Psychological Distance and Deviations from Rational Expectations

Harjoat S. Bhamra
Imperial College

Raman Uppal
EDHEC Business School

Johan Walden
UC Berkeley

AFA

January 2022

Outline

Motivation and objective

Motivation

- Traditional view: No real need to study expectation formation.
    - Assume rational expectations — Muth (1961), Lucas (1976)
- This view has recently been challenged
    - Manski (2004): We should test if rational expectations is true
    - Gennaioli and Shleifer (2018): Agents' forecast errors are predictable
    - Landier et al. (2017): Strongly reject rational expectations hypothesis

Objective: Address three fundamental questions

1. How to model deviations from rational expectations,
   while still imposing rigor in how beliefs are formed?

2. How to test the theory, given that beliefs are not observable?

3. What are the welfare implications of these beliefs?

Our contribution: (1) Theoretical

- Develop a theoretical framework, that provides a disciplined way for deviating from rational expectations
  - Households' beliefs are derived endogenously, based on their psychological distance from firms.
  - Link unobservable beliefs to observable portfolio weights so that we can take the model to the data.
  - Theory leads to a clean two-parameter specification that can be tested empirically.

Our contribution: (2) Empirical

- Establish empirically that our theoretical framework performs well in explaining belief formation using data on
  - portfolios of Finnish households and location of Finnish firms.
- Our empirical findings provide a microfoundation for results in Huberman (2001) and Grinblatt and Keloharju (2001), who focus on portfolio weights.
  - Provide a belief-based explanation for local bias or home bias.

Our contribution: (3) Welfare implications

- Derive welfare implications
  - Identify new channel of welfare loss based on household location.
  - Households more distant from firms have more distorted beliefs, and hence, suffer greater welfare losses.

Related literature

- Our paper is related to the following distinct literatures:
  1. Robust decision making
     - we use this framework to model deviations from rational expectations
  2. Construal level theory
     - we use psychological distance to model the penalty for deviating from rational expectations
  3. Portfolio choice
     - we use empirical observations on portfolio weights to make inferences about our proposed model of beliefs
  4. Wealth inequality and welfare
     - we use our model to derive effect of location of household on welfare and income inequality

Model and Theoretical Results

The Model: Overview

- Model has only two distinguishing features:
  1. Robust decision making (Hansen and Sargent, 2008)
     - Households uncertain about benchmark model for making decisions
     - Consider a range of models (beliefs) around benchmark
     - Deviation from benchmark model incurs a penalty
  2. Construal level theory (Trope and Liberman, 2010)
     - In contrast to Hansen and Sargent (2008), our penalty depends on psychological distance between households and firms.
- All other features of the model are standard.

Extending the model is straightforward

- Have chosen simplest possible setting to illustrate main point
- Straightforward to extend model to
  - Epstein-Zin preferences (see appendix)
  - General equilibrium model of a production economy

Firms

- There are N firms indexed by $n \in \{1, \dots, N\}$, with stock returns

$$dR_{n,t} = \alpha_n \, dt + \sum_{k=1}^{N} \sigma_{n,k} dZ_{k,t},$$

where

- α_n is the expected rate of return on firm n,
- $Z_{k,t}$ is a standard Brownian motion under the reference probability measure $\mathbb{P}$ that represents rational beliefs, and
- $\sigma_{n,k}$ is the loading of stock return n on the k'th Brownian motion.
- correlation given by $\rho_{nm} = 0$ ... to simplify exposition
- The parameters α_n, σ_n, and ρ_{nm} are constant over time.

Households

- There are H households, indexed by $h \in \{1, \dots, H\}$.
- Households can invest their wealth in $N+1$ assets:
  1. N risky stocks
  2. A risk-free asset that has an interest rate i, which is constant.
- Household h's portfolio return is given by

$$dR_{h,t} = \sum_{n=1}^{N} \omega_{hn,t}\, dR_{n,t} + \left(1 - \sum_{n=1}^{N} \omega_{hn,t}\right) i\, dt.$$

- Households have mean-variance preferences over portfolio returns
  - All results extend to Epstein and Zin (1989) preferences.

Construal level theory

- People experience only the here and now. How do we plan for the distant future and take into account hypothetical alternatives?
- Construal level theory (CLT) proposes that we do so by forming abstract mental construals of distal objects.
- Its reference point is the self, here and now.
- The different ways in which an object might be removed from that point—in time, space, and hypotheticality—constitute different dimensions of psychological distance.

Psychological distance and household beliefs

- We link household beliefs about stock returns to psychological distance between the household and the firm.
- The distance of household h from firm n is denoted by d_{hn}.
  - This could be a geographic distance or a more abstract measure of distance such as cultural, linguistic, or social distance.

Distance $\to$ Trust

Proposition

- We map the distance, d_{hn}, into a measure of trust, ϕ_{hn}, which lies in the interval $[0, 1]$:

$$\phi_{hn} = \begin{cases} e^{-\kappa\, d_{hn}} & , d_{hn} \leq \overline{d}, \\ 0 & , d_{hn} > \overline{d}, \end{cases}$$

  - κ is a measure of the sensitivity of ϕ_{hn} to d_{hn}, and
  - $\overline{d}$ is a constant denoting some threshold value.
  - Rational expectations (RE) when $\kappa = 0$ and $\bar{d} = \infty$.

