Democratic Aggregation: Issues and Implications for Consumer Price Indexes

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Outline

1. Introduction
2. Group Price Index Theory
3. Methods
4. Results
5. Discussion and Conclusion
Overview

- Current Consumer Price Index (CPI) methodology geared to measure inflation from macroeconomic perspective
  - Prices and expenditure weights for a “representative consumer”
  - Budget shares matches the “average dollar” of expenditure

- Different from inflation experienced by the average household?
  - Among other potential differences, like consumption vs. payments

- Main finding: Aggregation differences are three times larger when using Tornqvist formula than when using modified Laspeyres (Lowe)
Selected Literature


- **Common finding:** Plutocratic-Democratic gap tends to be small when using Lowe (a.k.a. modified Laspeyres) formula
Group Price Indexes

- Standard price index formulas use market-level prices and expenditures (i.e., a “representative consumer”)
  - Formulas used by BLS
    - Lowe/modified Laspeyres: For CPI-U, fixed biennial weights
    - Tornqvist: For Chained CPI-U, updating monthly weights
- These implicitly give more weight to households with higher expenditure
- Democratic price indexes give equal weight to each household
  - Building blocks are household-specific price indexes
Household Price Indexes

- Items \( i = 1, \ldots, N \); households \( h = 1, \ldots, H \)
- Time: months \( t = 1, \ldots \);
  - For Lowe: \( b \) is biennial weight reference period, \( v \) is pivot month
- Quantities \( q_{ith} \), prices \( p_{it} \)
  - **Data limitation**: assume households face common \( p_{it} \)
- Expenditure shares
  - For Lowe: \( s_{i\{v,b\}h} = p_{iv}q_{ibh} / \sum_{i=1}^{N} p_{iv}q_{ibh} \)
  - For Tornqvist: \( s_{ith} = p_{it}q_{ith} / \sum_{i=1}^{N} p_{it}q_{ith} \)
Household Price Indexes (2)

- Lowe: \( P_{Lo,h} = \sum_{i=1}^{N} S_{i\{v,b\}h} \frac{p_{it}}{p_{iv}} \)

- Tornqvist: \( P_{T,h} = \prod_{i=1}^{N} \left( \frac{p_{it}}{p_{i,t-1}} \right)^{w_{ith}}, w_{ith} = 0.5 \left( s_{i,t-1,h} + s_{ith} \right) \)

- In application, \( p_{it}/p_{iv} \) and \( p_{it}/p_{i,t-1} \) represent elementary CPI for 211 items and 32 geographic areas

- Household share of total expenditure
  - For Plut. Lowe: \( S_{\{v,b\}h} = \sum_{i=1}^{N} p_{iv}q_{ibh}/\sum_{h=1}^{H} \sum_{i=1}^{N} p_{iv}q_{ibh} \)
  - For Plut. Torn: \( S_{th} = \sum_{i=1}^{N} p_{it}q_{ith}/\sum_{h=1}^{H} \sum_{i=1}^{N} p_{it}q_{ith} \)
## Democratic and Plutocratic Price Indexes

<table>
<thead>
<tr>
<th></th>
<th>Lowe</th>
<th>Tornqvist</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Plutocratic</strong></td>
<td>$P_{PLO} = \frac{1}{H} \sum_{h=1}^{H} P_{LO,h}$</td>
<td>$P_{PT} = \prod_{h=1}^{H} \left( \frac{p_{it}}{p_{i,t-1}} \right)^{w_{ith}}$</td>
</tr>
<tr>
<td></td>
<td>$= \sum_{i=1}^{N} s_{i{v,b}} \frac{p_{it}}{p_{iv}}$,</td>
<td>$= \prod_{i=1}^{N} \left( \frac{p_{it}}{p_{i,t-1}} \right)^{w_{it}}$,</td>
</tr>
<tr>
<td></td>
<td>$s_{i{v,b}} = \sum_{h=1}^{H} s_{i{v,b}h} s_{i{v,b}h}$</td>
<td>$w_{ith} = .5(s_{t_{-1},h}s_{i_{-1},h} + s_{th}s_{ith}), w_{it} = \sum_{h=1}^{H} w_{ith}$</td>
</tr>
<tr>
<td><strong>Democratic</strong></td>
<td>$P_{DLO} = \frac{1}{H} \sum_{h=1}^{H} P_{LO,h}$</td>
<td>$P_{DT} = \frac{1}{H} \sum_{h=1}^{H} P_{T,h}$</td>
</tr>
<tr>
<td></td>
<td>$= \sum_{i=1}^{N} s_{i{v,b}} \frac{p_{it}}{p_{iv}}$,</td>
<td></td>
</tr>
</tbody>
</table>
The Plutocratic Gap

Gaps derive from differences in how price changes are weighted

- Lowe Plut. Gap: \( P_{PLO} - P_{DLO} = \sum_{i=1}^{N} (s_{i\{v,b\}} - \bar{s}_{i\{v,b\}}) \frac{p_{it}}{p_{iv}} \)

