Slowbalization, fragmentation and the Labor Market

To be completed

By

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Abstract: After a period of Hyper Globalization that ended around 2008 the growth rate of trade has declined. The persistence of this slowdown is remarkable. We relate this slowdown to changes in global supply chains. Key in the analyses are fragmentation costs. Changes in fragmentation costs are reflected in the length of supply chains. Our formalization explains the existence, length, and consequences of changes in fragmentation cost along global supply chains. In addition we can link changes in supply chains to developments on the labor market. We allow tasks within supply chains to be a combination of different occupations. While the model endogenizes production fragmentation, allowing for multiple production stages in multiple countries, it remains tractable. Analyzing the equilibria also allows us to track factor price changes around fragmentation points. From an empirical point of view, the model explains both, the period of Hyper Globalization and the subsequent Slowbalization in terms of changing fragmentation costs along global supply chains.

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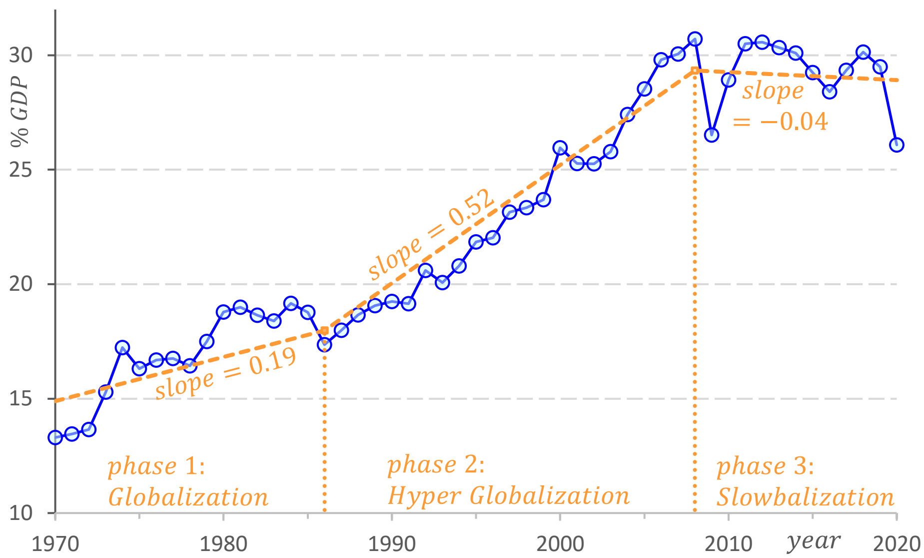
1 Introduction

Since 2008 the growth of trade has slowed down. This trend reversal, or slowbalization, is well-documented.[2] We link this development to changes in fragmentation costs. Shocks like the financial crisis, the blockage of the Suez canal by a container vessel, the corona pandemic The main contribution of our paper is to (i) develop a tractable model of occupations and tasks that endogenizes fragmentation into multiple stages with sourcing from various countries and helps understand (ii) the different phases of globalization (see below), (iii) some related labor market phenomena (see section 2), and (iv) the natural limits of fragmentation / globalization (even with zero fragmentation- and coordination costs) as the process involves a finite number of fragmentation steps only (the most important of which are taken at the early stages). This also implies that (even with zero fragmentation costs) the demand for certain occupations does not fall to zero for any country. [3]

Figure 1 Globalization and Slowbalization; world export of goods and services (% GDP), 1970-2020



Source: created using World Development Indicators, world exports of goods and services (% of GDP); dashed line is spline regression with knots at 1986 and 2008, which explains 96 percent of the variance in export share of GDP.

Figure 1 illustrates the slowdown of globalization using the world exports of goods and services as a percent of income (GDP). Antrás (2020), using a similar Figure, relates the recent slowbalization to earlier periods. The Figure illustrates three periods. First, since 1970 the ratio of world exports increased rom 13.3 percent in 1970 to some 17 percent in 1985. Second, a rapid increase between 1986 and 2008, from 17 percent to more than 30 percent (just before the Great Recession). Third, a notable stagnation after 2008. The slopes in Figure 1 indicate the different periods: in phase 1 of Globalization (1970-1986) relative exports rose by about 0.19 percentage points per year, in phase 2 about 0.52 percentage points per year, in phase 3 relative exports declined by 0.04 percentage points per year. If we exclude 2020 from phase 3 (the first Covid-19 year), relative exports rise by only 0.06 percentage points per year.

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[2] "Slowbalisation: The steam has gone out of globalisation." *The Economist* (24 Jan.2019): https://www.economist.com/leaders/2019/01/24/the-steam-has-gone-out-of-globalisation.

[3] Fragmentation may have complex implications not only for welfare gains in total but also in the distribution and the degree to which it may compromise progress towards greater equality between nations and within nations. The distribution to a large extent depends on the value added income that is created along supply chains, see Bohn et al. (2021).

Besides the financial crisis and the trade collapse of 2009 and the COVID-19 crisis of 2020 part of this stagnation can be explained by changes re-shoring within global supply chains. Goa et al. (2022) show, for instance – based on input/output data –, that after 2008 the top-5 re-shoring countries are China, USA, UK, South-Korea and Germany. The reason could be that shocks increase the awareness of supply chain sensitivity. Carvallo et al. (2021), for example, analyze the consequences of the Fukushima earthquake in Japan, using a calibrated general equilibrium model, find that next to the direct effect of the earthquake, firms whose suppliers were affected by the disaster also suffered sales losses as were firms whose customers were hit. Their calculations suggest that total losses in Japan amounted to 0.47 percentage point reduction in GDP growth. Grossman and Helpman (2021) point out that although firms experience set-backs from supply chain shocks the government could step in because firms do not always take the social surplus of uninterrupted supply of products into account and invest too little in supply chain resilience. Case studies also illustrate the costs of disruptions. According to the news agency Reuters, for example, carmaker Toyota realized after the Fukushima disaster, that the supply chains were too long and vulnerable. Because of the disaster, it took six months to restore normal production levels.[4] McKinsey (2020) calculates the total costs of six major supply chain disruptions: Pandemics, Large scale Cyber attacks, Geophysical events, Heat stress, Flooding, and Trade disputes. The estimated total costs are expected losses of 24% of a year's earnings before interest, taxes, depreciation and amortization in pharmaceuticals to 67% in aerospace, over a ten year period (Exhibit E5, p.12). It is this realization that motivates our analysis and formalization of global supply chains. Slowbalization and the shortening of supply chains, could thus be a first sign of this re-assessment of the costs involved with international trade. This altered assessment of costs changes the international organization of production processes and (global) *supply chains*.

Of course, other factors also explain the slowbalization after 2008. The American President Donald Trump (2016-2020), for example, translated his 'America first' slogan into import tariffs in order to protect US jobs. [5] Trading partners retaliated by installing import tariffs of their own. But these political factors could have contributed to shortening of supply chains as firms return to the home market behind 'tariff walls.' These disturbances together stimulated a debate to consider a 'decoupling' or 'repatriation' of global value chains, especially those that involved China (Baldwin and Evenett, 2020, Baldwin and Tomiura, 2020). The discussion focuses on the trade-off between efficiency and certainty of delivery.

We model the economic consequences of the increased awareness of the rise in supply chain costs on the organization of supply chains. Our formalization helps us to understand both, (hyper) globalization as well as slowbalization in terms of the development of the (perceived) fragmentation costs. In addition, changes in the supply chains have consequences for labor markets, in particular regarding labor market polarization. Our model illustrates how labor market polarization could be related to changes in supply chains. In section 2 we discuss some of the related literature. Section 3 introduces our theoretical framework and provides intuition on the associated relationship between coordination costs, fragmentation, and the labor market. Section 4 explains the structure of the model in more detail. Section 5 analyzes the limits of fragmentation. Section 6 concludes.

2 Globalization and Slowbalization

The evolution of global supply chains enables countries, especially emerging markets, to participate in the world economy by specializing in specific intermediate goods rather than building an entire industry from scratch.

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[4] See: https://www.reuters.com/article/us-japan-fukushima-anniversary-toyota-in-idUSKBN2B1005

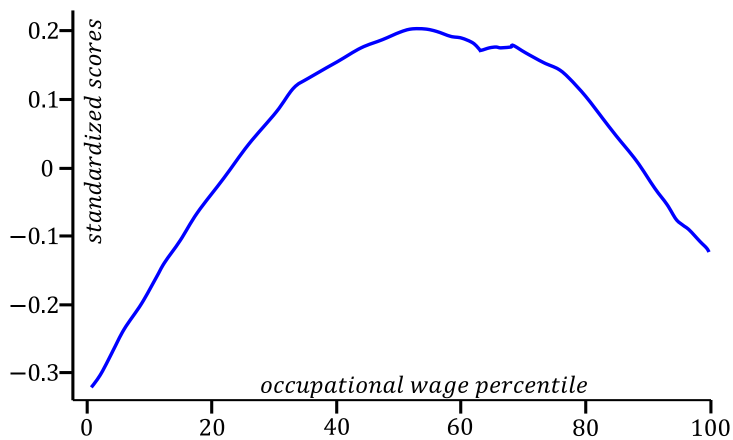
[5] To this date, US president Biden has continued the import tariffs installed by former president Trump.

According to Baldwin (2016) the ICT revolution, starting in the 1990s, made it possible to fragment the production process globally. The result was a global *Great Convergence* in terms of income per capita since the 1990s, but also a rapid increase of world trade (Baldwin, 2016, Rougoor and van Marrewijk, 2015). This happened on a grand scale from the 1990s onwards, and coincides with the period of hyper-globalization in Figure 1 (Baldwin, 2016). Grossman and Rossi-Hansberg (2008) analyze the consequences of dividing the production process into a variety of tasks, some of which are executed from abroad. The consequences of offshoring are – according to this model – threefold. First, offshoring has similar effects as technological progress making production factors more productive. Second, a price effect occurs that makes products less expensive because they are imported from low(er)-wage economies. Third, a labor supply effect arises because offshoring results in excess supply of labor that reduces the gains of fragmenting the production effect. The overall effect, due to a further global division of labor, seems positive. The key issue here is that changes in global fragmentation can have substantial labor market consequences.

In addition to these technological developments, world trade was stimulated by a reduction of protectionist measures; the number of regional trade agreements increased rapidly in this period and, in addition, China joined the World Trade Organization (WTO) in 2001 (Kohl et al., 2016). The labor market consequences during the period of Hyper Globalization are remarkable. The share in the total labor market of low-pay and high-pay occupations increased, but that of mid-pay occupations decreased (Autor et al., 2016). This is consistent with Grossman and Rossi-Hansberg (2008) that fragmentation leads to an excess supply of specific types of labor.

In Autor (2019) he shows that labor market polarization is a salient characteristic of the US labor market. What is also apparent from the data he provides (see for example, Figure 5, p.6) is that after the financial crisis in 2008/9 the speed of polarization in the 2010-16 period slows down compared to the speed of polarization in the 1990-2010 period. This slowdown is consistent with our model of fragmentation costs increases after a global crisis. Figure 2 indicates that the ICT revolution indeed potentially affects the mid-occupations more than occupations at the extreme end of the occupational distribution. The exposure to software developments is especially high for the mid-range of the occupational wage percentile. Note, that the exposure to ICT has two consequences. Domestic – ICT related - technological progress and globalization or offshoring both affect the labor market position of mid-pay jobs (see Terzidis et al. 2019 for a survey).