- Trust has the nice feature that
  - $\phi_{hn} = 1$ when distance measure is 0;
  - $\phi_{hn} = 0$ when distance measure exceeds threshold $\overline{d}$.

Household beliefs

- Each household h has its own beliefs,
  - represented by its personal probability measure denoted by $\mathbb{P}^h$,
  - which differs from the physical (objective) probability measure $\mathbb{P}$.

Distortion of household beliefs

- Household beliefs $\mathbb{P}^h$ are a distortion of rational beliefs $\mathbb{P}$.
- Household h's conditional expectation that event A could occur:

$$E_t^{\mathbb{P}^h}[I_A] = E_t^{\mathbb{P}}\left[\frac{M_{h,T}}{M_{h,t}} I_A\right], \qquad \text{where}$$

  - I_A is the indicator function for the event A;
  - $M_{h,t}$ is an exponential martingale:
    (Radon-Nikodym derivative of $\mathbb{P}^h$ with respect to $\mathbb{P}$)

$$\frac{dM_{h,t}}{M_{h,t}} = \sum_{n=1}^{N} \frac{\nu_{hn,t}}{\sigma_n} dZ_{n,t}$$

  - $\nu_{hn,t}$ are distortions of household beliefs from rational expectations.

Household beliefs: Intuition

- Under household h's beliefs the expected rate of return for firm n's stock is $\alpha_n + \nu_{hn,t}$. (If risk premium positive, then $\nu_{hn,t} < 0$.)
- From Girsanov's Theorem, choosing a vector of personal distortions is equivalent to changing objective measure to a new measure, $\mathbb{P}^h$.
- To discipline the distortion from $\mathbb{P}$, there is a penalty for deviating, which depends on psycholgical distance.

Household's optimization over beliefs and weights

- Under rational expectations (i.e., full trust, $\phi_{hn} = 1$ for all n), household h would solve the standard mean-variance problem

$$\max_{\boldsymbol{\omega}_h} E_t^{\mathbb{P}}[dR_{h,t}] - \frac{1}{2}\gamma_h \text{Var}_t[dR_{h,t}].$$

- When there is less than full trust by household h for some firms n, the household trades off
  - the benefits of choosing conservative beliefs against
  - the losses associated with deviating from rational expectations.
- Specifically, household h faces the following max-min problem:

$$\max_{\boldsymbol{\omega}_h} \min_{\boldsymbol{\nu}_h} \quad E_t^{\mathbb{P}^h}[dR_{h,t}] - \frac{1}{2}\gamma_h \text{Var}_t[dR_{h,t}] + \frac{1}{\gamma_h} L_h dt$$

Psychological-distance-weighted information loss

Definition (Psychological-distance-weighted information loss)
The psychological-distance-weighted loss for household h is

$$L_{h,t} = \frac{1}{2}\sum_{n=1}^{N} \underbrace{\frac{\phi_{hn}}{1-\phi_{hn}}}_{\text{weight}} \times \underbrace{\frac{\nu_{hn,t}^2}{\sigma_n^2}}_{\substack{\text{information}\\\text{discarded}}}$$

- The weight $\frac{\phi_{hn}}{1-\phi_{hn}}$ ensures that
  - an information loss impacts $L_{h,t}$ only when household's trust with respect to a particular firm is not zero;
  - the impact of the information loss increases with trust; and
  - becomes infinitely large when $\phi_{hn} = 1$, i.e., when distance is 0.
- It is in this manner that a household's location determines its psychological-distance-weighted information loss.

Optimal belief distortion from rational expectations

Proposition

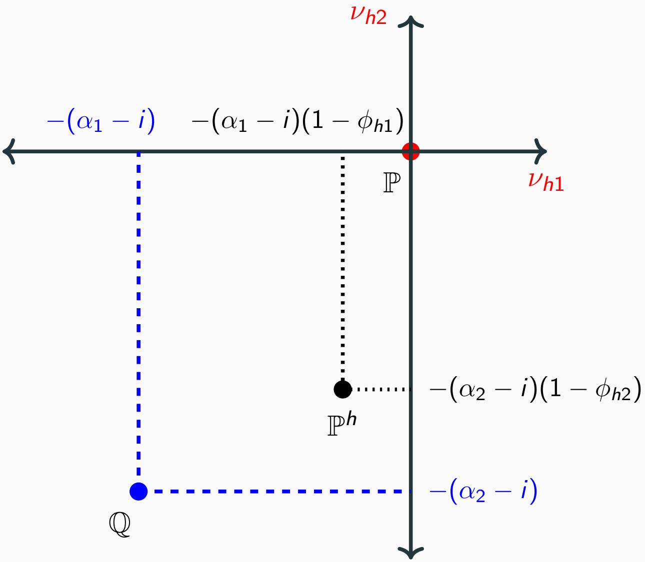
The optimal distortion in household h's beliefs about expected returns for firm n is

$$\nu_{hn} = -(\alpha_n - i)(1 - \phi_{hn}),$$

which, in terms of the household's psychological distance, is:

$$\nu_{hn} = \begin{cases} -(\alpha_n - i)(1 - e^{-\kappa\, d_{hn}}), & d_{hn} \leq \overline{d} \\ -(\alpha_n - i), & d_{hn} > \overline{d}. \end{cases}$$

Visualizing distortion to rational expectation beliefs



Linking beliefs to portfolio weights

- Because belief distortions are not observable, in order to test the model we link beliefs to portfolio weights.

