of the result that a more procyclical inflation is associated with a larger discount in times of no default risk (relative to times with positive default risk).

Column 1 reports the baseline results (from Tables 1 and 2 in the text). Columns 2 and 3 experiment with shorter and longer windows over which the moments of interest are computed. Column 4 shows the result of using median regression instead of standard OLS. Column 5 experiments with an alternative measure of rates, derived using yields on 10-year government bonds from Haver Analytics. Column 6 shows that the main findings are robust to using *ex post* realized inflation to computing real interest rates.

The first panel (line 1) shows that the coefficient on inflation consumption/covariance is always negative and significant, that is, there is always an inflation procyclicality discount. The second and third panels show that the procyclicality discount in times of no default risk (lines 2 and 4) is always statistically significant with a point estimate that is larger than the discount in times with positive default risk (lines 3 and 5). Moreover, the discount in times of positive default risk (lines 3 and 5) is significantly different from zero (at the 5 percent level) in only 2 out of 12 specifications.

Table 9: Robustness of main empirical findings

	Real yield on government debt					
	(1) baseline	(2) 8-year window	(3) 12-year window	(4) Median reg.	(5) Alt. yields	(6) Alt. real rate
1. Inflation-consumption covariance	−1.80**	−1.73***	−1.94**	−1.19***	−1.76**	−1.80**
	(0.64)	(0.58)	(0.79)	(0.23)[a]	(0.70)	(0.65)
adj. R^2	0.90	0.89	0.92	N/A[a]	0.92	0.88
N	1726	1838	1614	1764	1620	1726
2. Interaction term (No default risk: credit rating)	−2.70***	−2.21**	−2.73***	−1.85***	−2.32**	−2.61***
	(0.91)	(0.78)	(0.89)	(0.28)[a]	(1.01)	(0.94)
3. Interaction term (Positive default risk)	−1.31	−1.28*	−1.84	−1.63***	−0.84	−1.42*
	(0.79)	(0.68)	(1.13)	(0.28)[a]	(0.93)	(0.82)
adj. R^2	0.92	0.91	0.94	N/A[a]	0.92	0.92
N	1438	1524	1352	1463	1375	1438
4. Interaction term (No default risk: cons. growth)	−2.99***	−2.29***	−3.34***	−2.53***	−2.35**	−2.98***
	(0.70)	(0.65)	(0.69)	(0.22)	(0.94)	(0.75)
5. Interaction term (Positive default risk)	−1.16	−1.32**	−0.91	0.16	−0.97	−1.17*
	(0.68)	(0.63)	(0.77)	(0.21)[a]	(0.75)	(0.67)
adj. R^2	0.91	0.89	0.93	N/A[a]	0.92	0.89
N	1726	1838	1614	1764	1620	1726

Note: Standard errors are in parentheses and are clustered by country. All regressions include country and time fixed effects, averages and variances of the residuals of inflation and consumption growth in the window, lagged debt, and in panels 2 and 3, dummies for no default risk.
[a]: The median regression does not include lagged debt, and standard errors are not clustered.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

B.2 Evidence with annual data

In this section, we extend our empirical findings using annual data that spans from 1950 to 2016. Our dataset is assembled from the Jordà et al. (2017) Macrohistory database, which includes real consumption growth, inflation, interest rates on long-term government bonds, and government debt for a panel of 17 advanced economies: Australia, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

Because the annual data allows us to extend our analysis to a longer time horizon, we adopt two methods that allow for time-varying VAR estimates. In the first method, we estimate time-varying covariances from a bivariate VAR(1) on inflation and real consumption growth, using 20-year rolling windows à la Lunsford and West (2019). In the second, we use the Bayesian multivariate stochastic volatility TVP-VAR specified in Primiceri (2005) using the implementation codes in Koop et al. (2010).

B.2.1 Rolling window VAR

We follow Lunsford and West (2019) and compute 20-year rolling VARs of the following form:

$$\begin{bmatrix} \pi_{it} \\ g_{it} \end{bmatrix} = c_{it} + \mathbf{A_{it}} \begin{bmatrix} \pi_{it-1} \\ g_{it-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{it}^{\pi} \\ \varepsilon_{it}^{g} \end{bmatrix} \quad , \quad \varepsilon \sim \mathcal{N}(0, \Sigma_{it}) \tag{25}$$

where π_{it} is inflation, g_{it} is the change in log consumption in country i in period t, $\mathbf{A_{it}}$ and $\mathbf{\Sigma_{it}}$ are country- and window-specific matrices of coefficients and variance-covariance matrices, respectively. As in the baseline, we define the real interest rate as the nominal rate on 10-year government bonds minus the expected rate of inflation from the VAR.