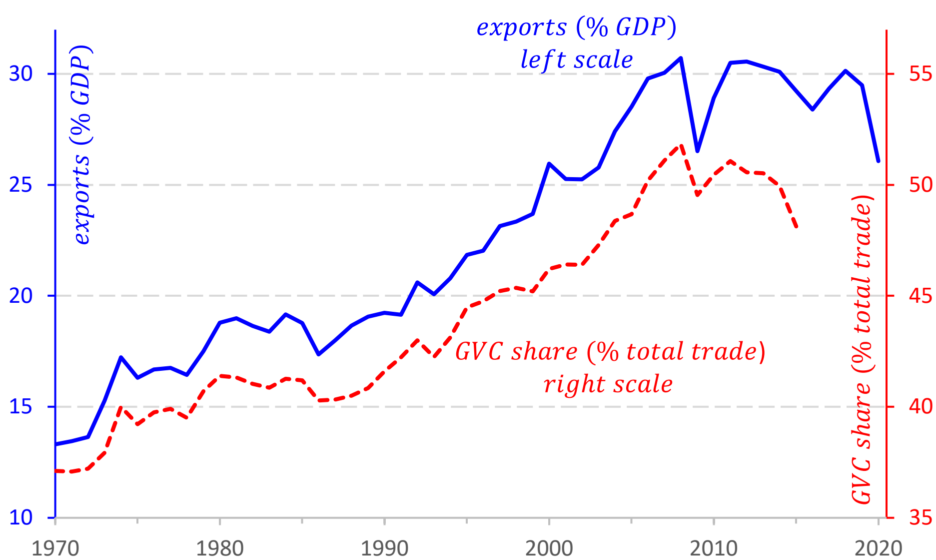
Figure 2 Exposure to software by occupational wage percentile; USA, 2016



Source: based on Webb (2020); the figure shows the average of standardized occupation-level exposure scores for software by occupational wage percentile rank using a locally weighted smoothing regression (bandwidth 0.8 with 100 observations); wage percentiles are measured as the employment-weighted percentile rank of an occupation’s mean hourly wage in the May 2016 Occupational Employment Statistics.

The recent global crises make, however, clear that being part of a global supply chain can also make countries more susceptible to shocks. The World Trade Organization (2009, p. 2), for example, notes with respect to the trade collapse of 2009: *"…the magnitude of recent declines relates to the increasing presence of global supply chains in total trade…— goods cross many frontiers during the production process and components in the final product are counted every time they cross a frontier..*" The evidence on the effects is mixed. Some studies show that the participation in supply chains slows down economic recovery after a major global shock and thus makes countries more vulnerable to economic shocks, while others only find a marginal effect. Altomonte et al. (2012), for example, observe that along a global supply chain the effects of shocks can be magnified due to 'inventory' effects (the so-called bullwhip effect). In contrast, Wagner and Gelübcke (2014) conclude for Germany that foreign multinationals are *not* relatively more severely hit than domestic firms following a negative economic shock. While Behrens et al. (2013) conclude that value chains play only a minor role in Belgium and, like Bems et al (2011), find a dampening effect of participation in supply chains to the sensitivity of the trade collapse in 2009. Brakman and Van Marrewijk (2019) look at a large group of countries to analyze the heterogeneity of country experiences that participate in global supply chains. Their main conclusion is straightforward; the stronger the involvement in global supply chains, the slower the recovery of countries to recessions, which is in line with the findings of Altomonte et al. (2012).

Figure 3 Globalization and Slowbalization; world exports (% GDP) and GVC share (% total trade)

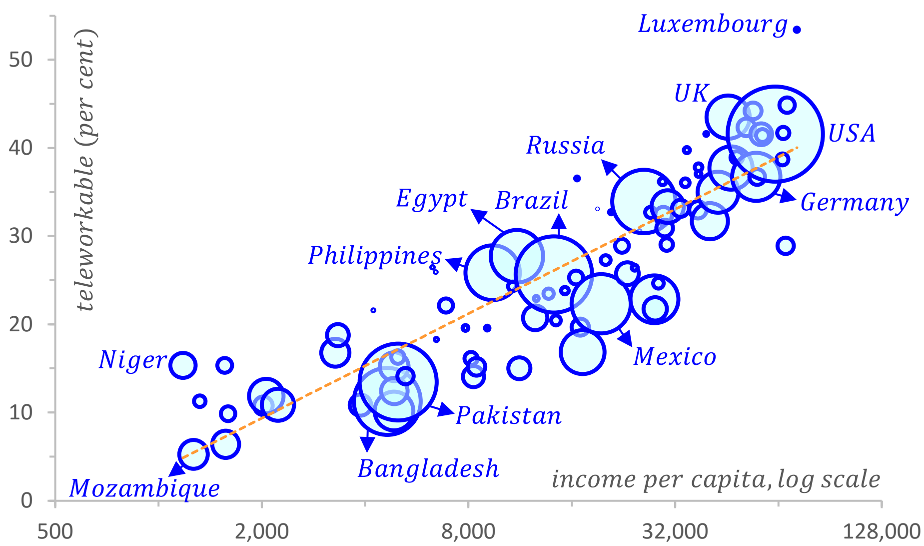


Source: created using World Bank data; world exports of goods and services (% of GDP) from World Development Indicators; GVC = Global Value Chain; GVC share (% of total trade) from World Development Report (2020).

The Covid-19 crisis has added to the increased cost awareness that affects trade. Governments and firms are confronted with unwanted international dependencies and vulnerabilities. The current shortage of electronic chips is a case in point as supply shortages affect the production of chip-intensive products (Wall Street Journal, 2021). Experiences like these indicate that becoming too dependent on global supply chains leads governments, firms, and consumers to re-evaluate the net benefits of long supply chains and the related costs (Brakman, Garretsen, and Van Witteloostuijn, 2020; Baldwin and Freeman, 2022). A possible response could be the re-shoring that we witness today, in order to reduce these costs .Figure 3 is a first and suggestive indication. The (red) dashed line shows the Global Value Chain (GVC) share in total trade. Following the crisis of 2008/9 the GVC share in total trade decreased. So, not only trade stalled in terms of gross-exports (the solid [blue] line), but the GVC share actually declined, which is consistent with shorter supply chains.

From a labor market perspective the Covid-19 epidemic also has disruptive effects. Figure 4 illustrates the relationship between income per capita in a country and the share of jobs that can be done from home. Human Resources (HR) experts expect that Covid-19 will increase the potential of working from home (WFH), which has seen a steep learning curve and the consequences will most likely persist even after the Covid-19 pandemic wears off (Kniffin et al.,2021). This raises the question what type of jobs can be done from home. Dingel and Neiman (2020), on which Figure 4 is based, classify the extent to which different types of jobs are teleworkable and use their classification to determine the extent to which different economies have teleworkable jobs. An example of a job that (mostly) *cannot* be done from home is cleaning. Jobs that (to a large extent) *can* be done from home include: computer and mathematical occupation, education, training, legal occupations, and business & financial operations. Figure 4 shows that the potential for working from home increases with income per capita (the regression line explains about 79 percent of the variance in teleworkability).

Figure 4 Teleworkable jobs (percent of total) and income per capita



Source: Own calculations based on data from Dingel and Neiman (2020) for teleworkable (percent) and World Development Indicators online for income per capita (GNI PPP in constant 2017 $, most recent 2017-2018) and population (million, 2018); 86 countries included; circle size proportional to population; dashed line is a regression with slope 8.5454 which explains 79 percent of the variance in teleworkability.

The new experience could result in a new and more optimal mix between online working and working from the office. The consequences for globalization are less clear. Once employers discover that online working is possible and efficient, there is no need to stop looking for new employees at the national border (Baldwin, 2019). Living at commuting distances from the office is no longer necessary. An increasing number of tasks will be performed and can be coordinated without the workers actually being located in the same building or in the same country. From the perspective of the international division of labor, the effective global labor supply for any given task performed by a firm may vastly increase if workers and the office are geographically separated from each other. Consequently, labour markets might become more global compared to product markets. For product markets, as discussed above, re-shoring could be a possible answer to an increased awareness of fragmentation risks, but for labour markets the opposite occurs as labor markets become more global because of online working.

Internationally oriented firms will increasingly continue to experiment with the newly discovered online options. Possible political responses could add to slowbalization (Autor et al. 2020).

In the next section we formalize and discuss the consequences of supply chains for global trade and the local labor market. The analysis builds on Kremer (1993) who models the organization of production as a sequence of tasks in which the quality of output is paramount for the successful completion of the whole production process. A matching process leads to a sorting of skills and tasks; advanced tasks concentrate in advanced economies. As the number of tasks rise and the global supply chains become longer the impact of uncertainty in the model rises, such that supply chains are re-organized and the traditional sorting breaks down. Less advanced tasks can concentrate in advanced economies. The increased risks affect trade along global supply chains for both emerging markets and advanced countries, as well as their vulnerability to shocks. Our approach explains both the period of (hyper) globalization and slowbalization as described in Figure 1 in terms of changes of fragmentation costs. Fragmentation also has labor market consequences. During the period of Hyper Globalization labor market polarization became an important characteristic of labor markets. Our formalization can explain this phenomenon. Moreover, we can explain slowbalization based on the natural limits of fragmentation and globalization within the structure of our model, see section 5.

3 Occupations, tasks, and fragmentation

Already in the early 1990s it became clear that fragmentation of production was reducing the cost of production and increased efficiency (see, for example, Jones and Kierzkowski, 2002, and Deardorff, 2001, who show the possible cost reductions in a Heckscher-Ohlin framework). Yi (2003) observes that standard trade models need implausible large elasticities to explain trade growth in the 1990s, and introduces vertical specialization as a more plausible explanation. These contributions analyzed the consequences for factor prices if fragmentation occurs. Here we do this in two steps. First we take factor prices as given, Subsequently we drop this assumption, which takes us a step closer to a complete general equilibrium analysis. Our model is related to theoretical work on supply chains that builds on Kremer (1993) – who takes factor prices a given – by assuming that various stages in the production chain follow upon each other. These stages can have specific characteristics. In Costinot et al. (2013), for example, it is assumed that productivities of stages fall along the value chain. This is consistent with a Ricardian model along supply chains. A consequences is that more efficient (high-skilled) countries specialize in downstream stages of the production process.[6] Supply chain models can quickly become complex in a multi-country, multi fragment world. The challenge is to minimize production costs along the supply chain; the number of possible paths towards final production rapidly becomes unwieldly.