Proposition
The optimal proportion of wealth invested by household h in firm n is

$$\omega_{hn} = \frac{1}{\gamma_h} \frac{\alpha_n - i + \nu_{hn}}{\sigma_n^2},$$

which, in terms of psychological distance, is given by

$$\omega_{hn} = \begin{cases} \frac{1}{\gamma_h} \frac{\alpha_n - i}{\sigma_n^2} e^{-\kappa\, d_{hn}}, & d_{hn} \leq \overline{d} \\ 0, & d_{hn} > \overline{d}. \end{cases}$$

Contrast with portfolio weights in information-based model

- The portfolio weights in our model differ from those arising in information models in at least three respects:
- In **our model**:

$$\omega_{hn} = \frac{1}{\gamma_h}\frac{\alpha_n - i + \nu_{hn}}{\sigma_n^2} = \begin{cases} \frac{1}{\gamma_h}\frac{\alpha_n - i}{\sigma_n^2} e^{-\kappa\, d_{hn}}, & d_{hn} \leq \overline{d} \\ 0, & d_{hn} > \overline{d}. \end{cases}$$

- In an **information-based model**

$$\omega_{hn} = \frac{1}{\gamma_h}\frac{(\alpha_n|\text{info}) - i}{(\sigma_n|\text{info})^2}.$$

- Information
  - can lead to an increase in the weight; in our model, only a decrease
  - can lead to short positions; in our model, "no" shorting
  - cannot lead to a weight of zero; in our model, portfolios are sparse.

Household welfare

Proposition
The welfare loss in deviating from rational expectations is:

$$\Delta U^{MV}(\mathbb{P}, \mathbb{P}^h) = U^{MV}\big(\omega_h(\mathbb{P})\big) - U^{MV}\big(\omega_h(\mathbb{P}^h)\big)$$

Household welfare

Proposition
The welfare loss in deviating from rational expectations is:

$$\Delta U^{MV}(\mathbb{P}, \mathbb{P}^h) = U^{MV}\big(\omega_h(\mathbb{P})\big) - U^{MV}\big(\omega_h(\mathbb{P}^h)\big)$$

$$= \frac{1}{\gamma_h} D^{KL}[\mathbb{P}|\mathbb{P}^h] \quad \dots \quad \textit{Kullback-Leibler divergence}$$

Household welfare

Proposition

The welfare loss in deviating from rational expectations is:

$$\Delta U^{MV}(\mathbb{P}, \mathbb{P}^h) = U^{MV}\left(\boldsymbol{\omega}_h(\mathbb{P})\right) - U^{MV}\left(\boldsymbol{\omega}_h(\mathbb{P}^h)\right)$$

$$= \frac{1}{\gamma_h} D^{KL}[\mathbb{P}|\mathbb{P}^h] \quad \ldots \quad \textit{Kullback-Leibler divergence}$$

$$= \frac{1}{\gamma_h} \frac{1}{2} \sum_{n=1}^{N} \left(\frac{\nu_{hn,t}}{\sigma_n}\right)^2$$

where

$$\nu_{hn} = \begin{cases} -(\alpha_n - i)(1 - e^{-\kappa\, d_{hn}}), & d_{hn} \leq \overline{d} \\ -(\alpha_n - i), & d_{hn} > \overline{d}. \end{cases}$$

Data and Empirical results

Data on household accounts

- Data for all accounts on Helsinki Stock Exchange, as of 2 Jan. 2003.
- Contains portfolio holdings and postal code information, as well as other characteristics (age, gender, and sector code classification).
- We include all accounts that are
    - classified as households,
    - are associated with a valid postal code, and
    - owned shares in at least one of the 125 stocks on January 2, 2003.
- This gives $H = 405,628$ households in $P = 2,923$ postal codes.
    - Assume households live at center of gravity of respective postal code
- We obtain geographic coordinates for each postal code area from the Finnish postal services company.

Data on companies

- We have data for 125 stocks.
  - Some of these stocks represent A- and B-shares in the same company.
- We exclude
  - companies headquartered outside of Finland
  - or whose shares were not traded in the previous month.
- We get information about postal codes of company headquarters from Thomson One Reuters.

Geographical distance

- We use geographic distance as a measure of trust between agents and companies.
- We normalize the distance function so that all geographic coordinates lie in the unit square, $[0, 1] \times [0, 1]$.
    - The household-stock that are farthest apart are therefore at a distance somewhere between 1 and $\sqrt{2}$ from each other
    - in our sample, the maximum distance is 1.175.