Following Lunsford and West (2019), we compute 10-year moving averages of all variables (real interest rates, covariance, etc.). Table 10, which is the analogue of Table 1, shows that the procylicality discount is robust to using this alternative data and method.

In the baseline empirical analysis, we proxied for default risk with credit ratings and residual consumption growth. Since we do not have credit rating data for most countries prior to 1985, we alternatively define a window as one with positive default risk if average debt accumulation is higher than the 67th percentile for average debt accumulation for that country or if average log consumption growth is lower than the 33rd percentile for average log consumption growth for that country. Table 11—the analogue of Table 2—shows that the conditional procylicality discount is also robust to this alternative data and method. For example, a two standard deviation increase in the covariance of inflation and consumption growth is associated with a 147 basis point reduction (=1.63×2×0.45) in the real interest in the absence of default risk, compared to an 82 basis point reduction with default risk.

Table 10: Inflation consumption growth comovement and real interest rates using rolling VARs

	Real yield on government debt		
	covariance		correlation
	(1)	(2)	(3)
Inflation consumption comovement	–0.29***	–0.35***	–1.38***
	(0.05)	(0.05)	(0.30)
Lagged government debt to GDP		1.54***	1.16***
		(0.35)	(0.33)
Average inflation		–0.25***	0.03
		(0.08)	(0.08)
Average cons. growth		0.24*	0.18
		(0.13)	(0.12)
Variance of inflation			–0.12***
			(0.02)
Variance of cons. growth			–0.05***
			(0.01)
standard deviation of comovement	1.62	1.63	0.29
adj. R^2	0.44	0.46	0.52
N	731	720	720

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Robust standard errors are in parentheses. All regressions include country and time fixed effects. All variables are 10-year moving averages.

B.2.2 Time-Varying Parameter VAR

The Multivariate Time Series Model We follow Primiceri (2005) and postulate the following vector auto-regressive model of lag order k for inflation (π_{it}) and consumption growth (g_{it}) in country i:

$$\begin{bmatrix} \pi_{it} \\ g_{it} \end{bmatrix} = c_{it} + \mathbf{B_{1,it}} \begin{bmatrix} \pi_{it-1} \\ g_{it-1} \end{bmatrix} + \cdots + \mathbf{B_{k,it}} \begin{bmatrix} \pi_{it-k} \\ g_{it-k} \end{bmatrix} + \begin{bmatrix} u_{\pi_{it}} \\ u_{g_{it}} \end{bmatrix} \quad t = 1, \ldots, T \tag{26}$$

The variance-covariance matrix $\mathbf{\Omega_{it}}$ of the innovations u_{it} satisfies the triangular reduction

$$\mathbf{A_{it}} \mathbf{\Omega_{it}} \mathbf{A'_{it}} = \mathbf{\Sigma_{it}} \mathbf{\Sigma'_{it}} \tag{27}$$

Table 11: Inflation procyclicality discount with and without default risk (rolling VAR)

	Real yield on government debt	
	(1)	(2)
Inflation consumption covariance	–0.35**	
	(0.05)	
Interaction term (No default risk)		–0.45***
		(0.07)
Interaction term (Positive default risk)		–0.25***
		(0.06)
Additional controls	Yes	Yes
adj. R^2	0.46	0.49
N	720	718

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Robust standard errors are in parentheses. Additional controls include country and time fixed effects, lagged government debt-to-GDP, average inflation and consumption growth, and, in column (2), dummies for no default risk. All variables are 10-year moving averages.

where $\mathbf{A_{it}}$ is a lower triangular matrix

$$\mathbf{A_{it}} = \begin{bmatrix} 1 & 0 \\ \alpha_{it} & 1 \end{bmatrix}$$

and $\mathbf{\Sigma_{it}}$ is a diagonal matrix with diagonal vector $\sigma_{it} = [\sigma_{\pi,it}, \sigma_{g,it}]$:

$$\mathbf{\Sigma_{it}} = \begin{bmatrix} \sigma_{\pi,it} & 0 \\ 0 & \sigma_{g,it} \end{bmatrix}.$$

