Motivation
- translog cost system with one input (simplified)
\[ \ln C = \ln (\theta(Z)) + \beta_1 \ln W + \frac{1}{2} \beta_2 (\ln W)^2 + u_1 \]
where \( S = \beta_1 + \beta_2 \ln W + u_2 \)
- C: total cost; W: input price
- \( \theta \): efficiency parameter of environmental factors \( Z \)
- \( u_1 \): input share obtained by Shephard’s lemma
- more efficient estimating the system as a whole.

Abstract
- motivated by estimation of a translog cost system
- propose more efficient estimators for a partially linear SUR model
- combine profile least-square (Robinson, 1988) and SUR (Zellner, 1962)
- establish asymptotic normality and efficiency for both the linear and nonparametric estimators
- covariance decomposition method remains in terms of nonparametric efficiency, i.e., Cholesky decomposition
- errors across equations are correlated so that

A partially linear SUR model
Consider a system of \( m \) equations
\[ y_i = \beta_0(x_{i1}, \ldots, x_{im}) + \epsilon_i \]
for \( i = 1, \ldots, n \) and \( x = 1, \ldots, m \).

Moment conditions
1. As \( \E(s_i) = 0 \) and \( \epsilon_i \) enters linearly
2. errors across equations are correlated s.t. \( \E(s_i \epsilon_j) = c_{ij} \) but not across time \( \E(s_i \epsilon_i) = 0 \)

Estimation
- by Robinson (1988), single-equation estimator for \( \beta_i \)
\[ \hat{\beta}_i = \left( \sum_{j=1}^{m} y_j \gamma_{ij} \right)^{-1} \sum_{j=1}^{m} y_j \gamma_{ij} \]
- by Zellner (1962), our SUR estimator for \( \beta_i \)
\[ \hat{\beta}_i = \left( \sum_{j=1}^{m} \gamma_{ij} \gamma_{ij} \right)^{-1} \sum_{j=1}^{m} \gamma_{ij} y_j \]

Asymptotic normality

Nonparametric estimator

Theorem 1. Under Assumptions A1-A4, we have
\[ \sqrt{n} \left( \hat{\beta}_i - \beta_i \right) \xrightarrow{d} N(0, V) \]
where
\[ V = \E \left( \E \left( \gamma_{ij} \gamma_{ij} \right)^{-1} \right) \]

Efficiency discussion
- by Robinson (1982), SUR estimator for \( \beta_i \) is efficient relative to \( \hat{\beta}_i \) as \( \E(\beta_i^2) = \E(\beta_i^2) \)
- but not across time

Simulations
Consider the following DGPs

Table 1: Finite Sample Performance with Cross-Equation Correlation (\( \rho_{xy} = 0.6 \))

References
Nankai University & Kai Sun
"Nankai University Shanghai University"