Model predictions that we test empirically

1. Center of gravity of stock ownership is influenced by the distance between households and firms.

Model predictions that we test empirically

1. Center of gravity of stock ownership is influenced by the distance between households and firms.

2. Households farther away from clusters of firms should hold portfolios that are less well diversified.

Model predictions that we test empirically

1. Center of gravity of stock ownership is influenced by the distance between households and firms.

2. Households farther away from clusters of firms should hold portfolios that are less well diversified.

3. Households in proximity should have similar beliefs, so should hold more similar portfolios than randomly chosen households.

Model predictions that we test empirically

1. Center of gravity of stock ownership is influenced by the distance between households and firms.
2. Households farther away from clusters of firms should hold portfolios that are less well diversified.
3. Households in proximity should have similar beliefs, so should hold more similar portfolios than randomly chosen households.
4. Household's belief distortions about firms included in its portfolio are lower than belief distortions about firms not included.

Model predictions that we test empirically

1. Center of gravity of stock ownership is influenced by the distance between households and firms.
2. Households farther away from clusters of firms should hold portfolios that are less well diversified.
3. Households in proximity should have similar beliefs, so should hold more similar portfolios than randomly chosen households.
4. Household's belief distortions about firms included in its portfolio are lower than belief distortions about firms not included.
5. The estimated decay factor κ is strictly positive, $\kappa > 0$.

Prediction 1:
Center of gravity of stock ownership influenced by distance

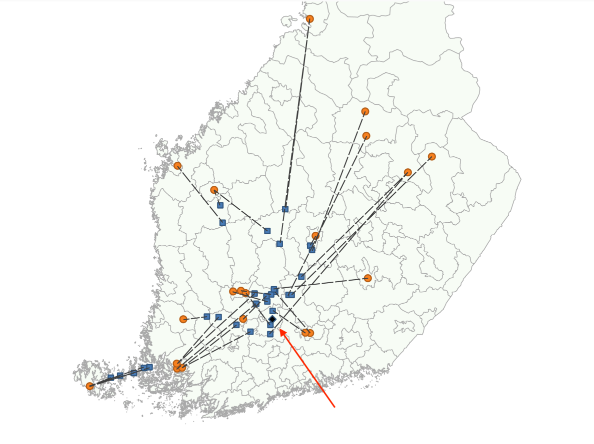
- We calculate the center of gravity of ownership in each stock,

$$x_n^O = \frac{1}{|\mathcal{H}_n|} \sum_{h \in \mathcal{H}_n} x_{p_h} \quad \text{and} \quad y_n^O = \frac{1}{|\mathcal{H}_n|} \sum_{h \in \mathcal{H}_n} y_{p_h},$$

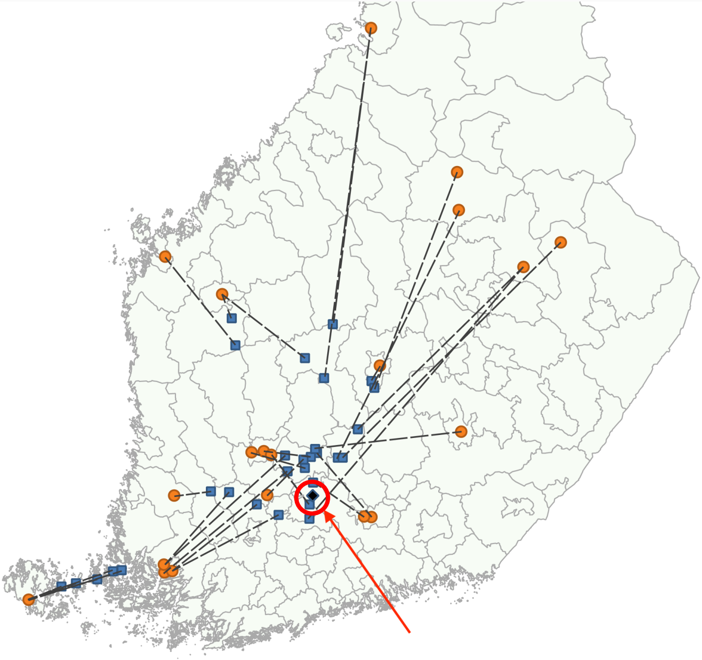
  where $\mathcal{H}_n$ is the set of households that have invested in stock n.

