Are Managers Paid for Market Power?*

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Abstract

To answer the question whether managers are paid for market power, we propose a theory of executive compensation in a macroeconomic environment where firms have market power, and the market for managers is competitive. We identify two distinct channels that contribute to manager pay in the model: market power and firm size. Both increase the profitability of the firm, which makes managers more valuable as it increases their marginal product. Using data on executive compensation from Compustat, we quantitatively analyze how market power affects manager pay and how it changes over time. We attribute on average 45.8% of manager pay to market power, from 38.0% in 1994 to 48.8% in 2019. There is also a lot of heterogeneity within the distribution of managers. For the top managers, 80.3% of their pay in 2019 is due to market power. Top managers are hired disproportionately by firms with market power, and they get rewarded for it, increasingly so.


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1 Introduction

In the last four decades, inequality has increased dramatically, mainly driven by the increase in top incomes (Piketty and Saez, 2003, 2006; Atkinson, Piketty, and Saez, 2011). Among the top incomes earners, many of them are managers. Managers do things differently than production and service workers. Most notably, successful managers render other workers more productive and firms pay a premium to hire those successful managers. And when firms become larger and more profitable, the impact of managers becomes more valuable to the firm, as every single decision now has far-reaching implications. Because their salaries are determined in a competitive labor market, this leads to higher pay for managers. This has been the seminal insight of Gabaix and Landier (2008) and Terviö (2008): the rise of the size of firms can explain why manager pay increased so much.

Yet, it remains an open question what determines the size and productivity of the firm. In this paper, we build on the insights from this literature by shedding new light on the contribution of managers to firm productivity and firm size and how this affects profitability. In particular, we focus on the role of market power. The recent literature documents that in the last four decades, there has been a rise in market power, and this evolution coincides remarkably with the rise in manager pay (see Figure 1 below). Pay was relatively stable until the 1980s, when it started to rise sharply, a pattern similar to that of markups. So we ask whether managers are paid for market power.

Our objective is to decompose the origins of manager pay that are due to market power and those due to firm size. Do firms with market power pay managers more because those managers make firms sell more, or produce more efficiently? Because they generate more profits? Do managers extract some of the rents that market power creates? To analyze the contribution of market power to manager pay and the underlying mechanism, we start from the premise that productivity and firm size are not determined in a competitive vacuum. Instead, firms exert market power in the goods market and managers who are hired in a competitive labor market contribute to their firm’s productivity. Therefore, managers are paid well not only due to the large firm size, but also because firms are competing for market power by hiring better managers.

Firms with market power also tend to be larger, and it is predominantly large superstar firms that

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1Overall, about one fifth of the workforce has a managerial position (Santamaria, 2018). Of course, not all managers are top earners, but one of the main determinants of higher wages is whether a worker supervises other workers. Because there are hierarchies, the managers of managers supervise most workers and hence become the top earners. In what follows, we use manager and CEO interchangeably.

2For tractability, in the baseline analysis we abstract from the role of agency, which has contributes to manager pay as well. In Appendix B.4 we extend the model as in Edmans and Gabaix (2011) to allow for incentive pay and show that agency problems, while relevant, do not alter the insights about the effect of market power on manager pay (see also the discussion in Section 3.2).

3See amongst others Grullon, Larkin, and Michaely (2016); Gutiérrez and Philippon (2017); De Loecker et al. (2020).
exert market power: firms that have market power and have higher markups tend to obtain a larger market share. Of course, firms are also large because they have superior technologies. The fact that market power and firm size correlate poses a serious challenge to tease out the role of each in determining the pay of managers. The correlation between markups and manager pay therefore does not elucidate our understanding of the causal determinants, due to, amongst other, reverse causality and omitted-variable bias.

To address this issue, we build a model with a small number of competitors, in an economy with many of these small markets as in Atkeson and Burstein (2008). The main determinants of market power in these small, oligopolistic markets is the market structure (the number of firms in each market) and the distribution of productivities of the competitors. As in standard oligopolistic markets, the fewer the number of firms, or the more dispersed the productivities, the more market power. But market power is not equally distributed. The more productive firms compete by setting lower prices, which yields a higher market share and hence a larger firm. However, due to incomplete pass-through of cost savings, those productive firms do not pass on all their productivity gains to the customer, so they set higher markups and generate more profits.

And here comes in the role of the manager. The manager raises the productivity of the firm in the sense of Lucas (1978) span of control. As in the canonical matching model of Gabaix and Landier (2008) and Terviö (2008), total factor productivity (TFP) is determined by the complementary inputs of the manager ability and the firm type. Hence, firms can become more productive by hiring good managers. But firms that become more productive will charge higher markups and extract more profits instead of passing on all the efficiency gain to consumers. In a competitive labor market, managers will therefore get rewarded for the extra market power they create. Incomplete passthrough in conjunction with a competitive market for managers who raise firm productivity is the key mechanism we introduce to answer the question this paper set out to ask.

The technology, and especially the differences in TFP between firms in the same market, determines market power. The positive sorting of managers to firms amplifies the heterogeneity across managers and firms. Due to assortative matching, top managers join top firms and less talented managers lead low productivity firms. The bigger the gap between firms, the larger the market power: the high

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4One of the robust drivers of the rise in market power is the reallocation of market share towards more efficient firms that have high markups. See Autor, Dorn, Katz, Patterson, and Van Reenen (2020) and De Loecker et al. (2020).

5This setup is consistent with the view that what skilled managers do is to increase the productivity of the firm (see for example, Bloom, Sadun, and Van Reenen, 2016). We refer readers to section 3.2 for further discussion.

6Even if the number of firms is small, when firms are identical in TFP, they have identical profits and market shares. Instead, when one firm has higher TFP, it achieves higher profits, and it obtains a larger market share. In the limit, as one firm is a lot more productive than all competitors in the market, it effectively behaves as a monopolist and obtains a market share close to one even if there are multiple competitors.
productivity firm faces less competition, and as a result passes on less of the productivity gains to the customer, a view put forward initially by Sutton (1991, 2001). Because managers reinforce the dominant position for high productive firms, firms that compete for top managers must compensate them for the manager’s contribution to profits. The incomplete passthrough leads to a disproportional effect of market power on managers with highest ability.

Using data on executive compensation from Compustat between 1994 and 2019, we quantify the production technology and the underlying distributions of productivities by matching moments on sales, markups, and compensation over time. Most remarkable is that the estimated model matches the manager pay distribution extremely well over the entire period of the sample, even though our estimation does not target the wage distribution directly. This provides external validation of the theory and indicates that the model, despite its simplifying assumptions, provides an accurate description of the executive wage setting mechanism.

We then use the model in which manager pay is determined in conjunction with market power to quantitatively decompose the sources of manager pay due to market power and firm size. We attribute on average 45.8% of manager pay to market power, the remainder is due to firm size. Over time, 57.8% of the growth in pay is due to market power. Consistent with the theory, we also find there is a lot of heterogeneity within the distribution of managers. For the top managers in 2019, 80.3% of their pay is due to market power, and so is nearly all their growth since 1994. For the lower ranked managers, pay and growth of pay is determined mainly by firm size.

Our main conclusion is that top managers are hired disproportionately by firms with market power, and they get rewarded for it. And while our focus due to data availability is on CEOs, the same logic applies to all managers who supervise other workers and other professions where sorting and superstar pay is a key determinant. And because one fifth of the workers supervise other workers, these findings have macroeconomic implications, especially at the top percentiles of the income distribution.

Related Literature. Our work builds on a large literature of prior work. The starting point is the body of work that introduces matching of managers of heterogeneous ability to firms of different size. This approach can explain why, in a competitive labor market, managers receive superstar pay and why it has increased so much in recent decades. See Gabaix and Landier (2008) and Terviö (2008), and also Edmans and Gabaix (2016) and Edmans, Gabaix, and Jenter (2017), for comprehensive surveys of the literature. For further evidence documenting the firm size hypothesis and its effect on compensation, see also Frydman and Saks (2010), Gabaix, Landier, and Sauvagner (2014) and Green, Heywood, and Theodoropoulos (2021).
There is also a growing literature documenting the rise of superstar firms and the effect this has on the capital and labor shares (Hartman-Glaser, Lustig, and Zhang, 2016; Kehrig and Vincent, 2017; Barkai, 2019; Autor, Dorn, Katz, Patterson, and Van Reenen, 2020). Much of this literature highlights the role of market power, and the reallocation of market share towards high markup firms. Firms that are large also tend to have high markups: Grassi (2017); Edmond, Midrigan, and Xu (2019); and De Loecker et al. (2021).

Our paper bridges these literatures on firm size and manager pay on the one hand, and firm size and market power on the other. We model market power in the tradition of the general equilibrium model of Atkeson and Burstein (2008), which allows for endogenous markups, a flexible market structure and firm heterogeneity. The theoretical novelty is to add a two-sided matching framework to this model with oligopolistic competition and endogenous markups in general equilibrium. Our analysis framework is also related to Jung and Subramanian (2017, 2021), who check the relationship between CEO compensation and product market competition. While their works are built on Dixit and Stiglitz (1977) with monopolistic competition and exogenous markups, we examine from a new perspective that managers are paid because they allow firms to exert larger market power.

For simplicity, we abstract from incentive provision. Our work complements the work that studies the effect of product market competition on incentive provision and optimal incentive contracts (Schmidt, 1997; Aggarwal and Samwick, 1999; Raith, 2003; Falato and Kadyrzhanova, 2012; Antón, Edgerer, Giné, and Schmalz, 2021). Key in our setup with matching are endogenous markups and our ultimate objective is to estimate the technology and the market structure and to measure the contribution of market power to manager pay. The inefficiency from imperfect competition in the output market leads to rent extraction, where the manager and the owner of the firm join forces to extract rents from customers and competitors.

Other recent studies also consider managerial compensation in a general equilibrium setup. Acemoglu, Akcigit, and Celik (2022) focuses on the choice between incremental and radical innovation, and investigates the sorting of managers of different ages and human capital across firms. Celik and Tian (2017) builds a general equilibrium with an agency problem to study the joint dynamics of corporate governance, managerial compensation, and disruptive innovations. As far as we know, no other work offers a theoretical and quantitative examination of the role of market power for manager compensation in general equilibrium.

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7See Section 3.2 for more discussion on risk aversion and agency issue. We also provide a theoretical framework in Appendix B.4 demonstrating that this assumption will not alter our key insights.

8In a sense, this is in line with the rent extraction in Bebchuk et al. (2002), but rather than extraction by the manager from owners, it is by manager and owners from customers and competitors.
Finally, there is also a growing literature linking economy-wide inequality to market power. Using micro data from the US Census, Deb et al. (2022a) document the effect of market power on the skill premium and the wage level of all workers. Kaplan and Zoch (2020) analyze the productivity of different occupations and the effect of markups. And Fernández-Villaverde, Mandelman, Yu, and Zanetti (2021) focus on the complementarities between firms and customers, which fosters market concentration, monopsony power, and wage inequality.

In the next section, we describe the data and perform a preliminary analysis on the correlation between pay and market power. In Section 3 we propose a theory that captures the mechanism that drives manager pay by market power and firm size, and derive analytical results for its properties. In Section 4, we quantify the model. We present our main results in Section 5. Finally, Section 6 concludes.

2 Data and Preliminary Analysis

Data. We use data from Compustat throughout the paper. The North America Fundamentals Annual data set (1950–2019) contains information on firm-level financial statements, including measures of sales, input expenditure, and industry classifications. We drop the finance, insurance, and real estate sectors (SIC between 6000 and 6799). The ExecuComp data set (1992–2019) has measures for manager pay. We use the variable TDC1 for manager pay, which include salary, bonus, restricted stock grants, and value of option grants. Although the ExecuComp data starts in 1992, we observe a substantial difference with the samples in 1992 and 1993, so our analysis will be carried out during the period 1994 to 2019. Finally, all the nominal variables are deflated by dollars in 2019. Appendix A.1 provides more details of the firm-level panel data used in our reduced-form and structural analysis.

Markups are a key component in our analysis. Because they are not directly observable in the data, we use firm-level markups obtained using the production approach or the demand approach. In our baseline model, we use the markups from De Loecker et al. (2020). We also find that our quantitative results are robust when using markups obtained with the demand approach from Deb et al.

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9 The Compustat Data has been used extensively in the literature related to executive compensation, for example, Gabaix and Landier (2008), which makes our results comparable with the literature.

10 The difference between TDC1 and the alternative measure TDC2 measures is that TDC1 includes the value of options at the time the options are awarded while TDC2 includes the value of options at the time they are exercised. Our quantitative results are robust with both definitions.

11 Details are documented in Figure A.1 of Appendix A.3, which is also mentioned in Terviö (2008).

12 The recent work by Traina (2018), Basu (2019), Syverson (2019), Bond, Hashemi, Kaplan, and Zoch (2021) and De Ridder, Grassi, and Morzenti (2022) has brought to the attention of the research community important methodological aspects of production function estimation. Most notably, estimates are biased due to endogeneity (first addressed by Olley and Pakes (1996) using the control function approach) and omitted price bias (first pointed out by Klette and Griliches (1996)). In the production function estimation to obtain markups, De Loecker et al. (2020) control for these biases using the techniques laid out in this literature. For a detailed discussion, see Appendix A in De Loecker et al. (2020).
Figure 1: The evolution of manager pay and markups

Notes: Panel A plots the average executive compensation and average markup from ExecuComp sample. Panel B shows the long-term evolution of manager pay and markups, where the red line is the median manager pay among top firms constructed by Frydman and Saks (2010) and the blue, dotted line is the average markup from Compustat sample. All of them are plotted in 2019 million dollars, in log scale, and in five-year centered moving average.

(2022b) where markups are estimated from a structural demand and supply model without estimating a production function.

Correlation between markups and executive salaries. We begin our analysis by looking at the correlation between manager pay and markups. As we have mentioned in the introduction, Figure 1.A depicts the evolution over time of average manager pay and average markups, and shows that the increase in manager pay correlates with the rise of markups. From 1994 to 2019, the average CEO salary more than doubled from $3.34 to $6.96 million, while the average markup also increased from 1.53 to 1.78. In Figure 1.B, we show the same series for markups for a longer time period (starting in 1955). Instead, for executive compensation, in the right panel we use data from Frydman and Saks (2010) who have constructed a longer time series dating back to pre-WWII and running until 2005. The Frydman and Saks (2010) measure of manager pay shows barely any increase between 1936 and the late 1970s, after which there is a sharp increase. The year 1980 is also when markups start to increase. Casual empiricism shows that there is a positive correlation between the markup and manager pay between 1955 and 2005.

Given the positive correlation between markups and executive compensation, we further analyze this relation including year and firm fixed effects. We are typically interested in the regression with interactions between year dummies and markup:\(^\text{13}\)

\[
\log \text{Manager Pay}_{it} = \sum_t \beta_t (\log \text{Markup}_{it} \times \text{Year}_t) + \delta_i + \kappa_t + e_{it},
\]  

\(^{13}\)Additional regression results with different specifications are reported in Appendix A.2.
Figure 2: The elasticity of markups on manager pay over time

Notes: This figure reports the coefficients $\beta_t$ in regression specification (1) across year. The 95% confidence interval (CI), which is constructed with robust standard errors under heteroscedasticity, is indicated by the shaded area.

where $i$ and $t$ represent for firm and year, respectively. We have to assume that the residual term, $e_{it}$, is independent from markups after controlling the fixed effects $\delta_i$ and $\kappa_t$. Figure 2 shows the evolution of manager pay elasticity of markup, $\beta_t$, across time. Overall, this elasticity is significantly positive, indicating that a higher markup will correlate with higher manager pay. Moreover, we see an increase in the importance of market power. In 2019, a one percent increase in the firm level markup increases manager pay by 0.41 percent, which is 58.2% larger than the effect in 1994.

While these regression results give an indication of the correlation between market power and managers’ pay, they do not inform us about causality. We are faced with a number of serious identification problems, including reverse causality and omitted-variable bias. The limited availability of data and instruments makes it extremely difficult to uncover a clean causal relationship behind the correlations we observe. Therefore, in the remainder of this paper we propose a theory that can explain the relation between market power and executive compensation. Then we structurally estimate the model using the data that we have analyzed in this section.