The presence of fragmentation costs makes this problem extra complicated; an efficient location that is far away might be by-passed for a less efficient location that is nearby if lower fragmentation costs make up for the cost difference. Calculating an optimal path can thus become challenging. Simplifying assumptions are necessary to make the models tractable (see, for example, the canonical model of Caliendo and Parro, 2015). Antras and De Gortari (2020), make simplifying assumptions in a Ricardian model where tasks coincide with occupations to solve this problem of finding a cost-minimizing path along supply chains; with an arbitrary number (which, however, is exogenous) of countries and fragments the calculations quickly become intractable. Antrás and Chor (2021) survey a wide range of models that deal with complications like these, and discuss the simplifying assumptions that have to be made. The literature on production networks also discusses how shocks propagate

[6] If there is a continuum of stages, the model is reminiscent of the Dornbusch et al. (1977) model in which productivity differences are modelled as a continuum.

through supply chains. The type of shocks that are discussed are different from the crisis shocks that are central in this paper. As extensively documented by Carvallo and Tahbaz-Salehi (2019), the shocks in the literature usually involve: policy, productivity, preference, and labor supply shocks. A large majority of applications using this model assumes that there are only two stages of production. This is a strong assumption, but necessary to derive analytical results. Antras and De Gortari (2020) is an example of a multi-stage extension, but as noted by Antras and Chor (2021, p.51), the computational restraints are large, which limits the number of stages in practice. A key characteristic of these models is often that the structure of the production network itself is given and that the length of the supply chains is not affected by shocks. Endogenizing a network rapidly becomes unwieldy because the complexity of analyzing all possible networks quickly becomes too large because of '*the combinatorial nature of graphs*', see Carvallo and Tahbaz-Salehi (2019, p. 648). [7]

Our model is related to these models in the sense that we assume, as a simplifying assumption, a sequence (a continuum) of fragments that have to be executed one after the other and combined in the final stage of production. This makes the model tractable and enables us to endogenize fragmentation. A feature of the model is that we do not have to assume that the structure of the production network is given; the number of fragments in the supply chain is endogenous.[8] This is the contribution of this paper, because it relates the periods, as described in Figure 1, to changes in fragmentation costs. Simplifying supply chains as a sequence of production steps by passes the 'combinatorial 'challenges.
Our approach, has four benefits: changes in fragmentation costs do not require a recalculation of the total sequence of production steps, each fragment in the production process represents a combination of occupations, fragmentation in our model is endogenous (and allows for no fragmention), and finally we can analyze the consequences on factor rewards, at fragmentation points, of changes in fragmentation costs. We assume that fragmentations costs cover all aspects of fragmentation, such as, the liability of foreignness, tariffs, or subsidies. Analyzing the consequences of changes in fragmentation costs on factor rewards relates our approach to Autor et al. (2003), who introduce a task-based approach to analyze changes in the labor market caused by automation. Cost increases of a particular supply chain can be incorporated in models like these by assuming that the fragmentation costs increase.

3.1 Demand, profit, and assumptions

This set-up is general in the sense that it encompasses various market forms and demand structures. It applies to all profit maximizing firms with separable profit functions. A simple example is a firm that maximizes profits $\pi(y) = p(y)y - C(y)$, where y is output, $p(y)$ is the inverse demand function and $C(y)$ is the cost function. If $'$

[7] The supply chain management literature in Operations Research has a different approach. Models in this field minimize total facility/inventory costs (with fixed facility costs per facility) within a network. The mathematical methodology aims to calculate the shortest paths within large networks which is computationally demanding (see for examples, Silva et al., 2021, or Jabbarzadeh et al., 2018). The models in Operations research and the literature mentioned in the main text are to a large extent developed separately. Antras and Chor (2021, p.77/8) mention in this respect that: " *The economics literature has largely been focused on finding suitable environments in which these decisions can be qualitatively characterized or computationally simplified, but it is hard to envision at this point that this agenda will lead to successful unified quantitative models of the decisions of lead firms. Our sense is that, sooner or later, this literature will need to close the gap with the parallel literature in supply chain management in the Operations Research field, which has long adopted heuristic methods to guide the optimal design of supply chains*."

[8] Fujita and Thisse (2013, p. 453) develop a supply chain model in a New Economic Geography framework in which Headquarters can be split from production in a two-region setting. In their model, fragmentation depends on communication costs that affect the marginal cost of production. Depending on these communication costs fragmentation can occur. Their model illustrates that globalization can have more extreme effects for regions and cities than for nations as a whole, which illustrates the importance of taking 'space' into account.

denotes a derivative, the first-order condition is $p'y + p - C' = 0$, more commonly known as $p(y)(1 - 1/\varepsilon(y)) = C'(y)$, where $\varepsilon(y) \equiv -p/p'y$ is the price elasticity of demand. Under perfect competition $\varepsilon(y) = \infty$ and price must be equal to marginal cost, which leads to a specific optimal output level y^* under decreasing returns to scale ($C" > 0$). Under perfect competition and constant returns to scale ($C" = 0$), we have a borderline case in which firms either produce nothing or output y^* is determined by market conditions.[9] In all other cases, profit maximization leading to output y^* can in principle also occur under increasing returns to scale ($C" < 0$). We can, of course, complicate the analysis. For example, if there is a range of output markets j, in which case we have $\pi = \sum_j p_j(y_j)y_j - C(y)$ with $y = \sum_j y_j$, we determine optimal output y_j^* in each market and hence total output y^* as its sum. Alternatively, we can have a complicated game-theoretic setting in which a firm determines it output level y^*. In all cases, firms maximize profits by minimizing total costs for the production of the given total output level y, which is the focus of our analysis. More specifically, we analyze the organization of the production process for a given output level y as fragmentation costs fall. As always, changes in market forms or changes in exogenous variables, such as fragmentation costs, may affect the cost function and thus the chosen output level. It is, however, important to realize that the conclusions we derive below hold for any given output level, including the adjusted output level.

The main assumptions underlying our model are:

- The production of output y requires the sequential completion of a continuum of tasks x, ordered from 0 to 1.
- The completion of task x requires the combination of a finite number of occupations $i \in I = \{1, .., \bar{I}\}$. The employment of occupation i for task x (given output y) is provided by the function $g_i(x|y)$. For ease of exposition, we assume the functions g_i to be differentiable.[10] Total employment for task x (given output y) is therefore: $\sum_i g_i(x|y)$. It is depicted in graphs like Figure 5 by stacking the functions g_i. We use G_i to denote the distribution function associated with g_i, such that $G_i(x|y) \equiv \int_0^x g_i(z|y)dz$. Its derivative (denoted by $'$) is thus: $G_i'(x|y) = g_i(x|y)$. Total employment L_i for occupation i is thus $L_i = G_i(1|y)$. If the wage rate in the Home country for occupation i is w_i, then total cost of production at Home is: $TC = \sum_i w_i L_i$.
- There is a finite number of countries $j \in J = \{0, .., \bar{J}\}$ with given wage rates $\bar{w}_{ij}$ (where Home is country 0). There can be Hicks-neutral technology differences between countries, such that $g_{ij}(x|y) = \lambda_j g_i(x|y)$. We can express the costs $\bar{w}_{ij}\lambda_j g_i(x|y)$ in terms of Home country efficiency units by defining $w_{ij} \equiv \bar{w}_{ij}/\lambda_j$, such that the costs are $w_{ij} g_i(x|y)$.
- Fragmentation occurs by producing a range of tasks sequentially in different countries.[11] Fragmentation into $p + 1$ parts requires determining p fragmentation points x_s, where the first fragment is produced in country $j_0 \in J$, the second fragment in country $j_1 \in J$, and so on, up to fragment p in country $j_p \in J$. At each step s we must have different countries, so: $j_s \neq j_{s+1}$. We denote the sequence of $p + 1$ countries $j_0 j_1 .. j_p$ by $\prod_{s=0}^p j_s$ and the p fragmentation points by x_s for $s = 1, .., p$; with the convention that $x_0 = 0$ and $x_{p+1} = 1$. The fixed fragmentation costs associated with this sequence are denoted by $F_{\prod_{s=0}^p j_s}$. Firms minimize the sum of production costs and fragmentation costs. We assume that the fragmentation costs rise rapidly as the number

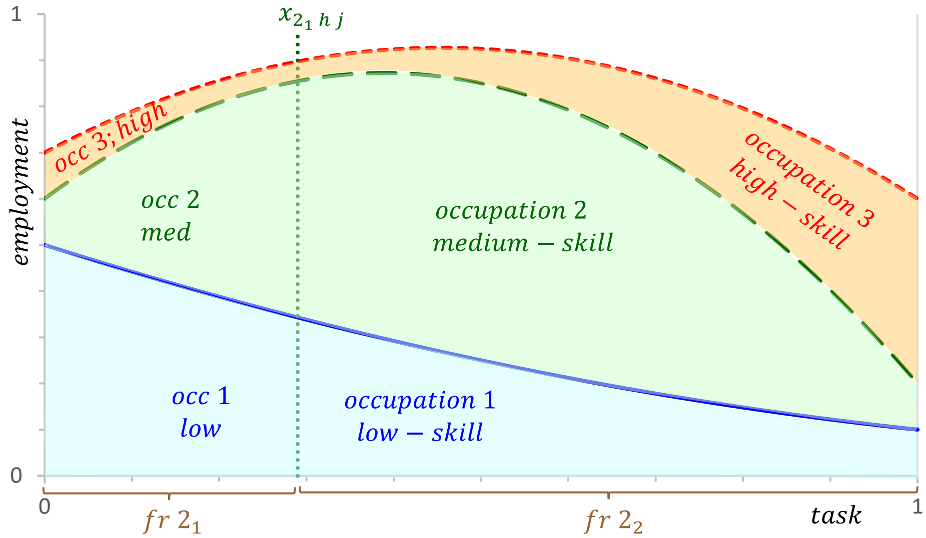
[9] The possibility of infinite profits is not economically viable.

[10] Most results hold, however, for well-behaved continuous functions g_i.

[11] Note, that we refer to countries. However, the regional aspects of fragmentation are also important, as is highlighted by Bolea et al. (2022). They show that regional supply chains are not independent and that spatial spill-overs are important and affect the position of a region in a supply chain. We abstract from these complications.

of fragments rises and also that these costs tend to fall over time, see the discussion in section 3.2. We also analyze the limiting case of zero fragmentation costs, see section 5.3.[12]

Figure 5 Example: tasks and occupations; fragmentation of the production process into two parts to country j:



Note: see Appendix 1 for details; occupation functions $g_i(x|y)$ are stacked on top of each other to depict total employment

As an example, the model is illustrated in Figure 5 using three occupations (see section 4 for further details, also on the fragmentation in two parts), where occupation 1 is more intensively used in the early stages of production (low task index), occupation 2 is more intensively used in the medium stages of production (medium task index), and occupation 3 is more intensively used in the later stages of production (high task index). For clarity, and without loss of generality, we label occupations 1, 2, and 3 as being low-skilled, medium-skilled, and high-skilled occupations, respectively. Appendix 1 lists and briefly discusses the numerical specification for all examples in the paper. Note that in our example each task requires a combination of all occupations for completion, while each occupation is involved in the production of all tasks. Neither observation is required, so a task may involve employment of only one occupation and an occupation may be involved in only a sub-range of tasks, rather than being required for all tasks.

3.2 Intuition 1: coordination costs and fragmentation

To illustrate the working of the model and provide some intuition we illustrate its key characteristics. We focus on changes in fragmentation costs, taking the rewards to factors of production (the wage rates for occupations in the different countries) as given. The main driving force of (hyper) globalization is the decline in coordination and fragmentation costs due to the technological (ICT) revolution, which makes it easier to monitor and coordinate the cooperation and interaction of tasks incorporated in different fragments across a rising number of individual countries. Similar to Kremer's (1993) O-ring theory, we can interpret the combination of fragments as a sequence of steps to be executed to produce a final product. Failing any singular step of the fragmentation process renders the final product as worthless and hence the emphasis on quality and reliability rises as the number of fragments rises, since we are dealing with a rapidly rising number of coordination costs as the number of fragments rises.