Model Dynamics Denote B_{it} the vector of stacked R.H.S. coefficients. That is: $B_{it} = [c_{it}, vec(\mathbf{B_{1,it}}), \ldots, vec(\mathbf{B_{k,it}})]$. Primiceri (2005) shows that the TVP-VAR formulation above is equivalent to

$$y_{it} = X'_{it}B_{it} + \mathbf{A_{it}^{-1}\Sigma_{it}}\varepsilon_{\mathbf{it}} \tag{28}$$

$$X'_{it} = \mathbf{I} \otimes [1, y'_{it-1}, \ldots, y'_{it-k}] \tag{29}$$

$$\mathbf{var}(\varepsilon_{it}) = \mathbf{I} \tag{30}$$

where

$$B_{it} = B_{it-1} + \nu_{it},$$

$$\log \sigma_{it} = \log \sigma_{it-1} + \eta_{it},$$

$$\alpha_{it} = \alpha_{it-1} + \zeta_{it},$$

and

$$\mathbf{var}\left(\begin{bmatrix} \varepsilon_{it} \\ \nu_{it} \\ \zeta_{it} \\ \eta_{it} \end{bmatrix}\right) = \begin{bmatrix} \mathbf{I} & 0 & 0 & 0 \\ 0 & \mathbf{Q_i} & 0 & 0 \\ 0 & 0 & \mathbf{S_i} & 0 \\ 0 & 0 & 0 & \mathbf{W_i} \end{bmatrix} \tag{31}$$

Bayesian Inference As noted above, we extend the benchmark calibration of Primiceri (2005) to our dataset. For each country, we have an annual sample from 1950 to 2016. We use $k = 2$ lags. The simulations are based on 200,000 iterations of the Gibbs sampler, discarding the first 20,000. We use the implementation programs released by Koop et al. (2010). We calibrate the prior distributions based on OLS point estimates and the variances in a time-invariant VAR from the first 10 years of data. We summarize the priors below and refer the interested reader to the original paper and implementation codes:

$$
\begin{aligned}
B_{i0} &\sim N\left(\widehat{B}_{i,OLS}, 4 * \mathbf{var}(\widehat{B}_{i,OLS})\right) \\
\mathbf{A}_{i0} &\sim N\left(\widehat{\mathbf{A}}_{i,OLS}, 4 * \mathbf{var}(\widehat{\mathbf{A}}_{OLS})\right) \\
\log \sigma_{i0} &\sim N\left(\log \widehat{\sigma}_{i,OLS}, \mathbf{I}\right) \\
\mathbf{Q}_i &\sim IW\left(k_Q^2 * 10 * \mathbf{var}(\widehat{B}_{i,OLS}), 10\right) \\
\mathbf{W}_i &\sim IW\left(k_W^2 * 4 * \mathbf{I}, 4\right) \\
\mathbf{S}_i &\sim IW\left(k_S^2 * 2 * \mathbf{var}(\widehat{A}_{i,OLS}), 2\right)
\end{aligned}
$$

The benchmark results are based on the following values: $k_Q = 0.01$, $k_S = 0.10$, $k_W = 0.01$ to allow for diffuse and uninformative priors as in Primiceri (2005).

We measure the expected inflation as the forward-looking predicted inflation from the estimated VAR, that is, $\mathbf{E}[\pi_{i,t+1}]$. We then derive real rates on government debt as nominal rates less expected inflation. Finally, we measure the conditional comovement between inflation and consumption growth as the covariance α_{it} between the two innovations, $u_{\pi it}$ and u_{git}.

Estimation Results We combine the TVP-VAR estimates with the Jordà et al. (2017) macrohistory database to estimate how the conditional covariance of inflation and consumption growth relates to interest rates faced by governments. We average all the variables in the estimation over 7-year centered overlapping windows. All specifications include a full set of country fixed effects and time fixed effects.

Table 12 reports the results from regressing the real interest rate on the conditional comovement between inflation and consumption growth. The main result from the table is that the coefficients in the first row of the table are always negative and significantly different from 0. This means that in periods with higher comovement between inflation and consumption growth (measured using either covariance in columns 1–3 or correlation in column 4), governments face lower real interest rates. This finding is robust to the inclusion of the lagged change in government debt-to-GDP ratio and average inflation and consumption growth in the period (columns 2, 3, and 4). This association is also robust to the inclusion of the variances of residual inflation and consumption growth as additional regressors (columns 3 and 4).