3 Model

We build a model of the macroeconomy where firms have market power, and each firm hires a manager. The imperfect competition is modeled in the fashion of Atkeson and Burstein (2008), while the allocation of managers to firms is within a Becker (1973) matching framework in the spirit of Gabaix and Landier (2008) and Terviö (2008). We will introduce the model setup in section 3.1, where the main
assumptions are discussed in section 3.2. Then, section 3.3 solves the model and section 3.4 further explores mechanisms that determine equilibrium manager pay.

3.1 Setup

Environment. The general equilibrium economy is populated by representative households and heterogeneous firms. A continuum of identical households consume goods, and they supply unskilled labor and managers. All surpluses generated in the economy revert to the households. The measure of firms is equal to \( M \). The measure of households is normalized to one, and contains a large measure of identical production workers and a measure \( M \) of heterogeneous managers whose ability is indexed by \( x \) with distribution \( F(x) \).\(^{14}\) The market structure contains a continuum of markets with measure \( J \), each indexed by \( j \in [0, J] \). Each market \( j \) contains a finite number of \( I_j \) firms, where \( I_j \) varies by market \( j \).\(^{15}\) A single firm produces a single good. We use the subscript \( ij \) to index firm \( i \) in market \( j \).

There are two stages. In stage 1, firms and managers match and the type of the manager and the type of the firm will contribute to total factor productivity. In stage 2, households choose their consumption bundles and make their labor supply decisions, and firms compete by choosing their production allocations.

Preferences. Households have preferences for consumption of all goods, within and between markets. The utility of consumption is represented by the double-nested Constant Elasticity of Substitution (CES) aggregator. The finite number of \( I_j \) goods are substitutes with elasticity \( \eta \), and the elasticity of substitution between markets is \( \theta \). We assume \( \eta > \theta > 1 \), indicating that households are more willing to substitute goods within a market (say Pepsi vs. Coke) than across markets (soft drinks vs. cars). The CES aggregates are defined as:

\[
C = \left[ \int_0^J J^{-\frac{1}{\theta}} \left( \sum_{i=1}^{I_j} I_j^{-\frac{1}{\eta}} c_{ij}^{\frac{\eta - 1}{\eta}} \right) \right]^{\frac{\theta}{\theta - 1}} \quad \text{and} \quad c_j = \left[ \sum_{i=1}^{I_j} I_j^{-\frac{1}{\eta}} c_{ij}^{\frac{\eta - 1}{\eta}} \right]^{\frac{1}{\theta - 1}},
\]

where \( c_{ij} \) is the consumption of good \( ij \), \( c_j \) is the consumption aggregate of market \( j \), and \( C \) is the economy-wide aggregate of consumption. We normalize the utility by the number of varieties to neutralize the love of variety effect, both within market \( j \) with size \( I_j \) and between markets with measure

\(^{14}\)The fact that the measure of managers equals the measure of firms is without loss of generality. A variation of the model can have occupational choice between becoming a manager and a production worker and where the number of managers is determined endogenously.

\(^{15}\)The measure \( J \) is endogenous, which is determined by \( M = J \times \mathbb{E}(I_j) \).
J.\textsuperscript{16} We represent the household’s preferences with the following utility function over the consumption bundle \( \{c_{ij}\} \) that aggregates to \( C \), and the supply of labor \( L \):

\[
U(C, L) = C - \frac{1}{\phi} \left( \frac{L^{1+\frac{1}{\phi}}}{1+\frac{1}{\phi}} \right),
\]

where utility is linear utility over aggregate consumption, and there is a constant elasticity disutility of labor with elasticity \( \phi \) and intercept \( \bar{\phi} \). We further assume without loss that the manager’s labor is supplied inelastically at zero cost.

Prices of the final consumption goods are denoted by \( p_{ij} \), wages for production labor by \( W \), salaries for managers by \( \omega(x) \), and profits by \( \pi_{ij} \). Manager salaries aggregate economy-wide to \( \Omega \) and profits to \( \Pi \), of which each household receives an equal share. Households face a budget constraints, where their spending on goods cannot exceed the income consisting of wage bill \( WL \), executive salaries \( \Omega \), and dividends \( \Pi \). We can thus summarize the household problem as follows:

\[
\max_{\{c_{ij}\}, L} U(C, L), \quad \text{s.t.} \quad \int_0^1 \left( \sum_{i=1}^J p_{ij} c_{ij} \right) dj \leq WL + \Omega + \Pi. \tag{3}
\]

An important feature here is that all output produced is equal to the total income of the households. Therefore, all the value general by the allocation of this economy stays in the economy.

**Technology.** Firms differ in two dimensions. First, each firm has its own type \( z_{ij} \), where \( z_{ij} \sim G(z_{ij}) \). Second, there is a productivity \( A_j \) that commonly affects all firms in the same market, which captures technology differences across markets, with \( A_j \sim H(A_j) \). Denoting the ability of the manager who matches with firm \( ij \) as \( x_{ij} \), the firm-specific Total Factor Productivity (TFP) \( A_{ij} \) is defined as:

\[
A_{ij} = A \left( x_{ij}, z_{ij}, A_j \right), \quad \text{with} \quad A_x > 0, \ A_z > 0, \ \text{and} \ A_{xz} > 0. \tag{4}
\]

We introduce managerial ability as an input that determines productivity, which can be interpreted as a Lucas (1978) model of span of control.\textsuperscript{17} Like the classic literature, we also assume that managers and

\textsuperscript{16}The love of variety adjustment ensures the households’ preferences remain fixed when the market structure changes over time. This assumption is not crucial to any of our results.

\textsuperscript{17}There is a large body of empirical literature documenting the importance of management for productivity (e.g. Ichniowski, Shaw, and Prennushi, 1995; Bertrand and Schoar, 2003). Most recently, Bloom, Sadun, and Van Reenen (2016) uses a structural analysis showing that management is indeed like a technology that raises TFP. Hence, manager ability is commonly interpreted as a TFP-enhancing (or equivalently, cost-reducing) technology in the literature (see also Jung and Subramanian, 2021; Acemoglu, Akcigit, and Celik, 2022). This is also consistent with the reverse channel championed by Sutton (1991, 2001), that firms create market power (whether it is through investment, or here through hiring skilled managers) by increasing the productivity of the firm. Kaplan and Zoch (2020) raises an alternative possibility that managers (they call it expansionary labor) may increase in product variety, which in our theoretical framework is equivalent to an increase in productivity (see
firms are complementary. We will specify a CES functional form when mapping this model into data in Section 4. Given the firm’s TFP $A_{ij}$, the technology that determines the quantity of output $y_{ij}$ as a function of inputs of production labor is linear:

$$y_{ij} = A_{ij}l_{ij}. \tag{5}$$

**Timing.** All types realize at the outset: $\{x, z_{ij}, A_j\}$. There are two stages. In stage 1, each firm hires one manager in a frictionless market with payoffs under perfectly transferable utility (TU). The salary $\omega(x)$ denotes the compensation function of manager type $x$. Therefore, the profit maximization problem for firm $ij$ at this stage is:

$$\max_{x_{ij}} \pi_{ij} = \tilde{\pi}_{ij}(A_{ij}|A_{-ij}) - \omega(x_{ij}), \tag{6}$$

where $\tilde{\pi}_{ij}$ is the firm’s gross profit coming from the next period. We use the ‘$\sim$’ to distinguish between gross profits $\tilde{\pi}$ before paying the manager compensation, and net profits $\pi$ after paying the manager compensation. Note that there is an externality in the problem (6), that the profit of the firm $ij$ depends not only on its own TFP $A_{ij}$ but also on the productivity of its competitors, $A_{-ij}$.\(^{18}\)

Once managers of type $x$ and firms of type $z_{ij}$ in markets $A_j$ have matched, the firm’s TFP $A_{ij}$ is common knowledge to all in the economy. In stage 2, firms then Cournot compete in quantity $y_{ij}$ with their rivals in the same market.\(^{19}\) The firms make production decisions to maximize gross profits:

$$\max_{l_{ij}} \tilde{\pi}_{ij} = p_{ij}y_{ij} - Wl_{ij}, \quad \text{s.t.} \quad y_{ij} = A_{ij}l_{ij}. \tag{7}$$

This is a problem with strategic interaction within each market $j$ through the Cournot game, so in equilibrium $l_{ij}$ depends on $l_{-ij}$. As we have described above, in the first period matching problem, the gross profits is then further partitioned into executive salaries, $\omega_{ij}$, and net profits, $\pi_{ij}$.

**Equilibrium.** We can now define the equilibrium of this economy in the two subgames, as first, a compensation function $\omega(x)$ that specifies the salary for all managers and an assignment function $\Gamma$ of manager abilities to firm productivities that is measure preserving and that maximizes (6) of the matching game, taking as given the stage two subgame, which includes prices $p_{ij}$, the wage $W$, and employment $l_{ij}$ that solve (7) for all firms.

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\(^{18}\)Competition only occurs within each market. As there is a continuum of markets, a single firm cannot influence the aggregates of the entire economy. Therefore, there is no externality between markets. In general, for a treatment of matching games in the presence of externalities, see Chade and Eeckhout (2020).

\(^{19}\)Cournot competition is not the crucial assumption. As is shown in Appendix D.2, all of our results extend when the firms Bertrand compete on price.
3.2 Illustration and discussion of model assumptions

Figure 3 summarizes the setup of this general equilibrium model. The representative households (marked in red) consume goods, supply labor and managers, and own the profits, which correspond to the four parts in budget constraint (3), respectively. Given heterogenous firms exogenously distributed across a continuum of markets, managers first match with firms in stage one and generate productivities. In stage two, firms Cournot compete in a setting with complete information about: (1) market structure, (2) all the productivities, (3) output demand, and (4) labor supply. The whole process generates equilibrium outcomes (marked in green) such as wages and gross profits, where the latter can be further divided into net profits and manager pay. We also provide a complete list of the model variables in Table 1. For tractability, we make several model assumptions, which we discuss now.

Representative households. Our general equilibrium model builds up on representative households following Atkeson and Burstein (2008), and more fundamentally, the CES structure from Dixit and Stiglitz (1977). This assumption gives us well-behaving output demand and labor supply function. To maintain the representativeness of the households, we assume that households own a perfectly diversified stake in economy-wide manager income and profits. Note that although households are
Table 1: Summary of the model variables

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Name</th>
<th>Meaning</th>
<th>Name</th>
<th>Meaning</th>
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<tbody>
<tr>
<td></td>
<td>$\theta$</td>
<td>Elasticity of sub. across markets</td>
<td>$\eta$</td>
<td>Elasticity of sub. within a market</td>
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<tr>
<td></td>
<td>$J$</td>
<td>Number of markets</td>
<td>$I_j$</td>
<td>Number of firms in market $j$</td>
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<td></td>
<td>$\varphi$</td>
<td>Labor supply shifter</td>
<td>$\varphi$</td>
<td>Labor supply elasticity</td>
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<td>$M$</td>
<td>Measure of firms and managers</td>
<td>$\omega_0$</td>
<td>Manager’s reservation utility</td>
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<td></td>
<td>$F(\cdot)$</td>
<td>CDF of manager ability</td>
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<td></td>
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<tr>
<td>Firm/Good</td>
<td>$x_{ij}$</td>
<td>Manager ability</td>
<td>$z_{ij}$</td>
<td>Firm type</td>
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<td>$p_{ij}$</td>
<td>Price</td>
<td>$\mu_{ij}$</td>
<td>Markup</td>
</tr>
<tr>
<td></td>
<td>$\bar{\pi}_{ij}$</td>
<td>Gross profit</td>
<td>$\pi_{ij}$</td>
<td>Net profit</td>
</tr>
<tr>
<td></td>
<td>$s_{ij}$</td>
<td>Sales share (within a market)</td>
<td>$r_{ij}$</td>
<td>Sale (revenue)</td>
</tr>
<tr>
<td></td>
<td>$\omega_{ij}$</td>
<td>Manager pay</td>
<td>$\varepsilon_{pij}$</td>
<td>Price elasticity of demand</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon_{lij}$</td>
<td>Markup elasticity of TFP</td>
<td>$\varepsilon_{lij}$</td>
<td>Employment elasticity of TFP</td>
</tr>
<tr>
<td>Market</td>
<td>$c_j$</td>
<td>CES aggregation of consumption</td>
<td>$y_j$</td>
<td>CES aggregation of output</td>
</tr>
<tr>
<td></td>
<td>$p_j$</td>
<td>CES price index</td>
<td>$\mu_j$</td>
<td>Average markup</td>
</tr>
<tr>
<td></td>
<td>$A_j$</td>
<td>Market type</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Economy</td>
<td>$C$</td>
<td>CES aggregation of consumption</td>
<td>$Y$</td>
<td>CES aggregation of output</td>
</tr>
<tr>
<td></td>
<td>$P$</td>
<td>CES price index (normalized)</td>
<td>$L$</td>
<td>Aggregation of labor</td>
</tr>
<tr>
<td></td>
<td>$\Pi$</td>
<td>Aggregation of profits</td>
<td>$\Omega$</td>
<td>Aggregation of manager pay</td>
</tr>
<tr>
<td></td>
<td>$W$</td>
<td>Wage</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The firm-level variables use subscript $ij$ that indexes for the firm $i$ in market $j$.

identical, we are still able to talk about the income inequality among managers within the households. Likewise for profits. The reason for building a model with a representative household is to keep the analysis tractable. The innovation of this model comes from the heterogeneity on the production side (firms and managers) and not come from the preferences and goods demand.

**Absence of agency problem.** For tractability, we focus on the matching problem in the main analysis of this paper. Although the literature on agency has emphasized the importance of incentive provision and risk aversion in determining manager pay (for example, see Gayle, Golan, and Miller, 2015; Antón, Ederer, Giné, and Schmalz, 2021), recent analysis that combines matching and agency problems

---

20 Alternatively, we can interpret managers as another group of agents in the economy that is independent of the representative households with the same preferences, which will not change any of our theoretical and empirical results. By excluding managers from the households, we only need to rewrite the budget constraint in household problem (3). But since this budget constraint is always automatically satisfied by the transfer of profits, it is never binding, and hence this change will not influence any equilibrium outcome.
suggests that the matching component plays an important role in explaining the rise of manager pay and its cross-sectional distribution. For example, according to Edmans, Gabalex, and Landier (2009) and Edmans and Gabaix (2011), the rise of manager pay over time cannot be due to the economy-wide increases in risk. Moreover, by decomposing the CEO compensation into the matching component and the incentive provision component, Chade and Eeckhout (2022) find that executives receive an incentive component that is remarkably constant in levels across the distribution of abilities. In Appendix B.4, we solve our baseline model where in addition managers are risk averse and owners face an agency problem, à la Edmans and Gabaix (2011). We find that the incentive payment (such as stock options) that elicits effort and compensates for risk aversion also goes through the market power and firm size channels. Therefore, the assumption to leave out incentive provision, while relevant, does not alter the insights about the changes and inequality in manager pay.

General skill of managers. Following Gabaix and Landier (2008), we model manager ability by a one-dimensional general skill. There are good reasons to believe that managerial experience is industry or even firm-specific. At the same time, there is substantial mobility of managers across firms and industries. In particular, recent studies show an increased importance of general managerial skills over firm-specific human capital (e.g., Murphy and Zabojnik, 2004, 2007; Custódio, Ferreira, and Matos, 2013). Most recently, Dupuy, Kennes, and Lyng (2022) empirically examine a multidimensional matching model and conclude that CEO productivity is not higher in their firms or industries of initial employment, suggesting that firms value general CEO skills rather than industry or firm-specific skills.

Frictionless matching market. The matching market in our setting is frictionless. In the real world there are all kinds of frictions: search, information, learning, ... At the same time, there is substantial turnover among top managers, with the average duration two and a half years, which is shorter than the average job duration economy wide (4.5 years). What frictions would introduce is some notion of mismatch: two-sided search would induce managers and firms to accept a less than ideal match (see for example, Shimer and Smith, 2000), and information frictions would lead to ex-post mismatch upon revelation of the information, even though ex ante there is no mismatch in expectation (see for example, Chade and Eeckhout, 2022). In these examples, the type of mismatch introduces noise around the frictionless allocation. Note that in our model we already have a form of “mismatch” that deviates from the perfect sorting allocation due to random realization of productivities of competitors in a market.