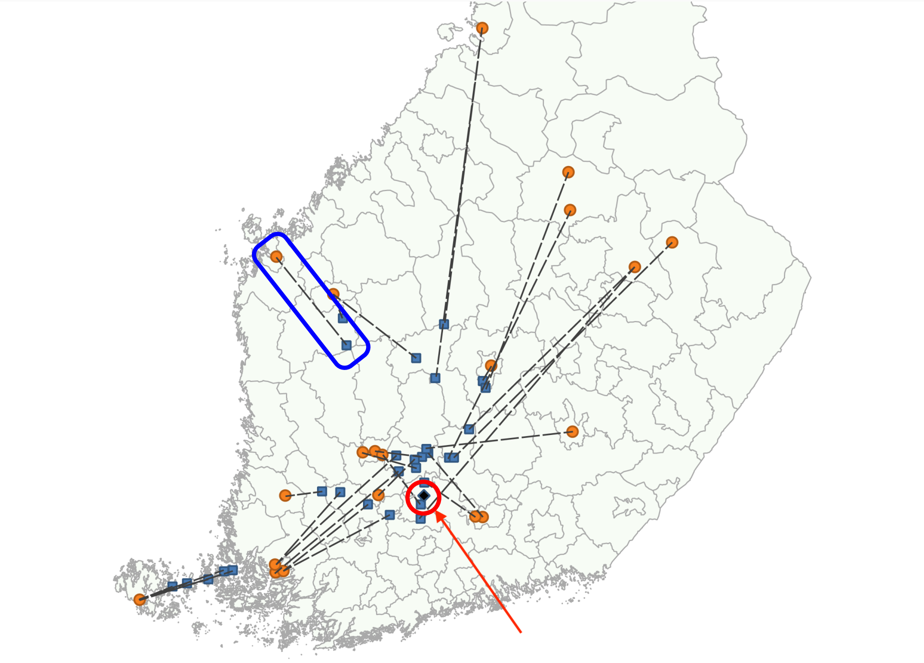
- Under rational expectations,
  - center of gravity of ownership should be the same across stocks,
  - coinciding with the market's total center of gravity of ownership.

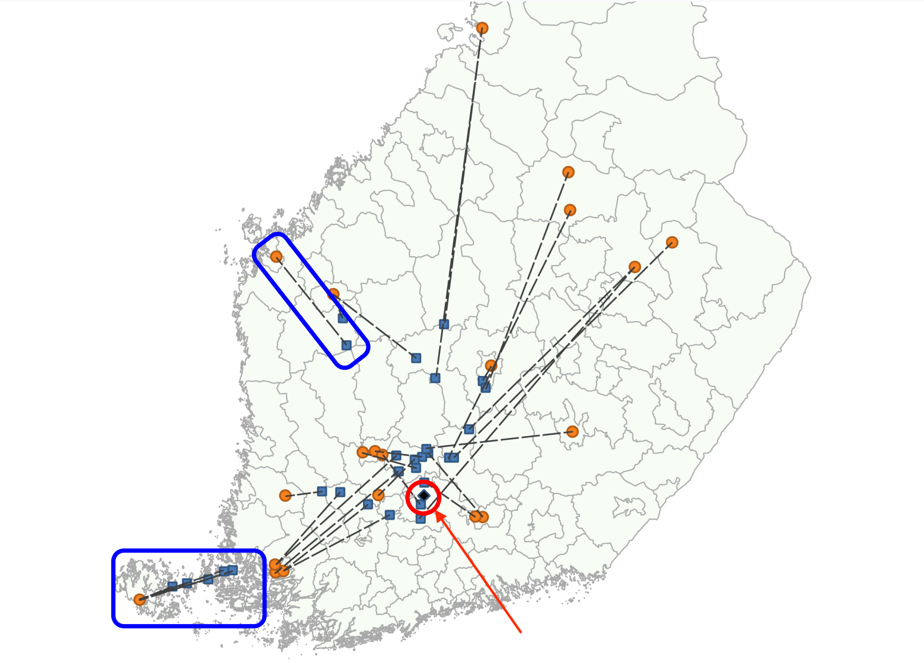
Prediction 1:
Center of gravity of stock ownership influenced by distance



Orange circles: firms outside of Helsinki
Blue squares: center of gravity of ownership for each stock
Black diamond: market's total center of gravity of ownership







Prediction 2: Relation between q_p and d_p
Households farther away from firms are less well diversified

- q_p = average number of stocks in households' portfolios
- $d_p = D((x_p, y_p), (x^C, y^C))$, the distance between households in postal code area p and average firm center of gravity (CoG)

	Equal weighted		Value weighted	
	(1)	(2)	(3)	(4)
Distance, d_p	−1.01 ***	−0.31 ***	−1.02 ***	−0.31 ***
Standard error	0.08	0.07	0.08	0.07
Portfolio size, $\ln(W_p)$		0.45***		0.45***
Standard error		0.01		0.01

Prediction 3:
Households in proximity have similar beliefs and portfolios

- We study the likelihood that households invest in the same stocks.
- Portfolio overlap between two households is defined as number of stocks held by both households.
- Simple test (ignoring firm size): Portfolio overlap within postal code is on average 14.8 times higher than if random.
- Sophisticated test (accounting for firm size): Portfolio overlap within postal code is on average 44%–139% higher than if random.

Prediction 4:
Belief distortions for firms included are lower than for those not included in portfolio

- We test if postal codes with zero holdings in a stock tend to lie geographically further away than postal codes with positive holdings.
- Perform two-sample t-test, comparing avg. distances of postal codes
- The average distance for postal codes with
  - zero holdings is 0.33, but for positive holdings is 0.22,
  - corresponding to a difference of 0.107—about 85 miles.
  - average t-statistic for the difference of means being positive is 13.3.

Size of trust region: Threshold for zero holdings

- We also estimate the cutoff distance, $\bar{d}$ for zero portfolio holdings.
  - Specifically, we choose the $\bar{d}$ that maximizes the number of correctly classified postal codes for the 125 stocks.
- The average estimated $\bar{d} = 0.1799$, equal to about 143 miles.
- Beyond this distance to a firm's headquarter, an investor completely avoids investing in a stock.