Table 12: Inflation consumption growth comovement and real interest rates using TVP-VAR estimates

	Real yield on government debt			
		covariance		correlation
	(1)	(2)	(3)	(4)
Inflation consumption comovement	–0.466***	–0.452***	–0.446***	–4.907***
	(0.040)	(0.029)	(0.033)	(0.507)
Lagged change in govt. debt to GDP	0.226***	0.257***	0.146***	0.127***
	(0.023)	(0.021)	(0.021)	(0.018)
Average expected inflation		–0.382***	–0.317***	–0.238***
		(0.032)	(0.044)	(0.045)
Average consumption growth		–0.067	–0.079 *	–0.081 *
		(0.045)	(0.045)	(0.045)
Variance of inflation residual			–0.279 **	–0.493 ***
			(0.125)	(0.138)
Variance of cons. growth residual			0.503 ***	0.814***
			(0.150)	(0.159)
standard deviation of comovement	0.907	0.907	0.907	0.089
adj. R^2	0.671	0.726	0.733	0.706
N	1003	1003	1003	1003

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parentheses. Standard errors are clustered by country. All regressions include country and time fixed effects. All variables are computed over 7-year overlapping centered windows.

Our second main finding is that this procyclicality discount is significantly larger in times when default on government debt is not an issue. Columns (2) and (3) of Table 13 report the results from a regression similar to the one from Table 12, with the difference that now the inflation-consumption covariance is interacted with a dummy for no default risk and with a dummy for its complement, positive default risk.

In column (2), in the absence of reliable historical credit ratings data, we define a window with no default risk for a country as a 7-year window in which the average debt accumulation is below the country's median debt accumulation of that country or in which the average

Table 13: Inflation procyclicality discount with and without default risk using TVP-VAR estimates

	Real yield on government debt		
	(1)	(2)	(3)
		Debt growth	Cons. growth
		as *default risk* measure	
Inflation consumption covariance	–0.452***		
	(0.029)		
Interaction term (No default risk)		–0.630***	–0.528***
		(0.079)	(0.045)
Interaction term (Positive default risk)		–0.376***	–0.326***
		(0.032)	(0.040)
Additional controls	Yes	Yes	Yes
adj. R^2	0.733	0.737	0.738
N	1003	1003	1003

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parentheses. Standard errors are clustered by country. Additional controls include country and time fixed effects, lagged government debt-to-GDP, the averages and variances of residual inflation and consumption growth, and, in columns (2)-(3), dummies for no default risk. All variables are computed over a 7-year overlapping centered window.

consumption growth for that country is above the country's median growth. In column (3), we experiment with an alternative measure of no default risk; that is, a 7-year window in which the average consumption growth for that country is above its median.

Both columns show that the interaction term between the inflation-consumption growth covariance and the no-default risk dummy is negative, statistically significant, and larger than the discount estimated on the full sample. The interaction of the same covariance with the indicator for times with positive default risk is also statistically significant, but smaller than in the full sample.

C Proofs

C.1 Proof of Theorem 1

Theorem 1. Inflation procyclicality discount

Assume that both borrowers and lenders have quasi-linear utility such that $u(c) = Ac$, *and* $v(c) = Ac - \frac{\phi}{2}c^2$ *with* $A > 0$, $\phi > 0$ *and* $\frac{A}{\phi} > \mu$. *Then, the equilibrium real interest rate* $r \equiv 1/q - 1$ *features an inflation procyclicality discount. That is,*

$$\frac{\partial r}{\partial \kappa} < 0. \tag{32}$$

Proof: Notice first that since $r(\kappa) \equiv \frac{1}{q(\kappa)} - 1$, $\dfrac{dr(\kappa)}{d\kappa} < 0 \Leftrightarrow \dfrac{dq(\kappa)}{d\kappa} > 0$.

Lender. The lender's first-order condition is given by

$$-qu'(1-qb) + \beta_\ell \mathbf{E}\left[v'\left(x + \frac{b}{1+\pi(x;\kappa)}\right)\frac{1}{1+\pi(x;\kappa)}\right] = 0, \tag{33}$$

which can be written as

$$qA = \beta_\ell \left[A - \phi(\mu + b) + \phi\kappa\sigma^2 - \phi b\kappa^2\sigma^2\right]. \tag{34}$$

Rearranging terms in equation (34) yields the optimal debt supply:

$$b_\ell(q;\kappa) = \frac{-\dfrac{A}{\phi}q + \beta_\ell\left(\dfrac{A}{\phi} - \mu + \kappa\sigma^2\right)}{\beta_\ell\left(1+\kappa^2\sigma^2\right)}. \tag{35}$$