---

21 See Cziraki and Jenter (2022) for evidence of frictions in the market for CEOs.
Absence of monopsony power. For tractability, we also assume the market for manager is perfectly competitive, which is therefore exempted from monopsony power. While excluding monopsony power has a direct impact on manager pay, recent work indicates that quantitatively the impact is small. Deb et al. (2022a,b) jointly estimate markups and markdowns based on the same structural framework from Atkeson and Burstein (2008), and find that while monopsony power exists, it matters significantly less than output market power. They find that markups have risen substantially drastically from 1997 to 2016, while markdowns are invariant over time. If we were to extrapolate those results to the manager market, then we can conclude that (1) monopsony power is quantitatively less important than output market power; and (2) omitting monopsony is likely to lead to little systematic bias explaining the growth of manager pay over time.

3.3 Solution

Stage 2. Production with market power. We solve the model backwards. In stage 2, we solve the canonical Atkeson and Burstein (2008) taking as given the TFP \( A_{ij} \) which depends on the allocation \( \Gamma \) determined in stage 1. The manager’s compensation is sunk, so it does not enter as a choice in this subgame. We first write down the solution to the household problem in Lemma 1 and then solve the firm’s profit maximization problem. Market clearing closes the economy.

Lemma 1 (Household Solution) The solution to the household problem (3) yields:

(a) Goods demand function:

\[
y_{ij} = \frac{1}{\int_I} \left( \frac{p_{ij}}{p_j} \right)^{-\eta} \left( \frac{p_j}{P} \right)^{-\theta} Y, \]

where

\[
p_j := \left[ \frac{1}{\int_I} \sum_{i=1}^{I_j} p_{ij}^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad \text{and} \quad P := \left[ \frac{1}{\int_0^I} p_j^{1-\theta} dj \right]^{\frac{1}{1-\theta}}.
\]

(b) Labor supply function:

\[
L = \bar{\varphi} W^\varphi.
\]

Proof. See Appendix B.1.

We now turn to the firm’s optimal production decision. The profit maximization problem (7) yields the first order condition:

\[
p_{ij}(y_{ij}) \left[ 1 + \frac{d p_{ij} y_{ij}}{dy_{ij} p_{ij}} \frac{dy_{ij}}{dl_{ij}} \right] = W \quad \Leftrightarrow \quad p_{ij} \left( 1 + \frac{\epsilon_{ij} p_j}{p_{ij}^{1\eta}} \right) A_{ij} = W. \quad (8)
\]
The markup $\mu_{ij}$ is defined as the ratio of the output price $p_{ij}$ to the marginal cost $W/A_{ij}$, which is also equal to the inverse of the price elasticity of demand according to equation (8). This is known as the inverse elasticity pricing rule in oligopolistic competition (or Lerner rule). Under the nested CES utility structure, this elasticity, and thus the markup, can be expressed simply by the elasticities of substitution, $\theta$ and $\eta$:

$$
\mu_{ij} = \left[ 1 - \frac{1}{\theta} s_{ij} - \frac{1}{\eta} (1 - s_{ij}) \right]^{-1},
$$

(9)

where $s_{ij} := p_{ij} y_{ij} / (\sum_{i'} p_{i'j} y_{i'j})$ is firm $i$'s sales share in market $j$. Equation (9) suggests that the markups contain the information on the elasticity of substitution within and between markets weighted by sales shares. For example, a monopolist’s markup only depends on the between-market elasticity because it has no competitors in its market. In contrast, a small business has to face fierce competition within its market, which determines its markup.

Finally, market clearing closes the economy. Lemma 2 summarizes the subgame equilibrium.

**Lemma 2 (Subgame Equilibrium)** Given TFP $A_{ij}$, the equilibrium markup is determined by equation (9), which can be further solved from:

$$
s_{ij} = \left( \frac{\mu_{ij}}{A_{ij}} \right)^{1-\eta} \sum_{i'} \left( \frac{\mu_{ij}}{A_{ij}} \right)^{1-\eta},
$$

The equilibrium wage $W$ and output $Y$ are pinned down by:

$$
\frac{W}{P} = \left[ \int_0^1 \sum \left( \frac{p_{ij}}{p_{i'j}} \right)^{-\eta} \frac{1}{p_{i'j}} \frac{1}{p_{i'j}} d_{i'} \right]^{1-\eta}, \quad Y = \left[ \int_0^1 \sum \frac{1}{p_{ij}} p_{ij}^{-\eta} \left( \frac{p_{ij}}{p_{ij}} \right)^{-\theta} d_{i'} \right]^{1-\eta} \bar{p} W^\theta,
$$

where $p_{ij} = \mu_{ij} W / A_{ij}$. Finally, the equilibrium outputs, employment and gross profits are:

$$
y_{ij} = \frac{1}{p_{ij}} \left( \frac{p_{ij}}{p_{i'j}} \right)^{-\eta} \left( \frac{p_{ij}}{p_{ij}} \right)^{-\theta} Y, \quad l_{ij} = \frac{y_{ij}}{A_{ij}}, \quad \text{and} \quad \tilde{\pi}_{ij} = (\mu_{ij} - 1) W l_{ij}.
$$

(10)

**Proof.** See Appendix B.2 for derivation and more intuition. ■

**Stage 1. Matching Managers to Firms.** Anticipating the gross profits in stage 2, firms compete for managers in a frictionless matching market. We define a stable match in Definition 1.

**Definition 1 (Stablility)** A match is stable if and only if, for any two firms $ij$ and $i'j'$, the total gross profits $\tilde{\pi}_{ij} + \tilde{\pi}_{i'j'}$ cannot be improved by swapping managers.
If this condition is not satisfied for two firms, then both firms can be made better off by matching and redistributing the surplus. Furthermore, the complementarity between manager ability and firm type assumed in the technology (4) indicates that the matching output, \( \tilde{\pi}_{ij} \), is supermodular. In a classical matching model, supermodularity is sufficient for positive assortative matching (PAM) (see for example, Becker, 1973; Chade et al., 2017), but in the presence of the externality from imperfect competition, here this is no longer the case — the profitability of a firm also depends on the TFP of its competitors. Consequently, we cannot explicitly find the stable match and have to rely on a computational algorithm to find it.\(^\text{22}\)

Given the stable matching \( \Gamma \), the firms’ stage 1 optimization problem (6) yields the FOC:

\[
\frac{\partial \tilde{\pi}_{ij}}{\partial A_{ij}} \frac{\partial A_{ij}}{\partial x_{ij}} = \frac{\partial}{\partial x_{ij}} \omega(x_{ij}),
\]

which requires the marginal benefit of hiring a higher ability manager (LHS) equals the marginal cost (RHS). It is instructive to point out that equation (11), joint with the gross profits (10), permit us to decompose the managers’ marginal contribution to gross profits.

**Proposition 1 (Marginal Contribution of Managerial Ability)** The marginal contribution to gross profits of managerial ability can be decomposed as follows:

\[
\frac{\partial \tilde{\pi}_{ij}}{\partial x_{ij}} = \left[ \frac{\partial \mu_{ij}}{\partial A_{ij}} Wl_{ij} + (\mu_{ij} - 1) W \frac{\partial l_{ij}}{\partial A_{ij}} \right] \frac{\partial A_{ij}}{\partial x_{ij}}. \tag{12}
\]

The marginal contribution of managerial ability to gross profits consists of the market power channel which increases the markups given output, and the firm size channel which in general increases output given markups. Another way to interpret this market power channel is through the pass-through rate. Basically, a better manager helps the firm reduces its marginal production cost \( W/A_{ij} \). However, the incomplete pass-through suggests that prices will not decline as fast as the marginal cost, which creates a larger markup and therefore leads to an increase in profitability. The term \( \partial \mu_{ij}/\partial A_{ij} \) in the market power channel captures this exact idea of the pass-through rate.

Note also that Proposition 1 relies on the specific way we write the gross profits in equation (10), i.e., \( \tilde{\pi}_{ij} = (\mu_{ij} - 1) Wl_{ij} \). The underlying thought experiment here is to think of profits as the product of two parts: (1) the marginal profit per dollar of inputs, that is, markup \( \mu_{ij} - 1 \); and (2) the total amount

\(^\text{22}\)The stable match is not necessarily efficient either, as firms fail to internalize this externality when making their matching decisions. In addition, in the presence of externalities, the stable matching may be mixed and there may be multiple stable equilibria. For further theoretical results, see as Chade and Eeckhout (2020).
of variable inputs, $W_{ij}$. Note also that the gross profits are the product of Lerner index, which is the marginal profit per dollar of revenue, and total revenue. Since revenues contain information about prices (and hence market power), the latter will quantitatively overstate the importance of firm size channel by looking at the marginal effect of managerial ability on revenues.

Since the marginal contribution of managers to gross profits can be decomposed, then under the frictionless matching assumptions we made for the manager market, manager pay can similarly be decomposed in Proposition 2 by solving the differential equation (12).

**Proposition 2 (Manager Pay)** Given stable matching $\Gamma$, the executive salary schedule $\omega(x)$ satisfies:

$$
\omega(x_{ij}) = \omega_0 + \int_{x_{ij}}^{\infty} \left[ \frac{\partial \mu_{i'j'}}{\partial A_{i'j'}} W_{i'j'} + (\mu_{i'j'} - 1) W \frac{\partial l_{i'j'}}{\partial A_{i'j'}} \right] \times \left[ \frac{\partial A_{i'j'}}{\partial x_{i'j'}} \right] dx_{i'j'},
$$

where $\omega_0$ is the reservation utility that determines the wage for the lowest-type manager.

Proposition 2 provides insights into the properties of executive compensation in this model. First, $\omega$ is increasing in $x$, since $\partial \pi_{ij} / \partial A_{ij} > 0$. Second, when $\alpha$ increases, manager pay (net of the reservation utility) increases proportionally. On the other hand, when managers are more complementary to firms, the salary schedule can become either steeper or flatter, depending on the distribution of the types of firms and managers. Finally, Proposition 2 suggests that manager pay can be decomposed into two separate channels: the market power channel and the firm size channel, which comes directly from the gross profits equation (12). The first channel shows that high-ability managers are valuable because they allow firms to exert greater market power and hence earn higher gross profit. The second effect is consistent with the conventional wisdom about firm size, that a firm can adjust its production decision to make more profit when it is more productive due to the manager ability.

We can also learn about the stable matching from Proposition 2. The match surplus is generally increasing in $z_{ij}$ and $A_j$, though not always due to the externalities from competition in the market. The same firm type $z_{ij}$ will make lower (higher) profits if all competitors $z_{-ij}$ are high (low) types. In the absence of those externalities, the matching pattern of managers $x$ to pairs $(z_{ij}, A_j)$ is illustrated in Figure 4. High type managers match with high $z_{ij}$ firms in high $A_j$ markets. But there is a trade-off as managers get the same wage for pairs with (low $z_{ij}$, high $A_j$) and (high $z_{ij}$, low $A_j$). This results in indifference maps that correspond to iso-wage curves for the manager. Given the match surplus (gross

---

23This statement depends on the assumption of constant return to scale in production, which ensures the marginal profit per cost is identical to the average profit per cost.

24For this reason, we focus on the decomposition (12) in the main body of this paper. In Appendix D.1, we show that our results are empirically robust between these two different interpretations.
Figure 4: Matching of Managers to firm-market pairs \((z_{ij}, A_j)\) with iso-wage curves

profits) is complementary in \(x\) and \((z_{ij}, A_j)\), those indifference curves are ordered in the equilibrium matching from high \(x\) to low \(x\) as illustrated in the Figure. When there are externalities, these indifference maps are “noisy” in the sense that they depend on the realization of productivities in a given market. In our quantitative analysis in Section 4.6, we plot the kernel of those indifference maps derived in the presence of externalities and confirm that high ability managers are more likely to match with high-type firms in both \(z_{ij}\) and \(A_j\).

### 3.4 Determinants of manager pay

To understand the determinants of manager pay, we first investigate the market power channel, that is, how managers influence the firms’ gross profits through markups. Using the implicit function theorem on the FOC (8), we can derive the markup elasticity of TFP:

\[
\varepsilon_{ij}^\mu := \frac{\partial \mu_{ij}}{\partial A_{ij}} \frac{A_{ij}}{\mu_{ij}} = \left[ \frac{(\eta - 1) (1 - \phi_{ij})}{1 + (\eta - 1) \left( \frac{1}{\theta} - \frac{1}{\eta} \right) \mu_{ij} s_{ij}} \right] \times \left[ \frac{1 - \frac{1}{\eta}}{1 + (\eta - 1) \left( \frac{1}{\theta} - \frac{1}{\eta} \right) \mu_{ij} s_{ij}} \right] \in [0, 1), \tag{13}
\]

where

\[
\phi_{ij} := \left[ \frac{s_{ij}}{1 + (\eta - 1) \left( \frac{1}{\theta} - \frac{1}{\eta} \right) \mu_{ij} s_{ij}} \right] \left/ \sum_{j'} \frac{s_{j'i}}{1 + (\eta - 1) \left( \frac{1}{\theta} - \frac{1}{\eta} \right) \mu_{j'i} s_{j'i}} \right.
\]

is a weight that measures the relative importance of the firm \(i\) in the market \(j\). The way we write this elasticity indicates that the impact of higher TFP can be decoupled into two components: (1) higher TFP leads to a higher share of sales; and (2) a higher share leads to a higher markup. Note that the first part is decreasing in \(A_{ij}\) because it is harder to make a giant firm bigger because of the CES demand structure. On the other hand, the second term is increasing in \(A_{ij}\) due to the convexity of the markup expression.
Thus, although higher TFP always contributes to a higher markup, the size of the markup elasticity depends on the trade-off between these two opposing effects.

Similarly, we can write the firm size channel as:

$$
\varepsilon'_{ij} := \frac{\partial l_{ij}}{\partial A_{ij}} \frac{A_{ij}}{l_{ij}} = \phi_{ij} \left[ \theta - 1 \right] + (1 - \phi_{ij}) \left[ \frac{\eta}{1 + \left( \frac{1}{\theta} - \frac{1}{\eta} \right) (\eta - 1) \mu_{ij}s_{ij} - 1} \right].
$$

which can be viewed as the $\phi_{ij}$-weighted sum of the monopolist’s elasticity, $\theta - 1$, and a term measuring strategic interaction. The first part is positive, which means that a monopolist will hire more labor when its TFP increases. In this case, only $\theta$ enters the elasticity because there is no competition within the market. The second term comes from the strategic interaction, which is decreasing in $A_{ij}$. For a small firm, better technology motivates it to grow so it can have a bigger share and exert a higher markup. However, strategic interaction makes a large firm less willing to produce because it is too expensive to raise shares due to the CES demand structure. The net effect of TFP on firm size depends on the trade-off between the monopolistic and the strategic interaction parts.

**Proposition 3 (Elasticities of TFP)** The markup and firm size elasticities of TFP are given by equation (13) and (14), respectively. They have following properties:

1. The markup elasticity first increases with sales share, then decreases, with

$$
\lim_{s_{ij} \to 0} \varepsilon''_{ij} = \lim_{s_{ij} \to 1} \varepsilon''_{ij} = 0;
$$

2. The firm size elasticity first decreases with sales share, then increases, with

$$
\lim_{s_{ij} \to 0} \varepsilon'_{ij} = \eta - 1 > 0 \quad \text{and} \quad \lim_{s_{ij} \to 1} \varepsilon'_{ij} = \theta - 1 > 0.
$$

In addition, $\varepsilon'_{ij}$ can be negative when $s_{ij}$ is moderately large.

**Proof.** See Appendix B.3 for the proof as well as an example under duopoly.  

We summarize the important properties of these elasticities in Proposition 3. Depending on the relative firm size within a market $s_{ij}$, a firm’s markup and employment will react differently to a TFP increase. Intuitively, for a small firm, the increase in gross profits is mainly due to the increase in employment, while for a large (but not monopolist) firm, the markup becomes the dominating channel that contributes to gross profits. Therefore, Proposition 3 suggests a heterogeneity of the markup and firm
size effects among managers who match with different sizes of firms, which we will further elaborate in the empirical part.

4 Quantitative Exercise

We quantify the model year by year using Simulated Method of Moments in this part. Section 4.1 documents the strategy we implement to solve the matching problem. We further map our theory to the data by generalizing the production function in Section 4.2. In Section 4.3, we parametrize the model. The targeted moments are presented in Section 4.4, based on which we estimate the parameters in Section 4.5. Finally, we investigate some key properties of the matching equilibrium in Section 4.6.

