Stock Returns, Market Trends, and Information Theory: 
A Statistical Equilibrium Approach

Emanuele Citera
Ph.D. Candidate
Department of Economics
New School for Social Research

www.emanuelecitera.com

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Introduction

The search for serial correlations in stock returns has been one the main criteria to assess market efficiency (Fama, 1965; Bhowmik and Wang, 2020).

Three main issues associated with serial correlations:

1. Sample size and power of econometric test (Shiller, 2015).
2. Sources of statistical dependence and randomness (Fama, 1965).
3. Informational component of stock prices (Mantegna and Stanley, 1999).
Objectives

- Analyzing the **statistical regularities** of stock returns through an **entropy-constrained statistical equilibrium model**.

- Explaining **randomness in stock prices** as the result of unintended consequence of investors seeking higher rates of return.

- Providing an **original assessment of the Efficient Market Hypothesis** by considering the role of **unfulfilled expectations of investors**, and how they impact stock market volatility.
Data Collection

S&P 500 individual companies’ daily adjusted stock prices (Yahoo Finance), from which we compute daily logarithmic returns:

\[ r_{i,t} = \log[p_{i,t}] - \log[p_{i,t-1}] \]  \hspace{1cm} (1)

Sample:

- Observations: 3,004,150.
- Divided into bull markets, bear markets, and corrections.

We consider the cross-sectional distributions of individual companies’ returns, and then analyze their statistical regularities.
Figure 1: Cross-sectional distributions over bull, bear markets, and corrections.
Figure 2: Empirical moments over bull, bear markets (red bars), and corrections (grey bars).
Quantal Response Statistical Equilibrium

The Quantal Response Statistical Equilibrium model (Scharfenaker and Foley, 2017) provides a theoretical framework which explains observed statistical regularities through a process of Smithian competition.

Entropy-constrained model, which derives equilibrium as an information theoretic probability distribution representing all possible states of the system.

Three main components of the model:

1. Quantal response behavior of market participants ($\mu, T$).
2. Negative feedback of individual actions on outcomes ($\alpha, S$).
3. Role of expectations ($\zeta = \mu - \alpha$).
Figure 3: Logit quantal response conditional probabilities for various values of $T$ and $\mu = 0$. 
Figure 4: Marginal and joint frequency distributions for different values of $\delta$. 

\[ \hat{f}(r) \]

\[ f(a, r) \]

\[ f(\text{sell}, r) \]
\[ f(\text{buy}, r) \]

$\delta = 0.005$  $\delta = 0.007$  $\delta = 0.011$
Expectations

(a) Symmetry ($\zeta = 0$).

(b) Positive skewness ($\zeta < 0$).

(c) Negative skewness ($\zeta > 0$).

Figure 5: Marginal, conditional, and joint frequency distributions for fulfilled and unfulfilled expectations.
Figure 6: Parameter estimates (%/day).
Figure 7: Bull market: 2003/03/11 – 2007/10/09.

<table>
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<tr>
<th>ID</th>
<th>μ</th>
<th>T</th>
<th>α</th>
<th>S</th>
<th>( \bar{r} )</th>
<th>ζ</th>
<th>δ</th>
<th>( f[\text{buy}] )</th>
<th>( f[\text{sell}] )</th>
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Table 1: Parameter estimates (%/day).
Figure 8: Bear market: 2007/10/09 – 2009/03/09.

Table 2: Parameter estimates (%/day).

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<th>ID</th>
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Conclusions

- We find evidence of punctuated statistical equilibrium over multiple market periods, disrupted by structural changes affecting the stock market.

- We find evidence of significant deviations of individual expectations from market outcomes over extended time periods, even though they remain consistent in the long-run.

- We show how the stochastic nature of stock prices can be explained as the spontaneous convergence of the system towards a market convention.
Thank You!