Borrower. The borrower's first-order condition is given by

$$qu'(1+qb) + \beta_b \mathbf{E}\left[u'\left(x - \frac{b}{1+\pi(x;\kappa)}\right)\frac{1}{1+\pi(x;\kappa)}\right] = 0, \tag{36}$$

which can be written as

$$qA = \beta_b \left[A - \phi(\mu - b) + \phi\kappa\sigma^2 + \phi b\kappa^2\sigma^2\right]. \tag{37}$$

Hence, the optimal debt demand is given by

$$b_b(q;\kappa) = \frac{\frac{A}{\phi}q - \beta_b\left(\frac{A}{\phi} - \mu + \kappa\sigma^2\right)}{\beta_b\left(1+\kappa^2\sigma^2\right)}. \tag{38}$$

Inflation Procyclicality Discount. The market clearing condition is

$$b_\ell(q;\kappa) = b_b(q;\kappa). \tag{39}$$

Substituting equations (35) and (38) and rearranging terms, we obtain

$$q = \frac{\phi}{A}\frac{2\beta_b\beta_\ell}{\beta_b+\beta_\ell}\left(\frac{A}{\phi} - \mu + \kappa\sigma^2\right). \tag{40}$$

Finally, taking the derivative of q with respect to κ yields the desired result. □

C.2 Proof of Theorem 2

Theorem 2. Inflation procyclicality and default

Assume that $-(\mu - x_{\min})^{-1} < \kappa < (x_{\max} - \mu)^{-1}$. *For* ψ *large enough, there exists a unique threshold,* $\widehat{x}(\kappa, b_b) \in [x_{\min}, \mu]$, *such that default occurs if and only if* $x \in [x_{\min}, \widehat{x}]$. *Furthermore, the default threshold is increasing in debt* (b_b) *and the cyclicality of inflation* (κ), *ceteris paribus. That is,*

$$\frac{\partial \widehat{x}(\kappa, b_b)}{\partial b_b} > 0 \tag{41}$$

$$\frac{\partial \widehat{x}(\kappa, b_b)}{\partial \kappa} > 0. \tag{42}$$

Proof: The borrower defaults when the cost of default is less than the cost of repayment, that is, when

$$C\left(x\right) \leq b_b[1+\pi\left(x;\kappa\right)]^{-1}$$

or

$$C\left(x\right)\left[1+\pi\left(x;\kappa\right)\right] \leq b_b. \tag{43}$$

The proof proceeds in the following steps. First, we show that if a solution exists, it is unique. Second, we show that the unique threshold is increasing in debt and the cyclicality of inflation.

Existence and uniqueness. If a solution exists, it is unique if the left-hand side of (43) is strictly increasing,

$$C_x\left[1+\pi\left(x;\kappa\right)\right]+C\left(x\right)\pi_x\left(x;\kappa\right)>0. \tag{44}$$

We know that

$$\begin{aligned}\pi\left(x;\kappa\right)&=\frac{-\kappa\left(\mu-x\right)}{1+\kappa\left(\mu-x\right)}\\ \Rightarrow\pi_x\left(x;\kappa\right)&=\frac{\kappa+\kappa\pi\left(x;\kappa\right)}{1+\kappa\left(\mu-x\right)}\\ &=\kappa\left[1+\pi\left(x;\kappa\right)\right]^2.\end{aligned}$$

Condition (44) then becomes

$$C_x>-C\left(x\right)\kappa\left[1+\pi\left(x;\kappa\right)\right],$$

which holds since

$$\begin{aligned}&C_x>-C\left(x\right)\kappa\left[1+\pi\left(x;\kappa\right)\right]\\ \Leftrightarrow&2\psi\left(x-x_{\min}\right)>-\psi\left(x-x_{\min}\right)^2\kappa\left[1+\pi\left(x;\kappa\right)\right]\\ \Leftrightarrow&2\left[1+\kappa\left(\mu-x\right)\right]>-\left(x-x_{\min}\right)\kappa\\ \Leftrightarrow&\kappa\left(\mu-\frac{x+x_{\min}}{2}\right)>-1\\ \Leftarrow&\frac{-1}{\mu-x_{\min}}<\kappa<\frac{1}{x_{\max}-\mu}.\end{aligned}$$

Hence if a solution exists, it is unique. Since $C(x)$ is continuous, by the intermediate value theorem, a solution exists in $x\in[x_{\min},\mu]$ if

$$C\left(x_{\min}\right)\left[1+\pi\left(x_{\min};\kappa\right)\right]\leq 0,$$

which holds since $C(x_{\min}) = 0$, and

$$C(\mu)[1 + \pi(\mu; \kappa)] \geq b_b,$$

which holds for ψ large enough.