4.1 Matching algorithm

In the presence of externalities, finding the stable matching equilibrium defined in Definition 1 is a problem that is known to require non-polynomial time. To verify stability, we have to check the condition for all pairs of firms in the economy. This verification grows exponentially with the number of firms in the economy. As such, for the large setting that we consider, there is no hope to find the exact solution for the stable matching.

In order to solve for the equilibrium matching, we use an algorithm that yields an approximate stable matching. Our algorithm uses a proxy for positive sorting between manager types and firm conditional profitability. The firm type now is no longer a sufficient statistic of the ranking of firms because the profitability of a firm also depends on its competitors’ types. Instead, we construct the ranking by assigning all firms with the average manager and calculating the marginal product of the manager to each firm. This approach gives us a good proxy for firm conditional profitability taking into account the externality across firms, based on which we construct a PAM thanks to the complementarity between firms and managers. Specifically, we follow these steps:

(a) Compute the marginal contribution of the manager ability on gross profits for each firm, assuming all firms are matched with the average manager \(\bar{x}: \frac{d\bar{\pi}_{ij}}{dx_{ij}}|_{\bar{x}}\).

(b) Construct the PAM allocation between the manager types \(x\) and firm’s conditional profitability, \(\frac{d\bar{\pi}_{ij}}{dx_{ij}}|_{\bar{x}}\). That is, a high-type manager matches the firm with high \(\frac{d\bar{\pi}_{ij}}{dx_{ij}}|_{\bar{x}}\).

In Appendix C.1 we verify the efficiency for a smaller sample with 200 markets where we can calculate the equilibrium allocation using brute force and show that our approximate stable matching obtained with our algorithm comes very close to the exact stable matching. We further show that this
finding is robust over different $J$, which ensures that we can generalize this verification to the large economy we consider here.

4.2 Production function

In the quantitative exercise, we use the CES specification for the TFP function (4):

$$A_{ij} = A_j \left[ \alpha x_{ij}^\gamma + (1 - \alpha) z_{ij}^\gamma \right]^{1/\gamma}. \quad (15)$$

Both the manager ability $x_{ij}$ and the firm type $z_{ij}$ determine the TFP of the firm, while $A_j$ is a market-level Hicks-neutral technology. The share $\alpha$ measures the importance of the manager relative to the firm type. The expression (15) allows for a CES functional form where $\gamma$ is the constant elasticity of substitution between manager ability and firm type. For example, when $\gamma < 1$, managers and firms become complementary. This CES form allows for a flexible specification of the TFP technology. When $\gamma = 0$, the expression (15) is the Cobb-Douglas function similar to Gabaix and Landier (2008). It turns out that this flexible CES setup plays an important role in matching the model to the data.

Furthermore, in order to reconcile our technology with the data, where we observe intermediate inputs and capital, we follow De Loecker et al. (2021) and extend our production function into a more general form:

$$y_{ij} = A_{ij} (l_{ij} + m_{ij})^z k_{ij}^{1-z}. \quad (16)$$

We assume that the material $m_{ij}$ is perfectly substitutable with labor, which allows us to estimate the production function without knowing the prices of the materials. The capital $k_{ij}$ in a standard Cobb-Douglas way. Furthermore, for tractability, we set the supply of capital and materials exogenously. Capital supply is assumed to be inelastic at the price $R$. Because materials are perfectly substitutable with labor, we do not explicitly specify its supply, but instead assume that it can be automatically adjusted so that the material share, $m_{ij} / (l_{ij} + m_{ij})$, is equal to an estimated parameter $\psi$ at equilibrium.

Lemma 3 The production function (16) can be equivalently expressed by a labor-only production function:

$$y_{ij} = \tilde{A}_{ij}l_{ij}, \quad \text{where} \quad \tilde{A}_{ij} := \frac{1}{\psi} \left[ \frac{W/\zeta}{R/(1-\zeta)} \right]^{1-\zeta} A_{ij} \quad \text{and} \quad m_{ij} = \frac{1}{\psi \zeta} \frac{W}{\tilde{A}_{ij}}.$$
The decomposition of manager pay can be therefore written as:

\[
\omega(x_{ij}) = \omega_0 + \frac{1}{\psi \zeta} \int_x x_{ij} \left[ \frac{\partial}{\partial A_{ij}} (\mu_{ij} + (\mu_{ij} - 1) W \frac{\partial}{\partial A_{ij}}) \right] \times \left[ \frac{\partial}{\partial x_{ij}} \right] dx_{ij}.
\]

(17)

**Proof.** See Appendix B.5. ■

As each single firm cannot influence aggregate wage \( W \), it will take the input-adjusted TFP, \( \hat{A}_{ij} \), as given. In the marginal cost expression, the term \( W / \hat{A}_{ij} \) is the marginal cost of labor, while \( 1 / \psi \) and \( 1 / \zeta \) adjust for the cost share of materials and capital, respectively. The manager pay (17) is therefore also scaled by these two inputs share. Lemma 3 demonstrates that there is a one-to-one mapping between this general production function (16) and the labor-only production function that our theory is built on. Therefore, this general model shares the same insights and can be solved in the same way as the simplified model in Section 3.

### 4.3 Parametrization

We assume that \( F(x_{ij}), G(z_{ij}), \) and \( H(A_j) \) are independent and lognormal. This rules out any negative realizations and has been shown to be consistent with the productivity distribution in the data.\(^ {25} \) Furthermore, as we will endogenously estimate \( \{\alpha, \gamma\} \), we are unable to distinguish between \( F(x_{ij}) \) and \( G(z_{ij}) \). Being aware that the distribution of manager ability should be relatively stable over time, we normalize its distribution throughout the quantitative exercise to \( \log x_{ij} \sim N(-0.5, 1) \) such that the mean of \( x_{ij} \) is 1. Moreover, we assume that the mean of \( z_{ij} \) is also normalized to 1. Its standard deviation \( \sigma_z \) will determine the lognormal distribution of \( z_{ij} \). The market component \( A_j \) therefore captures the TFP level of firms, whose distribution is determined by its mean and standard deviation, \( m_A \) and \( \sigma_A \).

To mimic the continuum of markets in the simulation, we set the number of markets equal to \( J = 10,000 \).\(^ {26} \) Furthermore, we assume that the number of firms in each market, \( I_j \), is random to capture the heterogeneity across markets that we see in the data: \( I_j \) is an integer drawn exogenously from a truncated normal distribution \( \mathcal{N}(m_I, \sigma_I^2) \) within the range \([1, 10]\).\(^ {27} \)

---

\(^{25}\)For example, using LBD data, Deb et al. (2022a) back out the firm-level productivity distribution which is close to lognormal.

\(^{26}\)Since we have neutralized the love of variety effect, a change in the number of markets does not make a systematic difference in our model. Our model is converging to the continuous case when \( J \to +\infty \).

\(^{27}\)Specifically, we first draw a number from the normal distribution within the range \([0, 10]\), then round it to the nearest integer greater than or equal to that number. The assumption that the distribution of \( I_j \) is truncated normal is not crucial to our analysis. We have also done the analysis with the log-normal distribution and the beta distribution, both of which give us robust results. Finally, the choice of the upper bound of the truncation comes from De Loecker et al. (2021), whose estimates for the number of potential entrants in each market is less than ten over this period. Our estimates in Section 4.5 show that the upper bound is slack and therefore not crucial.
Table 2: Endogenous, estimated parameters (time-varying)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Match</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>The importance of manager relative to firm type</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>The elasticity of substitutes between manager and firm type</td>
</tr>
<tr>
<td>II. Market</td>
<td></td>
</tr>
<tr>
<td>$m_I$</td>
<td>Market structure $I_j \sim \mathcal{N} \left( m_I, \sigma^2_I \right)$</td>
</tr>
<tr>
<td>$\sigma_I$</td>
<td>$I_j \in \mathbb{N}_+ \cap [1, 10]$</td>
</tr>
<tr>
<td>III. Firm</td>
<td></td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>Standard deviation of firm type $z_{ij}$</td>
</tr>
<tr>
<td>$m_A$</td>
<td>Mean of market-level productivity $A_j$</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>Standard deviation of market-level productivity $A_j$</td>
</tr>
<tr>
<td>IV. Aggregates</td>
<td></td>
</tr>
<tr>
<td>$\bar{\varphi}$</td>
<td>Aggregate labor supply level</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Factor share: labor in labor + material, or material supply</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>Reservation utility of managers</td>
</tr>
</tbody>
</table>

Table 3: Exogenous parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>Within-sector elasticity of demand</td>
<td>5.75</td>
<td>De Loecker et al. (2021)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Between-sector elasticity of demand</td>
<td>1.20</td>
<td>De Loecker et al. (2021)</td>
</tr>
<tr>
<td>$R$</td>
<td>User cost of capital</td>
<td>1.16</td>
<td>De Loecker et al. (2021)</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Labor supply elasticity</td>
<td>0.25</td>
<td>Chetty et al. (2011)</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Factor share: labor + material in variable cost</td>
<td>0.88</td>
<td>Compustat data</td>
</tr>
</tbody>
</table>

To summarize, Table 2 lists the endogenous parameters that we estimate. They are organized in four categories: I. Match; II. Market; III. Firm; IV. Aggregates. In addition, we take some exogenous parameters from the literature or calculate them directly from the data. Those are listed in Table 3. On the goods demand side, we take the elasticities of substitution, $\eta$ and $\theta$, from De Loecker et al. (2021) who quantify a model with a similar demand side, and we also use their user cost of capital $R$.\textsuperscript{28} We obtain the elasticity of labor supply, $\varphi$, from the meta study Chetty, Guren, Manoli, and Weber (2011), and we will estimate the intercept $\bar{\varphi}$ to match the level of employment of the model with the data. Given the Cobb-Douglas specification (16), the elasticity $\zeta$ is equal to the input share at equilibrium and is quite stable across years, so we compute it directly from the Compustat data.

\textsuperscript{28}In section 5.4 and Appendix D.3, we show that our quantitative results are robust for different values of $(\theta, \eta)$. All our insights continue to hold when we use $\theta = 1.5$ and $\eta = 10.0$ from Atkeson and Burstein (2008).
Table 4: Targeted Moments

<table>
<thead>
<tr>
<th>Category</th>
<th>Moment</th>
<th>Key Parameter(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Match</td>
<td>Average salary share</td>
<td>$\alpha$</td>
</tr>
<tr>
<td></td>
<td>Slope salary share-sales</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>II. Market</td>
<td>Average markup</td>
<td>$m_I$</td>
</tr>
<tr>
<td></td>
<td>Variance markup (between)</td>
<td>$\sigma_I$</td>
</tr>
<tr>
<td>III. Firm</td>
<td>Variance markup (within)</td>
<td>$\sigma_z$</td>
</tr>
<tr>
<td></td>
<td>Average worker’s wage</td>
<td>$m_A$</td>
</tr>
<tr>
<td></td>
<td>Variance sales</td>
<td>$\sigma_A$</td>
</tr>
<tr>
<td>IV. Aggregates</td>
<td>Average employment</td>
<td>$\bar{\sigma}$</td>
</tr>
<tr>
<td></td>
<td>Average manager salary</td>
<td>$\psi$</td>
</tr>
<tr>
<td></td>
<td>Manager salary, bot. 1%</td>
<td>$\omega_0$</td>
</tr>
</tbody>
</table>

Notes: We base all our moments on the data discussed in Section 2. For the construction of empirical moments, we take the direct observations of revenues, employment and CEO compensation from the data. We estimate markups using the production approach. Unlike the model, in the data there is not a single wage $W$ for the production workers, both within and between firms, so $\mathbb{E}(W)$ denotes the average wage across all production workers. An industry (market) is defined as four-digit NAICS code.

4.4 Identification

To capture the evolution of executive compensation and markups, we estimate the set of parameters listed in Table 2 that best matches the key moments of the data. We estimate the model annually: because the model is static, the estimates in different years are completely independent. We identify the endogenous parameters by minimizing the objective function:

$$
\min_{\theta} \mathcal{G}(\theta) := \left( \hat{M} - M(\theta) \right) \mathcal{W}^{-1} \left( \hat{M} - M(\theta) \right), \quad \theta := \{ \alpha, \gamma, m_I, \sigma_I, \sigma_z, m_A, \sigma_A, \bar{\sigma}, \psi, \omega_0 \}, \quad (18)
$$

where $\hat{M}$ is a vector of data moments and $M(\theta)$ are their model counterpart given a set of parameters $\theta$. The matrix $\mathcal{W}$ is the inverse of the variance-covariance matrix of the data moments.\(^{29}\)

Table 4 lists the 10 moments that we target. The targeted moments, like the parameters, can be categorized into the same four groups, those corresponding to the matching, to the market, to the firm, and to the aggregates. While all parameters affect all moments in this general equilibrium model, in the table we also list the corresponding key parameter that affects each of the moments most directly. Next, we motivate our choice of the targeted moments. We also refer further to Appendix C.3, where we report the comparative statics prediction of how the parameters affect the selected model moments.

\(^{29}\)Because our model is exactly identified, the choice of the weighting matrix is not crucial.
Notes: We plot log sales ($\log r_{ij}$) on the x-axis and log salary shares ($\log \chi_{ij}$) on the y-axis. Panel A shows the negative correlation in the data with 1144 observations in 2019. In Panel B and C, points with different colors represent for firms in different economy. As there are a larger number of CEOs in our model each year, we randomly select 500 representatives of them in each economy to plot. The baseline parameter is the estimates in 2019.

I. Match. We motivate our choice of moments on the matching side by showing how executive compensations are determined by $\{\alpha, \gamma\}$. Notice that manager ability $x_{ij}$ influences gross profits exclusively through TFP $A_{ij}$. The expression below, which comes from the CES technology function (15), further gives us an intuitive way to understand the payoff share of managers:

$$\frac{\partial A_{ij}}{\partial x_{ij}} x_{ij} + \frac{\partial A_{ij}}{\partial z_{ij}} z_{ij} = 1,$$

where $\frac{\partial A_{ij}}{\partial x_{ij}} A_{ij} = \alpha \left( \frac{x_{ij}}{A_{ij}} \right)^\gamma$ and $\frac{\partial A_{ij}}{\partial z_{ij}} A_{ij} = (1 - \alpha) \left( \frac{z_{ij}}{A_{ij}} \right)^\gamma$.

Assume for now that there is no reservation utility nor market power. Then in a Cobb-Douglas world (i.e., $\gamma = 0$), the manager share will be constant and equal to $\alpha$, which is commonly assumed in many matching literature (for example, Becker, 1973). The bigger $\alpha$ is, the more managers get. We therefore use the average log share of manager salary out of total sales, which we define as:

$$\chi_{ij} := \frac{\omega_{ij}}{r_{ij}} \quad \text{and} \quad r_{ij} := p_{ij} y_{ij}.$$  

While $\alpha$ pins down the average salary share of the manager, the salary share is not a constant in the data, as is shown in the panel A of Figure 5. This implies the case when $\gamma$ is non-zero. We therefore use the slope of the linear prediction of log $\chi_{ij}$ on log $r_{ij}$ to inform us about the elasticity of substitution, $\gamma$. Panel B and C of Figure 5 reiterates the logic of our choice of parameters by plotting the relationship between log $\chi_{ij}$ and log $r_{ij}$ when each of the parameters $\{\alpha, \gamma\}$ change.

II. Market. Equation (9) indicates a systematic relationship between the average markups and the number of firms in each market. In a representative economy where firms are identical, Figure 6a shows
Figure 6: Identification of parameters in category “II. Market”

Notes: This figure shows the determinant of markups in a representative economy where firms have the same TFP. From Equation (9), we have: $\mu_{ij} = \left[ 1 - \frac{1}{2} \right]^{-1} \left( \frac{1}{2} \right) \frac{1}{1} = \frac{1}{2}$ that is declining in $I_j$. The two panels demonstrate how the markup will respond when $m_I$ and $\sigma_I$ decline (from blue dots to red crosses), respectively.

that markups will increase monotonically as $I_j$ decreases, which helps us identify the average number of firms $m_I$. Furthermore, because the number of firms differs in different markets, this monotonicity also makes the distribution of markups across markets informative on $\sigma_I$. Figure 6b illustrates that, when $I_j$ gets less dispersed, market-level markups $\mu_j$ also become more concentrated. Therefore, we will exploit the between-market variance of markups to identify $\sigma_I$.