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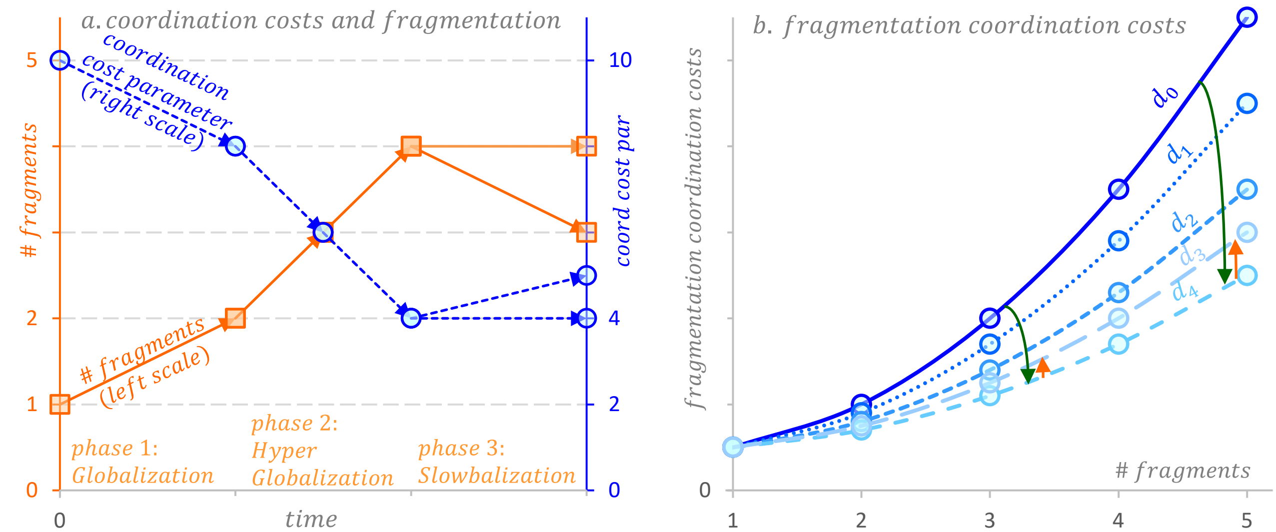
As already noted in section 2, many issues play a role in determining the fixed fragmentation costs, including transport costs, cultural differences, legal systems, language barriers, regional trade agreements, and so on. An important aspect of the fragmentation costs, however, is the coordination of the production process between the different fragments and the reliability and quality, as emphasized above. If there are p fragments, the number of different connections between the fragments is equal to $p(p-1)/2$. If we assume that the coordination and quality costs are a convex function of the number of connections, this implies that the coordination costs rise rapidly with the number of fragments. As a result, technological improvements are particularly important for lowering the coordination costs of more complicated production processes consisting of many fragments

Figure 6 Technological improvement, coordination costs, and fragmentation



Note: panel a depicts the number of fragments over time as the coordination costs parameter (= coord cost par) changes; panel b curves indicate total fragmentation coordination costs for different coordination cost parameters (consistent with panel a) with $d_0 > d_1 > d_2 > d_3 > d_4$, see the main text for details.[13]

We illustrate the discussion and its main implications in Figure 6 and Figure 7 (see section 3.3). Panel a of Figure 6 shows that the costs of coordination decline over time due to technological improvements (right scale of panel a). Panel b shows that the associated total fragmentation coordination costs (which in the example is proportional to the number of connections between the fragments) falls particularly fast if the number of fragments is larger (illustrated by the downward [green] arrows). Firms minimize total costs, which is the sum of production costs and fragmentation coordination costs. The optimal number of fragments is thus a trade-off in the gain from lower production costs and the loss in higher coordination costs. Initially, as coordination costs are very high, fragmentation is too costly and the firm produces one fragment at Home only, see Figure 6a and the top part of Figure 7. As coordination costs fall, it becomes optimal to fragment the production process into two parts, where the first fragment is produced at Home and the second part in country 1, see the second part of Figure 7. This starts the process of globalization in Figure 6a, which speeds up as coordination costs decline more rapidly during Hyper Globalization leading first to fragmentation into three parts (Home – country 1 – country 2) and then to

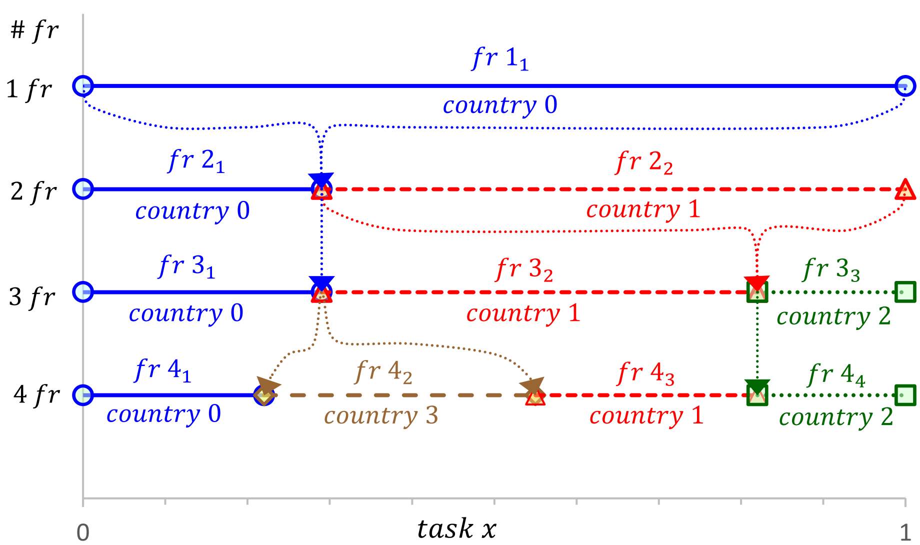
[13] If p is the number of fragments the number of connections is $p(p-1)2$; panel b makes the fragmentation coordination costs proportional to the number of connections for $d_0 = 10; d_1 = 8; d_2 = 6; d_3 = 5; d_4 = 4$. If we make the fragmentation coordinations costs a strictly convex function of the number of connections (see the main text), these costs rise even faster as the number of fragments rises.

fragmentation into four parts (Home – country 3 – country 1 – country 2), see Figure 7 and the discussion below. In the final phase of slowbalization coordination costs no longer decline or even rise (see Figure $6a, b$), leading to a stagnation in the number of fragments or even a decline (see Figure $6a$). This is in line with the GVC share information on the right-hand side of Figure 3.

3.3 Intuition 2: step-wise fragmentation

We discuss the step-wise fragmentation example formally in section 4. An important implication of our model is that it is *not* necessary to re-organize the entire production process as the next step is taken, because the previous fragmentation point is not affected if the same countries are involved in the range that includes this fragmentation point. Finding a new supply chain route – which involves a complete recalculation of the supply chain – is not necessary. We show this principle in Figure 7. Notation wise, when fragmentation is into p parts, we refer to these fragments as $fr\ p_1, fr\ p_2, .., fr\ p_p$, respectively. The optimal fragmentation points x depend on the number of fragments and the order of countries active at this fragmentation point, which we denote by sub-indices (see section 4). In this notation for the optimal fragmentation point $x_{2_1\,h\,j}$ the sub-index 2_1 indicates the first fragmentation point of fragmentation into 2 parts, while the sub-index $h\,j$ indicates that the first fragment is produced at home and the second in country j. Similarly for the other points.

Figure 7 Step-wise fragmentation



Note: see Appendix 1 for parameter details; # fr = number of fragments (inverse scale).

Step 1. Initially, as fragmentation costs are high, the firm produces all goods in the Home country; there is then only one fragment $fr\ 1_1$, produced in the Home country.

Step 2. As fragmentation costs are falling (because of lower coordination costs) it becomes feasible for the firm to fragment the production process into two parts. As discussed in section 4.1, the firm then analyses the optimal fragmentation point of $fr\ 1_1$ relative to each country, determines the minimal total costs, and evaluates if these costs are lower than production of all goods at Home. In the example, this implies the production of $fr\ 2_1$ in the

Home country and the production of $fr\ 2_2$ in country 1, with fragmentation point $x_{2_1\,0\,1} = 0.29$. We assume at this stage that the coordination costs are too high to make fragmentation into three parts an attractive option.[14]

Step 3. As fragmentation costs continue to fall, it becomes feasible for the firm to fragment the production process into three parts. Evaluating different options (see section 4.2), the firm fragments part $fr\ 2_2$ into two pieces at $x_{3_2\,1\,2} = 0.83$, thus producing $fr\ 3_2$ in country 1 and $fr\ 3_3$ in country 2. In this case, the first fragmentation point $x_{3_1\,0\,1} = x_{2_1\,0\,1}$ is not affected as the first order condition in this range between the Home country and country 1 is still fulfilled and the Home country thus continues to produce the same fragment: $fr\ 2_1 = fr\ 3_1$.

Step 4. As fragmentation costs continue to fall, Figure 7 illustrates a hypothetical continuation of the fragmentation process. We now assume that it is optimal for the firm to fragment the first part of $fr\ 3_2$ to country 3 and the second part to country 1 at fragmentation point $x_{4_2\,3\,1} = 0.55$. In this case, the last fragmentation point $x_{4_3\,0\,1} = x_{3_2\,0\,1}$ is not affected as the first order condition in this range between country 1 and country 2 is still fulfilled and country 2 thus continues to produce the same fragment: $fr\ 3_3 = fr\ 4_4$. In contrast, the first fragmentation point *is* affected as we now have to determine the optimal fragmentation point for the Home country relative to country 3 instead of country 1. We have assumed this to occur at $x_{4_1\,0\,3} = 0.22$. In this case, therefore, the expansion of fragmentation has squeezed a new fragment $fr\ 4_2$ in between the old fragments $fr\ 3_1$ and $fr\ 3_2$ without affecting fragment $fr\ 3_3$.

As we continue this process it becomes clear that falling coordination costs will make it profitable for firms to step-wise fragment the production process into smaller parts. During this process it is in many cases *not* necessary to revise the entire production process at each step, but rather to focus only on the part of the production process where continued fragmentation leads to lower costs, leaving the remainder of the production process unaffected. Obviously, the further firms are in this process, the larger the unaffected part and the smaller the re-organized part. Note, that changes in fragmentation costs affect the length of a supply chain, but only of that section for which the costs change. It is not necessary the re-calculate an entire new fragmentation route. If coordination costs rise again (see the right-hand part of Figure $6a$ or the upward arrows in Figure $6b$) the process of step-wise fragmentation may come to a halt or might be partially reversed. However, rising coordination costs is *not* a necessary condition for the fragmentation process to stop as our model implies limits on the extent and impact of fragmentation even if coordination costs continue to decline, see section 5.

4 Details of the fragmentation example

4.1 Two-part fragmentation

We first analyze the possibility of fragmentation of the production process into two parts: one part at Home and the other part abroad. The first fragment consists of all tasks from 0 to the point x_1 and the second fragment consists of all tasks from x_1 to 1, see Figure 5 for an illustration. Firms are free to decide on the range of tasks to be included in a fragment and thus choose the point x_1 optimally. Fragmentation abroad can be relative to any country $j \in J$, with associated costs for task x equal to $w_{ij} g_i(x|y)$. The firm has an incentive to fragment part of the production process abroad if the wage costs in terms of home efficiency units are lower for *some* occupation,

[14] The reader may wish to verify using the example in the Appendix that the optimal fragmentation point relative to country 2 for fragmentation into 2 parts would be around $x = 0.72$, which leads to substantially larger variable costs than fragmentation relative to country 1 (around 0.825 compared to 0.802). The ultimate optimal decision depends, of course, on the fixed fragmentation costs relative to these locations as well.

that is $w_{ij} < w_i$ for some i. This condition is, of course, necessary but not sufficient as the fragmentation of tasks also involves other occupations which may be more expensive abroad than at Home in terms of efficiency units. For ease of exposition, we assume that the Home country has a comparative advantage in the early stages of the production process (low task index), such that if fragmentation occurs Home produces fragment 1 and fragment 2 is produced abroad. Obviously, there are additional costs associated with fragmentation of the production process in terms of coordination, transport costs, cultural differences, legal systems, language barriers, regional trade agreements, and the like. In the case of fragmentation into two parts, we let $F_{h\ j}$ denote these, fixed, fragmentation costs if the first fragment is produced in Home and the second fragment in country j.