- Torn. Plut. Gap: \( \ln(P_{PT}) - \ln(P_{DT}) \approx \sum_{i=1}^{N} (w_{it} - \bar{w}_{it}) \ln\left(\frac{p_{it}}{p_{i,t-1}}\right) - \zeta^2 \)
  \[
  \bar{w}_{it} = \frac{1}{H} \sum_{h=1}^{H} w_{ith}, \quad \zeta \text{ is the coeff. of var. of } P_{T,h} \text{ across } h, \text{ captures use of arithmetic mean instead of geometric mean across } h
  \]

- Ley (2005): Gap determined by how spending patterns vary by household expenditure level and how they covary with price changes
Methods Overview

- Using CPI elementary indexes, Consumer Expenditure Survey (CE) for second-stage aggregation weights
  - Key limitation: same elementary price indexes (211 items) used for each household. Only variation is by 32 geographic areas
- CE = two independent surveys: Interview and Diary
- For Tornqvist, the democratic average is over the one-month index links, which are then chained together as in C-CPI-U
- For Lowe, to better mimic biennial weighting of CPI-U, using only households who completed four interviews
Interview-Diary Matching

- Interview households: Records for 3-12 months
  - **Roughly 75% of CPI expenditures**: most of Housing, Transportation, Medical Care, Education and Communication

- Diary households: Records for 1-2 weeks
  - **Roughly 25% of CPI expenditures**: Most of Food and Beverages, Apparel. Significant portion of Recreation and Other Goods

- For each Interview-month, select one Diary which has similar demographic characteristics, where similarity is based on predicted expenditures (similar to Hobijn, et. al. 2009)
Results
### Indexes for the Urban Population

**Avg. 12 mo. Percent Change**

<table>
<thead>
<tr>
<th>Index</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plutocratic Lowe</td>
<td>2.055%</td>
</tr>
<tr>
<td>Democratic Lowe</td>
<td>2.132%</td>
</tr>
<tr>
<td>Difference, Lowe</td>
<td>-0.077%</td>
</tr>
<tr>
<td>Plutocratic Tornqvist</td>
<td>1.831%</td>
</tr>
<tr>
<td>Democratic Tornqvist</td>
<td>2.073%</td>
</tr>
<tr>
<td>Difference, Tornqvist</td>
<td>-0.242%</td>
</tr>
</tbody>
</table>

#### Official Indexes

<table>
<thead>
<tr>
<th>Index</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPI-U (Lowe)</td>
<td>2.099%</td>
</tr>
<tr>
<td>C-CPI-U (Torn.)</td>
<td>1.845%</td>
</tr>
</tbody>
</table>

### Index Levels (Dec. 2001 = 1.0)

![Index Levels Chart](chart.png)
Tornqvist Aggregation Differences Over Time

<table>
<thead>
<tr>
<th>Avg. 12 mo. % ch., Urban</th>
<th>P. Torn.</th>
<th>D. Torn</th>
<th>Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001-04</td>
<td>1.910%</td>
<td>2.389%</td>
<td>-0.479%</td>
</tr>
<tr>
<td>2005-08</td>
<td>3.016%</td>
<td>3.508%</td>
<td>-0.492%</td>
</tr>
<tr>
<td>2009-12</td>
<td>1.540%</td>
<td>1.568%</td>
<td>-0.028%</td>
</tr>
<tr>
<td>2013-16</td>
<td>0.941%</td>
<td>0.959%</td>
<td>-0.018%</td>
</tr>
<tr>
<td>2017-19</td>
<td>1.679%</td>
<td>1.916%</td>
<td>-0.237%</td>
</tr>
</tbody>
</table>

Note: Margins of error are preliminary and conditional on elementary item-area indexes.
Contributions to Tornqvist Plutocratic Gap

By CPI Major Group (December 12 mo. log-change)

Bars represent $\sum_{t=jan}^{Dec.} \sum_{i \in MG} (w_{it} - \bar{w}_{it}) \ln \left( \frac{p_{it}}{p_{i,t-1}} \right)$ over items in the major group.
Tornqvist Indexes for Subgroups

Average 12-month Percent Change

Expenditure Quintiles*

Income Quintiles**


Summary and Conclusions

- Equally-weighting household has (on average) three times the impact when using the Tornqvist formula versus the Lowe
  - Similar results using only Interview expenditures or geo. mean across $h$
  - The gap is still small relative to overall dispersion in $P_{T,h}$
- The Tornqvist gap has trended smaller over time, but has ticked up recently
- Housing and Transportation are the most important categories in determining the gap
Future Research

- This is work in progress, so comments are welcome
- Ongoing: more practical approximations to democratic index (i.e., an average of indexes by expenditure quintile)
- Ongoing: impact of democratic aggregation on sampling variation
- Long-term: missing “within-stratum” heterogeneity (e.g., in prices paid) is likely very important (Jaravel, 2021)
Thank you!

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References (1)


References (2)