III. Firm. The variance of markups within each market $\mu_{ij}$ is in turn determined by the variance in firm type $\sigma_z$. As Figure 7a shows, a smaller $\sigma_z$ will reduce the difference in $s_{ij}$, which eventually reduces the within-market variance of markups according to Equation (9). On the other hand, the panel B shows that the level of $A_j$ influences the marginal revenue product of labor (MRPL), which shifts the labor demand function and eventually determines worker’s wage. Finally, as the revenue is monotonically increasing over productivity, less dispersion in $A_j$ leads to smaller variance in revenue, which becomes a good target for us to identify $\sigma_A$. This idea is shown in Figure 7c.

IV. Aggregates. Finally, we want to match the level of the variables in our model economy to the data. Specifically, we will use three aggregate parameters, $\{\varphi, \psi, \omega_0\}$, to match the firm-level average employment, and average executive compensation. In the model, $\varphi$ is the intercept of the labor supply (in log) that can match the employment level, and $\psi$ adjusts the level of the manager’s compensation. Given any set of parameters in the first three categories, $\{\varphi, \psi\}$ can be uniquely pinned down by the

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30 Some readers may think of using the information on the number of firms from the dataset instead of estimating it. However, the market definition in the data is kind of ambiguous. For example, a coffee house in New York does not compete with the one in California even if they have the same industry code.

31 Recall that Lemma 2 shows that the within-market distribution of markups is uniquely determined by the TFP.
4.5 Estimation

We estimate the ten endogenous parameters jointly, in a separate estimation each year. Figure 8 shows the estimated parameters and how they evolve over time. To further validate our estimation, in Figure 14 of section 5.1 we plot the manager pay distribution generated by the model, which matches the data distribution remarkably well in all years, even though we do not directly target the pay distribution.

I. Match. We first report the parameters that correspond to the match, \( \{\alpha, \gamma\} \), in the first column of Figure 8. Estimates of \( \alpha \), which measure the relative importance of managers, are minuscule around 0.001 all the time. This observation is consistent with Gabaix and Landier (2008), who show that managers make only a small difference on firms. However, our interpretations differ. In their setting, the small impact managers make stems from the fact that the dispersion in manager talent is very low. Instead, we show that the impact of managers is small due to the way the manager ability enters the production function, i.e., through the parameter \( \alpha \). Moreover, we find that \( \alpha \) is generally increasing over time, suggesting that managers play an increasingly important role.

The estimated elasticity of substitution \( \gamma \) is negative, which confirms the complementarity between firms and managers that is commonly assumed in the literature. Furthermore, \( \gamma \) was relatively stable, and then sharply declined from \( -2.22 \) in 2014 to \( -3.55 \) in 2019. This trend corresponds to the increasingly negative correlation between salary share and sales that is shown in Panel I-B of Figure C.3 after

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32 See Appendix C.4 for more details.
33 In Figure C.3 in Appendix Appendix C.2 we report the close fit of the model moments to those in the data.
II. Market. The second column in Figure 8 reports the estimated parameters $\{m_I, \sigma_I\}$ in the category of market from 1994 to 2019. Consistent with the rise of market power, we see an increasingly concentrated market structure over time from two perspectives. First, the average number of firms in each market steadily declines from 4.40 to 3.15, suggesting that there is less competition overall. Second, the dispersion in the number of competitors $\sigma_I$ is also decreasing, from 1.56 to 1.16. As a result, most markets will have a concentrated market structure and there are fewer markets that tend towards being competitive. This finding confirms the results in the literature that document the increase in concentration (Grullon, Larkin, and Michaely, 2016; Gutiérrez and Philippon, 2017; De Loecker, Eeckhout, and Mongey, 2021).

III. Firm. The results of our estimation also suggest an increasing dispersion in firm type. Panel III-A of Figure 8 shows that its standard deviation $\sigma_z$ increases from 0.51 to 0.77. This change mainly contributes to the increase in the variance of the log markups within and between markets. The same trend is documented in other literature as well (see for example, De Loecker et al., 2021; Deb et al., 2022a). One implication of this quantitative result is the rise of superstar firms, which is consistent with the findings of Autor, Dorn, Katz, Patterson, and Van Reenen (2020). Moreover, we also find that the managers have recently become more complementary to firms.
average annual production worker wage is slightly decreasing in this sample, from $65.8K to $59.3K, which is consistent with the real wage stagnation of production workers in the economy. The overall decline in $m_A$ matches this trend. We also document a huge difference across markets $\sigma_A$, which comes from the huge variance in sales and is within our expectation. Aggregating across widely different sectors implies there are huge productivity differences, say between labor-intensive sectors such as retail and sectors such as biotech.

IV. Aggregates. In order for our model to match the levels in the data, we scale the economy by aggregated parameters $\{\bar{\varphi}, \psi, \omega_0\}$. Column IV in Figure C.3 shows that the average number of production workers grows from 16.9 thousand to 25.1 thousand over time, which is much larger than the average firm size in the entire economy. This of course stems from the selection on large firms in the pool of publicly traded firms. The average executive compensation doubles from $3.34$ million to $6.96$ million, while the lowest manager pay is almost flat. The same trends show up in the aggregates in Figure 8. Note that we actually treat $\psi$ as a residual term that matches the manager pay from the model to the data. This term is quite stable and is within a reasonable range except for during the Great Recession, which indicates that our model predicts the evolution of manager pay over time quite well.

4.6 Matching

In this section, we analyze the properties of the equilibrium match in our estimated economy. Understanding these results is essential for interpreting executive compensation. All results are robust in different years, so we will take the year 2019 as the baseline in presenting the crosssectional results.

Figure 9 shows how managers are matched with firms. Panel A reports the manager’s iso-wage curves, which are consistent with the theoretical prediction in Figure 4. Basically, higher-type managers can earn more by working for firms with higher type $z_{ij}$ and $A_j$. Panel B to D support these insights by showing the correlation between managers’ type on the one hand, and firm type $z_{ij}$, market productivity $A_j$, and markups $\mu_{ij}$. Because there are multiple dimensions and because there are externalities, we do not expect to find perfect positive sorting. Still, we expect to find a strong positive correlation. On all three dimensions, better managers tend to match with more productive firms, they are in more productive markets, and they match with higher-markup firms. Moreover, consistent with the data, we observe that manager ability is more closely correlated to firm type than the market productivity, which suggests that managers are mainly hired for competition within markets. This phenomenon is increasingly significant over time as the correlation within markets increases from 0.81 in 1994 to 0.95
in 2019.\textsuperscript{34} The last panel reinforces this point by showing that better managers are in general hired by firms with larger market power.

In addition, we can also check the relationship between the type of CEOs and the elasticity of TFP on markup $\epsilon_{ij}^m$ and on employment $\epsilon_{ij}^l$, which has been discussed in Proposition 3. Figure 10 shows that the markup elasticity generally increases with manager type, which means that a high-type manager will contribute more to the corresponding firm’s profit through the markup. In contrast, the employment elasticity is decreasing in manager type and may even be negative for some high-ability managers. Top managers in top firms hire less labor, which is consistent with the lower labor shares in superstar firms.\textsuperscript{35} These different elasticities drive the heterogeneity in salaries between manager types, which is a topic that we will further elaborate on in Section 5.1.

## 5 Main Results

With the estimates of the technology in hand, we can now analyze the different determinants of manager pay, which are summarized in Figure 11. Our main focus is on the contribution to manager pay from two channels: market power and firm size.\textsuperscript{36} In section 5.1, we carry out decomposition exercises of manager pay on both the margin and the level. We then analyze in section 5.2 the contribution of different categories of parameters to manager pay through each of these two channels: 1. $\{\alpha, \gamma\}$ for the

\textsuperscript{34}See Appendix C.5 for a time-series plot of these rank correlation coefficients.

\textsuperscript{35}For example, see Autor, Dorn, Katz, Patterson, and Van Reenen (2020) and De Loecker et al. (2020).

\textsuperscript{36}Throughout this section, we will tease out the reservation utility as it is negligible and flat over time, as is shown in Panel IV-C of Figure 8.
match; 2. \( \{m_I, \sigma_I\} \) for market structure; 3. \( \{\sigma_Z, \mu_A, \sigma_A\} \) for firms; and 4. \( \{\varphi, \psi\} \) for inputs supply. Finally, we investigate a counterfactual economy without market power in section 5.3.

### 5.1 The rise of manager pay: market power vs. firm size

Our theory allows us to decompose equilibrium manager pay into two channels: market power and firm size. We will start our analysis with a decomposition on the margin based on Proposition 1, which requires the minimum level of assumptions. Then, by putting the same structure on the manager market as Gabaix and Landier (2008) and Terviö (2008), we extend our decomposition exercise into the level of manager pay according to Proposition 2. Finally, we look at the cross-sectional distribution and draw conclusions regarding inequality that are consistent with Proposition 3.
The marginal contribution of manager pay. We first find that market power is quantitatively important on the margin of manager pay, and increasingly so. Panel A of Figure 12 reports the time-series decomposition of marginal manager pay, where we yearly attribute the average marginal payment across managers to the market power and the firm size channels. It shows that the average contribution of the market power channel has been steadily increasing, from 36.4% in 1994 to 46.4% in 2019.

![Figure 12: The marginal contribution of market power on manager pay](image)

Notes: This figure reports the decomposition of manager pay at the margin according to Proposition 1. In panel A, we plot the average (across managers) share of market power channel in determining the marginal manager pay for each year, i.e., (markup channel)/(markup channel + firm size channel). The final plot takes five year centered moving average. Panel B plots the cross-sectional distribution of the share of market power in marginal manager pay for 1994 and 2019. We put percentiles of manager ability on the x-axis and plot the kernel median smoother as the solid lines. The shaded part indicates the area between the first and third quartiles of the market power share distribution that conditions on manager ability.

We also see a robust result that top managers benefit more from market power. In panel B of Figure 12, we plot the share of market power channel in marginal manager pay as a function of managerial ability, which is in general increasing. This finding suggests that, on the margin, a higher-ability manager gets rewarded more from the extra market power he or she creates than the larger firm size. Indeed, for the bottom manager, all the marginal salary comes through the firm size channel; contrarily, almost all the top manager’s marginal pay comes from the market power channel.37

By looking at the decomposition on the margin, we build a basic picture on how the two channels shape the manager pay both across time and across managers with different abilities. In order to learn more about those effects on the level, we will rely on the matching structure on the manager market and try to decompose the manager pay level according to Proposition 2.

The level of manager pay. In Figure 13, we plot the contribution to equilibrium manager pay of the market power and firm size channels. Consistent with the results on the marginal contribution

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37For some top managers, the firm size channel may even be negative when the market power share is greater than one, which means their employers have a tendency to reduce size on that margin.
of manager pay, panel A shows that both market power and firm size effects play important roles in determining executive salaries. Over the period, the average manager pay (net of reservation utility) increases from $2.94 million to $6.43 million, where the market power effect increases from $1.12 million to $3.14 million and the firm size effect increases from $1.82 million to $3.30 million. Panel B further shows that market power determines 45.8% of total manager pay on average. Moreover, its importance has been increasing over time, from 38.0% in 1994 to 48.8% in 2019. Correspondingly, the importance of the firm size channel is declining.

In addition, we can also decompose the change of manager pay over time and attribute it to each channel. Panel C shows the cumulative change in average manager pay and its components relative to the baseline year 1994. During this period, the manager pay increases by $3.49 million, of which $2.02 million is due to the increase in market power and $1.47 million due to the firm size effect. Panel D in Figure 13 further shows that 57.8% of the cumulative increase in executive compensation over this period is through the increase in market power, while the remaining 42.2% is through the change in firm size.

**Heterogeneity and inequality among managers.** We now analyze inequality in salaries among managers and how this heterogeneity is determined by each of these two channels. Panel A and B of Figure 14 plot the evolution of manager pay at different percentiles over time, both in the data and as predicted by the model. In the data, we see significant inequality among managers, which is also increasing over time. The pay of the lower-ranked managers is almost flat from 1994 to 2019, while the compensation for high-paid managers increases substantially, including p50, p75 and p90. Panel C and D further present the distribution of manager pay in both data and model. Even though the pay distribution is not a direct target in our estimation, our model captures the characteristic of the changes in distribution...
of manager pay remarkably well. It provides an external validation of the model. The remarkable is testament to the fact that our model, with all its simplifying assumptions, provides an accurate description of the income process of managers.

We now detail how the channels of market power and firm size contribute to the evolution of the distribution. In the theory, Proposition 3 predicts that for small, low TFP firms, the firm size channel dominates the market power channel, while the importance of the market power channel should generally increase in firms’ revenues. To understand the mechanism, we first analyze heterogeneity in the cross-section. In the first two panels of Figure 15, we plot the salary (net of reservation utility) of each percentile of managers, as well as the corresponding decomposition of the effects of market power and firm size in 2019. Panel A shows that the effect of market power channel is more convex than the firm size channel. Furthermore, Panel B demonstrates that for the lowest type of managers, almost all of their salary comes from the firm size effect. The market power channel, by contrast, becomes increasingly important when the manager is more talented due to the overall positive sorting. For the top managers, 80.3% of their salaries is due to market power. This discrepancy contributes to the huge inequality in manager pay.

We now zoom in on the change in manager pay over time. From the time-series results, we learn that manager pay is increasing faster through the market power channel than the firm size channel. Combining with the fact that the impact of the market power channel is larger for the high-ability managers, this result suggests that the larger inequality would be generated among managers across their types. For the same percentile of managers, Panel C and D in Figure 15 plots the difference in

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38 Observe also a peculiar feature of the largest firms in our model. There is a sharp decrease in the firm size effect among the very top managers. This is because the best managers are matched with superstar, but not monopolistic, firms whose employment elasticity of TFP (equation (14)) is negative. This insight is also confirmed by Figure 10 that the employment elasticity is negative for some high-ability managers.
5.2 Factor decomposition

Empowered by our structural model, we can also analyze the contribution of each primitive parameter to manager pay through the channels of market power and firm size. To do this, we keep all parameters fixed at their 1994 values, and then feed in one or more estimated, year-specific parameters, plotting the cumulative changes in the effects of market power and firm size on manager pay. Note that this decomposition is not perfect as we are only checking the stand-alone effects of changing each set of parameters without considering the indirect effect from their interaction with changes in other primitives. Despite this shortcoming, we can still see the direct effect of each parameter.

Market power channel. We first check the impact of the primitive parameters on manager pay through the market power channel. In the panel A of Figure 16, we decompose the gross change in the market power component (i.e., the blue, dashed line from Figure 13.A) into three objects: market structure \( \{m_I, \sigma_I\} \), productivity \( A_{ij} \), and the inputs supply \( \{\overline{\varphi}, \psi\} \). Over the entire time period, the changes in

---

39Once again, the sharp decrease in the effect of firm size for top managers stems from the presence of superstar firms. It suggests that there are more superstars in 2019 than there were in 1994. This observation aligns with the increasing variance in the types of firm \( c_z \).
technology is the dominating factor that raises manager pay through the market power channel, which contributes $1.33 million and account for 65.9% of the total growth. The changes in market structure and inputs alone have tiny effects on this market power component.

We then further decompose the TFP change into the match category \( \{\alpha, \gamma\} \) and the firm type category \( \{\sigma_z, m_A, \sigma_A\} \). Panel B shows that the evolution in match component is the dominating factor that contributes to the TFP change. Specifically, according to the panel C, the increasing importance of managers \( \alpha \) plays an important role that contributes to the growth of market power effect by $1.14 million, while the increasing complementarity becomes more important only most recently and contributes by $0.34 million. Finally, Panel D demonstrates that overall firm type has little influence on the market power channel, but it does have a significant impact during the Great Recession.

**Firm size channel.** We then do the same decomposition on the firm size effect. In Panel A of Figure 17, the baseline is the gross change in the firm size channel, i.e., the green, dotted line in Figure 13.A. It’s decomposition suggests that the change in TFP again is the dominating factor that accounts for 70.1% ($1.03 million) of the increase in firm size channel. The shifts in market structure and inputs supply have almost no influence on the firm size effect.