In terms of cost minimization, the firm now faces three decisions.

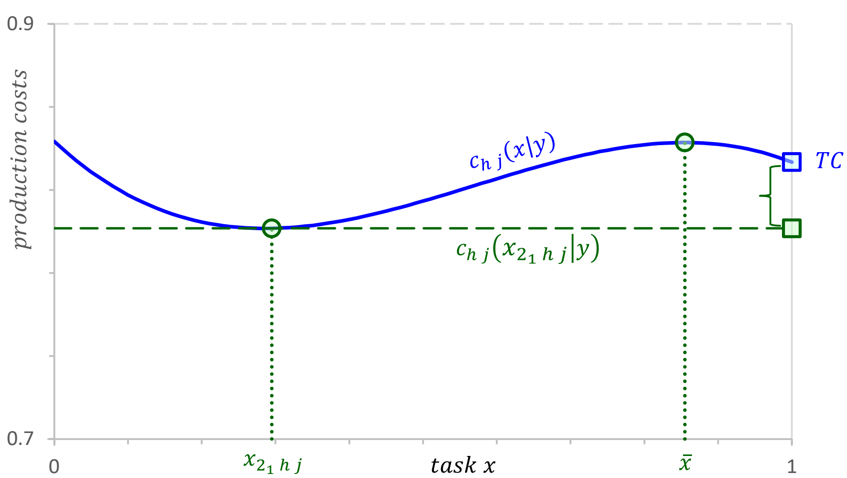
1. Given fragmentation into two parts relative to country j, what is the optimal fragmentation point $x_{2_1\ h\ j}$? In this notation, the sub-index 2_1 indicates the first fragmentation point of fragmentation into 2 parts, while the sub-index $h\ j$ indicates that the first fragment is produced at home and the second in country j.
2. Determine what is the best country to fragment to if fragmentation is into two parts. This requires calculating the minimum total costs of fragmentation into two parts relative to all countries j.
3. Determine if fragmentation into two parts leads to lower costs than producing at Home only.

We address these questions in turn below.

Relative to point 1, we start by determining the fragmentation point $x_{2_1\ h\ j}$, given that Home produces the first fragment and country j produces the second fragment. If fragmentation is into 2 parts, we refer to these fragments as $fr\ 2_1$ and $fr\ 2_2$, respectively. Total employment for occupation i at Home if the fragmentation point is at x is given by $G_i(x|y)$. These workers are paid the wage rate w_i. Total employment for occupation i in country j in terms of Home efficiency units if the fragmentation point is at x equals $G_i(1|y) - G_i(x|y)$. These workers are paid the wage rate w_{ij}. Exclusive of the fragmentation costs $F_{h\ j}$, the variable costs $c_{h\ j}(x)$ of fragmentation at point x in country j is given in equation (1), where the first term on the right-hand side reflects the costs of producing fragment 1 at Home and the second term reflects the costs of producing fragment 2 in country j.

$$(1) \qquad c_{h\ j}(x|y) = \underbrace{\left(\textstyle\sum_i w_i\, G_i(x|y)\right)}_{Home\ cost} + \underbrace{\left(\textstyle\sum_i w_{ij}\big(G_i(1|y) - G_i(x|y)\big)\right)}_{country\ j\ cost}$$

Figure 8 Determining the optimal fragmentation point $x_{2_1\ h\ j}$



Note: see Appendix 1 for parameter details.

If we differentiate equation (1) with respect to fragmentation point x and equate to zero, we get the first-order condition for the optimal fragmentation point given in equation (2), requiring orthogonality of the differences in wage rates using the density function g_i at the fragmentation point $x_{2_1 h j}$.

(2) $$\sum_i (w_i - w_{ij}) g_i (x_{2_1 h j} | y) = 0$$

If condition (2) is not fulfilled at any point in the domain of x, then there is no internal fragmentation point relative to country j. If there are multiple such points, these have to be evaluated to determine the point with minimum costs. This is illustrated in Figure 8 for the example with three occupations used above. The curve $c_{h j}(x|y)$ depicts the costs of equation (1) throughout the domain. It has two points satisfying the orthogonality condition of equation (2), of which $\bar{x}$ represents a local maximum and $x_{2_1 h j}$ represents the cost minimum.

The minimum costs with two-part fragmentation relative to country j exclusive of the fragmentation costs $F_{h j}$ are given by $c_{h j}(x_{2_1 h j}|y)$. Total costs relative to country j are thus given by $c_{h j}(x_{2_1 h j}|y) + F_{h j}$. Point 2 above is then choosing country $j \in J$ with the lowest costs. Point 3 above involves choosing two-part fragmentation instead of production at Home if: $F_{h j} + c_{h j}(x_{2_1 h j}|y) < TC$, provided j is the optimal choice in point 2. Note that a decline in fragmentations costs in terms of a reduction of $F_{h j}$ does not affect the fragmentation point, but may affect the decision to fragment.

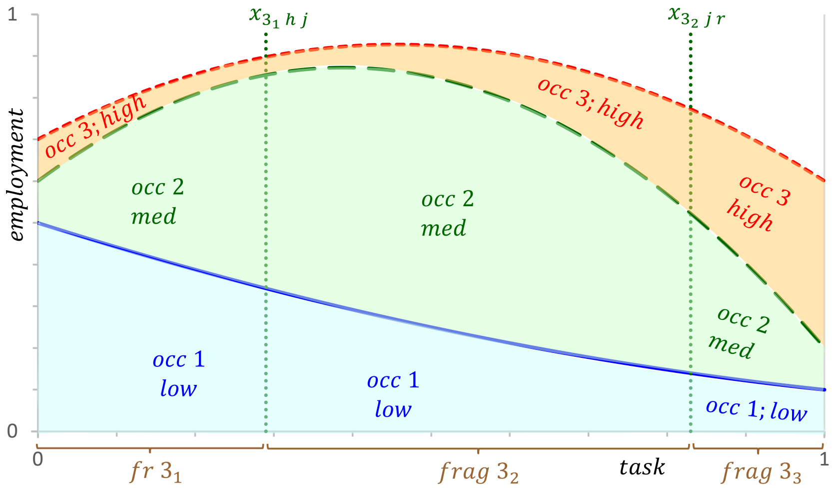
Result 1. *Fragmentation costs can affect the decision to fragment; the higher the costs, the less likely fragmentation becomes.*

4.2 Three-part and many-part fragmentation

The next step in our exposition of the model is to analyse three-part fragmentation, where the first part is produced at home, the second part in country j, and the third part in country r. As above, the first fragment consists of all tasks from 0 to the point x_1, the second fragment consists of all tasks from x_1 to x_2, and the third fragment consists of all tasks from x_2 to 1, see Figure 9. The firm chooses the fragmentation points optimally, taking into consideration the fixed costs of fragmentation and given the prices of occupations in the different locations. In the case of fragmentation into three parts, we let $F_{hj\,r}$ denote the fixed fragmentation costs if the first fragment is produced in Home, the second fragment in country j, and the third fragment in country r. In our discussion we assume for simplicity that the order of fragmentation does not affect the fixed costs F of fragmentation; only the involved countries are important, but this can easily be relaxed.[15]

[15] In the discussion we thus have: $F_{hjr} = F_{hrj} = F_{jrh} = F_{jhr} = F_{rhj} = F_{rjh}$.

Figure 9 Fragmentation of the production process into three parts to countries j and r



Note: see Appendix 1 for details; occupation functions $g_i(x|y)$ are stacked on top of each other to depict total employment

We start again with determining the optimal fragmentation points. Exclusive of the fragmentation costs $F_{h\,j\,r}$, the variable costs $c_{h\,j\,r}(x_1, x_2|y)$ of fragmentation at points x_1 and x_2 is given in equation (3). Note that $0 \leq x_1 \leq x_2 \leq 1$, where the first term on the right-hand side of equation (3) reflects the costs of producing $fr\ 3_1$ at Home, the second term reflects the costs of producing $fr\ 3_2$ in country j, and the third term reflects the cost of producing $fr\ 3_3$ in country r. Appendix 2 provides a surface plot and contour plot for the associated costs.[16]

$$(3) \qquad c_{h\,j\,r}(x_1, x_2|y) = \underbrace{\left(\textstyle\sum_i w_i\, G_i(x_1|y)\right)}_{Home\ cost} + \underbrace{\left(\textstyle\sum_i w_{ij}\big(G_i(x_2|y) - G_i(x_1|y)\big)\right)}_{country\ j\ cost} + \underbrace{\left(\textstyle\sum_i w_{ir}\big(G_i(1|y) - G_i(x_2|y)\big)\right)}_{country\ r\ cost}$$

If we differentiate equation (3) with respect to fragmentation points x_1 and x_2 and equate to zero, we get the first-order conditions for the optimal fragmentation points given in equation (4), implying orthogonality of the differences in wage rates using the density functions g_i at the fragmentation point. Note that the first-order condition for the *first* fragmentation point $x_{3_1\,h\,j}$ is identical to equation (2), such that this fragmentation point *does not change*, as discussed in section 3. More specifically, as long as the order of the first two countries (Home – country j) does not change, the first fragmentation point does not change and we are left to determine the optimal fragmentation point between country j and country r from then onwards.

$$(4) \qquad \textstyle\sum_i (w_i - w_{ij})\, g_i\big(x_{3_1\,h\,j}|y\big) = 0$$
$$\textstyle\sum_i (w_{ij} - w_{ir})\, g_i\big(x_{3_2\,j\,r}|y\big) = 0$$

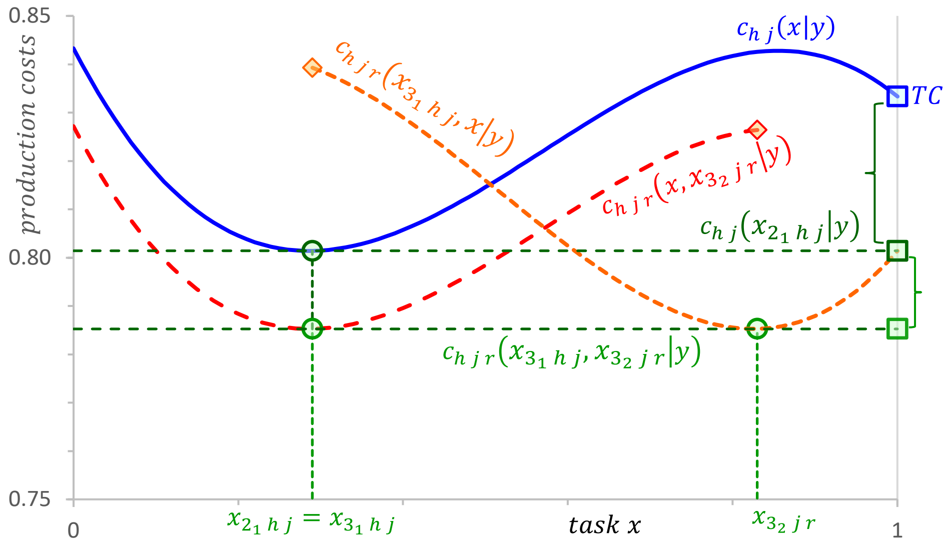
We can analyse the behaviour of cost function (3) in two dimensions by extending Figure 8, which determines the optimal two-part fragmentation point, to also determine the optimal three-part fragmentation points for sub-parts of the domain of x, see Figure 10. We do this in three steps.