Hence, there exists an output threshold

$$\hat{x} \in [x_{\min}, \mu]$$

such that the borrower defaults if and only if $x \leq \hat{x}$.

Comparative Statics. Let $G(\hat{x}; \kappa, b_b) = C(\hat{x}) - b_b(1 + \pi(\hat{x}; \kappa))^{-1} = 0$. By the implicit function theorem,

$$\frac{\partial G(\hat{x}; \kappa, b_b)}{\partial \hat{x}} \frac{d\hat{x}}{db_b} + \frac{\partial G(\hat{x}; \kappa, b_b)}{\partial b_b} = 0$$

and

$$\frac{\partial G(\hat{x}; \kappa, b_b)}{\partial \hat{x}} \frac{d\hat{x}}{d\kappa} + \frac{\partial G(\hat{x}; \kappa, b_b)}{\partial \kappa} = 0.$$

Hence,

$$\begin{aligned}
\frac{d\hat{x}}{db_b} &= -\frac{-(1 + \pi(\hat{x}; \kappa))^{-1}}{C_x(\hat{x}) + b_b(1 + \pi(\hat{x}; \kappa))^{-2}\pi_x(\hat{x}; \kappa)} \\
&= \frac{1}{C_x(\hat{x})[1 + \pi(\hat{x}; \kappa)] + b_b[1 + \pi(\hat{x}; \kappa)]^{-1}\pi_x(\hat{x}; \kappa)} \\
&= \frac{1}{C_x(\hat{x})[1 + \pi(\hat{x}; \kappa)] + C(\hat{x})\pi_x(\hat{x}; \kappa)} > 0
\end{aligned}$$

since

$$C_x[1 + \pi(x; \kappa)] + C(x)\pi_x(x; \kappa) > 0$$

from (44). We also have

$$\begin{aligned}
\frac{d\hat{x}}{d\kappa} &= -\frac{b_b[1+\pi(\hat{x};\kappa)]^{-2}\pi_\kappa(\hat{x};\kappa)}{C_x(\hat{x})+b_b(1+\pi(\hat{x};\kappa))^{-2}\pi_x(\hat{x};\kappa)} \\
&= -\frac{b_b[1+\pi(\hat{x};\kappa)]^{-1}\pi_\kappa(\hat{x};\kappa)}{C_x(\hat{x})[1+\pi(\hat{x};\kappa)]+b_b[1+\pi(\hat{x};\kappa)]^{-1}\pi_x(\hat{x};\kappa)} \\
&= -\frac{b_b[1+\pi(\hat{x};\kappa)]^{-1}\pi_\kappa(\hat{x};\kappa)}{C_x(\hat{x})[1+\pi(\hat{x};\kappa)]+C(\hat{x})\pi_x(\hat{x};\kappa)} > 0
\end{aligned}$$

since

$$\pi(x;\kappa) = \frac{-\kappa(\mu-x)}{1+\kappa(\mu-x)} \tag{45}$$

$$\Rightarrow \pi_\kappa(\hat{x};\kappa) = \frac{-(\mu-\hat{x})-(\mu-\hat{x})\pi(\hat{x};\kappa)}{1+\kappa(\mu-\hat{x})} \tag{46}$$

$$= \frac{-(\mu-\hat{x})(1+\pi(\hat{x};\kappa))}{1+\kappa(\mu-\hat{x})} \tag{47}$$

$$= -(\mu-\hat{x})[1+\pi(\hat{x};\kappa)]^2 < 0. \tag{48}$$

This concludes the proof of Theorem 2. □

D Sensitivity Analyses

Table 14: Robustness to lender's risk aversion

	Negative comovement (–1 s.d.)	Positive comovement (+1 s.d.)	Difference
Lower risk aversion ($\gamma_\ell = 8$)			
Spreads (percent)			
Overall	1.38	1.38	–0.00
No default risk (Low prob.)	0.85	0.78	–0.07
No default risk (High y)	1.10	0.85	–0.25
Positive default risk (High prob.)	4.64	5.50	+0.86
Positive default risk (Low y)	1.65	1.88	+0.24
Default prob. (percent)			
Overall	0.22	0.24	+0.02
No default risk (Low prob.)	0.00	0.00	+0.00
No default risk (High y)	0.00	0.00	+0.00
Positive default risk (High prob.)	0.44	0.55	+0.11
Positive default risk (Low y)	0.40	0.44	+0.04
Higher risk aversion ($\gamma_\ell = 120$)			
Spreads (percent)			
Overall	1.77	1.24	–0.53
No default risk (Low prob.)	1.36	0.56	–0.80
No default risk (High y)	1.54	0.61	–0.93
Positive default risk (High prob.)	5.70	5.96	+0.26
Positive default risk (Low y)	1.98	1.83	–0.16
Default prob. (percent)			
Overall	0.14	0.22	+0.08
No default risk (Low prob.)	0.00	0.00	+0.00
No default risk (High y)	0.00	0.00	+0.00
Positive default risk (High prob.)	0.50	0.50	+0.00
Positive default risk (Low y)	0.26	0.42	+0.15