Prediction 5: The estimated decay factor $\kappa > 0$

- The expression for optimal portfolio weights implies

$$\omega_{pn} = \frac{\text{amt}_{pn}}{W_p} = \begin{cases} \frac{1}{\gamma_p} \frac{\alpha_n - i}{\sigma_n^2} e^{-\kappa\, d_{pn}}, & d_{pn} \leq \overline{d} \\ 0, & d_{pn} > \overline{d}, \end{cases}$$

$$\text{amt}_{pn} = \begin{cases} \frac{W_p}{\gamma_p} \frac{\alpha_n - i}{\sigma_n^2} e^{-\kappa\, d_{pn}}, & d_{pn} \leq \overline{d} \\ 0, & d_{pn} > \overline{d}. \end{cases}$$

- Defining $a_{pn} = \ln(\text{amt}_{pn})$, $g_p = \ln(\gamma_p / W_p)$, $s_n = \ln\left(\frac{\alpha_n - i}{\sigma_n^2}\right)$, then implies that, when the psychological distance is less than $\overline{d}$,

$$a_{pn} = -g_p + s_n - \kappa\, d_{pn}.$$

Prediction 5: The estimated decay factor $\kappa > 0$

	(1)	(2)	(3)	(4)
Panel A				
Sensitivity coeff: κ	5.87***	3.86***	3.19***	3.18***
Standard error	0.04	0.06	0.06	0.43
log risk aversion, g				
-average		-4.33	-4.12	
-max		-0.38	-0.33	
-min		-12.89	-12.83	
log distribution, s				
-average			0	
-max			7.26	
-min			-2.66	
R^2	0.06	0.39	0.59	0.59
Adj. R^2	0.07	0.38	0.59	0.59
$N = 368,298$				

Statistical and economic significance of κ

- κ is highly statistically significant in all regressions
  - univariate,
  - including risk-aversion fixed effects,
  - including risk-aversion and stock-characteristic fixed effects, and
  - panel regression with robust standard errors clustered at stock level.
- The results are also economically significant.
  - Standard deviation of distance between firm and household is 0.312.
  - For $\kappa = 3.18$ (column 4), a one standard deviation decrease in distance to a firm predicts an increase in portfolio holdings by a factor of $e^{3.18 \times 0.312} = 2.645$.
  - The R^2 for univariate regression is 0.0654, implying a correlation between trust and log-portfolio holdings of about 0.26.

Robustness tests

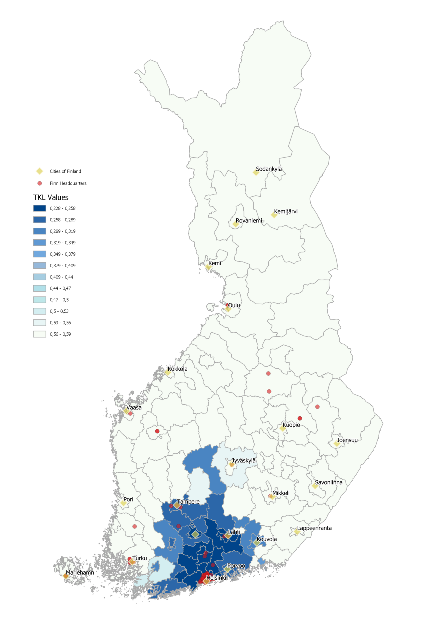
- Adjust for Helsinki:
  - Exclude households with Helsinki postal codes
  - Exclude households and firms with Helsinki postal codes
- Adjust for special stocks:
  - Exclude Nokia and Stockmann
- Adjust for employment effects:
  - Exclude observations with distance < 8 or < 24 miles
- Adjust for hedging demands:
  - Age and gender do not affect significance of κ

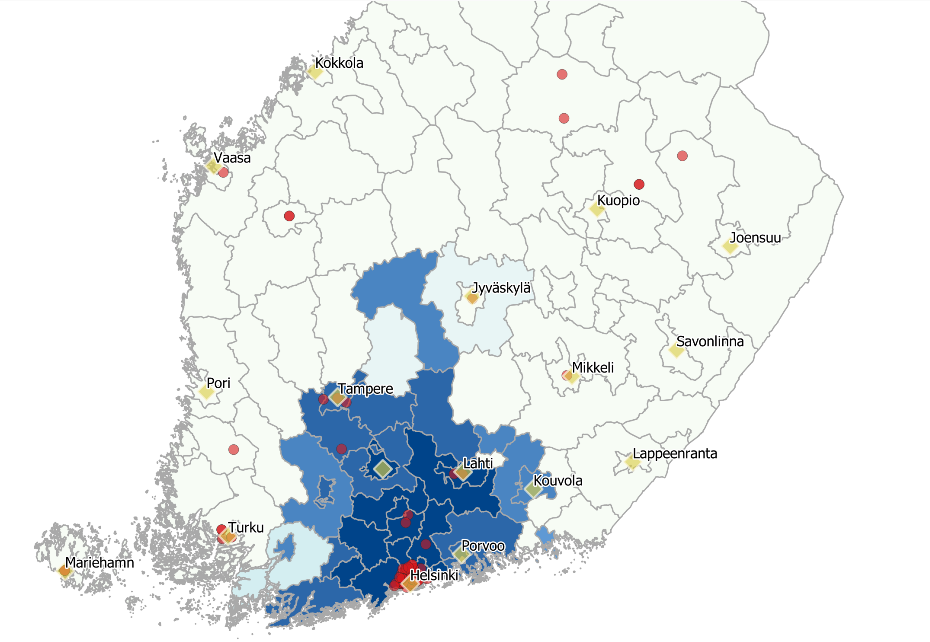
Welfare loss from deviating from rational expectations