Step 1 depicts the two-part fragmentation procedure by repeating the $c_{h\,j}$ cost function of equation (1) as a solid blue line. The cost minimum is achieved at point $x_{2_1\,h\,j}$, which leads to costs $c_{h\,j}\big(x_{2_1\,h\,j}\big)$. As we argued above,

[16] Note that in our discussion the order of countries is Home – country j – country r, while the illustrations in Appendix 2 include the order Home – country r – country j as well.

two-part fragmentation lowers total costs relative to producing exclusively at Home only if the fixed fragmentation costs are sufficiently low: $F_{h\,j} < TC - c_{h\,j}(x_{2_1\,h\,j}|y)$.

Figure 10 Determining the optimal fragmentation points $x_{3_1\,h\,j}$ and $x_{3_2\,j\,r}$



Note: see Appendix 1 for parameter details.

Step 2 is based on the observation that the optimal fragmentation point $x_{3_1\,h\,j}$ does not change if the order of countries does not change. To determine the optimal second fragmentation point $x_{3_2\,j\,r}$, we therefore depict the three-part fragmentation cost function of equation (3) in the domain $[x_{3_1\,h\,j}, 1]$, *given* the optimal choice for the first fragmentation, see the (orange) dashed curve $c_{h\,j\,r}(x_{3_1\,h\,j}, x|y)$. Its minimum cost level is reached at point $x_{3_2\,j\,r}$, which satisfies the second first-order condition of equation (4). The associated minimum cost level is $c_{h\,j\,r}(x_{3_1\,h\,j}, x_{3_2\,j\,r}|y)$, as depicted in Figure 10.

Step 3 confirms the optimal choice of the first fragmentation point for three-part fragmentation by depicting the three-part fragmentation cost function of equation (3) in the domain $[0, x_{3_2\,j\,r}]$, *given* the optimal choice for the second fragmentation point, see the (red) long-dashed curve $c_{h\,j\,r}(x, x_{3_2\,j\,r}|y)$. Its minimum cost level is reached at point $x_{3_1\,h\,j}$, which satisfies the first first-order condition of equation (4). The associated minimum cost level is again $c_{h\,j\,r}(x_{3_1\,h\,j}, x_{3_2\,j\,r}|y)$, as shown in Figure 10.

(5) $$\underbrace{(F_{h\,j\,r} - F_{h\,j})}_{rise\ fragmentation\ costs} < \underbrace{\left(c_{h\,j}(x_{2_1\,h\,j}|y) - c_{h\,j\,r}(x_{3_2\,h\,j}, x_{3_2\,j\,r}|y)\right)}_{reduction\ variable\ costs}$$

Finally, we note that three-part fragmentation leads to lower total costs of production if the rise in fixed costs relative to two-part fragmentation is lower than the variable cost reduction, as given in equation (5).

Appendix 2 briefly explains how this procedure can be extended to many-part fragmentation.

Result 2. *With many-part fragmentation, falling fragmentation costs affect the geography of supply chains.*

4.3 Fragmentation and the labor market (To be extended)

We now briefly discuss the (employment) consequences associated with the fragmentation process using the examples discussed above. We summarize the implications in *Table 1*, which provides an overview of the distribution of employment over the three occupations associated with each fragment if production is organized in one, two, or three fragments, where the top part (panel a) is in absolute terms and the bottom part (panel b) in relative terms. It is illustrated in Figure 11 (see section 5).

The top-left part of *Table 1* depicts absolute employment if there is no fragmentation, in which case there is one fragment produced in Home. Employment in occupation 1 – 2 – 3 is 0.267 – 0.433 – 0.133, respectively, which leads to 0.833 total employment and 0.833 variable costs (because all wage rates are normalized to one in the Home country). The bottom-left part of *Table 1* shows relative employment and indicates that 52 per cent is in medium-skilled occupation 2, followed by 32 per cent in low-skilled occupation 1 and 16 per cent in high-skilled occupation 3. These percentages indicate, of course, the average involvement of an occupation over the entire range of tasks.

The top-middle part of *Table 1* depicts absolute employment for the fragments produced in Home and country 1 if fragmentation is in two parts. We assumed that occupations 1 and 3 are more expensive in country 1 than in Home, whereas medium-skilled occupation 2 is cheaper. As a consequence, the Home country produces the first fragment $fr\ 2_1$ and country 1 produces the second fragment $fr\ 2_2$. As the bottom part of *Table 1* shows, fragment $fr\ 2_1$ produced in Home is relatively more intensive in the use of low-skilled occupation 1 (namely 51 per cent compared to the overall average of 32 per cent) and relatively less intensive in the use of medium-skilled occupation 2 (namely 41 per cent compared to the overall average of 52 per cent). The reverse holds for fragment $fr\ 2_2$ produced in country 1: it is more intensive in the use of medium-skilled occupation 2 and less intensive in the use of low-skilled occupation 1. Fragmentation thus allows countries to specialize according to their comparative advantage as medium-skilled occupation 2 is relatively cheap in country 1. Note that the bundling of occupations in fragments poses a limitation in this respect for high-skill occupation 3, as we discuss in section 5.

Table 1 Distribution of employment and percent of employment for fragmentation example

a. Employment

		Number of fragments					
		1	2	2	3	3	3
Frag	Occ	Home	Home	Country 1	Home	Country 1	Country 2
1	1 *low*	0.267	0.121		0.121		
	2 *med*	0.433	0.097		0.097		
	3 *high*	0.133	0.018		0.018		
2	1 *low*			0.145		0.125	
	2 *med*			0.336		0.294	
	3 *high*			0.115		0.061	
3	1 *low*						0.020
	2 *med*						0.043
	3 *high*						0.055
Total empl		0.833	0.236	0.597	0.236	0.480	0.118
Var costs		0.833	0.236	0.565	0.236	0.439	0.111

b. Percent of employment

		Number of fragments					
		1	2	2	3	3	3
Frag	Occ	Home	Home	Country 1	Home	Country 1	Country 2
1	1 low	32	51		51		
	2 med	52	41		41		
	3 $high$	16	8		8		
2	1 low			24		26	
	2 med			56		61	
	3 $high$			19		13	
3	1 low						17
	2 med						36
	3 $high$						47
Total empl		100	100	100	100	100	100

Data are based on the example discussed in Appendix 1; empl = employment; var = variable.

The top-right part of *Table 1* depicts absolute employment for the fragments produced in Home, country 1 and country 2 if fragmentation is in three parts. We assumed that low-skilled occupation 1 is more expensive in country 2 than in Home, while high-skilled occupation 3 is less expensive and medium-skilled occupation 2 is equally expensive. This gives country 2 a comparative advantage in the high-skill intensive end stages of the production process, which explains why Home produces fragment $fr\ 3_1$, country 1 produces fragment $fr\ 3_2$, and country 3 produces fragment $fr\ 3_3$. As the bottom part of *Table 1* illustrates, there is no change in the relative composition of occupations for Home, which is still most intensive in low-skill occupation 1. For country 1, however, the intensity of medium-skill occupation 2 (which is cheapest in country 1) rises substantially for the fragment it produces (from 56 to 61 per cent), while the intensity of high-skill occupation 3 (which is dearest in country 1) falls substantially (from 19 per cent to 13 per cent). Similarly, country 2 produces a fragment which intensively uses high-skill occupation 3 (namely 47 per cent), which is cheapest in country 2. All countries thus specialize according to their comparative advantages.[17]

The discussion above shows that a fall in coordination costs, which allows deeper fragmentation (into more parts), provides countries with better opportunities to benefit from their comparative advantages as reflected in the reward for a certain occupation. The driving force behind this process is the fact that fragments are more directly linked to a certain occupation as the fragmentation process continues. This raises the question if ever-continuing and more-detailed fragmentation will completely wipe out the demand for certain occupations in certain countries, as is widely argued. It does not. More-detailed fragmentation can lower the relative demand for certain occupations in certain countries compared to unfragmented production. In the example: for medium- and high-skill occupations in Home, for low- and high-skill occupations in country 1, and for low- and medium-skill occupations in country 2, while raising the relative demand for other occupations (low-skill occupations in Home, medium-skill occupations in country 1, and high-skill occupations in country 2). It is straightforward to construct a multi-product example in a general equilibrium setting in which fragmentation from the perspective of multiple countries raises the relative demand for both low-skill and high-skill occupations in Home as well as lowering the

[17] Note that if there is no country 2 and with sufficiently low fixed fragmentation costs, fragmentation into three parts: Home – country 1 – Home will occur, see also section 5.

relative demand for medium-skill occupations (see Figure 2, and Figure 4) while simultaneously raising the relative demand for medium-skill occupations in country 1. There are, however, both practical and theoretical limits to what fragmentation can achieve, as discussed next.

5 Possibilities and limitations of fragmentation

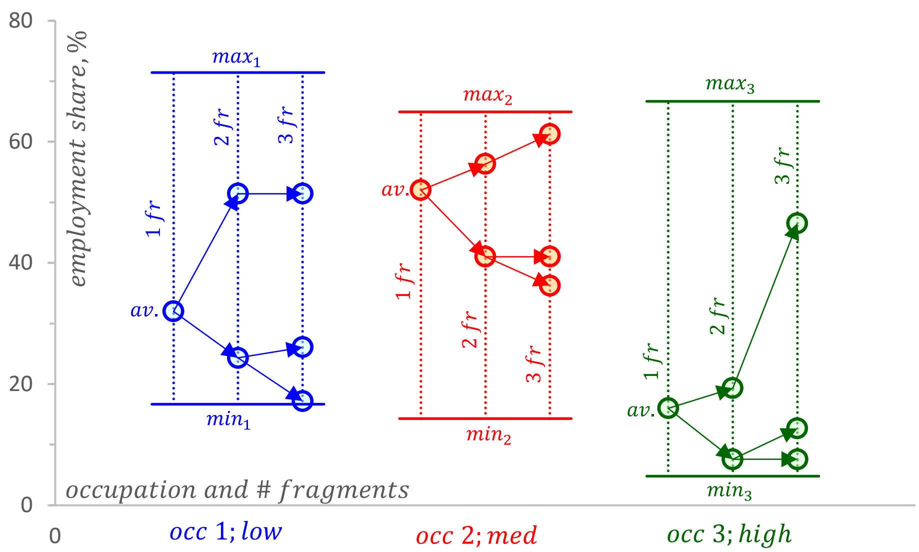
5.1 Possibilities of fragmentation

As already discussed in section 4, a disadvantage of unfragmented production in one location is the inability to take advantage of variations in occupation intensity across the production process. The cost of production at any particular location reflects the price of occupations for the *average* use of that occupation in the production process. In the example we discussed, this was 52 per cent for medium-skill occupation 2, followed by 32 per cent for low-skill occupation 1, and 16 per cent for high-skill occupation 3. This is illustrated in Figure 11 for the three occupations with the $av.$ balls if fragmentation is not possible (the 1 fr lines). When deciding where to produce if fragmentation is not possible, firms evaluate the attractiveness of locations based on these averages. In our example, unfragmented production is cheapest in the Home country, followed by production exclusively in country 1, while exclusive production in country 2 is most expensive. We thus initially assume production takes place in the Home country.