Moreover, from the subsequent panels, we learn that among TFP change, parameters from the match category has a huge and positive effect. The increase in \( \alpha \) makes managers more important, which directly drives up the manager pay by $2.22 million through the firm size channel. On the other hand, the complementarity \( \gamma \) determines the convexity of the managers’ wage schedule, whose recent change accounts for $2.29 million of increase in the firm size effect. However, the huge effect from
\[ \{ \alpha, \gamma \} \] will be mostly offset by the negative influence from the firm side that is mainly due to the increasing dispersion in firm type, \( \sigma_z \). The change in \( \gamma \) and market productivity \( A_j \) mainly lead to the hump in the TFP effect during the Great Recession.

5.3 Counterfactuals

In this section, we discuss the role of market power by illustrating counterfactual examples. We will first investigate the welfare effect of technology and market power, then further focus on the manager pay that is induced by those counterfactual exercises.

**TFP and market power.** Welfare is determined by many factors, among which we are especially interested in the roles of technology and market power in this paper.\(^{40}\) Panel A in Figure 18 reports the evolution of the aggregated productivity and welfare over time. We see a slight increase in technology overall, which peaks before the Great Recession. This trend is mainly driven by the wage stagnation in our sample (Figure C.3). However, the welfare is generally declining over the same period, as is shown by the pink, solid line. We observe a discrepancy between the increasing technology and declining social welfare, which is mainly attributed to the rise of market power by De Loecker et al. (2021) and Deb et al. (2022b).

We further show that market power indeed leads to a significant welfare loss. In a counterfactual economy where firms are forced to price at their marginal costs, the social welfare (the brown, dotted line in Panel B) can on average increase by 58.4\%.\(^{41}\) Moreover, this welfare loss due to market power is constantly increasing over the sample period. In Figure 18.C, we plot the TFP equivalence \( \lambda_t \) of this welfare loss, that is, firms have to increase their productivity to \( \lambda_t A_{ij} \) in order to realize the same welfare.

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\(^{40}\)We further refer readers to De Loecker et al. (2021) for a complete discussion.

\(^{41}\)The assignment of managers is still determined by the algorithm described in Section 4.1, given that the markups of all firms are \( 1 + \varepsilon \). The \( \varepsilon \) here makes sure that managers are not indifferent among all jobs.
Notes: Panel A plots the time series of the aggregate TFP and welfare in our model. Based on De Loecker et al. (2021), the TFP is defined in CES aggregates:

\[ A = \left[ \left( \int_0^J A^{\theta-1} dj \right)/J \right]^{1/(\theta-1)} \] and

\[ A_j = \left( \sum_i A_j^{\eta-1} / I_j \right)^{1/(\eta-1)}, \]

while welfare is the net utility of households defined in Equation (2). We normalize their value in year 1994 to 100 for both variables. Panel B presents the social welfare in the First Best economy without output market power. Panel C reports the TFP equivalence of our model, which is defined as how much the technology \( A_{ij} \) has to increase to compensate for the welfare loss due to market power.

Manager pay. We assume households are representative in our model, so manager pay plays no role in determining welfare. However, we are still curious about the income inequality among managers in those counterfactual economies. In Figure 19, we plot the evolution of Manager pay as well as the pay schedule (2019) in the baseline economy, the first-best economy, and the economy with a 30% increase in TFP.

Interestingly, when there is no market power (the first-best case), all managers will only earn the reservation utility, which is tiny compared to what they earn in the real world. The intuition is that firms will always earn zero profits no matter how productive they are, which makes a high-ability manager

\[ \times 10^7 A. \text{ Average Pay} \]

\[ \times 10^7 B. \text{ Schedule 2019} \]

as the first best case in year \( t \). It turns out that the welfare effect of market power can be compensated by a 33.8% increase of TFP in 1994, while this number has increased to 51.7% by year 2019. This evolution aligns with the rise of markup in the real economy.
“useless”. Notice that this conclusion also depends on the assumption that there is no incentive problem for managers. We also find that the expansion in firm type alone will lead to an increase in manager pay (the blue, dashed lines), even though managers’ ability is fixed. This finding confirms Gabaix and Landier (2008)’s insights that shifts in firm type account for a part of increase in manager pay over the past few decades.

**The role of managers.** To see the importance of managers more clearly, we run the same counterfactual experiment as Gabaix and Landier (2008), where we keep matching the same but replace the median manager by the best manager. We find that the profit of the firm who originally matches with the median manager will on average increases by 0.19% of profit (compared to 0.016% in Gabaix and Landier (2008)), increasing from 0.08% in 1994 to 0.15% in 2019, indicating that managers are getting more important over time.42 Yet, despite the seemingly small increase of profits as a percentage, the managers’ contribution to profits in levels is still huge, which translates into the high manager pay we observe in the data.

### 5.4 Robustness

**Measuring market power in different ways.** The literature has documented the rise of market power from various perspectives, including the rising markups, the higher market concentration, and the higher profit rates.43 In this paper, we treat markups as the measure of market power and estimate our model matching moments from the markup distribution. Our results are similar when we use the markups estimated by Deb et al. (2022b). They use a structural model instead of the production function approach and Census data for the universe of firms instead of Compustat, which generates a markup distribution that is similar to the one using the production function approach. It is worth noting that, although we mainly focus on the markup distribution during estimation, our estimated model is also able to replicate the rise in market concentration and profit rates. Hence, our framework is consistent with various observations related to market power.

Quantitatively, we can also measure market power by the Lerner index, which is $1 - 1/\text{Markup}$. In Appendix D.1, we replicate our main exercises by decomposing the margin, level and distribution of manager pay into the channel of Lerner index (market power) and revenues (firm size).44 The results

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42 This finding is consistent with the results that CEOs have a significant effect on profitability (e.g., Bennedsen, Pérez-González, and Wolfenzon, 2020).
43 See Syverson (2019), De Loecker et al. (2020), Bond, Hashemi, Kaplan, and Zoch (2021), and De Ridder, Grassi, and Morzenti (2022) for discussions on methods including production function estimation, the use of concentration measures, structural estimation,...
44 The detailed decomposition expression is documented in Appendix D.1 as well.
using the Lerner index show trivially that the results are identical to those using the markup. After all, the Lerner index is a monotone transformation of the markup. We also include in the Appendix the outcome of an exercise where we naively – though wrongly – interpret the impact of the Lerner index as being different from the impact of the markup.

**Bertrand competition.** An alternative way to model oligopolistic competition is to let firms set prices (Bertrand) instead of quantities (Cournot). In Appendix D.2, we replicate our main results under Bertrand competition and demonstrate the robustness. Indeed, we see that market power becomes more important across time, whose contribution to the manager pay margin (level) has been increasing from 26.5% (27.1%) in 1994 to 53.4% (56.8%) in 2019. The cross-sectional decomposition of manager pay distribution shows the same pattern where the top managers’ pay is determined predominantly by the market power channel.

**Elasticities of demand.** In our main analysis, we calibrate the parameters that determine the demand elasticity \((\theta, \eta) = (1.20, 5.75)\), based on the estimates from De Loecker et al. (2021). In Appendix D.3, we redo our quantification exercise with the elasticity \((\theta, \eta) = (1.50, 10.00)\), used by Atkeson and Burstein (2008) and show that our results are qualitatively robust over the different sets of elasticity of demand. We find that market power contributes to 43.4% of manager pay in 1994 and 51.5% in 2019, and these demand parameters also generates a similar pattern across managers. This is not a surprise because as long as competition is imperfect, market power will play a role in determining manager pay irrespective of the demand.

6 Conclusion

Market power in the goods market distorts the efficient allocation of resources. In this paper, we have shown that market power also distorts manager pay, as managers are paid in part for market power, due to the sorting of managers into top firms. Without market power, superstar managers would earn less. Currently, managers are paid to create profits, and profits derive from higher value of the firm as well as more market power. Better managers grow the firm which increases value, but they also increase market power, which is inefficient.

The main insight of this paper is to decompose the contribution of market power to manager pay as distinct from firm size. We estimate the model using Compustat data on executive compensation which allows us to quantify the contribution of market power. On average, 45.8% of pay is due to market power, growing from 38.0% in 1994 to 48.8% in 2019. Market power accounts for 57.8% of growth over
this period. Most striking is the large heterogeneity among managers. For the top managers, 80.3% of their pay is due to market power. The growth of their pay due to market power is even larger. The best managers are lured by large, high markup firms where they create high profits for the shareholders, but disproportionately little additional value to the economy due to the incomplete pass-through.

The mechanism that we identify and that is behind the rewards these managers receive crucially hinges on the competitive pressures within a market. In the presence of imperfect competition, the most productive firms extract higher rents than the less productive firms. Because of the complementarity between manager ability and firm productivity, the most productive firms can widen the gap even more by hiring a highly skilled manager. This increases their markups even further. The less productive firms have low markups and hence have little to gain from hiring a superstar manager. Because there is competition for managers, all top firms in their own market who benefit from having a top manager will bid up the top wages. Managers at top firms are paid predominantly for increasing the gap between their direct competitors.

The mechanism that we propose in this paper is not necessarily restricted to CEOs. The ability of the holders of all managerial positions in the firm that affect the productivity of the workers they supervise helps increase the gap between their own firm’s productivity and that of the competitors. The impact is highest the higher up in the management hierarchy, but since one in five workers supervises some workers, this has implications for the distribution of earnings. And because the rise in inequality resides mainly in the top percentiles of the income distribution, and managers tend to have top earnings, our mechanism can help explain the rise in income inequality at the top.

Finally, the central mechanism that links market power to compensation is not restricted to managers. A superstar coder who improves an algorithm for a dominant tech firm for example, will command a superstar salary as her code will help her firm outperform competitors. And in the sports leagues, there is strategic interaction that derives from the zero-sum nature of sports competitions which is similar to the strategic interaction in oligopoly. The team that attracts the top players is more likely to win games, and this will make them bid up the compensation for the top players.
Online Appendix

Appendix A  Data

Appendix A.1  Description

**Compustat.** We obtain firm-level financial variables of U.S. publicly listed companies active at any point during the period 1950-2019. We access the Compustat North America Fundamentals Annual and download the annual accounts for all companies through WRDS on October 28, 2021. We exclude firms that do not report an industry code, employees, cost-of-goods (COGS), SG&A, capital, or sales. All financial variables are deflated with the appropriate deflators. We do the following truncation to the data set: (1) we drop all firms that report negative sales, COGS, or SG&A; (2) we eliminate firms whose sales are lower than COGS; (3) we eliminate firms with estimated markups in the top and bottom 1%, where the percentiles are computed for each year separately.

**ExecuComp.** Our data for manager pay comes from ExecuComp during the period of 1992 to 2019. All financial variables are deflated with the appropriate deflators. We drop firms that have zero TDC1 or TDC2. We also annually eliminate firms with TDC1 and TDC2 in the top and bottom 1%. We are using TDC1 as our manager pay throughout the paper, but the results are robust over different definitions. This data can be mapped into Compustat data set by gvkey and year.

Appendix A.2  Regression

Given the positive correlation between firm level markups and executive compensation, we further analyze this relation including covariates about the firm characteristics (number of employees, sales, and SG&A) as well as year, firm and industry fixed effects. Table A.1 reports this exercise.

In the first three columns, we run the regressions only with fixed effects. Column (1) performs the exercise with year and firm fixed effects. We find that the average treatment effect is 0.310 and is highly significant. A one percent increase in the firm level markup increases manager pay by 0.310 percent. Next, we control for a fixed effects at the industry level in Column (2), with the assumption that firms are comparable within the same market in the same year. In this case, the elasticity of manager pay with respect to markups is 0.189 and is again statistically significant. Finally, in Column (3) we control for the finest set of fixed effects at both firm and industry-by-year level, which gives us a coefficient 0.251. We then introduce covariates in Column (4) to (6). In all three cases, the estimated elasticities are between 0.269 and 0.299, which are robust and significant.
Table A.1: Regression: the executive salary elasticity of markup

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<th>(3)</th>
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<td>0.251</td>
<td>0.284</td>
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<td>(0.033)</td>
<td>(0.030)</td>
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<td>-0.0270</td>
<td>-0.0499</td>
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<tr>
<td></td>
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<td>(0.0081)</td>
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<td>(0.012)</td>
<td>(0.023)</td>
<td></td>
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<tr>
<td>log SG&amp;A</td>
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<td>0.117</td>
<td>0.0541</td>
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<tr>
<td></td>
<td>(0.020)</td>
<td>(0.0109)</td>
<td>(0.023)</td>
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<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
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</table>

R-squared  | 0.668 | 0.199 | 0.720 | 0.695 | 0.567 | 0.741 |
Observations | 33263 | 31982 | 31850 | 33263 | 31982 | 31850 |

Notes: The robust standard errors under heteroscedasticity are documented in the parenthesis. In the fixed effects rows, “Y” stands for “yes” and “–” means this fixed effect has already been covered by some other fixed effects. Industry is defined at 4-digit NAICS code level. The number of observations when we control industry-by-year fixed effects drops because there are firms that are alone in an industry by a year, which does not provide any variation for these identifications.

Appendix A.3 Supplementary figures

Selection in ExecuComp data set. We observe substantial differences with the samples in 1992 and 1993. Specifically, the panel A of Figure A.1 shows that the average sales of sampling firms is more than $9 million in 1992, which is abnormally greater than the level in other years. The same problem also exists in 1993. Furthermore, the panel B shows that there is a systematic difference of sample selection in 1992 and 1993. The sampled firms in these two years are overall larger than firms in subsequent years. For this reason, we eliminate the year 1992 and 1993 from our analysis.

Lognormal distribution of manager share in data. Figure A.2 reports the kernel distribution of log $\chi_{ij}$ in the data. It demonstrates that $\chi_{ij}$ follows a lognormal distribution. Based on this property, we are
constructing moments with $\log \chi_{ij}$ instead of $\chi_{ij}$.

**Appendix B  Model appendix**

**Appendix B.1  Lemma 1: household solution**

Recall the household problem:

$$\max_{\{c_{ij}\},L} U (C, L), \quad \text{s.t.} \quad \int_0^L \left( \sum_{i=1}^{l_i} p_{ij} c_{ij} \right) \, dj \leq WL + \Omega + \Pi.$$  

Because there is a continuum of identical households, any single household cannot influence the aggregate manager pay, $\Omega$, and profits, $\Pi$. They will take those aggregates as given in optimizing their utility. We start our analysis by deriving the aggregate labor supply function.
Labor supply. Given any wage $W$ and price index $P$, the household chooses labor supply $L$ to maximize utility:

$$\max_L U = \frac{WL + \Omega + \Pi}{P} - \varphi^{-\frac{1}{\varphi}} \left( L^{1 + \frac{1}{\varphi}} \right)$$

which incurs first order condition:

$$\frac{W}{P} = \varphi^{-\frac{1}{\varphi}} L^{1 + \frac{1}{\varphi}} \iff L = \varphi^{\phi} \left( \frac{W}{P} \right)^{\phi} \quad (B.1)$$

Inverse demand function. We then derive the inverse demand function by solving households’ cost-minimization problem. Within each market $j$ and given utility $c_j$, the household will choose the consumption bundle to minimize the expenditure:

$$\min_{\{c_{ij}\}} E = \sum_i p_{ij}c_{ij} \quad \text{s.t.} \quad c_j(c_{ij}) = c_j.$$  

The FOC gives:

$$I_j^{-\frac{1}{\eta}} c_{ij} \frac{1}{\eta} = \lambda_j^{-1} p_{ij} c_{ij} \quad \Rightarrow \quad c_j = \lambda_j^{-1} \sum_i p_{ij} c_{ij},$$

where $\lambda_j$ is the shadow price for goods at market $j$. Hence, we further define $\lambda_j$ as the price index for this market. The FOCs lead to:

$$c_{ij} = I_j^{-\frac{1}{\eta}} \left( \frac{p_{ij}}{p_j} \right)^{-\eta} c_j \quad \text{and} \quad p_j = \left[ \sum_i \frac{1}{I_j} p_{ij}^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad (B.2)$$

Similarly, we can solve the expenditure minimizing problem at the economy level, which incurs:

$$c_j = J^{-\frac{1}{\theta}} \left( \frac{p_j}{P} \right)^{-\theta} \bar{C} \quad \text{and} \quad P = \left[ \int_0^1 \frac{1}{J} p_j^{1-\theta} d_j \right]^{\frac{1}{1-\theta}} \quad (B.3)$$

Combining equation (B.2) and (B.3), we get the demand system from the household side:

$$y_{ij} = \frac{1}{J} \frac{1}{I_j} \left( \frac{p_{ij}}{p_j} \right)^{-\eta} \left( \frac{p_j}{P} \right)^{-\theta} Y. \quad (B.4)$$
Appendix B.2  Lemma 2: sub-game equilibrium

In this section, we derive the output market equilibrium in second stage given any matching allocation $x_{ij}$ from the period one. To begin with, recall the firm-level FOC:

$$p_{ij}A_{ij} = \mu_{ij}W,$$

where $\mu_{ij} := \left[1 + \frac{dp_{ij}y_{ij}}{dy_{ij}p_{ij}}\right]^{-1} = \left[1 - \frac{1}{\theta}s_{ij} - \frac{1}{\eta}(1 - s_{ij})\right]^{-1}$, \hspace{0.5cm} (B.5)

where the second equality comes from the elasticity of demand function (B.4). The CES structure incurs following property:

$$s_{ij} = \frac{p_{ij}^{-\eta}}{\sum_{i'} p_{i'j}^{-\eta}}.$$

(B.6)

Combining equation (B.5) and (B.6), we can solve for markups $\mu_{ij}$ (or equivalently, sales shares $s_{ij}$) directly from TFP $A_{ij}$ by:

$$s_{ij} = \frac{(\mu_{ij}/A_{ij})^{1-\eta}}{\sum_{i'} (\mu_{i'j}/A_{i'j})^{1-\eta}}.$$

Therefore, we will take $\mu_{ij}$ and $s_{ij}$ as the primitives for the subsequent analysis.