Table 15: Robustness to debt maturity

	Negative comovement (–1 s.d.)	Positive comovement (+1 s.d.)	Difference
Shorter debt maturity (4 years)			
Spreads (percent)			
Overall	1.28	1.02	–0.26
No default risk (Low prob.)	0.88	0.48	–0.40
No default risk (High y)	1.04	0.51	–0.53
Positive default risk (High prob.)	4.25	4.67	+0.42
Positive default risk (Low y)	1.50	1.50	–0.00
Default prob. (percent)			
Overall	0.16	0.21	+0.05
No default risk (Low prob.)	0.00	0.00	+0.00
No default risk (High y)	0.00	0.00	+0.00
Positive default risk (High prob.)	0.51	0.56	+0.05
Positive default risk (Low y)	0.30	0.39	+0.09
Longer debt maturity (6 years)			
Spreads (percent)			
Overall	2.26	2.06	–0.19
No default risk (Low prob.)	1.59	1.19	–0.41
No default risk (High y)	1.98	1.33	–0.65
Positive default risk (High prob.)	7.20	7.96	+0.76
Positive default risk (Low y)	2.52	2.76	+0.24
Default prob. (percent)			
Overall	0.22	0.27	+0.06
No default risk (Low prob.)	0.00	0.00	+0.00
No default risk (High y)	0.00	0.00	+0.00
Positive default risk (High prob.)	0.57	0.65	+0.07
Positive default risk (Low y)	0.41	0.51	+0.10

Table 16: Robustness to the lender's utility function

	Negative comovement (–1 s.d.)	Positive comovement (+1 s.d.)	Difference
CRRA ($\gamma_\ell = 4$)			
Spreads (percent)			
Overall	1.63	1.45	–0.18
No default risk (Low prob.)	1.03	0.83	–0.19
No default risk (High y)	1.56	1.13	–0.43
Positive default risk (High prob.)	4.69	4.80	+0.10
Positive default risk (Low y)	1.71	1.76	+0.05
Default prob. (percent)			
Overall	0.23	0.24	+0.00
No default risk (Low prob.)	0.00	0.00	+0.00
No default risk (High y)	0.00	0.00	–0.00
Positive default risk (High prob.)	0.50	0.53	+0.03
Positive default risk (Low y)	0.41	0.46	+0.05
CRRA ($\gamma_\ell = 8$)			
Spreads (percent)			
Overall	2.00	1.57	–0.43
No default risk (Low prob.)	1.30	0.89	–0.41
No default risk (High y)	2.26	1.60	–0.66
Positive default risk (High prob.)	5.17	4.68	–0.49
Positive default risk (Low y)	1.74	1.56	–0.19
Default prob. (percent)			
Overall	0.24	0.26	+0.02
No default risk (Low prob.)	0.00	0.00	+0.00
No default risk (High y)	0.01	0.00	–0.01
Positive default risk (High prob.)	0.47	0.54	+0.07
Positive default risk (Low y)	0.48	0.48	+0.00

Table 17: Robustness to government's risk aversion

	Negative comovement (–1 s.d.)	Positive comovement (+1 s.d.)	Difference
Lower government risk aversion ($\gamma_g = 1$)			
Spreads (percent)			
Overall	1.99	1.89	–0.10
No default risk (Low prob.)	1.41	1.12	–0.29
No default risk (High y)	1.76	1.31	–0.45
Positive default risk (High prob.)	4.11	4.49	+0.38
Positive default risk (Low y)	2.21	2.44	+0.23
Default prob. (percent)			
Overall	0.23	0.32	+0.09
No default risk (Low prob.)	0.00	0.00	+0.00
No default risk (High y)	0.00	0.00	–0.00
Positive default risk (High prob.)	0.44	0.55	+0.11
Positive default risk (Low y)	0.43	0.60	+0.17
Higher government risk aversion ($\gamma_g = 3$)			
Spreads (percent)			
Overall	0.32	–0.34	–0.66
No default risk (Low prob.)	0.32	–0.34	–0.66
No default risk (High y)	0.32	–0.34	–0.66
Positive default risk (High prob.)	–	–	–
Positive default risk (Low y)	0.32	–0.33	–0.65
Default prob. (percent)			
Overall	0.00	0.00	+0.00
No default risk (Low prob.)	0.00	0.00	+0.00
No default risk (High y)	0.00	0.00	+0.00
Positive default risk (High prob.)	–	–	–
Positive default risk (Low y)	0.00	0.00	+0.00