The intensity with which occupations are used for producing the different tasks varies as the task ranges from 0 to 1. To provide an idea of this range, Figure 11 also provides the minimum and maximum employment share (as a percentage) of each occupation, as defined in equation (6). In general, the minimum will be larger than zero and the maximum will be smaller than 100, although both extremes are, of course, possible if an occupation is not necessary for a particular task or if a task requires only one occupation, respectively. Figure 11 shows that the average without fragmentation is somewhere along this range (the $av.$ balls).

(6) $$min_i = min\left\{\frac{g_i(x)}{\sum_r g_r(x)}\middle|x \in X\right\};\ \ max_i = max\left\{\frac{g_i(x)}{\sum_r g_r(x)}\middle|x \in X\right\}$$

Figure 11 On the limits and possibilities of fragmentation



Note: see Appendix 1 for details; occ = occupation; fr = fragment(s); av. = average; min = minimum; max = maximum.

As fragmentation into two parts is possible (the $2\ fr$ lines in Figure 11), the two fragments allow for a wider coverage of the range of intensities, although in discrete steps. For low-skill occupation 1, for example, the average intensity was 32 per cent without fragmentation, while fragmentation into two parts allows for an intensity of 24 per cent (in country 1) and 51 per cent (for Home), which is substantially closer to the minimum (of 16.7 per cent) and the maximum (of 71.4 per cent). Further fragmentation further widens the range for occupations in three discrete steps, namely 17 per cent (in country 2), 26 per cent (in country 1), and 51 per cent (in Home). In short, as fragmentation continues a wider part of the range is covered, although in discrete steps. Similar observations hold for the other occupations. As discussed above, it allows countries to benefit from their comparative advantages in occupations. As a consequence, fragmentation creates *possibilities* to expand the relative demand for occupations with comparative advantage and reduce the relative demand for other occupations. The question arises if there are any *limits* to the possibilities created as fragmentation continues? Without such limits, we can expect an ever-rising demand for occupations with a comparative advantage at the expense of an ever-declining demand for other occupations. There are, however, clear limits to the possibilities of fragmentation in two different ways, as is clear from the visualization of the possibilities created by fragmentation in Figure 11 as a wider coverage of the range in between minimum and maximum.

5.2 Limitations of fragmentation

The first limitation of fragmentation is the extent of the range depicted in Figure 11 itself: no matter how detailed fragmentation, the relative demand for any occupation in any country cannot be lower than the minimum over the entire range of tasks, nor can the relative demand for any occupation in any country be higher than the maximum over the entire range of tasks. This minimum and maximum thus provide clear limits on the extent to which fragmentation can raise the relative demand of an occupation or lower the relative demand of an occupation, no matter how detailed fragmentation itself may be. Only if the minimum is zero for all occupations and the maximum is 100 for all occupations does the range not imply a limitation for fragmentation, but these are clearly special circumstances.

The second limitation of fragmentation focuses on the coverage of the range in between minimum and maximum. In the example, the range for an occupation is a continuum taking on all values in between minimum and maximum. As Figure 11 shows, rising fragmentation of the production process implies a better coverage of the range in between minimum and maximum, but this only takes place in discrete steps as fragmentation occurs in discrete steps only. The discretization thus provides a limitation.

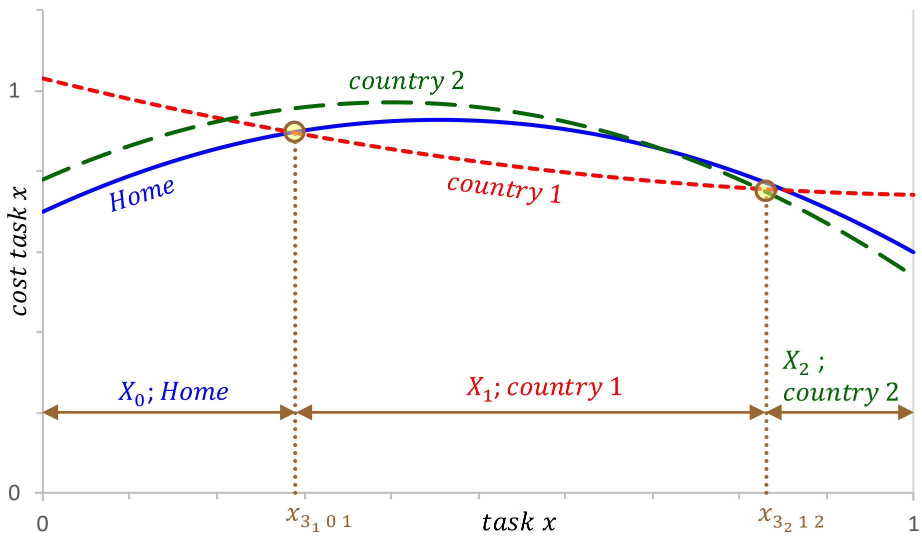
The second limitation is actually *much stronger* than one may realize at first sight. Surely, one might argue, as the fixed costs of fragmentation continue to fall towards zero, the fragmentation process continues indefinitely, covering the continuum in between the minimum and maximum more densely, and ultimately (without any fixed fragmentation costs) covering the entire range itself? Actually no, this is not true! As long as there is a finite number of countries, even with zero fixed fragmentation costs which allows for infinite fragmenation of the production process, there is in general only a finite number of fragmentation steps to be taken before fragmentation is complete. In the three-country example provided in Appendix 1 and illustrated in Figure 11, the fragmentation process is already completed after *two* steps (!), where the first step is from no fragmentation to two-part fragmentation and the second step is from two-part to three-part fragmentation. No matter how much the fixed fragmentation costs decline, all the way up to zero, there will be no more re-organization of the production process as this is already complete with the three fragments only. Section 5.3 explains why.

5.3 Infinite fragmentation and practical limits

Antras (2020) discusses slowbalization. He observes that the increase in fragmentation cannot continue forever. Our model explains why. There are practical limits to fragmentation even if there are no fixed fragmentation costs at all, which in principle makes infinite fragmentation possible. We assume (for this section only) that there are no fixed fragmenation costs ($F_{\prod_{s=0}^{p} j_s} = 0$ for all p). The firm will produce task x in country $r \in J$ if, and only if, production costs for task x are lowest in r. Let X_r denote the set of tasks for which this is the case, see eq. (7).

(7) $$X_r = \left\{ x \in X \,\middle|\, \textstyle\sum_i w_{ir} g_i(x|y) \le \sum_i w_{ij} g_i(x|y)\,;\; j \ne r;\; j, r \in J \;\right\}$$

Figure 12 Task allocation with zero fragmentation costs



Note: see Appendix 1 for parameter details.

The problem is illustrated for our three-country example in Figure 12. For each country j and all fragments x, the figure depicts the costs of producing the fragment in that country. These functions are continuous in x since the density functions g_i are continuous in x. In Figure 12, the sets of lowest cost tasks are intervals: $X_0 = \left[0, x_{3_1\,0\,1}\right]$; $X_1 = \left[x_{3_1\,0\,1}; x_{3_2\,1\,2}\right]$; and $X_2 = \left[x_{3_2\,1\,2}, 1\right]$. These sets coincide exactly with the fragments $fr\ 3_1$; $fr\ 3_2$; and $fr\ 3_3$ depicted in Figure 9 and illustrated in Figure 11. The reason is simple: at the borderline point $x_{3_1\,0\,1}$ in Figure 12, the costs for producing the task are the same in Home (country 0) and in country 1, such that from equation (7) we have:[18]

(8) $$\textstyle\sum_i w_{i0} g_i\left(x_{3_1\,0\,1}\middle|y\right) = \sum_i w_{i1} g_i\left(x_{3_1\,0\,1}\middle|y\right)$$

This implies that $\sum_i (w_{i0} - w_{i1}) g_i\left(x_{3_1\,0\,1}\middle|y\right) = 0$, which is the first-order-condition for determining the first fragmentation point in the three-part fragmentation discussion of section 4.2, see equation (4). Similarly for the second borderline point in Figure 12, which is equal to $x_{3_2\,1\,2}$. In this case, therefore, as soon as the fixed fragmentation costs are low enough to enable fragmentation into three parts, the production process is already

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[18] In the example and the discussion we assume the set of borderline points has measure zero. This is in general the case, for example if the rewards to occupations are different for all countries: $w_{ij} \ne w_{ir}$ for $j \ne r$. Otherwise, we would have to determine an allocation mechanism in the case of ties, without affecting the outcome of our main arguments below.

optimally organized. Any further lowering of the fixed fragmentation costs will lower total costs, but will not cause any re-organization of the production process. Infinite fragmentation in this case is equivalent to fragmentation into three parts.

Several observations are important regarding the limits of infinite fragmentation.

First, it is important to note that the optimal infinite fragmentation of tasks depends on the rewards to factors of production (in this case occupations) in each country. A change in these rewards in general requires a change in the fragmentation of tasks.

Second, in the 3-country example illustrated in Figure 12 fragmentation is completed when there are 3 fragments, but there can obviously be both more fragments than countries and more countries than fragments. An example of the former is to consider a world consisting of only 2 countries, namely Home and country 1. In this case infinite fragmentation is still in 3 parts, namely Home – country 1 – Home, where the last fragment for Home is a little bit smaller than the equivalent for country 2, see Figure 12. An example of the latter can be easily constructed by maintaining three countries and adjusting the rewards for occupations to ensure there are only two fragments.[19]

Third, and most importantly, it is not hard to see that optimal infinite fragmentation requires only a finite number of steps. Given a finite number of countries J, the rewards to occupations w_{ij} in each country $j \in J$, and continuous density functions g_i, the optimal infinite fragmentation of X consists of a finite number of closed intervals and the set of tasks allocated to country r is a finite union of such intervals (or the empty set if country r does not have the lowest cost for any task $x \in X$). In general, therefore, there is only a finite number of fragmentation steps required before we reach the production process associated with infinite fragmentation. This poses clear practical limits to the implications of infinite fragmentation in terms of raising the relative demand of occupations with a comparative advantage and reducing the relative demand for the other occupations.

Result 3. *Even if the costs of coordination and fragmentation continue to fall, the fragmentation process for a product is completed in a finite number of steps. Upon completion of this process, the demand for all occupations in all countries associated with this production process is constant.*

6 Evaluation

Recently and following a period of Hyper Globalization, growth in trade has slowed down. This is remarkable given that economists routinely point out that the global division of labor is welfare increasing. Despite the benefits of trade and the division of labor, a long period of trade growth has come to an end. Figure 1 illustrates these two periods of globalization: a period of Hyper Globalization from 1986-2008, followed by Slowbalization from 2008-present. In this paper we explain both phenomena – Hyper Globalization and Slowbalization – by linking trade to the developments of fragmentation costs of global supply chains. We develop a simple model that endogenizes fragmentation into multiple stages and sourcing from multiple countries. A feature of the model is that, while it endogenizes fragmentation into many fragments and many countries, it remains tractable. This formalization highlights important aspects of the current structure of trade. Central in the model are fragmentation costs that systematically change over time. During the period of Hyper Globalization fragmentation costs declined, but these costs increased following the financial crisis in 2008/9 and the recent Covid-19 pandemic.