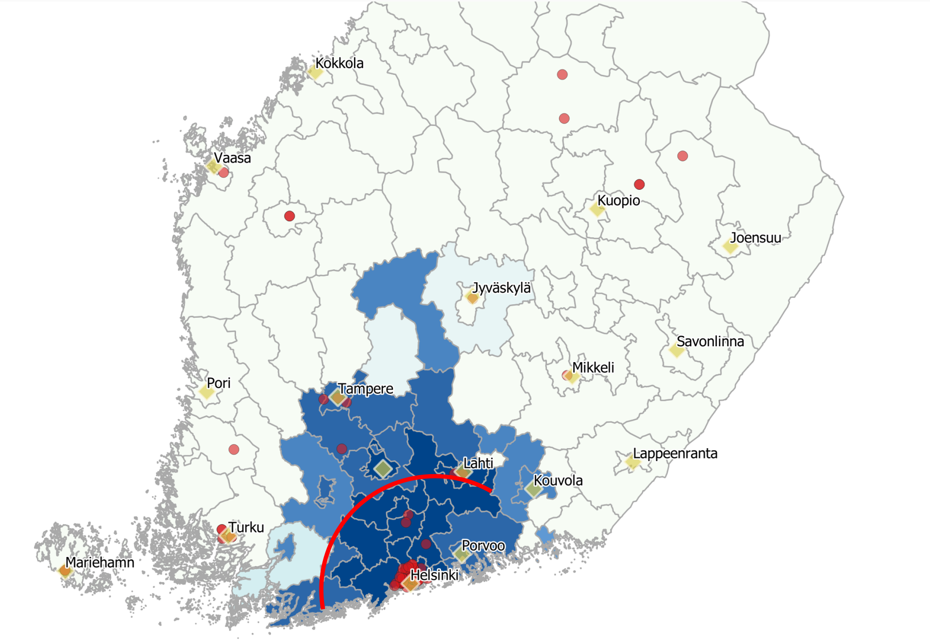
Welfare loss

- Measure welfare in terms of loss of Sharpe-ratio, (which is independent of household's risk aversion coefficient).
  - For the two extremes ($\mathbb{P}$ and $\mathbb{Q}$):
    - under Rational Expectations, this would be 0;
    - under completely distorted beliefs, it would be 0.59.
  - What we find:
    - it is about 0.25, for Helsinki region
    - it is 0.50 for postal codes greater than 20000
    - it is 0.57 for postal codes greater than 40000