Output market clearing. As we take the price index as the numeraire, the goods clearing condition simply requires the prices implied by markups are consistent with this normalization, i.e.,

$$\left[\int_0^I \frac{1}{J} \left(\frac{1}{I_j} \sum_i p_{ij}^{1-\eta}\right) \frac{1-\theta}{\eta} dj\right]^{1/\eta} = P \hspace{0.5cm} \text{where} \hspace{0.5cm} p_{ij} = \mu_{ij} \frac{W}{A_{ij}}.$$

This condition gives us the equilibrium wage:

$$\frac{W}{P} = \left[\left(\int_0^I \frac{1}{J} \left[\frac{1}{I_j} \sum_i \left(\frac{\mu_{ij}}{A_{ij}}\right)^{1-\eta}\right] \frac{1-\theta}{\eta} dj\right)^{1/\eta}\right]^{-1}.$$

(B.7)

The equilibrium wage is the marginal revenue product of labor \textit{without} markups. To see this more clearly, imagine a homogenous economy where $A_{ij} \equiv A$ and $\mu_{ij} \equiv \mu$. The equation (B.7) becomes $W = AP/\mu$, where the term $AP$ is marginal revenue of labor, while the markup $\mu$ puts a wedge that becomes the gross profit of the firms. Furthermore, the term $1/I_j$ and $1/J$ neutralize the effect of love of variety — it prevents the change in $I_j$ and $J$ from directly influencing equilibrium wage. As a result, all changes in wages $W$ are due to the evolution of markups and productivities.
**Labor market clearing.** Finally, labor market clearing pins down the aggregate labor supply \( L \), using the household’s labor supply decision (B.1) in conjunction with the equilibrium wage:

\[
\bar{\varphi}W^q = \int_0^J \left[ \sum_i \frac{1}{A_{ij}} \frac{1}{I_j} \left( \frac{p_{ij}}{P_j} \right)^{-\eta} \left( \frac{p_j}{P} \right)^{-\theta} \right] \mathrm{d}j.
\]

(B.8)

The LHS is the labor supply function and the RHS is the aggregate labor demand function. This condition eventually pins down the output level \( Y \). After pinning down aggregates \( W \) and \( Y \), other equilibrium objects can be further derived from the inverse demand function and production function.

**Appendix B.3 Proposition 3: markup and firm size elasticities of TFP**

In this section, we first present the proof for the two elasticities of TFP shown in the paper. We then give an illustration using an example of a duopoly market.

The method is implicit function theorem. By taking derivatives of both sides of the FOC (B.5) w.r.t. \( A_{ij} \) and \( A_{kj} \) \((k \neq i)\), we get:

\[
\frac{\partial \mu_{ij}}{\partial A_{ij}} A_{ij} \mu_{ij} = \left( \frac{1}{\theta} - \frac{1}{\eta} \right) (\eta - 1) \mu_{ij}s_{ij} \left[ \sum_i' s_{i'j} \frac{\partial \mu_{i'j}}{\partial A_{ij}} \right] + (1 - s_{ij})
\]

\[
\frac{\partial \mu_{kj}}{\partial A_{ij}} A_{ij} \mu_{kj} = \left( \frac{1}{\theta} - \frac{1}{\eta} \right) (\eta - 1) \mu_{kj}s_{kj} \left[ \sum_i' s_{i'j} \frac{\partial \mu_{i'j}}{\partial A_{ij}} \right] + (1 - s_{ij}).
\]

Sum them up with the sales weight, we get:

\[
\sum_{i'} s_{i'j} \frac{\partial \mu_{ij}}{\partial A_{ij}} A_{ij} \mu_{i'j} = s_{ij} - \Phi_{ij} \quad \text{where} \quad \Phi_{ij} = \frac{s_{ij}}{1 + \left( \frac{1}{\theta} - \frac{1}{\eta} \right) A_{ij} \sum_i' s_{i'j} \frac{\partial \mu_{i'j}}{\partial A_{ij}} s_{i'j}}.
\]

This equation in turn gives us the markup elasticity of TFP:

\[
\frac{\partial \mu_{ij}}{\partial A_{ij}} A_{ij} = \left( \frac{1}{\theta} - \frac{1}{\eta} \right) (\eta - 1) \mu_{ij}s_{ij} \left( 1 - \Phi_{ij} \right)
\]

(B.9)

\[
\frac{\partial \mu_{kj}}{\partial A_{ij}} A_{ij} = -\left( \frac{1}{\theta} - \frac{1}{\eta} \right) (\eta - 1) \mu_{kj}s_{kj} \Phi_{ij}.
\]

(B.10)
Notes: We plot the example of a duopoly market here. By construction, sales shares of the two firms are $s_{ij}$ and $1 - s_{ij}$, respectively. Every object has a closed-form expression. The elasticity of substitutes are set as: $\theta = 1.2$ and $\eta = 5.75$.

Furthermore, by using the inverse demand function and production function from the sub-game equilibrium, we can write the equilibrium employment $l_{ij}$ as:

$$l_{ij} = \frac{1}{A_{ij}} \left( \frac{\mu_{ij}}{A_{ij}} \right)^{-\eta} \left[ \frac{1}{I_{ij}} \sum_{j' \in j} \left( \frac{\mu_{i j'}}{A_{ij'}} \right)^{1-\eta} \right]^{\frac{\eta - \theta}{1-\eta}} \left( \frac{Y_{ij}}{I_{ij}} \right) \left( \frac{W}{P} \right)^{-\theta},$$

from which we get:

$$\frac{\partial l_{ij}}{\partial A_{ij}} l_{ij} = \left[ \frac{\eta}{1 + \frac{\eta}{\theta - 1} (\eta - 1) \mu_{ij} s_{ij} - 1} \right] (1 - \phi_{ij}) + (\theta - 1) \phi_{ij},$$  \hspace{1cm} (B.11)

A duopoly example. In a duopoly economy, we have analytical form for all the equilibrium objects, which makes it an ideal example for us to check the property of aforementioned elasticities. In Figure B.1, we plot the markup and employment elasticities of TFP against the sales share $s_{ij}$. Their behaviors follow the theoretical interpretation we made in the paper, that the markup elasticity first increases then declines over the firm size, while the employment one is decreasing over $s_{ij}$ until the firm converges to the monopolist.

Appendix B.4 Extension: agency issue

In this section, we introduce risk aversion and an agency problem into our matching framework à la Edmans and Gabaix (2011) (henceforth, EG). We rederive the main theoretical predictions of our baseline model, based on which we will reach the conclusion that ignoring agency issue does not affect the decomposition of manager pay into the effect of market power and firm size.

We follow the setup and notations from the baseline model. In addition, we assume managers have
CRRA utility function:

\[ U(b, a) = \begin{cases} 
\frac{(be^{-g(a)})^{1-\Phi}}{1-\Phi} & \text{for } \Phi \neq 1 \\
\ln b - g(a) & \text{for } \Phi = 1
\end{cases} \]

where \( b \) denotes the realized compensation and \( a \in [a, \bar{a}] \) denotes the effort level. In this specification, \( \Phi \geq 0 \) is the relative risk aversion that is essential for introducing the agency problem. The term \( g(a) \) captures the disutility of effort. Effort enters the gross profits in a multiplicative way:

\[ \hat{\pi} = \frac{\bar{\pi} e^{a - \pi + i}}{\mathbb{E}[e^i]} , \]

where \( \bar{\pi} \) is the baseline gross profit defined in the main body, \( i \) is mean-zero noise with standard deviation \( \sigma \) and bounded interval support, and \( \mathbb{E}[e^i] \) normalizes the matching output to ensure that it does not depend on the noise distribution. Note that we introduce the effort and shock in a multiplicative way that will not influence the output market competition, just like if it were a profit taxes. We assume the manager privately observes \( i \) before choosing \( a \), but after signing the contract. Hence, the manager remains exposed to risk.

A key assumption of EG is that the baseline matching outcome \( \bar{\pi} \) should be sufficiently large, or alternatively the cost of effort and the risk should be sufficiently small, such that the maximum productive effort \( \pi \) becomes optimal for the firm, because the benefits of eliciting effort outweigh the costs. Under this assumption, the matching outcome on the equilibrium path is:

\[ \hat{\pi} = \frac{\bar{\pi} e^i}{\mathbb{E}[e^i]} . \]

Given this setup, we can replicate the analysis by EG, which leads to simple, closed form expression for the optimal contract.

**Claim (Proposition 1 in EG)** Let \( u \) be the reservation utility of the manager. The optimal contract pays the manager an amount \( b \) defined by

\[ \ln b = \Lambda \ln \bar{\pi} + K, \tag{B.12} \]

where \( \Lambda := g'(\bar{\pi}) \) is the marginal cost of effort and \( K \) is a constant that makes the participation constraint bind, that is,

\[ \mathbb{E}\left[ \frac{(be^{-g(\pi)})^{1-\Phi}}{(1-\Phi)} \right] = u. \]

The optimal contract for firms can thus be implemented by giving the manager \( \Lambda \omega \) of stock and \( (1 - \Lambda) \omega \)
of cash, given the amount of expected payment $\omega$.

**Proof.** See Edmans and Gabaix (2011). ■

Furthermore, given this contract, we can write the risk premium demanded by a manager as $\Phi (\Lambda^2 \sigma^2)/2$, where we define

$$\Phi (\sigma^2) = 2 \left( \ln \mathbb{E} [e^r] - \frac{1}{1 - \Phi} \ln \mathbb{E} [e^{(1-\Phi)\lambda}] \right).$$

The expected utility of the manager can hence be written as:

$$U = \mathbb{E} \left[ \left( \frac{be^{-g(\pi)}}{1 - \Phi} \right)^{1 - \Phi} \right] = \frac{\omega e^{-g(\pi)} e^{-\Phi (\Lambda^2 \sigma^2)/2}}{1 - \Phi} \left( \frac{\omega e^{-\tau}}{1 - \Phi} \right)^{1 - \Phi}.$$

where $\tau := g(\bar{a}) + \Phi (\Lambda^2 \sigma^2)/2$ is the “equivalent variation” and $U$ is the certainty equivalence of the manager under this contract with expected compensation of $\omega$. Therefore, we can define $\nu := \omega e^{-\tau}$ as the effective wage of the manager.

In the matching market, firm $ij$’s problem then becomes:

$$\max_x \pi_{ij} = \mathbb{E} \left[ \tilde{\pi}_{ij} (A_{ij} | A_{-ij}) - \nu (x) e^\tau \right]$$

$$= \tilde{\pi}_{ij} (A_{ij} | A_{-ij}) - \nu (x) e^\tau$$

$$= e^\chi \left[ \tilde{\pi}_{ij} (A_{ij} | A_{-ij}) e^{-\tau} - \nu (x) \right],$$

which incurs the FOC:

$$e^{-\tau} \frac{\partial \tilde{\pi}_{ij}}{\partial A_{ij}} \frac{\partial A_{ij}}{\partial x_{ij}} = \frac{d}{dx} \nu (x_{ij}). \quad (B.13)$$

As before, we can numerically find the stable matching and solve for the equilibrium effective wage schedule $\nu(x)$ and do the same decomposition exercise as in the benchmark model. We now replicate the first two main propositions in the presence of agency issue, whereas the Proposition 3 is unchanged.

**Proposition 1’** The marginal contribution to gross effective profits of managerial ability can be decomposed as follows:

$$\frac{\partial}{\partial x_{ij}} (\tilde{\pi}_{ij} e^{-\tau}) = \left[ \frac{\partial \mu_{ij}}{\partial A_{ij}} Wl_{ij} + (\mu_{ij} - 1) W \frac{\partial l_{ij}}{\partial A_{ij}} \right] \frac{\partial A_{ij}}{\partial x_{ij}} e^{-\tau}. \quad (B.14)$$
Proposition 2’ Given stable matching $\Gamma$, the executive effective salary schedule $v(x)$ satisfies:

$$v(x_{ij}) = v_0 + e^{-\tau} \int_{x_{ij}}^{\infty} \left[ \frac{\partial \mu_{ij}}{\partial x_{ij}} Wl_{ij} + (\mu_{ij} - 1) W \frac{\partial l_{ij}}{\partial x_{ij}} \right] \times \left[ \frac{\partial A_{ij}}{\partial x_{ij}} \right] dx_{ij},$$

where $v_0$ is the reservation utility that determines the effective wage for the lowest-type manager.

The results in this section show that introducing risk aversion and incentive provision in a multiplicative way does not change the key insights demonstrated in this paper. The intuition is that, although managers are additionally compensated for effort elicitation and risk aversion, these compensation also goes through the two channels and can hence be attributed to market power and firm size. In the current setup where all firms have identical $\tau$, we can derive the same results because everything is simply scaled by a constant $e^{-\tau}$. This framework also allows us to introduce heterogenous risk $\sigma_{ij}^2$ and effort cost $g_{ij}(a)$, which can lead to richer heterogeneity in the quantitative exercise, yet it does not change the key mechanism through which market power determines manager pay.

Appendix B.5 Lemma 3: production transformation

We prove Lemma 3 by solving the cost minimization problem of firms. The Lagrangian problem can be written as:

$$\mathcal{L}(l_{ij}, m_{ij}, k_{ij}; y_{ij}) = Wl_{ij} + P^m m_{ij} + Rk_{ij} - \lambda_{ij} \left[ A_{ij} (l_{ij} + m_{ij})^\xi k_{ij}^{1-\xi} - y_{ij} \right],$$

with FOCs:

$$\frac{\partial \mathcal{L}}{\partial l_{ij}} = W - \frac{\lambda_{ij} \xi}{l_{ij} + m_{ij}} \left[ A_{ij} (l_{ij} + m_{ij})^\xi k_{ij}^{1-\xi} \right] = 0,$$

$$\frac{\partial \mathcal{L}}{\partial m_{ij}} = P^m - \frac{\lambda_{ij} \xi}{l_{ij} + m_{ij}} \left[ A_{ij} (l_{ij} + m_{ij})^\xi k_{ij}^{1-\xi} \right] = 0,$$

$$\frac{\partial \mathcal{L}}{\partial k_{ij}} = R - \frac{\lambda_{ij} (1-\xi)}{k_{ij}} \left[ A_{ij} (l_{ij} + m_{ij})^\xi k_{ij}^{1-\xi} \right] = 0,$$

where $P^m$ is the price for materials. This set of FOCs give us the optimal inputs choices:

$$m_{ij} = \frac{1 - \psi}{\psi} l_{ij} \quad \text{and} \quad k_{ij} = \frac{1}{\psi R/(1-\xi)} l_{ij},$$

(B.15)

where $\psi := l_{ij}/(l_{ij} + m_{ij})$ is an exogenous parameter for all firms. Note also that since labor and materials are perfectly substitutable, at equilibrium we must have $P^m = W$. 
Moreover, solving this cost minimization problem gives us the marginal cost of production:

$$mc_{ij} = \frac{1}{\psi} \frac{1}{\zeta} \frac{W}{A_{ij}},$$

(B.16)

which further leads to the gross profit:

$$\tilde{\pi}_{ij} = (\mu_{ij} - 1) mc_{ij} y_{ij} = \frac{1}{\psi} \frac{1}{\zeta} (\mu_{ij} - 1) W l_{ij}.$$ (B.17)

Compared to the labor-only model, the gross profit (B.17) is scaled by the production elasticity of material and capital, which indicates the final decomposition of manager pay in equation (17).