Table 18: Robustness to default cost threshold d_0

	Negative comovement (–1 s.d.)	Positive comovement (+1 s.d.)	Difference
Lower output threshold ($d_0 = -0.035$)			
Spreads (percent)			
Overall	1.52	1.30	–0.22
No default risk (Low prob.)	1.06	0.67	–0.39
No default risk (High y)	1.27	0.72	–0.55
Positive default risk (High prob.)	5.01	5.67	+0.67
Positive default risk (Low y)	1.74	1.80	+0.06
Default prob. (percent)			
Overall	0.15	0.23	+0.08
No default risk (Low prob.)	0.00	0.00	+0.00
No default risk (High y)	0.00	0.00	+0.00
Positive default risk (High prob.)	0.46	0.48	+0.02
Positive default risk (Low y)	0.28	0.41	+0.14
Higher output threshold ($d_0 = -0.020$)			
Spreads (percent)			
Overall	1.53	1.23	–0.30
No default risk (Low prob.)	1.07	0.64	–0.43
No default risk (High y)	1.30	0.71	–0.59
Positive default risk (High prob.)	5.28	5.71	+0.43
Positive default risk (Low y)	1.78	1.80	+0.02
Default prob. (percent)			
Overall	0.18	0.21	+0.03
No default risk (Low prob.)	0.00	0.00	+0.00
No default risk (High y)	0.00	0.00	+0.00
Positive default risk (High prob.)	0.48	0.52	+0.04
Positive default risk (Low y)	0.36	0.41	+0.06

Table 19: Robustness to government discount factor

	Negative comovement (–1 s.d.)	Positive comovement (+1 s.d.)	Difference
Lower government discount factor ($\beta_g = 0.985$)			
Spreads (percent)			
Overall	3.63	3.88	+0.25
No default risk (Low prob.)	2.36	2.27	–0.09
No default risk (High y)	3.05	2.67	–0.38
Positive default risk (High prob.)	7.33	8.08	+0.75
Positive default risk (Low y)	4.18	5.04	+0.86
Default prob. (percent)			
Overall	0.49	0.60	+0.11
No default risk (Low prob.)	0.00	0.00	+0.00
No default risk (High y)	0.01	0.00	–0.00
Positive default risk (High prob.)	0.57	0.58	+0.01
Positive default risk (Low y)	0.92	1.13	+0.21
Higher government discount factor ($\beta_g = 0.989$)			
Spreads (percent)			
Overall	0.75	0.38	–0.37
No default risk (Low prob.)	0.59	0.11	–0.48
No default risk (High y)	0.65	0.10	–0.56
Positive default risk (High prob.)	4.29	4.72	+0.43
Positive default risk (Low y)	0.84	0.64	–0.19
Default prob. (percent)			
Overall	0.06	0.09	+0.03
No default risk (Low prob.)	0.00	0.00	+0.00
No default risk (High y)	0.00	0.00	+0.00
Positive default risk (High prob.)	0.47	0.53	+0.05
Positive default risk (Low y)	0.11	0.16	+0.05

E Sensitivity for Inflation Volatility Counterfactual

Table 20: Robustness to default cost threshold d_0 under increased inflation risk

		Baseline inflation variance ($\sigma_\pi = 0.01$)	Higher inflation variance ($\sigma_\pi = 0.02$)	Difference
		Panel A: Baseline default cost threshold ($d_0 = 0.20$)		
Procyclical inflation	Spreads	1.31	1.35	+0.05
	Default prob.	0.21	0.31	+0.10
Countercyclical inflation	Spreads	1.57	1.73	+0.16
	Default prob.	0.16	0.17	+0.01
		Panel B: Lower default cost threshold ($d_0 = 0.16$)		
Procyclical inflation	Spreads	1.63	1.76	+0.13
	Default prob.	0.29	0.37	+0.08
Countercyclical inflation	Spreads	1.79	1.99	+0.20
	Default prob.	0.21	0.21	+0.00

Units: percent.