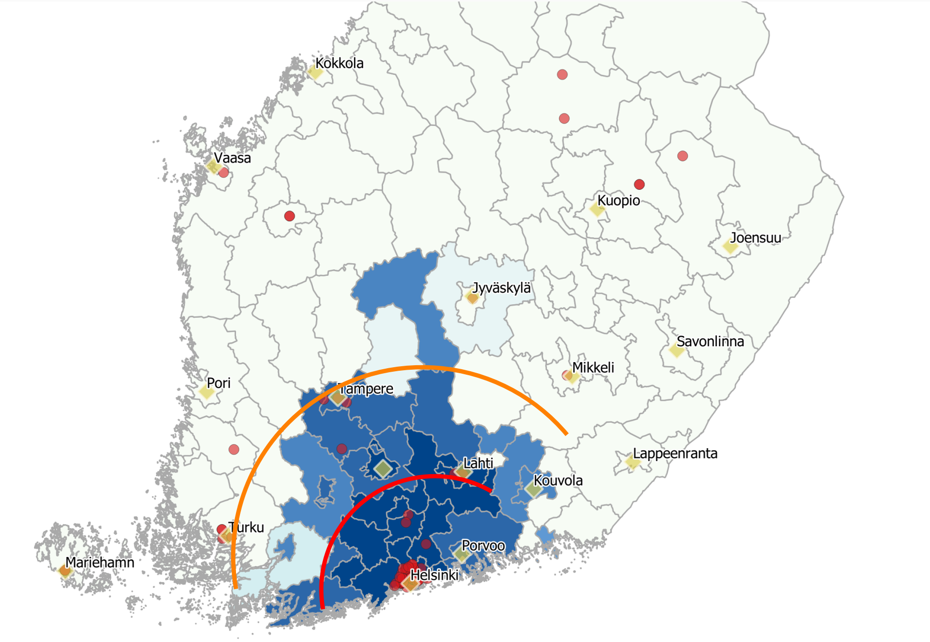
Welfare loss based on location

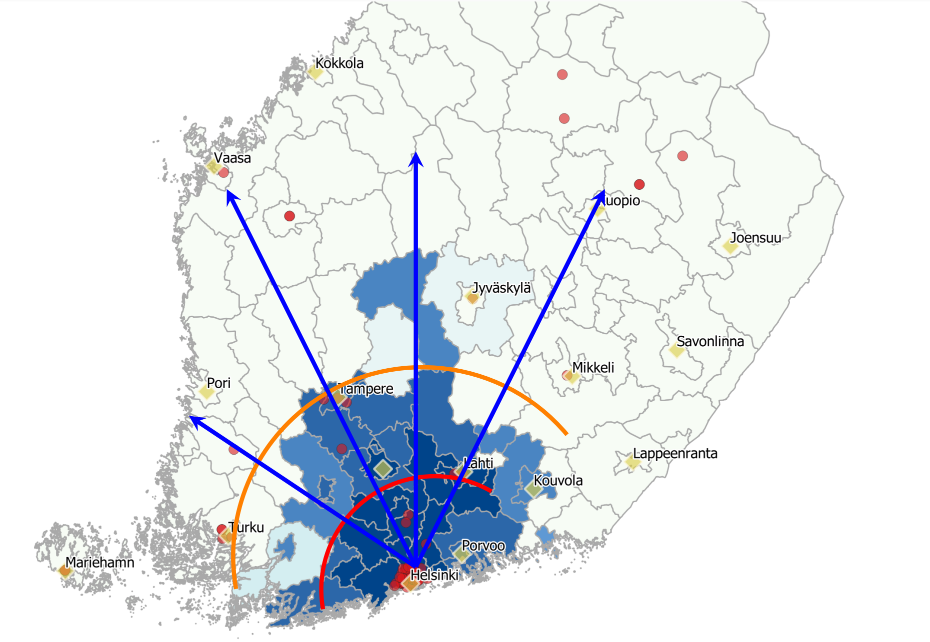



Kokkola
Vaasa
Kuopio
Joensuu
Jyväskylä
Savonlinna
Mikkeli
Pori
Tampere
Lappeenranta
Lahti
Kouvola
Turku
Porvoo
Mariehamn
Helsinki



Kokkola
Vaasa
Kuopio
Joensuu
Jyväskylä
Savonlinna
Mikkeli
Pori
Tampere
Lappeenranta
Lahti
Kouvola
Turku
Porvoo
Mariehamn
Helsinki


Kokkola
Vaasa
Kuopio
Joensuu
Jyväskylä
Savonlinna
Mikkeli
Pori
Tampere
Lappeenranta
Lahti
Kouvola
Turku
Porvoo
Mariehamn
Helsinki


Kokkola
Vaasa
Kuopio
Joensuu
Jyväskylä
Savonlinna
Mikkeli
Pori
Tampere
Lappeenranta
Lahti
Kouvola
Turku
Porvoo
Mariehamn
Helsinki

Concluding remarks

Concluding remarks

1. Develop a theoretical framework to demonstrate how one can deviate from rational expectations in a disciplined fashion.
   - Households' beliefs are derived endogenously, based on their psychological distance from firms.

2. Show empirically that this framework performs well in explaining belief formation of Finnish households.

3. Welfare implications: Households distant from firms disadvantaged because their beliefs distortions are larger.
   - If firms located close to big cities, then a rural-urban divide.
   - If firms located close to Helsinki, then a Helsinki-nonHelsinki divide.

Thank you!

h.bhamra@imperial.ac.uk
raman.uppal@edhec.edu
walden@haas.berkeley.edu

References i

Epstein, Larry G., and Stanley Zin, 1989, Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937–969.

Gennaioli, Nicola, and Andrei Shleifer, 2018, *A Crisis of Beliefs: Investor Psychology and Financial Fragility* (Princeton University Press).

Grinblatt, Mark, and Matti Keloharju, 2001, How distance, language, and culture influence stockholdings and trades, *Journal of Finance* 56, 1053–1073.

Hansen, Lars Peter, and Thomas J. Sargent, 2008, *Robustness* (Princeton University Press, Princeton, NJ).

Huberman, Gur, 2001, Familiarity breeds investment, *Review of Financial Studies* 14, 659–680.

Landier, Augustin, Yueran Ma, and David Thesmar, 2017, New experimental evidence on expectations formation, Working Paper, MIT.

Lucas, Jr., Robert E., 1976, Econometric policy evaluation: A critique, in Karl Brunner and Allan H. Meltzer, (eds.) *The Phillips Curve and Labor Markets: Carnegie-Rochester Conference Series on Public Policy*, volume 1, 19–46 (North-Holland).

Manski, Charles F., 2004, Measuring expectations, *Econometrica* 72, 1329–1376.

Muth, John F., 1961, Rational expectations and the theory of price movements, *Econometrica* 29, 315–335.

Trope, Yaacov, and Nira Liberman, 2010, Construal-level theory of psychological distance, *Psychological Review* 117, 440–463.