**Appendix C  Quantification**

**Appendix C.1  Verifying the efficiency of the approximate algorithm**

To check the efficiency of this approximation algorithm, we compare the exact stable matching to the approximate stable matching obtained with our approximate algorithm. We do this for an economy with $J = 200$ markets where we can still calculate the equilibrium stable matching exactly. Figure C.1 confirms first that, due to the externalities, the PAM allocation between the types of firms and the manager type $x$ (the diagonal line in the left panel) is no longer stable. More importantly, it shows that there is remarkable overlap between the allocations of the exact and the approximate stable matching. For our purpose, this naturally implies that the estimated salary schedule (in the right panel) under the approximate stable matching is virtually identical that under the exact stable matching. Moreover, the total revenue change (in absolute value) between the exact and approximate matching is 0.001% of the total revenue from the exact matching, and the total pay change (in absolute value) is 1.17% of the total manager pay, both of which are negligible.

To address the concern that the robustness we have observed in Figure C.1 may be due to the fact that $J$ is small, we further repeat this exercise over different values of $J$. The result is reported in Figure C.2. We see that as $J$ increases, the differences in revenue and manager pay between approximate and exact matching are robustly small, which suggests that our approximation does a good job regardless of the number of markets (firms). Furthermore, we can make a conclusion that the approximate stable matching is close to the exact one for a large economy that we are considering in the quantitative exercises.
Figure C.1: Comparison: Exact and Approximate Stable Matching

Notes: We set \( J = 200 \) in this exercise. The set of parameter is taken from the estimates in 2019, which is presented in Section 4.5. The PAM is derived in above algorithm. To find the stable matching, we iterate over all pairs of firms and shift managers if they can get better off, until all of them satisfy the condition in Definition 1. Panel C and D report the revenue difference for each firm as a share of the exact revenue, and the pay difference for each firm as a share of the exact pay.

Figure C.2: Robustness of Approximate Stable Matching over \( J \)

Notes: These figures report the gross revenue (manager pay) difference as a share of the exact gross revenue (pay) in absolute value for different number of markets \( J \).

Appendix C.2  Fit of model and data moments

Appendix C.3  Comparative static

Category I. Match

Importance of Managers - \( \alpha \). Figure C.4 reports the comparative static results for \( \{\alpha, \gamma\} \). The importance of the manager is measured by the share of the manager \( \alpha \). As is shown by Proposition 2, an increase in \( \alpha \) will proportionally raise the marginal contribution of managers for all firms. This leads to the two conclusions regarding moments: first, the average salary share of managers will increase; and second, the slope of salary share on sales will be constant.
Notes: Data moments are computed annually. Moments in categories “II. Market” and “III. Firm” are generated from Compustat sample, while the ones in category “I. Match” and “IV. Aggregates” are from ExecuComp sample due to data limitation. The latter sample is a sub-sample of the former one. We apply a five-year centered moving average in plotting both data and model moments.

**COMPLEMENTARITY - $\gamma$.** When $\gamma$ increases, manager ability and firm type become less complementary. The first implication is that managers will get paid less because they become less productive. This shows up in a declining average salary share in Panel 1.B. Furthermore, as we have discussed in Section 4.4, the slope of salary share on sales becomes flatter, which aligns with the results in Column two of Figure C.4.

**Category II. Market**

**Mean of Market Structure Distribution - $m_I$.** We first look at the effect of $m_I$, the average number of firms in each market, which is shown in the first column of Figure C.5. As the average number of firms increases, the economy becomes more competitive, so the markup level goes down. On the other hand, when there are more competitive, low-markup markets, the between-market variance of markups also decreases.

**Standard Deviation of Market Structure Distribution - $\sigma_I$.** An increase in $\sigma_I$ makes the distribution of $I_j$ more dispersed, so it mainly impacts the heterogeneity across markets. As expected, Column two in Figure C.5 shows that a larger $\sigma_I$ leads to a larger between-market variance of markups.
Notes: In this exercise, we move parameters and check how the model moments response. In each column, we move only one parameter while fixing all others. The baseline parameters are the estimates in 1994, which is presented in Section 4.5. The range of each parameter is chosen as the range of corresponding estimates from 1994 to 2019. To reduce the noise due to reservation utility, we fix its relative level $\omega_0/E(\omega)$ rather than the absolute level $\omega_0$.

The effect on the markup level is negligible.

**Category III. Firm**

**Standard Deviation of Firm Type - $\sigma_z$.** Figure C.6 presents the comparative static over firm-level parameters. The change in standard deviation of the $z_{ij}$ influences mainly the variance of firm types. Larger heterogeneity among firms will make the markup distribution more dispersed within each market. Moreover, as we fix the mean of $z_{ij}$ to 1, changes in its standard deviation will not heavily influence the level of technology, and thus the worker’s wage. Finally, this increase will also naturally show up in the increasing variance of the revenue distribution.

**Mean of Market Productivity - $\mu_A$.** Column B shows that the level of $z_j$ only shifts the unskilled wage level. Clearly, higher TFP induces greater marginal revenue product of labor, which leads to larger labor demand and hence drives the equilibrium wage up. It does not influence the within-market variance of markups because markups only depend on the relative productivity of firms in the same market. It also has negligible impact on the variance of revenue.
STANDARD DEVIATION OF MARKET PRODUCTIVITY - $\sigma_A$. The last parameter is the standard deviation of the market-level productivity shock, $A_j$. Since the markups are determined within each market according to Lemma 2, this market-level shock will not influence the markup distribution at all. Therefore, its only effect is on the firm size distribution. By making firms more different across markets, a larger $\sigma_A$ will drive the variance of firm size up. This intuition is confirmed by the column three in Figure C.6. Finally, $\sigma_A$ has a slightly positive impact on $W$ because an increase in the standard deviation of a lognormal distribution will also contribute to a larger expectation. A higher TFP level hence leads to higher wages, as is shown in Panel 2.C.

Appendix C.4 Rescaling

We simply take the reservation utility $\omega_0$ from data. This section documents the way we use the parameters $\{\bar{\varphi}, \psi\}$ to match the average employee and the average manager pay from model to the data. Note that the constant return to scale allows us to rescale the model without influencing any other moments we targeted in the first three categories.

First, the parameter $\bar{\varphi}$ can be simply derived from the labor supply function:

$$L = \bar{\varphi} \left( \frac{W}{P} \right)^\psi \iff \bar{\varphi} = \frac{L}{(W/P)^\psi}. \quad (C.1)$$
Then, because we match the exact wage and average employment, the revenue expression:

$$r_{ij} = \frac{\mu_{ij} W_{ij}}{\xi \psi}$$

indicates that revenue (and thus manager pay) are proportional to $1/\psi$, based on which we can easily find the right $\psi$ to match the level of manager pay.

**Appendix C.5 Matching correlation over time**

In this section, we show the Spearman rank correlation coefficient of firm and market types, $x_{ij}$ and $A_j$, with manager ability, $z_{ij}$, over time. These coefficients correspond to the numbers reported in Panel B and C of Figure 9. Figure C.7 specifies a clear trend that manager ability is getting more and more correlated with firm type than market type. It suggests that managers are hired by firms increasingly for competition within a market over time.

**Appendix D Robustness**
Appendix D.1 Revenue as a measure of firm size

In this section, we report our decomposition exercise where we interpret revenue $r_{ij}$ as firm size. We will first present the decomposition equation and detail why this way of decomposition will underestimate the effect of market power. Finally, we show the corresponding quantitative results.

**Decomposition equation.** We can write the equilibrium gross profit $\pi_{ij}$ as:

$$\pi_{ij} = \left(1 - \frac{1}{\mu_{ij}}\right) r_{ij}.$$  

Therefore, we get another way to decompose the marginal contribution of manager, i.e.,

$$\frac{\partial\pi_{ij}}{\partial x_{ij}} = \left[ \frac{\partial}{\partial A_{ij}} \left(1 - \frac{1}{\mu_{ij}}\right) r_{ij} + \left(1 - \frac{1}{\mu_{ij}}\right) \frac{\partial r_{ij}}{\partial A_{ij}} \frac{\partial A_{ij}}{\partial x_{ij}} \right] \frac{\partial A_{ij}}{\partial x_{ij}}$$

which further gives us the way to decompose manager pay:

$$\omega(x_{ij}) = \omega_0 + \int_{x_0}^{x_{ij}} \left[ \frac{1}{\mu_{ij}} \left( \frac{\partial \mu_{ij}}{\partial x_{ij}} W_{ij} \right) + \left(1 - \frac{1}{\mu_{ij}}\right) \frac{\partial r_{ij}}{\partial x_{ij}} \right] \frac{\partial A_{ij}}{\partial x_{ij}} \left( \frac{A_{ij}}{A_{ij}} \right)^{1-\gamma} dF(x_{ij}).$$

Notice that, compared to Proposition 2, the market power channel in Equation (D.2) is being rescaled by a factor $1/\mu_{ij}$. This difference comes from the fact that markups directly enter the expression for revenue, that is, $r_{ij} = \mu_{ij} W_{ij}$. Therefore, decomposition (D.2) ignores the contribution of market power on
By the same token, if we interpret market power as the Lerner index \( L_{ij} = 1 - \frac{1}{\mu_{ij}} \) instead of the markup, we would obtain the same result as when using revenue as the measure of firm size, since \( \tilde{\pi}_{ij} = \left(1 - \frac{1}{\mu_{ij}}\right) r_{ij} = L_{ij} r_{ij} \). But again, this is because revenue \( r_{ij} = \mu W L_{ij} \) includes part of the markup channel. Therefore, trivially, if we measure the impact of the Lerner index \( L_{ij} \) but correct for the effect of \( L_{ij} \) on \( r_{ij} \), we obtain the same result as the effect of the markup \( \mu_{ij} \). This follows immediately from the chain rule: \( \frac{\partial L_{ij}}{\partial \mu_{ij}} = \frac{1}{\mu_{ij}^2} \).

**Quantification.** Nevertheless, we find that even when we are underestimating the market power effect, we still quantify a significant influence from it and see a robust increase of this effect over time. In Figure D.1, we replicate the marginal decomposition and find that the market power channel contributes for 22.7% of manager pay on the margin in 1994, and 25.3% in 2019. The distribution across manager ability is also heterogenous: almost all the marginal manager pay comes from the firm size channel for bottom managers, while the top manager benefit nearly 40% on the margin in both 1994 and 2019.

Figure D.2 shows the decomposition of manager pay level. As we expect, the market power channel is less important in this case, which accounts for $0.68 million in 1994 and $1.66 million in 2019. Over time, the market power component still plays a slightly more important role, whose share increases from 23.1% to 25.8%. Furthermore, Panel C and D show that market power contributes to the growth in manager pay by $0.98 million (28.1% of the total growth). Our main results still hold in this specification, although this method actually underestimates the effect of market power.

We also revisit the results regarding distribution. Figure D.3 demonstrates that the heterogeneity in market power and firm size channels still hold true in this decomposition. Basically, market power contributes to the compensation of high-ability managers (30.9%) more than those low-ability ones, and
so does its contribution to growth (31.1% for the top manager).

**Appendix D.2  Bertrand equilibrium**

In this section, we replicate our main exercises under the assumption that firms are competing in prices. We will first adjust our theory accordingly, then report the quantitative results.

**Equilibrium under Bertrand.** First of all, price competition will lead to different demand elasticities, and hence different markups than quantity competition. To see this, the firms problem in stage 2, i.e., equation (7), becomes:

\[
\max_{p_{ij}} \pi_{ij} = p_{ij} y_{ij} - W_{ij},
\]

which incurs FOC:

\[
\left[ 1 + \frac{\partial y_{ij}}{\partial p_{ij}} \frac{p_{ij}}{y_{ij}} \right] \frac{\partial y_{ij}}{\partial p_{ij}} = \frac{W}{A_{ij}} \frac{\partial y_{ij}}{\partial p_{ij}} \quad \Leftrightarrow \quad \mu_{ij} = \frac{-\eta + (\eta - \theta) s_{ij}}{1 - \eta + (\eta - \theta) s_{ij}}.
\]
The subgame equilibrium of stage two can be solved in the same way but instead using this new FOC.

As firms are competing in different ways, the decomposition of manager pay has to be adjusted as well. Following the same approach, we can derive markup and firm size elasticities of TFP:

\[
\frac{\partial \mu_{ij}}{\partial A_{ij}} = \left( \frac{\eta - 1}{\eta - \theta} \frac{\mu_{ij} s_{ij}}{\eta - (\eta - \theta) s_{ij}} \right) \times \left( \frac{\eta - \theta}{\eta - (\eta - \theta) s_{ij}} \right)
\]

\[
\frac{\partial l_{ij}}{\partial A_{ij}} = \phi_{ij} \left[ \theta - 1 \right] + (1 - \phi_{ij}) \left[ \frac{\eta}{1 + (\eta - 1) (\eta - \theta) \mu_{ij} s_{ij}} \right] \frac{s_{ij}}{\eta - (\eta - \theta) s_{ij}}^2 - 1
\]

where

\[
\phi_{ij} := \left[ \frac{s_{ij}}{1 + \mu_{ij} (\eta - 1) (\eta - \theta) s_{ij}} \right] \left[ \sum_{j'} \frac{s_{i'j}}{1 + \mu_{j'} (\eta - 1) (\eta - \theta) s_{i'j}} \right].
\]

All of our theoretical results remain robust when switching to the Bertrand equilibrium.

**Quantification.** We report the quantitative exercise under the Bertrand equilibrium. Using the timeseries estimates of parameters shown in Figure 8, we decompose manager pay into the market power and firm size channels both across years and in crosssection. We report the marginal decomposition in Figure D.4, where we observe the same patterns as in Cournot competition. Figure D.5 shows that, under Bertrand competition, the market power channel contributes to manager pay by 27.1% in 1994 and 56.8% in 2019, which indicates an even larger increase across time compared to the results under Cournot setup. In Figure D.6, we show that at crosssection level our results are also robust. In 2019, almost all of the top manager pay is due to market power, while the bottom manager pay comes mainly from the firm size channel. This heterogeneity also shows up in the growth of the manager pay.

**Appendix D.3 Output elasticity \((\theta, \eta)\) from Atkeson and Burstein (2008)**

In this section, we redo our quantification exercise using the output elasticity in Atkeson and Burstein (2008), that is, \((\theta, \eta) = (1.5, 10.0)\). We reestimate the endogenous model parameters using the same set of data moments, based on which we generate our main decomposition results.

**Decomposition on the margin.** We first replicate the decomposition of marginal manager pay in Figure D.7. We can draw the same conclusions under as what we get in our main paper. First, the market power share has been increasing from 43.00% in 1994 to 51.28 in 2019. We also see an increasing re-
relationship between market power share and manager ability. The reason for the sharp decline for top managers in 2019 is that there are some monopolists firm in the economy who hire top managers. However, since they are the only firm producing in their markets, their markups are exogenously given by the elasticity $\theta - 1$ like a standard CES structure. Hence, those top managers contribute little to the market power, which drives the market power share down on the right tail.

**Decomposition on the level.** The decomposition of manager pay level over time in Figure D.8 generates exactly the same insights. We see an increasing contribution from the market power channel to the manager pay, and this channel also contributes a lot to the growth of manager pay over time.

**Inequality among managers.** Finally, we report the exercise on the heterogeneity among managers in Figure D.9. Same insights are generated, that the market power channel matters more for the high-ability managers. Again, the sharp decline of the contribution of market power (share) for the very top managers comes from the fact that more monopolists firms are generated in the economy.
Figure D.6: Distribution of manager pay and its decomposition: Bertrand

Figure D.7: The marginal contribution of market power on manager pay, with $(θ, η) = (1.5, 10)$
Figure D.8: Manager pay decomposition into market power and firm size, by year with $(\theta, \eta) = (1.5, 10)$

Figure D.9: Distribution of manager pay and its decomposition, with $(\theta, \eta) = (1.5, 10)$
References