[19] An adjustment in Appendix 1 of the reward for occupation 3 in country 1 from 1.3 to 0.8 would create only two fragments, the first in Home and the second in country 1, with country 2 not playing any role.

The model can also explain labor market consequences of these developments. Changes in fragmentation costs affect the length of supply chains and result in changes on the labor market. The reason is that each fragment represents a specific set of occupations, some of which are more skill-intensive than others. Importantly, the model answers the question if ever-continuing and more-detailed fragmentation can completely wipe out the demand for certain occupations in certain countries, as is widely argued. It does not. Fragmentation has a natural limit, even with zero fragmentation- and coordination costs, as the process involves only a finite number of fragmentation steps. This also implies that (even with zero fragmentation costs) the demand for certain occupations does not fall to zero for any country.

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Appendix 1 Example specification

The example graphs used for explaining the model are based on quadratic density functions $g_i(x|y) = (a_i x^2 + b_i x + c_i)y$, which leads to cumulative distributions $G_i(x|y) = \left(\frac{1}{3}a_i x^3 + \frac{1}{2}b_i x^2 + c_i x\right)y$. The constants a_i, b_i, and c_i are chosen such that the density function is strictly positive in the domain of x, while $y = 1$.

For occupation 1 we have: $a_1 = 0.2$, $b_1 = -0.6$, and $c_1 = 0.5$; this implies that the density function for occupation 1 is monotonically declining with task x, from 0.5 at $x = 0$ to 0.1 at $x = 1$. We think of this as a reflection of R&D intensive tasks at the early stages of the production process.

For occupation 2 we have: $a_2 = -2$, $b_2 = 2$, and $c_2 = 0.1$; this implies that the density function for occupation 2 is first rising and then declining in task x, starting at 0.1 for $x = 0$, reaching a peak of 0.6 at $x = 0.5$, and declining to 0.1 at $x = 1$. We think of this as a reflection of low-skilled labour intensive activities associated with assembly activities which are particularly important at the intermediate stages of the production process.

Finally, for occupation 3 we have: $a_3 = 0.7$, $b_3 = -0.4$, and $c_3 = 0.1$; this implies that the density function for occupation 3 is first declining and then rising in task x, starting at 0.1 for $x = 0$, reaching a low point of about 0.043 at $x = 0.29$, and rising to 0.4 at $x = 1$. We think of this as a reflection of sales promotion activities, which are important at the later stages of the production process.

Total employment in the production process is 0.833; of this 52 per cent is for occupation 2, followed by 32 per cent for occupation 1, and 16 per cent for occupation 3. When analysing the fragmentation process, we normalize all wage rates to unity at Home, so $w_1 = w_2 = w_3 = 1$. We assume that country 1 has a comparative advantage in occupation 2 and country 2 has a comparative advantage in occupation 3; more specifically, for country 1 we have: $w_{11} = 1.7$; $w_{21} = 0.5$; and $w_{31} = 1.3$, while for country 2 we have: $w_{12} = 1.2$; $w_{22} = 1$; and $w_{32} = 0.8$.

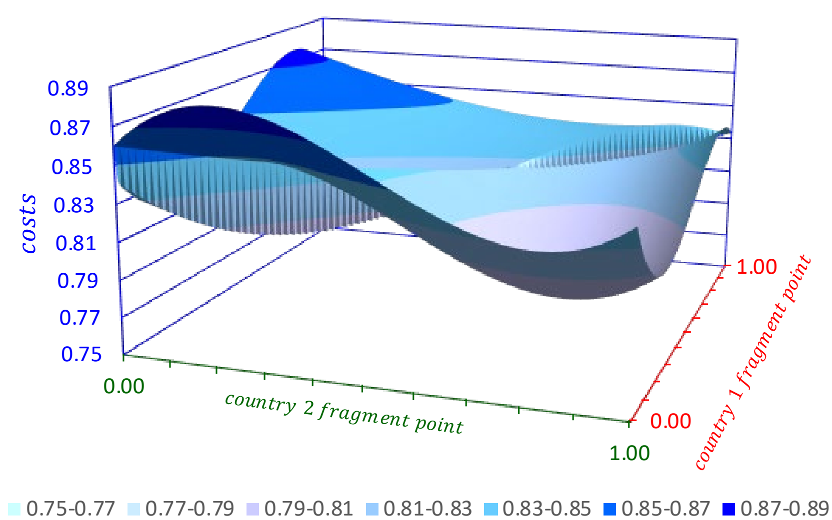
Appendix 2 Three-part and many-part fragmentation

Three-part fragmentation

We start again with determining the optimal fragmentation points. Exclusive of the fragmentation costs $F_{h\,j\,r}$, the variable costs $c_{h\,j\,r}(x_1, x_2|y)$ of fragmentation at points x_1 and x_2 is given in equation (3). Note that $0 \le x_1 \le x_2 \le 1$, where the first term on the right-hand side of equation (3) reflects the costs of producing $fr\ 3_1$ at Home, the second term reflects the costs of producing $fr\ 3_2$ in country j, and the third term reflects the cost of producing $fr\ 3_3$ in country r.[20]

$$c_{h\,j\,r}(x_1, x_2|y) = \underbrace{\left(\textstyle\sum_i w_i\, G_i(x_1|y)\right)}_{Home\ cost} + \underbrace{\left(\textstyle\sum_i w_{ij}\left(G_i(x_2|y) - G_i(x_1|y)\right)\right)}_{country\ j\ cost} + \underbrace{\left(\textstyle\sum_i w_{ir}\left(G_i(1|y) - G_i(x_2|y)\right)\right)}_{country\ r\ cost} \qquad (3)$$

Figure 13 Surface plot of costs $c_{h\,j\,r}$ and $c_{h\,r\,j}$ for three-part fragmentation

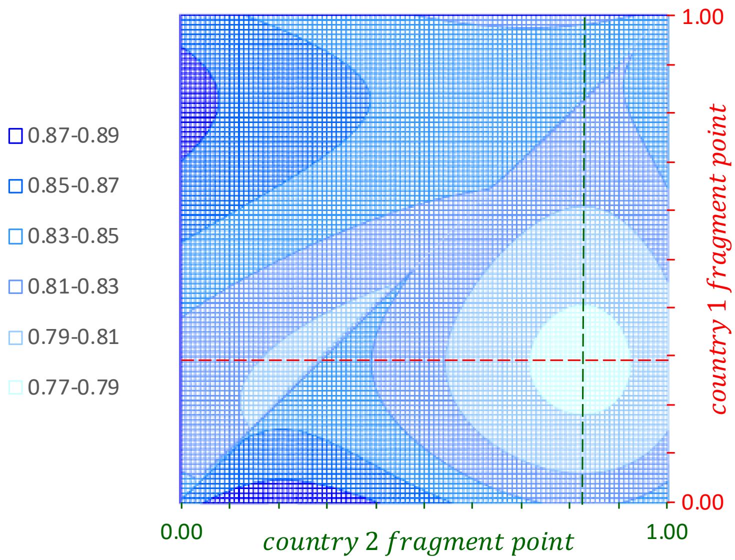


[20] Note that in our discussion the order of countries is Home – country j – country r, while in the illustrations of Figure 13 and Figure 14 we include the order Home – country r – country j as well.

Note: see Appendix 1 for parameter details.

Figure 13 provides a 3-dimensional surface plot of equation (3) for our example for the cost function $c_{h\,j\,r}$ (if the first fragment is produced at Home, the second fragment in country j and the third fragment in country r) as well as its counterpart $c_{h\,r\,j}$ (if the first fragment is produced at Home, the second fragment in country r and the third fragment in country j). Note that the function is continuous along the diagonal, but not differentiable. This is also visible in Figure 14, which provides the contour plots associated with Figure 13. Within the depicted domains, the firm wants to minimize costs, which happens approximately at fragmentation points $x_{3_1\,h\,1} = 0.29$ and $x_{3_2\,1\,2} = 0.83$, as we now discuss.

Figure 14 Contour plot of costs $c_{h\,j\,r}$ and $c_{h\,r\,j}$ for three-part fragmentation



Note: see Appendix 1 for parameter details; the horizontal and vertical lines are at minimum costs, see section 4.2.

Many-part fragmentation

To generalize our discussion of fragmentation into more than three parts, note that fragmentation into $p+1$ parts requires determining p fragmentation points. We assume that the first fragment is produced in country $j_0 \in J$, the second fragment in country $j_1 \in J$, and so on, up to fragment p in country $j_p \in J$. At each step s we must have different countries, so: $j_s \neq j_{s+1}$. We denote the sequence of $p+1$ countries $j_0 j_1 .. j_p$ by $\prod_{s=0}^{p} j_s$ and the p fragmentation points by x_s for $s = 1,..,p$; with the convention that $x_0 = 0$ and $x_{p+1} = 1$. The fixed fragmentation costs associated with this sequence are denoted by $F_{\prod_{s=0}^{p} j_s}$.

(9) $$c_{\prod_{s=0}^{p} j_s}(x_1,..,x_p|y) = \sum_{s=0}^{p} \underbrace{\left[\sum_i w_{ij_s}\left(G_i(x_{s+1}|y) - G_i(x_s|y)\right)\right]}_{fragment\ s+1\ cost\ in\ country\ j_s}$$

(10) $$\sum_i \left(w_{ij_{s-1}} - w_{ij_s}\right) g_i\left(x_{(p+1)_s\, j_{s-1}\, j_s}|y\right) = 0 \text{ for } s = 1,..,p$$

(11) $$\underbrace{\left\{F_{\prod_{s=0}^{p} j_s} - F_{\prod_{r=0}^{p-1} j_r}\right\}}_{rise\ fragmentation\ costs} < \underbrace{\left\{c_{\prod_{r=0}^{p-1} j_r}\left(x_{p_1\, j_0\, j_1},..,x_{p_{(p-1)}\, j_{p-2}\, j_{p-1}}\middle|y\right) - c_{\prod_{s=0}^{p} j_s}\left(x_{(p+1)_1\, j_0\, j_1},..,x_{(p+1)_p\, j_{p-1}\, j_p}\middle|y\right)\right\}}_{reduction\ variable\ costs}$$

To determine the optimal fragmentation points x_s, we look at the variable costs $c_{\prod_{s=0}^{p} j_s}(x_1,..,x_p)$ exclusive of the fixed fragmentation costs as given in equation (9), where the term in square brackets reflects the costs of producing fragment $s + 1$ in country j_s and we sum over all fragments. If we differentiate equation (9) with respect to fragmentation points x_s and equate to zero, we get the first-order conditions for the optimal fragmentation points given in equation (10), implying orthogonality of the differences in wage rates using the density functions f_i at the optimal fragmentation points $x_{(p+1)_s\, j_{s-1}\, j_s}$. This leads to the minimum cost level $c_{\prod_{s=0}^{p} j_s}\left(x_{(p+1)_1\, j_0\, j_1}, \ldots, x_{(p+1)_p\, j_{p-1}\, j_p} \middle| y\right)$ for fragmentation into $p + 1$ parts. We conclude this discussion by pointing out that fragmentation into $p + 1$ parts leads to lower total costs of production if the rise in fixed costs relative to fragmentation into p parts is lower than the variable cost reduction, as given in equation (11). We could illustrate this in an increasingly complicated 2-dimensional diagram similar to Figure 10.